Abstract

A limit on the tau neutrino mass is obtained using all the data $Z^0 \rightarrow \tau^+\tau^-$ collected by the DELPHI detector between 1992 and 1995. In this analysis events have been used in which one of the taus decays into a charged particle, while the second $\tau$ decays into five charged pions (1-5 topology).

The neutrino mass is determined from a bidimensional fit of the invariant mass spectrum $m_{5\pi}$ and of the energy spectrum $E_{5\pi}$ of the five pion system. The result found is $m_{\nu_\tau} < 54.8$ MeV/$c^2$ at 95% of the confidence level.

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1 Introduction

The question of whether neutrinos are massive is one of the outstanding issues in particle physics, astrophysics and cosmology.

Among the possible frameworks the “see-saw” mechanism [1] is considered to be the most interesting because it explains the smallness of neutrino masses by connecting them to the scale of new physics. In fact the see-saw mechanism assumes a neutrino mass hierarchy similar to that of quarks or leptons.

On the basis of cosmological arguments a stable tau neutrino $\nu_\tau$ with a mass larger than a few eV/$c^2$ cannot exist; however unstable neutrinos can be more massive, so a direct neutrino mass measurement of the order of a few MeV/$c^2$ would suggest that the $\nu_\tau$ is unstable.

The clearest evidence of neutrinos having mass comes from the oscillation experiments, even if this kind of experiment cannot give the neutrino scale mass. Only direct measurements can do it and the most recent experiments [2] have fixed the mass of the electron neutrino to be about 2 eV/$c^2$; as a consequence, there is no room for a massive $\nu_\tau$. However the results of the oscillation experiments must be confirmed before concluding that massive unstable neutrinos cannot exist.

Several direct experimental measurements have been done of the $\nu_\tau$ mass. The lowest limit was obtained by the ALEPH detector [3] [4] combining the analysis of $\tau^\pm \rightarrow 3\pi^\pm 2\pi^0 (\pi^0)\nu_\tau (\nu_\tau)$ and $\tau^\pm \rightarrow 2\pi^\pm 3\pi^- (\pi^-)\nu_\tau (\nu_\tau)$ events: the final upper limit is $m_{\nu_\tau} < 18.2$ MeV/$c^2$ at 95% of the confidence level.

2 The DELPHI detector and the data sample

The DELPHI detector and its performances are described in detail in [5] [6]. We give here a brief description of the subdetectors used for this analysis. The DELPHI detector is composed by a main central part (barrel) and by two forward regions (end-caps) and it covers the full solid angle. A superconducting solenoid provides a highly uniform magnetic field of 1.23 T parallel to the $z$ axis\(^1\).

Charged particles are tracked in the barrel region of DELPHI using a combination of four cylindrical subdetectors: the silicon Vertex Detector (VD), the Inner Detector (ID), the Time Projection Chamber (TPC) and the Outer Detector (OD).

The VD has three cylindrical layers of silicon micro-strip modules at the average radii of 6.3 cm, 9.0 cm and 10.9 cm from the beam axis. The full azimuth angle is covered by each layer with 24 partially overlapping sections. The space point precision is 8 $\mu$m in $r\phi$. From 1994 onwards, the inner and the outer layers were equipped with double sided silicon detectors, and a measurement of $rz$ was possible with a precision of about 15 $\mu$m. The two track resolution is 100 $\mu$m in $r\phi$ and 200 $\mu$m in $rz$. The ID, consisting of a jet chamber tracker and of a wire chamber used for trigger purposes, has an inner radius of 12 cm and an outer radius of 28 cm. The inner jet chamber part lay at radii below 22 cm. The ID has a two tracks resolution of 1 mm in $r\phi$ and a precision of about 50 $\mu$m. The TPC is the main tracking system. It is placed just outside the ID. The

\(^1\)In the standard DELPHI coordinate system, the $z$ axis is along the electron direction, the $x$ axis points towards the centre of LEP, and the $y$ axis points upwards. The coordinates $r$, $\theta$, $z$ form a cylindrical coordinate system, while $\theta$ is the polar angle with respect to the $z$ axis.
detector provides up to 16 space points per particle trajectory. The TPC can also help in charged particle identification by measuring the $dE/dx$: the sense wires of its proportional chambers provide up to 192 ionization measurements per track. Every $60^\circ$ in $\phi$ there is a boundary region between read-out sectors about $1^\circ$ wide without instrumentation; at $\cos \theta = 0$ there is the cathode plane which causes a reduced tracking efficiency in the polar angle range $|\cos \theta| < 0.035$. The TPC has a two track resolution of about 3 mm in $r \phi$ and 1.5 cm in $z$. The OD consists of five layers of drift tubes, operating in the limited streamer mode, located between radii of 197 cm and 206 cm. Its main purpose is to give a measurement of the momenta of the most energetic particles. The single point precision is 110 $\mu$m in $r \phi$, while the precision in the $z$ coordinate is 3.5 cm.

Electron and photon identification is provided by the High density Projection Chamber (HPC) located outside the OD. It covers the polar angle region $|\cos \theta| < 0.75$, while in the forward region $0.800 < |\cos \theta| < 0.985$ the calorimeter consists of lead-glass blocks. The smaller polar angles, essential for luminosity measurement, are covered by the Small angle TTile Calorimeter (STIC). Hadron and muon identification are performed by an hadron calorimeter and by muon chambers that lay outside the iron return of the superconducting magnet.

For this analysis, all the data collected by the DELPHI detector in the years 1992 through 1995 have been used. The centre-of-mass energy $\sqrt{s}$ of the $e^+e^-$ system was between 89 and 93 GeV. The integrated luminosity of the data sample was 135 pb$^{-1}$, about 100 pb$^{-1}$ of which has been taken at $\sqrt{s}$ equal to 91.3 GeV.

In section 3 the method used in this analysis is described. In section 4 the data selection is presented, based on variables defined using samples of simulated events which had been passed through a detailed simulation of the detector response and reconstructed with the same program used for the data. Section 5 describes the Maximum Likelihood Method used to give the upper limit on the $\nu_\tau$ mass, while in section 6 the systematic uncertainties are considered.

### 3 The method

The multi-hadronic decays of the tau lepton [7] [8] are due to the couplings of the charged weak current to hadrons, which in the case of the decay $\tau^\pm \rightarrow 3\pi^\pm 2\pi^0 \nu_\tau (\nu_\tau)$ reduces to the coupling of the $W^\pm$ boson to the $ud'$ current.

We can describe the tau decay as a two-body decay:

$$
\tau^- (E_\tau, \vec{p}_\tau) \rightarrow h^- (E_h, \vec{p}_h) + \nu_\tau (E_{\nu}, \vec{p}_{\nu})
$$

(1)

where the hadronic system $h^-$ is composed by five pions. In the tau rest frame, the energy of the hadronic system is used to compute the value of $m_{\nu_\tau}$ because of its dependence on $m_\tau$, $m_h$ and $m_{\nu_\tau}$:

$$
E_h^* = \frac{m_\tau^2 + m_h^2 - m_{\nu_\tau}^2}{2m_\tau}
$$

(2)

In the laboratory frame this energy becomes:
Figure 1: two hypothetical events have been plotted in the $E_{5\pi}/E_{\text{beam}}$ vs $m_{5\pi}^*$ plane with their error ellipses. The coloured lines bound the allowed kinematic regions for different values of $m_{\nu_e}$.

$$E_h = \gamma (E_h^* + \beta p_h^* \cos \theta)$$

(3)

where $\beta = \sqrt{(E_\tau^2 - m_\tau^2)/E_\tau^2}$ is the tau velocity, $\gamma = \sqrt{1/(1 - \beta^2)}$ is the boost factor and $\theta$ is the angle between the direction of the $\tau$ and that of the hadronic system in the tau rest frame. We assume here that the $\tau$ energy, $E_\tau$, is equal to the beam energy, $E_{\text{beam}}$. Initial (ISR) and final (FSR) state radiation can reduce $E_\tau$ and will be considered later in section 6.

Because of the undetected neutrino, the direction of the $\tau$ is not determined, so $m_{\nu_e}$ cannot be computed directly. However, the energy of the hadronic system is helpful because it partially recovers the loss of information. In fact $E_h$ depends on $\cos \theta$ and it must fall inside the interval $E_{\text{min}} \leq E_h \leq E_{\text{max}}$:

$$E_{h,\text{min,max}}^* = \gamma (E_h^* \pm \beta p_h^*)$$

(4)

In this way kinematic allowed regions are defined for different values assumed by $m_{\nu_e}$, as showed in Fig. 1 obtained with the KORALZ 4.0 [9] Montecarlo event generator for $e^+e^- \to Z^0 \to \tau^+\tau^-$ events. The KORALZ 4.0 simulator incorporated the TAUOLA 2.4 [10] package for modelling the $\tau$ lepton decays. The probability to find an event in the allowed region clearly depends on the kinematic of the decay and on the resonance structure. Because both the invariant mass $m_{5\pi}^*$ and $E_{5\pi}/E_{\text{beam}}$ for the hadronic system are functions of the measured momenta of the five charged tracks, a positive correlation arises between these two quantities. The error ellipses for each event is taken into account, so that, as shown in Fig. 1, event 2 constraints the neutrino mass $m_{\nu_e}$ much more than event 1, even if the latter has an higher hadronic mass. This is the advantage of fitting the distribution of the two variables $m_{5\pi}^*$ and $E_{5\pi}/E_{\text{beam}}$, rather than $m_{5\pi}^*$ alone.
4 Data selection

At centre-of-mass energies of $\sqrt{s} \simeq M_{Z^0}$, $\tau^+$ and $\tau^-$ are produced back-to-back (ignoring radiative effects). Each $\tau$ (with a lifetime $\tau_\tau = 290.6 \pm 1.1$ fs) decays producing one, three or more charged particles in addition to one or two neutrinos and, possibly, neutral mesons. All particles apart from neutrinos can be detected by DELPHI. So $e^+e^- \to Z^0 \to \tau^+\tau^-$ decays are easy to recognize because the two back-to-back jets are narrow and with low charged multiplicity. The undetected neutrinos imply that not all the energy in the event is seen and the invariant mass of the final state particles $m^*\tau$ is less than $m_{\tau\tau}$.

The background from multihadron production $e^+e^- \to Z^0 \to q\bar{q}$ at LEP energies can easily be reduced since it is characterized by a relatively high charged multiplicity and by an high invariant mass. Backgrounds from $e^+e^- \to Z^0 \to e^+e^-$ and $e^+e^- \to Z^0 \to \mu^+\mu^-$ leave very characteristic signatures in DELPHI and are effectively discriminated.

4.1 Preselection criteria

The first step was to reject all those events in which there is not a $Z^0$ decay (cosmic rays, secondary interactions in the vacuum pipe, etc.). Each event has been split into two hemispheres defined by the plane perpendicular to the thrust axis. It was required that the reconstructed topology was one charged particle in one side and five in the other (1-5).

Then events were accepted if $E_{\text{vis}}$, defined as the sum of the momenta of all the charged particles plus the total neutral electromagnetic energy, was greater than 0.09 $\sqrt{s}$. This cut removes most of the hadronic $Z^0$ decays, $\gamma\gamma$ interactions and beam gas interactions. To reject cosmics and to reduce further the contamination from beam gas events it was required that, defining particles 1 and 2 as the most energetic in each hemisphere, $r_{1,2} < 1.5$ cm and $z_{1,2} < 4.5$ cm, where $r_j$ and $z_j$ are their transverse and longitudinal distances of closest approach to the average beam spot. Distributions of the $E_{\text{vis}}$ variable for data and simulation ($\tau^+\tau^-$ and backgrounds) is given in Fig. 2 as an example.

4.2 External background

Most of $e^+e^- \to Z^0 \to q\bar{q}$ events were rejected by the requirement on the topology and on the impact parameters as described above. The low multiplicity $q\bar{q}$ and $\gamma\gamma$ events can be easily rejected by requiring that the isolation angle $\theta_{\text{iso}}$ be greater than 160°. The isolation angle is defined as the minimum angle between any pair of tracks belonging to opposite hemispheres.

Another discriminating variable is the total momentum of the 5-prong system, $P_3$. It must be greater than 30 GeV/c. We point out that either $\theta_{\text{iso}}$ and $P_3$ are related to the invariant mass $m_{5\pi}$ of the hadronic system, but the reader can see in section 4.4 that the invariant mass spectrum is not affected in the region of the end point by these selections. To reduce further $q\bar{q}$ we required that the total number of neutrals must be less than six. The hadronic $Z^0$ decays usually have a large number of neutrals, whereas the signal we are looking for should have no or few neutrals.

If 1 and 2 are the most energetic particles in each hemispheres, then:
are useful to reduce the contamination from $\mu^+\mu^-(\gamma)$ and $e^+e^-(\gamma)$. Here, $\vec{p}_1$ and $\vec{p}_2$ are the momenta of the highest momentum charged particles, and the quantity $p'_1$ is given by the following relation:

\[
p'_1 = \sqrt{s} \frac{\sin \theta_2}{\sin \theta_1 + \sin \theta_2 + |\sin (\theta_1 + \theta_2)|}
\]  

(similarly $p'_2$, with indices 1 and 2 interchanged). The angles $\theta_1$ and $\theta_2$ are the polar angles of the particles; $E_1$ and $E_2$ are the total electromagnetic energies deposited in cones of half-angle 30° around the momentum vectors $\vec{p}_1$ and $\vec{p}_2$ respectively.

Four-fermion background was suppressed by the requirement on the topology. Fig. 3, 4 and 5 show the spectra of all these variables.

### 4.3 Internal background

The minimum momentum among the five charged particles can help to reject those events with a false 5-prong multiplicity. In fact there could be $\tau^\pm \rightarrow 2\pi^\pm \pi^\mp \nu_\tau (\nu_\tau) \geq 1\pi^0$ events that are reconstructed with a 5-prong multiplicity because of a secondary interaction or a $\pi^0$ Dalitz decay ($\pi^0 \rightarrow \gamma e^+e^-$). For this kind of events, secondary interactions or successive decays cause a degradation of the outgoing momenta. Events with $P_{\text{min}} < 0.5 \text{ GeV}/c^2$ were rejected.

The minimum invariant mass of pairs of oppositely charged particles in the hemisphere containing five charged particles, $m_{ee}$ (assuming the electron mass for both), was required to be greater than 0.05 GeV/$c^2$, to reject events with a $\gamma \rightarrow e^+e^-$ conversion. The decay $\tau^\pm \rightarrow 3\pi^\pm 2\pi^\mp \nu_\tau (\nu_\tau) \geq 1\pi^0$ is rejected requiring that the maximum value of the electromagnetic energy to be less than 4 GeV.

### 4.4 Final data sample

Table 1 presents a summary of the effects of the selection on simulation, and Table 2 the same on data. The simulated sample at the end consists of 264 events, with the composition listed in Table 3.

A total of 47 $\tau^\pm \rightarrow 3\pi^\pm 2\pi^\mp \nu_\tau (\nu_\tau)$ candidates have been selected in the data. This sample is made up by the signal and some background events. In particular Fig. 6 shows the distribution of the hadronic invariant mass $m_{5\pi}$ for the simulation (signal and background) and a comparison between simulation and data. $Z^0 \rightarrow q\bar{q}$ events have typically values of the invariant mass greater than 2 GeV/$c^2$, while $\tau^\pm \rightarrow 5\pi^\pm \nu_\tau$ events have invariant mass values lower than $m_{\tau}$. Before proceeding to the analysis of the selected data, a cross check on the five prong branching ratio has been performed. The result:
\[ Br(\tau^\pm \rightarrow 5\pi^\pm\nu_\tau(\nu_\tau)) = (0.8 \pm 0.2) \cdot 10^{-3} \] (7)

is in good agreement with the branching ratio quoted by the Particle Data Group [11],
\[ Br = (0.99 \pm 0.7) \cdot 10^{-3}. \]

<table>
<thead>
<tr>
<th>Montecarlo</th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5\pi^\pm</td>
<td>5\pi^\pm\pi^0</td>
<td>3\pi^\pm</td>
<td>3\pi^\pm n\pi^0</td>
<td>other</td>
<td>q\bar{q}</td>
<td>\gamma\gamma</td>
<td>e\bar{e}\tau</td>
</tr>
<tr>
<td>1-5 topology</td>
<td>516</td>
<td>103</td>
<td>606</td>
<td>2147</td>
<td>684</td>
<td>3133</td>
<td>912</td>
<td>51</td>
</tr>
<tr>
<td>( P_{vis} &gt; 0.09\sqrt{s} \text{ GeV/c} )</td>
<td>516</td>
<td>103</td>
<td>605</td>
<td>2147</td>
<td>678</td>
<td>2756</td>
<td>139</td>
<td>51</td>
</tr>
<tr>
<td>( r_{1,2} &lt; 1.5 \text{ cm} ) (</td>
<td>z_{1,2}</td>
<td>&lt; 4.5 \text{ cm} )</td>
<td>509</td>
<td>102</td>
<td>587</td>
<td>2065</td>
<td>581</td>
<td>2581</td>
</tr>
<tr>
<td>( \theta_{\text{iso}} &gt; 160^\circ )</td>
<td>502</td>
<td>99</td>
<td>392</td>
<td>1880</td>
<td>435</td>
<td>436</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>( P_{\text{rad}} &lt; 1 ) ( E_{\text{rad}} &lt; 1 )</td>
<td>421</td>
<td>78</td>
<td>318</td>
<td>1490</td>
<td>345</td>
<td>371</td>
<td>0</td>
<td>18</td>
</tr>
<tr>
<td>( P_3 &gt; 30 \text{ GeV/c}^2 ) ( N^\text{tot}_{\text{neutrals}} &lt; 6 )</td>
<td>350</td>
<td>50</td>
<td>155</td>
<td>613</td>
<td>80</td>
<td>52</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>( P_{\text{min}} &gt; 0.8 \text{ GeV/c} ) ( NVD_{\text{min}} \geq 1 )</td>
<td>248</td>
<td>36</td>
<td>18</td>
<td>57</td>
<td>0</td>
<td>26</td>
<td>0</td>
<td>7</td>
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<tr>
<td>( m_{ee} &gt; 0.05 \text{ GeV/c}^2 ) ( \text{Max}(E_{\text{em}}) \leq 4 \text{ GeV} )</td>
<td>201</td>
<td>22</td>
<td>6</td>
<td>9</td>
<td>0</td>
<td>26</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 1: effects of selection on the signal and on different sources of background. In the column “other” we summarize all the other channels of the internal background.

<table>
<thead>
<tr>
<th>Data</th>
<th></th>
<th></th>
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<td>1-5 topology</td>
<td>1518</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>( P_{vis} &gt; 0.09\sqrt{s} \text{ GeV/c} )</td>
<td>1222</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r_{1,2} &lt; 1.5 \text{ cm} ) (</td>
<td>z_{1,2}</td>
<td>&lt; 4.5 \text{ cm} )</td>
<td>1137</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \theta_{\text{iso}} &gt; 160^\circ )</td>
<td>631</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{\text{rad}} &lt; 1, E_{\text{rad}} &lt; 1 )</td>
<td>515</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_3 &gt; 30 \text{ GeV/c}^2 ) ( N^\text{tot}_{\text{neutrals}} &lt; 6 )</td>
<td>204</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{\text{min}} &gt; 0.8 \text{ GeV/c} ) ( NVD_{\text{min}} \geq 1 )</td>
<td>66</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>( m_{ee} &gt; 0.05 \text{ GeV/c}^2 ) ( \text{Max}(E_{\text{em}}) \leq 4 \text{ GeV} )</td>
<td>47</td>
<td></td>
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Table 2: effects of selection on the data.

<table>
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<tr>
<td>5\pi^\pm</td>
<td>76.14%</td>
</tr>
<tr>
<td>5\pi^\pm\pi^0</td>
<td>8.33%</td>
</tr>
<tr>
<td>3\pi^\pm</td>
<td>2.27%</td>
</tr>
<tr>
<td>3\pi^\pm n\pi^0</td>
<td>3.41%</td>
</tr>
<tr>
<td>q\bar{q}</td>
<td>9.85%</td>
</tr>
</tbody>
</table>

Table 3: composition of the data sample.
Figure 2: total visible energy $E_{\text{vis}}$: the picture on the left shows the spectra of this variable for different simulated events; on the right, there is a comparison between Montecarlo (blue hatched histogram) and data (dots).

Figure 3: angle of isolation $\theta_{\text{iso}}$: the picture on the left shows the spectra of this variable for different simulated events; on the right, there is a comparison between Montecarlo (blue hatched histogram) and data (dots).
Figure 4: energy of the five charged tracks $P_5$: the picture on the left shows the spectra of this variable for different simulated events; on the right, there is a comparison between Montecarlo (blue hatched histogram) and data (dots).

Figure 5: total number of neutrals $N_{\text{neutrals}}$: the picture on the left shows the spectra of this variable for different simulated events; on the right, there is a comparison between Montecarlo (blue hatched histogram) and data (dots).
Figure 6: hadronic invariant mass $m_{5\pi}^*$ for the final data sample: on the left the spectra of this variable for different channels; on the right, there is a comparison between Montecarlo (hatched histogram) and data (dots).
5 The Likelihood function

An unbinned likelihood fit has been performed in order to determine an upper limit on $m_{\nu_{\tau}}$. The likelihood function depends on the invariant mass $m_{5\pi}$ and on the normalized energy $E_{5\pi}/E_{\text{beam}}$ of the hadronic system:

$$\mathcal{L} = \prod_{i=1}^{N_{\text{obs}}} P(m_i^*, E_i|m_{\nu_{\tau}})$$

where $N_{\text{obs}}$ is the number of selected candidates, while $P(m_i^*, E_i|m_{\nu_{\tau}})$ is the probability for observing each selected event $i$ at the position $(m_i^* \equiv m_{5\pi}, E_i \equiv E_{5\pi}/E_{\text{beam}})$ in the kinematic plane defined as a function of the tau neutrino mass $m_{\nu_{\tau}}$:

$$P(m_i^*, E_i|m_{\nu_{\tau}}) = \frac{1}{N} \frac{d^2 N (m_i^*, E_i|m_{\nu_{\tau}})}{dm \, dE} \otimes \mathcal{R}(m, E, \rho, \sigma_m, \sigma_E) \otimes \epsilon(m, E)$$

$$= \frac{1}{N} \int_{0}^{+\infty} \int_{0}^{+\infty} \frac{dmdE \, d^2 N (m_i^*, E_i|m_{\nu_{\tau}})}{dm \, dE} \mathcal{R}(m - m_i^*, E - E_i) \epsilon(m, E)$$

$\mathcal{R}(m, E, \rho, \sigma_m, \sigma_E)$ is the experimental resolution function, and $\epsilon(m, E)$ is the selection efficiency. Both $\mathcal{R}$ and $\epsilon$ them could present a dependence on either the invariant mass or the normalized energy. The shape of the resolution function $\mathcal{R}$ has been determined from simulation. Since the two variables are correlated, the experimental resolution function is a 2D gaussian depending on the three parameters $\sigma_m, \sigma_E$ and $\rho$. The invariant mass resolution is about 16 MeV/c^2 (in average), and the normalized energy resolution is of the order of $10^{-2}$ ($\sigma_E \simeq 500$ MeV). If the resolution is constant, the experimental resolution function can be factorized, and the likelihood $\mathcal{L}$ becomes simpler. In this analysis each event is considered with its proper resolution.

The selection efficiency $\epsilon(m, E)$ was found to have no dependence on either the invariant mass or the energy, as shown in Fig. 7. It is constant for both $m_{5\pi} > 1.6$ GeV/c^2 and $E_{5\pi}/E_{\text{beam}}$ greater than 0.85. The fit will be restricted then to this region of the $(m_{5\pi}, E_{5\pi}/E_{\text{beam}})$ plane.

The theoretical prediction $P(m_i^*, E_i|m_{\nu_{\tau}})$ have been calculated as a function of $m_{\nu_{\tau}}$ including initial (ISR) and final (FSR) radiation.

The 1/N factor in equation (9) ensures that the probability density used in the event likelihood is normalized for any neutrino mass because the kinematic allowed region in the $(m_{5\pi}, E_{5\pi}/E_{\text{beam}})$ plane depends on the value of $m_{\nu_{\tau}}$.

The event likelihood can be then expanded as:

$$\mathcal{L} = \alpha \mathcal{L}_{\text{signal}} + (1 - \alpha) \mathcal{L}_{\text{bgd}}$$

where the first term is the neutrino mass likelihood for a pure signal and the second term describes the background shape. $\alpha$ is the fraction of background. In the fit region the
background is reasonably flat and equal to about 6% of the signal, as shown in Fig. 6. 15 of the 47 selected events are in the fit region. The fit gives an upper limit of $m_{\nu_{\tau}} < 48.0$ MeV/c$^2$ at 95% CL; Fig. 8 shows the log-likelihood for these 15 events. A possible bias in the analysis method has been investigated by fitting Montecarlo samples with massive neutrinos. The fit was repeated with high statistics Montecarlo samples with three different values of $m_{\nu_{\tau}}$, 0, 30 and 60 MeV/c$^2$. In all cases the best fit results are in
good agreement with the input values. The corresponding values are listed in Table 4, the log-likelihood distributions for these three samples are shown in Fig. 9. The expected limit for \( m_{\nu_\tau} = 0 \text{ MeV}/c^2 \) is about 30 \( \text{MeV}/c^2 \).

A comparison between a 1D and a 2D analysis was also made. The 1D analysis consists on fitting the distribution of the hadronic invariant mass alone. The upper limit found in this case is \( m_{\nu_\tau} < 63 \text{ MeV}/c^2 \) at 95% CL.

### 6 Systematic uncertainties

Four sources of systematic errors have been considered in this analysis. For each source a new fit has been performed after having changed the parameter of that particular source by one standard deviation. The difference between the 95% CL upper limit on the tau neutrino mass computed in section 5 (\( m_{\nu_\tau} < 48 \text{ MeV}/c^2 \)) and the one obtained by the modified likelihood has been considered as the systematic error due to that particular source. All these variations have been then summed in quadrature to obtain the global systematic error which is added linearly to the upper limit of the original fit.

The systematic sources considered are:

- the tau mass \( m_\tau \) and its energy \( E_\tau \). In the Montecarlo program used to calculate the probability for observing each selected event \( i \), \( P(m_\tau^i, E_i|m_{\nu_\tau}) \), the value of \( m_\tau \),

<table>
<thead>
<tr>
<th>( m_{\nu_\tau} ) (MeV/c^2)</th>
<th>Best Fit (MeV/c^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>( 7.3^{+30.7}_{-7.3} )</td>
</tr>
<tr>
<td>30.0</td>
<td>34.0±17.2</td>
</tr>
<tr>
<td>60.0</td>
<td>55.2±19.0</td>
</tr>
</tbody>
</table>
Table 5: systematic variations of the 95% CL upper limit on \( m_{\nu_e} \).

<table>
<thead>
<tr>
<th>Source</th>
<th>Variation of ( m_{\nu_e} ) limit (MeV /c(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_\tau )</td>
<td>0.3</td>
</tr>
<tr>
<td>( E_{\text{beam}} )</td>
<td>(&lt; 0.1)</td>
</tr>
<tr>
<td>experimental resolution</td>
<td>2.5</td>
</tr>
<tr>
<td>mass calibration</td>
<td>1.0</td>
</tr>
<tr>
<td>fitted region</td>
<td>0.7</td>
</tr>
<tr>
<td>total</td>
<td>2.8</td>
</tr>
</tbody>
</table>

was taken from the Particle Data Group [11], \( m_\tau = 1777.03^{+0.30}_{-0.26} \) MeV/c\(^2\).

The tau energy \( E_\tau \) was assumed to be equal to the beam energy \( E_{\text{beam}} \). After having varied \( E_\tau \) as explained in [12], the impact of this systematic on the tau neutrino mass limit is negligible.

- detector effects such as the hadronic scale mass and the experimental resolution function. The systematics due to detector effects concern two aspects: the parametrization of the resolution function \( R \) and the calibration of the experimental apparatus.

The shape of the resolution function \( R \) has been studied with Montecarlo. Since the two variables \( m_{\pi^+\pi^-} \) and \( E_{\text{beam}}/E_{\text{beam}} \) are partially correlated, the function used for \( R \) is a two-dimensional Gaussian depending on three parameters, \( \sigma_m \), \( \sigma_E \) and the correlation \( \rho \). To determine the systematic error from \( R \) we have fitted the candidates fixing the average resolutions \( \overline{\sigma_m}, \overline{\sigma_E} \) instead of the proper \( \sigma_m \), \( \sigma_E \) of each event. The variation of the neutrino mass limit in negligible. In a second step we have considered the small non-Gaussian tails of the distributions of \( \sigma_m \) and \( \sigma_E \).

The resulting variation of the upper limit is 2.5 MeV/c\(^2\).

The invariant mass of the hadronic system is sensitive to possible miscalibration of momentum reconstruction. To investigate this effect, the decay \( D^0 \to K^-\pi^+\pi^-\pi^+ \) has been used. This topology is quite similar to that of the tau decay considered in this analysis, the opening angle between the kaon and the pions is large with respect to the angular resolution of the tracking system, no neutrinos are present and the value of \( m_{D^0} \) has been measured by several experiments [11]. The DELPHI hadronic scale mass is shifted by +4 MeV/c\(^2\), with a resolution of about one MeV/c\(^2\) [13].

- the selection efficiency and the effect of the size of the fitted region. The selection efficiency is independant both from the invariant mass and the normalized energy of the hadronic system: \( \epsilon (m, E) \) can be considered constant in the fitted region and factorized.

The systematic errors arising from the size of the fitted region have been investigated by varying the boundaries by 20 MeV/c\(^2\) in mass and by 1 GeV in energy. The corrisponding variations are small, less than 1 MeV/c\(^2\).

- the intermediate structure of the decay. There are very few studies of the spectral functions of the \( \tau^\pm \to 3\pi^\pm 2\pi^\mp \nu_\tau (\nu_\tau) \) decay. As explained in section 3, the multi-hadronic tau decays are due to the coupling of the charged weak current to hadrons.
Figure 10: distributions of the hadronic invariant mass for data (dots) and four models of decay. The black hatched histogram indicates the pure phase space model, while the red, the green and the blue lines are the distributions obtained by means of three different intermediate resonance structures.

<table>
<thead>
<tr>
<th>Model</th>
<th>$m_{\nu\tau}$ (MeV/$c^2$)</th>
<th>CL</th>
</tr>
</thead>
<tbody>
<tr>
<td>pure phase space</td>
<td>&lt; 48</td>
<td>95%</td>
</tr>
<tr>
<td>$a_1(1260)\pi\pi$</td>
<td>&lt; 47.9</td>
<td>95%</td>
</tr>
<tr>
<td>$\rho(770)\pi\pi\pi\pi$</td>
<td>&lt; 46.3</td>
<td>95%</td>
</tr>
<tr>
<td>$\rho(770)\rho(770)\pi$</td>
<td>&lt; 44.5</td>
<td>95%</td>
</tr>
</tbody>
</table>

Table 6: 95% CL tau neutrino mass limits for different decay structures.

The hadronic invariant mass spectrum $m_{5\pi}^*$ depends on the resonance structure. Possible resonance structures are:

- $\tau^\pm \rightarrow a_1^{\mp}(1260)\pi^\pm\pi^\mp\nu_\tau(\nu_\tau)$;
- $\tau^\pm \rightarrow \rho(770)2\pi^\mp\nu_\tau(\nu_\tau)$;
- $\tau^\pm \rightarrow 2\rho(770)\pi^\pm\nu_\tau(\nu_\tau)$.

In all cases the most evident effect is that including intermediate resonances the shape of the hadronic invariant mass distributions are shifted to higher values of $q^2$, nearer to the kinematic limit $m_{\tau}$. Fig. 10 shows the invariant mass distribution of the five-prong candidates compared with those predicted by a pure phase space model and by the models with intermediate resonances. A model with a $2\rho(770)\pi^\pm$ seems to be preferred. However, because of the small number of observed events, it is not possible to choose which is the dominant resonance structure.

The numerical effects on the neutrino mass limit are different for each different resonance structure. Table 6 shows the 95% CL upper limits obtained for the resonance structures considered above.
For what concerns this last systematic, we quote the most conservative result, $m_{\nu_e} < 48\text{ MeV}/c^2$, given by simple five-pion phase space for the $h(q^2)$ function. Table 5 summarises all the variations studied in section 6.

7 Conclusion

The kinematic study of the decay $\tau^\pm \rightarrow 3\pi^\pm 2\pi^\mp \pi^0(\nu_\tau)$ yields information on $m_{\nu_e}$. In this analysis the $e^+e^- \rightarrow Z^0 \rightarrow \tau^+\tau^-$ events collected by the DELPHI detector between 1992 and 1995 have been used.

The final sample of 47 events have been fitted in the $(m_{5\pi}^*, E_{5\pi}/E_{\text{beam}})$ plane. An upper limit from the region $m_{5\pi}^* > 1.6\text{ GeV}/c^2$ and $0.85 < E_{5\pi}/E_{\text{beam}} < 1.10$ is obtained:

$$m_{\nu_e} < 48.0\text{ MeV}/c^2 \quad 95\% \text{ C.L.}$$

assuming pure phase space for the $5\pi^\pm$ system. The sensitivity of DELPHI was expected to be about 30 MeV/$c^2$.

References


