A Precise Measurement of the $B^+$, $B^0$ and Mean 
b-Hadron Lifetime 
with the DELPHI Detector at LEP I.

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Abstract
Final results for the lifetime of $B^+$ and $B^0$ mesons and the mean b-hadron lifetime, 
are presented using the data collected at the $Z^0$ peak by the DELPHI detector 
in 1994 and 1995. Elaborate, inclusive, secondary vertexing methods have been 
employed to ensure a B-hadron reconstruction with good efficiency. To separate 
samples of $B^+$ and $B^0$ mesons, high performance neural network techniques are 
used that achieve excellent signal purity and the lifetimes are extracted by a binned 
chi-square fit to the data.

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1 Motivation and overview

In addition to testing models of b-hadron \(^1\) decay, knowledge of b-hadron lifetimes is of importance to the determination of other Standard Model quantities such as the CKM matrix element \(V_{cb}\) and in measurements of the time dependence of \(B^0 \rightarrow \bar{B}^0\) oscillations.

The Spectator Model provides the simplest description of b-hadron decay. Here, the lifetime depends only on the process \(b \rightarrow cW^-\) with the other light-quark constituent(s) playing no role in the decay dynamics. This in turn leads to the prediction that all b-hadron species have the same lifetime.

However non-spectator effects such as quark interference, W exchange and weak annihilation can induce lifetime differences among the different b-hadron species in a similar way to that already experimentally established in the charm hadron system. Due to the relatively high b-quark mass, the lifetime differences between b-hadrons are expected to be smaller than in the charm system. Models of B decay based on expansions in \(1/m_b\) predict the hierarchy, \(\tau(A_b) < \tau(B^0) \sim \tau(B_s) < \tau(B^+)\), and there is a growing consensus between models that a difference in lifetime of order 5% should exist between the \(B^+\) and \(B^0\) meson [1]. Precise measurements of all B-species are needed to thoroughly test the validity of these predictions.

This note reports on the measurement of \(B^+\) and \(B^0\) meson lifetimes from the DELPHI data set at LEP I in data sub-samples enriched in \(B^+\) and \(B^0\) mesons respectively. In addition, we report a measurement of the mean b-hadron lifetime, \(\tau_b\), from the full data sample which is a quantity of importance for many b-physics analyses at LEP e.g. in the extraction of CKM matrix element \(|V_{cb}|\). The approach used is highly inclusive and based strongly on the DELPHI inclusive B-physics package, BSAURUS [2]. Aspects of BSAURUS directly relating to the analysis are presented in a summarised form but the full reference should be consulted for details of the package.

After describing parts of the DELPHI detector essential for this measurement in Section 2, and details of the event selection in Section 3, we highlight relevant aspects of BSAURUS in Section 4. Section 5 describes the reconstruction of the B-candidate proper time from measurements of decay length and momentum. Samples with a \(\sim 70\%\) purity in \(B^+\) or \(B^0\) mesons are achieved by use of a sophisticated neural network approach which is described in Section 6. The extraction of B-lifetimes from the data is achieved by a simple \(\chi^2\) minimisation fit to the reconstructed proper time distribution detailed in Section 7. Finally, the systematic uncertainties on the measurement are dealt with in Section 8.

2 The DELPHI detector

A complete overview of the DELPHI detector [3] and its performance [4] have been described in detail elsewhere. This analysis depends crucially on precision charged particle tracking performed by the Vertex Detector (VD), the Inner Detector, the Time Projection Chamber (TPC) and the Outer Detector. A highly uniform magnetic field of 1.23 T parallel to the \(e^+e^-\) beam direction, was provided by the superconducting solenoid throughout the tracking volume. The momenta of charged particle tracks were reconstructed with a precision of \(\sigma_p/p < 2.0 \times 10^{-3}\) \(p\) (in GeV/c) in the polar angle region \(25^\circ < \theta < 155^\circ\).

\(^1\)The corresponding charge conjugate state is always implied when discussing b-hadron lifetimes.
The VD was of particular importance for the reconstruction of the decay vertices of short-lived particles. In the years 1991-1993 the VD was composed of three layers of single-sided silicon microstrip detectors at radii of 6.3 cm, 9 cm and 11 cm from the beam line and called the Closer, Inner and Outer layers respectively [5].

In 1994 the VD was upgraded to become a three-dimensional device [6] instrumented with double-sided microstrip detectors in the Closer and Outer layers providing coordinates in both the $R\phi$ and the $Rz$ planes. For polar angles of $44^\circ \leq \theta \leq 136^\circ$, a track crosses all three silicon layers of the VD. The measured intrinsic resolution is about 8 $\mu$m for the $R\phi$ coordinate for both the old and the upgraded VD, while for $Rz$ it depends on the incident polar angle of the track and reaches about 9 $\mu$m for tracks perpendicular to the modules. For charged tracks with hits in all three $R\phi$ VD layers the impact parameter resolution was $\sigma_{R\phi}^2 = ([61/(P \sin^{3/2} \theta)]^2 + 20^2) \mu m^2$ for both the old and the upgraded VD and for tracks with hits in both $Rz$ layers and with $\theta \approx 90^\circ$, $\sigma_{Rz}^2 = ([67/(P \sin^{5/2} \theta)]^2 + 33^2) \mu m^2$. Before the start of data taking in 1995 the ID was replaced with a similar device but with a larger polar angle coverage in preparation for LEP 2 running. The impact of this change on the current analysis, in comparison to the 1994 upgrade, is relatively minor.

Calorimeters detected photons and neutral hadrons by the total absorption of their energy. The High-Density Projection Chamber (HPC) provided electromagnetic calorimetry coverage in the polar angle region $46^\circ < \theta < 134^\circ$ giving a relative precision on the measured energy $E$ of $\sigma_E/E = 0.32/\sqrt{E} \oplus 0.043$ ($E$ in GeV). In addition, each HPC module is essentially a small TPC which can chart the spatial development of showers and so provide an angular resolution exceeding that of the detector granularity alone. For high energy photons the angular precisions were ±1.7 mrad in the azimuthal angle $\phi$ and ±1.0 mrad in the polar angle $\theta$.

The Hadron Calorimeter was installed in the return yoke of the DELPHI solenoid and provided a relative precision on the measured energy of $\sigma_E/E = 1.12/\sqrt{E} \oplus 0.21$ ($E$ in GeV).

Powerful particle identification was possible by the combination (see Section 4.2) of $dE/dx$ information from the TPC (and to a lesser extent from the VD) with information from the DELPHI Ring Imaging Cherenkov counters (RICH) in both the forward and barrel regions. The RICH devices utilised both liquid and gas radiators in order to optimise coverage across a wide momentum range: liquid was used for the momentum range from 0.7 GeV/c to 8 GeV/c and the gas radiator for the range 2.5 GeV/c to 25 GeV/c.

3 General Event Selection

Multihadronic $Z^0$ decays were selected by the following requirements:

- at least 5 reconstructed charged particles,

- The summed energy in charged particles (with momentum > 0.2 GeV/c) had to be larger than 12% of the centre-of-mass energy, with at least 3% of it in each of the forward and backward hemispheres defined with respect to the beam axis.

Due to the evolution of the DELPHI tracking detectors, detailed above, and run-specific details such as the RICH efficiency, the data were treated throughout this analysis as
two independent data sets for the periods 1994 and 1995. The multihadronic event cuts selected approximately 1.4 million events in 1994 and 0.7 million events in 1995.

The same multihadronic selection was applied to the Monte Carlo samples used consisting of $Z \to q\bar{q}$ and $Z \to b\bar{b}$ events generated with JETSET 7.3 [7] (with tunings optimized to DELPHI data [8]) and passed through a full detector simulation [4]. For each data period, separate, dedicated Monte Carlo samples were produced.

The remaining stages of event selection were performed by BSAURUS. Each event was split into two hemispheres using the plane perpendicular to the event thrust axis. In addition, each hemisphere was assigned a reference axis which was essentially the axis of the jet most likely to contain the weakly decaying b-hadron. Jets were reconstructed via the routine LUCLUS [7] using a transverse momentum cutoff value of $d_{\text{join}}=\text{PARU}(44)=5.0$ GeV/c. A first estimate of the B-candidate four-vector was obtained by calculating the track rapidity, with respect to the reference axis, and summing all tracks with rapidity greater than 1.6.

For each hemisphere an attempt was made to fit a secondary vertex to tracks with rapidity greater than 1.6 passing the following track selection criteria:

- Impact parameter in the $r\phi$ plane $|\delta_{r-\phi}| < 4.0$ cm
- Impact parameter in the $z$ plane $|\delta_z| < 6.0$ cm
- $|\cos \theta| < 0.94$
- $\frac{p_T}{E} < 1.0$
- At least one $r - \phi$ track hit from the silicon vertex detector (VD)
- Tracks must not have been flagged as originating from interactions with detector material by the standard DELPHI interaction vertex reconstruction package, described in [9].

From this class of track, additional criteria were applied with the aim of selecting tracks for the vertex fitting that were likely to have originated from the decay chain of a weakly decaying b-hadron e.g. good lepton or kaon candidates (see Section 4.2). Using this track list a secondary vertex fit was performed in 3-dimensions constrained to the direction of the B-candidate momentum vector.

This vertex information formed part of the TrackNet neural network output (see Section 4.1) that distinguishes between a track originating from the fragmentation process and a track from the decay of a b-hadron. In the final stage of the fit, the TrackNet information was used to add any remaining b-hadron decay candidate tracks to the secondary vertex definition. Finally, hemispheres were rejected for which either of the following criteria were true:

- the secondary vertex fit did not converge (according to limits set on the fit $\chi^2$ and the number of iterations taken),
- $|\cos \Theta_{T H R U S T}|$ was greater than 0.65.
4 General Tools

This section describes briefly some general tools for inclusive B-physics that were used extensively in the analysis:

4.1 The TrackNet

The TrackNet implemented in BSAURUS, and shown in Figure 1(a), supplied for each track in the hemisphere a probability of whether that track originated from a weakly decaying b-hadron or not. The main inputs to the network were impact parameter-based probabilities that the track belonged to the primary or secondary vertex and track rapidity.

4.2 Particle Identification

The MACRIB package [10] provided separate neural networks for the tagging of kaons and protons which combined the various sources of particle identification in DELPHI. An efficiency for the correct identification of $K^\pm$ of 90% (70%) was attained with a contamination of 15% (30%) for $p < 0.7$ GeV/c ($p > 0.7$ GeV/c).

Electron and muon candidates were defined according to the standard DELPHI lepton identification scheme for candidates with energy larger than 3 GeV.

4.3 The BD-Net

The BSAURUS BD-Net, shown in Figure 1(b), was designed to discriminate between tracks whose origin was the weakly decaying b-hadron and tracks that originated from the subsequent cascade D-meson decay. To achieve this, input variables including the impact parameter of the track with respect to the secondary vertex, the momentum of the track in the B-rest frame and particle identification information were utilised.

5 Proper Decay Time Reconstruction

The proper time was defined by the following relationship,

$$\tau_B = \frac{Lm_0}{pc}$$

where $L$ and $p$ are the reconstructed decay length and momentum of the B-candidate respectively and $m_0$ is the B-rest mass which was taken to be 5.2789 GeV/$c^2$[11]. The magnitude of the B-candidate momentum vector was fixed by the relationship $E^2 = p^2 + m_0^2$, where $E$ was the reconstructed B-candidate energy.

5.1 B-candidate energy reconstruction

This section begins by describing how an initial estimate of the weakly decaying b-hadron energy was attained by use of the rapidity algorithm and then corrected for sources of missing energy. The section concludes with a presentation of how this corrected energy was combined, with other correlated variables, in a neural network to produce a more optimal estimator.
Figure 1: Data (points) compared to simulation (histograms) for the (a) TrackNet and (b) BD-Net outputs. The TrackNet simulation distribution is split into 'signal' peaking near 1.0, consisting of tracks originating from the b-hadron decay chain, and 'background' peaking near 0.0 consisting of tracks from fragmentation or excited b-hadron decay. The BD-Net simulation distribution illustrates the discrimination possible of tracks originating at the B decay vertex from those originating at the subsequent D-cascade decay vertex. The secondary peak at low values in the B-distribution is due to high p_t leptons tagging the B-decay.

5.2 The Rapidity Algorithm

Events were split into two hemispheres using the plane perpendicular to the thrust axis. Each hemisphere was assigned a reference axis defined by reconstructing jets via the routine LUCLUS [7] with $p_T$ as a distance measure and the cutoff parameter $d_{join} = \text{PARU}(44) = 5.0 \text{ GeV}/c$. In simulation studies this was found to give the best reconstruction of the initial b-hadron direction. For hemispheres where 2 or more jets were reconstructed (about 16% of the cases) the standard DELPHI b-tag [12] applied at the jet level was used to discriminate the b-jet from the gluon jet. With this scheme, the probability to select correctly the two b-jets in a three-jet event was about 70%.

With the b-jets defined, the rapidity $y = \frac{1}{2} \log \left( \frac{(E + P_t)}{(E - P_t)} \right)$ with respect to the reference axis was calculated for each charged and neutral particle assuming a pion mass for charged particles. The $y$ distribution of particles originating from the decay of a b-hadron is peaked to higher values than is the case for particles originating from the hadronisation process and can be efficiently separated with a cut at $y > y_{cut} = 1.6$. The sum of particle 4-momenta with $y > y_{cut}$ defined the rapidity energy $E_y$ and rapidity mass $M_y$. 
5.3 A First Estimate of the $B$ Energy: $E_{raw}$

In each hemisphere a secondary vertex was fitted to charged particle tracks with $y > y_{cut}$ that satisfied the following track selection criteria:

- Impact parameter in the $R\phi$ plane $|\delta_{R\phi}| < 4.0$ cm.
- Impact parameter in the $z$ plane $|\delta_{z}| < 6.0$ cm.
- $|\cos \theta| < 0.94$
- $\frac{dE}{dx} < 1.0$
- At least one $R\phi$ track hit from the VD.
- Tracks must not have been flagged as originating from interactions with detector material by the standard DELPHI interaction vertex reconstruction package, described in [9].

From this class of track, additional criteria were applied with the aim of selecting tracks for the vertex fitting that were likely to have originated from the decay chain of a $b$-hadron e.g. good lepton or kaon candidates. Using this track list, a secondary vertex fit was performed in 3-dimensions constrained to the direction of the $B$-momentum vector from the rapidity algorithm. This vertex information was an essential input in the construction of the TrackNet, a neural network built to distinguish between tracks originating from the fragmentation process and tracks from the decay of a $b$-hadron.

Following studies based on the simulation, a first estimate of the $b$-hadron 4-momentum and mass $\mathbf{P}_{raw} = (\mathbf{P}_{raw}, E_{raw}, M_{raw})$ was constructed based on the information available. For 1-jet hemispheres, $\mathbf{P}_{raw}$ consisted of the sum of 4-momenta of TrackNet-weighted charged particles and rapidity-weighted neutral particles. For hemispheres containing 2 or more jets and where $x_{hem} = \frac{E_{hem}}{E_{beam}} > 60\%$, the result of the rapidity algorithm was taken.

5.4 Reconstructing $b$-Hadron Energy by a Neural Network

A neural network was trained to reconstruct the energy of the weakly decaying $b$-hadron, and a Bayesian interpretation to the network output was applied in order to return a conditional probability density function for the energy on a hemisphere-by-hemisphere basis. In a first step the algorithm took as input the true, i.e. generated, $b$-hadron energy distribution of a training Monte Carlo sample and transformed it into a uniform distribution in the interval $[0,1]$. The transformed distribution was then sampled at $N$ equidistant levels, each containing the same number of entries. For each of these levels, a separate output node of a simple feed-forward neural network was trained using back-propagation to the binary classification problem: “is the true energy value above the level threshold value” vs “is the true energy value below the level threshold value”. The network outputs were filtered through a sigmoid transfer function and a cubic B-spline fitted through the $N$ net output values. Finally, this spline was interpreted as an estimator of the cumulative probability distribution function (p.d.f.) of the true energy value for a given event and can be regarded as a hemisphere-by-hemisphere unfolding of the input.
$b$-hadron energy distribution. The median (and associated error) of the estimated p.d.f.
then defined the $B$-energy (and error on the energy).

For the network training, sixteen input variables were chosen, which included different
estimators of the energy available in the hemisphere together with some measures of the
expected quality of such estimators e.g. as given by such quantities as hemisphere track
multiplicity and hemisphere reconstructed energy.

The most powerful input variable, with a correlation of 73% to the true $B$-energy,
was the corrected energy $E_{\text{corr}}$. This variable adjusts $E_{\text{raw}}$ to account for sources of
missing energy e.g. neutrinos and inefficiencies. The correction function was determined
from simulation by first dividing the events into 20x20 equally populated bins in the
$\Delta(M_y) = ((M_{\text{raw}} - \langle M_{\text{raw}} \rangle, x_{\text{hem}})$ plane. In each bin the median energy residual $\langle \Delta E \rangle = \langle E_{\text{raw}} - E_{\text{true}} \rangle$ was determined and, to obtain a smooth correction function, interpolating
polynomials were fitted to the $\langle \Delta E \rangle$ distribution. Separate corrections were calculated
according to whether $x_{\text{hem}}$ was above or below the value of 0.6 and also according to
whether the event was a two, or multi-jet event. For completeness, the full list of variables
input to the energy network were:

- The scaled hemisphere energy $x_{\text{hem}}$.
- The raw energy estimate $E_{\text{raw}}$.
- The rapidity mass $M_y$.
- The corrected energy $E_{\text{corr}}$.
- The total hemisphere energy normalised by an estimate of the c.o.m. energy $E_{\text{c.o.m.}}$, given by considering the $Z^0$ to decay into the two-body final state of a $B$-jet with
mass $M_{B\text{-jet}}$ recoiling against all other particles in the event with mass $M_{\text{recoil}}$ i.e.
in the rest frame of the $Z^0$,

$$E_{\text{c.o.m.}} = \frac{M_{Z^0}^2 - M_{\text{recoil}}^2 + M_{B\text{-jet}}^2}{2 \cdot M_{Z^0}} \quad (1)$$

- An estimate of $E_{\text{c.o.m.}}$ as given by Eqn 1 where $M_{\text{recoil}}$ is based on charged tracks
only. In the hemisphere containing the $B$-candidate, only tracks likely to have
originated from the fragmentation process are selected (by use of the TrackNet)
whereas all tracks are used in the opposite hemisphere.

- A further estimate of $E_{\text{c.o.m.}}$ as given by Eqn 1 where neutrals from the opposite
hemisphere are also included in the formulation of $M_{\text{recoil}}$.

- An estimate of the missing $p_T$ between the $B$-candidate direction and the thrust axis calculated using only fragmentation tracks in the same hemisphere as the $B$-
candidate (via use of the TrackNet) and all tracks in the opposite hemisphere. The
calculation is repeated for two different definitions of the $B$-direction: one being
the vector pointing from reconstructed primary vertex to reconstructed secondary
vertex and the other given by the vector $p_{\text{raw}}$.

- The mass of the reconstructed secondary vertex.
• The polar angle of the $B$-candidate momentum vector.

• The difference between the number of tracks in the event passing the selection cuts (described in Section 5.3) and the number of such tracks that, in addition, pass a TrackNet cut of 0.5.

• The probability that the best electron or muon candidate in the hemisphere, with the correct charge correlation, originates from the $B$-candidate.

• The gap in rapidity between the track of highest rapidity with TrackNet value less than 0.5 and the track of lowest rapidity and TrackNet value greater than 0.5.

• The (binomial) error of the vertex charge measurement, $Q_v$, defined as,

\[
\sigma_{Q_v} = \sum_{i}^{\text{tracks}} \sqrt{P_B(i)(1 - P_B(i))}
\]

where $P_B(i)$ is the TrackNet value for track $i$.

• A hemisphere ‘quality flag’ constructed by counting the number of tracks in the hemisphere likely to be badly reconstructed i.e. the number failing the selection cuts of Section 3, the number of ambiguous \(^2\) tracks, the number of reconstructed hadronic interactions and the number of tracks failing the quality cuts of the standard DELPHI b-tag package.

The training sample was required to pass a $b$-tagging cut of $g_{\text{amb}} > 0.5$ and lie in the DELPHI barrel region i.e. $|\cos \Theta_{THRUSTR}| < 0.75$. The final $B$-energy resolution from Monte Carlo is plotted in Figure 2 and shows a Gaussian peak with non-Gaussian tails. The fit is to a double-Gaussian with the central, narrow, Gaussian covering 71% of the total area with standard deviation of 2.3 GeV.

5.5 Decay length reconstruction

Starting from the standard secondary vertex described in Section 3, four independent algorithms were implemented in BSAURUS, based on the BD-Net, with the aim of improving the decay length resolution and minimising any forward bias resulting from the inclusion of tracks from the cascade D-decay vertex in the B-decay vertex reconstruction:

1) In the **Strip-down method**, candidate tracks were selected if they had a TrackNet output bigger than 0.5 and BD-Net value less than $-0.1$. A secondary vertex fit was made if there were 2 or more tracks selected. If the fit failed to converge (within the same criteria as for the standard BSAURUS secondary vertex fit described in Section 3) and more than two tracks were originally selected, the track with the highest $\chi^2$ contribution was removed and the fit repeated. This procedure continued iteratively until convergence was reached or only two tracks were left. The fit was constrained by the direction estimated from the $b$-hadron energy reconstruction and the starting point of the fit was the secondary vertex coordinates of the standard fit.

\(^2\)An ambiguous track is defined to be one containing hit information that could equally well be associated with a neighbouring track.
Figure 2: Double Gaussian fits to the corrected $B$-energy residual (left plot) and the reconstructed $B$-candidate decay length (right plot). The resolutions indicated are the standard deviation of the narrow and broader Gaussian where the narrow contribution is 71% and 67% of the total respectively for the energy and decay length distributions.

2) In the **D-rejection method**, a cascade D-candidate vertex was built by applying the opposite BD-Net selection criteria to the Strip-down method i.e. candidate tracks were selected if they satisfied TrackNet output bigger than 0.5 and BD-Net value greater than $-0.1$. The B-candidate vertex was then fitted using the Strip-down algorithm but applied to all tracks except those already selected for the D-vertex.

3) In the **Build-Up method** those two tracks with TrackNet bigger than 0.5 and smallest BD-Net values were chosen to form a seed vertex. If the invariant mass of all remaining tracks with TrackNet $> 0.5$ exceeded the D-mass, that track with the lowest BD-Net output was also fitted to a common vertex with the two seed tracks. This process continued iteratively until either the fit failed to converge or the mass in remaining tracks dropped below the D-meson mass.

4) The **Semileptonic algorithm** attempted to improve the vertex resolution for the case of semileptonic decays of b-hadrons where energy has been carried away by the associated neutrino. When there was a clear lepton candidate in the hemisphere, the algorithm reconstructed a cascade D-candidate vertex in a similar way to the D-rejection method but with the lepton track excluded. The tracks associated with the vertex were then combined to form a ‘D-candidate track’ which was extrapolated back to be vertexed with the lepton track and so make the B-candidate vertex.

The actual choice of decay length for the decay time calculation was dictated by optimising the resolution and minimising any bias while still retaining the best possible
efficiency. The choice from one of the four algorithms potentially available, was made in the following way:

- the Strip-down method was chosen if the algorithm worked successfully and had a decay length error smaller than 1mm,
- next, the D-rejection method was used, if successful, and had a decay length error smaller than 1mm,
- next, the Build-up vertex was chosen, if successful, and had a decay length error smaller than 200\(\mu\text{m} \),
- lastly, the Semileptonic algorithm was used, if the algorithm was successful, and the angle between the lepton and D-candidate satisfied \(|\cos\Theta_{LD}| < 0.99\) and if the decay length error was smaller than 1mm.

6 Selection of \(B^+\) and \(B^0\) Enhanced Samples

The enrichment of \(B^0\) and \(B^+\) mesons was part of a general attempt, implemented in BSARUS, to provide a probability for an event hemisphere to contain a b-hadron of a particular type. The result was implemented in a neural network (\(NN(B_x)\)) consisting of 16 input variables and a 4-node output layer. Each output node delivered a probability for the hypothesis it was trained on: the first supplied the probability for \(B^0\) mesons to be produced in the hemisphere, the second for \(B^0\) mesons, the third for charged B-mesons and the fourth for all species of b-Baryons. The method relied heavily on the reconstruction of the following quantities:

- The b-hadron flavour (i.e. the charge of the constituent b-quark) both at the fragmentation \(F_{frag.}\) and decay time \(F_{dec.}\): This was determined by first constructing with neural network techniques, the conditional probability for each track in the hemisphere to have the same charge as the b-quark in the b-hadron and was repeated separately for each of the four possible b-hadron type scenarios i.e. \((B^0, B^+, B^0_b \text{ and } b\text{-baryon})\). The network was trained on a target value of +1(-1) if the track charge was correlated(uncorrelated) to the b-quark charge. The main input variables concerned the identification of kaons, protons, electrons and muons together with quantities sensitive to the B-D vertex separation in the hemisphere. Tracks originating from the fragmentation (decay) phase are discriminated by checking the TrackNet value is less(greater) than 0.5. In a final step, these track level probabilities were combined via a likelihood ratio into a hemisphere quantity.

- b-hadron type probabilities \(P(B_x)\): Supplied by an auxiliary neural network constructed to supply inputs to the more optimal \(NN(B_x)\) network. In common with the \(NN(B_x)\), there were four output nodes trained to return the probability that the decaying b-hadron state was \(B^+, B^0, B^0_b \text{ or } b\text{-baryon} \text{ respectively. There were 15 input variables in total, the most powerful of which included the hemisphere TrackNet-weighted charge sum, which discriminates charged from neutral states, and variables that exploit the presence of particular particles produced in association with b-hadron states. Examples of this include } B^0_b \text{ mesons, which are normally}}\)
produced with a charged kaon as the leading fragmentation particle with a further kaon emerging from the weak decay, and in $B^+$ and $B^0$ production where the decay is associated with a larger multiplicity of charged pions than is the case for $B^0_s$ and $b$-baryons which on average will produce a higher proportion of neutrons, protons and kaons.

The inputs to the $NN(B_x)$ were constructed to optimally exploit all of the information that the $b$-hadron production and decay process reveals. The basic construct for input variables were, ignoring details of variable transformation,

$$F_{\text{dec.}}(B_x) \cdot F_{\text{frag.}}(B_x) \cdot P(B_x).$$  \hspace{1cm} (3)

The upper plots of Figure 3 show the output of the $B^+$ and $B^0$ output nodes of the $NN(B_x)$ for the different $B$-species overlaid with the data. The lower plots trace the change in purity per bin for the different $B$-species as a function of the network output value at the $B^+$ and $B^0$ output nodes respectively.

7 Extraction of $B^+$ and $B^0$ Lifetimes

7.1 Monte Carlo Weighting

The first stage of the fitting procedure involved weighting quantities in the Monte Carlo to agree with recent experimental measurements. Weights were constructed to account for the following effects:

- The current world average measurements of $B^0_s$ and $\Lambda_b$ lifetimes and $B$-species production fractions (see the systematics summary tables of Section 8 for the actual values).

- The $b$-fragmentation function. The value of $\langle x \rangle$ in the default Monte Carlo was weighted to agree with the value obtained from a recent DELPHI analysis of the 1994 data set [13].

- The ‘hemisphere quality’, described in Section 5.4, as a function of hemisphere track multiplicity so ensuring that the overall track multiplicity is essentially invariant under the application of the weight.

7.2 The Lifetime Fit

The $B^+$ and $B^0$ lifetimes were extracted by a simultaneous fit to proper lifetime distributions reconstructed in the $B^+$ and $B^0$-enhanced samples, using a binned $\chi^2$ method.

Nominally 100 bins were chosen but the exact binning was determined by the requirement that at least 10 entries be present in all bins of the data distribution. The fit range was chosen to be between 0.0 ps and 10.0 ps. The upper end of the range was positioned to avoid the worst effects of spurious very long reconstructed lifetimes while still accepting the vast majority of the data available.
Figure 3: The upper plots show the output of the $B^+$ and $B^0$ output nodes of the $NN(B_s)$ in the 1994 data. Overlaid is the $b$-hadron composition as seen in the Monte Carlo and the lower plots trace the change in purity per bin. The background, labelled 'bg' consists of light and charm quark events. The lower plots show the purity against efficiency performance attained by making sequential cuts on the $NN(B^+)$ and $NN(B^0)$ outputs.
To avoid the need to generate many separate Monte Carlo samples of different B-lifetimes, weighting factors were formed for each lifetime measurement from the ratio of exponential decay probability functions. Specifically, the weight,

\[ w_i = \frac{\tau_{old}}{\tau_{new}} \exp \left( \frac{t_i (\tau_{new} - \tau_{old})}{\tau_{old} \tau_{new}} \right), \]

for measurement \( i \) and true B-lifetime \( t_i \), effectively transforms the Monte Carlo lifetimes generated with a mean lifetime \( \tau_{old} \) to be distributed with a new mean value of \( \tau_{new} \). The \( \chi^2 \) function given below was then minimised with respect to the the \( B^+ \) and \( B^0 \) lifetimes,

\[ \chi^2 = \sum_{B^0, B^+} \sum_{i=1}^{N_{\text{bins}}} \frac{(W_i^{MC} - N_i^{\text{data}})^2}{(\sigma_i^{MC})^2 + (\sigma_i^{\text{data}})^2}. \]

Here, \( N_i^{\text{data}} \) is the number of data entries in bin \( i \) and \( W_i^{MC} \) is the corresponding sum of weights.

In addition, a one parameter fit was made to the not enhanced data sample (i.e. a sample containing the natural mix of b-hadron species) for the mean b-hadron lifetime \( \tau_b \). As was noted in Section 1, the mean lifetime is a quantity of physics interest and was also a useful tool for cross-checking the analysis during the development phase.

### 7.3 Fit working point and results

The selection conditions imposed on the data samples used for the lifetime fits were motivated by the wish to minimise the total error on the final results. Systematic error contributions due to inexact detector resolution simulation and the physics modelling of \( u, d, s \) and charm production, mean that relatively high b-hadron purities were required while still keeping the selection efficiency above a level where the statistical error would begin to significantly degrade.

With these considerations in mind, the criteria to select the final data samples to be used in the fitting procedure were as follows:

- An enhancement in \( Z^0 \rightarrow b\bar{b} \) events, attained by a cut on the DELPHI b-tagging variable.

- The \( N.N(B_c) \) neural network outputs, described in Section 6, were then cut at > 0.52 and > 0.6 to obtain enhanced samples in \( B^+ \) and \( B^0 \) respectively. These cut values corresponded to a purity in both \( B^+ \) and \( B^0 \) of \( \sim 70\% \) according to the Monte Carlo.

In addition, to ensure that the correlation between the two lifetimes was kept to a minimum for the final result, it was demanded that the two samples were statistically independent. This was achieved, e.g. for the case of selecting a \( B^+ \) hemisphere, by first requiring that the \( B^+ \) output node value passed a cut at 0.52. If however for this hemisphere, the \( B^0 \) output node value was also larger than the selection cut at 0.6, the hemisphere went into the \( B^+ \) sample only if the \( B^+ \) output node value was larger than the \( B^0 \) output node value. If this was not the case, the hemisphere entered into the \( B^0 \) sample. Following this procedure and after all selection cuts already described, the \( B^+(B^0) \) enhanced sample
consisted of 38988(11573) hemispheres in 1994 data and 18822(5366) hemispheres in 1995 data. This represented a selection efficiency, with respect to the selected multihadronic sample total, of about 14%(4.6%) in 1994 and 14%(4.7%) in 1995.

In addition to the weights (described in Section 7.1) and the sample selection cuts listed above, a final correction to the Monte Carlo was applied to account for residual physics and detector modelling deficiencies in the region of small reconstructed proper lifetimes i.e. < 1.0 ps. This 'acceptance correction' was formed by the ratio of the data to the simulation in this small lifetime region and was parameterised as a parabola. Monte Carlo studies showed that the form of the acceptance correction was very similar for B+ and B0 enhanced samples and so the same correction was applied in both cases. No such correction was applied for the case of the mean b-hadron lifetime, instead the lifetime fit was started at τ > 1.0 ps to avoid the badly modelled region. The stability of the fit above the τ = 1.0 ps point is illustrated in Figure 5.

![Figure 4](image1.png)

**Figure 4:** The result of the fit in the B+(top left) and B0 (top right) samples compared to data. The b-hadron composition of the B+(B0) sample is shown bottom left (bottom right). Here, 'bg' refers to the background from non-bB Z0 decays.

The results from all lifetime fits, after imposing the working point conditions and following the procedure described in Section 7.2, are listed in Table 1.

<table>
<thead>
<tr>
<th>b-State</th>
<th>Fitted Lifetime</th>
<th>Sample Size</th>
<th>χ²/d.f.</th>
</tr>
</thead>
<tbody>
<tr>
<td>B+</td>
<td>1.6364 ± 0.0155 (1.6285 ± 0.0230) ps</td>
<td>36088 (17367)</td>
<td>207/179 (160/161)</td>
</tr>
<tr>
<td>B0</td>
<td>1.5823 ± 0.0238 ps (1.5142 ± 0.0358) ps</td>
<td>10790 (5065)</td>
<td></td>
</tr>
<tr>
<td>τb</td>
<td>1.5766 ± 0.0059 ps (1.5518 ± 0.0085) ps</td>
<td>179635 (86631)</td>
<td>76/88 (70/88)</td>
</tr>
</tbody>
</table>

**Table 1:** The results of the lifetime fits in the 1994(1995) data samples where the errors quoted are statistical only. The correlation coefficient between τ(B+) and τ(B0) was -0.49 (-0.50)

The B+ and B0 fits, for the case of 1994 data, are shown in Figure 4 and the stability...
Figure 5: Results from the mean b-hadron lifetime fit: the left-hand plot shows the fitted value of $\tau_0$ as a function of the fit start point. The result is stable above a starting value of about $\tau = 1$ ps. The right-hand plot shows the fit (histogram) to the data (points) at the working point together with the sample composition from Monte Carlo.

of the fit procedure around the working point can be gauged from Figure 6 as a function of varying key cuts used to define the working point.

For the case of the mean b-hadron lifetime fit, only the $Z^0 \rightarrow b\bar{b}$ enhancement cut was applied. The fit to the data is shown in Figure 5. A crosscheck was also made to test the validity of fitting for a single mean lifetime component when it is predicted that the sample could contain (at least) four different lifetime components. The crosscheck involved fitting the sample to four separate lifetime components i.e. $\tau(B^+)$, $\tau(B^0_d)$, $\tau(B^0_s)$ and $\tau(b - baryon)$, using a $\chi^2$ formalism that constrained $\tau(B^+)$ and $\tau(B^0_d)$ to be the values obtained from this analysis and constrained $\tau(B^0_s)$ and $\tau(b - baryon)$ to be their current world average values of 1.464 ps and 1.208 ps respectively. The results of this fit, when combined with the current world average values for b-hadron production fractions $F(B_i)$ (see Table 2), to form a mean lifetime, $\langle \tau_B \rangle = \sum_i F(B_i) \cdot \tau(B_i)$, gave a value that was consistent, to much better than the statistical error, with the result of the single parameter fit. The inclusive sample used for the mean b-hadron lifetime fit, contained a background of 3.4% and the $Z^0 \rightarrow b\bar{b}$ component had the following composition: 43.7% ($B^+$), 42.3% ($B^0$), 6.7% ($B^0_s$) and 7.2% (b-baryons). The corresponding fractions present in the Monte Carlo generator were: 41.8% ($B^+$), 41.8% ($B^0$), 7.3% ($B^0_s$) and 9.1% (b-baryons).

8 Systematic Uncertainties

Tables 3,4 and 5 present the final results for $\tau(B^+)$, $\tau(B^0)$ and $\tau_b$ together with the full systematic error breakdown for 1994 and 1995 data respectively.

Where possible, B-physics modelling uncertainties were estimated by varying central values by plus and minus one standard deviation and taking half of the observed change in the fitted lifetime value as the resulting systematic uncertainty from that source.
Figure 6: Plots from 1994 data, showing the variation in the fitted ratio $\tau(B^+)/\tau(B^0)$ as a function of the $Z^0 \rightarrow b\bar{b}$ purity (top left), the enhancement cuts on the $NN(B_\tau)$ network output (top right), the starting point of the fit (lower left) and the end-point of the fit (lower right). Note that the spread of the points for each bin of $B^0$ purity represents changing the $B^+$ purity in the range 65% – 75%. The upper and lower shaded bands represent the statistical one standard deviation error.
Table 2: Values for the $b$-hadron production fractions, taken from [14], used to weight the Monte carlo.

The $b$-hadron production fractions were varied within their errors taking account of the covariance matrix listed in Table 2. Half of the full variation seen when applying the recalculated weight was assigned as a systematic. The uncertainty from D-topological branching fractions was estimated from the difference in the fit result obtained when weighting according to the results from [15]. The systematic error due to uncertainty in the b-fragmentation function was set to the change in the fit result seen when the fragmentation weight was put in and out of the analysis.

<table>
<thead>
<tr>
<th>b-hadron species</th>
<th>fraction</th>
<th>correlation with $f(B_s)$</th>
<th>correlation with $f(b - \text{baryon})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_s^-$</td>
<td>$f(B_s^-) = 0.097 \pm 0.011$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b-baryons</td>
<td>$f(b - \text{baryon}) = 0.104 \pm 0.017$</td>
<td>+0.034</td>
<td></td>
</tr>
<tr>
<td>$B_d^-$ or $B^+$</td>
<td>$f(B_d^-) = f(B^+) = 0.399 \pm 0.010$</td>
<td>-0.577</td>
<td>-0.836</td>
</tr>
</tbody>
</table>

Table 3: Summary of systematic uncertainties in the $B^+$ and $B^0$ lifetimes and their ratio for 1994 data. Systematic errors are assumed independent and added in quadrature to give the final total error.

<table>
<thead>
<tr>
<th>Source of Systematic Error</th>
<th>Range</th>
<th>Physical Modelling</th>
<th>Analysis Method</th>
<th>Detector Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\Delta \tau_{B^+}$</td>
<td>$\Delta \tau_{B^0}$</td>
<td>$\Delta \frac{\tau_{B^+}}{\tau_{B^0}}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6364</td>
<td>1.5823</td>
<td>1.0342</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0155</td>
<td>0.0238</td>
<td>0.0226</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B_s$ lifetime</td>
<td>1.464 $\pm$ 0.057 ps</td>
<td>0.0007</td>
<td>0.0079</td>
<td>0.0048</td>
</tr>
<tr>
<td>b-baryon lifetime</td>
<td>1.208 $\pm$ 0.051 ps</td>
<td>0.0008</td>
<td>0.0028</td>
<td>0.0030</td>
</tr>
<tr>
<td>b-hadron prod. fractions</td>
<td>See text</td>
<td>0.0034</td>
<td>0.0038</td>
<td>0.0003</td>
</tr>
<tr>
<td>frag. function weight</td>
<td>on/off</td>
<td>0.0001</td>
<td>0.0011</td>
<td>0.0006</td>
</tr>
<tr>
<td>B $\rightarrow$ D branching fractions</td>
<td>See text</td>
<td>0.0085</td>
<td>0.0069</td>
<td>0.0008</td>
</tr>
<tr>
<td>BR(B $\rightarrow$ wrong-sign charm)</td>
<td>11% $\rightarrow$ 22%</td>
<td>0.0097</td>
<td>0.0135</td>
<td>0.0029</td>
</tr>
<tr>
<td>BR($B_s$ $\rightarrow$ D$_s$)</td>
<td>35% $\rightarrow$ 70%</td>
<td>0.0019</td>
<td>0.0075</td>
<td>0.0061</td>
</tr>
<tr>
<td>D$^+$, D$^0$ topo. branching ratios</td>
<td>[15]</td>
<td>0.0002</td>
<td>0.0100</td>
<td>0.0065</td>
</tr>
<tr>
<td>B meson mass</td>
<td>$m_B = 5.2789 \pm 0.0018$ GeV</td>
<td>0.0002</td>
<td>0.0017</td>
<td>0.0010</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b and c efficiency correction</td>
<td>on/off</td>
<td>0.0054</td>
<td>0.0036</td>
<td>0.0059</td>
</tr>
<tr>
<td>$N(N(B_s))$ cuts</td>
<td>65%-75% purity</td>
<td>0.0093</td>
<td>0.0216</td>
<td>0.0196</td>
</tr>
<tr>
<td>$N(N(B_s))$ shape</td>
<td>See text</td>
<td>0.0008</td>
<td>0.0099</td>
<td>0.0059</td>
</tr>
<tr>
<td>Binning</td>
<td>50-250 bins</td>
<td>0.0045</td>
<td>0.0094</td>
<td>0.0049</td>
</tr>
</tbody>
</table>

Close attention was paid to possible systematic effects on the analysis due to the
modelling of $b \to$ charm branching ratios, where current experimental knowledge is scarce. The charm content impacts on the performance of the $B^+$ and $B^0$ enhancement networks and can pull the reconstructed $B$-vertex position to longer decay lengths. The size of this pull in turn depends on whether a $D^0$ or $D^-$ was produced since $\tau(D^+) \sim 2.5$ times larger than $\tau(D^0)$. Specific aspects of the Monte Carlo that were found to warrant systematic error contributions were:

(a) Branching ratios for $\bar{B}^0 \to (D^+)D^0X$ and $B^- \to (D^-)D^0X$ were adjusted in the Monte Carlo according to a fit using all currently available measurements from [11] as constraints.

(b) the standard Monte Carlo data set used was generated with a wrong-sign charm rate of only 11% and contained no wrong-sign $D^0$ or $D^+$ production. It is now known (see e.g. [16]) that there is an additional wrong-sign contribution, of a comparable size, due to the production of $D^0$ and $D^+$ at the $W$ or upper-vertex. To estimate the effect of this omission a dedicated Monte Carlo sample containing wrong-sign $D^0$ and $D^+$ production was combined with the standard Monte Carlo data set raising the overall wrong-sign branching ratio to 22%. Since as described above, this has the effect of increasing the reconstructed lifetime (or equivalently decreasing the fitted lifetime), the final fit central values were corrected downwards by half of the observed shift in the fitted lifetime. The full change in the resulting lifetime fit was taken as a systematic error from this source.

(c) BR($b \to D_sX$) is currently known to, at best, $\pm 30\%$ [11] and was varied in the Monte Carlo by a factor two. Half of the observed change in the fitted lifetime was then assigned as a systematic error.

The efficiency for reconstructing $b\bar{b}$ and $c\bar{c}$ events (as a function of the event $b$-tag) has been evaluated by a double hemisphere tagging technique [17] on the DELPHI data set. The results suggest that while the reconstruction efficiency for $b\bar{b}$ events might be wrong in the simulation by a few percent, the efficiency for $c\bar{c}$ events could be up to 20% underestimated in the simulation compared to data. To account for this possible source of error, the efficiencies were changed in the simulation according to these results and the difference seen in the fit results assigned as a systematic error. Since a large part of the discrepancy between simulation and data in the $c\bar{c}$ event reconstruction efficiency is probably due to a relatively poor modelling of D-physics, this error contribution has already, in part, been accounted for by the explicit D-physics systematics detailed above. Given the current level of uncertainty in this sector we prefer to take the conservative approach of quoting both error contributions.

Uncertainties resulting from the method itself have been checked by firstly scanning regions around critical cut values to check for stability. Figure 6 illustrated the effect of cut scans in the $Z^0 \to b\bar{b}$ event purity and the upper and lower lifetime range of the fit, showing a good stability over a wide range of the cut values. In addition, the binning used for the $\chi^2$ formulation was varied and half of the maximum variation observed in the fitted lifetime taken as a systematic.

Rather critical to the analysis is the assumption that the $B^+$ and $B^0$ purity as given by the simulation, well models the situation in data. The effect in the result of scanning over the $B^+$ and $B^0$ enhancement purities was also illustrated in Figure 6. A systematic error will arise if the composition of the $B^+$ and $B^0$ simulated samples differ from the data and/or the overall shape of the $NN(B^+)$ and $NN(B^0)$ network outputs differ. To account for the first effect, half of the maximum variation in the fitted lifetime while scanning the purity range $[65\%, 75\%]$ was assigned as an error. A separate error contribution was also
assigned to take account of any residual difference in shape between data and simulation in the $NN(B_x)$ distribution. Assuming that the difference can be wholly accounted for by a change in the $B^+$ or $B^0$ composition in the sample, it was found that the maximum error made in calculating the $B^+$ or $B^0$ purities in the samples fitted was of order 2\% and 4\% respectively. The effect of these changes were then propagated into errors on the extracted lifetimes.

The acceptance correction, described in Section 7.3, ensured that the scan over different choices of start point for the fit (see Figure 6) was stable. Since modelling all details of the low decay length region is a complex task we choose to assign a generous systematic for the acceptance correction based on the maximum change seen in the fit results when scanning around the start point with the acceptance correction turned off.

In order to account for uncertainties in the simulation originating from detector response modelling, the effect was studied of switching on and off the following corrections:

- the hemisphere quality weight, described in Section 7.1,
- an attempt to better match the track impact parameter and error (with respect to the primary vertex) between simulation and data according to the prescription detailed in [18].

Since in general, knowledge of detector modelling uncertainties are not at the same level of certainty as e.g. the knowledge that $B$-production fractions in our Monte Carlo generator differ with the world average, we have taken the following approach to assigning systematic values for these effects: All four combinations of switching these corrections on/off in the analysis were made and the mean fitted lifetimes of the four possibilities recorded. The central results were then chosen to be these mean values, and the resulting systematic error from detector response modelling was assigned to be half of the full spread of the values from the four combinations.

9 Summary and Conclusion

The lifetimes of $B^+$, $B^0$ (plus their ratio) and the mean b-hadron lifetime have been measured. The analysis isolated b-hadron candidates with neural network techniques trained to exploit the physical properties of inclusive b-hadron decays. Binned $\chi^2$ fits to the resulting DELPHI data samples collected in 1994 and 1995 yielded the results presented in Table 3 and 4 for $B^+$ and $B^0$ and the result for the mean b-hadron lifetime is presented in Table 5.

A combination of the results, treating all systematics as 100\% uncorrelated apart from physics-modelling systematics gives:

$$\tau_{B^+} = 1.633\pm0.013 \text{ (stat)}\pm0.020 \text{ (syst)} \text{ ps}$$

$$\tau_{B^0} = 1.560\pm0.020 \text{ (stat)}\pm0.036 \text{ (syst)} \text{ ps}$$

$$\frac{\tau_{B^+}}{\tau_{B^0}} = 1.045\pm0.019 \text{ (stat)}\pm0.023 \text{ (syst)}$$

And for the average b-hadron lifetime:

$$\tau_b = 1.571\pm0.005 \text{ (stat)}\pm0.006 \text{ (syst)} \text{ ps}$$
Table 4: Summary of systematic uncertainties in the B⁺ and B⁰ lifetimes and their ratio for 1995 data. Systematic errors are assumed independent and added in quadrature to give the final total error.

References


Table 5: Summary of systematic uncertainties in the average b-hadron lifetime for 1994 and 1995 data. Systematic errors are assumed independent and added in quadrature to give the final total error.

<table>
<thead>
<tr>
<th>Source of Systematic Error</th>
<th>Range</th>
<th>$\Delta \tau_B$ for 1994</th>
<th>$\Delta \tau_B$ for 1995</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physics Modelling</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b-hadron prod. fractions</td>
<td>See text</td>
<td>0.0012</td>
<td>0.0010</td>
</tr>
<tr>
<td>frag. function weight</td>
<td>on/off</td>
<td>0.0014</td>
<td>0.0018</td>
</tr>
<tr>
<td>BR($B \rightarrow \text{wrong-sign charm}$)</td>
<td>11% → 22%</td>
<td>0.0034</td>
<td>0.0035</td>
</tr>
<tr>
<td>BR($B_s \rightarrow D_s$)</td>
<td>35% → 70%</td>
<td>0.0011</td>
<td>0.0015</td>
</tr>
<tr>
<td>B → D branching fractions</td>
<td>See text</td>
<td>0.0005</td>
<td>0.0015</td>
</tr>
<tr>
<td>$D^+, D^0$ topo. branching ratios</td>
<td>[15]</td>
<td>0.0011</td>
<td>0.0003</td>
</tr>
<tr>
<td>B meson mass</td>
<td>$m_B = 5.2789 \pm 0.0018$ GeV</td>
<td>0.0002</td>
<td>0.0009</td>
</tr>
<tr>
<td>Analysis Method</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b and c efficiency correction</td>
<td>on/off</td>
<td>0.0013</td>
<td>0.0028</td>
</tr>
<tr>
<td>Binning</td>
<td>50-250 bins</td>
<td>0.0024</td>
<td>0.0040</td>
</tr>
<tr>
<td>Detector Effects</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Resolution and hemisphere quality</td>
<td>On/off</td>
<td>0.0035</td>
<td>0.0083</td>
</tr>
<tr>
<td>Total Systematic Error</td>
<td></td>
<td>0.0061</td>
<td>0.0107</td>
</tr>
</tbody>
</table>


[12] G. V. Borisov, Lifetime tag of events $Z^0 \rightarrow \bar{b}b$ with the DELPHI detector. AABTAG program., DELPHI internal note, 94-125 PROG 208, (1994);


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