CMS Physics Analysis Summary

Measurement of Momentum Scale and Resolution using Low-mass Resonances and Cosmic Ray Muons

The CMS Collaboration

Abstract

The momentum measurement of charged tracks is affected by systematic uncertainties due to the limited knowledge and modeling of the detector material, the magnetic field, the alignment and the reconstruction algorithms used to fit the track trajectory. In this paper it is shown that studying the mass of resonances, improvements in the modeling of the detector and in the track reconstruction can be achieved. A novel method to correct the track momentum measurement and to determine with precision its resolution is presented. Results at 7 TeV center-of-mass energy show deviations of order of per mille from the expectation, with an increase only at very low momentum \( p_T < 1 \text{ GeV} \) and high pseudorapidity. For \( J/\psi \) mesons a 0.2% shift in the reconstructed dimuon mass is observed in addition to a slight parabolic dependence on the pseudorapidity. Both the shift and the pseudorapidity dependence are corrected within the statistical uncertainty. The resolution on muon transverse momentum is also measured and found to be within 5% of the MC predictions. A method that uses cosmic ray muons to determine the momentum scale in the range of transverse momentum above 200 GeV, is also presented. With cosmic ray muons above 200 GeV a constant curvature offset is measured to be \( \delta q / p_t = -0.044 \pm 0.022 \text{ TeV}^{-1} \).
1 Introduction

The early data collected by CMS are most valuable towards studying detector performance, calibrating detector subsystems and reconstruction tools. In particular the determination of the transverse momentum is heavily affected by the details of the detector integration and operation, and highly sensitive to the precise alignment of the silicon sensors of the tracker and of the muon chambers, to the material composition and distribution inside the tracking volume, and to the detailed map of the magnetic field inside and outside the solenoid volume.

The basis of the method presented in this paper is the reconstruction of decaying particles which are copiously produced in collisions and have a narrow intrinsic width. Such particles are readily available in early data and the observed widths of the mass distributions are dominated by detector effects. We have studied $J/\psi$ decaying to two muons, $K_S$ decaying to two charged pions and $\phi$ decaying to two charged kaons. These particles provide a validation over different kinematic regimes and allow understanding of detector effects.

A two step approach is followed: first a study of the average mass and width as a function of the single track kinematics both in simulation and data reveals potential biases in the tracking. If the origin of the bias is understood, an appropriate modification of the algorithm, the amount of the detector material, the alignment, or the magnetic field map can be envisaged. Secondly, the remnant bias is corrected on a track-by-track basis, by comparing and fitting the lineshape of the resonance, parametrized by the kinematics of the single tracks. This procedure is applied to dimuon resonances to provide a precise determination of track momentum resolution due to these irreducible biases. The early collision data does not contain statistically sufficient high momentum muon samples, and even when a large sample of $Z$ bosons is available to calibrate the detectors up to the 100 GeV scale, the extrapolation to the TeV momentum range will not be straightforward. The measurement of the charge ratio of cosmic ray muons [1] has shown a surprisingly high sensitivity to the determination of the absolute momentum scale at the TeV scale, as small variations in momentum are easily observable given the steeply falling cosmic ray muon momentum spectrum.

At low momentum the track resolution is dominated by the inner tracker, therefore the first part of this paper focuses on the possible biases coming from the imperfect modeling of the silicon tracker. Low momentum muons are identified with a single hit in the muon chambers (“Tracker Muons”) but still the track fit is performed without using the muon chamber spatial information. At high momentum, the muon chambers play an important role in the resolution of the muon transverse momentum [2, 3], and thus the full muon track is considered (“Global Muon”).

Tracks from the decay of the $K_S$ and $\phi$ have typical momenta lower than one GeV, while the ones from $J/\psi$ average several GeV. With sufficient statistics, the dimuon decay of the $Z$ boson will provide muon samples with momenta on the 50 GeV scale. By measuring the momentum scale using all these particles it will be possible to calibrate the detector in a very wide momentum interval and determine a possible linear dependence in order to allow the extrapolation towards higher momenta.

This paper presents a method to understand the detector by using the two tracks from the decay of particles; firstly with simulated events, then with the data collected with the CMS detector during the first months of 2010 at 7 TeV center-of-mass energy. A method to extract the momentum scale for TeV muons from the shape of the muon flux from cosmic ray interactions in the atmosphere is also presented, using cosmic-ray data collected during the years from 2008 to 2010. In all cases data is compared to the expected detector response as simulated
using the GEANT4 program [4].
2 Selection of the Decay Channels

2.1 The $K_S$ analysis

The most readily available source of particles decaying into two tracks in the early LHC running arises from the $K_S \rightarrow \pi^+ \pi^-$ decay due to the copious QCD production of neutral kaons. The reconstruction of the $K_S$ particle provides a handle on the track momentum scale for low momentum tracks. The displaced nature of the decays provides an additional handle for reducing the combinatorial background. In contrast to the effective minimum cut of $\sim 3$ GeV needed to reach the muon chambers inherent in the dimuon analysis, the $K_S$ selection does not place a lower bound on the momentum, allowing the reconstruction of very low $p_T$ tracks (see Figure 1). The displacement of the decays into the tracker volume also allows study of the dependence of the mass reconstruction on $L_{xy}$, the transverse distance between the primary vertex and the $K_S$ decay vertex along the momentum direction.

![Figure 1: The $p_T$ spectrum for $K_S$ on left and the $p_T$ spectrum of daughter pions of the $K_S \rightarrow \pi^+ \pi^-$ decay on the right for data and simulation.](image)

Vertex fitting of track pairs is necessary to find and measure $K_S$ candidates. Requiring the vertex to be displaced from the primary interaction point provides a powerful means of reducing background. In addition, the fit is needed so that the track parameters can be taken at the point of intersection in order to get an accurate mass calculation for the pair. The selection proceeds in two steps. Candidate track pairs are identified considering track helix intersections in the $r - \phi$ plane with opposite sign tracks. The initial vertex must pass the preselection:

- $380 \text{ MeV} < m(\pi^+ \pi^-) < 620 \text{ MeV}$
- decay radius $> 1$ mm
- $z$-distance between tracks at $xy$ intersection $< 1$ mm

Selected track pairs are fit using the CTVMFT package [5] interfaced to CMSSW. This fit is a full three dimensional fit. If the fit converges, the resulting $K_S$ candidate is required to pass a tighter selection:

- $p_T(K_S \rightarrow \pi^+ \pi^-) > 500$ MeV
- $L_{xy} > 3$ mm
2 Selection of the Decay Channels

- 2D impact parameter $|d_{xy}(K_S \rightarrow \pi^+\pi^-)| < 1 \text{ mm}$
- no hit on either track inside of the fit vertex

![Figure 2: The dipion invariant mass for $K_S \rightarrow \pi^+\pi^-$ decay. A clear $K_S$ peak can be seen.](image)

The overall mass distribution of the reconstructed $K_S \rightarrow \pi^+\pi^-$ is shown in Figure 2. The final candidates are binned in $\eta, p_T, \phi$, or $L_{xy}$, and the resulting mass distribution is fit to a Gaussian, which results in an average mass for the particle for that kinematic point. The background is modeled with a quadratic polynomial.

The simulation used for these studies is a sample of about 50 Million of Minimum Bias events generated with Pythia version 6 [6], with the D6T tuning [7, 8] and with alignment and calibration conditions as expected at the startup of data taking (see Section 3).

2.2 The $\phi$ analysis

The decay of the $\phi(1020)$ meson into $K^+K^-$ provides a copious and well recognizable source of prompt charged kaons, and allows an estimation of the tracker momentum scale performance. The main way to discriminate the prompt kaons from pions is through the energy loss $dE/dx$ inside the active tracker material. As shown in [9], the discrimination between $K$ and $\pi$ is possible only for $p < 1 \text{ GeV}$, while for tracks having higher momenta the full sample is employed for the $\phi$ reconstruction.

The requirements applied to select charged kaon candidates from the full track sample are:

- $p_T > 0.2 \text{ GeV}$
- $|\eta| < 2.4$
2.3 The $J/\psi$ analysis

- Track fit $\chi^2/N_{dof} < 2$
- $N_{dof} > 5$
- $d_{xy} < 0.3$ cm
- highPurity flag [9]
- Mass $m$ estimated from $dE/dx$ such that $|m - m_K| < 200$ MeV if $p < 1$ GeV

More details about the highPurity flag and mass selection can be found in Ref. [9].

Opposite sign kaon candidates are then combined to create $\phi$ candidates. The dikaon mass peak, after applying all the above cuts, is shown in Figure 3. Since the $dE/dx$ is used only for the small momentum region, a sizable combinatorial background is present in the distribution.

![Figure 3: The reconstructed mass peak for the $\phi \to K^+K^-$ decay.](image)

2.3 The $J/\psi$ analysis

$J/\psi \to \mu^+\mu^-$ decays represent the largest source of dimuons in this early data taking era. Due to the low luminosity, a muon trigger with low threshold is exploited to collect a large sample of $J/\psi \to \mu^+\mu^-$ events at low transverse momentum. The reconstruction of low momentum muons is a major challenge for the CMS detector: to be identified, the muons must reach the muon stations placed outside the solenoid, that implicitly imposes a transverse momentum threshold of about 3 GeV in the barrel. Therefore the transverse momentum and the pseudorapidity of the muons coming from low momentum $J/\psi$ are highly correlated, with the low transverse momentum muons populating only the endcap region.

To be reconstructed as a Global Muon, the muon must register in at least two muon stations.
This algorithm provides high quality and purity muon reconstruction but low efficiency at low momentum. Tracks which do not pass the Global Muon requirements are passed to the Tracker Muon algorithm [2], which requires signal in only one muon station, thus giving better efficiency at low momentum but larger background from punch-through and decays-in-flight events.

Global Muons are generally built with a combined fit of hits in the inner tracker and in the muon chambers but in the present analysis only the track built with the silicon hits is considered for the momentum scale and resolution measurement. In this way the Global and Tracker Muons can be analysed together. When more statistics of high momentum J/ψ will be available, a dedicated study of Global Muons, considering the combined track, will be possible.

The inner tracker track of each muon should pass this selection:

- number of silicon hits > 11
- track fit $\chi^2/N_{dof} < 4.0$
- number of pixel layers with at least one hit > 1
- the transverse impact parameter < 3 cm and the longitudinal impact parameter < 30 cm.

In order to ensure a high purity muon sample, the combined track of Global Muons must have $\chi^2/N_{dof} < 20.0$, and satisfactory matching in direction and position is required between the track extrapolated from the inner tracker and the track segment reconstructed in the muon stations. Subsequently pairs of opposite sign muon helices are fit for a common vertex and the resulting J/ψ candidates are retained if the fit $\chi^2/N_{dof}$ probability is larger than 0.1%.

Finally to clean the data sample from beam background the following quality cuts are applied to each event:

- at least one vertex of good quality
- if many tracks (>10) are reconstructed, at least 25% of them must be of highPurity quality

The dimuon invariant mass spectrum obtained with $\sim 40$ nb$^{-1}$ integrated luminosity is shown in Figure 4. In Figure 5 the transverse momentum and pseudorapidity distributions for the muons from the J/ψ decay and from the reconstructed J/ψ are shown, for events that have an invariant mass between 2.9 and 3.2 GeV.

The selected muons are binned in $p_T$ and $\eta$ and the resulting mass distribution is fit with a Gaussian (or a Crystal Ball [10] in case of high statistics) summed with an exponential for the background. The Gaussian (or Crystal Ball) mean and standard deviation for each bin provide an estimation of the scale and the resolution, respectively, for that kinematic configuration. For a more precise determination of the scale and resolution for single tracks, a multiparameter unbinned likelihood fit is performed, as explained in Sec. 4. Throughout this document in all distributions where J/ψ quantities are shown with respect to the muons $p_T$ or $\eta$, each bin is filled twice per event, using both muons.
2.3 The $J/\psi$ analysis

Figure 4: The dimuon invariant mass spectra for a $L_{\text{int}} \sim 40 \text{ nb}^{-1}$ at $\sqrt{s} = 7 \text{ TeV}$.

Figure 5: The $p_T$ (left) and $\eta$ (right) of the muons from the $J/\psi$ decay and of the $J/\psi$ (i.e. dimuon) resonance. The curve from simulation represents the muons from the $J/\psi$ decay.
3 Studies of Track Parameter Bias from Simulation

The effects of material budget modeling, magnetic field, alignment, and reconstruction algorithms on the momentum scale can be studied using simulation. The assessment of each potential cause of scale bias is studied individually using systematically altered detector configurations.

Detector material description. A detailed description of the material budget of the Inner Tracker is implemented in the simulation. Measurements of the tracker material performed with nuclear interactions and conversions [11] show excellent agreement with the expectation. In contrast, the algorithm to reconstruct the tracks is based on an approximated description of the material budget to speed up the processing. The effects of possible discrepancies between the simulated and the real detector are expected to be much smaller than the systematics due to the approximation in the reconstruction algorithm. Before the arrival of LHC collision data, studies with simulated data revealed mass biases resulting from a convolution of effects from the material model implementation and the track reconstruction algorithm. A re-parametrization of the detector material with finer granularity and a re-estimation of the error of the track fit in the bending direction led to an improvement of the mass reconstruction of particles decaying in the barrel. Sizable (up to 0.1%) and complicated biases are still observable in the transition and endcap regions where the amount of material crossed by a particle is larger and its distribution is not uniform (see Figure 6).

Misalignment. To study the related systematics, a simulated scenario has been developed which represents the misalignment expected at the start of data-taking (the “startup scenario”). This scenario was generated by performing an alignment on a Monte Carlo sample of atmospheric muons generated with design geometry, and using the geometry obtained on data from cosmic-rays as starting point for the reconstruction [12]. The same strategy and track selection applied on the data were used. The final geometry is somewhat optimistic, since along the degrees of freedom not aligned in data, the modules remain in their known design position. On the other hand the geometry may embed global movements and distortions, inherited from starting geometry, which are not easily detected using only the cosmic-ray track topology. Therefore this scenario represents in a more realistic way the actual uncorrected displacement of the modules inside the geometry. The comparison of a design alignment scenario (i.e. with a perfectly aligned and calibrated detector) with the startup scenario described above results in a small bias (< 1 MeV) for the J/ψ mass through lower mass values in the high |η| (see Figure 6) and low p_T region and a large bias (up to ∼ 5 MeV) as a function of the azimuthal angle. The effect is not visible for the K_S analysis within the considered statistics, indeed the alignment impact is expected to increase with momentum. The misalignment also degrades the mass resolution in the high |η| region, by about 0.5 MeV at low momentum, up to about 1 MeV at higher momentum. The mass resolution shows a modulation as a function of φ, also for the design scenario. The current hypothesis is that the bias is due to the approximated distribution of the material in the reconstruction algorithm.

The residual misalignment can also contain non-trivial transformations, the so called weak modes: these kind of distortions leave the χ^2 of the tracks unchanged thus they can survive even after the track based alignment. If uncorrected, these modes would produce large systematic biases in physics measurements: for instance, a systematic rotation of the layers of the tracker would introduce a charge-dependent asymmetry in the tracks momentum. Weak modes were widely studied with data from cosmic-rays during CMS 2008 commissioning [12] but tracks from LHC collisions can provide better sensitivity to these systematic deformations. Various systematic distortions have been considered in ∆r, ∆ϕ, and ∆z as a function of r, ϕ, and
While the effect is less than 0.5 MeV on the mass and less than 1 MeV on the mass resolution of the $J/\psi$, it induces a modulation of the mass versus $\phi$ with amplitude as large as 2 MeV, opposite for positive and negative muons. The available statistics in data is still too low to be sensitive to such modulation.

**Magnetic Field.** To study the effect of the description of the magnetic field, the results obtained with two different field maps have been compared. The first map is obtained with a finite-element computation of a detailed model of the CMS solenoid and yoke and is the standard one used in CMS; the second is a parametrization of field measurements taken during a dedicated field mapping campaign. The difference between the two maps varies with $r$ and $z$ up to a maximum of about 0.1% [13, 14] and provides a realistic estimation of the uncertainty on the magnetic field. For this exercise, the scenario with the modified magnetic field has been built using the standard map in the simulation of the events and the parametrization for the reconstruction. The modified magnetic field gives a bias on the mass which increases with $|\eta|$ (from about 0.01% in the barrel, up to about 0.1% in the endcaps as expected, see [13, 14]). No effects are visible as a function of the azimuthal angle. Finally, the modified magnetic field does not affect sizably the mass resolution.
4 Calibration strategies using $J/\psi$

Dedicated studies, requiring high statistics samples, can be developed to search in data and possibly correct separately each of the effects listed in Sec. 3. A parallel approach can be pursued to correct possible residual mass shifts exploiting the precise knowledge of the $J/\psi$ mass and width. A method based on a likelihood fit to the measured $J/\psi$ lineshape has been developed to extract possible muon momentum scale biases and to estimate the muon momentum resolution.

A scale bias can manifest itself as a shift of the $J/\psi$ reconstructed mass and a widening of the mass distribution. The effect of the resolution is also to change the width of the mass distribution. Since the $J/\psi$ mass is not a per-track variable, a probabilistic approach is necessary in order to relate the difference between expected and observed mass with a hypothetical bias on the measured parameters of either or both daughter tracks.

The connection between scale and resolution in the determination of measured biases has to be taken into account: a potential bias in the momentum scale can be meaningfully inferred from the measured dimuon mass on an event-by-event basis only if a value is assumed for the uncertainty in the four-momentum of the daughter tracks, such that an estimate of mass resolution is possible. The information on the scale is hidden by the finite mass resolution, but can be fully recovered by a fit.

A set of functions describing the expected dependence on track kinematics of the biases and of the measurement resolution is defined inspecting the average mass and the mass width of the dimuon resonance as a function of the muon kinematics in data. The best estimate of the parameters of those functions is then determined from a likelihood minimization, provided a set of homogeneous data with enough statistics.

4.1 Likelihood construction

In order to perform the minimization, a model of the $J/\psi$ mass lineshape is required. Given the narrow $J/\psi$ natural width ($\sim$ 90 KeV) compared to the mass resolution (> 10 MeV), the relativistic Breit-Wigner shape of $J/\psi$ production cross-section is approximated with a narrow peak of 0.2 MeV. The Final State Radiation tail is described by a triple exponential function obtained from the fit of the parton-level invariant mass generated with Photos [15]. This lineshape is convoluted with a Gaussian function to describe the resolution effects. Finally this model provides a probability value for each value of mass and mass resolution.

The observed mass $m$ and the expected mass resolution $s$ are functions of the momentum scale correction and the resolution on the track parameters:

$$ p_{T}^{\text{corr}} = F(\vec{x}; \vec{a}) p_{T} $$
$$ m = m(p_{T1}^{\text{corr}}, \phi_1, \cot \theta_1; p_{T2}^{\text{corr}}, \phi_2, \cot \theta_2) $$
$$ s = \sqrt{\sum_{i=1}^{2\text{muons}} \left( \frac{\partial m}{\partial p_{T_i}} \right)^2 \sigma_{p_T}^2 + \left( \frac{\partial m}{\partial \phi_i} \right)^2 \sigma_{\phi}^2 + \left( \frac{\partial m}{\partial \cot \theta_i} \right)^2 \sigma_{\cot \theta}^2} $$

where $\alpha, \beta, \gamma$ and $\delta$ are free parameters. The importance of the correlation terms in the mass resolution has been found to be negligible. Finally a simple exponential shape is used to model the background, introducing additional parameters $\vec{e}$. 
4.2 The misalignment start-up scenario

A sample of simulated J/ψ corresponding to about 13 pb⁻¹ is reconstructed with the detector alignment and calibration as at the moment of the start of data taking (as described in Section 3). The fit is performed on this realistic sample to estimate the momentum bias and to measure the momentum resolution. The resolution function used in the fit is:

\[
\frac{\sigma(p_T)}{p_T} = \begin{cases} 
    c + b_1 \eta^2 & \text{for } |\eta| \leq b_0 \\
    b_2 + b_3(|\eta| - b_4)^2 & \text{for } |\eta| > b_0
\end{cases},
\]

where \(c\) is such that the function is continuous. The resolution on the polar angle is parametrized from Monte Carlo (\(\sigma(\cot \theta) = -0.005 + 0.03 \text{ GeV/c } p_T\)) and the resolution on the azimuthal angle is fixed to the average value in Monte Carlo (\(\sigma(\phi) = 0.001\)). This function will be applied also on data in Section 5.3. Table 1 show the parameter values and the corresponding errors resulting from the fit. The J/ψ mass has large dependencies on the muon \(\eta\) and \(\phi\). The function used for the momentum correction is:

\[
p_T' = p_T \cdot (1 + A + B f(|\eta|) + C |\phi| \sin(2\phi + D)),
\]

where \(f(|\eta|)\) is a by-point function derived from the actual mass distribution vs muon \(\eta\), but symmetrized between positive and negative \(\eta\) and the mean of the function over \(|\eta|\) is subtracted, so that the distribution is centered around zero. The parameters \(C\) and \(D\) are different for positive and negative charge and for \(\phi > 0\) or \(\phi \leq 0\). The total number of free parameters is 10. The fit strategy is to first fit the resolution and, in a second iteration, the scale correction.

The biases as a function of muon \(\eta\) and \(\phi\) (up to \(\sim 5\) MeV and \(\sim 4\) MeV, respectively) are reduced to \(\sim 1\) MeV. The limit on the corrections are due to the choice of rather simple functions. The results can be improved by further refining the description of the biases. In Figure 6 the J/ψ mass is shown as a function of the \(\eta\) before and after the correction for the momentum scale.  

<table>
<thead>
<tr>
<th>parameter</th>
<th>value±error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b_0)</td>
<td>1.305 ± 0.012</td>
</tr>
<tr>
<td>(b_1)</td>
<td>((1.5833 ± 0.0019) \times 10^{-2})</td>
</tr>
<tr>
<td>(b_2)</td>
<td>((6.35 ± 0.23) \times 10^{-3})</td>
</tr>
<tr>
<td>(b_3)</td>
<td>((1.7465 ± 0.0085) \times 10^{-2})</td>
</tr>
<tr>
<td>(b_4)</td>
<td>1.5278 ± 0.0014</td>
</tr>
</tbody>
</table>

Table 1: Results of the resolution fit on the full MC sample.
Figure 6: The $J/\psi$ invariant mass distribution as a function of $\eta$ computed before and after the calibration procedure. In the simulation the alignment scenario at the start of data taking is considered.
5 Results in early 7 TeV data

The events collected by the CMS experiment during the first data taking period at 7 TeV center-of-mass energy are selected as described in Section 2. The analyzed luminosity correspond to 10-40 \( \text{nb}^{-1} \), depending on the channel discussed.

5.1 \( K_S \) decays in data

The \( K_S \) sample corresponds to \( \sim 10 \text{ nb}^{-1} \) of data taken with the minimum bias trigger, nearly 500 million events, though not all containing \( K_S \) particles. With this sample the mass dependence of the \( K_S \) versus different kinematic quantities is studied. Figure 7 displays the dependence on \( K_S \eta \), which is scattered around the true mass at the sub-MeV scale for the barrel region \( |\eta| < 1 \). The simulation is shifted upward by \( \sim 0.3 - 0.5 \text{ MeV} \). Moving outward in \( |\eta| \), there is an \( \approx 0.5 \text{ MeV} \) shift downward in the transition region \( 1 < |\eta| < 1.5 \), both in data and simulation. This region comprises the section of tracker volume where the bulk of the inactive material resides as well as the highest rate of change of this material, making it difficult to model. In addition, tracks traversing this region have high probability of complicated hit patterns involving multiple tracker subdetectors, where both track reconstruction algorithms and alignment can contribute to this abrupt shift. The bias increases linearly with \( |\eta| \) in the endcaps, \( |\eta| > 1.5 \). The simulation follows this trend, although the mass is shifted upwards, similar to the barrel region.

![Figure 7: The \( K_S \) mass dependence on \( \eta \) for 10 nb\(^{-1}\) of data, compared to expectations from simulation.](image)

Motivated by the structure in Figure 7 the mass dependence on \( K_S p_T \) is split between barrel
and endcap, as shown in Figure 8. In the barrel there is a very weak dependence, with the $\sim 0.5$ MeV shift between simulation and data, while in the endcap there is a stronger dependence, again with simulation reproducing the shape of the data distribution. The same division between barrel and endcap but plotted versus $\phi$ is shown in Figure 9. In these plots the same overall shift between data and simulation is present, and both show small but significant structure.

Figure 8: The $K_S$ mass dependence on $p_T$, for 10 nb$^{-1}$ of data, compared to expectations from simulation. On the left is the barrel ($|\eta| < 1$), on the right, the endcap ($|\eta| > 1.5$).

The $p_T$ spectrum of the pions in $K_S$ decays is considerably softer than the corresponding muon $p_T$ from $J/\psi$ decays. In order to further quantify the track resolution as well as provide a more equitable comparison with the dimuon sample, a “high $p_T$” sub sample is generated via a cut on pion momentum $p_T > 1$ GeV. Figure 10 shows the mass dependence for the high $p_T$ sample superimposed on the full sample result, for both the $\eta$ distribution and the $L_{xy}$ distribution in the endcap, where the effects are greatest. The conclusion is that a significant portion of the disagreement with the nominal mass in the endcap comes from the low pion $p_T$ component of the spectrum. In addition, while in the barrel region the $L_{xy}$ distribution in the high $p_T$ sample reproduces that of the full sample to within $\sim 0.2$ MeV, in the endcap, this bias is significantly reduced and more uniform in the high $p_T$ sample, which is consistent with the particular importance of the material quantity and variation in the endcap region. Overall, above $p_T = 1$ GeV the data reproduces the $K_S$ mass to within $\sim 0.3$ MeV over the full $\eta$ range.

5.2 $\phi$ decays in data

Similar to the $K_S$ analysis, the final $\phi$ candidates are binned in $p_T$ and $\eta$ and the mass distribution is fit with the convolution of a relativistic Breit-Wigner and a Gaussian, while an arctangent is used for background.

The distribution of the resolution $\sigma$ coming from the fit as a function of the pseudorapidity of the $\phi$ is shown in Figure 11. A degradation of the resolution with increasing $\eta$ is seen in both data and simulation. Simulation reproduces well the behavior of data in the central ($|\eta| \lesssim 1.6$) region, while there a significant discrepancy at high $\eta$.

The dependence of the mass on the $p_T$ of the $\phi$ meson for the barrel is shown in Figure 12.
5.2 $\phi$ decays in data

Figure 9: The $K_S$ mass dependence on $\phi$, for 10 nb$^{-1}$ of data, compared to expectations from simulation. On the left is the barrel, $|\eta| < 1$, on the right, the endcap, $|\eta| > 1.5$

Figure 10: The $K_S$ mass dependence on $\eta$ on the left, and $L_{xy}$ for the endcap on the right for 10 nb$^{-1}$ of data, comparing the distribution with a $p_T > 1$ GeV cut compared to no cut. These two quantities show the largest change between the two samples.
Figure 11: The dependence of the resolution $\sigma$ coming from the $\phi$ fit on $\eta$, for 10 nb$^{-1}$ of data, compared to expectations from simulation.
The bias seen at low $p_T$ is compared with the one coming from the simulation and with the $K_S$ results. To make this comparison, the value of $(M_{\text{fit}} - M_{\text{PDG}}) / (M_{\text{PDG}} - 2 \cdot M_{K/\pi})$ is plotted instead of the value coming from the fit.

Figure 12: Left: the $\phi$ mass dependence on $p_T$ for barrel, for 10 nb$^{-1}$ of data, for data and simulation. Right: the comparison between the shift from the PDG mass for $\phi$ and $K_S$, divided by the respective $Q$ value from the PDG.

5.3 $J/\psi$ decays in data

The $J/\psi$ mass and mass resolution are studied as a function of the muon kinematics and the calibration procedure described in Section 4 is applied to extract the momentum scale and measure the momentum resolution. The data used correspond to $\sim 40$ nb$^{-1}$ of integrated luminosity.

In Figure 13 the measured $J/\psi$ mass in data and MC is plotted as a function of the $\eta$ and $p_T$ of the muons. The dashed line corresponds to the nominal mass and the MC sample is the one described in Section 4.2. A shift up to 7 MeV in the high pseudorapidity and low momentum region is observed in data.

In Figure 14 the $J/\psi$ mass resolution is plotted as a function of $\eta$ and $p_T$ of the muons for data and Monte Carlo.

At high pseudorapidity and low momentum the resolution in data is up to 10% worse than in MC. In the barrel region and at high momentum, the statistics is still too low to draw any conclusion. Actually, the $p_T$ and $\eta$ spectra are slightly different in data and Monte Carlo, as shown in Figure 5. Therefore a correct comparison between data and Monte Carlo could be done only taking into account the dependence of the scale and the resolution on $p_T$ and $\eta$ at once, as done with the likelihood fit described in Sec. 4.

The likelihood fit is performed assuming this functional correction form:

$$p_T' = (1 + a_0 + a_1|\eta| + a_2\eta^2 + a_3p_T) \cdot p_T,$$

(7)

The fit is done following this sequence: first the background is determined using an exponential shape. Then the muon momentum resolution is determined using Eq. 5 and finally the muon
Figure 13: The $J/\psi$ invariant mass as function of the $\eta$ and $p_T$ of the muons.

Figure 14: The $J/\psi$ invariant mass resolution as function of the $\eta$ of the muons for data and simulation.
momentum scale parameters $a_i$ are fitted. The parametrization of the scale (Eq. 7) and the resolution are motivated by the largest effects seen in Monte Carlo studies (Section 3) and in the analysis of the mass average as a function of the muon kinematics in data (Figure 13 and 14). When more statistics will be available additional dependence can be probed, e.g. describing an azimuthal dependent bias like the one described in Section 4.2 and adding a $p_T$ dependence to the resolution parametrization.

The results of the scale measurement are reported in Table 2. Figure 15 shows the dimuon invariant mass before and after the corrections. The peak is shifted in the right direction recovering the discrepancy of 5 MeV with respect to the nominal value.

The main systematics in the scale corrections is due to the uncertainty on the Final State Radiation tail description in the lineshape model. The uncertainty has been estimated, in a conserva-
tive way, by comparing the results of a Crystal Ball fit to the used lineshape and to a simplified model, where the FSR tail is described by a double-exponential function. The difference has been found to be $< 1$ MeV (corresponding to an effect $< 1.5 \times 10^{-4}$ on the momentum scale).

Figure 16 shows the mass as a function of $\eta$ before and after the scale corrections, compared with the expected values from a Crystal Ball fit to the lineshape model smeared with the resolution for each $\eta$ value. The mass values after the corrections are much more in agreement with the expected ones but a larger data set will be needed to completely remove the bias.

The results for the transverse momentum resolution are shown in Figure 17 and the parameter values for the fit function of Eq. 5 are shown in Table 3. A discrepancy between data and simulation is visible in the high pseudorapidity region, especially in the transition region between barrel and endcaps, as already noticed in Figure 14. The systematics due the choice of the parametrization function can be estimated from the Monte Carlo exercise shown in Section 4.2: the largest relative difference between the resolution obtained in Monte Carlo from the fit and from MC truth being 20% in the barrel and $\sim 5\%$ in the endcaps.

The measured scale and resolution have large uncertainties due to the low statistics. In particular the $p_T$ dependence of the scale corrections suffers of the restricted momentum distribution available for probing. The extracted scale corrections seem not applicable to the pions from $K_s$. When additional statistics will be available for higher mass resonances ($\psi(2s)$, $Y(1s)$, $Y(2s)$, $Y(3s)$ and $Z$) the scale dependence on the transverse momentum, as well as on other variables, will be studied more precisely.
Table 3: Results of the resolution fit on ~ 40 nb$^{-1}$ of integrated luminosity using $J/\psi$ resonances.

<table>
<thead>
<tr>
<th>parameter</th>
<th>value±error</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_0$</td>
<td>1.61 ± 0.11</td>
</tr>
<tr>
<td>$b_1$</td>
<td>$(5.0 ± 0.6) \times 10^{-3}$</td>
</tr>
<tr>
<td>$b_2$</td>
<td>$(1.9 ± 0.3) \times 10^{-2}$</td>
</tr>
<tr>
<td>$b_3$</td>
<td>$(1.4 ± 0.7) \times 10^{-2}$</td>
</tr>
<tr>
<td>$b_4$</td>
<td>1.5 ± 0.3</td>
</tr>
</tbody>
</table>

Figure 17: Resolution on transverse momentum as measured with ~ 40 nb$^{-1}$ of integrated luminosity (black line) compared to the Monte Carlo resolution computed from Monte Carlo truth (red points) and from the fit as described in Section 4.2 (black squares). The gray band in data represents the error on the fitted function for data computed from the errors on the parameters.

Finally, it should be noticed that these corrections have been applied for the measurement of the $J/\psi$ differential cross section [16] and the measured resolution has been exploited to estimate the related systematics both in this analysis and in the measurement of the W and Z cross sections [17].
6 TeV Muons from Atmospheric Interactions

The measurement of the charge ratio of cosmic ray muons with the CMS detector [1] in the momentum range from 5 GeV to 1 TeV has been found to be very sensitive to absolute momentum scale shifts. The reason for this sensitivity lies in the shape of the momentum spectrum of cosmic ray muons. The energy of the cosmic ray muons at the surface of the Earth has a falling shape proportional to $E^{-2.7}$ with $E$ the energy of the cosmic ray muons. This is illustrated in Fig. 18, where one can see the spectra for positive and negative cosmic ray muons. For an accurate charge ratio measurement, any charge-asymmetric effect has to be considered. Here the focus is on the effect that some alignment weak modes can have on the charge ratio measurement. Some of the weak modes can introduce a constant curvature offset, with the curvature defined as $\kappa = q / p_T$. The existence of such a constant curvature offset (or bias) would have the effect shown in Figure 18. In this example the induced bias corresponds to the curvature of a 10 TeV negative track, a practically straight track. Due to this small shift, negative tracks are measured as having systematically lower momentum, and positive tracks are measured as having systematically higher momentum. A constant curvature offset is then charge asymmetric, and the cosmic ray muons can be used to quantify it. The idea behind the method that is presented here is based on the fact that no cosmic ray muons are expected to have zero curvature (or infinite momentum). The measurement of cosmic ray muons in the vicinity of zero curvature would then be a clear sign of a curvature offset.

![Figure 18: Absolute curvature bias impact on the observed positive and negative muon differential rates.](image-url)
6.1 Data sample and event simulation

To estimate the high momentum scale the Cosmic Run At Four Tesla in 2008 [18], 2009 and 2010 data are analysed. The selection criteria are kept simple - only good runs are read, and a minimum of 14 tracker hits for the muon candidates is required. Besides, the normalized track-fit $\chi^2$ has to be smaller than 10. The study is limited to the $|\eta| < 2$ region. The available yields after these selection criteria have been applied are 2.2 million, 0.6 million and 1.5 million, for the 2008, 2009 and 2010 data-taking periods, respectively.

Single cosmic ray muons are simulated using the Monte Carlo event generator CMSGEN [19, 20], which makes use of parametrization of the distributions of the muon energy and incidence angle based on the air shower program CORSIKA [21].

6.2 Cosmic Ray muons end-point method

Figure 19 shows the spectra of positive and negative muons as a function of transverse curvature. The spectra both die off as the momentum approaches infinity. The shape of this distribution will be significantly distorted by a constant curvature bias, as illustrated by introducing a curvature bias equivalent to 50% of the curvature of a 1 TeV track. Such bias is introduced preserving the $\chi^2$ of the track. The distinct shape of this distribution can be used to fit for the curvature bias in high momentum tracks. At the same time the output of the fit is needed to decouple as much as possible from the value of the charge ratio observed in the data.

![Figure 19: Illustrations of the cosmic ray muons end-point method: (left) distribution of the transverse curvature for 2010 data from cosmic-rays and (right) inducing a bias into the cosmic ray-muons Monte Carlo by hand.](image)

A direct comparison of the data to Monte Carlo binned histograms is performed. The $\chi^2$ for the shape of the positive muons and negative muons is separately computed, while forcing the relevant histogram normalizations to match. In other words, the curvature distributions of data and Monte Carlo for muons with negative curvature is taken, the Monte Carlo curvature distribution is normalized to the data and the $\chi^2$ between data and Monte Carlo is computed. This process is repeated for different curvature offsets, obtaining at the end a curve of $\chi^2$ versus curvature offset. Exactly the same process is repeated for positive cosmic ray muons, and at
the end both $\chi^2$ curves are combined into a single one. This separation of positive and negative cosmic ray muons decouples the curvature bias from the charge ratio. An illustration of the obtained $\chi^2$ curves is shown in Figure 20, for 2010 data from cosmic-rays.

The minimum of the $\chi^2$ distribution is jagged due to statistical fluctuations, since this is not a comparison of smooth shapes. The effect is mastered by fitting the $\chi^2$ distribution with a polynomial and using the minimum of the fitted polynomial as the bias estimator. The minimum of the polynomial is very close to being parabolic, with the uncertainties computed from second derivatives practically identical to those computed by stepping up the $\chi^2$ distribution by one.

Figure 20: (Left) $\chi^2$ computed separately for the shapes of the positive and the negative muons for 2010 data from cosmic-rays. (Right) Minima of the $\chi^2$ distributions for 2008, 2009 and 2010 data.

Figure 20 shows the fit projection for the 2010 data from cosmic-rays. The fitted value is $-0.044 \pm 0.022 \, \text{TeV}^{-1}$. In Figure 20 one can also see the stability of the momentum scale between different data-taking periods.
Summary and future prospects

This paper presents the current understanding of the momentum scale and resolution of the CMS detector. Detailed studies of the invariant mass of known resonances, as a function of the track kinematic variables, are presented. The effects of several possible sources of bias are investigated with dedicated MC samples. This paper focuses on resonances (K_s, ϕ and J/ψ) copiously produced during the first period of data taking, and thus on a relative low momentum region, where the silicon tracker is dominating the resolution of the track transverse momentum.

The reconstructed masses in data are always lower than the simulated values. In addition, in the forward region the difference with respect to the expected value increases with pseudorapidity.

A likelihood fit to the J/ψ lineshape is performed to measure the scale of muon momenta. After the extracted scale correction is applied, the J/ψ mass is shifted to the expected value, with a statistical uncertainty of 1 MeV and a systematic uncertainty of 1 MeV. The muon resolution is simultaneously fitted, and it is found to be in quite good agreement with the simulated one. (The largest difference being of about 5% in the transition region between barrel and endcaps.)

Once additional data are collected, other known resonances such as the Z boson will be used to determine the momentum scale and resolution, up to a momentum of 100 GeV. To determine the momentum scale in the high momentum range, i.e., above 100 GeV, the properties of the cosmic ray muons are exploited. Prior to the start of the LHC, the CMS detector has collected millions of cosmic ray muons with momenta up to the TeV scale. These muons have been used to align the CMS detector. With cosmic ray muons above 200 GeV a constant curvature offset is measured to be $\delta q/p_t = -0.044 \pm 0.022 \text{ TeV}^{-1}$.

In summary, already with the cosmic ray muons and first collision data taken by CMS, it has been possible to do important steps in the understanding of the detector. With future data, especially with a large sample of Z bosons, it will be possible to precisely determine the momentum scale and resolution in the momentum region relevant for precision Electroweak measurements.
References


