Derivation of Coulomb’s Law of Forces Between Static Electric Charges Based on Spherical Source and Sink Model of Particles

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We speculate that the universe may be filled with a continuum which may be called aether. Based on a spherical source and sink model of electric charges, we derive Coulomb’s law of interactions between static electric charges in vacuum by methods of hydrodynamics. A reduced form of the Lorentz’s force law of static electric charges is derived based on a definition of electric field.

Keywords: Coulomb’s Law; hydrodynamics; spherical source; spherical sink; aether.

I. INTRODUCTION

Coulomb’s law of interactions between static electric charges in vacuum can be written as (refer to, for instance, 1)

\[ \mathbf{F}_{21} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{r}_{21}, \]  

where \( q_1 \) and \( q_2 \) are the electric quantities of two electric charges, \( r \) is the distance between the two electric charges, \( \varepsilon_0 \) is the dielectric constant of vacuum, \( \mathbf{F}_{21} \) is the force exerted on the electric charge with electric quantity \( q_2 \) by the electric charge with electric quantity \( q_1 \), \( \hat{r}_{21} \) denotes the unit vector directed outward along the line from the electric charge with electric quantity \( q_1 \) to the electric charge with electric quantity \( q_2 \).

The main purpose of this paper is to derive Coulomb's law of interactions between static electric charges in vacuum by means of fluid mechanics based on spherical source and spherical sink model of particles.

The motive of this paper is to seek a mechanism of Coulomb's law. The reasons why new mechanical interpretations of Coulomb’s law are interesting may be summarized as follows.

Firstly, Coulomb’s law is an elementary and profound law in physics and play various roles in the fields of electromagnetism, electrodynamics, quantum mechanics, cosmology and thermodynamics, etc., for instance, see 2 3 and the references cited there. From the point view of reductionism, the fundamental importance of Coulomb’s law in all branches of physics urges the reductionists to provide it a proper mechanical interpretation.

Secondly, the mechanism of this action-at-a-distance Coulomb’s law remains an unsolved problem in physics for more than 200 years after the law was put forth by Coulomb in 1785 2 3 4 5 6. From the point view of reductionism, we need a satisfactory mechanical interpretation in the framework of Descartes’ scientific research program.

Thirdly, although the Maxwell’s theory of electromagnetic phenomena is a field theory 1, the concept of field is different from that of continuum mechanics 2 8 9 10 because of the absence of a continuum. Thus, the Maxwell’s theory can only be regarded as a phenomenological theory. New mechanical interpretations of Coulomb’s law may help us to establish a field theory of electromagnetic phenomena 3 11.

Fourthly, there exists some inconsistencies and inner difficulties in the classical electrodynamics 8 12 13 14. New theories of Coulomb’s law may help to resolve such difficulties.

Finally, one of the tasks of physics is the unification of the four fundamental interactions in the universe. New theories of interactions between static electric charges may shed some light on this puzzle.

To conclude, it seems that new considerations on Coulomb’s law is needed. It is worthy keeping an open mind with respect to all the theories of interactions between static electric charges before the above problems been solved.

Now let us briefly review the long history of mechanical interpretations of electromagnetic phenomena.

Descartes was the first to bring the aether concept into science by suggesting that it has mechanical properties according to E. T. Whittaker 2. He believed that every physical phenomenon could be interpreted in the construction of a mechanical model of the universe.

William Watson and Benjamin Franklin introduced the one-fluid theory of electricity independently in 1746 5. Henry Cavendish attempted to explain some of the principal phenomena of electricity by means of an elastic fluid in 1771 5. Not contented with the above mentioned one-fluid theory of electricity, du Fay, Robert Symmer and C. A. Coulomb developed a two-fluid theory of electricity from 1733 to 1789 5. John Bernoulli introduced a fluid aether theory of light in 1752 5. Euler believed that all electrical phenomena is caused by the same aether that propagates light.

In 1821, in order to explain polarisation of light, A. J. Frensnel proposed an aether model which is able to transmit transverse waves. Inspired by Frensnel’s luminiferous aether theory, numerous dynamical theories of elastic solid aether were established by Stokes, Cauchy, Green, MacCullagh, Boussinesq, Riemann and William Thom-
son (refer to, for instance, [3]).

In 1861, in order to obtain a mechanical interpretation of electromagnetic phenomena, Maxwell established a mechanical model of a magneto-electric medium. Maxwell’s magneto-electric medium is a cellular aether, looks like a honeycomb. In a remarkable paper published in 1864, Maxwell established a group of equations which were named after his name later.

In a remarkable paper published in 1905, Einstein abandoned the concept of aether[15]. However, Einstein’s assertion did not cease the exploration of aether, for instance, see [5 16 17 18 20 21 22 23 24 25]. I regret to admit that it is impossible for me to mention all the works related to this field in history.

Inspired by the above mentioned works, we show that Coulomb’s law of interactions between static electric charges can be derived based on a continuum mechanics model of vacuum and a spherical source and sink model of electric charges.

II. DEFINITIONS OF SPHERICAL SOURCES AND SPHERICAL SINKS IN IDEAL FLUIDS

The purpose of this section is to establish a definition of spherical sources and spherical sinks in fluids.

If there exists a velocity field which is continuous and finite at all points of the space, with the exception of individual isolated points, then these isolated points are called velocity singularities usually. Point spherical sources and spherical sinks are examples of singularities.

To avoid singularities, we may introduce a mechanical model of spherical sources and spherical sinks with finite radii in fluids.

**Definition 1** Suppose there exists a hard sphere with a finite radius \( a \) at point \( P_0 = (x_0, y_0, z_0) \). If the velocity field near the hard sphere at point \( P = (x, y, z) \) is

\[
\mathbf{u}(x, y, z, t) = \frac{Q}{4\pi r^2} \mathbf{r},
\]

where \( r = \sqrt{(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2} \), \( r \geq a \), is the distance between the point \( P_0 \) and the point \( P \). \( \mathbf{r} \) denotes the unit vector directed outward along the line from the point \( P_0 \) to the point \( P \), then we call this hard sphere a spherical source if \( Q > 0 \) or a spherical sink if \( Q < 0 \). \( Q \) is called the strength of the spherical source or the spherical sink.

For convenience, we may regard a spherical sink as a negative spherical source.

Suppose a static spherical source with strength \( Q \) and a radius \( a \) locates at the origin \( (0, 0, 0) \). In order to calculate the volume leaving the source per unit time, we may enclose the source with an arbitrary spherical surface \( S \) with radius \( b > a \). A calculation shows that

\[
\iint_S \mathbf{u} \cdot n \, dS = \iint_S \frac{Q}{4\pi b^2} \mathbf{r} \cdot n \, dS = Q,
\]

where \( n \) denotes the unit vector directed outward along the line from the origin of the coordinates to the field point \((x, y, z)\). Equation (3) shows that the strength \( Q \) of a spherical source or spherical sink evaluates the volume of the fluid leaving or entering a control surface per unit time.

III. FORCES ACTING ON SPHERICAL SOURCES AND SPHERICAL SINKS IN IDEAL FLUIDS

The purpose of this section is to calculate the forces between spherical sources and spherical sinks in inviscid incompressible fluids which is called ideal fluids usually.

Suppose the velocity field \( \mathbf{u} \) of an ideal fluid is irrotational, then we have (refer to, for instance, [28 29 30 31 32 33 34]),

\[
\mathbf{u} = \nabla \phi,
\]

where \( \phi \) is the velocity potential, \( \nabla = \mathbf{i} \frac{\partial}{\partial x} + \mathbf{j} \frac{\partial}{\partial y} + \mathbf{k} \frac{\partial}{\partial z} \) is the Hamilton operator.

It is known that the equation of mass conservation of a fluid is

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{u} = 0,
\]

and \( \mathbf{u} = \mathbf{v} \) is the velocity field, \( \nabla \cdot \mathbf{u} = 0 \) is the Laplace operator.

Using spherical coordinates \((r, \theta, \varphi)\), a general form of solution of Laplace’s equation \( \phi \) can be obtained by separation of variables as \[34\]

\[
\phi(r, \theta, \varphi) = \sum_{l=0}^{\infty} \left( A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos \theta),
\]

where \( A_l \) and \( B_l \) are arbitrary constants, \( P_l(\cos \theta) \) are Legendre’s function of the first kind which is defined as

\[
P_l(x) = \frac{1}{2^l l!} \frac{d^l}{dx^l} (x^2 - 1)^l.
\]

From Eq.(2) and Eq.(4), we see that the velocity potential \( \phi(r, \theta) \) of a spherical source or spherical sink is a solution of Laplace’s equation \[34\].

The following lemma is useful in the proof of the main theorems in this section.

**Lemma 2** Suppose (1) the velocity field \( \mathbf{u} \) of a fluid is irrotational, i.e., we have \( \mathbf{u} = \nabla \phi \), where \( \phi \) is the velocity potential; (2) there is an arbitrary closed surface \( S \) fixed
in the space without any bodies or singularities inside \( S \); (3) the velocity field \( \mathbf{u} \) is continuous in the closed surface \( S \). Then, we have
\[
\frac{D}{Dt} \int_S \rho \phi dS = \frac{\partial}{\partial t} \int_S \rho \phi dS + \int_S \mathbf{u} \cdot \mathbf{n} dS. \tag{8}
\]
where \( \frac{D}{Dt} \) represents the material derivative in the Lagrangian system.

**Remark.** For the proof of Lemma 2 refer to, for instance, Appendix 2 in [35].

**Theorem 3** Suppose (1) there exists an ideal fluid (2) the ideal fluid is irrotational and barotropic, (3) the density \( \rho \) is homogeneous, that is \( \frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = 0 \) (4) there are no external body forces exerted on the fluid, (5) the fluid is unbounded and the velocity of the fluid at the infinity is approaching to zero. Suppose a spherical source or spherical sink is stationary and is immersed in the ideal fluid. Then, there is a force
\[
\mathbf{F}_Q = \frac{4}{3} \rho Q \mathbf{u}_0 + \frac{4 \pi \rho^3}{3} \frac{\partial \mathbf{u}_0}{\partial t} \tag{9}
\]
exerted on the spherical source or the spherical sink by the fluid, where \( \rho \) is the density of the fluid, \( Q \) is the strength of the spherical source or the spherical sink, \( \mathbf{u}_0 \) is the velocity of the fluid at the location of the spherical source induced by all means other than the spherical source itself.

**Proof.** Only the proof of the case of a spherical source is needed. Let us select the coordinates \( \{0, x, y, z\} \) or \( \{0, x_1, x_2, x_3\} \) that is attached to the static fluid at the infinity.

We set the origin of the coordinates at the center of the spherical source. Let \( S_1 \) denotes the spherical surface of the spherical source. We surround the spherical source by an arbitrary spherical surface \( S_2 \) with radius \( R \) centered at the center of the spherical source. The outward unit normal to the spherical surface \( S_1 \) and \( S_2 \) is denoted by \( \mathbf{n} \). Let \( \tau(t) \) denotes the mass system of fluid enclosed in the volume between the surface \( S_1 \) and the surface \( S_2 \) at time \( t \).

Let \( \mathbf{F}_Q \) denotes the hydrodynamic force exerted on the spherical source by the mass system \( \tau \). Then according to Newton’s third law, a reacting force of the force \( \mathbf{F}_Q \) must act on the fluid enclosed in the mass system \( \tau \). Let \( \mathbf{F}_1 = -\mathbf{F}_Q \) denotes this reacting force acted on the mass system \( \tau \) by the spherical source through the surface \( S_1 \). Let \( \mathbf{F}_2 \) denotes the hydrodynamic force exerted on the mass system \( \tau \) due to the pressure distribution on the surface \( S_2 \). Let \( \mathbf{K} \) denotes momentum of the mass system \( \tau \).

As an application of Newton’s second law of motion to the mass system \( \tau \), we have
\[
\frac{D\mathbf{K}}{Dt} = \mathbf{F}_1 + \mathbf{F}_2 = -\mathbf{F}_Q + \mathbf{F}_2, \tag{10}
\]
where \( \frac{D}{Dt} \) represents the material derivative in the Lagrangian system (see, for instance, [29, 30, 31, 32, 33, 34]).

In order to calculate \( \mathbf{F}_Q \), let us calculate \( \mathbf{DK}/Dt \) and \( \mathbf{F}_2 \) respectively. The expressions of the momentum \( \mathbf{K} \) and the force \( \mathbf{F}_2 \) are
\[
\mathbf{K} = \iiint \rho \mathbf{v} dV, \quad \mathbf{F}_2 = \iiint (-p) ndS, \tag{11}
\]
where the first integral is volume integral, the second integral is surface integral, \( \mathbf{n} \) denotes the unit vector directed outward along the line from the origin of the coordinates to the field point \((x, y, z)\).

Since the velocity field is irrotational, we have the following relation
\[
\mathbf{u} = \nabla \phi, \tag{12}
\]
where \( \phi \) is the velocity potential.

According to Ostrogradsky–Gauss theorem (see, for instance, [29, 30, 31], and using Eq. (12), we have
\[
\iiint \rho v dV = \iiint \rho \nabla \phi dV = \iiint \rho n dS - \iiint \rho \phi n dS. \tag{13}
\]
Therefore, from Eq. (11) and Eq. (13), we have
\[
\frac{D}{Dt} \iiint \rho v dV = \frac{D}{Dt} \left[ \iiint \rho n dS - \iiint \rho \phi n dS \right]. \tag{14}
\]
Applying Lemma 2 to the second integral in Eq. (14), we have
\[
\frac{D}{Dt} \iiint \rho n dS = \frac{\partial}{\partial t} \iiint \rho n dS + \iiint \rho \mathbf{u} \cdot \mathbf{n} dS. \tag{15}
\]
Putting Eq. (15) into Eq. (14), we have
\[
\frac{DK}{Dt} = \frac{\partial}{\partial t} \iiint \rho n dS + \iiint \rho \mathbf{u} \cdot \mathbf{n} dS - \frac{D}{Dt} \iiint \rho \phi n dS. \tag{16}
\]
Now, let us calculate \( \mathbf{F}_2 \). According to Lagrange–Cauchy integral (see, for instance, [29, 30, 31, 32, 34]), we have
\[
\frac{\partial \phi}{\partial t} + \frac{(\nabla \phi)^2}{2} + \frac{p}{\rho} = f(t), \tag{17}
\]
where \( f(t) \) is an arbitrary function of time \( t \). Since the velocity \( \mathbf{u} \) of the fluid at the infinity is approaching to zero, and noticing (3), \( \phi(t) \) must be of the following form
\[
\phi(r, \theta, t) = \sum_{l=0}^{\infty} \frac{B_l(t)}{r^{l+1}} P_l(\cos \theta), \tag{18}
\]
where $B_l(t), l \geq 0$ are functions of time $t$. Thus, we have the following estimations at the infinity of the velocity field

$$\phi = O \left( \frac{1}{r} \right), \quad \frac{\partial \phi}{\partial t} = O \left( \frac{1}{r} \right), \quad r \to \infty,$$  \hspace{1cm} (19)

where $\varphi(x) = O(\psi(x)), x \to a$ stands for $\lim_{x \to a} | \varphi(x) | / \psi(x) = k, (0 \leq k < +\infty)$.

Applying (17) at the infinity and using (19), we have $|u| \to 0, \partial \phi/\partial t \to 0$ and $p = p_\infty$, where $p_\infty$ is a constant. Thus, $f(t) = p_\infty/\rho$. Therefore, according to (17), we have

$$p = p_\infty - \rho \frac{\partial \phi}{\partial t} - \frac{\rho(u \cdot u)}{2}.$$  \hspace{1cm} (20)

Using (11) and (20), we have

$$F_2 = \iint_{S_2} \rho \phi \, dS + \iint_{S_2} \frac{\rho(u \cdot u)n}{2} \, dS.$$  \hspace{1cm} (21)

Now, putting (16) and (21) into (10), we have

$$F_Q = \iint_{S_2} \left[ \frac{1}{2} \rho(u \cdot u)n - \rho u(u \cdot n) \right] \, dS + \frac{D}{Dt} \iint_{S_1} \rho \phi \, dS.$$  \hspace{1cm} (22)

For convenience, let us introduce the following definition

$$F = \iint_{S_2} \left[ \frac{1}{2} \rho(u \cdot u)n - \rho u(u \cdot n) \right] \, dS.$$  \hspace{1cm} (23)

Since the radius $R$ of the spherical surface $S_2$ is arbitrary, we may let $R$ to be large enough. Now, making use of the result (5.13) in [34], we have

$$F = 0.$$  \hspace{1cm} (24)

Therefore, using (24), (22) becomes

$$F_Q = \frac{D}{Dt} \iint_{S_1} \rho \phi \, dS.$$  \hspace{1cm} (25)

Applying Lemma 2, (25) becomes

$$F_Q = \frac{\partial}{\partial t} \iint_{S_1} \rho \phi \, dS + \iint_{S_1} \rho u(u \cdot n) \, dS.$$  \hspace{1cm} (26)

For convenience, let us introduce the following definitions

$$I_1 = \frac{\partial}{\partial t} \iint_{S_1} \rho \phi \, dS, \quad I_2 = \iint_{S_1} \rho u(u \cdot n) \, dS.$$  \hspace{1cm} (27)

Thus, Eq.(26) becomes

$$F_Q = I_1 + I_2.$$  \hspace{1cm} (28)

Now let us calculate the two terms in Eq.(28) respectively. Firstly, let us calculate the integral $I_1$ in Eq.(29). Since the velocity field induced by the spherical source with strength $Q$ is Eq.(2), then according to the superposition principle of velocity field of ideal fluids, the velocity $u$ on the spherical surface $S_1$ is

$$u = \frac{Q}{4\pi a^2}n + u_0,$$  \hspace{1cm} (29)

where $n$ denotes the unit vector directed outward.

Since the velocity field $u$ is irrotational, we have

$$\phi = -\frac{Q}{4\pi a} + \phi_0,$$  \hspace{1cm} (30)

where $\phi_0$ is the velocity potential respect to $u_0, i.e., u_0 = \nabla \phi_0$.

Since the density $\rho$ is homogeneous, we have

$$\frac{\partial \rho}{\partial t} = 0.$$  \hspace{1cm} (31)

Therefore, using Eq.(31), we have

$$I_1 = \iint_{S_1} \rho \phi_0 \, dS.$$  \hspace{1cm} (32)

Suppose that $\partial Q/\partial t = 0$. Then, using Eq.(30), we have

$$\frac{\partial \phi_0}{\partial t} = \frac{1}{4\pi a} \frac{\partial Q}{\partial t} = \frac{\partial \phi_0}{\partial t}.$$  \hspace{1cm} (33)

Using Eq.(38) and Ostrogradsky–Gauss theorem, Eq.(32) becomes

$$I_1 = \frac{\partial}{\partial t} \iint_{S_1} \rho \phi_0 \, dS$$
$$= \frac{\partial}{\partial t} \iiint_{V_1} \rho \nabla \phi_0 \, dV$$
$$= \frac{\partial}{\partial t} \iiint_{V_1} \rho u_0 \, dV.$$  \hspace{1cm} (34)

We speculate that the radius a of the spherical source may be so small that the velocity $u_0 \, dS$ at any point of the spherical surface may be treated as a constant. Thus, Eq.(34) becomes

$$I_1 = \frac{\partial}{\partial t} \iiint_{V_1} \rho \phi_0 \, dS$$
$$= \frac{\partial}{\partial t} \iiint_{V_1} \rho \phi_0 \, dV$$
$$= \frac{4\pi \rho a^3 \phi_0}{3} \frac{\partial}{\partial t}.$$  \hspace{1cm} (35)

Now let us calculate the integral $I_2$ in Eq.(37).

Noticing Eq.(20), we have

$$I_2 = \rho \iint_{S_1} \left[ \frac{Q^2}{16\pi a^4}n + \frac{Q}{4\pi a^2}u_0 + \frac{Q}{4\pi a^2} \frac{(u_0 \cdot n)n + (u_0 \cdot n)u_0}{dS}. \right.$$  \hspace{1cm} (36)
For convenience, let us introduce the following definitions

\[ J_1 = \int \int \frac{\rho Q^2}{4 \pi a^2} \hat{n} dS, \]
\[ J_2 = \int \int \frac{\rho Q}{4 \pi a^2} u_0 dS, \]
\[ J_3 = \int \int \frac{\rho Q}{4 \pi a^2} (u_0 \cdot n) n dS, \]
\[ J_4 = \int \int (u_0 \cdot n) u_0 dS. \] (37)

Thus, Eq. (36) becomes

\[ I_2 = J_1 + J_2 + J_3 + J_4. \] (38)

Suppose that the radius \( a \) of the spherical source may be so small that the velocity \( u_0 \) at any point of the spherical surface \( S_1 \) may be treated as a constant. Therefore, the four integral terms in Eq. (38) turn out to be

\[ J_1 = 0, \]
\[ J_2 = \rho Q u_0, \]
\[ J_3 = \frac{1}{3} \rho Q u_0, \]
\[ J_4 = 0. \] (42)

Therefore, using Eq. (39) - (42), we have

\[ I_2 = \frac{4}{3} \rho Q u_0. \] (43)

Putting Eq. (43) and Eq. (44) into Eq. (28), we arrive at Eq. (29). This completes the proof. \( \square \)

Theorem 3 only considers the situation that the spherical sources or spherical sinks are at rest in fluids. Now let us consider the case that the spherical sources or spherical sinks are moving in fluid.

**Theorem 4** Suppose the presuppositions (1), (2), (3), (4) and (5) in Theorem 3 are valid and a spherical source or a spherical sink is moving in the fluid with a velocity \( \mathbf{v}_s \), then there is a force

\[ \mathbf{F} = \frac{4}{3} \rho Q (\mathbf{u}_f - \mathbf{v}_s) + \frac{4 \pi \rho a^3}{3} \frac{\partial}{\partial t} (\mathbf{u}_f - \mathbf{v}_s) \] (44)

is exerted on the spherical source or the spherical sink by the fluid, where \( \rho \) is the density of the fluid, \( Q \) is the strength of the spherical source or the spherical sink, \( a \) is the radius of the spherical source or the spherical sink, \( \mathbf{u}_f \) is the velocity of the fluid at the location of the source induced by all means other than the spherical source itself.

**Proof.** The velocity of the fluid relative to the spherical source at the location of the spherical source is \( \mathbf{u}_f - \mathbf{v}_s \). Let us select the coordinates that is attached to the spherical source and set the origin of the coordinates at the center of the spherical source. Then Eq. (41) can be obtained following the same procedures in the proof of Theorem 3. \( \square \)

Applying Theorem 4 to the situation that a spherical source or spherical sink is exposed to the velocity field of another spherical source or sink, we have

**Corollary 5** Suppose the presuppositions (1), (2), (3), (4) and (5) in Theorem 3 are valid and a spherical source or spherical sink with strength \( Q_2 \) is exposed to the velocity field of another static spherical source or spherical sink with strength \( Q_1 \), then the force \( \mathbf{F}_{21} \) exerted on the spherical source or spherical sink with strength \( Q_2 \) by the velocity field of the spherical source or spherical sink with strength \( Q_1 \) is

\[ \mathbf{F}_{21} = \frac{4}{3} \rho Q_2 \cdot \mathbf{r}_{21} - \mathbf{v}_s, \]

where \( \mathbf{r}_{21} \) denotes the unit vector directed outward along the line from the spherical source or spherical sink with strength \( Q_1 \) to the spherical source or spherical sink with strength \( Q_2 \), \( \mathbf{r}_s \) is the moving velocity of the spherical source or spherical sink with strength \( Q_2 \).

If the spherical source with strength \( Q_2 \) is static in the aether, then Eq. (45) reduces to

\[ \mathbf{F}_{21} = \frac{\rho Q_1 Q_2}{3 \pi r^2} \mathbf{r}_{21} + \frac{4 \pi \rho a^3}{3} \frac{\partial}{\partial t} \left( \frac{Q_1}{4 \pi r^2} \mathbf{r}_{21} - \mathbf{v}_s \right), \] (46)

IV. A SPHERICAL SOURCE AND SPHERICAL SINK MODEL OF ELECTRIC CHARGES

The purpose of this section is to establish a spherical source and spherical sink model of electric charges.

It is an old idea that the universe may be filled with a kind of continuously distributed material which may be called aether (refer to, for instance, [5]).

In order to compare fluid motion with electric fields, Maxwell introduced an analogy between sources or sinks and electric charges [3]. Recently, we derive the Maxwell’s equations in vacuum by methods of continuum mechanics based on a continuum mechanical model of vacuum and a point source and sink model of electric charges [3].

In 1892 [37], Lorentz established an electromagnetic theory in order to derive the Fresnel convection coefficient. There are only two kinds of entities in Lorentz’s theory: movable electrons and a stagnant aether. To avoid singularities, the electrons were not designed to be singularities as Larmor’s electrons in the aether field, but were extremely small hard spheres with a finite radius.

Inspired by Maxwell [3] and Lorentz [37], we speculate that electric charges may not be singularities, but may be...
extremely small hard spherical sources or spherical sinks with finite radii.

In [36], we introduced a hypothesis that the constitutive relation of the aether satisfies
\[
\frac{de_{ij}}{dt} = \frac{1}{2\eta} s_{ij} + \frac{1}{2G} \frac{ds_{ij}}{dt}, \tag{47}
\]
where \(s_{ij}\) is the stress deviator, \(e_{ij}\) is the strain deviator, \(G\) is the shear modulus, \(\eta\) is the dynamic viscosity.

Now let us introduce the following definition of Maxwellian relaxation time \(\tau\)
\[
\tau = \frac{\eta}{G}. \tag{48}
\]

Therefore, using Eq. (48), Eq. (17) becomes
\[
s_{ij} + \frac{ds_{ij}}{dt} = 2G \frac{de_{ij}}{dt}. \tag{49}
\]

Let \(T_0\) be the characteristic time of a observer of an electric charge. We may suppose that the observer’s time scale \(T_0\) is very large compares to the the Maxwellian relaxation time \(\tau\). So the Maxwellian relaxation time \(\tau\) is a relatively a small number and the stress deviator \(s_{ij}\) changes very slowly. Thus, the second term in the left of Eq. (19) may be neglected. Therefore, the observer concludes that the aether behaves like the Newtonian-fluid. According to this observer, the constitutive relation of the aether may be written as
\[
s_{ij} = 2\eta \frac{de_{ij}}{dt}. \tag{50}
\]

We see that Eq. (50) is the constitutive relation of a Newtonian-fluid. Therefore, the observer concludes that the aether behaves like a Newtonian-fluid under his time scale.

Now let us introduce the following hypothesis.

**Hypothesis 6** Suppose all the electric charges in the universe are small hard spherical sources or spherical sinks with finite radii in the aether. We define a spherical source as a negative electric charge. We define a spherical sink as a positive electric charge. The electric charge quantity of an electric charge is defined as
\[
q_e = -k_Q \rho Q, \tag{51}
\]
where \(\rho\) is the density of the aether, \(k_Q\) is a positive dimensionless constant, \(Q\) is called the strength of the spherical source or spherical sink.

A calculation shows that the mass \(m\) of a electric charge is changing with time as
\[
\frac{dm}{dt} = -\rho Q = \frac{q_e}{k_Q}, \tag{52}
\]
where \(q_e\) is the electric charge quantity of the electric charge.

We may introduce a hypothesis that the masses of electric charges are changing so slowly relative to the time scaler of human beings that they can be treated as constants approximately.

**V. DERIVATION OF COULOMB’S LAW OF INTERACTIONS BETWEEN STATIC ELECTRIC CHARGES IN VACUUM**

The purpose of this section is to derive Coulomb’s law of interactions between static electric charges in vacuum.

It is known that the motion of an incompressible Newtonian-fluid is governed by the Navier-Stokes equations (refer to, for instance, [29, 30, 31, 32, 33, 34]),
\[
\rho \left[ \frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} \right] = -\nabla p - \eta \nabla^2 \mathbf{u}, \tag{53}
\]
where \(\mathbf{u}\) is the velocity field of the fluid, \(p\) is the pressure field, \(\rho\) is the density field, \(\eta\) is the dynamic viscosity coefficient, \(t\) is time.

The definition of Reynolds number \(Re\) of a fluid field is
\[
Re = \frac{\rho_0 U_0 L_0}{\eta_0}, \tag{54}
\]
where \(\rho_0\) is the characteristic density, \(U_0\) is the characteristic velocity, \(L_0\) is the characteristic length, \(\eta_0\) is the characteristic dynamic viscosity coefficient.

**Hypothesis 7** We speculate that the characteristic velocity \(U_0\) of an electric charge is so high compares to the characteristic dynamic viscosity coefficient \(\eta_0\) of the aether that the Reynolds number \(Re\) of the fluid field is a large number.

Under this hypothesis, we may treat the aether as an inviscid incompressible fluid when we study the motion of electric charges. Therefore, according to Hypothesis 7, the motion of the aether is governed by the Euler equations (refer to, for instance, [29, 30, 31, 32, 33, 34]),
\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \mathbf{u} = -\frac{1}{\rho} \nabla p. \tag{55}
\]

Now based on Hypothesis 6 and Hypothesis 7, we can make use of Theorem 4 and Theorem 1 to study the motions of electric charges.

**Theorem 8** Suppose a static electric charge with an electric charge quantity \(q_1\) is exposed to the electric field of another static electric charge with an electric charge quantity \(q_2\), then the force \(\mathbf{F}_{21}\) exerted on the electric charge with electric charge quantity \(q_2\) by the electric field of the electric charge with electric charge quantity \(q_1\) is
\[
\mathbf{F}_{21} = \frac{4}{3k_E k_Q} \frac{1}{4\pi\epsilon_0} \frac{1}{4\pi r^2} \hat{r}_{21} \frac{q_1 q_2}{r^3} \mathbf{r}_{21} - \frac{4\pi\epsilon_0}{3k_Q} \frac{\partial}{\partial t} \left( \frac{q_1}{4\pi r^2} \right) \mathbf{F}_{21}, \tag{56}
\]
where \(\hat{r}_{21}\) denotes the unit vector directed outward along the line from the electric charge with electric charge quantity \(q_1\) to the electric charge with electric charge quantity \(q_2\).
\( q_2, \ a_2 \) is the radius of the electric charge with electric charge quantity \( q_2 \), \( r \) is the distance between the two electric charges, \( k_Q \) and \( k_E \) are two positive dimensionless constants.

**Proof.** From Hypothesis 6 we have

\[
Q_1 = -\frac{q_1}{k_Q \rho}, \quad Q_2 = -\frac{q_2}{k_Q \rho},
\]

where \( Q_1 \) and \( Q_2 \) are the strengths of the electric charges respectively. From a definition in Eq.(65) in [36], we have

\[
e_0 = \frac{k_Q \rho}{k_E}.
\]

where \( e_0 \) is the dielectric constant of vacuum. Putting Eq.(67) and Eq.(68) into Eq.(60), we get Eq.(69). □

If we ignore the second term in Eq.(63), then we get

\[
F_{21} = \frac{4}{3k_E k_Q} \frac{1}{4\pi e_0} \frac{q_1 q_2}{4\pi r^2} r_{21}.
\]

Compare Eq.(69) with Eq.(1), it is natural to introduce the following hypothesis.

**Hypothesis 9** Suppose we have the following relation

\[
k_E k_Q = \frac{4}{3}.
\]

Now based on Hypothesis 9 from Eq.(59) we see that we have arrived at Coulomb's law (11) of interactions between static electric charges in vacuum.

Theorem 8 only states the force exerted on a static electric charge by the electric field of another static electric charge. We may generalize this result to the case of an static electric charge exposed to any electric field of the aether.

In [36], we introduced the following definition

\[
E = -k_E \frac{\partial u}{\partial t}.
\]

where \( u \) is the displacement of the visco-elastic aether, \( \frac{\partial u}{\partial t} \) is the velocity field of the aether, \( E \) is the electric field intensity, \( k_E \) is a positive dimensionless constant.

Since the observer of an electric charge concludes that the aether behaves as a Newtonian-fluid under his time scale, we may define the electric field intensity as the velocity field of the aether. Therefore, we may introduce the following definition.

**Definition 10** We define the electric field intensity in the aether as

\[
E = -k_E v_a,
\]

where \( v_a \) is the velocity field of the aether at the location of a testing electric charge induced by all means other than the testing electric charge itself, \( E \) is the electric field intensity, \( k_E \) is a positive dimensionless constant.

**Theorem 11** Suppose a static electric charge with an electric charge quantity \( q \) is exposed to an electric field \( E \) of the aether, then the force \( F \) exerted on the electric charge by the electric field \( E \) is

\[
F = qE - \frac{4\pi k_0 e_0}{3k_Q} \frac{\partial E}{\partial t}.
\]

where \( q \) is the electric charge quantity of the electric charge, \( a \) is the radius of the electric charge, \( e_0 \) is the dielectric constant of vacuum, \( k_Q \) is a positive dimensionless constant, \( E \) is the electric field at the location of the electric charge induced by all means other than the electric charge itself.

**Proof.** From Hypothesis 9 we have

\[
Q = -\frac{q}{k_Q \rho}.
\]

where \( Q \) is the strength of the electric charge. Putting Eq.(62) and Eq.(63) into Eq.(61), we get Eq.(64). □

If we ignore the second term in Eq.(63), then we get

\[
F = qE.
\]

We know that the Lorentz’s force law can be written as

\[
F = qE + qv_e \times B,
\]

where \( q \) is the electric charge quantity of a electric charge, \( F \) is the force exerted on the electric charge, \( v_e \) is the velocity of the electric charge, \( E \) and \( B \) are the electric field intensity and the magnetic induction respectively at the location of the electric charge induced by all means other than the electric charge itself.

We see that Eq.(65) is a reduced form of the Lorentz’s force law (60) for static electric charges in vacuum.

**VI. SUPERPOSITION PRINCIPLE OF ELECTRIC FIELD OF STATIC ELECTRIC CHARGES**

From the definition 10 we see that the electric field intensity is a linear function of the velocity field of the aether. Therefore, the superposition principle of electric fields of static electric charges in vacuum is deduced from the superposition theorem of the velocity field of fluids.

**VII. CONCLUSION**

We show that the forces between spherical sources and spherical sinks in ideal fluids coincide with Coulomb’s inverse-square-law of interactions between static electric charges. It is an old idea that the universe may be filled
with a kind of continuously distributed material which may be called aether. Based on a continuum mechanical model of vacuum and a spherical source and spherical sink model of electric charges, we derive Coulomb’s law of interactions between static electric charges in vacuum. Furthermore, we define the electric field intensity as a linear function of velocity field of the aether at the location of an testing electric charge induced by all means other than the testing electric charge itself. Thus, we obtain a reduced form of the Lorentz’s force law for static electric charges in vacuum.

VIII. DISCUSSION

There exists some interesting theoretical, experimental and applied problems in the fields of fluid mechanics, continuum mechanics, classical electrodynamics, quantum electrodynamics and other related fields involving this theory of Coulomb’s law of interactions between static electric charges. It is an interesting task to generalize this work further to describe the forces exerted on a electric charge moving with a velocity in an electromagnetic field.

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