Analysis of two-body charmless
decays for branching
fraction and $CP$ violating
asymmetry measurements with
the BABAR experiment

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Introduction

The main goal of the BaBar experiment is studying CP violation in neutral B meson decays. CP symmetry violation is an expected consequence of the Standard Model with three quark generation. The Standard Model accommodates this violation through the presence of a single complex phase in the mixing CKM matrix [2, 3]. Experimental measurements in this field are important tests of the Standard Model. CP violation has been first observed in K decays [1] in 1964. Recently, the BaBar [4] and Belle experiment [5] have observed evidence of CP violation in neutral B meson decays, making a 4σ significant measurement of the parameter sin 2β, β being one of the angles of the Unitary Triangle.

Hadronic charmless two-body decays are important because they provide information on the other angles of the Unitary Triangle. In the SM the time-dependent CP-violating asymmetry in the channel \( B^0 \rightarrow \pi^+\pi^- \) is related to the angle \( \alpha \). Moreover rates asymmetries in both neutral and charged B decays into charmless final states like \( K^{\pm}\pi^{\pm}, K^0_S h^{\pm}, \pi^0 h^{\pm} \) (\( h \) being a π or a K) are evidence of direct CP violation. Also ratios of branching fractions can lead to bounds on the angle \( \gamma \).

This thesis presents measurements of branching fractions of the charmless two-body modes \( B^0 \rightarrow K^0_S h^{\pm} \) and \( B^0 \rightarrow K^0_S K^0_S \) and of the time integrated CP violating asymmetry in the charged B decays. In addition, a study of the the time-dependent asymmetry is performed in the channel \( B^0 \rightarrow \pi^+\pi^- \).

The first chapter describes the theoretical background of the CP violation and physical meaning of the measurements presented in the following chapters.

The second chapter is a description of the BaBar detector with details on the tracking system and the particle identification. The third chapter presents an analysis on inclusive \( K^0_S \) reconstruction which has been performed in order to provide an estimate of the \( K^0_S \) absolute reconstruction efficiency with the BaBar detector.

The fourth chapter is an overview on the common issues of the hadronic charmless two-body analyses, while the fifth presents the actual analysis of the decay \( B^\pm \rightarrow K^0_S h^\pm \) whose result has been presented in Ref. [6]. The sixth chapter describes the analysis of the decay \( B^0 \rightarrow K^0_S K^0_S \) [7].

The seventh chapter present a preliminary result of the time-dependent analysis in \( B^0 \rightarrow \pi^+\pi^- \) decays together with a measurement of the rate asymmetry in \( B^0 \rightarrow K^{\pm}\pi^{\mp} \) decays: this analysis is going to be published in Ref. [8].
Introduzione


I decadimenti adronici senza charm negli stati finali sono di notevole importanza perché possono essere ricondotti ad un altro angolo del Triangolo di Unitarietà. Nel MS la misura dell’asimmetria dipendente dal tempo nel canale $B^0 \rightarrow \pi^+ \pi^-$ è collegata all’angolo $\alpha$. Inoltre eventuali asimmetrie nelle ampiezze di decadimento di $B$ carichi o neutri in stati finali senza charm come $K^\pm \pi^\pm$, $K^0 h^\pm$, $\pi^0 h^\pm$ (essendo $h$ un $\pi$ od un $K$) sarebbero evidenze di violazione diretta di $CP$. Anche i rapporti tra i vari $branching fractions$ possono stabilire dei vincoli sull’angolo $\gamma$.

Questa tesi presenta misure di $branching fractions$ dei canali di decadimento $B^\pm \rightarrow K^0 h^\pm$ e $B^0 \rightarrow K^0_s K^0_s$ e della asimmetria non dipendente dal tempo nei decadimenti dei $B$ carichi. E’ stata poi sviluppata l’analisi della asimmetria dipendente dal tempo nel canale $B^0 \rightarrow \pi^+ \pi^-$. 

Il primo capitolo descrive le basi della teoria e della fenomenologia della violazione di $CP$ ed il significato fisico delle misure riportate nei capitoli successivi. Il secondo capitolo è una panoramica sul rivelatore di BaBar con particolare attenzione al sistema di tracking e di identificazione di particella. Il terzo capitolo presenta un’analisi sulla ricostruzione inclusiva dei $K^0_s$: questa analisi ha lo scopo di fornire una misura dell’efficienza di ricostruzione dei $K^0_s$ con il rivelatore di BaBar.

Il quarto capitolo tratta le strategie e gli strumenti di analisi comuni allo studio di tutti i decadimenti adronici senza charm, mentre il quinto capitolo descrive in dettaglio l’analisi del canale $B^\pm \rightarrow K^0 h^\pm$, il cui risultato è stato pubblicato in [6]. Il sesto capitolo espone l’analisi del decadimento $B^0 \rightarrow K^0_s K^0_s$ [7].

Infine il settimo capitolo presenta un risultato preliminare dell’analisi dipendente dal tempo nel canale $B^0 \rightarrow \pi^+ \pi^-$ insieme ad una misura della asimmetria nelle ampiezze di decadimento nei canali $B^0 \rightarrow K^\pm \pi^\mp$: questa analisi sarà pubblicata in [8].

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CP Violation in the $B\bar{B}$ System

$CP$ symmetry violation is an expected consequence of the Standard Model with three quark generations (see Sec. 1.4.1): as a matter of fact, the $CP$ violation that shows up in a small fraction of weak decays is accommodated simply in the three-generation Standard Model Lagrangian. All it requires is that $CP$ is not imposed as a symmetry.

Some experiments have proved that $CP$ violation occurs in neutral $K$ decays [1]. The $K$-decay observations, together with other measurements, place constraints on the parameters of the Standard Model mixing matrix (the $CKM$ matrix [2, 3]) but do not yet provide any test about whether the pattern of $CP$ violation predicted by the minimal Standard Model is the one found in nature. A multitude of $CP$-violating effects are expected in $B$ decays, some of which are very cleanly predicted by the Standard Model.

If enough independent observations of $CP$ violation in $B$ decays can be made then it will be possible to test the Standard Model predictions for $CP$ violation. Either the relationships between various measurements will be consistent with the Standard Model predictions and fully determine the $CKM$ parameters or there will be no single choice of $CKM$ parameters that is consistent with all measurements. This latter case would indicate that there is a contribution of physics beyond the Standard Model: so the main goal for the BaBar experiment is to measure enough quantities to impose redundant constraints on Standard Model parameters, including particularly the convention-independent combinations of $CP$-violating phases of $CKM$ matrix elements.

Since the Standard Model accommodates $CP$-violation, no extension of the Standard Model can be $CP$-conserving and thus many extensions have additional sources of $CP$-violating effects, or effects which change the relationship of the measurable quantities to the $CP$-violating parameters of the Standard Model: $B$ Factories like BaBar can play an important role in measuring most of these parameters.

1.1 $P$, $C$ and $T$ symmetries

The fundamental point is that $CP$ symmetry is broken in any theory that has complex coupling constants in the Lagrangian which cannot be removed by any choice of phase redefinition of the fields in the theory. Three discrete operations are potential symmetries of a field theory Lagrangian [9]: two of them, parity and time reversal are space-time symmetries. Parity, denoted by $P$, sends $(t,\mathbf{x}) \rightarrow (t,-\mathbf{x})$, reversing the handedness of space. Time reversal, denoted by $T$, sends $(t,\mathbf{x}) \rightarrow (-t,\mathbf{x})$, interchanging the forward and backward light-cones. A third (non-space-time) discrete operation is charge conjugation, denoted by $C$. This operation interchanges particles and anti-particles. The combination $CP$ replaces a particle by its anti-particle and reverses momentum and helicity.
The combination $CPT$ is an exact symmetry in any local Lagrangian field theory: the $CPT$ theorem is based on general assumptions of field theory and relativity and states that every Hamiltonian that is Lorentz invariant is also invariant under combined application of $CPT$, even if it is not invariant under $C$, $P$ and $T$ separately. One of the consequences of this theorem is that particles and anti-particles should have exactly the same mass and lifetime.

From experiment, it is observed that electromagnetic and strong interactions are symmetric with respect to $P$, $C$ and $T$. The weak interactions violate $C$ and $P$ separately, but preserve $CP$ and $T$ to a good approximation. Only certain rare processes, all involving neutral $K$ mesons, have been observed to exhibit $CP$ violation. All these observations are consistent with exact $CPT$ symmetry.

The operators associated to these symmetries have different properties: $P$ and $C$ operators are unitary (and thus they satisfy the relation $U^T = U^{-1}$) and linear (and thus $U(\alpha |a\rangle + \beta |b\rangle) = \alpha U |a\rangle + \beta U |b\rangle$). Otherwise, $T$ operator is anti-unitary, that means that satisfies the unitary relation $\tilde{A}^T = (A^{-1})^*$, but it is anti-linear $(A(\alpha |a\rangle + \beta |b\rangle)) = \alpha^* A |a\rangle + \beta^* A |b\rangle$: because of this, $T$ operator can be written as the product of two operators $UK$ where $U$ is unitary and $K$ transforms every complex number in its conjugate.

Taking into account a generic fermionic state, some quantic numbers $\alpha$ are associated to it, together with a polarization $J_z$ and a momentum $\mathbf{p}$: an anti-particle with same polarization and momentum have opposite quantic numbers, $-\alpha$. Defining $| f(t, x) \rangle$ or $| f(\mathbf{p}, J_z) \rangle$ the generic fermionic state and applying $P$, $C$ and $T$, we would get:

$$P| f(t, x) \rangle = \eta_P | f(t, -x) \rangle = \eta_P | f(-\mathbf{p}, J_z) \rangle$$

$$C| f(t, x) \rangle = \eta_C | \tilde{f}(t, x) \rangle = \eta_C | \tilde{f}(\mathbf{p}, J_z) \rangle$$

$$T| f(t, x) \rangle = \eta_T | f(-t, x) \rangle = \eta_T | f(-\mathbf{p}, -J_z) \rangle$$

and applying $CP$:

$$CP| f(t, x) \rangle = \eta_{CP} | \tilde{f}(t, -x) \rangle = \eta_{CP} | \tilde{f}(-\mathbf{p}, J_z) \rangle$$

Taking into account the Lorentz invariance and hermiticity of the Lagrangian, $CP$ transformation rules imply that each of the combinations of fields and derivatives that appear in the Lagrangian transforms under $CP$ to its Hermitian conjugate. However, there are coefficients in front of these expressions which represent either coupling constants or particle masses and which do not transform under $CP$. If any of these quantities are complex, then the coefficients in front of $CP$-related terms are complex conjugates of each other. In such a case, $CP$ is not necessarily a good symmetry of the Lagrangian. When the rates of physical processes that depend on these Lagrangian parameters are calculated, there can be $CP$-violating effects, namely rate differences between pairs of $CP$ conjugate processes.
Note, however, that not all Lagrangian phases are physically meaningful quantities. Consider the Lagrangian that contains the most general set of complex coupling constants consistent with all other symmetries in the theory. That is to say CP symmetry is not imposed and hence any coupling is allowed to be complex (unless the Hermitian structure of the Lagrangian automatically requires it to be real). Now any complex field in the Lagrangian can be redefined by an arbitrary phase rotation; such rotations will not change the physics, but will change the phases of some set of terms in the Lagrangian. Some set of couplings can be made real by making field re-definitions. However if any non-zero phases for couplings remain after all possible field re-definitions have been used to eliminate as many of them as possible, then there is CP violation. It is a matter of simple counting for any Lagrangian to see whether this occurs. If all phases can be removed in this way then that theory is automatically CP-conserving. In such a theory it is impossible to introduce any CP violations without adding fields or removing symmetries so that additional couplings appear. (This is the case for the Standard Model with only two generations and a single Higgs multiplet.) Choosing to make certain terms real and leave others complex has no physical meaning and so a different choice, related to the first by field re-definitions, has the same physical consequences: only those differences between pairs of phases that are unchanged by such re-definitions are physically meaningful.

1.2 Neutral B Mesons

There are two possible pairs of mesons involving b quarks: B⁻ mesons, made from one b type quark (or anti-quark) and one d type, and B_s mesons from one b and one s. Like the neutral K mesons, the neutral B mesons are characterized by the fact that different neutral states are relevant to the discussion of different physical processes. There are two flavor eigenstates, which have definite quark content and are most useful when treating particle production, and there are eigenstates of the Hamiltonian, namely states of definite mass and lifetime. Assuming CP as a good symmetry for the weak Hamiltonian, the mass eigenstates would also be CP eigenstates which under a CP transformation would transform into themselves with a definite eigenvalue ±1. On the contrary, considering CP not a good symmetry, the mass eigenstates can be different from CP eigenstates. In any case the mass eigenstates are not flavor eigenstates, and so the flavor eigenstates are mixed with one another as they propagate through space. The flavor eigenstates for B⁻ are B⁻ = b̄d and B_⁻ = b̄b. The B⁻ meson is the isospin partner of B⁺: therefore it contains the b quark\(^1\). The conventional definitions for the B_s system are B_s = b̄s and B̅_s = b̄s.

1.2.1 Phenomenology of the decay processes with the Wigner-Weisskopf perturbative method

Given a system described by a Hamiltonian \(H\) that can be written like this:

\[
H = H_0 + H^I
\]

\(^1\)This is similar to the K mesons, where \(K^0\), the isospin partner of \(K^+\), contains the \(\bar{\tau}\) quark, and the corresponding anti-particle doublet is \((\bar{K}^0, \bar{K}^-)\).
where $H_0$ is the strong and electromagnetic Hamiltonian $H_0 = H_s + H_{em}$, while $H'$ is a small perturbation that represents the weak-interaction Hamiltonian $H' = H_w$. The system governed by this Hamiltonian must be solution of the time dependent Schrödinger equation:

$$i \frac{d}{dt} \langle \psi(t) \rangle_s = H \langle \psi(t) \rangle_s$$

We can define a set of discreet eigenstates $|g\rangle$ and a set of eigenstates in the continuum $|f\rangle$ for which we can write:

$$H_0 |g\rangle = E_0 |g\rangle \quad \text{and} \quad H_0 |f\rangle = E_f |f\rangle$$

At this point, using the Schrödinger picture, the generic system can be written like:

$$\langle \psi(t) \rangle_s = \sum_g \alpha_g(t) |g\rangle + \sum_f \beta_f(t) |f\rangle$$

where $|f\rangle$ represent the states to which the mesons $|P^0\rangle$ and $|\bar{P}^0\rangle$ (or in a more general way the states $|g\rangle$) can decay. For $t = 0$, the generic state is:

$$\langle \psi(t = 0) \rangle_s = \sum_g \alpha^0_g |g\rangle.$$

Using the interaction picture, we have:

$$\langle \psi(t = 0) \rangle_I = \langle \psi(t = 0) \rangle_s, \quad \langle \psi(t) \rangle_I = e^{iH_0t} \langle \psi(t) \rangle_s = \sum_g \alpha_g(t) |g\rangle + \sum_f \beta_f(t) |f\rangle$$

the dynamic equation being written:

$$i \frac{d}{dt} \langle \psi(t) \rangle_w = H'_w \langle \psi(t) \rangle_w$$

that depends only on the weak Hamiltonian redefined like $H'_w = e^{iH_0t}H'e^{-iH_0t}$. In terms of amplitude we have:

$$i \frac{da_g(t)}{dt} = \sum_{g'} \langle g | H'_w | g' \rangle a_{g'}(t) + \sum_{f'} e^{i(E_0 - E_f)t} \langle g | H'_w | f' \rangle b_{f'}(t)$$

$$i \frac{db_f(t)}{dt} = \sum_{g'} e^{i(E_f - E_0)t} \langle f | H'_w | g' \rangle a_{g'}(t) + \sum_{f'} e^{i(E_f - E_{f'})t} \langle f | H'_w | f' \rangle b_{f'}(t)$$

(1.1)

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Applying the Wigner-Weisskopf method [10], we introduce an approximation by leaving out in Eq. 1.1, the last term: this corresponds to neglecting the weak interaction for those particles to which the initial mesons can decay. Therefore the decay products are considered to be stable. With this method and with this approximation, we can write these two equations as functions of $\alpha(t)$ that are a finite number $n$ of functions. Defining the vector:

$$a(t) = \begin{pmatrix} a_1(t) \\ \vdots \\ a_n(t) \end{pmatrix}$$

that satisfies $a(t) = e^{-iWt}a^0$, where $a^0 = \begin{pmatrix} a^0_1 \\ \vdots \\ a^0_n \end{pmatrix} = \alpha(t = 0)$

and where $W$ is defined as:

$$W = (W_{gg'}) = \left( \langle g | H' | g' \rangle + \sum_f \frac{\langle g | H' | f \rangle \langle f | H' | g' \rangle}{(E_0 - E_f)} - i\pi \sum_f \delta(E_0 - E_f) \langle g | H' | f \rangle \langle f | H' | g' \rangle \right).$$

(1.2)

and going back to the Schrödinger picture, one obtains:

$$\alpha(t) = e^{-iE_0 t}a(t) = e^{-i(E_0 + W)t}a^0 = e^{-iHt}a^0$$

(1.3)

where $H = E_0 + W$. This matrix $H$ is called the non-Hermitian mass (or energy) matrix. At this point, One can define two Hermitian matrices $M = M^\dagger$ and $\Gamma = \Gamma^\dagger$:

$$M = \frac{i}{2}(H + H^\dagger) \quad \Gamma = i(H - H^\dagger) \quad \text{from which one obtains} \quad H = M - \frac{i}{2} \Gamma$$

The elements of these matrices can be extracted from Eq. 1.2:

$$M_{gg'} = E_0 \delta_{gg'} + \langle g | H' | g' \rangle + \sum_f \frac{\langle g | H' | f \rangle \langle f | H' | g' \rangle}{(E_0 - E_f)},$$

$$\Gamma_{gg'} = \sum_f 2\pi \delta(E_0 - E_f) \langle g | H' | f \rangle \langle f | H' | g' \rangle$$

The $CPT$ invariance guarantees the equality $H_{gg} = H_{\bar{g}\bar{g}}$ with the state $|\bar{g}\rangle$ that represents the charge conjugate of $|g\rangle$ (they both belong to the same eigenvalue): in fact, from

$$\langle g | H | g \rangle = \langle g | (CPT)^{-1}H(CPT)^{-1} | g \rangle = |\eta_{CPT}|^2 \langle \bar{g} | H | \bar{g} \rangle = \langle \bar{g} | H | \bar{g} \rangle.$$

supposing $(CPT)H(CPT)^{-1} = H$ and knowing that $CPT(CPT)^{-1} = 1$ (therefore the eigenvalues can be $|\eta_{CPT}|^2 = 1$ exclusively) and that $CPT|g\rangle = \eta_{CPT} |\bar{g}\rangle$.

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Moreover one can demonstrate that, given the invariance for $CP$ symmetry, the off-diagonal terms of $M$ and $\Gamma$ should be real. As a matter of fact defining a Hermitian matrix $H$ (which represents $M$ or $\Gamma$):

$$\langle g | H | \overline{\varphi} \rangle = \langle g | (CP)^{-1}(CP)H(CP)^{-1}(CP) | \overline{\varphi} \rangle = |\eta_{CP}|^2 \langle \overline{\varphi} | H | g \rangle = \langle \langle g | H | \overline{\varphi} \rangle \rangle^*,$$

(1.4)

assuming that $(CP)H(CP)^{-1} = H$ and remembering that $CP(CP)^{-1} = 1$ (from which the eigenvalues satisfy the $|\eta_{CP}|^2 = 1$) and that $CP | g \rangle = \eta_{CP} | \overline{\varphi} \rangle$.

We can obtain one more constraint if we consider again the $CPT$ invariance: the off-diagonal terms of $M$ and $\Gamma$ have to be one the complex conjugate of the other. This can be shown using the generic Hermitian matrix $H$:

$$\langle g | H | \overline{\varphi} \rangle = \langle g | (CPT)^{-1}(CPT)H(CPT)^{-1}(CPT) | \overline{\varphi} \rangle = |\eta_{CPT}|^2 \langle \overline{\varphi} | H | g \rangle = \langle \langle g | H | \overline{\varphi} \rangle \rangle^*$$

where we have been using $(CPT)H(CPT)^{-1} = H$ and $CPT | g \rangle = \eta_{CPT} | \overline{\varphi} \rangle$.

The $H$ eigenvalues (complex in the most general case) can be written as $\eta - \frac{i}{2} \Gamma_i$ where one can demonstrate that $\Gamma_i \geq 0$: the matrix $\Gamma$ is defined the decay matrix. As a matter of fact, if one defines the eigenstates $\phi_i$ of the non-Hermitian matrix $\mathcal{H}$, the evolution in time of such a state is given by (as Eq. 1.3 shows):

$$\psi(t) = e^{-i\mathcal{H}t} \phi_i = e^{-im_0 t - (\frac{1}{2})\Gamma_i t} \phi_i$$

from which one can extract the probability of the initial particle not to be decayed yet at a given time $t$:

$$|\psi(t)|^2 = e^{-\Gamma_i t} \phi_i^\dagger \phi_i.$$ 

This quantity depends only from $\Gamma_i$ that can be considered the decay rate of the given $\mathcal{H}$ eigenstate $\phi_i$. On the other hand, the matrix $M$ is called the Hermitian part of the mass matrix.

### 1.2.2 The $B$ system: general formalism

A generic neutral meson $|P^0\rangle$ together with its anti-particle $|\overline{P}^0\rangle$\(^2\) can be considered as a set of eigenstates of the imperturbed Hamiltonian $H_0$ with eigenvalues $m_0$ and $\overline{m}_0$, respectively: assuming that $H_0$ conserves $CPT$, $m_0$ can be considered equal to $\overline{m}_0$ and thus:

$$H_0 |P^0\rangle = m_0 |P^0\rangle, \quad H_0 |\overline{P}^0\rangle = m_0 |\overline{P}^0\rangle.$$ 

These two states (particle and anti-particle) belong to $m_0$ that is the degenerate eigenvalue of $H_0$. Thus if an arbitrary linear combination of them is considered:

\(^2\)Here $P^0$ and $\overline{P}^0$ label each neutral meson anti-meson pair.
$$|P\rangle = a|P^0\rangle + b|P^\prime\rangle$$

this gives another $H_0$ eigenstate. In this particular case, this state must satisfy the Schrödinger equation where the matrix $\mathcal{H}$ is a $2 \times 2$ matrix (together with the matrices $M$ and $\Gamma$):

$$i\frac{d}{dt} \begin{pmatrix} a \\ b \end{pmatrix} = \mathcal{H} \begin{pmatrix} a \\ b \end{pmatrix} \equiv (M - i\frac{\Gamma}{2}) \begin{pmatrix} a \\ b \end{pmatrix}.$$ (1.5)

Taking into account the $B$ system and the relation $|B\rangle = p|B^0\rangle + q|\overline{B}^0\rangle$ that satisfies:

$$i\frac{d}{dt} \begin{pmatrix} p \\ q \end{pmatrix} = \mathcal{H} \begin{pmatrix} a \\ b \end{pmatrix} \equiv \left( \begin{pmatrix} M_0 & M_{12} \\ M_{12}^* & M_0 \end{pmatrix} - i\frac{\Gamma}{2} \begin{pmatrix} \Gamma_0 & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma_0 \end{pmatrix} \right) \begin{pmatrix} p \\ q \end{pmatrix}.$$ 96/10/24 11.13

![Feynman's box diagrams describing $B^0 \overline{B}^0$ oscillations.](image)

The off-diagonal terms should be one the complex conjugate of the other, since the matrices are Hermitian. $CP$ conservation would imply also the reality of those terms. The off-diagonal terms in these matrices, $M_{12}$ and $\Gamma_{12}$, are particularly important in the discussion of $CP$ violation: they are the dispersive and absorptive parts respectively of the transition amplitude from $B^0$ to $\overline{B}^0$. $M_{12}$ contributes to the transition amplitude from $B^0$ to $\overline{B}^0$ through intermediate states described by box diagrams (see Fig. 1-1). The box diagrams have four vertices and so they are fourth order diagrams: in the Standard Model, they correspond to second

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order terms with respect to $\alpha_W$ expansion, the weak interaction coupling constant. These contributions arise from the box diagrams with two $W$ exchanges: the quark contribution can come from $u$, $c$ or $t$ exchanges. Theoretically the matrix element $M_{12}$ can be related to the squared mass of the exchanged quark $q$, to the squared mass of the $B$ mesons and to the product $(V_{td} V_{tb}^*)^2$ where the $V_{ij}$ terms are the CKM matrix elements (1.46). Thus, since the elements $V_{ub}$ and $V_{cb}$ are strongly suppressed with respect to $V_{tb}$ (more than how $V_{td}$ is suppressed with respect to $V_{ud}$ and $V_{cd}$), and the $t$ mass dominates on the $c$ and $u$ masses, the dominant contribution is due to $t$ exchange. Therefore we can write [11]:

$$M_{12}^{B^0} = \frac{G_F^2 f_B^2 m_B}{12\pi^2} \left( [V_{td} V_{tb}]^2 \left( m_q^2 + \frac{1}{3} m_B^2 + m_B^2 \ln(\frac{m_q^2}{m_B^2}) \right) + O(m_q^2, \frac{m_q^2}{m_B^2}) \right)$$

where $G_F$ is the Fermi constant for the weak interaction when considered point-like and $f_B$ is a constant called $B$ decay constant which rules the purely leptonic decays and represents the probability of the two constituent quarks to annihilate. Thanks to the fact that the QCD effective coupling constant becomes smaller in the processes where high values of momentum are transferred (if we call $k$ the transferred momentum, this happens for $k \gg \Lambda_{QCD}$ where $\Lambda_{QCD}$ is a typical scale for QCD interaction), these quark diagrams are, to a good approximation, the main contribution to $M_{12}$, according to the Standard Model. $\Lambda_{QCD} \sim 0.2$ GeV corresponds to the order of magnitude according to which one can distinguish the small coupling constant region from the strong coupling constant region, the latter being non-perturbative: if the quark $Q$ mass satisfy the $m_Q \gg \Lambda_{QCD}$, $Q$ is then called a heavy quark. According to this criterion, quarks $u$, $d$ and $s$ are light quarks, while $c$, $b$ and $t$ are heavy. For heavy quarks, the effective coupling constant $\alpha_s (m_Q)$ is small, so strong interactions can be considered perturbative and can be treated in a similar way as we do with the electromagnetic interactions. In the $B$ system, long-distance contributions are expected to be negligible (unlike in the $K$ system), so that a good approximation is taking into account only the leading orders of the expansion with respect to the strong coupling.

The matrix $\Gamma$ can be related to the decay amplitude of the $H$ eigenstates: it describes the processes that rule the meson decay. The off-diagonal element $\Gamma_{12}$ represents the absorptive part of processes like $B^0 \rightarrow X \rightarrow \overline{B}^0$ and $\overline{B}^0 \rightarrow X \rightarrow B^0$, where $X$ is an on-shell intermediate state. In the case of decays through on-shell intermediate states, the top quark cannot contribute (because of the energy conservation) and so the leading contribution becomes the term containing the mass $m_B$ of the $B$ mesons. Theoretically, one obtains the expression [11]:

$$M_{ARCELLA BONA}$$
\[
\Gamma_{12}^{B_{d}} = -\frac{G_{F}^{2} f_{B}^{2} m_{B}^{2}}{6\pi} \left\{ [V_{td} V_{tb}]^2 m_{B}^2 \left( 1 - 4 \frac{m_{s}^2}{m_{B}^2} \right)^{\frac{3}{2}} + 2[V_{td} V_{tb}] [V_{td} V_{tb}] \cdot \right. \\
\left. \cdot \left( m_{B}^2 \left( 1 - 3 \frac{m_{s}^2}{m_{B}^2} + 2 \frac{m_{s}^6}{m_{B}^6} \right) - m_{c}^2 \left( 1 - \frac{m_{s}^2}{m_{B}^2} \right)^{3} \right) + [V_{td} V_{tb}]^2 m_{c}^2 \right\} 
\]

(1.7)

that contains the contributions of \(c\) and \(u\). Using one of the unitary relation of the CKM matrix (see Sec. 1.4.2), it can be simplified into [11]:

\[
\Gamma_{12}^{B_{d}} = -\frac{G_{F}^{2} f_{B}^{2} m_{B}^{2}}{6\pi} \left\{ [V_{td} V_{tb}]^2 m_{B}^2 + 2[V_{td} V_{tb}] [V_{td} V_{tb}] m_{c}^2 + O\left( \frac{m_{c}^4}{m_{B}^2} \right) \right\}. 
\]

So now the leading term is the one containing mass \(m_{B}\), that is of the same order of the mass \(m_{b}\) of quark \(b\): talking about orders of magnitude, we can write:

\[
\frac{[\Gamma_{12}^{B_{d}}]}{[M_{12}^{B_{d}}]} \sim \frac{m_{b}^2 [V_{td} V_{tb}]^2}{m_{t}^2 [V_{td} V_{tb}]^2} \sim \frac{m_{b}^2}{m_{t}^2} \sim 10^{-3}. 
\]

(1.8)

This allows to write also:

\[
[\Gamma_{12}^{B_{d}}] \ll [M_{12}^{B_{d}}]. 
\]

(1.9)

This relation is crucial for the \(B\) system and it will be used in Sec. 1.2.5.

1.2.3 The \(B\) system: mass eigenstates

The states with definite mass and lifetime are eigenstates of the whole Hamiltonian \(\mathcal{H}\) and they can be written like \(B_{L}\) (the lighter) and \(B_{H}\) (the heavier), linear combination of the \(B\) flavour eigenstates:

\[
\begin{align*}
|B_{L}\rangle &= p |B^{0}\rangle + q |\bar{B}^{0}\rangle \\
|B_{H}\rangle &= p |B^{0}\rangle - q |\bar{B}^{0}\rangle
\end{align*}
\]

with the normalization condition: \(|q|^2 + |p|^2 = 1. \)

(1.10)

where \(p\) and \(q\) are complex coefficients. Being these eigenstates of \(\mathcal{H}\), they correspond to two eigenvalues that can be written as:

\[
\mu_{L,H} = M_{L,H} - \frac{i}{2} \Gamma_{L,H}, 
\]

eigenvalues of \(|B_{L}\rangle\) and \(|B_{H}\rangle\), respectively. The mass difference \(\Delta m_{B}\) and the width difference \(\Delta \Gamma_{B}\) between the neutral \(B\) mesons are defined as follows:
\[ \Delta m_B \equiv M_H - M_L, \quad \Delta \Gamma_B \equiv \Gamma_H - \Gamma_L, \]

so that \( \Delta m_B \) is positive by definition and so that, through those definitions, one can also define the difference between eigenvalues \( \mu_{L,H} \):

\[ \Delta \mu \equiv \Delta m_B - \frac{i}{2} \Delta \Gamma_B. \tag{1.11} \]

Going back to the Schrödinger equation (1.5), one can extract the eigenvalues \( \mu_{L,H} \) and from these, one can also obtain constraints for the weights \( p \) and \( q \) as functions of the Hamiltonian matrix elements:

\[
\begin{pmatrix}
\mathcal{H}_0 & \mathcal{H}_{12} \\
\mathcal{H}_{21} & \mathcal{H}_0
\end{pmatrix}
\begin{pmatrix}
p \\
\pm q
\end{pmatrix}
= \mu_{L,H}
\begin{pmatrix}
p \\
\pm q
\end{pmatrix}.
\]

from this, for the two eigenvalues one gets:

\[
\mu_L = \mathcal{H}_0 + \mathcal{H}_{12} \frac{q}{p} = \mathcal{H}_0 + \mathcal{H}_{21} \frac{p}{q} \quad \text{and thus the relation:} \quad \left( \frac{q}{p} \right)^2 = \frac{\mathcal{H}_{21}}{\mathcal{H}_{12}}. \tag{1.12}
\]

Looking at the difference \( \Delta \mu \), the diagonal term \( \mathcal{H}_0 \) vanishes and one gets:

\[ \Delta \mu = \Delta m_B - \frac{i}{2} \Delta \Gamma_B = 2 \mathcal{H}_{12} \frac{q}{p}. \tag{1.13} \]

Now squared:

\[ (\Delta \mu)^2 = (\Delta m_B)^2 - \frac{1}{4} (\Delta \Gamma_B)^2 - i \Delta m_B \Delta \Gamma_B = 4 \mathcal{H}_{12}^2 \left( \frac{q}{p} \right)^2 = 4 \mathcal{H}_{12}^2 \frac{\mathcal{H}_{21}}{\mathcal{H}_{12}} = 4 \mathcal{H}_{12} \mathcal{H}_{21}.
\]

where, to get to the last equalities, Eqs. 1.12 has been used. Now from the latter expression, one obtains:

\[
\Delta m_B = 2 \Re \left[ \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M^*_{12} + \frac{i}{2} \Gamma^*_{12} \right) \right]^{1/2}
\]

\[
\Delta \Gamma_B = -4 \Im \left[ \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M^*_{12} + \frac{i}{2} \Gamma^*_{12} \right) \right]^{1/2}
\]

By substituting the elements of the matrix \( \mathcal{H} \) with the elements of the two matrices \( M \) and \( \Gamma \), one can split \( (\Delta \mu)^2 \) in its real and imaginary parts:

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\[(\Delta \mu)^2 = 4 \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right) = 4 \left[ |M_{12}|^2 - \frac{1}{4} |\Gamma_{12}|^2 - \frac{i}{2} (\Gamma_{12} M_{12}^* + M_{12} \Gamma_{12}^*) \right] \]

from which one obtains:

\[(\Delta m_B)^2 - \frac{1}{4} (\Delta \Gamma_B)^2 = 4(|M_{12}|^2 - \frac{1}{4} |\Gamma_{12}|^2), \tag{1.14} \]

\[\Delta m_B \Delta \Gamma_B = 2 [\Gamma_{12} M_{12}^* + M_{12} \Gamma_{12}^*] = 4 \mathcal{R}e(M_{12} \Gamma_{12}^*). \tag{1.15} \]

From the expressions 1.12, 1.11 and 1.13, one can rewrite the \( q/p \) ratio:

\[
\frac{q}{p} = \frac{\Delta \mu}{2 \mathcal{H}_{12}} = \frac{\Delta m_B - \frac{i}{2} \Delta \Gamma_B}{2(M_{12} - \frac{i}{2} \Gamma_{12})} = \frac{2(M_{12}^* - \frac{i}{2} \Gamma_{12}^*)}{\Delta m_B - \frac{i}{2} \Delta \Gamma_B}. \tag{1.16} \]

Also the time-dependent Schrödinger equation becomes:

\[
i \frac{d}{dt} B_{L,H} = \left( m_{L,H} - \frac{i}{2} \Gamma_{L,H} \right) B_{L,H} \]

and thus the solutions are

\[B_{L,H} = A e^{i M_{L,H} t} e^{-\Gamma_{L,H} \frac{t}{2}} \]

where \( t \) is the proper time of the \( B_{L,H} \) meson.

### 1.2.4 Phase Conventions

The states \( \bar{B}^0 \) and \( \bar{B}^0 \), as already shown in the previous section (Eq. 1.4), are related through \( CP \) transformation:

\[CP |\bar{B}^0 \rangle = e^{i \xi_B} |\bar{B}^0 \rangle \quad CP |\bar{B}^0 \rangle = e^{-i \xi_B} |\bar{B}^0 \rangle. \tag{1.17} \]

The phase \( \xi_B \) is arbitrary since flavour conservation is a symmetry of the strong interactions and a phase transformation [12]

\[|\bar{B}^0 \rangle = e^{-\zeta} |\bar{B}^0 \rangle \quad |\bar{B}^0 \rangle = e^{+\zeta} |\bar{B}^0 \rangle, \tag{1.18} \]

has no physical effects. In the new basis, \( CP \) transformations take the form:

\[CP |\bar{B}^0 \rangle = e^{2i(\xi_B - \zeta)} |\bar{B}^0 \rangle \quad CP |\bar{B}^0 \rangle = e^{-2i(\xi_B - \zeta)} |\bar{B}^0 \rangle \]

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and the various quantities defined change as:

\[ M_{12}^\xi = e^{2i\xi} M_{12} \quad \Gamma_{12}^\xi = e^{2i\xi} \Gamma_{12} \quad (q/p)_\xi = e^{-2i\xi}(q/p). \] (1.19)

Also decay amplitudes are affected by the phase transformation in 1.18:

\[
\frac{A_f}{\overline{A}_f} = \langle f | H | B^0 \rangle \quad \Rightarrow \quad \frac{(A_f)_\xi}{(\overline{A}_f)_\xi} = e^{-i\xi} A_f \quad e^{+i\xi} \overline{A}_f
\] (1.20)

where \( |f\rangle \) is the physical final state. From transformation in 1.18, and the transformation of \( q/p \) in 1.19, one gets:

\[
|B_{L\xi}\rangle = e^{i\xi} |B_L\rangle \quad |B_{H\xi}\rangle = e^{i\xi'} |B_H\rangle,
\] (1.21)

since one can always define the ratio \( (p)_\xi/p \) as a pure phase. As a matter of fact only the ratio \( q/p \) and its transformation are defined and one can write \( (p)_\xi/p e^{-i\xi} = e^{i\xi'}. \) Therefore equations 1.21 mean that both the eigenstates are rotated by a common phase factor with no physical meaning.

Similar phase freedom exists in defining the \( CP \) transformation law for a possible final state \( f \) and its \( CP \) conjugate \( e^{2i\xi/\overline{f}} \); the quantity \( \xi_f \) depends on the flavour content of \( f \) and is related to the quark flavour symmetries (\( c, u, s, d \)) of the strong interactions.

However, the freedom in defining the phase of the flavour eigenstates (which are defined through strong interactions only) does not mean that the full Lagrangian, which involves also weak interactions, is invariant under such phase re-definitions. Indeed, the differences of flavour redefinition phases appear as changes in the phases of the quark mixing matrix elements and of the Yukawa couplings of quarks to Higgs fields (or any other Lagrangian terms that cause couplings between different flavour eigenstates in more general models).

While both \( q/p \) and \( A_f \) acquire overall phase re-definitions when these phase rotations are made, the quantity:

\[ \lambda = \frac{q \overline{A}_f}{p A_f} \] (1.22)

has a convention independent phase that has physical meaning.

### 1.2.5 Time Evolution of Neutral \( B_d \) Mesons

The two neutral \( B_d \) mesons are expected to have difference in lifetime at the level of \( \mathcal{O}(10^{-2}) \) (see Sec. 1.2.2): \( \Delta \Gamma_{B_d} \) has not been measured yet, but from general considerations it can be considered negligible with respect to \( \Gamma_{B_d} \). As a matter of fact, the difference in width is produced by decay channels common to \( B^0 \)
and $\overline{B}^0$ and the branching ratios for such channels are at or below the level of $10^{-3}$; since various channels contribute with differing signs, their sum is not expected to exceed the individual level so $\Delta \Gamma_{B_d, s} \ll \Gamma_{B_d}$ is a rather safe and model independent assumption [13]. On the other hand, $\Delta m_{B_d}$ has been measured [14].

$$\Delta \Gamma_{B_d}/\Gamma_{B_d} = O(10^{-2})$$

$$x_d \equiv \Delta m_{B_d}/\Gamma_{B_d} = 0.723 \pm 0.032 \implies \Delta \Gamma_B \ll \Delta m_B. \quad (1.23)$$

Equations 1.23 imply that, to $O(10^{-2})$ accuracy, Eqs. 1.14, 1.15 and 1.16 simplify into:

$$\Delta m_B \simeq 2|M_{12}|$$

$$\Delta \Gamma_B \simeq 2 \Re(M_{12}\Gamma_{12}^s)/|M_{12}|$$

$$q/p \simeq -|M_{12}|/M_{12}$$

Any $B$ state can then be written as a combination of the states $B_H$ and $B_L$, and the amplitudes of this combination evolve in time as

$$a_H(t) = a_H(0)e^{-iM_H t}e^{-\frac{1}{2}\Gamma_H t}$$

$$a_L(t) = a_L(0)e^{-iM_L t}e^{-\frac{1}{2}\Gamma_L t}.$$ 

A state which is created at time $t = 0$ as initially pure $B^0$, is denoted $|B^0_{\text{phys}}\rangle$: it has $a_L(0) = a_H(0) = 1/(2p)$. Similarly an initially pure $\overline{B}^0$ can be called $|\overline{B}^0_{\text{phys}}\rangle$, and has $a_L(0) = -a_H(0) = 1/(2q)$. The time evolution of these states is thus given by

$$|B^0_{\text{phys}}(t)\rangle = g_+(t)|B^0\rangle + (q/p)g_-(t)|\overline{B}^0\rangle \quad (1.24)$$

$$|\overline{B}^0_{\text{phys}}(t)\rangle = (p/q)g_-(t)|B^0\rangle + g_+(t)|\overline{B}^0\rangle, \quad (1.25)$$

where

$$g_+(t) = e^{-iMt}e^{-\Gamma t/2} \cos(\Delta m_B t/2)$$

$$g_-(t) = e^{-iMt}e^{-\Gamma t/2} i \sin(\Delta m_B t/2)$$

$$M = \frac{1}{2}(M_H + M_L)$$

$$\Gamma = \frac{1}{2}(\Gamma_H + \Gamma_L) \simeq \Gamma_H \simeq \Gamma_L$$

remembering equations 1.23. Furthermore, it is useful to go beyond the leading approximation for the ratio $q/p$, from Eqs. 1.12 the relevant expression is:

$$q/p = \left(\frac{M_{12} - \frac{1}{2}\Gamma_{12}}{M_{12} - \frac{1}{2}\Gamma_{12}}\right)^\frac{1}{2} \sim \left(\frac{M_{12}^s}{M_{12}}\right)^\frac{1}{2} \left[1 - \frac{i}{4} \left(\frac{\Gamma_{12}^s}{M_{12}^s}\right)\right] \left[1 + \frac{i}{4} \left(\frac{\Gamma_{12}}{M_{12}}\right)\right]$$

and the first two orders of the expansion are

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\[ \frac{q}{p} \sim \frac{M_{12}^2}{|M_{12}|} \left[ 1 - \frac{1}{2} \Im \left( \frac{\Gamma_{12}}{M_{12}} \right) \right]. \] (1.26)

From Eqs. 1.12 and 1.9, one can then deduce that

\[ \left| \frac{q}{p} \right| \sim 1 \]

is a good approximation.

### 1.2.6 Time Formalism for Coherent \( B\bar{B} \) States

In an \( e^+e^- \) collider operating at the \( \Upsilon(4S) \) resonance (called a \( B \) factory, see the following chapter 2), the \( B^0 \) and \( \bar{B}^0 \) mesons produced from the decay of the \( \Upsilon \) are in a coherent \( L = 1 \) state. The \( \Upsilon(4S) \) is a resonant \( b\bar{b} \) state with quantum numbers \( J^{PC} = 1^- \) and it can decay into \( B^0 \bar{B}^0 \) or \( B^+ \bar{B}^- \) pairs: \( B \) mesons are scalars \((J^P = 0^-)\) and so, because of the total angular momentum conservation, the \( B\bar{B} \) pair has to be produced in a \( L = 1 \) state. Since the \( \Upsilon(4S) \) decays strongly, \( B \) mesons are produced in the two flavour eigenstates \( B^0 \) and \( \bar{B}^0 \).

After the production, one can imagine that each of the two particles evolve in time as described above for a single \( B \). However they evolve in phase, so that at any time there is always exactly one \( B^0 \) and one \( \bar{B}^0 \) present, at least until one particle decays. As a matter of fact, if at a given time \( t \) one \( B \) could oscillate independently from the other, they could become a state made up of two identical mesons: but this cannot happen since the \( L = 1 \) state is anti-symmetric, while a system of two identical mesons (that are bosons) must be completely symmetric for the two particle exchange. However once one of the particles decays the other continues to evolve, and thus there are possible events with two \( B \) or two \( \bar{B} \) decays, whose probability is governed by the time between the two decays.

Identifying the two particles from the \( \Upsilon(4S) \) decay by the angle \( \theta \) that they form with the \( e^- \) beam direction in the \( \Upsilon(4S) \) rest frame (and thus calling them a backward and a forward \( B \)), the two-\( B \) state can be written as [12]:

\[
S(t_f, t_b) = \frac{1}{\sqrt{2}} e^{-(\frac{t_f + t_b}{2} + iM)} \left\{ B^0(t_f, \theta, \phi) \bar{B}^0(t_b, \pi - \theta, \phi + \pi) + 
- \bar{B}^0(t_f, \theta, \phi) B^0(t_b, \pi - \theta, \phi + \pi) \right\} \sin(\theta) \] (1.27)

where \( t_f \) is the proper time of the \( B_f \), the \( B \) particle in the forward half-space at angle \( (\theta_f < \pi/2, \phi_f) \) and \( t_b \) is the proper time for the backward-moving \( B_b \), at \((\pi - \theta_f, \phi_f + \pi)\).

In Eq. 1.27, it has been taken into account that the \( \Upsilon(4S) \) is a spatially asymmetric state (it is a parity eigenstate with eigenvalue \(-1\)) and it is a \( C \) eigenstate with eigenvalue \(-1\). Therefore also the system to which it decays, must be spatially anti-symmetric as well as charge-conjugation anti-symmetric at a
given time \((t_f = t_b)\): in this case (a particle-anti-particle state), \(P\) and \(C\) transformations correspond to the same one. As a matter of fact, through parity \(B^0(\theta, \phi)\) goes to \(B^0(\pi - \theta, \phi + \pi)\) and therefore the state \(B^0(\theta, \phi)\overline{B}^0(\pi - \theta, \phi + \pi) - \overline{B}^0(\theta, \phi)B^0(\pi - \theta, \phi + \pi)\) results in a spatially anti-symmetric one. As a consequence, the spatial contribution coming from the \(L = 1\) condition in the spherical functions \(\sum_{m}^{\infty}\) must result symmetric: \(\sin(\theta)\) has been included. On the other hand, by applying \(C\), \(B^0(\theta, \phi)\) goes into \(\overline{B}^0(\theta, \phi)\) so that the state in the Eq. 1.27 is asymmetric for particle-antiparticle exchange as requested by the negative \(C\) eigenvalue of the \(T(4S)\).

Since the coherent time evolution of the two particles can be treated like a single particle evolution, in equations 1.24 and 1.25 one can substitute

\[
B^0(t_f, b) \rightarrow B^0_{\text{phys}}(t_f, b)
\]

\[
\overline{B}^0(t_f, b) \rightarrow \overline{B}^0_{\text{phys}}(t_f, b).
\]

After this substitution, Eq. 1.27 can be written extracting the time dependence (and using addition and subtraction trigonometric rules):

\[
S(t_f, t_b) = \frac{1}{\sqrt{2}} e^{-i(\frac{\pi}{2} + M)(t_f + t_b)} \left\{ \cos \left[ \frac{\Delta m_B |t_f - t_b|}{2} \right] \left( B^0_{\text{phys}} \overline{B}^0_{\text{phys}} - \overline{B}^0_{\text{phys}} B^0_{\text{phys}} \right) + i \sin \left[ \frac{\Delta m_B |t_f - t_b|}{2} \right] \left( \frac{q}{p} B^0_{\text{phys}} \overline{B}^0_{\text{phys}} - \overline{B}^0_{\text{phys}} B^0_{\text{phys}} \right) \right\} \sin(\theta_f).
\]

(1.28)

Since the \(B\)'s have equal (though back-to-back) momenta in the center-of-mass frame, before the decay of the first of the two \(B\)'s, \(t_f\) is equal to \(t_b\) and Eq. 1.28 contains one \(B^0\) and one \(\overline{B}^0\). However decay stops the clock for the decayed particle so the terms that depend on \(\sin[\Delta m_B |t_f - t_b|/2]\) begin to play a role. From Eq. 1.28 one can derive the amplitude for decays where one of the two \(B\)'s decays to any state \(f_i\) at time \(t_1\) and the other decays to \(f_2\) at time \(t_2\):

\[
A(t_1, t_2) = \frac{1}{\sqrt{2}} e^{-i(\frac{\pi}{2} + M)(t_1 + t_2)} \zeta(t_1, t_2) \left\{ \cos \left[ \frac{\Delta m_B |t_1 - t_2|}{2} \right] \left( A_1 \overline{A}_2 - \overline{A}_1 A_2 \right) + i \sin \left[ \frac{\Delta m_B |t_1 - t_2|}{2} \right] \left( \frac{q}{p} A_1 A_2 - \frac{q}{p} \overline{A}_1 \overline{A}_2 \right) \right\} \sin(\theta_1),
\]

(1.29)

where \(A_i\) is the amplitude for a \(B^0\) to decay to the state \(f_i\), \(\overline{A}_i\) is the amplitude for a \(\overline{B}^0\) to decay to the same state \(f_i\) (see Eqs. 1.20). To keep signs consistent with Eq. 1.28, the symbol

\[
\zeta(t_1, t_2) = \begin{cases} +1 & t_1 = t_f, \ t_2 = t_b, \\ -1 & t_1 = t_b, \ t_2 = t_f \end{cases}
\]

is introduced, but this overall sign factor will disappear in the rate. Any state that identifies the flavor of the parent \(B\) (tagging) has either \(A_f\) or \(\overline{A}_f = 0\). In Eq. 1.29, the sum \(t_f + t_b\) remains only in the factorized exponential and is vanished from \(\text{sine}\) or \(\text{cosine}\) arguments.
The time dependent rate for producing the combined final states $f_1, f_2$ (applying trigonometric rules like Werner’s and duplication ones) is then:

$$R(t_1, t_2) = C e^{-\Gamma(t_1 + t_2)} \left\{ \left| A_1 \right|^2 + \left| \overline{A}_1 \right|^2 \right\} \left\{ \left| A_2 \right|^2 + \left| \overline{A}_2 \right|^2 \right\} - 4 \Re \left( \frac{q}{p} A_1^{*} \overline{A}_1 \right) \Re \left( \frac{q}{p} A_2^{*} \overline{A}_2 \right) +$$

$$- \cos (\Delta m_B (t_1 - t_2)) \left[ \left| A_1 \right|^2 - \left| \overline{A}_1 \right|^2 \right] \left[ \left| A_2 \right|^2 - \left| \overline{A}_2 \right|^2 \right] + 4 \Im \left( \frac{q}{p} A_1^{*} \overline{A}_1 \right) \Im \left( \frac{q}{p} A_2^{*} \overline{A}_2 \right) +$$

$$+ 2 \sin (\Delta m_B (t_1 - t_2)) \left[ \Im \left( \frac{q}{p} A_1^{*} \overline{A}_1 \right) \left[ \left| A_2 \right|^2 - \left| \overline{A}_2 \right|^2 \right] - \left[ \left| A_1 \right|^2 - \left| \overline{A}_1 \right|^2 \right] \Im \left( \frac{q}{p} A_2^{*} \overline{A}_2 \right) \right]\}

where an integral over all directions for both $B$s has been performed, so the angular dependence has been removed from the expressions, and an overall normalization factor $C$ has been introduced. The approximation $|q/p| = 1$ has also been used.

In order to measure $CP$ asymmetries, one can look for events where one $B$ decays to a final $CP$ eigenstate $f_{CP}$ at time $t_f$, while the second decays to a tagging mode, that is a mode which identifies its $b$-flavor, at time $t_{tag}$. For example, one can consider a tagging mode with $A_2 = 0, \overline{A}_2 = \overline{A}_{tag}$. This identifies the other $B$-particle as a $B^0$ at time $t_2 = t_{tag}$ at which the tagging decay occurs. This is true even when the tag decay occurs after the $CP$ eigenstate decay: in this case the state of the other $B$ at any time $t_f < t_{tag}$ must be just that mixture that, if it had not decayed, would have evolved to become a $B^0$ at time $t_f = t_{tag}$. The double time expression reduces to the form:

$$R(t_{f_{CP}}, t_{tag}) = C e^{-\Gamma(t_{tag} + t_{f_{CP}})} \left\{ \left| A_{f_{CP}} \right|^2 + \left| \overline{A}_{f_{CP}} \right|^2 \right\} \left| \overline{A}_{tag} \right|^2 +$$

$$- \cos [\Delta m_B (t_{f_{CP}} - t_{tag})] \left[ \left| A_{f_{CP}} \right|^2 - \left| \overline{A}_{f_{CP}} \right|^2 \right] \left( -\left| \overline{A}_{tag} \right|^2 \right) +$$

$$+ 2 \sin [\Delta m_B (t_{f_{CP}} - t_{tag})] \left[ \Im \left( \frac{q}{p} A_{f_{CP}} \overline{A}_{f_{CP}} \right) \right] \left( -\left| \overline{A}_{tag} \right|^2 \right) \right\}

where defining

$$\lambda_{f_{CP}} = \frac{q}{p} \frac{\overline{A}_{f_{CP}}}{A_{f_{CP}}} \quad (1.31)$$

and substituting it in the expression 1.30, one obtains:

$$R(t_{f_{CP}}, t_{tag}) = C e^{-\Gamma(t_{tag} + t_{f_{CP}})} \left| \overline{A}_{tag} \right|^2 \left| A_{f_{CP}} \right|^2 \left\{ 1 + \left| \lambda_{f_{CP}} \right|^2 +$$

$$+ \cos [\Delta m_B (t_{f_{CP}} - t_{tag})] \left( 1 - \left| \lambda_{f_{CP}} \right|^2 \right) +$$

$$- 2 \sin [\Delta m_B (t_{f_{CP}} - t_{tag})] \Im (\lambda_{f_{CP}}) \right\} \quad (1.32)$$

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In case the tag final state has $\mathcal{A}_2 = 0$, $A_2 = \mathcal{A}_{\text{tag}}$, which identifies the second particle as a $\mathcal{B}$ at time $t_{\text{tag}}$, an expression similar to Eq. 1.32 applies, except that the signs of both the cosine and the sine terms are reversed. The fact that $|q/p| = 1$ means that the amplitudes for the two opposite tags are the same. Thus the difference of these rates divided by their sum, which measures the time-dependent $CP$ asymmetry, is given by

$$a_{fCP} = \frac{R(A_2 = 0, \mathcal{A}_2 = \mathcal{A}_{\text{tag}}) - R(A_2 = \mathcal{A}_{\text{tag}}, \mathcal{A}_2 = 0)}{R(A_2 = 0, \mathcal{A}_2 = \mathcal{A}_{\text{tag}}) + R(A_2 = \mathcal{A}_{\text{tag}}, \mathcal{A}_2 = 0)} = \frac{1 - |\lambda_{fCP}|^2 \cos(\Delta m_B t) - 2 \Im \lambda_{fCP} \sin(\Delta m_B t)}{1 + |\lambda_{fCP}|^2} \tag{1.33}$$

where $t = t_{fCP} - t_{\text{tag}}$. The above expressions has lost its dependence from the variable $(t_1 + t_2)$ or $t_{CP} + t_{\text{tag}}$: this means that now one can fit the dependence on the variable $t_1 - t_2$ without having to measure the $\mathcal{B}$ decay time. This can be done also from equation 1.32 that can be integrated over the variable $(t_1 + t_2)$, substituting the variables ($T = t_1 + t_2$ and $t = t_1 - t_2$) and integrating over $T$ which for $t_1 \geq 0$ and $t_2 \geq 0$ can take values between $|t|$ and infinity. This way, one gets an expression of $R(t)$ which is only a function of the time difference $t_{CP} - t_{\text{tag}}$ and not of the $\mathcal{B}$ decay time:

$$R(t_{fCP} - t_{\text{tag}}) \propto e^{-1|t_{\text{tag}} - t_{fCP}|} \cdot \{1 + |\lambda_{fCP}|^2 + (1 - |\lambda_{fCP}|^2) \cos(\Delta m_B (t_{fCP} - t_{\text{tag}})) - 2 \sin(\Delta m_B (t_{fCP} - t_{\text{tag}})) \Im (\lambda_{fCP})\}. \tag{1.34}$$

The fact that the variable $t_1 - t_2$ can be related to the distance between the decay vertices of the two $B$’s is the main reason for building an energy-asymmetric collider for this kind of measurements (see Sec. 2.1). By integrating also over this variable, all information on the coefficient of $\sin(\Delta m_B (t_1 - t_2))$ would be lost and the experiment would be sensitive only to those $CP$-violating effects that give $|\lambda| \neq 1$. This is a consequence of the coherent production of the two $B$ states: in a hadronic environment, where the $B$’s are produced incoherently, time-integrated rates are always integrals from $t = 0$ to infinity so that they retain information about the $\sin(\Delta m_B t)$ term.

### 1.3 The Three Types of $CP$ Violation in $B$ Decays

$CP$ violation can manifest itself in three different ways:

- **$CP$ violation in decay**: also called direct $CP$ violation, it occurs when a decay and its $CP$ conjugate process have different amplitudes. It can be studied in both charged and neutral decays.

- **$CP$ violation in mixing**: also called indirect $CP$ violation, it occurs when mixing provides interfering amplitudes. In this case, the two neutral mass eigenstates cannot be $CP$ eigenstates too.
• \textit{CP} violation in the interference between mixing and decay: it occurs in decays into final states that are common to $B^0$ and $\bar{B}^0$. It often occurs in combination with the other two types but there are cases when, to a very good approximation, it is the only effect.

1.3.1 \textit{CP} Violation in Decay

In order to study this type of \textit{CP} violation, for any final state $f$, the quantity $\frac{\overline{A_f}}{A_f}$ is defined since it is independent of phase conventions and physically meaningful. There are two types of phases that may appear in the amplitudes: complex parameters in any Lagrangian term that contributes to the amplitude will appear in complex conjugate form in the \textit{CP} conjugate amplitude. Therefore these phases appear in $A_f$ and $\overline{A_f}$ with opposite signs. In the Standard Model these phases appear in the CKM matrix and are called weak phases. The weak phase of any single term is dependent on the convention, but the difference between the weak phases in two different terms in the amplitudes is convention independent. A second type of phase can appear even when the Lagrangian is real: such phases come from the possible contribution from intermediate on-shell states dominated by strong interactions and so they are called strong phases. Since strong interactions conserve \textit{CP} these phases appear in $A_f$ and $\overline{A_f}$ with the same sign. Again only the relative strong phases of different terms have physical meaning.

Contributions to the amplitudes can be factorized as:

- the magnitude $A_i$;
- the weak phase term $e^{i\phi_i}$;
- the strong phase term $e^{i\delta_i}$.

If several amplitudes contribute to $B^0 \rightarrow f$, the amplitude $A_f$ (see Eq. 1.20) and the \textit{CP} conjugate amplitude $\overline{A_f}$ are given by:

$$A_f = \sum_i A_i e^{i(\delta_i + \phi_i)}, \quad \overline{A_f} = e^{2i(\xi_f - \xi_B)} \sum_i A_i e^{i(\delta_i - \phi_i)},$$

(1.35)

where $\xi_f$ and $\xi_B$ are defined in expressions like 1.17: $\text{CP}[B^0] = e^{-2i\xi_B} |B^0\rangle$ and $\text{CP}[f] = e^{-2i\xi_f} |f\rangle$ (one should consider the complex conjugate of the latter expression $\langle f | (\text{CP})^{-1} = e^{2i\xi_f} \langle f |$). If $f$ is a \textit{CP} eigenstate then $e^{2i\xi_f} = \pm 1$ being its \textit{CP} eigenvalue. The convention-independent quantity is then

$$\left| \frac{\overline{A_f}}{A_f} \right| = \left| e^{2i(\xi_f - \xi_B)} \sum_i A_i e^{i(\delta_i - \phi_i)} \right| \sum_i A_i e^{i(\delta_i + \phi_i)}. \quad (1.36)$$

\textit{CP} is conserved in decays when the magnitude of this ratio is 1, that means the rate of the decay must be equal to the rate of the \textit{CP} conjugate decay. This can happen only if all weak phases $\phi_i$ are the same phase or if all the strong phases $\delta_i$ are the same one. Therefore, from Eq. 1.36 one sees that

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1.3 The Three Types of \( CP \) Violation in \( B \) Decays

\[
\left| \frac{\overline{A}_f}{A_f} \right| = \left| e^{2i(\xi_f - \xi_n)} \right| = 1.
\]

Instead \( CP \) is violated in decays if both the weak phases \( \phi_i \) and the strong ones \( \delta_i \) are different one from the others. In this case the ratio cannot be reduce to a pure phase:

\[
\left| \frac{\overline{A}_f}{A_f} \right| \neq 1 \quad \Rightarrow \quad CP \text{ violation.} \quad (1.37)
\]

This type of \( CP \) violation is here called \( CP \) violation in decay. It is often also called direct \( CP \) violation. It results from the \( CP \)-violating interference among various terms in the decay amplitude. From Eq. 1.36 it can be shown that, in this case, \( CP \) violation requires at least two terms that have different weak phases and different strong phases:

\[
|A|^2 - \overline{|A|}^2 = -2 \sum_{i,j} A_iA_j \sin(\phi_i - \phi_j)\sin(\delta_i - \delta_j).
\]

Any \( CP \) asymmetries in charged \( B \) decays are from \( CP \) violation in decay. One can redefine:

\[
a_f = \frac{\Gamma(B^+ \to f) - \Gamma(B^- \to \overline{f})}{\Gamma(B^+ \to f) + \Gamma(B^- \to \overline{f})},
\]

from which, in terms of the decay amplitudes, one has:

\[
a_f = 1 - \frac{|\overline{A}/A|^2}{1 + |\overline{A}/A|^2}.
\]

The latter expression can be useful for neutral \( B \) mesons also: as a matter of fact \( CP \) violation in decays can also occur for neutral meson decays, where it competes with the other two types of \( CP \) violation effects described below. Since the amplitudes differ from their \( CP \) conjugate ones at most for a phase factor, in case only one amplitude contributes to a given decay process, no direct \( CP \) violation effect can be observed.

1.3.2 \( CP \) Violation in Mixing

A quantity which is useful in the study of this type of \( CP \) violation is:

\[
\left| \frac{q}{p} \right|^2 = \left| \frac{M_{12}^0 - \frac{i}{2}\Gamma_{12}^+}{M_{12}^0 - \frac{i}{2}\Gamma_{12}^-} \right|^2. \quad (1.38)
\]

\( CP \) Violation in the \( B\overline{B} \) System
This ratio, as already mentioned, is phase-convention independent. When $CP$ is conserved, the mass eigenstates must be $CP$ eigenstates. Taking into account the subspace spanned by $|B^0\rangle$ and $|\bar{B}^0\rangle$, one can write the matrix representation of $CP$ operator as [15]:

$$CP = \begin{pmatrix} 0 & e^{-2\xi_B} \\ e^{2\xi_B} & 0 \end{pmatrix}$$

If $CP$ is conserved, then $[CP, \mathcal{H}] = 0$ and also $(CP)^{-1}\mathcal{H}(CP) = \mathcal{H}$: expressing these conditions using matrices, one gets:

$$\begin{pmatrix} 0 & e^{-2\xi_B} \\ e^{2\xi_B} & 0 \end{pmatrix} \begin{pmatrix} \mathcal{H}_0 & \mathcal{H}_{12} \\ \mathcal{H}_{21} & \mathcal{H}_0 \end{pmatrix} \begin{pmatrix} 0 & e^{-2\xi_B} \\ e^{2\xi_B} & 0 \end{pmatrix} = \begin{pmatrix} e^{2\xi_B} \mathcal{H}_0 & \mathcal{H}_{12} \\ e^{4\xi_B} \mathcal{H}_{12} & e^{-2\xi_B} \mathcal{H}_0 \end{pmatrix} = \mathcal{H}$$

Expanding the terms of $\mathcal{H}$ as functions of $M$ and $\Gamma$, in order to have $\mathcal{H}_{CP} = \mathcal{H}$ one has to write:

$$e^{4i\xi_B} \left( M_{12} - \frac{i}{2} \Gamma_{12} \right) = \left( M_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)$$

and so this $CP$ conserving condition can be substitute in Eq. 1.38:

$$\left| \frac{q}{p} \right|^2 = \frac{M_{12}^* - \frac{i}{2} \Gamma_{12}^*}{M_{12} - \frac{i}{2} \Gamma_{12}} = e^{4i\xi_B} = 1.$$

On the other hand, one can write:

$$\left| \frac{q}{p} \right| \neq 1 \implies CP \text{ violation.} \quad (1.39)$$

This type of $CP$ violation is called $CP$ violation in mixing, but it is often referred to as indirect $CP$ violation. It results from the mass eigenstates being different from the $CP$ eigenstates. $CP$ violation in mixing has been observed in the neutral kaon system.

This $CP$ violation can be observed through the tagging modes, i.e. those decays in which the $B$ flavour can be unambiguously identified: for the neutral $B$ system, this effect could be observed through the asymmetries in semileptonic decays (they are tagging modes since a positive charged lepton identifies a $B^0$ and a negative charged lepton identifies a $\bar{B}^0$). In this case one can write:

$$\alpha_{st} = \frac{\Gamma(B^0_{\text{phys}}(t) \to \ell^- \nu X) - \Gamma(\bar{B}^0_{\text{phys}}(t) \to \ell^+ \nu X)}{\Gamma(B^0_{\text{phys}}(t) \to \ell^- \nu X) + \Gamma(\bar{B}^0_{\text{phys}}(t) \to \ell^+ \nu X)}$$

$$= \frac{|\langle \bar{B}^0 |{\cal L}_{\text{phys}}(t)|B^0\rangle|^2 - |\langle B^0 |{\cal L}_{\text{phys}}(t)|\bar{B}^0\rangle|^2}{|\langle \bar{B}^0 |{\cal L}_{\text{phys}}(t)|B^0\rangle|^2 + |\langle B^0 |{\cal L}_{\text{phys}}(t)|\bar{B}^0\rangle|^2}, \quad (1.40)$$

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To obtain the asymmetry in 1.40 as function of \(|q/p|\), one can derive from the Eqs. 1.24 and 1.25 the expressions:

\[
|\langle B^0 \rangle |_{\text{phys}(t)}| = \left| \frac{q}{p}g_-(t) \right| \quad \text{and thus} \quad a_{\text{al}} = \frac{|q/p|^4 - 1}{|q/p|^4 + 1}.
\]

Effects of \(CP\) violation in mixing in neutral \(B_d\) decays, such as the asymmetries in semileptonic decays, are expected to be small, \(O(10^{-2})\). In addition, to calculate the deviation of \(q/p\) from a pure phase, one needs to estimate \(\Gamma_{12}\) and \(M_{12}\), but they involve large hadronic uncertainties, in particular in the hadronization models for \(\Gamma_{12}\). The overall uncertainty can be even a factor of 2–3 in \(|q/p| - 1\) [13]. Thus even if such asymmetries are observed, it will be difficult to relate their rates to fundamental \(CKM\) parameters.

Going back to the general case, Eq. 1.26 can be rewrite as [15]:

\[
\frac{q}{p} = \left[ 1 + \frac{1}{2} \frac{|\Gamma_{12}|}{|M_{12}|} \sin(\theta_{M_{12}} - \theta_{\Gamma_{12}}) \right] e^{-i(\theta_{M_{12}} + \pi)}
\]

The order of magnitude of the term \(\sin(\theta_{M_{12}} - \theta_{\Gamma_{12}})\) is \(\propto (m_e/m_b)^2\). Remembering Eq. 1.8, the main contribution to \(|q/p| - 1\) can be evaluated as:

\[
|q/p| - 1 \propto \left( \frac{m_h}{m_t} \right)^2 \left( \frac{m_e}{m_b} \right)^2 \left( \frac{m_e}{m_t} \right)^2 \propto O(10^{-4}),
\]

and thus the effect of \(CP\) violation in mixing in neutral \(B_d\) decays are supposed to be rather small.

### 1.3.3 \(CP\) Violation in the Interference Between Decays With and Without Mixing.

Taking into account neutral \(B\) decays into final \(CP\) eigenstates, \(f_{CP}\) [16, 17, 18], these states are accessible from both \(B^0\) and \(\bar{B}^0\) decays. The quantity that can be used in studying this type of \(CP\) violation, is \(\lambda\) of Eq. 1.31,

\[
\lambda = \frac{q \overline{A}_{f_{CP}}}{p \overline{A}_{f_{CP}}} = \eta_{f_{CP}} |\lambda| e^{-2i\phi_{f_{CP}}}. \tag{1.41}
\]

where \(\eta_{f_{CP}}\) is the \(CP\) eigenvalue of the \(f_{CP}\) state \((CP|f_{CP}\rangle = \eta_{f_{CP}}|f_{CP}\rangle\) and \(\phi_{f_{CP}}\) represents the weak \(CKM\) phase of the \(A_{f_{CP}}\) amplitude. When \(CP\) is conserved, both \(|q/p| = 1\) and \(|\overline{A}_{f_{CP}}/A_{f_{CP}}| = 1\), as seen in the previous sections: also the relative phase between \(|q/p|\) and \(|\overline{A}_{f_{CP}}/A_{f_{CP}}|\) vanishes (as one can see in Eq. 1.33 where \(\text{Im} \, \lambda\) is the coefficient of the \(sine\) term). Thus, from the definition of \(\lambda\) in Eq. 1.31, one can obtain the condition:

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where $\pm 1$ depends on the eigenstate $\eta_{CP}$.

Both $CP$ violation in decays (1.37) and $CP$ violation in mixing (1.39) lead to the condition 1.42 through $|\lambda| \neq 1$. But even in the case in which, to a good approximation, $|q/p| = 1$ and $|A/A| = 1$, yet there can be $CP$ violation if:

$$|\lambda| = 1, \quad \Im \lambda \neq 0.$$ 

This type of $CP$ violation is called $CP$ violation in the interference between decays with and without mixing or more briefly “interference between mixing and decay”. This type of $CP$ violation has also been observed in the neutral kaon system.

![Diagram](image)

**Figure 1-3.** $CP$-violating asymmetries result from interference effects involving phases that change sign under the $CP$ operator. The weak phase of the CKM matrix has this property. One way to observe $CP$ violation is to use the interference between the direct decay $B^0 \to f_{CP}$ and the process $B^0 \to \bar{B}^0 \to f_{CP}$: the Standard Model predicts substantial asymmetries between this process and the one in which the initial meson is a $\bar{B}^0$.

For the neutral $B$ system, $CP$ violation in the interference between decays with and without mixing can be observed by comparing:

- direct decays $B^0 \to f$, where $f$ is a final state accessible in both $B^0$ and $\bar{B}^0$ decays;
- $B^0 \to \bar{B}^0$ mixing followed by the $\bar{B}^0 \to f$ decay.

The state $f$ can be a $CP$ eigenstate, but that’s not a necessary condition. From the analysis proposed in Sec. 1.2.6, one gets:

$$\alpha_{f_{CP}} = \frac{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) - \Gamma(\bar{B}^0_{\text{phys}}(t) \to f_{CP})}{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) + \Gamma(\bar{B}^0_{\text{phys}}(t) \to f_{CP})}. \quad (1.43)$$

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As shown in Eq. 1.33, the value of the time-dependent asymmetry is given by:

\[
\alpha_{f_{\text{CP}}} = \frac{(1 - |\lambda_{f_{\text{CP}}}|^2) \cos(\Delta m_B t) - 2 \text{Im} \lambda_{f_{\text{CP}}} \sin(\Delta m_B t)}{1 + |\lambda_{f_{\text{CP}}}|^2}.
\]  

(1.44)

This asymmetry will be non-vanishing if any of the three types of \textit{CP} violation are present. In the particular case of \( f_{\text{CP}} \) being the \textit{CP} eigenstate \( J/\psi K_S^0 \) (the so-called \textit{golden mode}), one can measure \textit{CP} violation effect given by \( \sin 2\beta \). In such decays for which \( |\lambda| = 1 \), the expression 1.33 simplifies considerably:

\[
\alpha_{f_{\text{CP}}} = - \text{Im} \lambda_{f_{\text{CP}}} \sin(\Delta m_B t)
\]  

(1.45)

with no hadronic uncertainties from the strong interactions. In case no \textit{CP} violation in decay is present,\textit{CP} violation in the interference between mixing and decay can be cleanly related to CKM parameters: in particular, if decays are dominated by a single \textit{CP}-violating phase, \( \alpha_{f_{\text{CP}}} \) is cleanly translated into a value for \( \text{Im} \lambda \) (see Eq. 1.45) which, in this case, is easily interpreted in terms of purely electroweak Lagrangian parameters.

On the other hand, when \textit{CP} violation in decay is present, the asymmetry in 1.43 depends also on the ratio of the different amplitudes and their relative strong phases, and thus the result is not cleanly interpreted because of the hadronic uncertainties. In some cases, however, it is possible to remove any large hadronic uncertainties by measuring several isospin-related rates and extract a clean measurement of \( CKM \) phases (this is the case of two pion decays, see Sec. 1.5.2).

There are also many final states for \( B \) decay that have \textit{CP} self-conjugate particle content but are not \textit{CP} eigenstates because they contain admixtures of different angular momenta and hence different parities. In certain cases angular analyses of the final state can be used to determine the amplitudes for each different \textit{CP} contribution separately (this is the case of \( B \to J/\psi K^* \) decays [24]).

\section{1.4 \textit{CP} Violation in the Standard Model}

\subsection{1.4.1 The \textit{CKM} Picture of \textit{CP} Violation}

The Standard Model [19] is the theory describing the electromagnetic, weak and strong interactions. It is based on a \( SU(3)_C \times SU(2)_L \times U(1)_Y \) gauge symmetry with three fermion generations. \textit{CP} violation is accommodated in this model through a phase in the mixing matrix for quarks [3]. Each quark generation consists of three multiplets:

\[
Q_L^I = \begin{pmatrix} U_L^I \\ D_L^I \end{pmatrix} = (3, 2)_{+1/6}, \quad u_R^I = (3, 1)_{+2/3}, \quad d_R^I = (3, 1)_{-1/3},
\]

\( CP \) \textit{VIOLATION IN THE \( B \overline{B} \) SYSTEM}
where, for example, $(3, 2)_{+1/6}$ denotes a triplet of $SU(3)_C$, doublet of $SU(2)_L$ with hypercharge $Y = Q - T_3 = +1/6$. The left-handed components of the quark and lepton families can be represented as $SU(2)_L$ doublets:

\[
\begin{pmatrix}
  u_L \\
  d_L
\end{pmatrix}
\begin{pmatrix}
  c_L \\
  s_L
\end{pmatrix}
\begin{pmatrix}
  t_L \\
  b_L
\end{pmatrix}
\begin{pmatrix}
  \nu_e \\
  \nu_L
\end{pmatrix}
\begin{pmatrix}
  \nu_\mu \\
  \nu_\tau
\end{pmatrix}
\]

while the right-handed components are described like $SU(2)_R$ singlets:

\[
\begin{align*}
  e_R & \quad \mu_R & \quad \tau_R \\
  u_R & \quad c_R & \quad t_R \\
  d_R & \quad s_R & \quad b_R
\end{align*}
\]

The Standard Model deals with flavour-changing quark transitions in term of a V-A charged weak current operator $J^\mu$ that couples to the $W$-boson according to the interaction Lagrangian [15]:

\[
\mathcal{L}_{int} = -\frac{g}{\sqrt{2}}(J^\mu W^+_\mu + J^\mu W^-_\mu)
\]

where for quark transitions:

\[
J^\mu = \sum_{i,j} V_{ij} J^\mu_{ij} = \sum_{i,j} u_i \gamma^\mu \frac{1}{2}(1 - \gamma_5)V_{ij}d_j.
\]

The $V_{ij}$ are the elements of the $CKM$ matrix [2, 3] (see the representation in 1.46) and the indices $i$ and $j$ run over the three quark generations ($u_1 = u$, $u_2 = c$, $u_3 = t$, $d_1 = d$, $d_2 = s$, $d_3 = b$). The field operators $u_i$ annihilate $u$, $c$ and $t$ or create their anti-quarks and the $d_i$ annihilate $d$, $s$ and $b$ (or create $\bar{d}$, $\bar{s}$ or $\bar{b}$). In a similar way, the field operator $W_{\mu}^+$ annihilates a $W^+$ or creates a $W^-$ while the reverse is true for $W_{\mu}^-$. The amplitudes for the processes in which a $W^+$ is radiated are proportional to $V_{ij}$, while the amplitudes for the process in which a $W^-$ is radiated are proportional to $V_{ij}^\dagger$.

The $CKM$ matrix can be considered as a rotation transformation from the quark mass eigenstates $d$, $s$ and $b$ to a set of new states $d'$, $s'$ and $b'$ with diagonal couplings to $u$, $c$ and $t$. The more general representation is:

\[
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix}
= \begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}.
\]

(1.46)
To a first approximation, the CKM matrix is simply the unit matrix, because the dominant transitions are $u \rightarrow d$, $c \rightarrow s$ and $t \rightarrow b$. In reality, none of the off-diagonal elements is exactly zero, leading to generation-changing transitions between quarks and to the possibility of a $CP$-violating phase. The values of both fermion masses and CKM matrix elements cannot be predicted since they are input parameters of the Standard Model originating in the Higgs field.

In order to have a complete representation of the CKM matrix, only four real and independent parameters are necessary: all nine CKM matrix elements can be expressed as functions of these four parameters. Generally speaking, an $n \times n$ unitary matrix has $n^2$ real and independent parameters: a generic $n \times n$ matrix would have $2n^2$ and the unitary condition imposes $n$ normalization constraints and $n(n-1)$ conditions from the orthogonality between each pair of columns: thus $2n^2 - n - n(n-1) = n^2$.

In the CKM matrix, not all of these parameters have a physical meaning since, given $n$ quark generations, $2n-1$ phases can be absorbed by the freedom to select the phases of the quark fields. A phase factor can be applied to every quark operator so that the current $\mathcal{J}^\mu$ could be written as:

$$\mathcal{J}^\mu = (ue^{-i\theta_u}, ce^{-i\theta_c}, te^{-i\theta_t}) \frac{1}{2} \gamma^\mu (1 - \gamma_5) \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} de^{i\theta_d} \\ se^{i\theta_s} \\ be^{i\theta_b} \end{pmatrix}.$$  

Each $u$, $c$ or $t$ phase allows for multiplying a row of the CKM matrix by a phase, while each $d$, $s$ or $b$ phase allows for multiplying a column by a phase: the $u$, $c$ and $t$ phases can be chosen in order to make real one element of each of the three rows (for example $V_{us}$, $V_{cs}$ and $V_{ts}$). Therefore all three elements of a column (the second in the example) can be made real. In a similar way, the $d$, $s$ and $b$ phases can be chosen in order to make real one element of each of the three columns (for example $V_{ud}$ and $V_{cb}$). At the end of this redefinition procedure, five of the CKM matrix phases have been re-absorbed with six quarks: in general, with $n$ quark families, $2n-1$ phases can be removed. So it is: $n^2 - (2n-1) = (n-1)^2$. From the latter, given 3 quark families, 4 real and independent parameters are necessary.

If the CKM matrix were simply real and orthogonal, it would have $n^2$ degrees of freedom from which one should subtract the $n$ normalization conditions and $\frac{1}{2}n(n-1)$ orthogonality conditions (the factor $\frac{1}{2}$ is due to the fact that $\sum_i V_{1i}V_{2i} = 0$ is the same condition as $\sum_i V_{2i}V_{1i} = 0$): in this case, one would obtain a number of degrees of freedom corresponding to $\frac{1}{2}n(n-1)$ real independent rotation parameters.

<table>
<thead>
<tr>
<th>$n$(families)</th>
<th>Total indep. params.</th>
<th>Real rot. angles</th>
<th>Complex phase factors</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$(n-1)^2$</td>
<td>$\frac{1}{2}n(n-1)$</td>
<td>$\frac{1}{2}(n-1)(n-2)$</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>9</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

**CP Violation in the $B\Phi$ System**
So going back to the $n^2$ real independent parameters of a generic unitary matrix, $\frac{1}{2}n(n - 1)$ of these parameters can be associated to real rotation angles: among the remaining $n^2 - \frac{1}{2}n(n - 1)$ phases, $(2n - 1)$ can be removed with quark field re-definitions, as already said. So the number of independent phase factors is:

$$n^2 - \frac{1}{2}n(n - 1) - (2n - 1) = \frac{1}{2}(n - 1)(n - 2).$$

In table 1-1, the cases with 2, 3 or 4 quark families are shown: note that at least three quark generations are necessary in order to have, in the CKM matrix, a CP-violating phase factor. A two-generation theory would not be able to accommodate CP violation without the addition of extra fields. It was this observation that led Kobayashi and Maskawa to suggest a third quark generation long before there was any experimental evidence for it [2, 3].

The unitarity of the CKM matrix can be made more explicit using a particular parameterization. There are various useful ways to parameterize it, but the standard choice is based on the definition of four angles $\theta_{12}$, $\theta_{13}$, $\theta_{23}$ and $\delta_{13}$ [14, 20]:

$$V = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - s_{12}s_{23} s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23} s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23} s_{13}e^{i\delta} & -s_{12}s_{23} - c_{12}c_{23} s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix},$$

(1.47)

where $c_{ij} \equiv \cos \theta_{ij}$ and $s_{ij} \equiv \sin \theta_{ij}$, $\theta_{ij}$ is the mixing angle between the $i$ and $j$ quark families and the phase $\delta_{13}$ is responsible for CP violation.

Considering that $V \rightarrow 1$ when CP violation effect goes to zero, the representation 1.47 can be simplified: for example one can use the fact that $|V_{ub}| = s_{13} \approx 0.003$ is very small and so $c_{13}$ is extremely close to unity. As a consequence, one can neglect terms proportional to $a_3$ relative to terms of order unity and obtain:

$$V = \begin{pmatrix}
    c_{12} & s_{12} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} & c_{12}c_{23} & s_{23} \\
    s_{12}s_{23} & -s_{12}c_{23} & c_{23}c_{13}
\end{pmatrix}.$$  

(1.48)

In this approximation, only $V_{ub}$ and $V_{td}$ carry phases. Another useful expansion of the CKM matrix has been first given by Wolfenstein [21] in the small parameter $\lambda = \sin \theta_C \approx 0.22$ where $\theta_C$ is the Cabibbo angle:

$$V = \begin{pmatrix}
    1 - \frac{1}{2}\lambda^2 & \frac{\lambda}{2} & A\lambda^3(\rho - i\eta) \\
    -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\
    A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + O(\lambda^4).$$

(1.49)

In this form, only four independent parameters remain: $A$, $\lambda$, $\rho$ and $\eta$. To $O(\lambda^4) \approx 10^{-3}$, the $2 \times 2$ upper-left portion of the CKM matrix is the matrix associated with Cabibbo rotations of the $d$ and $s$ quarks and it is constructed to be nearly unitary.
\[(1 - \frac{1}{2}\lambda^2)^2 + \lambda^2 = 1 + \mathcal{O}(\lambda^4)\]

accordingly with measurements which give \(\sqrt{|V_{ud}|^2 + |V_{us}|^2} = 0.998\).

### 1.4.2 Unitarity of the CKM Matrix

The unitarity of the CKM matrix implies various relations among its elements. Three of them are related to the study of CP violation within the Standard Model:

\[
V_{ud}V_{us} + V_{cd}V_{cs} + V_{td}V_{ts} = 0, \\
V_{us}V_{ub} + V_{cs}V_{cb} + V_{ts}V_{tb} = 0, \\
V_{ud}V_{ub} + V_{cd}V_{cb} + V_{td}V_{tb} = 0. \\
\tag{1.50}
\]

Each of these three relations corresponds to an orthogonality condition between columns and requires the sum of three complex quantities to vanish: as a consequence, it can be geometrically represented in the complex plane as a triangle. These are the unitarity triangles: the term Unitarity Triangle is traditionally reserved for the relation 1.50 only. The latter is the one involving the two smaller elements of the CKM matrix and every single element of the sum is of the order of \(\lambda^3\), as in the parameterization in 1.49.

In the parameterization in 1.47, \(V_{cd}, V_{cb}\) and \(V_{tb}\) are real and, using the approximations \(V_{ud} \simeq V_{tb} \simeq 1\) and the fact that \(V_{cd} < 1\), the relation 1.50 can be re-written as:

\[
\frac{V_{ub}}{|V_{cd}V_{cb}|} + \frac{V_{tb}}{|V_{cd}V_{cb}|} = 1.
\]

In terms of the Wolfenstein parameterization, the coordinates of this triangle are \((0, 0), (1, 0)\) and \((\rho, \eta)\) (as a matter of fact, two sides are \((\rho - i\eta)\) and \((1 - \rho + i\eta))\).

All the three triangles can be drawn knowing the experimental values (within errors) for the various \(|V_j|\): this has been done in Fig. 1-5 in a common scale. This figure can be understood by looking at the order of magnitude:

\[
V_{ud}V_{us} + V_{cd}V_{cs} + V_{td}V_{ts} \simeq \mathcal{O}(<\lambda) + \mathcal{O}(\lambda^2) = 0, \\
V_{us}V_{ub} + V_{cs}V_{cb} + V_{ts}V_{tb} \simeq \mathcal{O}(\lambda) + \mathcal{O}(\lambda^2) = 0, \\
V_{ud}V_{ub} + V_{cd}V_{cb} + V_{td}V_{tb} \simeq \mathcal{O}(\lambda^3) + \mathcal{O}(\lambda^2) = 0.
\]

\section*{CP Violation in the \(B\overline{B}\) System}
In the first two triangles, one side is much shorter than the other two, and so they almost collapse to a line. This can give an intuitive explanation of why CP violation is small in the K system (the first triangle) and in the B_s system (the second triangle). The openness of the third triangle predicts large CP asymmetries in B decays.

All triangles have the same area. One can define the quantity:

\[ J_{CP} = |\Im(V_{ij}V_{kl}^*)|; \quad i \neq k, j \neq l, \]

where no sums over indices are implied. The term \( z_1 = V_{ij}V_{il}^* \) is one term in the sum of terms that gives the inner product between column \( j \) and column \( l \), while, in the same way, \( z_2^* = V_{kj}V_{kl}^* \) is the complex conjugate of another one of these terms. Since \( z_1 \) and \( z_2 \) are two of the sides of the unitary triangles, the quantity \( \Im(z_1 z_2^*) \) is proportional to the sine of the angle between \( z_1 \) and \( z_2 \) (this can be demonstrated using polar coordinates: \( z_1 = k_1 e^{i \theta_1} \) and \( z_2 = k_2 e^{i \theta_2} \) and thus \( \Im(z_1 z_2^*) = k_1 k_2 \sin(\theta_1 - \theta_2) \)) and so the area of the unitary triangles can be written like:

\[ \text{Area} = \frac{1}{2} |\Im(z_1 z_2^*)| \]

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Figure 1-5. The three unitarity triangles: a) $V_{td}V_{ts}^* = 0$, b) $V_{td}V_{tb}^* = 0$, and c) $V_{td}V_{tb}^* = 0$, drawn to a common scale.

The area of the unitary triangles is constant: as a matter of fact from Eq. 1.47 one can get that $J_{CP}$ is always

$$J_{CP} = c_{12}c_{23}c_{13}^2 s_{12} s_{23} s_{13} \sin \delta$$

or, using the Wolfenstein’s parameterization in 1.49,

$$J_{CP} \simeq A^2 \eta \lambda^6.$$  (1.51)

In the Wolfenstein’s parameterization, measurements of $|V_{cb}|$ and $|V_{ub}|$ provide the constraints

$$\left(\frac{V_{cb}}{V_{us}}\right) = A \quad \left|\frac{V_{ub}}{V_{cd}V_{cb}}\right| = \sqrt{\rho^2 + \eta^2}.$$  

Thus, measurements of $|V_{cb}|$ essentially determine $A$, while the constraint from $|V_{ub}|$ defines a circle in the $(\rho, \eta)$ plane and when errors are taken into account this constraint becomes an annulus. Considering $B\bar{B}$ mixing, since the mixing rates are dominated by virtual $t$ intermediate states (as seen in Sec. 1.2.2), $B\bar{B}$ measurements constrain

$$\Delta m_d \propto |V_{td}|^2 = A^2 \lambda^6 [(1 - \rho)^2 + \eta^2].$$

$CP$ violation in the $B\bar{B}$ system
For a given value of $\Delta m_d$, this constraint corresponds to a circle centered at the point $(\rho = 1, \eta = 0)$. Fig.

**Figure 1-6.** Individual constraints on the $(\rho, \eta)$ plane arising from measurement of $|V_{ub}|$, $B^0\overline{B}^0$ mixing (from $\Delta m_d$), CP violation in kaon decays ($\epsilon$) and $|V_{cb}|$ [23].

$X_u^{(U)}, |V_{td}|$ (from $B^0\overline{B}^0$ mixing) and $|\epsilon_K|$ (from CP violation in the neutral $K$ system).

Considering the unitarity triangle in Fig. 1-4, the three angles are denoted by $\alpha, \beta$ and $\gamma$ [22] and, using the ratio $z_2/z_1 = (k_2/k_1)e^{i(\theta_3 - \theta_1)}$, one can extract the expressions for these angles:

$$\frac{V_{ub}^*V_{ud}/|V_{ub}||V_{ud}|}{V_{tb}^*V_{td}/|V_{tb}||V_{td}|} = e^{i(\pi - \alpha)} = -e^{-i\alpha}$$

$$\frac{-V_{cb}^*V_{cd}/|V_{cb}||V_{cd}|}{V_{tb}^*V_{td}/|V_{tb}||V_{td}|} = e^{i\beta}$$

$$\frac{V_{ub}^*V_{ud}/|V_{ub}||V_{ud}|}{-V_{cb}^*V_{cd}/|V_{cb}||V_{cd}|} = e^{i\gamma}$$

and then:

$$\alpha \equiv \arg \left(-\frac{V_{tb}^*V_{td}}{V_{ub}^*V_{ud}}\right) \quad \beta \equiv \arg \left(-\frac{V_{cb}^*V_{cd}}{V_{tb}^*V_{td}}\right) \quad \gamma \equiv \arg \left(-\frac{V_{ub}^*V_{ud}}{V_{cb}^*V_{cd}}\right).$$

In reality, in the discussion of CP violation, one has to use the quantities $\Im m(z_2/z_1z_2) = \sin(2(\theta_1 - \theta_2))$ and so:

$$\Im m \left(\frac{V_{tb}^*V_{td}V_{ub}^*V_{ud}}{V_{tb}^*V_{td}V_{ub}^*V_{ud}}\right) = \sin 2\alpha,$$

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\[ \text{Im} \left( \frac{V_{td}^* V_{td} V_{tb} V_{tb}^*}{V_{td} V_{td}^* V_{td} V_{td}^*} \right) = \sin 2\beta, \]  

(1.54)

\[ \text{Im} \left( \frac{V_{ub}^* V_{ud} V_{cd} V_{cd}^*}{V_{ub} V_{ud}^* V_{cd} V_{cd}^*} \right) = \sin 2\gamma, \]

In the parameterization 1.47, the terms \( V_{ud}, V_{cd}, V_{cb}, \) and \( V_{tb} \) are chosen to be real and \( V_{cd} < 0 \) so that (from Eqs. 1.52):

\[ \frac{V_{td}}{V_{ub}} = - \frac{|V_{td}|}{|V_{ub}|} e^{i\alpha}, \quad V_{td} = |V_{td}| e^{-i\beta}, \quad V_{ub} = |V_{ub}| e^{i\gamma}, \]

and also:

\[ \sin 2\alpha = \text{Im} \left( \frac{V_{ub} V_{td}}{V_{ub}^* V_{td}^*} \right), \quad \sin 2\beta = \text{Im} \left( \frac{V_{td}^*}{V_{td}} \right), \quad \sin 2\gamma = \text{Im} \left( \frac{V_{ub}^*}{V_{ub}} \right). \]

The same relations can be written through the Wolfenstein parameters:

\[ \tan g\alpha = \frac{\eta}{\eta^2 - \rho(1 - \rho)}, \quad \tan g\beta = \frac{\eta}{1 - \rho}, \quad \tan g\gamma = \frac{\eta}{\rho}. \]

### 1.4.3 Measuring CKM Parameters with CP Conserving Processes

Six of the nine absolute values of the CKM elements are measured directly, basically from tree level processes. (All numbers below are taken from Review of Particle Physics [14].)

A high precision measurement of \( |V_{ud}| \) is obtained by comparing the rates for \( \beta \) nuclear decays to the muon decay rate: it has been obtained

\[ |V_{ud}| = 0.9740 \pm 0.0010. \]

Semileptonic kaon decays and strange barion (hyperon) decays give:

\[ |V_{us}| = 0.2205 \pm 0.0018. \]

Values for \( |V_{cd}| \) and \( |V_{cs}| \) can be determined with various techniques: the rate of charm production in neutrino interactions with valence quarks in nucleons gives

\[ |V_{cd}| = 0.224 \pm 0.016. \]

A similar way cannot be used for \( |V_{cs}| \) because of the lack of knowledge regarding the population of \( \Xi^- \) pairs in the nucleon “sea”. Alternatively, \( |V_{cs}| \) can be determined from the decay \( D \to K^+ \nu_e \), giving

\[ |V_{cs}| = 1.01 \pm 0.18. \]
Semileptonic exclusive and inclusive $B$ decays give:

$$\begin{align*}
|V_{cb}| &= 0.038 \pm 0.003, \\
|V_{ub}| &= 0.0033 \pm 0.0008.
\end{align*}$$

(1.55)\]

For both $|V_{cd}| \in |V_{cs}|$ another technique can be used: assuming the unitarity of the $CKM$ matrix, as required in the Standard Model, the unitarity of the third row and the third column implies (substituting the values in Eq. 1.55):

$$\sqrt{1 - |V_{td}|^2} = \sqrt{|V_{td}|^2 + |V_{ts}|^2} = \sqrt{|V_{ub}|^2 + |V_{cb}|^2} \approx 0.04$$

and therefore $|V_{td}|$ and $|V_{ts}|$ must be both less than about 0.04 that implies:

$$|V_{ud}|^2 + |V_{cd}|^2 = 1 - |V_{td}|^2 \approx 1 \quad |V_{us}|^2 + |V_{cs}|^2 = 1 - |V_{ts}|^2 \approx 1.$$  

(1.56)

In other words, assuming the unitarity, the known very small values of $|V_{cb}|$ and $|V_{ub}|$ imply that $|V_{cd}| \in |V_{cs}|$ are essentially determined from $|V_{ud}|$, $|V_{us}|$ and equations 1.56.

Information on $|V_{td}|$ and $|V_{ts}|$ can be extracted from $B^0 \Bar{B}^0$ and $B^0 \Bar{B}_s^0$ mixing and from $B \to X_s \gamma$ processes: from the $B^0 \Bar{B}^0$ mixing rate one extracts these values:

$$\Delta m_{B_d} = 0.472 \pm 0.018 \text{ ps}^{-1} \quad x_d = \frac{\Delta m_{B_d}}{\Gamma_{B_d}} = 0.723 \pm 0.032,$$

where $\Delta m_{B_d} \approx 2|M_{12}|$ (see Eq. 1.6 and Sec. 1.2.5), and thus the condition:

$$|V_{td}^* V_{td}| = 0.0084 \pm 0.0018$$

which shows that $|V_{td}|$ is of the same order as $|V_{ub}|$.

Using unitarity constraints, one can narrow some of the above ranges and put constraints on the top mixing $|V_{tb}|$. The full information on the absolute values of the $CKM$ elements (as given by [14]) from both direct measurements and three generation unitarity is summarized by

$$|V| = \begin{pmatrix}
0.9745 - 0.9760 & 0.217 - 0.224 & 0.0018 - 0.0045 \\
0.217 - 0.224 & 0.9737 - 0.9753 & 0.036 - 0.042 \\
0.004 - 0.013 & 0.035 - 0.042 & 0.9991 - 0.9994
\end{pmatrix}.$$  

(1.56)

The only large uncertainties are in $|V_{ub}|$ and $|V_{td}|$: however, the two are related through Eq. 1.50. The measured ranges for the $V_{ij}$'s give the following 90% CL range for the $CP$-violating measure $|J|$:

$$|J| = (3.0 \pm 1.3) \times 10^{-5} \sin \delta.$$  

(1.57)
1.5 Determination of $\alpha$

The angle $\alpha$ can be extracted from decays $b \to d\bar{u}\bar{a}$, for example decays with two pions in the final state. The measurement of $\alpha$ is expected to be complicated if the penguin contribution is not negligible.

1.5.1 CP Violation using $B^0$ decays into non CP eigenstates and Extraction of $\alpha$ ignoring penguins

The angle $\alpha$ can be obtained by the measurements of CP-violating asymmetries in decays to final states that can be either CP eigenstates or not. In case of a CP eigenstate, Sec. 1.3 shows the relation between the asymmetry and the CKM elements. If a single weak amplitude contributes to the decay taken into account and if the penguins are negligible, then $|A_{f_{CP}}/A_{f_{CP}}| = 1$ and so:

$$a_{f_{CP}} = -\text{Im} \lambda_{f_{CP}} \sin(\Delta m_B t). \tag{1.58}$$

Then, Im$\lambda$ is clearly related to one of the angle of the unitary triangle. In the particular case of $B \to \pi\pi$, Im$\lambda_{\pi\pi} = \sin 2\alpha$.

In case the final state is not a CP eigenstate, four separate amplitudes can be defined [30]:

$$A(B^0 \to f) \equiv A_f = |A_f| e^{i\theta_f} \quad A(\bar{B}^0 \to f) \equiv \bar{A}_f = |\bar{A}_f| e^{i\bar{\theta}_f}$$

$$A(B^0 \to \bar{f}) \equiv A_f = |A_f| e^{i\theta_f} \quad A(\bar{B}^0 \to \bar{f}) \equiv \bar{A}_f = |\bar{A}_f| e^{i\bar{\theta}_f} \tag{1.59}$$

Using the expressions in 1.24 and 1.25 for the physical states $B^0_{\text{phys}}(t)$ and $\bar{B}^0_{\text{phys}}(t)$ together with the expressions of the coefficients $g_+(t)$ and $g_-(t)$, one can evaluate the amplitudes:

$$\langle f | H | B^0_{\text{phys}}(t) \rangle = e^{-\frac{t}{2}} e^{-i\lambda M t} \left[ e^{\Delta m_B t} \langle f | H | B^0 \rangle + i \sin \frac{\Delta m_B t}{2} e^{2i\phi_M} \langle f | H | \bar{B}^0 \rangle \right]$$

$$\langle f | H | \bar{B}^0_{\text{phys}}(t) \rangle = e^{-\frac{t}{2}} e^{-i\lambda M t} \left[ \cos \frac{\Delta m_B t}{2} \langle f | H | B^0 \rangle + i \sin \frac{\Delta m_B t}{2} e^{-2i\phi_M} \langle f | H | B^0 \rangle \right]$$

where $\phi_M$ is the phase of $B^0 - \bar{B}^0$ mixing coming from $q/p = e^{2i\phi_M} e^{2i\theta_{CP}}(B^0)$. From these time-dependent amplitude and substituting Eq. 1.59 one can obtain the rates for $B^0_{\text{phys}}(t)$ and $\bar{B}^0_{\text{phys}}(t)$ decaying into $f$:

$$\Gamma(B^0_{\text{phys}}(t) \to f) = e^{-\Gamma t} A^2 \times \left\{ 1 + R \cos(\Delta m_B t) \right\}$$

$$\Gamma(\bar{B}^0_{\text{phys}}(t) \to f) = e^{-\Gamma t} A^2 \times \left\{ 1 - R \cos(\Delta m_B t) \right\} \tag{1.60}$$

where

$$A^2 = \frac{1}{2} \left( |A_f|^2 + |\bar{A}_f|^2 \right) \quad R = \frac{|A_f|^2 - |\bar{A}_f|^2}{|A_f|^2 + |\bar{A}_f|^2} \quad D = 2 \frac{|A_f||\bar{A}_f|}{|A_f|^2 + |\bar{A}_f|^2} \tag{1.61}$$

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At the same way, for $B^0_{\text{phys}}(t)$ and $\bar{B}^0_{\text{phys}}(t)$ states decaying into $\bar{f}$:

$$
\Gamma(B^0_{\text{phys}}(t) \to \bar{f}) = e^{-\Gamma t} \mathcal{A}^2 \times \left\{ \begin{array}{l}
1 - R \cos(\Delta m_B t) - i \mathcal{D} \sin(2\phi_M + \theta_f - \theta_f) \sin(\Delta m_B t) \\
1 + R \cos(\Delta m_B t) + i \mathcal{D} \sin(2\phi_M + \theta_f - \theta_f) \sin(\Delta m_B t) \end{array} \right. 
$$

where

$$
\mathcal{A}^2 = 1 \left( |A_f|^2 + |\bar{A}_{\bar{f}}|^2 \right), \quad R = \frac{|A_f|^2 - |\bar{A}_{\bar{f}}|^2}{|A_f|^2 + |\bar{A}_{\bar{f}}|^2}, \quad \mathcal{D} = \frac{2|A_f||\bar{A}_{\bar{f}}|}{|A_f|^2 + |\bar{A}_{\bar{f}}|^2}.
$$

$CP$ invariance requires that $CP$ conjugated processes should have identical rates, this leading to these conditions:

$$
|A_f| = |\bar{A}_{\bar{f}}|, \quad |\bar{A}_{\bar{f}}| = |A_f|,
$$

(1.64)

$$
\sin(2\phi_M - \theta_f + \bar{\theta}_{\bar{f}}) = \sin(2\phi_M + \bar{\theta}_{\bar{f}} - \theta_f).
$$

(1.65)

$CP$ violation arises if any of these equalities is not satisfied.

Eqs. 1.63 are completely general. Assuming that one single amplitude contributes to the decay taken into account and that penguins can be neglected, we can express the phases of the amplitudes on the basis of their $CP^3$ relationships as:

$$
\theta_f = \phi_{D_f} + \delta, \quad \bar{\theta}_{\bar{f}} = -\phi_{D_{\bar{f}}} + \delta
$$

(1.66)

$$
\theta_{\bar{f}} = \phi_{D_{\bar{f}}} + \delta', \quad \bar{\theta}_f = -\phi_{D_f} + \delta'.
$$

where $\phi_{D_f}$ and $\phi_{D_{\bar{f}}}$ represent the weak phases, while $\delta$ and $\delta'$ are the strong phases. Substituting in Eq. 1.65, one gets:

$$
\sin(2\phi_M - \theta_f + \bar{\theta}_{\bar{f}}) = \sin(2\phi_M - \phi_{D_f} - \phi_{D_{\bar{f}}} - \Delta \delta),
$$

$$
\sin(2\phi_M + \bar{\theta}_{\bar{f}} - \theta_f) = \sin(2\phi_M - \phi_{D_{\bar{f}}} - \phi_{D_f} + \Delta \delta),
$$

(1.67)

where $\Delta \delta \equiv \delta - \delta'$.

The $CP$-violating weak phase is given by $2\Phi \equiv 2\phi_M - \phi_{D_f} - \phi_{D_{\bar{f}}}$. From measurements of the time-dependent decay distributions one can obtain the quantities:

$$
S \equiv \sin(2\Phi + \Delta \delta), \quad \bar{S} \equiv \sin(2\Phi - \Delta \delta)
$$

and from these one can extract $\sin^2 2\Phi$:

$$
\sin^2 2\Phi = \frac{1}{2} \left[ 1 + S\bar{S} \pm \sqrt{(1 - S^2)(1 - \bar{S})^2} \right].
$$

(1.68)

The two solutions correspond to $\sin^2 2\Phi$ and to $\cos^2 \Delta \delta$: this ambiguity can be removed analyzing other decays with final states which have the same weak phase $2\Phi$, but different strong phases.
Assuming negligible the penguins contributions, the previous method can be used to extract the angle $\alpha$ through measurements of decays of $B^0_{\text{phys}}(t)$ and $B^0_{\text{phys}}(t)$ to final states like $\rho^+\pi^-$ or $a_1^+\pi^-$. When penguins are not negligible, this method measures a quantity, denoted $\kappa_{\text{eff}}$, which differs from the true $\alpha$ by the unknown amount $\kappa_f/2$. This quantity is mode-dependent because it depends on the ratio of tree-dominated to penguin-only contributions.

### 1.5.2 Extraction of $\alpha$ in the Presence of Penguins

In most of the decays modes, more than one amplitude is present and in the expression of the total amplitude, contributions from tree or penguin diagrams can be split: in the particular case of the channels of interest here, the weak phase difference between these terms is $\alpha$.

Going back to the case of a final state $f$ being a CP eigenstate, we can factorize the decay amplitudes:

$$
A(B \to f_{\text{CP}}) = A_{f_{\text{CP}}} = T e^{i\phi_T} e^{i\delta_T} + P e^{i\phi_P} e^{i\delta_P}
$$

$$
A(\overline{B} \to f_{\text{CP}}) = \overline{A}_{f_{\text{CP}}} = T e^{-i\phi_T} e^{i\delta_T} + P e^{-i\phi_P} e^{i\delta_P}
$$

(1.69)

where $T$, $\phi_T$ and $\delta_T$ ($P$, $\phi_P$ and $\delta_P$) are the magnitude, the weak phase and the strong phase of the tree-dominated (penguin-only) amplitude.

Thus, assuming the presence of penguin contributions, if $\phi_T \neq \phi_P$, $\lambda_{f_{\text{CP}}}$ (Eq. 1.22) becomes a function of tree and penguin diagram parameters and as a consequence it does not correspond to a clean measure of the CKM phase. The presence of non-negligible penguin contributions also leads to direct CP violation (see Sec. 1.5.3), that is $|\lambda_{f_{\text{CP}}}| \neq 1$. In the presence of direct CP violation, the time-dependent CP asymmetry contains a $\cos(\Delta m_B t)$ term, the coefficient of which can also be measured. In case the strong phases are equal, $\delta_T = \delta_P$, then $\lambda_{f_{\text{CP}}}$ is a pure phase (i.e., $|\lambda_{f_{\text{CP}}}| = 1$) and so no direct CP violation is present. However, like in the previous case, this phase depends on both tree and penguin parameters, so that there is still a shift in $\alpha$ due to penguin contributions, even though there is no direct CP violation.

The method to separate the tree and penguin contributions is isospin analysis. Isospin amplitudes $K_{I,I_f}$ can be labeled by the $\Delta I$ value of the $b$-quark decay and by the $I_f$ of the final state, which includes the spectator quark. A gluon is pure $I = 0$, so that the dominant gluonic $b \to d$ penguin diagrams are pure $\Delta I = \frac{1}{2}$. On the other hand, the tree-level $b \to u\bar{c}d$ decays have both $\Delta I = \frac{3}{2}$ and $\Delta I = \frac{1}{2}$ components. If the $\Delta I = \frac{1}{2}$ part can be isolated, then the tree contribution, which contains the weak phase to be measured, can be isolated. The inclusion of the spectator quark gives the final isospin value of $0$ or $1$ for the gluonic penguin contributions, but $0$, $1$ or $2$ for the tree contributions. The same arguments apply to $b \to s$ penguins and $b \to u\bar{c}s$ tree amplitudes.

Table 1-2 lists the isospin amplitudes for all relevant channels for these states. Note that, in all cases, there is at least one isospin amplitude which can be reached only via tree diagrams: $A_{h/2,2}$ for $B \to \pi\pi$ and $A_{1,3/2}$ for $B \to \pi K$. Isolation of such isospin amplitudes allows the removal of penguin pollution.
Hadronic decays resulting from $b \to u$ transitions are highly suppressed due to the factor $|V_{ub}|^2$. Decays such as $B \to \pi\pi$ and $B \to K\pi$ can proceed either through a $b \to u$ spectator diagram or through a gluonic penguin.

In a gluonic penguin, a $b \to d$ or $b \to s$ transition occurs through a virtual loop containing a $W$ and either a $t$, $c$ or $u$ quark with the radiation of a gluon. The dominant contribution is expected to arise from the $t$-quark intermediate state, but effects from the $c$-quark are not necessarily negligible [32, 34].

A simple argument gives a rough idea of the possible relative contribution of the tree and penguin amplitudes in $B \to \pi\pi$ and $B \to K\pi$. Assuming that $t$-quark dominates the loop, the penguin contribution to $\overline{B}^0 \to \pi^+\pi^-$ is suppressed relative to that for $\overline{B}^0 \to K^-\pi^+$ by $|V_{td}/V_{ts}|^2 = \mathcal{O}(\lambda^2)$ in the rate. Furthermore the tree contribution to $\overline{B}^0 \to K^-\pi^+$ is suppressed relative to that $\overline{B}^0 \to \pi^+\pi^-$ by a factor $|V_{us}/V_{ud}|^2 \simeq \lambda^2$ in the rate. By itself, all this tells us is that the tree diagram contributes more to $\pi\pi$ than to $K\pi$ and the penguin contributes more to $K\pi$ than to $\pi\pi$. Suppose that $\mathcal{B}(\overline{B}^0 \to \pi^+\pi^-) \approx \mathcal{B}(\overline{B}^0 \to K^-\pi^+)$, then $\overline{B}^0 \to K^-\pi^+$ must be mainly penguin or else the $\pi^+\pi^-/K^-\pi^+$ ratio would be larger. Even if all of the $\overline{B}^0 \to K^-\pi^+$ rate were penguin, the assumed near equality of the branching fractions implies that the penguin contribution to $\overline{B}^0 \to \pi^+\pi^-$ must be fairly small.

The tree process should contribute more to $\overline{B}^0 \to \pi^+\pi^-$ than to $\overline{B}^0 \to K^-\pi^+$ while the penguin process (with intermediate $t$ or $c$ quarks) should contribute more to $\overline{B}^0 \to K^-\pi^+$ than to $\overline{B}^0 \to \pi^+\pi^-$. Both the tree and penguin contributions to $\overline{B}^0 \to \pi^+\pi^-$ are $\mathcal{O}(\lambda^2)$. In a scenario where $\mathcal{B}(\overline{B}^0 \to \pi^+\pi^-) \approx \mathcal{B}(\overline{B}^0 \to K^-\pi^+)$, we would conclude that $\overline{B}^0 \to K^-\pi^+$ was predominantly penguin and that $\overline{B}^0 \to \pi^+\pi^-$ was predominantly a tree process. That is, the upper-left and lower-right diagrams must dominate. For, if the $\overline{B}^0 \to K^-\pi^+$ were mainly a tree process (upper-right diagram, $\mathcal{O}(\lambda^3)$), then the $\overline{B}^0 \to \pi^+\pi^-$ tree contribution (upper-left, $\mathcal{O}(\lambda^3)$) would be even larger and we would observed $\mathcal{B}(\overline{B}^0 \to \pi^+\pi^-) > \mathcal{B}(\overline{B}^0 \to K^-\pi^+)$. 

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Figure 1-7. Diagrams for $B^0 \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow K^-\pi^+$: both modes have contributions from tree $b \rightarrow u$ and penguin processes. The figure shows also the dependence of each amplitude on $\lambda = \sin \theta_C$.

$K^-\pi^+)$. Similarly, if $B^0 \rightarrow \pi^+\pi^-$ were mainly penguin (lower-left, $O(\lambda^3)$), then $B^0 \rightarrow K^-\pi^+$ penguin contribution (lower-right, $O(\lambda^2)$) would be even larger.

1.5.3.1 $B^0 \rightarrow \pi^+\pi^-$

From previous section, one can assume that penguins can be present in $B^0 \rightarrow \pi^+\pi^-$ channel. Since the weak phase of the penguin diagram is different from that of the tree diagram, penguin pollution can affect the clean extraction of $\alpha$ from this process. As already said, the isospin analysis can be used to eliminate the penguin pollution in this case [25]. Knowing the isospin decomposition of the amplitude from Table 1-2, we can define the single isospin components: $A^{+0} \equiv A(B^+ \rightarrow \pi^+\pi^0)$, $A^{+\pi^-} \equiv A(B^0 \rightarrow \pi^+\pi^-)$ and $A^{00} \equiv A(B^0 \rightarrow \pi^0\pi^0)$. Because of Bose statistics the $J = 0$ two-pion state produced in $B$ decay has no

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$I = 1$ contribution. Therefore the three two-pion decay amplitudes depend only on two isospin amplitudes:

$$\frac{1}{\sqrt{2}} A^{+\pm} + A^{00} = A^{+0} \quad \frac{1}{\sqrt{2}} \tilde{A}^{+\pm} + \tilde{A}^{00} = A^{-0}$$  \hspace{1cm} (1.70)$$

where the amplitudes for the $CP$-conjugate processes $B^+ \rightarrow \pi^+\pi^0$, $B^0 \rightarrow \pi^+\pi^-$ and $\bar{B}^0 \rightarrow \pi^0\pi^0$ are obtained from the $A$ amplitudes by simply changing the sign of the CKM phases; the strong phases remain the same. Expressions in 1.70 can be seen as two triangles, as drawn in Fig. 1-8.

To have $|A^{+0}|$ the measurement of $B^+ \rightarrow \pi^+\pi^0$ rate is needed, while $|A^{-0}|$ is taken from the $B^- \rightarrow \pi^-\pi^0$ rate. $|A^{+\pm}|$ and $|\tilde{A}^{+\pm}|$ come from the measurement of the time-dependent decay rates for $B^0(t) \rightarrow \pi^+\pi^-$ and $\bar{B}^0(t) \rightarrow \pi^+\pi^-$. This measurement also allows the extraction of the asymmetry and thus $\text{Im} \lambda_{+\pi^-}$. Finally $|A^{00}|$ and $|\tilde{A}^{00}|$ can be obtain from the time-independent rate of $B^0(t) \rightarrow \pi^0\pi^0$. The six magnitudes determine the shapes of two isospin triangles.

Since the penguin diagram is purely $\Delta I = 1/2$, the $A^{+0}$ amplitude takes contributions only from the tree diagram and so $|A^{+0}| = |\tilde{A}^{00}|$. This means that the two triangles have a base in common. However, due to the fact that both the tree and penguin diagrams contribute to the $I = 0$ final state, $|A^{+\pm}| \neq |\tilde{A}^{+\pm}|$ and $|A^{00}| \neq |\tilde{A}^{00}|$.

One can superimpose the triangles defined in Eqs. 1.70 by introducing

$$\tilde{A}^{ij} \equiv e^{2i\phi_T} A^{ij} \quad \text{i.e.} \quad \tilde{A}^{-0} \equiv e^{2i\phi_T} A^{-0}$$  \hspace{1cm} (1.71)$$

where $\phi_T$ is the CKM phase of the tree diagram. This way one gets $\tilde{A}^{-0} = A^{+0}$ and can draw Fig. 1-8. The angle $\kappa_{\pi\pi}$ between $A^{+\pm}$ and $\tilde{A}^{+\pm}$ can be determined up to a fourfold discrete ambiguity corresponding to the choice of orientation of each of the triangles with respect to the other. Another two-fold ambiguity comes from the fact that only the sine of the angle $2\alpha + \kappa_{\pi\pi}$ is measured: this leads to an eight-fold ambiguity in the value of $\alpha$. If a small error on each possible choice of $\kappa_{\pi\pi}$ can be obtained, isospin analysis will be able to significantly reduce the uncertainty in $\alpha$ extraction.

**Figure 1-8.** Isospin analysis of $B \rightarrow \pi\pi$ decays.
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Using the above notations, the $CP$ asymmetry in $B^0 \rightarrow \pi^+\pi^-$ can be expressed as

$$\text{Im} \lambda_{\pi^+\pi^-} = \text{Im} \left[ e^{-2\alpha} \frac{\hat{A}^{+-}}{A^{+-}} \right].$$

The ratio $\hat{A}^{+-}/A^{+-}$ is therefore the measure of the penguin pollution on the relationship between the angle $\alpha$ and the measured asymmetry. The two-triangle construction gives the magnitude and phase, $\kappa_{\pi^+\pi^-}$, of this quantity, so that $\alpha$ can in principle be extracted cleanly, even in the presence of penguins.

One of the main issues in this analysis is the rate for $B^0 \rightarrow \pi^0\pi^0$. Some theoretical predictions of this branching ratio are below or at the level of $O(10^{-6})$: a branching ratio at this order of magnitude would make its measurement very challenging [33, 34, 31]. These estimates assume that color suppression is significant in $B^0$ decays to light mesons and small penguins, but if large rescattering effects are also present in $B \rightarrow \pi\pi$, the branching ratio of $B^0 \rightarrow \pi^0\pi^0$ may be considerably larger than expectations and the isospin analysis could then yield accurate results.

Theoretically, a problem that can arise in the isospin method is the presence of electroweak penguins (EWP’s) [26]: the main EWP contributions to $B \rightarrow \pi\pi$ come from diagrams with virtual $Z$ exchange (see Fig. 1-9). The couplings of the $Z$ contain both $I = 1$ and $I = 0$ terms and thus these diagrams contribute also to $\Delta I = \frac{3}{2}$ so that their effects pollute the tree contributions. Though, the effects of these electroweak penguins are expected to be small in this channel. Both the $W$ and the $Z$ are color singlet particles and thus two contributions, a color-allowed and a color-suppressed one, have to be considered for each tree or electroweak-penguin diagram. If one includes both color-allowed and color-suppressed EWP’s, the $B \rightarrow \pi\pi$ amplitudes become [27]:

$$
\begin{align*}
A(B^+ \rightarrow \pi^+\pi^0) &= -\frac{1}{\sqrt{2}}(T + C + P_{EW} + P_{EW}^C) \\
A(B^0 \rightarrow \pi^+\pi^-) &= -(T + P + E + \frac{2}{3}P_{EW}^C) \\
A(B^0 \rightarrow \pi^0\pi^0) &= -\frac{1}{\sqrt{2}}(C - P - E + P_{EW}) + \frac{1}{3}P_{EW}^C
\end{align*}
$$

where $T$ is the color-allowed tree contribution, $C$ the color-suppressed tree contribution, $P$ the gluonic penguin, $E$ the color-suppressed tree contribution, $P_{EW}$ the color-allowed EWP contribution and $P_{EW}^C$ the color-suppressed EWP contribution. The relative orders of magnitude of all these contributions are expected to be:

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Isospin analysis of $B \to \pi \pi$ decays with the inclusion of electroweak penguins.

\[
|T| \to 1 \quad |C|, |P| \to \mathcal{O}(\bar{\lambda}) \quad |E|, |P_{EW}| \to \mathcal{O}(\bar{\lambda}^2) \quad |P_{EW}^C| \to \mathcal{O}(\bar{\lambda}^3)
\]  

where $\bar{\lambda} \sim 20\%$ and it is simply a size-counting factor. These newly defined amplitudes still form the triangles of Eq. 1.70, but there are now two amplitudes, with different weak phases, which contribute to $B^+ \to \pi^+ \pi^0$ and to its CP conjugate. This means that the two triangles no longer have a common base: Fig. 1-10 shows the new triangles. Thus the presence of EWP’s introduces another theoretical uncertainty $\Delta \theta_f$ in the extraction of the angle $\kappa_{\pi\pi}$ and then in the determination of $\alpha$ [27]:

\[
\Delta \alpha \approx \frac{1}{2} \Delta \theta_f \leq \left| \frac{P_{EW} + P_{EW}^C}{T + C} \right|.
\]  

However, this uncertainty is small (of the order $O(\bar{\lambda}) \sim 5\%$) and so taking into account the electroweak penguins does not significantly pollute the isospin analysis. Moreover this conclusion is largely independent of assumptions about the size of color suppression in $B \to \pi \pi$ decays, since if the $C$ (and $P_{EW}^C$) contributions turn out to be larger than expected, the uncertainty in $\alpha$ is still at most about 5%.

1.5.3.2 $B^0 \to \pi K$

An isospin analysis can be performed for $B \to \pi K$ decays as well as in the previous channels [28, 29]. By measuring the rates for $B^+ \to \pi^0 K^+, B^+ \to \pi^+ K^0$ and $B^0 \to \pi^- K^+$, along with the rates and $CP$ asymmetry in $B^0 \to \pi^0 K_S$, it is possible in principle to remove the penguin pollution and measure $\alpha$. This analysis though is strongly based on the assumption of negligible electroweak-penguin contributions. $B \to \pi K$ decays have contributions from both the $b \to u \pi s$ tree amplitude and the $b \to s$ penguin amplitude. For example, in case one considers the $b \to s$ penguin amplitude being comparable with the
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$b \to u\bar{m}d$ tree amplitude, the Cabibbo-suppressed $b \to u\bar{n} s$ tree amplitude results in being smaller by a factor of about 0.2 (the Cabibbo angle) than the $b \to s$ penguin amplitude. On the other hand, the $b \to s$ electroweak penguin is also suppressed by about 0.2 relative to the $b \to s$ gluonic penguin. Therefore, the $b \to s$ electroweak penguin and the $b \to u\bar{n}s$ tree amplitude could be comparable in magnitude. So the electroweak-penguin contributions to $B \to \pi K$ could be non-negligible and an isospin analysis would not be able to isolate the tree contribution [27].

In the case of $B^- \to \bar{K}^0 \pi^-$, the penguin diagram produces the appropriate set of quarks, $\bar{s}d\bar{n}n$, while the tree diagram produces $s\bar{u}u\bar{n}\bar{n}$: so in principle one could think that there can be absolutely no tree contribution, in which case this signal would represent an unambiguous observation of a gluonic penguin process. In reality, final-state interactions are likely to be small one can still consider $B^- \to \bar{K}^0 \pi^-$ channel a pure gluonic penguin process. Moreover, the amplitude for the tree diagram is $O(\lambda^4)$, while that for the penguin is $O(\lambda^2)$, so it seems even more unlikely that the tree contribution could be significant.

1.5.4 Direct CP Violation

As already said in Sec. 1.3.1, direct CP violation can occur in processes involving charged or neutral $B$’s. In general, though, it is difficult to convert experimental observation of an asymmetry in a specific channel into a quantitative determination of the basic parameters of the Standard Model.

We can observe CP-violating effects by comparing the amplitude $\Gamma(P \to f)$ with $\Gamma(\bar{P} \to \bar{f})$ only if there are both CP-violating and non-CP-violating phases: one could compare $\Gamma(\bar{B}^0 \to K^- \pi^+)$ with $\Gamma(B^0 \to K^+ \pi^-)$. These decays have both tree and penguin contributions which have different weak (and presumably different strong) phases. Unfortunately it is not possible at present to calculate the strong phases and the value of the weak phase would be ambiguous.

An example of what can be done with direct CP violation is the Fleischer-Mannel bound which is based on an analysis of branching fractions for the various charged and neutral $B \to K \pi$ modes. The decay $B^0 \to K^+ \pi^-$ has both penguin and tree contributions, while, as previously discussed, to a good approximation, $B^+ \to K^0 \pi^+$ is entirely a penguin process. Moreover, the penguin amplitudes for these processes should be essentially identical, since the corresponding decays differ only in the isospin of the spectator quark. One can therefore write the decay rates as

$$\Gamma(B^0 \to K^+ \pi^-) \propto |A_P - A_T e^{-i\gamma} e^{i\delta}|^2$$
$$\Gamma(\bar{B}^0 \to \bar{K}^- \pi^+) \propto |A_P - A_T e^{i\gamma} e^{i\delta}|^2$$
$$\Gamma(B^+ \to K^0 \pi^+) = \Gamma(B^- \to \bar{K}^0 \pi^-) \propto |A_P|^2$$

where $A_P$ and $A_T$ contain no CKM phases and $\delta$ is the relative strong phase shift between the penguin and tree processes. Computing the ratio:

$$\frac{\Gamma(B^0 \to K^+ \pi^-) + \Gamma(\bar{B}^0 \to \bar{K}^- \pi^+)}{2} \propto |A_P|^2 (1 - 2r \cos \gamma \cos \delta + r^2)$$

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where $r = A_T/A_P$, one can get:

$$R \equiv \frac{\Gamma(B_d \to K^\pm \pi^\mp)}{\Gamma(B^\pm \to K^{0}\pi^\pm)} = 1 - 2r \cos \gamma \cos \delta + r^2.$$  

The minimum value of $R$ as a function of $r$ is obtained when $r = \cos \gamma \cos \delta$ and is given by

$$R \geq 1 - \cos^2 \gamma \cos^2 \delta \geq \sin^2 \gamma$$

Assuming that $\sin^2 \gamma < 1$ we have a constrain in the $\rho - \eta$ plane:

$$\sin^2 \gamma = \frac{\eta^2}{\rho^2 + \eta^2} \leq R$$

which can be used to obtain a bound on the CKM angle $\gamma$.

### 1.5.5 $B$ Decays into $K^0 \bar{K}^0$

Although the decay rate for $B^0 \to K^0 \bar{K}^0$ is expected to be small ($\sim 10^{-7}$) in the Standard Model, final state rescattering effects can lead to enhancement of the branching fraction and the possibility of large strong phases, with correspondingly large $CP$-violating charge asymmetries [35, 36]. Such rescattering effects may also have consequences for constraints on $\gamma$ derived from $B \to K \pi$ decays [37]. Observation of the $K^0 \bar{K}^0$ decay mode would provide important information about the strength of final state rescattering in charmless $B$ decays.

Recent studies [38] have also suggested that it is possible to derive a bound on $|2\alpha - 2\alpha_{\text{eff}}|$ depending on the $K^0 \bar{K}^0$ branching fraction. This channel is a pure $b \to d$ penguin process and in the SU(3) limit, this penguin diagram can be related to the $\pi^+\pi^-$ penguin. Considering the expression 1.73 for the $\pi^+\pi^-$ amplitude and defining $|P| = |P_{K^0 \bar{K}^0}|$ from the [38]:

$$|P|^2 = \frac{B_{\pi^+\pi^-}}{1 - \cos 2\alpha} \left[ 1 - \sqrt{1 - C^2} \cos(2\alpha - 2\alpha_{\text{eff}}) \right]$$

one get the bound:

$$|2\alpha - 2\alpha_{\text{eff}}| \leq \arccos \left[ \frac{1}{\sqrt{1 - C^2}} \left( 1 - 2\frac{B_{K^0 \bar{K}^0}}{B_{\pi^+\pi^-}} \right) \right]$$

where $C$ is related to the time-dependent asymmetry and it is defined in Eqs. 7.2. This bound holds under in the assumptions of SU(3) flavour symmetry of the strong interactions and neglecting the electroweak penguin contributions.

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2.1 Physics at $e^- e^+$ B Factory operating at the $\Upsilon(4S)$ resonance

2.1.1 $CP$ asymmetry experimental measure

The primary goal of the BaBar experiment is the study of $CP$-violating asymmetries in the decay of neutral $B$ meson. Secondary goals are precision measurement of decays of bottom and charm mesons and of $\tau$ leptons, searches for rare processes accessible because of the high luminosity of PEP II B factory.

Through equation (1.44), in the previous chapter, a $CP$ asymmetry measurement can be related to a measurement of $\Delta t$, the time interval between the $B$ decays: $\Delta t = t_{CP} - t_{tag}$. In case the $B$ momenta are known, $\Delta t$ can be measured by the decay point distance $\Delta z$. In a symmetric $e^+ e^-$ collider operating at the $\Upsilon(4S)$ resonance, the two $B$ mesons are created almost at rest and the decay point distance can be calculated taking the small phase space left once $B$ masses have been subtracted from the $\Upsilon(4S)$ mass

$$E_{\Upsilon(4S)} = 10,580 \text{ GeV} \rightarrow E_B = \frac{E_{\Upsilon(4S)}}{2} = 5,290 \text{ GeV}.$$ 

From the total energy of each $B$, the kinetic energy is

$$\sqrt{E_B^2 - m_B^2} = p = m_B \beta \gamma = 0.341 \text{ GeV}$$

getting:

$$\beta \gamma = 0.065 \sim \beta$$

where it has been considered that $\gamma$ is almost 1 since the system is non-relativistic. The $B$ decay length is then
\[ \lambda_B = \beta \tau c = 0.065 \cdot 468 \mu m \approx 30.42 \mu m \]

which is a quite small value with respect to a typical vertex detector resolution (\(\sim 50 \mu m\)).

**Figure 2-1.**  Applied boost in the B\(\bar{B}\)AR laboratory system.

If a boost is applied along the \(z\) axis, it results in a larger value of \(\beta \gamma\) so that the average \(B\) meson decay distance \(\Delta z\) is increased to values within the detector resolution. In order to produce a boost, PEP-II has two rings, one for 9 GeV electrons and one for 3.1 GeV positrons: therefore in the laboratory frame, the \(\Upsilon(4S)\) resonance has a non-zero momentum \(z\) component

\[ (\beta \gamma)_{\Upsilon(4S)} = \left( \frac{E_{ta}^2 - m_{\Upsilon(4S)}^2}{m_{\Upsilon(4S)}^2} \right) = 0.56. \]

The \(B^0\bar{B}^0\) system moves in the boost direction and considering the time expansion effect, the decay vertex distance increases up to \(\beta \gamma c \tau \approx 0.56 \cdot 468 \mu m \approx 260 \mu m\), a value the B\(\bar{B}\)AR detector can measure with good resolution.

The \(z\) component of a \(B\) decay position is related to the meson decay time: in the center of mass frame (CM), the relation is

\[ z_{CM} = \pm \beta_{CM} c t_{CM} \cos \theta_{CM} \]

where \(\theta_{CM}\) is the angle between the \(B\) decay direction and the \(z\) axis direction and where \(\hat{t}_{CM}\) is the meson velocity.

In the laboratory frame, the \(z\) component of the decay position becomes

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\[ z_{\text{lab}} = \gamma z_{\text{CM}} + \beta \gamma c t_{\text{CM}} \]

from which the decay vertex distance along \( z \) can be calculated

\[ \Delta z = z_{\text{lab}}^{(2)} - z_{\text{lab}}^{(1)} = \gamma \left[ \beta_{\text{CM}} c \left( t_{\text{CM}}^{(2)} + t_{\text{CM}}^{(1)} \right) \cos \theta_{\text{CM}} \right] + \beta \gamma c \left( t_{\text{CM}}^{(2)} - t_{\text{CM}}^{(1)} \right) \] (2.1)

Defining \( \Delta t = \left( t_{\text{CM}}^{(2)} - t_{\text{CM}}^{(1)} \right) \), equation (2.1) becomes

\[ \Delta z = \gamma \beta_{\text{CM}} c \left( 2t_{\text{CM}}^{(1)} + \Delta t \right) \cos \theta_{\text{CM}} + \beta \gamma c \Delta t \] (2.2)

where \( \Delta t \) is the quantity to be measured. This expression can be simplified considering that, in the \( \Upsilon(4S) \) rest frame, the \( B \) mesons have \( \beta_{\text{CM}} \sim 0.065 \) so that the first addend in (2.2) is small and can be neglected: it can be written

\[ \Delta z \simeq \beta \gamma c \Delta t \]

from which one has

\[ \Delta t = \frac{\Delta z}{\beta \gamma c} \]

This is a linear dependence on \( \Delta t \), the quantity that has to be measured, from \( \Delta z \) which is the quantity that can be measured.

### 2.1.2 PEP-II.

PEP II is an \( e^+e^- \) asymmetric machine running at a center of mass energy of 10.58 GeV corresponding to the mass of the \( \Upsilon(4S) \) resonance. The electron beam (in the High Energy Ring HER) has 9.0 GeV and the positron beam (in the Low Energy Ring LER) has 3.1 GeV. Some PEP-II parameters are shown in Tab. 2-1. PEP-II has surpassed its design goals both in term of instantaneous and integrated daily luminosity, with significantly fewer bunches than anticipated [39].

While most of the data are recorded at the peak of the \( \Upsilon(4S) \) resonance, about 12\% are taken at a center of mass energy 40 MeV lower to allow for studies of non-resonant background.

PEP-II measures radiative Bhabha scattering to provide a fast monitor of the relative luminosity for operations. BaBar derives the absolute luminosity offline from other QED processes, mainly \( e^+e^- \) and \( \mu^+\mu^- \) pairs: the systematic uncertainty on the absolute value of the luminosity is estimated to be about 1.5\%. This error is dominated by uncertainties in the Monte Carlo generator and the simulation of the detector.
The beam energies of the two beams are calculated from the total magnetic bending strength and the average deviations of the accelerating frequencies from their central values. The systematic error on the PEP-II calculation of the absolute beam energies is estimated to be $\pm 10$ MeV, while the relative energy setting for each beam is accurate and stable to about $\pm 1$ MeV.

Cross sections for the production of fermion pairs at the $\Upsilon(4S)$ mass energy (which is $\sqrt{s} = M_{\Upsilon(4S)} = 10.58$ GeV) are shown in table 2-2.

<table>
<thead>
<tr>
<th>e$^+$e$^-$ $\rightarrow$</th>
<th>cross sections (nb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b}$ ($\sigma_{bb}$)</td>
<td>1.05</td>
</tr>
<tr>
<td>$c\bar{c}$ ($\sigma_{cc}$)</td>
<td>1.30</td>
</tr>
<tr>
<td>$s\bar{s}$ ($\sigma_{ss}$)</td>
<td>0.35</td>
</tr>
<tr>
<td>$u\bar{u}$ ($\sigma_{uu}$)</td>
<td>1.39</td>
</tr>
<tr>
<td>$d\bar{d}$ ($\sigma_{dd}$)</td>
<td>0.35</td>
</tr>
<tr>
<td>$\tau^+\tau^-$ ($\sigma_{\tau\tau}$)</td>
<td>0.94</td>
</tr>
<tr>
<td>$\mu^+\mu^-$ ($\sigma_{\mu\mu}$)</td>
<td>1.16</td>
</tr>
<tr>
<td>$e^+e^-$ ($\sigma_{ee}$)</td>
<td>$\sim 40$</td>
</tr>
</tbody>
</table>

| Table 2-2. Cross sections $\sigma$ for the production of fermion pairs at the $\Upsilon(4S)$ mass energy. |

BaBar has accumulated 23.76 fb$^{-1}$ in year 2000 and 32.72 fb$^{-1}$ in 2001 until November: Fig. 2-2 shows the integrated luminosity including 1999, 2000 and 2001 periods and the BaBar efficiency in the same periods.
2.2 The BaBar detector.

The BaBar detector has been optimized to reach the primary goal of the $CP$ asymmetry measurement. This measurement needs the complete reconstruction of a $B$ decay in a $CP$ eigenstate (possibly with good efficiency since the branching fraction is so small), the flavour identification (tagging) of the non-$CP$ $B$ and a measure of the distance of the two decay vertices. To fulfill these needs, the detector is provided with a magnetic field to measure charged particles momenta, it is able to reconstruct tracks coming from the production vertex, to recognize leptons and $\pi$ and $K$ mesons and to measure photon energy and direction.

The BaBar detector is shown in figure 2-3 and it includes the following subsystems:

- a silicon vertex detector: SVT (Silicon Vertex Tracker);
- a drift chamber: DCH;
- a particle identification system: DIRC (Detector of Internally Reflected Čerenkov light);
- an electromagnetic calorimeter: EMC;
- a muon and neutral hadron identification system: IFR (Instrumented Flux Return).

---

**Figure 2-2.** Left plot: integrated luminosity plot including 1999, 2000 and 2001 periods: the red line represents the total BaBar recorded luminosity. The green line shows the off peak luminosity taken. Right plot: BaBar efficiency.
The $CP$ decay modes of interest generally have $BR$ below $10^{-4}$ and reconstructing them requires observing anywhere two to six charged particles: in order to get a good efficiency, the B$ar{B}$AR detector has to cover as much solid angle as possible. As PEP-II is an asymmetric collider, particular care must be taken to cover the forward region: the applied boost implies that, on average, half the produced particles are in the region with $\cos \theta_{lab} > 0.5$. The accelerator bending magnets limit the maximum acceptance to $17.2^\circ$, in both forward and backward directions, but to allow the maximum forward coverage, machine components such as cooling systems etc, are located in the backward region. The active parts of the silicon vertex tracker cover the polar angle between $20.1^\circ$ and $150.2^\circ$ in the laboratory frame. This region in the lab frame corresponds to $-0.95 < \cos \theta_{CM} < 0.84$ where $\theta_{CM}$ is the polar angle in the center-of-mass frame.

Another important parameter is the minimum measurable momentum value for both charged and neutral particles: taking into consideration that charged pions have minimum momentum values of about 130 MeV/c and that tagging kaons have minimum momentum values of about 0.3 GeV/c, the tracking system has a minimum acceptance value of 60 MeV/c for momenta. In case of photons, the energy spectra have shown that the minimum measurable energy value must be $\sim 20$ MeV.
The detector geometry is cylindrical in the inner zone and hexagonal in the outermost zone: the central part of the structure is called barrel and it’s closed forward and backward by end caps. The BaBar coordinate system has the $z$ axis along the boost direction (or the beam direction): the $y$ axis is vertical and the $x$ axis is horizontal and goes towards the external part of the ring.

### 2.2.1 The Silicon Vertex Tracker: SVT.

The charged particle tracking system is based on the vertex detector and the drift chamber: the main purpose of this charged particle tracking system is the efficient detection of charged particles and the measurement of their momentum and angles with high precision. These track measurements are important for the extrapolation to the DIRC, the EMC and the IFR: at lower momenta, the DCH measurements are more important while at higher momenta the SVT dominates. The vertex detector is the only tracker within a radius of 20 cm from the primary interaction region: it is placed inside the support tube of the beam magnets and consists of five layers to provide five measurements of the positions of all charged particles with polar angles in the region $20.1^\circ < \theta < 150^\circ$. Because of the presence of a 1.5 $T$ magnetic field, the charged particle tracks with transverse momenta lower than $\sim 100$ MeV/c cannot reach the drift chamber active volume. So the SVT has to provide stand-alone tracking for particles with transverse momentum less than 120 MeV/c, the minimum that can be measured reliably in the DCH alone: this feature is essential for the identification of slow pions from $D^0$—meson decays. Because of these, the SVT has to provide redundant measurements.

Beyond the stand-alone tracking capability, the SVT provides the best measurement of track angles which is required to achieve design resolution for the Čerenkov angle for high momentum tracks. The SVT is very close to the production vertex in order to provide a very precise measure of points on the charged particles trajectories on both longitudinal ($z$) and transverse directions. The longitudinal coordinate information is necessary to measure the decay vertex distance, while the transverse information allows a better separation between secondary vertices coming from decay cascades.

More precisely, the design of the SVT was carried out according to some important guidelines:

- The number of impact points of a single charged particle has to be greater than 3 to make a stand-alone tracking possible, and to provide an independent momentum measure.
- The first three layers are placed as close as possible to the impact point to achieve the best resolution on the $z$ position of the $B$ meson decay vertices.
- The two outer layers are close to each other, but comparatively far from the inner layers, to allow a good measurement of the track angles.
- The SVT must withstand 2 MRad of ionizing radiation: the expected radiation dose is 1 Rad/day in the horizontal plane immediately outside the beam pipe and 0.1 Rad/day on average.
- Since the vertex detector is inaccessible during normal detector operations, it has to be reliable and robust.
These guidelines have led to the choice of a SVT made of five layers of double-sided silicon strip sensors: the spatial resolution, for perpendicular tracks must be $10 - 15 \mu m$ in the three inner layers and about $40 \mu m$ in the two outer layers. The three inner layers perform the impact parameter measurement, while the outer layers are necessary for pattern recognition and low $p_t$ tracking. The silicon detectors are double-sided (contain active strips on both sides) because this technology reduces the thickness of the materials the particles have to cross, thus reducing the energy loss and multiple scattering probability compared to single-sided detectors. The sensors are organized in modules (see right drawing in fig. (2-4)). The SVT five layers contain 340 silicon strip detectors with AC-coupled silicon strips.

Each detector is 300 $\mu m$-thick but sides range from 41 mm to 71 mm and there are 6 different detector types. Each of the three inner layers has a hexagonal transverse cross-section and it is made up of 6 detector modules, arrayed azimuthally around the beam pipe, while the outer two layers consist of 16 and 18 detector modules, respectively. The inner detector modules are barrel-style structures, while the outer detector modules employ the novel arch structure in which the detectors are electrically connected across an angle. This arch design was chosen to minimize the amount of silicon required to cover the solid angle while increasing the solid angle for particles near the edges of acceptance: having incidence angles on the detector closer to 90 degrees at small dip angles insures a better resolution on impact points. One of the main features of the SVT design is the mounting of the readout electronics entirely outside the active detector volume.

The strips on the two sides of the rectangular detectors in the barrel regions are oriented parallel ($\phi$ strips) or perpendicular ($z$ strips) to the beam line: in other words, the inner sides of the detectors have strips oriented perpendicular to the beam direction to measure the $z$ coordinate ($z$-size), whereas the outer sides, with longitudinal strips, allow the $\phi$-coordinate measurement ($\phi$-side). In the forward and backward regions of the...
two outer layers, the angle between the strips on the two sides of the trapezoidal detectors is approximately 90° and the φ strips are tapered.

The inner modules are tilted in φ by 5°, allowing an overlap region between adjacent modules: this provides full azimuthal coverage and is convenient for alignment. The outer modules are not tilted, but are divided into sub-layers and placed at slightly different radii (see left drawing in fig. (2-4)).

The total silicon area in the SVT is 0.94 m² and the number of readout channels is about 150,000. The geometrical acceptance of SVT is 90% of the solid angle in the c.m. system and typically 80% are used in charged particle tracking.

The z-side strips are connected to the read-out electronics with flexible Upilex fanout circuits glued to the inner faces of half-modules: as a matter of fact, each module is divided into two electrically separated forward and backward half-modules. The fanout circuits consist of conductive traces on a thin flexible insulator (copper traces on Kapton): the traces are wire-bonded to the end of the strips.

In the two outer layers, in each module the number of z strips exceeds the number of read-out channels, so that a fraction of the strips is “ganged”, i.e., two strips are connected to the same read-out channel. The “ganging” is performed by the fanout circuits. The length of a z strip is about 50 μm (case of no ganging) or 100 μm (case of two strip connected): the ganging introduces an ambiguity on the z coordinate measurement, which must be resolved by the pattern recognition algorithms. The φ strips are daisy-chained between detectors, resulting in a total strip length of up to 26 cm. Also, for the φ-side, a short fanout extension is needed to connect the ends of the strips to the read-out electronics.

<table>
<thead>
<tr>
<th>Table 2-3. Parameters of the SVT layout: these characteristics are shown for each layer.</th>
</tr>
</thead>
<tbody>
<tr>
<td>radius (mm)</td>
</tr>
<tr>
<td>modules/layer</td>
</tr>
<tr>
<td>wafers/module</td>
</tr>
<tr>
<td>read-out pitch (μm)</td>
</tr>
</tbody>
</table>

The signals from the read-out strips are processed using a new technique, bringing in several advantages. After amplification and shaping, the signals are compared to a preset threshold and the time they exceed this threshold (time over threshold, or ToT) is measured. This time interval is related to the charge induced in the strip by the charged particle crossing it. Unlike the traditional peak-amplitude measurement in the shaper output, the ToT has the advantage of an approximately logarithmic relation of the time interval to the charge signal. This compresses the active dynamic range of the signal, ensuring a good sensitivity in the lower range. When a particle crosses a silicon detector a cluster of adjoining strips producing a signal...
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is formed. The good signal resolution in the lower range ensures a good determination of the tails of the
cluster thus improving the resolution on the impact point measurement.

The electronic noise measured is found to vary between 700 and 1500 electrons ENC (equivalent noise
charge), depending on the layer and the readout view: this can be compared to the typical energy deposition
for a minimum ionizing particle at normal incidence, which is equivalent to \( \sim 24000 \) electrons.

During normal running conditions, the average occupancy of the SVT in a time window of 1 \( \mu \)s is about 2%
for the inner layers, where it is dominated by machine backgrounds, and less than 1% for the outer layers,
where noise hits dominate.

The cluster reconstruction is based on a cluster finding algorithm: first the charge pulse height of a single
pulse is calculated form the ToT value and clusters are formed grouping adjacent strips with consistent times.
The position \( x \) of a cluster formed by \( n \) strips is evaluated with an algorithm called “head-to-tail” algorithm:

\[
x = \frac{(x_1 + x_n)}{2} + \frac{p (Q_n - Q_1)}{2 (Q_n + Q_1)}
\]

where \( x_i \) and \( Q_i \) are the position and the collected charge of i-th strip and \( p \) is the read-out pitch. This
formula always gives a cluster position within \( p/2 \) of the geometrical center of the cluster. The cluster pulse
height is simply the sum of the strip charges, while the cluster time is the average of the signal times.

The SVT efficiency can be calculated for each half-module by comparing the number of associated hits to
the number of tracks crossing the active area of the half-module. Excluding defective readout sections (9
over 208), the combined hardware and software efficiency is \( 97% \) (see fig. (2-5)).

The spatial resolution of SVT hits is calculated by measuring the distance (in the plane of the sensor)
between the track trajectory and the hit, using high-momentum tracks in two prong events: the uncertainty
due to the track trajectory is subtracted from the width of the residual distribution to obtain the hit resolution.
The track hit residuals are defined as the distance between track and hit, projected onto the wafer plane
and along either the \( \phi \) or \( z \) direction. The width of this residual distribution is then the SVT hit resolution. Fig.
(2-6) shows the SVT hit resolution for \( z \) and \( \phi \) side hits as a function of the track incident angle, for each of
the five layers: the measured resolutions are in very good agreement with the Monte Carlo expected ones.
Over the whole SVT, resolutions are raging from \( 10 - 15 \mu m \) (inner layers) to \( 30 - 40 \mu m \) (outer layers)
for normal tracks.

For low-momentum tracks (\( p_t < 120 \) MeV/c), the SVT provides the only particle identification information.
The measure of the ToT value enables to obtain the pulse height and hence the ionization \( dE/dx \): the
value of ToT are converted to pulse height using a look-up table computed from the pulse shapes. The
double-sided sensors provide up to ten measurements of \( dE/dx \) per track: with signals from at least four
sensors, a 60% truncated mean \( dE/dx \) is calculated. For MIPs, the resolution on the truncated mean \( dE/dx \)
is approximately 14%: a 2\( \sigma \) separation between kaons and pions can be achieved up to momentum of
500 MeV/c and between kaons and protons beyond 1 GeV/c.

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2.2.2 The drift chamber $DCH$.

The drift chamber is the second part of BaBar tracking system: its principal purpose is the efficient detection of charged particles and the measurement of their momenta and angles with high precision. The $DCH$ complements the measurements of the impact parameter and the directions of charged tracks provided by the $SVT$ near the impact point (IP). At lower momenta, the $DCH$ measurements dominate the errors on the extrapolation of charged tracks to the $DIRC$, $EMC$ and $IFR$. The reconstruction of decay and interaction vertices outside of the $SVT$ volume, for instance the $K^0_s$ decays, relies only on the $DCH$. For these reasons, the chamber should provide maximal solid angle coverage, good measurement of the transverse momenta and positions but also of the longitudinal positions of tracks with a resolution of $\sim 1\, mm$, efficient reconstruction of tracks at momenta as low as $100\, \text{MeV}/c$ and it has to minimally degrade the performance of the calorimeter and particle identification devices (the most external detectors). The $DCH$ also needs to supply information for the charged particle trigger. For low momentum particles, the $DCH$ is required to provide particle identification by measuring the ionization loss ($dE/dx$). A resolution of about $7\%$ will allow $\pi/K$ separation up to $700\, \text{MeV}/c$. This particle identification (PID) measurement is complementary to that of the $DIRC$ in the barrel region, while in the extreme backward and forward region, the $DCH$ is the only device providing some discrimination of particles of different mass. The $DCH$ should also be able to operate in presence of large beam-generated backgrounds having expected rates of about $5\, kHz/cell$ in the innermost layers.

To meet the above requirements, the $DCH$ is a $280\, cm$-long cylinder (see left plot in fig. (2-7)), with an inner radius of $23.6\, cm$ and an outer radius of $80.9\, cm$: it is bounded by the support tube at its inner radius.
Figure 2-6. SVT hit resolution in the $z$ and $\phi$ coordinate in microns, plotted as functions of the track incident angle in degrees. Each plot corresponds to a different layer of the SVT.

and the particle identification device at its outer radius. The flat end-plates are made of aluminum: since the B\Bar\Bar events will be boosted in the forward direction, the design of the detector is optimized to reduce the material in the forward end. The forward end-plate is made thinner (12 mm) in the acceptance region of the detector compared to the rear end-plate (24 mm), and all the electronics is mounted on the rear end-plate. The device is asymmetrically located with respect to the IP: the forward length of 174.9 cm is chosen so that particles emitted at polar angles of $\pm 18^\circ$ traverse at least half of the layers of the chamber before exiting through the front end-plate. In the backward direction, the length of 101.5 cm means that particles with polar angles down to $\pm 152^\circ$ traverse at least half of the layers.

The inner cylinder is made of 1 mm beryllium and the outer cylinder consists of two layers of carbon fiber on a Nomex core: the inner cylindrical wall is kept thin to facilitate the matching of SVT and DCH tracks, to improve the track resolution for high momentum tracks and to minimize the background from photon conversions and interactions. Material in the outer wall and in the forward direction is also minimized in order not to degrade the performance of the DIRC and the EMC.

The region between the two cylinders is filled up by a gas mixture consisting of Helium-isobutane (80% : 20%); the chosen mixture has a radiation length that is five times larger than commonly used argon-based gases. 40 layers of wires fill the DCH volume and form 7104 hexagonal cells with typical dimensions of $1.2 \times 1.9 cm^2$ along the radial and azimuthal directions, respectively (see right plot in fig. 2-7). The hexagonal cell configuration has been chosen because approximate circular symmetry can be achieved over a large portion of the cell. Each cell consist of one sense wire surrounded by six field wires: the sense wires are 20 $\mu m$ gold-plated tungsten-rhenium, the field wires are 120 $\mu m$ and 80 $\mu m$ gold-plated aluminum. By
2.2 The BaBar detector.

Figure 2-7. Side view of the BaBar drift chamber (the dimensions are in mm) and isochrones (i.e. contours of equal drift time of ions) in cells of layer 3 and 4 of an axial super-layer. The isochrones are spaced by 100 ns.

using the low-mass aluminum field wires and the helium-based gas mixture, the multiple scattering inside the DCH is reduced to a minimum, representing less than 0.2% $X_0$ of material. The total thickness of the DCH at normal incidence is 1.08% $X_0$.

The drift cells are arranged in 10 super-layers of 4 cylindrical layers each: the super-layers contain wires oriented in the same direction: to measure the $z$ coordinate, axial wire super-layers and super-layers with slightly rotated wires (stereo) are alternated. In the stereo super-layers a single wire corresponds to different $\phi$ angles and the $z$ coordinate is determined by comparing the $\phi$ measurements from axial wires and the measurements from rotated wires. The stereo angles vary between $\pm 45$ mrad and $\pm 76$ mrad.

While the field wires are at ground potential, a positive high voltage is applied to the sense wires: an avalanche gain of approximately $5 \times 10^4$ is obtained at a typical operating voltage of 1960 V and a 80:20 helium:isobutane gas mixture.

In each cell, the track reconstruction is obtained by the electron time of flight: the precise relation between the measured drift time and drift distance is determined from sample of $e^+e^-$ and $\mu^+\mu^-$ events. For each signal, the drift distance is estimated by computing the distance of closest approach between the track and the wire. To avoid bias, the fit does not include the hit of the wire under consideration. The estimated drift distances and the measured drift times are averaged over all wires in a layer.

The DCH expected position resolution is lower than 100 $\mu$m in the transverse plane, while it is about 1 mm in the $z$ direction. The minimum reconstruction and momentum measure threshold is about 100 MeV/c and it is limited by the DCH inner radius. The design resolution on the single hit is about 140 $\mu$m while the achieved weighted average resolution is about 125 $\mu$m. Left plot in fig. (2-8) shows the position resolution as a function of the drift distance, separately for the left and the right side of the sense wire. The resolution is taken from Gaussian fits to the distributions of residuals obtained from unbiased track fits: the results are based on multi-hadron events for data averaged over all cells in layer 18.

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The specific energy loss \( (dE/dx) \) for charged particles through the DCH is derived from the measurement of the total charge collected in each drift cell: the specific energy loss per track is computed as a truncated mean from the lowest 80\% of the individual \( dE/dx \) measurements. Various corrections are applied to remove sources of bias: these corrections include changes in gas pressure and temperature (±9\% in \( dE/dx \)), differences in cell geometry and charge collection (±8\%), signal saturation due to space charge buildup (±11\%), non-linearities in the most probable energy loss at large dip angles (±2.5\%) and variation of cell charge collection as a function of the entrance angle (±2.5\%).

Right plot in fig. (2-8) shows the distribution of the corrected \( dE/dx \) measurements as a function of track momenta: the superimposed Bethe-Bloch predictions have been determined from selected control samples of particles of different masses. The achieved \( dE/dx \) rms resolution for Bhabha events is typically 7.5\%, limited by the number of samples and Landau fluctuations, and it is close to the expected resolution of 7\%.

### 2.2.3 The charged particle tracking system.

As already said, the BaBar tracking system is based on SVT and DCH detectors: charged particle tracking has been studied with large samples of cosmic ray muons, \( e^+e^- \), \( \mu^+\mu^- \) and \( \tau^+\tau^- \) events, as well as multi-hadrons.

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2.2 The BBar detector.

 Charged tracks are defined by five parameters ($d_0$, $\phi_0$, $\omega$, $z_0$ and $\tan \lambda$) and their associated error matrix: these parameters are measured at the point of closest approach to the $z$-axis and $d_0$ and $z_0$ are the distances of this point from the origin of the coordinate system (in the $x-y$ plane and on the $z$ axis, respectively). The angle $\phi_0$ is the azimuth of the track, $\lambda$ is the dip angle relative to the transverse plane and $\omega$ is the curvature. $d_0$ and $\omega$ have signs that depend on the particle charge.

The track finding and the fitting procedure make use of the Kalman filter algorithm that takes into account the detailed description of material in the detector and the full map of the magnetic field. First of all, tracks are reconstructed with $DCH$ hits through a stand-alone $DCH$ algorithm: the resulting tracks are then extrapolated into the $SVT$ and $SVT$ track segments are added and a Kalman fit is performed to the full set of $DCH$ and $SVT$ hits. Any remaining $SVT$ hits are then passed to the $SVT$ stand-alone track finding algorithms. Finally, an attempt is made to combine tracks that are only found by one of the two tracking systems and thus recover tracks scattered in the material of the support tube.

The efficiency for track reconstruction in the $DCH$ has been measured as a function of transverse momentum, polar and azimuthal angles in multi-track events. These measurement rely on specific final states and exploit the fact that the track reconstruction can be performed independently in the $SVT$ and the $DCH$. The absolute $DCH$ tracking efficiency is determined as the ratio of the number of reconstructed $DCH$ tracks to the number of tracks detected in the $SVT$ with the requirement that they fall within the acceptance of the $DCH$. Left plot in fig. (2-9) shows the efficiency in the $DCH$ as a function of transverse momentum in multi-hadron events.

![Figure 2-9. Track reconstruction efficiency in the DCH at operating voltages of 1960 V and 1900 V as a function of transverse momentum (left plot) and of polar angle (right plot). The efficiency is measured in multi-hadron events.](image-url)
At design voltage of 1960 V, the efficiency averages 98 ± 1% per track above 200 MeV/c: the data recorded at 1900 V show a reduction in efficiency by about 5% for tracks almost at normal incidence, indicating that the cells are not fully efficient at this voltage (see right plot in fig. (2-9)).

**Figure 2-10.** Left plot: Monte Carlo studies of low momentum tracks in the SVT on $D^+ \rightarrow D^0\pi^+$ events. a) comparison with data in $B\bar{B}$ events and b) efficiency for slow pion detection derived from simulated events. Right plot: resolution in the parameters $d_0$ and $z_0$ for tracks in multi-hadron events as a function of the transverse momentum.

The stand-alone SVT tracking algorithms have a high efficiency for tracks with low transverse momentum: to estimate the tracking efficiency for these low momentum tracks, a detailed Monte Carlo study was performed. The pion spectrum was derived from simulation of the inclusive $D^*$ production in $B\bar{B}$ events and Monte Carlo events were selected in the same way as the data: since the agreement with MC is very good, the detection efficiency has been derived from MC simulation. The SVT extends the capability of the charge particle reconstruction down to transverse momenta of $\sim 50$ MeV/c (see left plot in fig. (2-10)).

The resolution in the five track parameters is monitored using $e^+e^-$ and $\mu^+\mu^-$ pair events: the resolution is derived from the difference of the measured parameters for the upper and lower halves of the cosmic ray tracks traversing the DCH and the SVT. On this sample with transverse momenta above 3 GeV/c, the resolution for single tracks is 23 $\mu$m in $d_0$ and 29 $\mu$m in $z_0$. To study the dependence of resolution from transverse momentum, a sample of multi-hadron events is used: the resolution is determined from the width of the distribution of the difference between the measured parameters ($d_0$ and $z_0$) and the coordinates of the vertex reconstructed from the remaining tracks in the event: right plot in fig. (2-10) shows the dependence of the resolution in $d_0$ and $z_0$ as a function of $p_T$. The measured resolutions are about 25 $\mu$m in $d_0$ and 40 $\mu$m in $z_0$ for $p_T$ of 3 GeV/c: these values are in good agreement with the Monte Carlo studies and in reasonable agreement also with the results from cosmic rays.
2.2.4 The Čerenkov-based detector *DIRC*.

The particle identification system is crucial for *BaBar* since the *CP* violation analysis requires the ability to fully reconstruct one of the *B* meson and to tag the flavour of the other *B* decay: the momenta of the kaons used for flavour tagging extend up to about 2 GeV/c with most of them below 1 GeV/c. On the other hand, pions and kaons from the rare two-body decays $B^0 \rightarrow \pi^+ \pi^-$ and $B^0 \rightarrow K^+ \pi^-$ must be well separated: they have momenta between 1.7 and 4.2 GeV/c with a strong momentum-polar angle correlation of the tracks (higher momenta occur at more forward angles because of the c.m. system boost). So the particle identification system should be:

- thin and uniform in term of radiation lengths to minimize degradation of the calorimeter energy resolution,
- small in the radial dimension to reduce the volume (cost) of the calorimeter,
- with fast signal response,
- able to tolerate high background.

*DIRC* stands for Detection of Internally Reflected Čerenkov light and it refers to a new kind of ring-imaging Čerenkov detector which meets the above requirements. The particle identification in the *DIRC* is based on the Čerenkov radiation produced by charged particles crossing a material with a speed higher than light speed in that material. The angular opening of the Čerenkov radiation cone depends on the particle speed:

$$\cos \theta_c = \frac{1}{n\beta}$$

where $\theta_c$ is the Čerenkov cone opening angle, $n$ is the refractive index of the material and $\beta$ is the particle velocity over $c$. The principle of the detection is based on the fact that the magnitudes of angles are maintained upon reflection from a flat surface.

Since particles are produced mainly forward in the detector because of the boost, the *DIRC* photon detector is placed at the backward end: the principal components of the *DIRC* are shown in fig. (2-11). The *DIRC* is placed in the barrel region and consists of 144 long, straight bars arranged in a 12-sided polygonal barrel. The bars are 1.7 cm-thick, 3.5 cm-wide and 4.90 m-long: they are placed into 12 hermetically sealed containers, called *bar boxes*, made of very thin aluminum-hexcel panels. Within a single bar box, 12 bars are optically isolated by a $\sim$ 150 $\mu$m air gap enforced by custom shims made from aluminum foil.

The radiator material used for the bars is synthetic fused silica: the bars serve both as radiators and as light pipes for the portion of the light trapped in the radiator by total internal reflection. Synthetic silica has been chosen because of its resistance to ionizing radiation, its long attenuation length, its large index of refraction, its low chromatic dispersion within its wavelength acceptance.

The Čerenkov radiation is produced within these bars and is brought, through successive total internal reflections, in the backward direction outside the tracking and magnetic volumes: only the backward end
of the bars is instrumented. A mirror placed at the other end on each bar reflects forward-going photons to the instrumented end. The Čerenkov angle at which a photon was produced is preserved in the propagation, modulo some discrete ambiguities (the forward-backward ambiguity can be resolved by the photon arrival-time measurement, for example). The DIRC efficiency grows together with the particle incidence angle because more light is produced and a larger fraction of this light is totally reflected. To maximize the total reflection, the material must have a refractive index (fused silica index is $n = 1.473$) higher than the surrounding environment (the DIRC is surrounded by air with index $n = 1.0002$).

Once photons arrive at the instrumented end, most of them emerge into a water-filled expansion region, called the Standoff Box: the purified water, whose refractive index matches reasonably well that of the bars ($n_{H_2O} = 1.346$), is used to minimize the total internal reflection at the bar-water interface.

The standoff box is made of stainless steel and consists of a cone, cylinder and 12 sectors of PMTs: it contains about 6000 liters of purify water. Each of the 12 PMTs sectors contains 896 PMTs in a close-packed array inside the water volume: the PMTs are linear focused 2.9 $cm$ diameter photo-multiplier tubes, lying on an approximately toroidal surface.

The DIRC occupies only 8 $cm$ of radial space, which allows for a relatively large radius for the drift chamber while keeping the volume of the CsI Calorimeter reasonably low: it corresponds to about 17% $X_0$ at normal incidence. The angular coverage is the 94% of the $\phi$ azimuthal angle and the 83% of $\cos \theta_{CM}$.

Čerenkov photons are detected in the visible and near-UV range by the PMT array. A small piece of fused silica with a trapezoidal profile glued at the back end of each bar allows for significant reduction in the area requiring instrumentation because it folds one half of the image onto the other half. The PMTs are operated directly in water and are equipped with light concentrators: the photo-multiplier tubes are about 1.2 $m$ away

Marcella Bona
from the end of the bars. This distance from the bar end to the PMTs, together with the size of the bars and PMTs, gives a geometric contribution to the single photon Čerenkov angle resolution of about $7_{\text{mrad}}$. This is a bit larger than the resolution contribution from Čerenkov light production (mostly a $5.4_{\text{mrad}}$ chromatic term) and transmission dispersions. The overall single photon resolution expected is about $9_{\text{mrad}}$.

The image from the Čerenkov photons on the sensitive part of the detector is a cone cross-section whose opening angle is the Čerenkov angle modulo the refraction effects on the fused silica-water surface. In the most general case, the image consists of two cone cross-sections out of phase one from the other by a value related to an angle which is twice the particle incidence angle. In order to associate the photon signals with a track traversing a bar, the vector pointing from the center of the bar end to the center of each PMT is taken as a measure of the photon propagation angles $\alpha_x$, $\alpha_y$ and $\alpha_z$. Since the track position and angles are known from the tracking system, the three $\alpha$ angles can be used to determine the two Čerenkov angles $\theta_C$ and $\phi_C$. In addition, the arrival time of the signal provides an independent measurement of the propagation of the photon and can be related to the propagation angles $\alpha$. This over-constraint on the angles and the signal timing are useful in dealing with ambiguities in the signal association and high background rates.

The expected number of photo-electrons ($N_{\text{pe}}$) is $\approx 28$ for a $\beta = 1$ particle entering normal to the surface at the center of a bar and increases by over a factor of of two in the forward and backward directions.

The time distribution of real Čerenkov photons from a single event is of the order of $50 \, \text{n.s}$ wide and during normal data taking they are accompanied by hundreds of random photons in a flat background distribution within the trigger acceptance window. The Čerenkov angle has to be determined in an ambiguity that can be up to 16-fold: the goal of the reconstruction program is to associate the correct track with the candidate PMT.
signal with the requirement that the transit time of the photon from its creation in the bar to its detection at
the PMT be consistent with the measurement error of about 1.5 ns.

An unbinned maximum likelihood formalism is used to take into account all information provided by the
\textit{DIRC}: the reconstruction routine provides a likelihood value for each of the five stable particle types \((e, \mu, \pi, K \text{ and } p)\) if the track passes through the active volume of the \textit{DIRC}. These likelihood probabilities
are calculated in an iterative process by maximizing the likelihood value for the entire event while testing
different hypotheses for each track. If enough photons are found, a fit of \(\theta_C\) and the number of observed
signal and background photons are calculated.

In the absence of correlated systematic errors, the resolution \(\sigma_{C,\text{track}}\) on the track Čerenkov angle should
scale as

\[
\sigma_{C,\text{track}} = \frac{\sigma_{C,\gamma}}{\sqrt{N_{pe}}}
\]

where \(\sigma_{C,\gamma}\) is the single photon angle resolution. This angular resolution (obtained from di-muon events) can
be estimated to be about 10.2 mrad, in good agreement with the expected value (see left plot in fig. 2-12).
The measured time resolution is 1.7 ns close to the intrinsic 1.5 ns time spread of the PMTs. In di-muon
event data, the number of photo-electrons varies between 20 for small polar angles at the center of the barrel
and 65 at large polar angles: this variation is well reproduced by Monte Carlo and can be understood by
the fact that the number of Čerenkov photons varies with the path length of the track in the radiator (smaller
path length at perpendicular incidence at the center of the barrel). Also the fraction of photons trapped by
total internal reflection rises with larger values of \(|\cos(\theta_{\text{track}})|\): this increase in the number of photons for
forward going tracks corresponds also to an increase in momentum of the tracks and thus an improvement
of the \textit{DIRC} performance.

The width of the track Čerenkov angle resolution for di-muon events is 2.4 mrad compared to the design
gold of 2.2 mrad (see right plot in fig. (2-12)). From the measured single track resolution versus momentum
in d-muon events and from the difference between the expected Čerenkov angles of charged pions and kaons,
the pion-kaon separation power of the \textit{DIRC} can be evaluated: the expected separation between pions and
kaons at 3 GeV/c is about 4.2\(\sigma\), within 15\% of the design goal.

Left plot in fig. (2-13) shows an example of use of PID from \textit{DIRC}: the \(K\pi\) invariant mass spectra are shown with and without the use of the \textit{DIRC} for kaon identification and the peak corresponds to the \(D^0\)
particle. Note how the \textit{DIRC} contribution can reduced the background level.

The efficiency for correct identifying a charged kaon passing through the radiator and the probability of
wrongly identifying a pion as a kaon are determined using \(D^0 \rightarrow K^-\pi^+\) decays selected kinematically
from inclusive \(D^+\) production: fig. (2-13) shows kaon identification efficiency and pion mis-identification
as functions of the track momentum. The mean kaon identification efficiency is 96.2 \(\pm 0.2\\%(\text{stat})\) and the
mean pion mis-identification is 2.1 \(\pm 0.1\\%(\text{stat})\).
2.2 The B\Bar{B}AR detector.

Figure 2-13. Left plot: $K\pi$ invariant mass spectrum with and without the use of the DIRC for kaon identification. Right plot: the selection efficiency and mis-identification for kinematically identified kaon tracks from the $(D^{+} \rightarrow D^{0}\pi^{+} , D^{0} \rightarrow K^{-}\pi^{+})$ sample are plotted as a function of track momentum.

2.2.5 The electromagnetic calorimeter $EMC$. 

The understanding of $CP$ violation in the $B$ meson system requires the reconstruction of final state containing a direct $\pi^0$ or that can be reconstructed through a decay chain containing one or more daughter $\rho$’s. The electromagnetic calorimeter is designed to measure electromagnetic showers with excellent efficiency and energy and angular resolution over the energy range from 20 MeV to 9 GeV. This capability should allow the detection of photons from $\pi^0$ and $\eta$ decays as well as from electromagnetic and radiative processes. By identifying electrons, the $EMC$ contributes to the flavour tagging of neutral $B$ mesons via semi-leptonic decays. The upper bound of the energy range is given by the need to measure QED processes like $e^+e^-\rightarrow e^+e^- (\gamma)$ and $e^+e^-\rightarrow \gamma\gamma$ for calibration and luminosity determination. The lower bound is set by the need for highly efficient reconstruction of $B$-meson decays containing multiple $\rho$’s and $\eta$’s. The measurement of very rare decays containing $\pi^0$’s in the final state (for example, $B^0 \rightarrow \pi^0\pi^0$) puts the most stringent requirements on energy resolution, expected to be of the order of $1-2\%$. Below 2 GeV energy, the $\rho$ mass resolution is dominated by the energy resolution, while at higher energies, the angular resolution becomes dominant and it is required to be of the order of few mrad. The $EMC$ is also used for electron identification and for completing the IFR output on $\mu$ and $K^0_L$ identification. It also has to operate in a 1.5 $T$ magnetic field.

The $EMC$ has been chosen to be composed of a finely segmented array of thallium-doped cesium iodide (CsI(Tl)) crystals. The crystals are read out with silicon photodiodes that are matched to the spectrum of scintillation light. The energy resolution of a homogeneous crystal calorimeter can be described empirically in terms of a sum of two terms added in quadrature:
where $E$ and $\sigma_E$ refer to the energy of a photon and its rms error, measured in GeV. The energy dependent term $a$ arises basically from the fluctuations in photon statistics, but also from the electronic noise of the photon detector and electronics and from the beam-generated background that leads to large numbers of additional photons. This first term dominates at low energy, while the constant term $b$ is dominant at higher energies (> 1 GeV). It derives from non-uniformity in light collection, leakage or absorption in the material in front of the crystals and uncertainties in the calibration.

The angular resolution is determined by the transverse crystal size and the distance from the interaction point: it can be empirically parameterized as a sum of an energy dependent and a constant term

$$\frac{\sigma_{\theta E}}{E} = \frac{a}{\sqrt{E \text{ (GeV)}}} \oplus b$$

where $E$ is measured in GeV. In CsI(Tl), the intrinsic efficiency for the detection of photons is close to 100% down to a few MeV, but the minimum measurable energy in colliding beam data is about 20 MeV for the EMC: this limit is determined by beam and event-related background and the amount of material in front of the calorimeter. Because of the sensitivity of the $\sigma_{\theta E}$ efficiency to the minimum detectable photon energy, it is extremely important to keep the amount of material in front of the EMC to the lowest possible level.

Thallium-doped CsI has high light yield and small Molière radius in order to allow for excellent energy and angular resolution. It is also characterized by a short radiation length for shower containment at BBar.
energies. The transverse size of the crystals is chosen to be comparable to the Molière radius achieving the required angular resolution at low energies while limiting the total number of crystals and readout channels.

The BaBar EMC consists of a cylindrical barrel and a conical forward end-cap: it had a full angle coverage in azimuth while in polar angle it extends from 15.8° to 141.8° corresponding to a solid angle coverage of 90% in the CM frame. Radially the barrel is located outside the particle ID system and within the magnet cryostat: the barrel has an inner radius of 92 cm and an outer radius of 137.5 cm and it’s located asymmetrically about the interaction point, extending 112.7 cm in the backward direction and 180.1 cm in the forward direction. The barrel contains 5760 crystals arranged in 48 rings with 120 identical crystals each: the end-cap holds 820 crystals arranged in eight rings, adding up to a total of 6580 crystals. They are truncated-pyramid CsI(Tl) crystals: they are tapered along their length with trapezoidal cross-sections with typical transverse dimensions of 4.7 × 4.7 cm² at the front face, flaring out towards the back to about 6.1 cm². All crystals in the backward half of the barrel have a length of 29.6 cm: towards the forward end of the barrel, crystal lengths increase up to 32.4 cm in order to limit the effects of shower leakage from increasingly higher energy particles. All end-cap crystals are of 32.4 cm length. The barrel and end-cap have total crystal volumes of 5.2 m³ and 0.7 m³, respectively. The CsI(Tl) scintillation light spectrum has a peak emission at 560 nm: two independent photodiodes view this scintillation light from each crystal. The readout package consists of two silicon PIN diodes, closely coupled to the crystal and to two low-noise, charge-sensitive preamplifiers, all enclosed in a metallic housing.

A typical electromagnetic shower spreads over many adjacent crystals, forming a cluster of energy deposit: pattern recognition algorithms have been developed to identify these clusters and to differentiate single clusters with one energy maximum from merged clusters with more than one local energy maximum, referred to as bumps. The algorithms also determine whether a bump is generated by a charged or a neutral particle. Clusters are required to contain at least one seed crystal with an energy above 10 MeV: surrounding crystals are considered as part of the cluster if their energy exceeds a threshold of 1 MeV or if they are contiguous neighbors of a crystal with at least 3 MeV signal. The level of these thresholds depends on the current level of electronic noise and beam-generated background.

A bump is associated with a charged particle by projecting a track to the inner face of the calorimeter: the distance between the track impact point and the bump centroid is calculated and if it is consistent with the angle and momentum of the track, the bump is associated with this charged particle. Otherwise it is assumed to originate from a neutral particle.

On average, 15.8 clusters are detected per hadronic event: 10.2 are not associated to any charged particle. Currently, the beam-induced background contributes on average with 1.4 neutral clusters with energy above 20 MeV.

At low energy, the energy resolution of the EMC is measured directly with the radiative calibration source yielding $\sigma_{E}/E = 5.0 \pm 0.8\%$ at 6.13 MeV. At high energy, the resolution is derived from Bhabha scattering where the energy of the detected shower can be predicted from the polar angle of the electrons and positrons. The measured resolution is $\sigma_{E}/E = 1.9 \pm 0.1\%$ at 7.5 GeV.

The measurement of the angular resolution is based on the analysis of $\phi^{0}$ and $\eta$ decays to two photons of approximately equal energy: the resolution varies between about 12 mrad at low energy and 3 mrad at high energies.
Left plot in fig. (2-15) shows the two-photon invariant mass in $B \bar{B}$ events: the reconstructed $\phi$ mass is measured to be 135.1 MeV/$c^2$ and is stable to better than 1% over the full photon energy range. The width of 6.9 MeV/$c^2$ agrees well with the prediction obtained from detailed Monte Carlo simulation. In low occupancy $\tau^+\tau^-$ events, the width is slightly smaller, 6.5 MeV/$c^2$, for $\pi^0$ energies below 1 GeV.

The EMC electron identification is based on the shower energy, lateral shower moments and track momentum to separate electrons from charged hadrons. In addition, the $dE/dx$ energy loss in the DCH and the DIRC Čerenkov angle are required to be consistent with an electron. The most important variable for the discrimination of hadrons is the ratio of the shower energy to the track momentum ($E/p$). Right plot in fig. (2-15) shows the efficiency for electron identification and the pion mis-identification probability as functions of momentum. The electron efficiency is measured using radiative Bhabha’s and $\phi\pi^+$ events, while the pion mis-identification for selected charged pions from $\phi$ decays and three-prong $\tau$ decays: a tight selector results in an efficiency plateau at 94.8% and a pion mis-identification probability of the order of 0.3%.

### 2.2.6 The magnet and the muon and neutral hadron detector IFR.

The Instrumented Flux Return (IFR) was designed to identify muons with high efficiency and good purity and to detect neutral hadrons (mainly $K_L^0$ and neutrons) over a wide range of momenta and angles. Muon identification is important for the flavour tagging of the neutral $B$ mesons via semileptonic decays, for the reconstruction of vector mesons ($J/\psi$ for instance) and for analyses of semileptonic and rare decays involving leptons of $B_s, D_s$ and $\tau$s. $K_L^0$ detection allows the study of exclusive $B$ decays (the golden mode $J/\psi K_L^0$ for example). The IFR can also help in vetoing charm decays and improve the reconstruction of neutrinos.
The main requirements for the *IFR* are large solid angle coverage, good efficiency and high background rejection for muons down to momenta below 1 GeV. For neutral hadrons, the most important requirements are high efficiency and good angular resolution. The momentum range in which the *IFR* can work, starts from about 450 MeV/c (limit due to the barrel magnetic field: lower momentum particles cannot enter the *IFR*), while in the forward and backward regions the lower limit is 250 MeV/c. The upper limit is of order of some GeV, but, since even direct muons cannot have momentum values higher than 6 GeV/c, one can say that the *IFR* does not have an upper limit for muons from $\Upsilon(4S)$ decays.

![Diagram of the Babar detector](image)

**Figure 2-16.** The *IFR* detector and schematic representation of RPC components.

The *IFR* uses the steel flux return of the magnet as muon filter and hadron absorber: the uniform magnetic field of $1.5 T$ is generated by a superconducting solenoid and the large iron structure needed as magnet yoke is segmented and instrumented with Resistive Plate Chambers (RPCs) with two-coordinate readout. The RPCs are installed in the gaps of the finely segmented steel of the barrel and the end doors of the flux return (see fig. (2-16)). The steel segmentation has been chosen on the basis of Monte Carlo studies of muon penetration and charged and neutral hadron interactions: the steel is segmented into 18 plates increasing in thickness from $2 \text{ cm}$ for the inner nine plates to $10 \text{ cm}$ for the outermost plates. The nominal gap between the steel plates is $3.5 \text{ cm}$ in the inner layers of the barrel and $3.2 \text{ cm}$ elsewhere. There are 19 RPC layers in the barrel and 18 in the end-caps. Each end-cap consists of hexagonal plates, divided vertically into two parts to allow opening of the detector and has a central hole for the beam components and the magnetic shields. In addition, two layer of cylindrical RPCs are installed between the *EMC* and the magnet cryostat, in order to detect particle exiting the *EMC*.

RPCs detect streamers from ionizing particles via capacitive readout strips. The position resolution depends on the segmentation of the readout: a value of a few mm is achievable. A cross section of an RPC is shown schematically in fig.(2-16): the planar RPC consists of two bakelite sheets, $2 \text{ mm}$ thick and separated by a gap of $2 \text{ mm}$. The gap width is kept uniform by policarbonate spaces that are glued to the bakelite, spaced

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**THE BaBar EXPERIMENT**
at distances of about 10 cm. The external surface of the bakelite are coated with graphite surfaces that are connected to high voltage (~ 8 kV) and ground and protected by an insulating Mylar film.

The RPC is essentially a gas gap at atmospheric pressure enclosed between two 2 mm-thick bakelite (phenolic polymer) plates: the gas mixture is based on comparable quantities of Argon and Freon and a small amount of Isobutane. A crossing charged particle produces a quenched spark that produces signals on external pick-up electrodes. The RPCs are operated in limited streamer mode and the signal are read out capacitively on both sides of the gap by external electrodes made of aluminum strips on a Mylar substrate. The $I_{FR}$ consists of a central part (barrel) and two plugs (end-caps) which complete the solid angle coverage down to 300 mrad in the forward direction and 400 mrad in the backward direction. The barrel extends radially from 1.78 m to 3.01 m and is divided into sextants: the length of each sextant is 3.75 m and the width varies from 1.94 m to 3.20 m.

The $I_{FR}$ detectors cover a total active area of about 2000 m$^2$: there are a total of 806 RPC modules, 57 in each of the six barrel sections, 108 in each of the four half end-doors and 32 in the two cylindrical layers. The modules of each chamber are connected to the gas system in series, while the high voltage is supplied separately to each module.

Barrel modules have 32 strips running perpendicular to the beam axis to measure the $z$ coordinate and 96 strips in the orthogonal direction extending over three modules to measure $\phi$. The readout strips are separated from the ground aluminum plane by a 4 mm-thick foam. The strips are connected to the readout electronics at one end and even and odd strips are connected to different front-end cards so that a failure of a card does not result in a total loss of signal, since a particle crossing the gap typically generates signals in two or more adjacent strips.

The cylindrical RPC is divided into four sections, each covering a quarter of the circumference: each of these sections has four sets of two single gap RPCs with orthogonal readout strips, the inner with helical $u - v$ strips that run parallel to the diagonals of of the module, and the outer with strips parallel to $\phi$ and $z$.

The efficiency of the RPCs is evaluated both for normal collision data and for cosmic ray muons: every week cosmic ray data are recorded at different voltage settings and the efficiency is measured chamber-by-chamber as a function of the applied voltage (the typical voltage is 7.6 kV). To calculate the efficiency in a given chamber, nearby hits in a given layer and hits in different layers are combined to form clusters. Two algorithms are used: the first relies only on $I_{FR}$ information, while the second tries to match $I_{FR}$ clusters with the tracks reconstructed in the DCH. They both start from one-dimensional $I_{FR}$ clusters defined as groups of adjacent hits in one of the two readout coordinates. The first algorithm consists of joining one-dimensional clusters (of the same readout coordinate) in different layers, in order to form two-dimensional clusters and then these two-dimensional clusters in different coordinates are combined into three-dimensional clusters. The second algorithm extrapolates DCH charged tracks to be combined with the $I_{FR}$ clusters to form two- and three-dimensional clusters.

The residual distribution from straight line fits to two-dimensional clusters typically have an rms width of less than 1 cm. An RPC is considered efficient if a signal is detected at a distance of less than 10 cm from the fitted straight line in either of the two readout planes: 75% of the active RPCs modules exceed an efficiency of 90%. The RPC dark current is very temperature dependent: this current increases $14 - 20\%$ per
During the first summer of operation, the daily temperature in the IR hall was 28°C and the maximum hall temperature frequently exceeded 31°C: the temperature inside the steel rose to more than 37°C so that the dark currents in many modules exceeded the capabilities of the HV system and some RPCs had to be temporarily disconnected. A water cooling was installed on the barrel and end door steel.

During operation at high temperature, a large fraction of the RPCs showed very high dark currents, but also some reduction in efficiency compared to earlier measurement: the cause of the efficiency loss remains under investigation. After the cooling was installed and the RPCs reconnected, some of them continued to deteriorate while others remained stable, some of them (> 30%) at full efficiency.

![Figure 2-17. Left plot: muon efficiency (left scale) and pion mis-identification probability (right scale) as a function of the laboratory track momentum. Right plot: difference between the direction of reconstructed neutral hadron cluster and the missing transverse momentum in events with a reconstructed J/ψ decay. The Monte Carlo simulation is normalized to the luminosity of the data.](image)

Muon identification relies almost entirely on the IFR: charged particles are reconstructed in the SVT and DCH and muon candidates are required to meet the criteria for minimum ionizing particles in the EMC. Charged tracks are extrapolated to the IFR taking into account the non-uniform magnetic field, multiple scattering and the average energy loss. The projected intersection with the RPC planes are computed and all clusters within a predefined distance from the predicted intersection are associated with the track.

The performance of the muon identification has been tested on samples of muons from $\mu\mu ee$ and $\mu\mu\gamma$ final states and pions from $K_{S}^{0}$ and three-prong $\tau$ decays: the muon detection efficiency is about 90% in the momentum range of $1.5 < p < 3.0$ GeV/c with a fake rate for pions of about $6 - 8\%$ (see left plot in fig. (2-17)).

$K_{L}^{0}$ and other neutral hadrons interact in the steel of the IFR and can be identified as clusters that are not associated with a charged track: Monte Carlo studies predict that about 64% of $K_{L}^{0}$ of more than 1 GeV/c momentum, produce a cluster in the cylindrical RPC or a cluster with hits in two or more planar RPC layers.
The direction of the neutral hadron is determined from the event vertex and the centroid of the neutral cluster: no information on the energy of the cluster can be obtained.

Information from EMC and the cylindrical RPCs is combined with the IFR cluster information: the angular resolution of the neutral hadron cluster can be derived from a sample of $K^0_L$ produced in the reaction $e^+e^- \rightarrow \phi\gamma \rightarrow K^0_L K^0_S\gamma$. The $K^0_L$ direction is calculated from the missing momentum computed from the measured particles in the final state. The angular resolution of the $K^0_L$ is of the order of $60\text{ mrad}$: for $K^0_L$ interacting in the EMC the resolution is about twice better. Right plot in fig. (2-17) shows the angular difference $\Delta\phi$ between the missing momentum and the direction of the nearest neutral hadron cluster. the $K^0_L$ detection efficiency increases almost linearly with momentum: it varies between $20\%$ and $40\%$ in the momentum range from $1$ GeV/c to $4$ GeV/c.

### 2.2.7 The trigger.

The PEP-II high luminosity is also due to the $1.2m$ bunch spacing: the bunch time spacing is $4.2ns$ corresponding to a cross frequency of $238\text{ MHz}$. At design luminosity, beam-induced background rates are typically about $20\text{ kHz}$ each for one or more tracks in the drift chamber with $p > 120\text{ MeV/c}$ or at least one EMC cluster with $E > 100\text{ MeV}$. This rate is to be contrasted with the desired logging rate of less than $120\text{ Hz}$. The trigger and data acquisition subsystems are designed to record data at no more than the latter rate: the purpose of the trigger is to reject backgrounds while selecting a wide variety of physics processes. The total trigger efficiency is required to exceed $99\%$ for all $B\bar{B}$ events and at least $95\%$ for continuum events. The trigger should also contribute no more than $1\%$ to dead time.

The B\Bar trigger has two levels: Level 1 which executes in hardware and Level 3 which executes in software after the event assembly. The Level 1 trigger system is designed to achieve very high efficiency and good understanding of the efficiency. During normal operation, the L1 trigger is configured to have an output rate of typically $1\text{ kHz}$, while the L3 filter acceptance for physics is $\approx 90\text{ Hz}$.

Event signatures are used to separate signal from background. Combinations of the following global event properties are used in the L1 trigger as general event selection criteria: charged track multiplicity, calorimeter cluster multiplicity and event topology. These selection criteria have associated thresholds for the following parameters: charged-track transverse momentum ($p_T$), energy of the calorimeter clusters ($E_{clus}$), solid angle separation ($\delta$) and track-cluster match quality. The trigger definition can contain selection criteria that differ only by the values of thresholds. A small fraction of random beam crossings and events that failed to trigger are also selected for diagnostic purposes.

For a given trigger level, the global selection is a logical OR of a number of specific trigger selection lines, where each line is the result of a boolean operation on any combination of trigger objects: table 2-4 shows some examples of trigger objects.

Table 2-5 shows some trigger lines together with their L1 trigger rates and their efficiencies for various physics processes: the star (*) symbol next to a trigger object indicates that a minimum angular separation was required in order to count more than one object (typically $90\degree$). Back-to-back topologies among clusters
Table 2-4. Trigger objects for the Level 1 trigger.

<table>
<thead>
<tr>
<th>object description</th>
<th>threshold</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B ) Short track reaching ( DCH ) super-layer 5</td>
<td>120 MeV/c</td>
</tr>
<tr>
<td>( A ) Long track reaching ( DCH ) super-layer 10</td>
<td>180 MeV/c</td>
</tr>
<tr>
<td>( A' ) High ( p_T ) track</td>
<td>800 MeV/c</td>
</tr>
<tr>
<td>( M ) All-( \theta ) MIP energy</td>
<td>100 MeV/c</td>
</tr>
<tr>
<td>( G ) All-( \theta ) intermediate energy</td>
<td>250 MeV/c</td>
</tr>
<tr>
<td>( E ) All-( \theta ) high energy</td>
<td>700 MeV/c</td>
</tr>
</tbody>
</table>

Table 2-5. Trigger efficiencies and rates at a luminosity of \( 2.2 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1} \) for selected triggers applied to various physics samples. Symbols refer to the counts for each object in table 2-4.

<table>
<thead>
<tr>
<th>Level 1 Trigger</th>
<th>( \epsilon_B ) (%)</th>
<th>( \epsilon_C ) (%)</th>
<th>( \epsilon_{\text{int}} ) (%)</th>
<th>( \epsilon_{\text{ee}} ) (%)</th>
<th>Rate (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A \geq 3 &amp; B^* \geq 1 )</td>
<td>97.1</td>
<td>88.9</td>
<td>81.1</td>
<td>-</td>
<td>180</td>
</tr>
<tr>
<td>( A \geq 1 &amp; B^* \geq 1 &amp; A' \geq 1 )</td>
<td>95.0</td>
<td>89.2</td>
<td>85.2</td>
<td>98.6</td>
<td>410</td>
</tr>
<tr>
<td>( M \geq 3 &amp; M^* \geq 1 )</td>
<td>99.7</td>
<td>98.5</td>
<td>94.7</td>
<td>-</td>
<td>160</td>
</tr>
<tr>
<td>( E - M \geq 1 )</td>
<td>71.4</td>
<td>77.1</td>
<td>79.5</td>
<td>97.8</td>
<td>150</td>
</tr>
<tr>
<td>( B \geq 3 &amp; A \geq 2 &amp; M \geq 2 )</td>
<td>99.4</td>
<td>94.8</td>
<td>87.8</td>
<td>-</td>
<td>170</td>
</tr>
<tr>
<td>( M^* \geq 1 &amp; A \geq 1 &amp; A' \geq 1 )</td>
<td>95.1</td>
<td>90.1</td>
<td>87.0</td>
<td>97.8</td>
<td>250</td>
</tr>
<tr>
<td>( E \geq 1 &amp; B \geq 2 &amp; A \geq 1 )</td>
<td>72.1</td>
<td>77.7</td>
<td>79.2</td>
<td>98.3</td>
<td>140</td>
</tr>
<tr>
<td>Combined Level 1 Triggers</td>
<td>&gt; 99.9</td>
<td>99.9</td>
<td>98.2</td>
<td>&gt; 99.9</td>
<td>970</td>
</tr>
</tbody>
</table>

are written like \( E - M \) meaning an \( E \) cluster back to back to an \( M \) cluster, while the \& symbol denotes requiring clusters and tracks in coincidence, a non-orthogonal selection criterion.

Level 3 trigger is part of the online farm and consists of a network of commercial processors: input are the L1 trigger data and the full event data for events that passed the L1 trigger. Output to mass storage is the full event and trigger data of events accepted by L3. L3 trigger algorithms have all event information available and they operate by refining and improving the selection methods used by L1: better \( DCH \) tracking (vertex resolution) and \( EMC \) clustering filters allow for greater rejection of beam backgrounds and Bhabha events. A cut on the vertex position can be made to reject events that did not originate at the interaction point. L3 trigger also includes a variety of filters to perform event classification and background reduction: the logging decision is based on two orthogonal filters, one relying exclusively on \( DCH \) data and the other relying only on \( EMC \) data.
The drift chamber filters select events with one tight \((p > 600 \text{ MeV}/c)\) track or two loose \((p > 250 \text{ MeV}/c)\) tracks originating from the IP: track selection is based on the \(x - y\) closest approach distance \((d_0)\) to the IP and the corresponding \(z\) coordinate for that closest approach point \((z_0)\). The IP is a fixed location close to the average beam position over many months. Tight (loose) tracks have to satisfy a vertex condition defined as \(|d_0| < 1.0 \text{ cm}\) \((|d_0| < 1.5 \text{ cm})\) and \(|z_0 - z_{IP}| < 7.0 \text{ cm}\) \((|z_0 - z_{IP}| < 10.0 \text{ cm})\).

The calorimeter filters select events with either high energy deposits \((E_{\text{CM}} > 350 \text{ MeV})\) or high cluster multiplicity (at least 4 clusters): they also require a high effective mass \((> 1.5 \text{ GeV}/c^2)\) calculated from the cluster energy sums and the energy weighted centroid positions of all clusters in the event assuming mass-less particles. A Bhabha veto filter is also used: it selects one-prong (only a positron in the back part of the detector) and two-prong events (with both \(e^+\) and \(e^-\) detected) and it applies stringent criteria on \(EMC\) energy deposits relying on the track momenta and \(E/p\) values.

During a typical run on the \(Y(4S)\) peak with an average luminosity of \(2.6 \times 10^{33} \text{ cm}^{-2}\text{s}^{-1}\), the physics events represent the \(13\%\) of the total L3 output (with a rate of 16 Hz), while the calibration and diagnostic samples comprise \(40\%\) (with a rate 49 Hz): the total output rate is 122 Hz.
$K^0_S$ reconstruction and efficiency studies

An efficient and accurate reconstruction of $K^0_S$ decaying into $\pi^+\pi^-$ is crucial to several analyses in BABAR including the measurement of $CP$ violation$^1$.

3.1 Reconstruction

$K^0_S$ are reconstructed pairing all possible tracks of opposite sign, and looking for the 3D point (vertex) which is more likely to be common to the two tracks. In BABAR there are three tools for vertex reconstruction: a so called “plain” vertexer, a “Fast” vertexer and the “GeoKin” fitter. The differences among the algorithms are described in the Ref. [43]. In summary, “Plain” and “Fast” are used as vertexers, that is they are just capable of imposing geometric constraints. “GeoKin” is instead used as a fitter since this is the configuration needed when building an exclusive $B$ decay tree, imposing mass constraints together with the geometric ones. The underlying algorithms are very similar (based on a $\chi^2$ minimization) and the differences can be summarized as follows: “Plain” and “GeoKin” use the position-momentum representation and use as a starting point for the vertex finding the closest approach in 3D. “Fast” uses the helix representation and starts from the closest approach in 2D.

The performances of the three algorithms are extremely similar (see figure 3-1) and in the following the algorithms are used without distinctions.

The analysis is performed after the events have already been reconstructed and therefore the track helix is stored in the event database at the point of closest approach to the origin (0,0,0). The effect of this approximation on long living $K^0_S$ is still under study.

3.1.1 $K^0_S$ candidate lists available for the physics analyses

There are three lists of $K^0_S$ candidates that can be used in the various analyses:

- “KsLoose” list is made of pairs of opposite charge tracks with no vertexing. Candidates are accepted if their invariant mass is between 300 and 700 MeV. The mass resolution is extremely broad (~ 15 MeV), given the fact that the track parameters are taken at the origin.

$^1$Most of the BABAR analysis involving $K^0_S$ reconstruct exclusively $K^0_S$ decays to $\pi^+\pi^-$. 
Figure 3-1. Comparison among invariant mass distributions for $\pi^+\pi^-$ reconstructed as $K^0_S$ using the three algorithms, superimposed (a) and with three separate fits (b).

- “KsDefault” is a refinement of the “KsLoose” list: the vertex is performed and only candidates within 25 MeV of the PDG mass are accepted. In this case the mass is recomputed at the vertex.
- “KsTight” refines the “KsDefault” list applying the mass constraint.

Note that the selection that creates these list is only based on mass windows.

3.2 Study on MC truth

In order to understand the reconstruction efficiency of the $K^0_S$ and possible sources of inefficiency, a study at MC truth level has been performed. Good part of the effort has been spent in understanding where do the $K^0_S$ actually decay and with which momentum. Figure 3-2 details the acceptance regions for the $K^0_S$ reconstruction and it is very helpful in understanding which topics we should be concentrating upon. It appears clear, for instance, that the outer region of the DCH, albeit challenging, is not the top priority.

Starting with the $K^0_S$ in the list from the Monte Carlo truth, we check whether the $K^0_S$ is in the KsDefault list or not. The geometrical acceptance requires $0.3 < \theta_{\pi^+} < 2.706$. We subdivided our MC $K^0_S$ in the following categories:

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3.3 Studies on data

The main goal of this analysis is though to provide a way to correct the efficiencies found on Monte Carlo samples to agree with data. In order to understand the efficiency\(^2\) on the data, our study has been performed on a sample of consistently processed data.

\(^2\)In the following, if not otherwise stated, efficiencies are quoted on the whole solid angle: they include acceptance.
Table 3-1. Number of (un)reconstructed $K^0_S$ for certain event categories.

<table>
<thead>
<tr>
<th>Event Category</th>
<th>Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>$# K^0_S$</td>
<td>2980</td>
</tr>
<tr>
<td>$# K^0_S$ associated</td>
<td>2007</td>
</tr>
<tr>
<td>(67.3% of associated)</td>
<td></td>
</tr>
<tr>
<td>$# K^0_S$ within geometrical acceptance</td>
<td>2553</td>
</tr>
<tr>
<td>$# K^0_S$ associated within geometrical acceptance</td>
<td>1995</td>
</tr>
<tr>
<td>(78.1% of associated)</td>
<td></td>
</tr>
<tr>
<td>$# K^0_S$ unassociated within geometrical acceptance</td>
<td>194</td>
</tr>
<tr>
<td>-two pions reconstructed-</td>
<td></td>
</tr>
<tr>
<td>(35.8% of unassociated)</td>
<td></td>
</tr>
<tr>
<td>$# K^0_S$ unassociated within geometrical acceptance</td>
<td>164</td>
</tr>
<tr>
<td>-$\pi^-$ reconstructed-</td>
<td></td>
</tr>
<tr>
<td>(30.3% of unassociated)</td>
<td></td>
</tr>
<tr>
<td>$# K^0_S$ unassociated within geometrical acceptance</td>
<td>158</td>
</tr>
<tr>
<td>-$\pi^+$ reconstructed-</td>
<td></td>
</tr>
<tr>
<td>(29.2% of unassociated)</td>
<td></td>
</tr>
<tr>
<td>$# K^0_S$ unassociated within geometrical acceptance</td>
<td>26</td>
</tr>
<tr>
<td>-no pion reconstructed-</td>
<td></td>
</tr>
<tr>
<td>(4.8% of unassociated)</td>
<td></td>
</tr>
</tbody>
</table>

3.3.1 Data samples

Data used for this analysis correspond to a subsample of the Run1 data set. In order to limit the underlying systematics another sample of off-peak (continuum) data has been used from Run 1 data set. All samples are split into the so called block 1 and block 2 subsamples: they differ in different DCH operating high voltage (HV). Block 1 corresponds to the DCH HV set at 1900 V, while block 2 corresponds to 1960 V. The HV difference in the DCH configuration is expected to lead to different tracking efficiencies and thus to different $K^0_S$ reconstruction efficiencies. The data sample consists of:

- on-resonance data from block 1, corresponding to a luminosity of 533.4 pb$^{-1}$ and to a number of $b\overline{b} = 590505.6 \pm 9898.7$(stat+syst) giving an average cross-section of $1.1072 \pm 0.0055$(stat) $\pm 0.0376$(syst) nb;

- on-resonance data from block 2, corresponding to a luminosity of 531.5 pb$^{-1}$ and to a number of $b\overline{b} = 593094.3 \pm 9953.2$(stat+syst) giving an average cross-section of $1.1158 \pm 0.0056$(stat) $\pm 0.0379$(syst) nb

- off-resonance data from block 1, corresponding to a luminosity of 330.3 pb$^{-1}$
• off-resonance data from block 2, corresponding to a luminosity of 392.3 pb⁻¹

• Monte Carlo simulation dataset
  - \( B\bar{B} \): 1 million events
  - \( c\bar{c} \): 1 million events
  - \( uds \): 2 million events

This Monte Carlo dataset has been produced for both the block 1 equivalent Monte Carlo sample and the block 2 equivalent sample:

- block 1 MC sample corresponds to 900.9 pb⁻¹ (assuming the \( b\bar{b} \) cross section of 1.11 nb) for \( b\bar{b} \) Monte Carlo and 776.9 pb⁻¹ for continuum Monte Carlo.

- block 2 MC sample corresponds to 892.9 pb⁻¹ (assuming the \( b\bar{b} \) cross section of 1.12 nb) for \( b\bar{b} \) Monte Carlo and 769.2 pb⁻¹ for continuum Monte Carlo.

The \( K_S^0 \) are reconstructed pairing all the opposite charged tracks of the event and using “GeoKin” fitter algorithm. The charged tracks used are requested to be in the fiducial region \( 0.41 < \theta_{ab} < 2.54 \) (i.e. inside the active region of the SVT). No selection is applied apart from the so-called hadronic selection that consists of:

- BGFMultiHadron³ tag bit set
- number of ChargedTracks in the fiducial region in the event \( \geq 4 \)
- \( R_2 < 0.74 \)
- the sum of energy of charged tracks in the fiducial region plus calorimeter clusters greater than 5 GeV
- primary vertex within 0.5 cm of beam spot in x and y.

This selection is the one used for \( B \) counting analysis: it is described and discussed in Ref. [44]. The efficiency of this hadronic selection is shown in table 3-2.

No other selection is applied in the analysis: the number of observed \( K_S^0 \) and their average invariant mass are evaluated from a fit to the invariant mass plot, using a double Gaussian with a linear background. The resolution on the \( K_S^0 \) mass is evaluated using a single Gaussian and linear background fit.

### 3.3.2 Mass and Resolution Studies

Possible dependencies of the reconstructed \( K_S^0 \) mass and resolution on various quantities were investigated.

The \( \theta \) and the \( \phi \) angles of the \( K_S^0 \) daughters have been considered in figures (3-3) and (3-4). The data-Monte Carlo comparison of the reconstructed invariant mass and resolution is shown as function of those angles. It is to be noted that Monte Carlo reconstructed invariant mass is shifted from the simulated one and that the

³the BGFMultiHadron selection consists of requiring at least 3 charged tracks and \( R_2 < 0.98 \).
⁴see Eq. (4.4) for the definition of this variable.
Table 3-2. Event selection efficiency of the hadronic selection in generic Monte Carlo events used in this analysis and the number of $K_S^0$ per event after the hadronic selection from the Monte Carlo truth.

<table>
<thead>
<tr>
<th>modes</th>
<th>efficiency (%) block 1</th>
<th># $K_S^0$ per event (b1 MC)</th>
<th>efficiency (%) block 2</th>
<th># $K_S^0$ per event (b2 MC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0\bar{B}^0$</td>
<td>93.9</td>
<td>0.52</td>
<td>94.9</td>
<td>0.52</td>
</tr>
<tr>
<td>$B^+\bar{B}^-$</td>
<td>94.0</td>
<td>0.46</td>
<td>95.3</td>
<td>0.46</td>
</tr>
<tr>
<td>$b\bar{b}$</td>
<td>94.0</td>
<td>0.49</td>
<td>95.1</td>
<td>0.49</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>77.8</td>
<td>0.43</td>
<td>77.1</td>
<td>0.43</td>
</tr>
<tr>
<td>$uds$</td>
<td>67.5</td>
<td>0.27</td>
<td>63.9</td>
<td>0.27</td>
</tr>
</tbody>
</table>

resolution on Monte Carlo is better than on data. In figure 3-5, data show a $\phi$ dependence of the number of reconstructed $K_S^0$ which is also well reproduced by Monte Carlo.

Then, the reconstructed decay length of the $K_S^0$ is considered both on MC and on real data samples. Defining the two-dimensional decay length:

$$dr = \sqrt{(x_{K_s} - x_{beam})^2 - (y_{K_s} - y_{beam})^2}$$

(where $x_{K_s}$ and $y_{K_s}$ are the reconstructed $x$ and $y$ position of the $K_S^0$ decay vertex and $x_{beam}$ and $y_{beam}$ come from the reconstructed impact point from Bhabhas), figures 3-6 show the invariant mass and the resolution obtained as functions of $dr$. Monte Carlo and data have similar structures with $dr > 3$, but they look pretty different within the beam pipe ($dr < 3$ cm): the Monte Carlo does not show any slope, while the data do. This behaviour is still under investigation.

In the range of $dr$ between 3 and 15 cm, less and less material gets traversed and therefore the fact that the data used for the analysis store the helix parameters at the origin causes an increase in the measured mass: the energy loss correction is applied although the material has not been traversed by the tracks.

Another drop is visible towards the end of the SVT: this might be due to the transition between DCH only and SVT only tracks, but the real source of the effect is not yet understood.

Another variable that has been considered in this analysis is the reconstructed momentum of the $K_S^0$ candidate, see figures 3-7. Data show a flat distribution, while Monte Carlo presents a positive slope at low momenta.
3.3 Studies on data

Figure 3-3. Top plots: invariant mass as function of the $\theta$ angle of the $K_S^0$ daughters in both $b1$ (left) and $b2$ (right) data-sets of on-resonance data. Bottom plots: invariant mass resolution as function of the $\theta$ angle of the $K_S^0$ daughters in both $b1$ (left) and $b2$ (right) data-sets of on-resonance data. The data-Monte Carlo comparison is presented: the empty dots come from the Monte Carlo sample and the black points come from the on resonance data.

3.3.3 Efficiency Studies

To evaluate the absolute efficiency, we need to estimate the hadronic efficiency and assuming the number of $K_S^0$ per event from the Monte Carlo. From the Monte Carlo truth, one can get the number of $K_S^0$ per events in $b\bar{b}$, $c\bar{c}$ and $uds$ events, but the hadronic selection is applied to the data sample and, since the hadronic selection efficiency differs from one event typology to another, the number of $K_S^0$ per event in the on-resonance data has to be calculated weighting the Monte Carlo truth information through the effective cross-sections. Using results from table 3-2, one can extract the cross section corrected for the hadronic selection efficiency and the effective number of $K_S^0$ per event in the on-resonance sample we use. A systematic uncertainty due to the number of expected $K_S^0$ per hadronic event given by the Monte Carlo needs to be evaluated. To get the corrected cross section:

$K_S^0$ RECONSTRUCTION AND EFFICIENCY STUDIES
Figure 3-4. **Top plots:** invariant mass as function of the φ angle of the $K^0_S$ daughters in both b1 (left) and b2 (right) data-sets of on-resonance data. **Bottom plots:** invariant mass resolution as function of the φ angle of the $K^0_S$ daughters in both b1 (left) and b2 (right) data-sets of on-resonance data. The data-Monte Carlo comparison is presented: the empty dots come from the Monte Carlo sample and the black points come from the on resonance data.

\[
\sigma_{\text{corr}}^t = \sum_t \sigma_{t}^{_{b\bar{b}}} \cdot \epsilon_t^H
\]

where $t$ runs over $\omega,uds$ and $b\bar{b}, \sigma_{t}^{_{b\bar{b}}}$ is the non-corrected cross section (see Tab. 2-2) and $\epsilon_t^H$ is the hadronic efficiency for each sample.

The effective number of $K^0_S$ per events ($F_{K_S}$) can be calculated:

\[
F_{K_S}^{\text{corr}} = \sum_t \sigma_{t}^{_{b\bar{b}}} \cdot \epsilon_t^H \cdot F_{K_S}^t / \sigma_{\text{corr}}^t
\]
3.3 Studies on data

![Graph showing observed Ks candidates](image)

**Figure 3-5.** On resonance data: number of reconstructed $K^0_S$ as function of the $\phi$ angle of the $K^0_S$ daughters in both b1 (left) and b2 (right) data-sets. The data-Monte Carlo comparison is presented: the empty dots come from the Monte Carlo sample and the black points come from the on resonance data.

where $F_{K_S}^T$ is the number of $K^0_S$ per event in each sample. From table 3-2, we can evaluate $\sigma^{\text{corr}}$ and $F_{K_S}^{\text{corr}}$ for both block 1 and block 2 Monte Carlo samples: this estimate can be found in Tab. 3-3. From the corrected cross section we can estimate the expected number of hadronic events in the on-resonance data, while from the number of $K^0_S$ per event we can calculate the expected number of $K^0_S$ in the data samples. The efficiency can be evaluated in both Monte Carlo and data. The same technique is used on both samples: an invariant mass window between 0.45 and 0.55 GeV/c$^2$ is taken into account and a double Gaussian fit with linear background is performed on the invariant mass distribution of $\pi^+\pi^-$ pairs without any selection apart from the hadronic one described in Sec. 3.3. The number of the reconstructed $K^0_S$ is taken from the area under the two Gaussians. The reason of the fit with a double Gaussian distribution can be understood looking at the distribution of the invariant mass of true Monte Carlo $K^0_S$: in Fig. 3-8 the tails of the second Gaussian can be clearly seen.

Table 3-3 contains the number of observed $K^0_S$: this is the result of the fits to the plots in Fig.3-9. Therefore the efficiency can be calculated:

<table>
<thead>
<tr>
<th>sample</th>
<th>variable</th>
<th>block 1</th>
<th>block 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monte Carlo</td>
<td>$\sigma^{\text{corr}}$</td>
<td>3.47 nb</td>
<td>3.40 nb</td>
</tr>
<tr>
<td></td>
<td>$F_{K_S}^{\text{corr}}$</td>
<td>0.38 ks/ev</td>
<td>0.39 ks/ev</td>
</tr>
<tr>
<td>on-res data</td>
<td># of expected $K^0_S$</td>
<td>755230</td>
<td>768087</td>
</tr>
<tr>
<td></td>
<td># of reconstructed $K^0_S$</td>
<td>372400 ± 3023</td>
<td>382200 ± 2900</td>
</tr>
</tbody>
</table>

**Table 3-3.** Number of $K^0_S$ per event, number of expected and reconstructed events.
Efficiency in Monte Carlo sample is slightly below the data values. The absolute efficiency is therefore 49.3% in the block 1 on-resonance sample and 49.8% in the block 2 sample. We have also evaluated the efficiency on Monte Carlo: in block 1 MC sample, the efficiency is 48.1%, while in block 2 MC sample, it is the 49.7%.

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3.3 Studies on data

At the same way, we can look at the efficiency as function of both $d\tau$ and the reconstructed momentum of the $K_S^0$: see Fig. 3-10.

To find out what is causing the drop between 2.0 and 3.0 cm in the efficiency as function of the decay length, the same study has been done using a $d\tau$ value taken from the Monte Carlo truth. The figure (3-11) shows two gaps: the first between 2 and 3 cm that is the same we see in data and Monte Carlo without using the true $d\tau$, while the second is between 12 and 15 cm due to a the Monte Carlo association failing in that region. This result shows that the vertexing is not introducing this gap in the efficiency shape.

In order to check the momentum dependence of the efficiency the shape of the $K_S^0$ momentum in $b\bar{b}$ events has been checked: figure (3-12) shows a nice agreement between $b\bar{b}$ from data and from Monte Carlo in the $K_S^0$ reconstructed momentum. The $K_S^0$ momentum in $b\bar{b}$ events in data is obtained from a side-band subtraction and then an off-resonance subtraction from the on-resonance distribution.

$K_S^0$ RECONSTRUCTION AND EFFICIENCY STUDIES

Figure 3-7. Top plots: invariant mass as function of the reconstructed momentum of the $K_S^0$ in block 1 (left) and in block 2 (right). Bottom plots: invariant mass resolution as function of the reconstructed momentum of the $K_S^0$ in block 1 (left) and in block 2 (right). The on-resonance data-Monte Carlo comparison is presented: the empty dots come from the Monte Carlo sample and the black points come from the on-resonance data.
3.3.4 Correction for the Monte Carlo efficiencies

To evaluate the $K_{S}^{0}$ efficiency in data with respect to the Monte Carlo estimate, a set of corrections is produced with the inclusive analysis described above. The corrections are given in 40 bins of the already defined 2-dimensional flight length: and they are simply the bin-by-bin ratios between the $K_{S}^{0}$ efficiency in the on-resonance data over the $K_{S}^{0}$ efficiency in the MC samples. This ratio does not depend on luminosity of the two samples. A correction $1 \pm 1$ is assigned in case there are no reconstructed $K_{S}^{0}$ in the $dr$ bin in data or in MC (i.e. high flight length values). They are also normalized to the first bin: that means that in principle one could get the same correction values from the ratio between the number of reconstructed $K_{S}^{0}$ in the on-resonance data over the number of reconstructed $K_{S}^{0}$ in the MC samples. The normalization to the first bin is due to the fact that $K_{S}^{0}$ reconstruction has to be correlated to the tracking efficiency. Since the $K_{S}^{0}$ daughter candidates are selected from the list of all the charged tracks, the overall correction should take into account also the differences of the charged track reconstruction in data and in Monte Carlo.

This tracking correction is studied in different control samples and these analyses provide the correction value and the associated systematic error. For the ChargedTracks list (no selection cuts at all), it has been found that no correction is necessary but a systematic error of 1% per track has to be included into the efficiency calculations. This leads to a systematic of 2% per $K_{S}^{0}$ candidate.

Thus, since the normalization is done to the first bin, one should scale the pure $K_{S}^{0}$ correction for the tracking efficiency correction that has to be applied within 1 cm of flight length.

The bin-by-bin number of reconstructed $K_{S}^{0}$ candidates is obtained with the usual double Gaussian fit with linear background to bin-by-bin invariant mass distributions: some examples of these fits are given in Fig. 3-13.

Two sets of corrections are provided for each period (block 1 and block 2): the first set comes from $K_{S}^{0}$ reconstruction with no cuts except for the hadronic selection on the events, while the second set has an additional momentum cut ($p_{K_{S}^{0}} > 1$ GeV/c). This method is used in order to provide corrections which are independent from the momentum range of the $K_{S}^{0}$ candidates taken into account in the specific analyses.
3.3 Studies on data

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**Figure 3-9. Top plots:** $K^0_S$ candidate invariant mass in both block 1 (left) and block 2 (right) samples of the on-resonance data. **Bottom plots:** $K^0_S$ candidate invariant mass in both block 1 (left) and block 2 (right) of the Monte Carlo samples. The superimposed curves are the results of the fit with a double Gaussian and a linear background.

Fig. 3-14 shows the values of the corrections as function of the 2-dimensional flight length: the red dots represent the corrections with the 1GeV momentum cut, while the black one are the values obtained from the no momentum-cut sample.

To evaluate the correction and its error in a specific analysis, one should apply both sets of corrections separately and then take into account the difference as a systematic error on this correction. The central value of the correction should be the one given from the set with the momentum cut, since in that case the extraction of the correction is cleaner (statistical error on the correction is smaller) due to a lower background.

In the case of a single $K^0_S$ and given the $k$ sample where $k$ runs over block 1 and block 2, the correction $C^k$ is given by:

$$C^k = \sum_i (f_i \cdot x_i^k)$$

---

$K^0_S$ RECONSTRUCTION AND EFFICIENCY STUDIES
Figure 3-10. On resonance data: \( K_S^0 \) reconstruction efficiency as function of decay length of the \( K_S^0 \)'s in both block 1 (left) and 2 (right) samples. The data-Monte Carlo comparison is presented: the empty dots come from the Monte Carlo sample and the black points come from the on resonance data.

where \( f_i = N_i / N_{\text{tot}_i} \), \( N_i \) is the number of \( K_S^0 \) in the bin number \( i \), \( N_{\text{tot}_i} \) is the total number of \( K_S^0 \) candidates in the \( k \) sample and \( s_i^k \) are the correction values for the \( k \) sample. The error \( \sigma_C^k \) on the correction is then given by:

\[
\sigma_C^k = \sqrt{\sum_i [f_i \cdot s_i^k]^2}
\]

where \( s_i^k \) are the errors on the correction values for the \( k \) sample. Taking into account both samples (block 1 and block 2), the final correction should be calculated from a weighted average of \( C_1 \) and \( C_2 \) values where the weights should be the relative luminosities of the two samples.

An example of the final correction can come from the analysis \( B^\pm \rightarrow K_S^0 \ell \pm \): from the first set of corrections, one gets \( 1.02 \pm 0.18 \) while from second set of corrections with the momentum cut, one gets \( 1.05 \pm 0.06 \). The final correction would be:

\[
C_{K_S^0 \ell \pm} = 1.05 \pm 0.06(\text{stat}) \pm 0.03(\text{syst})
\]

3.3.5 Run 2 data sample: first look at the \( K_S^0 \) reconstruction

These runs correspond to a subsample of the so-called block 1 data set in Run 2. The data sample consists of:

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3.3 Studies on data

Figure 3-11. Monte Carlo: $K_S^0$ invariant mass (left) and reconstruction efficiency (right) as function of decay length from the Monte Carlo truth: the empty dots correspond to block 2 Monte Carlo sample, while the black points are the block 1 Monte Carlo sample.

- on-resonance data corresponding to a luminosity of $2090.4$ pb$^{-1}$ and to a number of $l^0 = 2345375.2 \pm 72045.3$ giving an average cross-section of $1.1214 \pm 0.0028$(stat) $\pm 0.0381$(syst) nb

No further selection is applied apart from the hadronic selection which has been updated with respect to the Run 1 selection (see Sec. 3.3.1). The new selection consists of:

- BGFMutiHadron tag bit set
- number of GoodTrackLoose$^5$ in the event $\geq 3$
- $R_2 < 0.7$
- the sum of energy of charged tracks plus calorimeter clusters in the fiducial region $> 4.5$ GeV
- primary vertex within 0.5 cm of beam spot in x and y
- primary vertex within 6.0 cm of beam spot in z.

This results in a slightly looser selection with respect to the previous one. This selection is the one used for B counting analysis: it is described and discussed in [45]. A first estimate of the efficiency of this hadronic selection is shown in table 3-4: the Monte Carlo sample used is the Run 1 equivalent one so these are just preliminary values to be checked with the appropriate Run 2 Monte Carlo.

As in the Run 1 analysis, no other selection is applied: the number of observed $K_S^0$ and their average invariant mass are evaluated from a fit to the invariant mass plot, using a double Gaussian with a linear background. The resolution on the $K_S^0$ mass is evaluated using a single Gaussian and linear background fit. The invariant mass fit on this Run 2 data sample gives $1727300 \pm 5209$(stat) number of reconstructed $K_S^0$ (see Fig. 3-15).

$^5$GoodTrackLoose definition: more than 11 drift chamber hits, $d_0$ within 1.5 cm, $z_0$ within 10 cm and transverse momentum greater than 100 MeV.

$K_S^0$ RECONSTRUCTION AND EFFICIENCY STUDIES
Figure 3-12. $K_S^0$ reconstructed momentum: the $K_S^0$ momentum in $b\bar{b}$ events in data is obtained from a side-band and off-resonance subtraction and compared with the $K_S^0$ momentum in $b\bar{b}$ Monte Carlo events. The comparison is done with both a luminosity normalization and an area normalization.

Again possible dependencies of the reconstructed $K_S^0$ mass and resolution on flight length and momentum are investigated: Fig. 3-16 shows the invariant mass and resolution of the reconstructed $K_S^0$ as function of

<table>
<thead>
<tr>
<th>modes</th>
<th>efficiency (%)</th>
<th># $K_S^0$ per event</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0\bar{B}^0$</td>
<td>96.0</td>
<td>0.32</td>
</tr>
<tr>
<td>$B^+\bar{B}^-$</td>
<td>96.2</td>
<td>0.29</td>
</tr>
<tr>
<td>$c\bar{c}$</td>
<td>84.8</td>
<td>0.27</td>
</tr>
<tr>
<td>$uds$</td>
<td>75.7</td>
<td>0.17</td>
</tr>
</tbody>
</table>

Table 3-4. Event selection efficiency of the hadronic selection in generic Monte Carlo events used in this analysis and the number of $K_S^0$ per event after the hadronic selection from the Monte Carlo truth.
3.3 Studies on data

Figure 3-13. Values of the $K^0$ efficiency corrections for block 1 (left) and block 2 (right) data. The red dots represent the corrections with the 1GeV momentum cut, while the black one are the values obtained from the no momentum-cut sample.

the 2-dimensional flight length. Note that, with respect to Run 1 distributions (see Fig. 3-6), there is not any more a drop at the end of the SVT (around 13 cm). This should be due to an improved alignment of the SVT itself.

Fig. 3-17 show the invariant mass and resolution of the reconstructed $K^0_S$ as function of the $K^0_S$ momentum. The high statistics allows for very stable fits in almost all bins and the mass distribution shows a very good agreement with the PDG mass [14], with the exception of the low momentum values where the Run 1 behaviour with a positive slope is still present (compare to Fig. 3-7).

$K^0_S$ RECONSTRUCTION AND EFFICIENCY STUDIES
Figure 3-14. Values of the $K^0_S$ efficiency corrections for block 1 (left) and block 2 (right) data. The red dots represent the corrections with the 1GeV momentum cut, while the black one are the values obtained from the no momentum-cut sample.

Figure 3-15. $K^0_S$ candidate invariant mass in Run 2 on-resonance data. The superimposed curve is the result of the fit with a double Gaussian and a linear background.
3.3 Studies on data

Figure 3-16. Invariant mass (left) and resolution (right) as function of the 2-dimensional flight length of the $K_S^0$ in Run 2 data.

Figure 3-17. Invariant mass (left) and resolution (right) as function of the reconstructed momentum of the $K_S^0$ in Run 2 data.

$K_S^0$ RECONSTRUCTION AND EFFICIENCY STUDIES
This chapter describes the requirements, the techniques and the variables common to all the hadronic charmless two-body analyses. These analyses refer to those $B$ decays that do not include $c$ quarks in the final states and whose final states are made up of two particles among charged and neutral $\pi$ and charged and neutral $K$.

Crucial issues in the hadronic charmless two-body modes are background fighting and particle identification (where applicable). The main background to these decays is due to fake $B$ candidates reconstructed in the continuum $e^+e^- \rightarrow q\overline{q}$ production (see Tab. 2-2). This background contamination together with the small expected branching fractions and the relatively large $\mathcal{B}$ cross section, would not allow for high purity values while keeping reasonable efficiencies.

In order to reach a good discriminating power against background, CLEO approach has been adopted [46]: a Fisher discriminant [47] has been developed and studied to separate signal from background on a statistical basis. Since the charged tracks resulting from charmless two-body $B$ decays have relatively high momenta (approximately $1.7 - 4.2 \text{ GeV}/c$), the Čerenkov angle $\theta_c$, determined from the DIRC (Sec. '2.2.4), is the only measurement which provides good $K - \pi$ discrimination.

A maximum likelihood fit is used to measure the yields in the various channels from the data sample. The fit incorporates the Fisher output and kinematic variables of the $B$ candidate, which are used to separate signal and background, as well as the Čerenkov angle (where applicable), which is used to distinguish between the channels containing a $\pi$ or a $K$.

A second method, a counting analysis, is used as a cross check in the measurement of the decay rates (see Sec. 4.6).

### 4.1 Data samples

The full so called Run 1 data-set is used in the analyses described in the following chapters. The detailed data sample used is:

- $20.6 \pm 0.3 \text{ fb}^{-1}$ of on-resonance data corresponding to $(22.6 \pm 0.4) \times 10^6 B\overline{B}$ events.
- $2.61 \pm 0.04 \text{ fb}^{-1}$ off-resonance
10.1 million events (4.85 fb$^{-1}$) $u\pi, d\bar{d}$ and $s\pi$ Monte Carlo

- 6.2 million events (4.77 fb$^{-1}$) $c\bar{c}$ Monte Carlo

- 4.0 million events (4.3 fb$^{-1}$) $\tau^+\tau^-$ Monte Carlo

- 0.94 million events (18 fb$^{-1}$) generic $B$ charmless Monte Carlo

- 16k events of each signal Monte Carlo sample

The $B^- \to D^0\pi^-$ and $c\bar{c} \to D^*X$ decays have also been analyzed for control sample studies. All cuts are tuned using Monte Carlo simulation and off-resonance data samples together with the sidebands of the on-resonance data. Cuts were defined before analyzing the signal band of the on-resonance data-set (blind analysis).

4.2 Event selection

The preselection is done checking the value of a number of tagbits based on some event variables: each event should pass the so called BGFMultiHadron selection which consists of requiring at least 3 charged tracks and $R2 < 0.98$ (see Eq. (4.4) for the definition of this variable).

Then, a specific selector has been studied to set a tagbit (named TagTwoBody) for a very efficient exclusive selection of charmless two-body $B$ decays. It is designed to look separately for $h^+h^-$, $h^\pm K_s^0, \pi^0 h^\pm, \pi^0\pi^0, \pi^0 K_s^0$ and $K_s^0K_s^0$, selected among the charged tracks and the $K_s^0$ candidates reconstructed with loose criteria (see Sec. 3.1.1). For this selector we consider as a $\pi^0$ every cluster in the EMC (where a cluster is a connected set of EMC crystals with an energy deposit) with a raw (not calibrated) energy greater than 1.5 GeV/c$^2$. In order to take into account $\pi^0$’s split in two clusters, we use also a list of pseudo-clusters created by every pair of clusters having their centroids closer than 300 mrad. In order to reduce combinatorics, we loop over charged candidate or neutral cluster lists without considering tracks (clusters) with momentum (energy) less than 1.5 GeV/c (GeV/c$^2$).

A pair of candidate $h^+h^-$ ($h^\pm K_s^0, K_s^0K_s^0$) is accepted when both tracks (track and $K_s^0$, both $K_s^0$) have a momentum between 2.0 and 2.9 GeV/c$^3$, when the sum of their momenta ($p_T^B$ of the $B$ candidate) is in the range 0–0.860 GeV/c, and when the angle $\theta^*$ between the two tracks (track and $K_s^0$, both $K_s^0$) is such that $\cos(\theta^*) < 0.9$. All quantities ($p^*$ and $\theta^*$) are calculated in the center-of-mass (CM) rest-frame.

A pair of candidate $h^\pm\pi^0$ ($\pi^0 K_s^0$) is defined when the sum of the energies of a track and a cluster in the CM rest-frame is between 4.8 and 5.5 GeV and the angle $\theta^*$ between track direction and cluster centroid is such that $\cos(\theta^*) < 0.9$. A pair of candidate $\pi^0\pi^0$ is defined when the sum of the energies of the two clusters in the CM rest-frame is between 4.3 and 5.4 GeV and the angle $\theta^*$ between the cluster centroids is such that $\cos(\theta^*) < 0.9$.

---

1. we use $\pi^+$ ($\pi^0$) mass hypothesis for charged track (neutral cluster)
Table 4-1. Requirements implemented in $TagTwoBody$ selector for the charmless two-body modes.

<table>
<thead>
<tr>
<th>channel</th>
<th>selection</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0 \to h^+h^-$</td>
<td>lists: ChargedTracks, KSLoose ($K^0_S \to \pi^+\pi^-$)</td>
</tr>
<tr>
<td>$B^{\pm} \to h^{\pm}K^0_S$</td>
<td>$p &gt; 1.5 \text{ GeV}/c$</td>
</tr>
<tr>
<td>$B^0 \to K^0_SK^0$</td>
<td>select two oppositely charged tracks (one track and a $K^0_S$)</td>
</tr>
<tr>
<td></td>
<td>if each $2. &lt; p^* &lt; 2.9 \text{ GeV}/c$</td>
</tr>
<tr>
<td></td>
<td>and if $0. &lt; (p_1 + p_2)^* &lt; 0.860 \text{ GeV}/c$</td>
</tr>
<tr>
<td></td>
<td>and if $\theta^<em>$ relative angle between tracks $\cos \theta^</em> &lt; -0.9$</td>
</tr>
<tr>
<td>$B^{\pm} \to h^{\pm}\pi^0$</td>
<td>lists: ChargedTracks, pseudo-EmcCluster and EmcCluster, KSLoose</td>
</tr>
<tr>
<td>$B^0 \to \pi^0\pi^0$</td>
<td>$E &gt; 1.5 \text{ GeV}/c^2$</td>
</tr>
<tr>
<td>$B^0 \to \pi^0 K_S$</td>
<td>select two EmcClusters (one tracks/$K^0_S$ and EmcCluster)</td>
</tr>
<tr>
<td></td>
<td>$4.3 \ (4.8) &lt; (E_1 + E_2)^* &lt; 5.4 \ (5.5)$</td>
</tr>
<tr>
<td></td>
<td>$\theta^<em>$ relative angle between candidates $\cos \theta^</em> &lt; -0.9$</td>
</tr>
</tbody>
</table>

Table 4-2. The tagbit $TagTwoBody$ efficiency evaluated with Monte Carlo simulated samples of signal events.

<table>
<thead>
<tr>
<th>channel</th>
<th>efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^+\pi^-$</td>
<td>$0.86 \pm 0.01$</td>
</tr>
<tr>
<td>$K^0_S\pi^\pm$</td>
<td>$0.68 \pm 0.01$</td>
</tr>
<tr>
<td>$\pi^0\pi^\pm$</td>
<td>$0.71 \pm 0.01$</td>
</tr>
<tr>
<td>$\pi^0\pi^0$</td>
<td>$0.59 \pm 0.01$</td>
</tr>
<tr>
<td>$\pi^0 K^0_S$</td>
<td>$0.56 \pm 0.01$</td>
</tr>
<tr>
<td>$K^0_SK^0$</td>
<td>$0.59 \pm 0.01$</td>
</tr>
</tbody>
</table>

Table 4-1 contains the summary of the above described conditions: the tagbit $TagTwoBody$ is the logical OR of these requirements. In Table 4-2 the efficiencies evaluated with fully simulated Monte Carlo signal events are listed.

The sample passing these tagbit selections remains primarily background. Further selection criteria are employed to greatly reduce the background while maintaining a high efficiency for signal events. Additional cuts are used to ensure that the PID information for each of the candidates is of sufficient quality to be used in the analysis.
4.2.1 Topological Variables

Most of the background can be eliminated by simple kinematic cuts which make use of the differences between $B\bar{B}$ and continuum events, which are the primary source of background. In the CM frame this background typically exhibits a two-jet structure. In contrast, the low momentum and pseudo-scalar nature of $B$ mesons in the decay $\Upsilon(4S) \rightarrow B\bar{B}$ leads to a more spherically symmetric event. Some topological variables can be used in order to distinguish between signal and background events. First of all, the event sphericity $S$ can be defined like [48]:

$$S = \min_{\mathbf{n}} S(\mathbf{n}) = \min_{\mathbf{n}} \frac{3}{N} \sum_{i=1}^{N} \frac{p_{i\perp}^2}{\sum_{i=1}^{N} \frac{p_{i\perp}^2}{}}$$

(4.1)

where index $i$ runs over $N$ tracks of the event, the versor $\mathbf{n}$ spans over all the directions, $p_{i\perp}$ is the component of the momentum that is perpendicular to the versor $\mathbf{n}$ evaluated in the CM rest-frame of the $\Upsilon(4S)$. The versor $\mathbf{n}$ that satisfies the equation 4.1 is called sphericity axis. This function $S(\mathbf{n})$ contains the information on the way momenta are spatially distributed in the event.

Another useful discriminating variable is the event thrust, $T$, defined as:

$$T = \max_{\mathbf{n}} T(\mathbf{n}) = \max_{\mathbf{n}} \frac{\sum_{i \in C_+(\mathbf{n})} \frac{p_i^\parallel}{}}{\sum_{i=1}^{N} \frac{p_i^\parallel}{}}$$

(4.2)

where $p_i^\parallel$ is the component of the momentum that is parallel to the versor $\mathbf{n}$ and $C_+(\mathbf{n})$ is the entire set of tracks whose momenta have the component $p_i^\parallel$ greater than zero. As before, momenta are evaluated in the CM rest-frame of the $\Upsilon(4S)$. The function $T(\mathbf{n})$ represents the preferred direction of the momenta of the tracks in the event: this variable contains information on the jet direction.

In the $e^+e^- \rightarrow q\bar{q}$ production at high energies, the event in the CM rest-frame tends to assume a two-jet-like structure: since all the hadrons in the final state come from hadronization of the high energy $\mathfrak{g}\mathfrak{g}$ state, they have to conserve the four-momentum and thus their flight directions are correlated to the initial $q\bar{q}$ line of flight. This is the reason why variables like sphericity or thrust can be used to discriminate between signal and background events.

These functions though are optimized in case the events really have a two-jet structure. On the other hand, results from QCD show that more than 30% of the events produced in a $e^+e^-$ annihilation in the continuum and at high energies should produce three or more jets in the CM rest-frame. Therefore topological variables that do not depend on one specific event axis can be used to reject also non-two-jet continuum background: an example of such variables are the Fox-Wolfram moments that can be written as [49]:

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\[ H_l = \sum_{ij} \left| \frac{p_i \cdot p_j}{E_{\text{tot}}} \right| P_l(\cos \phi_{ij}) \]  \hspace{1cm} (4.3)

where indices \( i \) and \( j \) run over all the hadrons produced in the event, \( \phi_{ij} \) represents the angle between the particles \( i \) and \( j \) and \( P_l(\cos \phi_{ij}) \) is the Legendre polynomial of the \( l \)-th order. The energy and momentum conservation imposes the conditions \( H_0 \simeq 1 \) and \( H_1 = 0 \). Thus the already quoted variable \( R_2 \) is the ratio of the second order Fox-Wolfram moment over the zero order one:

\[ R_2 = \frac{H_2}{H_0} \]  \hspace{1cm} (4.4)

Left plot in Fig. 4-1 shows the \( R_2 \) distribution for Monte Carlo background and signal events: a cut on \( R_2 \) is useful to reduce the contribution from the \( \tau \tau \) background events.

**Figure 4-1.** Left plot: \( R_2 \) distribution for Monte Carlo events. Right plot: \( |\cos \theta_5| \) distribution for Monte Carlo events.

The first cuts applied are the one in the following *multi-hadron* selection:

- \( R_2 < 0.95 \)
- \( S > 0.01 \).

They remove the majority of two-prong events and, in particular, the sphericity cut rejects additional \( \tau \tau \) background.

After applying the multi-hadron selection cuts, continuum background suppression is achieved also by requiring

- \( |\cos \theta_5| < 0.9 \)
where \( \theta_s \) is the angle between the sphericity axis of the \( B \) candidate and the one of the rest of the event (using all charged tracks and neutral particle candidates which are not used in the \( B \) candidate). Right plot in Fig. 4-1 shows the \(|\cos \theta_s|\) distribution for Monte Carlo background and signal events: a cut at 0.9 removes the background peaking at 1 in this variable. All the cuts previously described are called the two-body standard selection.

### 4.2.2 \( B \) candidate selection: kinematic Variables

Candidate \( B \) mesons are reconstructed by forming all pairs of oppositely charged tracks, or a charged track and a \( K_0^0 \) or \( \pi^0 \) candidate, or two \( K_0^0 \) or \( \pi^0 \) candidates. The charged tracks used to form a \( B \) candidate are selected on the basis of the GoodTracksAccLoose criteria:

- \( N(DCH \text{ hits}) \geq 12 \)
- \( d_0 < 1.5 \text{ cm}, \ z_0 < 10 \text{ cm} \)
- \( p_T > 100 \text{ MeV/c} \)
- \( 0.410 < \theta < 2.540 \text{ rad} \)

where \( N(DCH \text{ hits}) \) is the number of \( DCH \) hits, \( d_0 \) is the distance in the \( (x, y) \) plane of the POCA (Point Of Closest Approach) of the track from the measured beam-spot, \( z_0 \) is the \( z \) position of the POCA, \( p_T \) is the transverse momentum of the track and \( \theta \) is its polar angle.

Instead, the \( K_0^0 \) daughters are selected as described in Sec. 3.3. The vertex algorithm is used to estimate the decay vertex of the candidate \( B \). The momentum vectors of the daughter particles are recalculated using this point as their production vertex. Simple four-vector addition, assuming the pion mass for the charged tracks, is then used to form the \( B \) candidate four-vector. A loose mass cut of \( \pm 600 \text{ MeV}/c^2 \) around the PDG \( B^0 \) mass value [14] is applied as part of the preselection cuts.

We define the beam energy-substituted mass [50]:

\[
m_{ES} = \sqrt{\left(\frac{1}{2} s + \mathbf{p}_0 \cdot \mathbf{p}_B\right)^2 / E_0^2 - p_B^2},
\]

where \( \sqrt{s} \) and \( E_0 \) are the total energies of the \( e^+e^- \) system in the CM and lab frames, respectively; \( \mathbf{p}_0 \) and \( \mathbf{p}_B \) are the momentum vectors in the lab frame of the \( e^+e^- \) system and the \( B \) candidate, respectively; and \( p_B \) is the magnitude of the \( B \) candidate momentum in the lab frame. Evaluated in the CM frame, this variable looks like:

\[
m_{ES} = \sqrt{(\sqrt{s}/2)^2 - p_B^2}
\]

which clarifies its physical meaning. The advantage of using the definition in the lab frame with respect to the one computed in the CM frame is that the first does not require assigning mass hypotheses to the charge tracks.

The mean value of \( m_{ES} \) and its Gaussian width \( \sigma(m_{ES}) \) are determined from a sample of fully reconstructed \( B^- \rightarrow D^0\pi^- \) decays (see next section). The values used are \( m_{ES} = 5.2800 \pm 0.0005 \text{ GeV}/c^2 \) and

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4.2 Event selection

Figure 4-2. Correlation between $m_{ES}$ and $\Delta E$ variables for Monte Carlo $B^0 \rightarrow \pi^+\pi^-$ signal events.

$\sigma(m_{ES}) = 2.6 \pm 0.2$ MeV/$c^2$. To an excellent approximation, the shapes of the $m_{ES}$ distributions for all fully-reconstructed $B$ decays to final states with charged tracks only are identical. The preselection requires $5.20 < m_{ES} < 5.30$ GeV/$c^2$.

The energy difference $\Delta E$ is defined as

$$\Delta E = E_B^* - \sqrt{s}/2,$$

where $E_B^*$ is the $B$ candidate energy in the CM frame. Signal events are Gaussian distributed in $\Delta E$ with a mean near zero, while the continuum background events fall roughly linearly over the region of interest. For those analyses including charged tracks in the final states, since the pion mass is assigned to the charged tracks, the $B \rightarrow K^+\pi^-$, $K^+K^-$ decays together with the $B \rightarrow \pi K^0_s$ and $K K^0_s$ decays have $\Delta E$ shifted from zero by an amount depending on the momenta of the tracks. From Monte Carlo simulation we find average shifts of $-45(-41)$ and $-91$ MeV for the $K^+\pi^-(K^0_sK^\pm)$ and $K^+K^-$ decays, respectively (this is described in detail in Sec 4.6.1). The resolution on $\Delta E$ is mode dependent and dominated by momentum resolution: the estimate of the width is taken from Monte Carlo simulated signal data and the observed difference in widths between data and Monte Carlo in $B^{-} \rightarrow D^0\pi^-$ decays is used to scale the Monte Carlo value of all the charmless channels to agree with data.

This pair of kinematic variables is chosen because it satisfies two criteria: it maximizes the use of the available information and minimizes the correlation between the two variables [51]. The main reason for requiring $m_{ES}$ and $\Delta E$ not to be correlated is the use of these variables in the maximum likelihood fit. Fig. 4-2 shows the correlation between the two variables.

4.2.2.1 Control Sample $B^\pm \rightarrow D^0\pi^\pm$

In order to study shape variables and mass resolutions, $B^- \rightarrow D^0\pi^- (D^0 \rightarrow K\pi)$ candidates have been reconstructed in the on-resonance data sample and compared to Monte Carlo simulated data.

Strategy and Tools for Charmless Two-body B Decays Analysis
For consistency, the same charmless two-body selection is applied to the $B^- \rightarrow D^0 \pi^-$ decays. $B$ and $D^0$ candidates are reconstructed using the vertex algorithm and the mass constraint is applied to the $D^0$. The kaon from the $D^0$ has been required to be selected by the loose kaon selector. Figure 4-3 shows the $m_{ES}$ and $\Delta E$ distributions of the selected events in the signal region ($|\Delta E| < 140$ MeV). Fits to these distributions indicate approximately 575 $B$’s in the $m_{ES}$ peak.

When background subtraction is required, the signal region is defined as $2.5 \sigma(m_{ES})$ around $B$ $m_{ES}$ of 5.280 GeV ($\sigma(m_{ES}) = 2.6$ MeV). The side-band is taken as $5.20 < m_{ES} < 5.27$ GeV. Proper normalization of this side-band to the signal region is obtained using the ARGUS background parameterization (see Eq. 4.12).

For validation of $\Delta E$ resolution, a Gaussian plus first other polynomial fit to $\Delta E$ in data indicates a resolution of $22.3 \pm 0.2$ MeV. In Monte Carlo the resolution is found to be $17.10 \pm 0.04$ MeV.

### 4.3 Background fighting

In addition to the previous defined topological variables, a Fisher discriminant technique is used to separate signal from background. The Fisher discriminant $F$ is calculated from a linear combination of $N$ discriminating variables $x_i$,

$$F = \sum_{i=1}^{N} \alpha_i x_i,$$

where the coefficients $\alpha_i$ are called Fisher coefficients. They are chosen to maximize the statistical separation between signal ($S$) and background ($B$) events through the function $(S-B\hat{F})/(S+B)$. The coefficients

---

**Figure 4-3.** $m_{ES}$ and $\Delta E$ distributions for $B^- \rightarrow D^0 \pi^-$ candidates.
4.3 Background fighting

Figure 4-4. Comparison of CLEO cones between off-resonance data and the Monte Carlo sample used to train the Fisher discriminant.

are defined:

\[
\alpha_i = \sum_{j=1}^{N} (U_{ij}^b + U_{ij}^s)^{-1}(\mu_j^b - \mu_j^s)
\]

where \(U_{ij}^b\) and \(U_{ij}^s\) are the elements of the covariance matrices for the background (b) and signal (s) events, and \(\mu_j^b\) and \(\mu_j^s\) are the mean values that the \(x_j\) variables assume for background (b) and signal (s). The coefficients (the covariant matrix elements and the mean values) have to be determined training the algorithm on large samples of Monte Carlo simulated events (or off-resonance data or side-bands for the background components).

In this analysis, the discriminating variables \(x_j\) have been chosen to be nine energy cones, the same variables used in the CLEO analysis. The energy cones are the scalar sum of the momenta of all charged and neutral particles in the rest of the events (i.e. excluding the \(B\) decay products) flowing into nine concentric cones centered on the \(B\) candidate thrust axis in the CM frame. Each cone subtends an angle of 10° and is folded to combine the forward and backward intervals (see drawing in Fig. 4-4). More energy will be found in the cones nearer the candidate thrust axis in jet-like continuum background events than in the more isotropic \(B\overline{B}\) events. A variety of discriminating variables have been considered in addition to the cones, but detailed comparisons show no significant gain.

The Fisher algorithm is trained on a 1.4 fb\(^{-1}\) sample of continuum Monte Carlo events: Fig. 4-4 shows good agreement in all nine cones between off-resonance and Monte Carlo data. A sample of 2000 \(B^0 \rightarrow \pi^+ \pi^-\)
Monte Carlo events is used to train the signal Fisher output. The training is validated in Fig. 4-5, where the Fisher output for $B^0 \rightarrow \pi^+ \pi^-$ (solid histogram) is compared to a sample of $B^- \rightarrow D^0 \pi^-$ reconstructed in a 7.6 fb$^{-1}$ on-resonance sample (filled squares), and off-resonance data (open squares) is compared to a sample of continuum Monte Carlo events independent from the training sample (dashed histogram). The comparison is made after applying the standard selection cuts (Sec. 4.1), the plots are normalized to equal area and the curves are double Gaussian fits. The $B^- \rightarrow D^0 \pi^-$ distribution has been background subtracted using $m_{ES}$ side-band. Note that the two signal distributions are consistent and this demonstrates that Fisher variable distribution is mode independent and just related to the event topology (jet-like or isotropic).

The performance of the Fisher discriminant is demonstrated in the right plot in Fig. 4-5 by plotting total signal efficiency in the mode $B^0 \rightarrow h^+ h^-$ vs. the number of background events expected in a signal region $M_B = M_B$(PDG)$ \pm 2.5 \sigma$ for 1 fb$^{-1}$ of data. For example, with a signal efficiency of 30% one would expect approximately 10 background events in the signal region per fb$^{-1}$.

Many cross-checks and systematic studies have been performed to test the robustness of the Fisher output. The Fisher output has also been compared with the neural net and likelihood methods. No significant difference is observed. In summary, the Fisher technique is robust and effective in separating signal from background.
In the case of the global maximum likelihood fit (see Sec. 4.6), no cuts are applied to the Fisher discriminant. Instead, signal-background discrimination is achieved by using $\mathcal{F}$ in the fit itself. Comparison of $\mathcal{F}$ for signal and background events is described in detail in section 4.6.1.

In case of the counting analysis, a cut on the Fisher discriminant output is applied and it is chosen in order to optimize the statistical significance $S^2/(S + B)$, where $S$ and $B$ are the number of expected signal and background events, respectively. After this cut, one should check that side-band background shape is well-modeled by the ARGUS function fitted before applying these cuts, giving confidence that the same function can be used (see Sec. 4.6).

### 4.4 PID selection

The difference between a $K$ or a $\pi$ in the final state taken here into account is apparent only in the reconstructed $\Delta E$, for which there is a separation of less than $2\sigma$. The particle identification capabilities of the BABAR detector provide additional means to distinguish the two decays. Of primary importance is the $\text{DIRC}$ information since the momenta of the two daughter tracks in these decays are in a region where the mean $dCH/dE$ for kaons and pions differ by only about $1\sigma$. In principle, the $\text{DIRC}$ can provide better than $3\sigma$ separation of pions and kaons throughout the momentum region of the daughter tracks.

The maximum likelihood fit makes direct use of the Čerenkov angle, $\theta_c$, reconstructed by the $\text{DIRC}$. Each track is assigned a likelihood to be a pion or kaon based on the value of the reconstructed $\theta_c$. As a cross check, a second complementary method is pursued. It employs particle selector algorithms which provide lists of kaon and pion tracks that are used to separately identify the different final state modes.

A cut on the number of signal photons observed in the $\text{DIRC}$ is used to improve the $\theta_c$ resolution and reduce the size of non-Gaussian tails. The cut $N_{\text{sig}}(\gamma) > 5$ is used, where $N_{\text{sig}}(\gamma)$ is the number of observed signal photons for the track. Protons are explicitly removed with the cut $\theta_c(p) > 10\text{ mrad}$, where $\theta_c(p)$ is the expected mean value of $\theta_c$ for a proton of a given momentum. Electrons are removed by rejecting tracks which pass a tight selector criteria.

The performance of the PID cuts are studied in the actual data using a pure sample of kaons and pions obtained from a control sample of $D^{*+} \rightarrow D^0\pi^+$, with $D^0 \rightarrow K^-\pi^+$.

### 4.4.1 $e^+ e^- \rightarrow D^{*+}X$ control sample

In order to assess and parameterize the performance of the particle ID methods, without relying on Monte Carlo simulation, one must identify a source of pions and kaons, the selection of which does not utilize particle ID from the $\text{DIRC}$. An ideal control sample consists of the daughter tracks from $D^0 \rightarrow K^-\pi^+$ decays in the reaction $D^{*+} \rightarrow D^0\pi^+ \rightarrow (K^-\pi^+)\pi^+$. The $\pi$ ($K$) track is always the one with the same (opposite) charge as the $D^*$ and cutting on the small $D^{*+} - D^0$ mass difference ensures that there is virtually no contamination from incorrectly reconstructed $D^0$ candidates (i.e. a true $D^0 \rightarrow K^-\pi^+$ reconstructed as a $D^0 \rightarrow \pi^-K^+$ candidate and then combined with a random track to form a $D^{*-}$

---

**Strategy and Tools for Charmless Two-body B Decays Analysis**
candidate). Candidates in $D^0$ mass side-band regions are used for background subtraction. The resulting sample is used to parameterize the $\theta_c$ distributions for pions and kaons.

A simple set of cuts were used to select a very clean sample of $D^{*+}$ decays in which either the $K$ or $\pi$ track from the $D^0$ decay has a momentum in the range $1.75 - 4.25 \text{ GeV}/c$, covering 90% of the momentum range of the daughters of the charmless two-body decays. The cuts are summarized in Table 4-3. The loose good-track$^2$ definition is used for selecting $D^0$ daughter tracks and a very-loose good-track$^3$ definition is used for the slow pion from the $D^{*+}$ decay. $\Delta M$ is the measured mass difference between the $D^{*+}$ candidate and the $D^0$ candidate, $\sigma_{\Delta M}$ is the measured resolution on this quantity, $M(K\pi)$ is the reconstructed $D^0$ mass and $\sigma_{M}$ is the resolution on $M(K\pi)$. The mass resolutions are measured to be $0.6 \text{ MeV}/c^2$ for $\sigma_{\Delta M}$ and $9 \text{ MeV}/c^2$ for $\sigma_{M}$. The quantity $\cos \theta^*_K$ is the cosine of the angle of the kaon track with respect to the $D^0$ flight direction, measured in the $D^0$ center-of-mass system. For signal decays, this distribution is flat, whereas the combinatorial background is peaked in the forward and backward directions.

The same set of cuts is used to select a sample of $D^{*+}$ in the Monte Carlo simulated data. The $\theta_c$ parameterizations obtained from this sample are used in constructing the PDFs for fits to Monte Carlo events. This sample is also used to check that the Monte Carlo accurately simulates the efficiency of the PID cuts. This is demonstrated in Figs. 4-6 which display the efficiencies of the PID $\theta_c > 0$ cut used in the $B \rightarrow \pi^+\pi^-$ analysis for kaons and pions, respectively, for both the data and Monte Carlo control samples. There is good agreement between the two samples. Figs. 4-7 compare the efficiencies obtained from the Monte Carlo $D^{*+}$ control sample to those obtained directly from Monte Carlo $B \rightarrow \pi^+\pi^-$ decays.

Good agreement is observed between the efficiency of the PID cuts in Monte Carlo simulated events and that obtained from this control sample. Thus, the PID efficiencies obtained from Monte Carlo signal events are used without any corrections.

<table>
<thead>
<tr>
<th>Cut</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
</tr>
<tr>
<td>$</td>
</tr>
<tr>
<td>$p^*(D^0) &gt; 2.5 \text{ GeV}/c$</td>
</tr>
<tr>
<td>$\cos \theta^*_K &lt; 0.8$</td>
</tr>
<tr>
<td>$1.75 &lt; p(\pi) &lt; 4.25 \text{ GeV}/c$ or $1.75 &lt; p(K) &lt; 4.25 \text{ GeV}/c$</td>
</tr>
</tbody>
</table>

Table 4-3. Cuts used to select a control sample of $D^{*+} \rightarrow K^-\pi^+\pi^+$ decays.

---

$^2$see Sec. 4.2.2

$^3$The very loose good track selection does not include the $p_T$ and $N(D\text{CH})$ cuts with respect to the loose good track one in Sec. 4.2.2.
4.4 PID selection

Figure 4-6. Left plot: the efficiency versus momentum of the $\theta_c > 0$ cut for kaons in the $D^{*+}$ control sample in data (filled circles) and Monte Carlo simulation (open diamonds). Right plot: the efficiency versus momentum of the $\theta_c > 0$ cut for pions in the $D^{*+}$ control sample in data (filled circles) and Monte Carlo simulation (open diamonds).

Figure 4-7. Left plot: the efficiency versus momentum of the $\theta_c > 0$ cut for kaons in the Monte Carlo $D^{*+}$ control sample (filled circles) and Monte Carlo simulated $B \rightarrow h^+ h^-$ decays (open diamonds). Right plot: the efficiency versus momentum of the $\theta_c > 0$ cut for pions in the Monte Carlo $D^{*+}$ control sample (filled circles) and Monte Carlo simulated $B \rightarrow h^+ h^-$ decays (open diamonds).

4.4.2 Selector-based PID

The selector method of particle ID attempts to identify kaons and pions on a per-track basis by cutting on a likelihood function derived using information from the SVT, DCH and DIRC subdetectors. The standard BaBar selector is called KaonSMSSelector (hereafter referred to as SMS) and provides decisions based on the comparison of the likelihoods for different mass hypotheses: $K$, $\pi$, and proton. Each likelihood is composed of products of individual subdetector likelihoods for the given hypothesis:

$$L(h) = L^{SVT}(h) \ast L^{DCH}(h) \ast L^{DIRC}(h),$$

where $h = (\pi, K, p)^4$. The $SVT$ and $DCH$ likelihoods are calculated assuming Gaussian $dE/dx$ distributions while the $DIRC$ likelihood is the product of the Čerenkov angle Gaussian likelihood and the Poissonian likelihood for the number of Čerenkov photons measured compared to expected for each hypothesis. In addition, the $DIRC$ is used in veto mode for particles below the Čerenkov threshold for kaons.

The SMS selector provides several levels of purity:

$p$ stands for proton.
Figure 4-8. **Left plots:** $K$-efficiency identification and $\pi$-contamination for SMS Loose selector as a function of the momentum for tracks within DIRC acceptance. **Right plots:** $\epsilon^{k \rightarrow k}$ for low momenta ($< 1.8 \text{ GeV}/c$) and high momenta ($> 1.8 \text{ GeV}/c$) as a function of $\cos(\theta)$ for SMS Loose selector.

- Very Loose: $L(\pi) > L(K)$
- Loose: $L(K) > rL(\pi)$ and $L(K) \geq L(p)$
- Tight and Very Tight: $L(K) > rL(\pi)$ and $L(K) > L(p)$
- NotAPion: $L(\pi) > rL(K)$ and $L(\pi) > rL(p)$,

where parameterization of $r$ as function of momentum depends on the criteria. The counting analysis used here as a cross check has been performed using the Loose selection. In the momentum region of interest (above $1.8 \text{ GeV}/c$) this selection uses only the DIRC with two different $r$ values: $r = 1$ for momenta $< 2.7 \text{ GeV}/c$ and $r = 80$ for momenta $> 2.7 \text{ GeV}/c$. A track is considered a kaon when it satisfies the Loose selection, otherwise it is considered to be a pion. To eliminate protons, an additional cut on the Čerenkov angle $(\theta_c - \theta_c(p))/\sigma(\theta_c) > 2$ is applied, where $\theta_c(p)$ is the expected angle for a proton.

In order to measure the branching fraction of the $K\pi$ and $\pi\pi$ decays the efficiency and contamination of $K$-identification need to be well known. These quantities are obtained using the $D^+$ control sample described in Sec. 4.4.1.

Left plots in Fig. 4-8 show the efficiencies $\epsilon^{k \rightarrow k}$ and $\epsilon^{\pi \rightarrow k}$ for the Loose selector as a function of the momentum for tracks inside the DIRC acceptance. Table 4-4 reports the integrated efficiencies assuming a flat momentum distribution between 1.8 and 4.0 GeV/c.

**Marcella Bona**
<table>
<thead>
<tr>
<th>Selector</th>
<th>$\varepsilon_{k \rightarrow k}$</th>
<th>$\varepsilon_{\pi \rightarrow k}$</th>
<th>$\varepsilon_{k \rightarrow \pi}$</th>
<th>$\varepsilon_{\pi \rightarrow \pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>SMS(Loose)</td>
<td>89.0±0.4%</td>
<td>6.3±0.3%</td>
<td>11.0±0.3%</td>
<td>93.7±0.4%</td>
</tr>
</tbody>
</table>

Table 4-4. Selector efficiencies for a single track within the DIRC acceptance.

With exception of $\theta$-momentum correlation, the efficiency shows no significant $\theta$ angle dependence for high momenta. The fall in efficiency for low momenta vertical tracks almost disappears above 1.8 GeV/c (see the right plots in Fig. 4-8).

### 4.5 Tracking Corrections

The difference in track reconstruction efficiency between data and Monte Carlo simulated events is taken into account by following the standard procedure outlined by the B\(\bar{B}\) AR Tracking Efficiency Task Force [52]. Look-up tables are used to scale the reconstruction efficiency for each track in the Monte Carlo sample. The scale factors are functions of the $p_T$, polar and azimuthal angles of the track, as well as the track multiplicity of the event. The overall correction factor to the efficiency is estimated on a mode by mode basis.

### 4.6 Analysis methods

The analyses are based on an **unbinned maximum likelihood fit** to determine from the data yields and asymmetries. The signal yields are divided by the efficiency estimates and by the number of neutral $B$ mesons produced in the data-set in order to obtain branching ratio measurements.

The distributions for $m_{ES}$, $\Delta E$ and $F$ provide good discrimination between signal and background, while the use of the Čerenkov angles, $\theta_c$, allows the fitter to measure the particle ID content of the $B$ candidates. The quantity $\Delta E$ provides additional separation power between signal modes which differ for PID contents of their final states.

The likelihood, $\mathcal{L}$, for a given candidate $j$ is obtained by summing the product of event yield $n_k$ and probability $P_k$ over all possible signal and background hypotheses $k$. The $n_k$ are determined by maximizing the extended likelihood function $\mathcal{L}$

$$\mathcal{L} = e^{-\sum_{i=1}^{M} n_i} \prod_{j=1}^{N} \left[ \sum_{k=1}^{M} n_k P_k(\bar{x}_j; \bar{\alpha}_k) \right]$$  \hspace{1cm} (4.8)

where $P_k(\bar{x}_j; \bar{\alpha}_k)$ is the probability for candidate $j$ to belong to category $k$ (of $M$ total categories), based on its characterizing variables $\bar{x}_j$ and parameters $\bar{\alpha}_k$ that describe the expected distributions of these variables. The probabilities $P_k(\bar{x}_j; \bar{\alpha}_k)$ are evaluated as the product of probability density functions (PDFs) for each of the independent variables $\bar{x}_j$, given the set of parameters $\bar{\alpha}_k$:

$$P_k = P_k^{M_{ES}} P_k^{\Delta E} P_k^F P_k^{\theta_c}.$$  \hspace{1cm} (4.9)

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The Poisson factor in Eq. 4.6 is the probability of observing \( M \) total events (the number of events used in the fit) when \( M' \) are expected. The quantity \(-2 \log \mathcal{L}\) is minimized, which is equivalent to maximizing \( \mathcal{L} \) itself, with respect to the fit variables.

As a cross-check, a counting analysis is performed: this is very similar to the likelihood one, but differs in its treatment of PID (see Sec. 4.4.2). Standard BaBar particle selector algorithms are used to separate the selected sample into subsamples which have identified \( \pi \)'s and \( K \)'s in the final states. A cut is placed on \( \mathcal{F} \). The fit includes events passing all cuts except the requirement that the tracks have an associated \( q \) measurement.

A maximum likelihood fit which uses all quantities except \( q \) and \( \mathcal{F} \) is then used to determine the signal yields in each of the subsamples. These yields are corrected by an efficiency/cross-feed matrix which takes into account both the selector efficiencies and residual cross feed of the other signal decays into each of the subsamples. The corrected number of candidates are then normalized to the total efficiency of the selection cuts and to the total number of \( B \bar{B} \) pairs: the branching fraction is therefore determined. The results from this analysis are compared with the official results, described above.

In the following section, descriptions of the PDFs, as well as the samples used to estimate them, are presented.

### 4.6.1 Sample definitions

The functional forms of the PDFs of the variables introduced in the previous section are derived from data samples that are independent of the sample used in the fit. These include: off-resonance data, on-resonance data from \( \Delta E \) side-bands, control samples of fully reconstructed \( B^- \to D \pi^- \) decays, control samples of \( c \bar{c} \to D^{**}X \) decays and Monte Carlo simulated events. The definitions of the samples used in this analysis are described below, followed by descriptions of how the PDFs used in the fit are derived from these samples.

**Monte Carlo simulated events**

A large sample of Monte Carlo simulated events is used to study both background and signal distributions and selection efficiencies.

**\( \Delta E \) side-band data:**

\( B \) candidates are selected in a \( \Delta E \) range which is mode dependent. Let’s consider the example of the \( h^+ h^- \) decay mode in which case the range considered is \( |\Delta E| < 0.42 \) GeV. The \( \Delta E \) variable is used to subdivide the data into two samples:

\[
0.15 < |\Delta E| < 0.42 \text{ GeV} \quad \text{(Side – band)} \quad (4.10)
\]

\[
-0.15 < \Delta E < 0.15 \text{ GeV} \quad \text{(Signal)} \quad (4.11)
\]

The same can be done in each mode. The signal range defines the region in which 100% of the signal lies. The side-band region is used to study characteristics of the background.
**$m_{ES}$ side-band data:**

$B$ candidates are selected in the range $5.2 < m_{ES} < 5.3 \text{ GeV}/c^2$. The $m_{ES}$ side-band sample is defined to be all candidates which are in the signal $\Delta E$ region, given above, and have $5.20 < m_{ES} < 5.27 \text{ GeV}/c^2$.

**charmless $B\bar{B}$ Monte Carlo:**

The charmless $B\bar{B}$ Monte Carlo sample is used to estimate the amount of feed-down from other charmless $B$ decays. It is found to be negligible\(^5\) in the $\Delta E$ signal region.

**Off-resonance data:**

Data taken 40 MeV below the $\Upsilon(4S)$ resonance is used to study continuum $e^+e^- \rightarrow \phi\phi$ background, free from any $B\bar{B}$ contamination.

**$B^- \rightarrow D^0\pi^-$ control sample:**

The resolution of $m_{ES}$ and $\Delta E$ for charmless two-body decays can be studied using a sample of fully reconstructed $B^- \rightarrow D^0\pi^-$ decays, where $D^0 \rightarrow K^-\pi^+$ (see Sec. 4.2.2.1). The $m_{ES}$ resolution is dominated by the spread in the beam energies for $B$ decays involving only charged tracks in the final state. The relatively large statistics of the $B^- \rightarrow D^0\pi^-$ signal can be used to accurately measure both the mean and resolution of $m_{ES}$ for the $B \rightarrow l^+l^-$ or $B \rightarrow K^0_S\pi^-\pi^+$ signals. The $\Delta E$ resolution, on the other hand, is dominated by the track momentum resolution and differs between the control sample and the signal, due to the softer momentum spectra of the tracks in the control sample. However, a comparison of the $\Delta E$ resolutions obtained in data and Monte Carlo simulated $B^- \rightarrow D^0\pi^-$ decays can be used to estimate the amount of additional momentum smearing that should be applied to Monte Carlo simulated decays in order to accurately represent what is expected in the data.

**$c\bar{c} \rightarrow D^{*+}X$ control sample:**

A very pure sample of kaon and pion tracks is derived from reconstructed $D^{*+} \rightarrow D^0\pi^+ \rightarrow (K^-\pi^+)\pi^+$ decays, as already described in Sec. 4.4.1. The $\pi(K)$ track is always the one with the same (opposite) charge as the $D^*$. The control sample used is limited to those decays for which one of the $D^0$ daughter tracks is in the momentum range relevant for two-body decays: 1.75−4.25 GeV/c. This sample is used to evaluate and parameterize the $\theta_e$ measurement from the DIRC for high momentum tracks.

Table 4-5 summarizes the functional forms used for the PDFs and the samples from which they are derived. Details of the PDFs are given in the following subsections. In all cases, reliance on Monte Carlo simulated data was avoided as much as possible.

### 4.6.2 Beam energy-substituted mass $m_{ES}$

The background shape in $m_{ES}$ is parameterized using the ARGUS function [53]:

$$
\frac{dN}{dN_{m_{ES}}} = N \cdot m_{ES} \cdot \sqrt{1 - x^2} \cdot \exp \left( -\xi \cdot (1 - x^2) \right),
$$

(4.12)

where $x = m_{ES}/m_{\text{max}}$ and the parameter $\xi$ is determined from a fit. The end-point of the ARGUS curve, $m_{\text{max}}$, is determined in a mode-independent way by finding the value which minimizes the $\chi^2$ of the $\xi$

\(^5\)This is not true in the $\pi^0\pi^0$ and $\pi^0\pi^0$ decay modes, which are not taken into account in the following.
<table>
<thead>
<tr>
<th>Fit Variable</th>
<th>Shape</th>
<th>Samples Used</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal $m_{ES}$</td>
<td>Gaussian</td>
<td>$D^0\pi^-$ (signal MC)</td>
</tr>
<tr>
<td>Background $m_{ES}$</td>
<td>ARGUS</td>
<td>Side-band (Off-res, MC $q\overline{q}$)</td>
</tr>
<tr>
<td>Signal $\Delta E$</td>
<td>Gaussian</td>
<td>$D^0\pi^-$ (signal MC)</td>
</tr>
<tr>
<td>Background $\Delta E$</td>
<td>Quadratic</td>
<td>Side-band (Off-res, MC $q\overline{q}$)</td>
</tr>
<tr>
<td>Signal Fisher</td>
<td>Double Gaussian</td>
<td>signal MC ($D^0\pi^-$)</td>
</tr>
<tr>
<td>Background Fisher</td>
<td>Double Gaussian</td>
<td>Side-band (Off-res, MC $q\overline{q}$)</td>
</tr>
<tr>
<td>Kaon $\theta_c$</td>
<td>Gaussian</td>
<td>$D^{<em>+}$ (MC $D^{</em>+}$, signal MC)</td>
</tr>
<tr>
<td>Pion $\theta_c$</td>
<td>Gaussian</td>
<td>$D^{<em>+}$ (MC $D^{</em>+}$, signal MC)</td>
</tr>
</tbody>
</table>

Table 4-5. Summary of functional forms of PDFs used in the fit and the samples used to obtain them. The samples in parentheses represent additional samples which were used as consistency checks and provide alternative parameterizations that can be used for studies of systematics.

fit: the chosen value is $m_{\text{max}} = 5.2895$ GeV/c$^2$. The off-resonance and Monte Carlo simulated $q\overline{q}$ data samples are used to demonstrate that the $m_{ES}$ distribution obtained from the on-resonance side-band sample accurately represents the shape of the background in the on-resonance signal region.

As discussed above, the shape of the $m_{ES}$ distribution for signal events can very reliably be taken directly from the $m_{ES}$ distribution of fully reconstructed $B^- \rightarrow D^0\pi^-$ decays. This is demonstrated in the left plot in Fig. 4-9, which displays the $m_{ES}$ distribution for Monte Carlo simulated $B \rightarrow \pi^+\pi^-$ and $B^- \rightarrow D^0\pi^-$ decays. Since there is good agreement between the two modes, we use the $m_{ES}$ distribution from $B^- \rightarrow D^0\pi^-$ decays in data, displayed in the right plot in Fig. 4-9, to parameterize the $m_{ES}$ PDFs. The distribution is fitted with a Gaussian for the signal and an ARGUS function for the background. The fit result gives $\langle m_{ES} \rangle = 5.280$ GeV/c$^2$ and $\sigma(m_{ES}) = 2.6$ MeV/c$^2$.

4.6.3 Energy difference $\Delta E$

As was done for $m_{ES}$, the on-resonance side-band data are used to determine the shape of $\Delta E$ for background in the signal region. A second order polynomial is found to give the best fit results: an example is given in Fig. 4-10. Also shown are the distributions of $\Delta E$ for off-resonance data and Monte Carlo simulated continuum events. There is good agreement between the shapes of all three samples.

The $B^- \rightarrow D^0\pi^-$ control sample is used to understand the $\Delta E$ resolution. The $\Delta E$ distributions for $B^- \rightarrow D^0\pi^-$ decays in data and Monte Carlo simulated data are shown in Fig. 4-11. The Monte Carlo distribution is best fit by the sum of two Gaussians, but the statistics are not large enough in the data sample to perform a reliable double Gaussian fit. Thus, in fitting the data $\Delta E$ distribution, the relative area of the wider Gaussian and its width are fixed to the values obtained from the Monte Carlo distribution. The combinatorial background in the $\Delta E$ distribution is subtracted off using $m_{ES}$ side-band data. What remains
is the $B^- \rightarrow D^0\pi^-$ signal as well as a large “shoulder” to the left of the signal, which is due primarily to fake $B$ decays.

The widths of the narrower Gaussians are $15.35 \pm 0.25$ MeV and $19.13 \pm 0.75$ MeV for Monte Carlo simulated decays and for Run 1 data, respectively. From this comparison, one estimates a 25% degradation of the $\Delta E$ resolution for data, with respect to the Monte Carlo simulation. In addition, the data $\Delta E$ distribution is observed to be offset from zero by $-5.4 \pm 0.9$ MeV. Offsets of the order $-2$ to $-5$ MeV are observed in other fully reconstructed $B$ decays as well.

Because this is such a considerable correction factor, a large range of possible $\Delta E$ resolutions is used in computing the associated systematic uncertainty: the lower bound is chosen from using twice the uncertainty on the correction factor, while the upper bound is chosen to be conservative adding the entire correction factor. We also assume that in data the reconstructed $\Delta E$ is shifted downward by 5 MeV, the same amount that is observed for the $B^- \rightarrow D^0\pi^-$ data sample.

As was described in section 4.2.1, the pion mass is assigned to the charged tracks when forming a $B$ candidate and calculating $\Delta E$. Therefore, modes with a $K$ in the final state will have a $\Delta E$ value which is not centered at zero, but is shifted to negative values by a quantity which depends on the momenta of the kaon track(s). This is due to the fact that the candidate energies are calculated in the CM system and the boost to that frame depends on the mass hypotheses of the tracks. On average, the mean $\Delta E$ value for one or two $K$ in the final state is $-45(-91)$ MeV. The variation due to the boost effect is of the order $\sim \pm 15$ MeV.

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The distribution of $\Delta E$ for the entire region of on-resonance data, side-band and signal, is given by the points with error bars in both plots. The solid curve represents a fit of a second order polynomial to the side-band regions only ($|\Delta E| > 0.15$). The histogram in the upper plot is off-resonance data and the histogram in the lower plot is continuum Monte Carlo data (both normalized to the same area as the on-resonance distribution).

The $\Delta E$ PDF for a decay with a $K$ in the final state consists of a Gaussian with a mean given by the following analytical form [54]:

$$
\langle \Delta E \rangle = -\gamma_{\text{boost}} \times \left( \sqrt{M_K^2 + p^2} - \sqrt{M_{\pi}^2 + p^2} \right),
$$

where $p$ is the momentum of the assumed kaon track.

### 4.6.4 Fisher output $\mathcal{F}$

The distribution for $\mathcal{F}$ for the background is determined from events in the $m_{ES}$ side-band region of the on-resonance sample since the statistics are large and the sample is free from contamination from $B\bar{B}$ events. The distribution is parameterized by the sum of two Gaussians, as shown in Figs. 4-12. The points are the $m_{ES}$ side-band data and the overlaid histograms are the distributions obtained from off-resonance and Monte Carlo simulated continuum events, using the full $m_{ES}$ region (5.2-5.3 GeV/$c^2$). These alternative parameterizations are used in computing systematic uncertainties due to the $\mathcal{F}$ parameterization.
4.6 Analysis methods

Figure 4-11. The $\Delta E$ distributions for fully reconstructed $B^+ \to D^0 \pi^- (D^0 \to K^- \pi^+)$ decays in (left) Monte Carlo simulated data, and (right) Run 1 data. The fits are described in the text.

The distribution of $F$ for the signal is very similar for all the signal modes, as well as for $B^+ \to D^0 \pi^-$ events, and is found to be well-modelled by the Monte Carlo simulation (see Fig. 4-5). Two Gaussians are used to describe the distribution. Monte Carlo simulated signal decays are used to describe the Fisher PDF for signal.

4.6.5 Pion and kaon $\theta_c$

The $\theta_c$ PDFs are determined by forming the $\theta_c - \theta'_c$ distributions for the kaon and pion tracks of the $D^0$ decays in the $D^{*+}$ control sample, in $\cos \theta$ slices in the range $-1.0 \to 1.0$, where $\theta$ is the polar angle of the track and $\theta'_c$ is the expected Čerenkov angle: $\theta'_c = \cos^{-1} (1/\langle n_\beta \rangle)$ ($n = 1.473$ is the mean index of refraction of the quartz bars of the DIRC). Only tracks in the momentum range 1.75–4.25 GeV/c are used. These distributions are fitted to single Gaussians and the widths ($\sigma_{\theta_c}$) and offsets from zero of the means are tabulated. Fig. 4-13 displays the offsets and widths of the aforementioned Gaussian fits.

The distribution of $\theta_c$ versus track momentum and measured $K-\pi$ separation are given in Fig. 4-14. The $K-\pi$ separation is defined as $(\langle \theta_c(K) \rangle - \langle \theta_c(\pi) \rangle)/\langle \sigma_{\theta_c} \rangle$. The separation is greater than 2.5$\sigma$ throughout the momentum range.

There is a small amount of cases where a true kaon(pion) is assigned a $\theta_c$ measurement consistent with a pion(kaon). This is due to biases in the $\theta_c$ reconstruction algorithm and not due to poorly reconstructed $\theta_c$ measurements which lead to long, non-Gaussian tails. The size of the effect is determined by plotting $\theta_c - \theta'_c$ in bins of momentum and observing a “satellite” peak centered at the expected $\theta_c$ difference for pions and kaons. To good approximation, the satellite peak constitutes 3% (2%) of the total number of selected events.
Figure 4-12. **Left plot:** the $F$ distribution for on-resonance $m_{ES}$ side-band data (points) and off-resonance data (histogram). **Right plot:** the $F$ distribution for on-resonance $m_{ES}$ side-band data (points) and Monte Carlo simulated continuum events (histogram). All the distributions are normalized to the same, arbitrary, area. The curves represent double Gaussian fits to the on-resonance $m_{ES}$ side-band distribution. The bottom plots are identical, but with a log scale on the y-axis.

Figure 4-13. **Left plots:** the offsets (left) of the measured mean $\theta_c$ from the expected value, and resolutions on $\theta_c$ (right) for kaons (top) and pions (bottom). **Right plots:** the distributions of $\theta_c - \theta'_c$ for kaon tracks selected from the $D^{*+}$ control sample in selected bins of momentum.
Figure 4-14. The Čerenkov angle (a) and \(K-\pi\) separation (b) as functions of momentum for tracks in the \(D^0\) control sample. Both tracks must be in the same momentum bin to achieve a given \(\theta_c\) separation. Tracks below the dashed line in (a) are rejected as proton candidates.

kaon(pion) tracks and has a width three times that of the primary peak. Left plots in Fig. 4-13 display the quantity \(\theta_c - \theta_c'\) for kaon tracks in four representative momentum bins. Overlayed on the distributions are the results of the double Gaussian fits to both the primary peaks and the satellite peaks. No significant momentum or \(\cos \theta\) dependence is observed. The effect is present in Monte Carlo simulated decays at a somewhat smaller level (~1%).

The satellite peak is included in the \(\theta_c\) PDFs as a second Gaussian with width and relative area fixed to the values described above, and centered at the opposite particle hypothesis. As will be demonstrated later, the presence of these satellite peaks has a very small effect on the fit results.

The \(\theta_c\) measurements for pions and kaons in Monte Carlo simulated \(B \to h^+h^-\) decays have been compared with those derived from the Monte Carlo \(D^{*+}\) control sample and good agreement is found. This validates the use of the \(D^{*+}\) decays in the data as a means of parameterizing the \(\theta_c\) resolutions and offsets for the signal samples.

### 4.6.6 Correlations between PDFs

The PDFs described in the previous sections are assumed to be uncorrelated in the maximum likelihood fit. To check this assumption, in each decay mode, the linear correlation coefficient \(\xi_{jk}\) between the PDFs for
variables $j$ and $k$ is calculated. The definition of $c_{jk}$ is [55]

$$c_{jk} = \frac{s_{jk}^2}{s_j s_k}, \quad (4.14)$$

where

$$s_{jk}^2 \equiv \frac{1}{N - 1} \sum [(x_{ij} - \bar{x}_j)(x_{ik} - \bar{x}_k)], \quad (4.15)$$

and $s_j \equiv s_{jj}$. 
Measurement of Branching Fractions for $B^\pm \rightarrow K^0 h^\pm$ decays

This chapter analyzes the two modes containing a $K^0_s$ combined with a charged pion or kaon. Table 5-1 shows the latest results from CLEO using an integrated luminosity of 9.13 fb$^{-1}$ [58].

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N_s$</th>
<th>Sig. $\epsilon$</th>
<th>BR $\times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi^\pm$</td>
<td>$25.2^{+6.4}_{-5.6}$</td>
<td>7.6$\sigma$</td>
<td>$18.2^{+4.6}_{-4.0} \pm 1.6$</td>
</tr>
<tr>
<td>$K^\pm K^0$</td>
<td>$1.4^{+2.4}_{-1.3}$</td>
<td>1.1$\sigma$</td>
<td>&lt; 5.7</td>
</tr>
</tbody>
</table>

5.1 Data samples and event selection

The analyses presented in this chapter use the data samples described in Sec. 4.1 and the selection described in Sec. 4.2. In addition, a control sample of $D^\pm \rightarrow K^0_s \pi^\pm$ decays is used to determine the efficiency of the $K^0_s$ mass cut.

Candidate $B$ mesons are selected corresponding to this specific modes $K^0_s h^\pm$. The kinematic variables used are the already described energy-substituted mass, $m_{ES}$, and energy difference $\Delta E_x$, where the notation with the subscript $\pi$ indicates that the candidate energy is calculated assuming the pion mass hypothesis for the charged track. We select candidates in the region $5.20$ GeV/$c^2 < m_{ES} < 5.30$ GeV/$c^2$ and $-0.305$ GeV < $\Delta E < 0.265$ GeV: this 2-dimensional area is called the grand side-band.

In order to define the $\Delta E$ side-bands together with the signal box, conservative values for $\Delta E$ shift and resolution have to be considered. The shift has to be taken into account since, in the case $B^\pm \rightarrow K^0_s K^\pm$, $\Delta E$ is calculated using the $\pi$ mass and thus the mean value of the $\Delta E$ distribution of those events is shifted of about $\mu_E = -41$ MeV. If one considers a conservative value of $\sigma_E = 25$ MeV for the $\Delta E$ resolution (see the complete discussion in Sec. 5.4.1), the $\Delta E$ signal box can be defined by:

$$\Delta E_{\text{max}} = 3 \times \sigma_E = 75 \text{ MeV}$$
$$\Delta E_{\text{min}} = 3 \times \sigma_E + \mu_E = -115 \text{ MeV}$$
$$\delta E = \Delta E_{\text{max}} - \Delta E_{\text{min}} = 190 \text{ MeV}$$

and thus
Measurement of Branching Fractions for $B^{\pm} \rightarrow K^0 h^\pm$ decays

**Figure 5-1.** Left plot: definition of the grand side-band, the $m_{ES}$ and $\Delta E$ side-bands and signal box.

\[
\begin{align*}
\text{Signal Box} & \quad \Rightarrow \quad -115 < \Delta E < 75 \text{ MeV} \\
\Delta E \text{ upper side-band} & \quad \Rightarrow \quad 75 < \Delta E < 265 \text{ MeV} \\
\Delta E \text{ lower side-band} & \quad \Rightarrow \quad -305 < \Delta E < -115 \text{ MeV},
\end{align*}
\]

where all the three $\Delta E$ band have the same 190 MeV width. The $m_{ES}$ side-band and signal box are defined in a mode independent way (see Sec. 4.2.2):

\[
m_{ES} \text{ side-band} \quad \Rightarrow \quad 5.20 < m_{ES} < 5.27 \text{ GeV}/c^2.
\]

### 5.2 $K^0_s$ reconstruction

Details of $K^0_s$ reconstruction can be found in Chapter 3. Candidate $K^0_s$ mesons are constructed using a preliminary loose cut $|M(\pi^+\pi^-) - M(K^0_s)| < 0.025 \text{ GeV}/c^2$. The $K^0_s$ candidates are then vertexed and the daughter momenta are recalculated at the best-fit $K^0_s$ decay vertex.

To select $K^0_s$ we use two cuts: a cut on the invariant mass cut and a cut on the lifetime significance which is defined as $t_{K^0_s}/\sigma t_{K^0_s}$, where $t_{K^0_s}$ is the measured 2-dimensional decay time and $\sigma t_{K^0_s}$ is its error.

Figure 5-3 shows the invariant mass distribution of the $K^0_s$ candidates for $B^+ \rightarrow K^0_s \pi^+$ Monte Carlo, while Fig. 5-4 shows the same variable in continuum Monte Carlo and off-resonance data. All distributions are fitted with a double Gaussian on a linear background. The resolution is 3.3 MeV/$c^2$ in signal Monte Carlo and 4.3 MeV/$c^2$ in data. To reduce contamination from fake $K^0_s$ candidates, the cut

\[
|m_{K_s} - m_{PDG}| < 11.2 \text{ MeV}/c^2
\]
5.2 $K_S^0$ reconstruction

is applied. Given the different $K_S^0$ mass resolution in Monte Carlo and data, a detailed study has been performed to measure the efficiency of the mass window using real data.

The efficiency of the mass cut is determined using a data control sample of $D^+ \rightarrow K_S^0 \pi^+$ decays on the full Run1 data-set. We select $D$ mesons from continuum $c\bar{c}$ events and we require $p_\pi > 2.5$ GeV/c, where $p_\pi$ is the momentum of the $D$ meson in the CM frame. For events with multiple candidates, we choose the one with invariant $D$ mass closest to the PDG value. A cut of $p_\pi > 0.5$ GeV/c is applied to suppress combinatorial background. Only high momentum ($p > 1.5$ GeV/c) $K_S^0$ candidates are considered and the requirement $t_{K_S^0}/\sigma_{K_S} > 5$ cut is applied in order to have a $K_S^0$ sample compatible with the one from charmless two-body analysis. Left plot in Fig. 5-2 shows efficiency as a function of the $K_S^0$ mass cut in this sample. For each cut, the efficiency is defined by fitting the $D^+$ mass distribution and dividing the yield found by the one obtained when no cut is applied to the $K_S^0$ mass. Right plot in Fig. 5-2 shows a typical fit.

We find an efficiency of $(97 \pm 1)\%$ cutting at the default value of $3.5\sigma$, where $\sigma = 3.2$ MeV/$c^2$. To evaluate the error on this efficiency, we used the signal MC of all the two-body charmless modes involving $K_S^0$: the same invariant mass is applied and the efficiency of the cut is evaluated. The quoted error of 1% is the greatest difference between the efficiencies estimated from MC signal events and the value found from $D^+ \rightarrow K_S^0 \pi^+$ control sample.

The second cut used to reduce contamination from fake $K_S^0$ candidates is the lifetime significance one: left plot in figure 5-5 shows the lifetime significance $t_{K_S^0}/\sigma_{K_S}$, which is peaked at zero for fake $K_S^0$ and has a flat distribution for true $K_S^0$. The data-MC agreement for the distribution of this variable has been checked on $K_S^0$ in $b\bar{b}$ events (see plots in [60]).

**Figure 5-2.** Study to measure the efficiency of the $K_S^0$ mass using $D^+ \rightarrow K_S^0 \pi^+$ control sample: efficiency for a cut of $\sigma$ on the $K_S^0$ mass in on-resonance $D^+ \rightarrow K_S^0 \pi^+$ decays (left); a typical fit to the $D^\pm$ mass distribution (right).
Measurement of Branching Fractions for $B^\pm \to K^{0*\pm}$ decays

Figure 5-3. $K^{0}_s$ invariant mass in $B^+ \to K^{0*}_s\pi^+$ Monte Carlo simulated data.

Figure 5-4. $K^{0}_s$ invariant mass in continuum Monte Carlo (left) and off-resonance data (right).

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5.3 Analysis Strategy

Figure 5-5. The variable $t_{K_S^0}/\sigma_{t_{K_S^0}}$ distributions. Left plot: lifetime significance for fake (histogram) and true (dots) $K_S^0$ continuum Monte Carlo data. Right plot: Data-MC agreement in $J/\psi K_S^0$ events in data (dots) and in the $J/\psi K_S^0$ signal MC (histogram). The plots are background subtracted ($K_S^0$ mass side-bands) and the histograms are normalized to equal area.

Since $K_S^0$'s in $b \bar{b}$ events have typically lower momenta with respect to $K^0$ in charmless two-body decays, we have also used, as a control sample for our channel, a sample of reconstructed $J/\psi K_S^0$ events: the momentum spectrum of these $K_S^0$ goes from 1 GeV up to 3 GeV. Right plot in figure 5-5 shows the good agreement of the lifetime significance distribution for data and MC in this control sample.

The statistical significance $S^2/(S+B)$ ($S$ and $B$ are the number of expected signal and background events respectively) of a cut on $t_{K_S^0}/\sigma_{t_{K_S^0}}$ is shown in Fig. 5-6. For this exercise a branching fraction of $B_R(B^0 \rightarrow h^+K_S^0) = 1.8 \times 10^{-5}$ is assumed.

From this optimization, we require $t_{K_S^0}/\sigma_{t_{K_S^0}} > 5$: the uncertainty on this cut will be taken into account in the systematic study, where we vary the lifetime significance cut and calculate from the global likelihood fit the difference of the yields found with respect to the nominal results (see Sec. 5.4.5).

5.3 Analysis Strategy

Signal yields in $B^+ \rightarrow K_S^0\pi^+$ and $B^+ \rightarrow K_S^0K^+$ channels are determined using an unbinned maximum likelihood technique. The background suppression variables and parameterization of the probability density functions (PDFs) are discussed in Sec. 5.4. The results of a fit to the full Run1 data-set are presented in Sec. 5.5. As a crosscheck, background suppression and particle identification cuts have been applied to
isolate samples of events that are consistent with the $K^0\pi$ and $K^0\bar{K}$ hypotheses, and signal yields are then obtained from an unbinned maximum likelihood fit (Sec. 5.5).

### 5.4 Background Suppression and PDF Parameterization

#### 5.4.1 $\Delta E$ PDF and Definition of Signal and Side-band Regions

Signal events are Gaussianly distributed in $\Delta E$ with a mean near zero as one can see from the distribution of Monte Carlo signal events (Fig. 5-7), while the continuum background events fall quadratically over the signal region (Fig. 5-8).

For this analysis, since the pion mass is assigned to all tracks, the $K^0\bar{K}$ decays have $\Delta E_\pi$ shifted from zero by an amount depending on the momentum of the tracks. From Monte Carlo simulation we find an average shift of $-41$ MeV for the $K^0\bar{K}$ decays (Fig. 5-7). In the global likelihood fit we take into account the shift depending on the momentum of the tracks using a $\Delta E$ PDF for $B \rightarrow K^0\bar{K}$ decays that consists of a Gaussian with a mean given by the analytical form in 4.13.

The $\Delta E_\pi$ distribution was fitted with a Gaussian width of 22.6 MeV for $K^0\bar{K}$ and 19.3 MeV for $K^0\pi$ Monte Carlo. A comparison of $B^- \rightarrow D^0\pi^-$ decays in data and Monte Carlo indicates that the Monte Carlo resolution should be scaled by a factor $1.24 \pm 0.06$ to agree with data (see study in Sec. 4.2.2.1). As a consequence, in case of $K^0\bar{h}$ decays, we have estimated the resolution on $\Delta E$ in real data to be $24 \pm 5$ MeV.

Monte Carlo shows that the mean in $\Delta E$ for $K^0\pi$ signal events is around $3.5$ MeV$^1$. The estimated mean of $\Delta E$ in data is therefore taken from the $B^- \rightarrow D^0\pi^-$ control sample ($-5 \pm 5$ MeV, see Sec. 4.2.2.1).

---

$^1$This effect in MC is not understood yet, but goes in the same direction as the shift seen in MC in the reconstructed $K^0_s$ mass. The reconstructed value in MC is higher than the PDG value, while in data the $K^0_s$ mass is in perfect agreement with the PDG (see Sec. 2).
5.4 Background Suppression and PDF Parameterization

**Figure 5-7.** Distributions of $\Delta E$ in $B \to K^0K$ (left) and $B \to K^0\pi$ Monte Carlo (right). The resolution is $22.6 \pm 0.2$ MeV for the former and $19.3 \pm 0.2$ MeV for the latter.

**Figure 5-8.** Distributions of $\Delta E$ in off-resonance data (left), comparison of continuum Monte Carlo and off-resonance data (center) and comparison of on-resonance side-bands and off-resonance data (right).

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5.4.2 Parameterizations of $m_{ES}$ Distributions

The background shape in $m_{ES}$ is parameterized by the ARGUS function in Eq. 4.6.2, where we use $m_{\text{max}} = 5.2895\text{ GeV}/c^2$ and where the parameter $\xi$ is determined from a fit. A fit to the on-resonance side-band region after applying the standard selection (see Sec. 4.2.1) gives $\xi = 23.1 \pm 1.3$ (Fig. 5-9).

A similar fit performed on off-resonance grand side-band region gives $\xi = 21.0 \pm 2.7$ and we find $\xi = 23.0 \pm 2.1$ in continuum Monte Carlo events (Fig. 5-10). All the values obtained from these samples are well compatible with each other and with the on-resonance fit. We use the value $\xi = 23.1 \pm 1.3$ in the rest of this analysis.

The $m_{ES}$ PDF for the signal is parameterized as a Gaussian with mean of $5.280\text{ GeV}/c^2$ and width of $2.6\text{ MeV}/c^2$ taken from the $B^- \rightarrow D^0\pi^-$ control sample (4.2.2.1).

5.4.3 Fisher Discriminant

The Fisher discriminant has already been defined in Sec. 4.3. For the parameterization of the Fisher variable in signal events we have used signal $K^0\pi$ MC, while for the Fisher variable in the background events, we have used the on-resonance $m_{ES}$ side-band ($5.20 < m_{ES} < 5.27\text{ GeV}$).

This parameter is determined in a mode independent way: see Sec.4.6.2
The Fisher distribution in on-resonance \( m_{ES} \) side-band has been validated against continuum MC and off-resonance data, both in the entire signal band and in the \( m_{ES} \) side-band. Figure 5-11 shows comparisons of the Fisher variable in on-resonance data with continuum Monte Carlo and off-resonance data in the \( m_{ES} \) side-band.

Figure 5-12 shows the parameterization for background events and signal Monte Carlo events respectively. Left plot shows also the separation power of the Fisher variable and the good agreement between Fisher variable evaluated in signal \( K^{0}_{s}\pi \) MC events and signal \( D^{0}\pi \) MC events. Therefore, the Fisher variable distribution from \( D^{0}\pi \) control sample will be used as a systematic check for the signal Fisher distribution included in the likelihood fit.

### 5.4.4 Particle ID Selection

We use the measured (\( \theta_{c} \)) minus expected (\( \theta_{c}(\text{exp}) \)) Čerenkov angle for the charged pion or kaon to separate the two signal modes on a statistical basis. The distribution of \( \theta_{c} - \theta_{c}(\text{exp}) \) is parameterized by a central Gaussian plus a satellite Gaussian that accounts for the few percent of tracks that are mis-reconstructed. A detailed description of the DIRC PDF’s can be found in Sec. 4.4. To have a clean sample of tracks with well measured \( \theta_{c} \) we require the already described particle ID (or PID) cuts: \( \theta_{c} > 0 \), number of signal photons > 5 and proton veto \( (\theta_{c} - \theta_{c}(p)) > 10 \) mrad where \( \theta_{c}(p) \) is the expected Čerenkov angle for a proton with the given momentum.
Figure 5-11. Fisher discriminant output for continuum Monte Carlo superimposed on the on-resonance distribution of the Fisher variable (left) and off-resonance data compared to on-resonance (right). These distributions correspond to the $m_{ES}$ side-band ($m_{ES} < 5.27$ GeV).

Figure 5-12. Left plot shows the Fisher output comparison: on the left side, signal $K^{0}\pi$ MC with the signal $D^{0}\pi$ MC (black dots) superimposed, on the right side the on-resonance $m_{ES}$ side-band with the parameterization for background events used in the global likelihood fit. Right plot shows the Fisher parameterization for signal events: signal $K^{0}\pi$ MC is fitted.

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5.4.5 Efficiency

In summary, after the standard selection, the following cuts are applied:

- \( |m_{K_S} - m_{PDG}| < 11.2 \text{ MeV}/c^2 \)
- \( t_{K_S}/\sigma_{t_{K_S}} > 5 \)
- PID cuts

Monte Carlo is used to estimate the efficiency of different cuts with the exceptions of track reconstruction, the \( K_S^0 \) reconstruction and the \( K_S^0 \) invariant mass cut. In this last case, we consider the efficiency (and the relative error) found in the \( D^+ \to K_S^0\pi^+ \) (see Section 5.2). Table 5-2 summarizes the efficiencies of the different cuts.

To correct the MC final efficiency for the tracking uncertainty (see Sec. 4.5), we use the results from detailed studies on various control samples [61]: the correction factor comes out to be \( G_{\text{trk}} = 0.99 \) with an uncertainty of 1.2\% per track.

In the \( K_S^0 \) case, the correction on the efficiency is done on the base of the correction tables from inclusive \( K_S^0 \) reconstruction described in Sec. 3.3.4: the correction factor is \( 1.05 \pm 0.07 \). No correction is applied for the \( K_S^0 \) daughter tracks since they are taken from the list ChargedTracks and the Tracking Efficiency Working Group has found agreement with MC, but the 2\% uncertainty per \( K_S^0 \) (1\% per ChargedTracks track) has to be taken into account to evaluate the error on the efficiency.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Efficiency ( K_S^0 \pi^+ )</th>
<th>Efficiency ( K_S^0 K^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>reco + tag bits + R2 + Sph + GoodTracksAccLoose</td>
<td>54.4%</td>
<td>53.1%</td>
</tr>
<tr>
<td>(</td>
<td>c\bar{c}\theta_S</td>
<td>&lt; 0.9 )</td>
</tr>
<tr>
<td>(</td>
<td>m_{K_S} - m_{PDG}</td>
<td>&lt; 11.2 \text{ MeV} )</td>
</tr>
<tr>
<td>( t_{K_S}/\sigma_{t_{K_S}} &gt; 5 )</td>
<td>97.6%</td>
<td>97.7%</td>
</tr>
<tr>
<td>( \theta_c &gt; 0 )</td>
<td>91.4%</td>
<td>91.3%</td>
</tr>
<tr>
<td>proton veto</td>
<td>98.0%</td>
<td>97.8%</td>
</tr>
<tr>
<td>( n_{\gamma} &gt; 5 )</td>
<td>97.8%</td>
<td>97.2%</td>
</tr>
<tr>
<td>all previous cuts</td>
<td>40.3%</td>
<td>38.9%</td>
</tr>
</tbody>
</table>

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In the systematic error on the efficiency, we include a $\cos \theta_k$ cut uncertainty of 1% which is the difference between the expected efficiency of 0.9 given from the flat distribution and the efficiency taken from the signal MC.

In the case of the lifetime significance cut, we varied the cut from 5 to 5 ± 1: the difference from the final result on the branching ratio is $^{+1.6}_{-1.0}$%. Since this systematic error is strongly correlated with the correction on the $K_{S}^{0}$ reconstruction efficiency, which is function of the $K_{S}^{0}$ flight length, we decide to take into account the largest of the two systematic errors. The error included in the efficiency evaluation is therefore the one coming from the $K_{S}^{0}$ reconstruction correction which is 3%.

The corrected efficiency is $41.7 \pm 1.6$% for $K_{S}^{0}\pi^{+}$ channel and $40.3 \pm 1.5$% for $K_{S}^{0}K^{+}$.

5.5 Maximum likelihood analysis

We use the unbinned maximum likelihood fit technique described in Sec. 4.6 to determine a total of eight parameters from the data:

- $N_{S}^{K_{S}^{0}\pi^{+}}$, the number of $B \rightarrow K_{S}^{0}\pi^{+}$ decays;
- $N_{S}^{K_{S}^{0}K^{+}}$, the number of $B \rightarrow K_{S}^{0}K^{+}$ decays;
- $A_{S}^{K_{S}^{0}\pi^{+}}$, the observed asymmetry between $B^{+} \rightarrow K_{S}^{0}\pi^{+}$ and $B^{-} \rightarrow K_{S}^{0}\pi^{-}$ decays, $(N_{K_{S}^{0}\pi^{+}} - N_{K_{S}^{0}\pi^{-}})/(N_{K_{S}^{0}\pi^{+}} + N_{K_{S}^{0}\pi^{-}})$;
- $A_{S}^{K_{S}^{0}K^{+}}$, the observed asymmetry between $B \rightarrow K_{S}^{0}K^{+}$ and $B \rightarrow K_{S}^{0}K^{-}$ decays;
- $N_{B}^{K_{S}^{0}\pi^{+}}$, the number of background $K_{S}^{0}\pi^{+}$ candidates;
- $N_{B}^{K_{S}^{0}K^{+}}$, the number of background $K_{S}^{0}K^{+}$ candidates;
- $A_{B}^{K_{S}^{0}\pi^{+}}$, the observed asymmetry between the number of background $K_{S}^{0}\pi^{+}$ and $K_{S}^{0}\pi^{-}$ candidates;
- $A_{B}^{K_{S}^{0}K^{+}}$, the observed asymmetry between the number of background $K_{S}^{0}K^{+}$ and $K_{S}^{0}K^{-}$ candidates.

The usual four quantities are used in the fit to distinguish between the various components: to summarize

- $m_{ES}$, the beam energy substituted mass of the $B^{0}$ candidate: it is parameterized as an ARGUS function with $\xi$ fixed to 23.1 for the background and as a Gaussian, with mean and width fixed to 5.280 GeV/$c^{2}$ and 2.6 MeV/$c^{2}$, respectively, for the signal;
- $\Delta E$, the difference between the $B^{0}$ candidate’s energy, using the pion mass for the charged particle, and $\sqrt{s}/2$: it is parameterized as a Gaussian for the signal and as a second order polynomial for the background;

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5.5 Maximum likelihood analysis

Table 5-3. Summary of functional form of PDFs used in the fit and of the sample used to obtain them. The samples in parentheses were used as a cross-check or to provide alternate parameterization to evaluate systematics.

<table>
<thead>
<tr>
<th>Signal (m_{ES})</th>
<th>Bkg (m_{ES})</th>
<th>Gaussian</th>
<th>ARGUS</th>
<th>(K_0^0 h^+) MC ((B^- \rightarrow D^0 \pi^-)) on-res side-band (off-res, cont MC)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal (\Delta E)</td>
<td>Bkg (\Delta E)</td>
<td>Gaussian</td>
<td>Quadratic</td>
<td>(K_0^0 h) MC with (B^- \rightarrow D^0 \pi^-) scale factor cont MC (off-res, on-res side-band)</td>
</tr>
<tr>
<td>Signal Fisher</td>
<td>Bkg Fisher</td>
<td>Double Gaussian</td>
<td>Double Gaussian</td>
<td>(K_0^0 h^+) MC ((B^- \rightarrow D^0 \pi^-)) on-res (m_{ES}) side-band (off-res, cont MC)</td>
</tr>
<tr>
<td>Kaon (\theta_c)</td>
<td>Gaussian</td>
<td></td>
<td></td>
<td>(D^*) control sample</td>
</tr>
<tr>
<td>Pion (\theta_c)</td>
<td>Gaussian</td>
<td></td>
<td></td>
<td>(D^*) control sample</td>
</tr>
</tbody>
</table>

- \(\mathcal{F}\), the value of the Fisher discriminant for the event: it is parameterized as a double Gaussian for both background and signal;
- \(\theta_c - \theta_c^{\text{exp}}\), the difference between the Čerenkov angle of the \(h^\pm\), measured by the DIRC, and the expected Čerenkov angle for a particle of that momentum: it is parameterized as a main Gaussian, whose width and mean depend on the polar angle of the track, with a satellite peak parameterized by a second Gaussian with width and mean fixed.

Table 5-3 summarizes the functional form of PDFs and the samples used to obtain them. The likelihood, \(\mathcal{L}\), for the selected sample is given by the product of the PDFs for each individual candidate and a Poisson factor. The quantity \(L = -2 \log \mathcal{L}\) is minimized, which is equivalent to maximizing \(\mathcal{L}\) itself, with respect to the eight fit parameters. The PDF for a given event \(i\) is the sum of signal and background terms:

\[
P_i(m_{ES,i}, \Delta E_i, \mathcal{F}_i, \theta_{c,i}) = \frac{N^S}{2N'} \left(1 + A^S_{K_0^0 h}\right) P_i^{K_0^0 h^+} + \frac{N^B}{2N'} \left(1 + A^B_{K_0^0 h}\right) P_i^{hK_0^0 h^+} + \frac{N^S}{2N'} \left(1 - A^S_{K_0^0 h}\right) P_i^{K_0^0 h^-} + \frac{N^B}{2N'} \left(1 - A^B_{K_0^0 h}\right) P_i^{hK_0^0 h^-} \tag{5.1}
\]

where \(N' \equiv \sum_k N_k\) and \(A_{K_0^0 h}\) is the charge asymmetry defined as

\[
A_{K_0^0 h} = \frac{N_{K_0^0 h^-} - N_{K_0^0 h^+}}{N_{K_0^0 h^-} + N_{K_0^0 h^+}}.
\]

Measurement of Branching Fractions for \(B^\pm \rightarrow K_0^0 h^\pm\) decays
Table 5-4. Linear correlation coefficients between variables used in the $K^0_{S\pi}$ maximum likelihood fit. Results are obtained from $K^0_{S\pi}$ Monte Carlo.

<table>
<thead>
<tr>
<th>Variables</th>
<th>correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{ES}$, $\Delta E$</td>
<td>-0.069</td>
</tr>
<tr>
<td>$m_{ES}$, $\mathcal{F}$</td>
<td>0.012</td>
</tr>
<tr>
<td>$m_{ES}$, $\theta_c$</td>
<td>-0.009</td>
</tr>
<tr>
<td>$\Delta E$, $\mathcal{F}$</td>
<td>0.012</td>
</tr>
<tr>
<td>$\Delta E$, $\theta_c$</td>
<td>-0.014</td>
</tr>
<tr>
<td>$\mathcal{F}$, $\theta_c$</td>
<td>-0.000</td>
</tr>
</tbody>
</table>

The PDF for each component, in turn, is the product of the PDFs for each of the fit input variables:

$$\mathcal{P}^k = \mathcal{P}^k_{m_{ES}} \mathcal{P}^k_{\Delta E} \mathcal{P}^k_{\mathcal{F}} \mathcal{P}^k_{\theta_c}. \quad (5.2)$$

Including the Poisson factor, the likelihood is:

$$\mathcal{L} = e^{-N'} (N')^N \prod_{i=1}^{N} \mathcal{P}_i. \quad (5.3)$$

Note that the factor $(N')^N$ in the above equation cancels an identical factor in the denominator that arises from equation 5.1.

### 5.5.1 Correlations between PDFs

The PDFs described in the previous section are assumed to be uncorrelated in the maximum likelihood fit. To check this assumption we have calculated the linear correlation coefficient $g_{jk}$ between the PDFs for variables $j$ and $k$. The definition of $g_{jk}$ is in Sec. 4.6.6. Table 5-4 summarizes the correlation coefficients obtained from signal $K^0_{S\pi}$ Monte Carlo.

### 5.5.2 Event yields and asymmetries

The fit, performed on the 3623 candidates in the full Run 1 data-set, returns:

$$N^S_{K^0_{S\pi}} = 58.7^{+10.7}_{-9.8} \text{ (stat)}$$

$$N^S_{K^0_{S\pi}} = 0^{+2.4}_{-0} \text{ (stat)}$$

$$A^S_{K^0_{S\pi}} = 0.21^{+0.17}_{-0.16} \text{ (stat)}$$

**Marcella Bona**
5.5 Maximum likelihood analysis

\[ A_{K^0_{S\pi}}^S = 0 \text{ (fixed)} \]
\[ N_{K^0_{S\pi}}^B = 1984.8^{+47.7}_{-47.0} \text{ (stat)} \]
\[ N_{K^0_{S\pi}}^B = 1579.5^{+42.7}_{-42.0} \text{ (stat)} \]
\[ A_{K^0_{S\pi}}^B = 0.000 \pm 0.024 \text{ (stat)} \]
\[ A_{K^0_{S\pi}}^B = -0.001 \pm 0.027 \text{ (stat)} \]

where \( K^0_{S\pi} \) signal has a significance of 9.8 standard deviations, determined by fixing that component to zero and recording the change in \( L = -2 \log \mathcal{L} \): the significance is \( \sigma = \sqrt{\mathcal{L}_0 - \mathcal{L}_{\text{fit}}} \) [14]. The asymmetry \( A_{K^0_{S\pi}}^S \) has a significance of 1.2 standard deviations.

**Table 5-5. Correlation Matrix between the fitted variables**

<table>
<thead>
<tr>
<th></th>
<th>( N_{K^0_{S\pi}}^S )</th>
<th>( N_{K^0_{S\pi}}^S )</th>
<th>( A_{K^0_{S\pi}}^S )</th>
<th>( N_{K^0_{S\pi}}^B )</th>
<th>( N_{K^0_{S\pi}}^B )</th>
<th>( A_{K^0_{S\pi}}^B )</th>
<th>( A_{K^0_{S\pi}}^B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{K^0_{S\pi}}^S )</td>
<td>1.000</td>
<td>-0.003</td>
<td>-0.106</td>
<td>-0.092</td>
<td>-0.005</td>
<td>0.002</td>
<td>0.000</td>
</tr>
<tr>
<td>( N_{K^0_{S\pi}}^S )</td>
<td>-0.003</td>
<td>1.000</td>
<td>0.001</td>
<td>0.000</td>
<td>-0.005</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>( A_{K^0_{S\pi}}^S )</td>
<td>-0.106</td>
<td>0.001</td>
<td>1.000</td>
<td>0.022</td>
<td>0.001</td>
<td>-0.092</td>
<td>-0.005</td>
</tr>
<tr>
<td>( N_{K^0_{S\pi}}^B )</td>
<td>-0.092</td>
<td>0.000</td>
<td>0.022</td>
<td>1.000</td>
<td>-0.106</td>
<td>-0.001</td>
<td>0.001</td>
</tr>
<tr>
<td>( N_{K^0_{S\pi}}^B )</td>
<td>-0.005</td>
<td>-0.005</td>
<td>0.001</td>
<td>-0.106</td>
<td>1.000</td>
<td>0.001</td>
<td>-0.001</td>
</tr>
<tr>
<td>( A_{K^0_{S\pi}}^B )</td>
<td>0.002</td>
<td>0.000</td>
<td>-0.092</td>
<td>-0.001</td>
<td>0.001</td>
<td>1.000</td>
<td>-0.106</td>
</tr>
<tr>
<td>( A_{K^0_{S\pi}}^B )</td>
<td>0.000</td>
<td>0.000</td>
<td>-0.005</td>
<td>0.001</td>
<td>-0.001</td>
<td>-0.106</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The systematic correlation matrix of the fit parameters is shown in Table 5-5.

In order to test the goodness of fit, we ran 1000 toy Monte Carlo pseudo-experiments taking the result of the fit as the mean number of signal and background events produced: we plot \(-2 \log \mathcal{L}\) from the fit of each pseudo-experiment in Fig. 5-13. The arrow indicates the value obtained from the fit to the Run1 data-set: from this, we estimate the probability to find a greater value for \(-2 \log \mathcal{L}\) to be 14%. This value can be considered a measurement of the goodness of fit.

5.5.3 Cross-check and systematics

Letting also \( N_{K^0_{S\pi}}^S \) and \( A_{K^0_{S\pi}}^S \) free in the fit, we obtains:

\[ N_{K^0_{S\pi}}^S = 58.7^{+10.7}_{-9.8} \text{ (stat)} \]
\[ N_{K^0_{S\pi}}^S = 0^{+3.2}_{-0.0} \text{ (stat)} \]

**Measurement of Branching Fractions for \( B^\pm \rightarrow K^0 h^\pm \) decays**
Figure 5-13. The value of $-2\log L$ from the fit of 1000 toy Monte Carlo pseudo-experiments. The arrow indicates the value obtained from the fit to the Run1 data-set.

\[
\begin{align*}
A_{K^0\pi}^S &= 0.21^{+0.17}_{-0.38} \text{ (stat)} \\
A_{K^0K}^S &= -0.47 \pm 1.00 \text{ (stat)} \\
N_{K^0\pi}^B &= 1984.8^{+47.7}_{-47.0} \text{ (stat)} \\
N_{K^0K}^B &= 1579.5^{+42.7}_{-42.0} \text{ (stat)} \\
A_{K^0\pi}^B &= 0.000 \pm 0.024 \text{ (stat)} \\
A_{K^0K}^B &= -0.001 \pm 0.027 \text{ (stat)}
\end{align*}
\]

While fixing all the asymmetries to zero and re-fitting:

\[
\begin{align*}
N_{K^0\pi}^S &= 59.2^{+10.7}_{-9.9} \text{ (stat)} \\
N_{K^0K}^S &= 0^{+2.3}_{-0} \text{ (stat)} \\
N_{K^0\pi}^B &= 1984.3^{+47.7}_{-47.0} \text{ (stat)} \\
N_{K^0K}^B &= 1579.5^{+42.7}_{-42.0} \text{ (stat)}
\end{align*}
\]

the result differs of less than 1% from the nominal one.

Marcella Bona
5.5 Maximum likelihood analysis

We have split the sample of on-resonance candidates in two according to the charge of the reconstructed B meson. Fitting the yield of signal and background events, we find

\[
\begin{align*}
N^S_{K^0\pi} &= 35.4^{+8.0}_{-7.2} \text{ (stat)} \\
N^S_{K^0\pi} &= 0^{+1.8}_{-0} \text{ (stat)} \\
N^B_{K^0\pi} &= 992.0^{+33.8}_{-33.1} \text{ (stat)} \\
N^B_{K^0\pi} &= 788.6^{+30.3}_{-29.6} \text{ (stat)}
\end{align*}
\]

on the sample of 1816 positive B candidates and:

\[
\begin{align*}
N^S_{K^0\pi} &= 23.3^{+7.3}_{-6.4} \text{ (stat)} \\
N^S_{K^0\pi} &= 0^{+2.7}_{-0} \text{ (stat)} \\
N^B_{K^0\pi} &= 992.8^{+33.9}_{-33.2} \text{ (stat)} \\
N^B_{K^0\pi} &= 790.9^{+30.3}_{-29.6} \text{ (stat)}
\end{align*}
\]

on the sample of 1807 negative B candidates. The resulting \( A_{K^0\pi} \) (0.21) is in agreement with the finding of the fit on the total sample.

To cross-check the effect of the PID parameterization, a sample of 2114 candidates were selected by requiring the \( h^\pm \) track to fail the SMS kaon selector\(^3\) criteria. With our likelihood fit applied taking out the DIRC PDF, the results are:

\[
\begin{align*}
N^S_{K^0\pi} &= 57.1^{+10.4}_{-9.6} \text{ (stat)} \\
A^S_{K^0\pi} &= 0.16^{+0.17}_{-0.18} \text{ (stat)} \\
N^B_{K^0\pi} &= 2057.0^{+46.2}_{-45.5} \text{ (stat)} \\
A^B_{K^0\pi} &= 0.005 \pm 0.022 \text{ (stat)}
\end{align*}
\]

The same exercise is done requiring the \( h^\pm \) track to satisfy the Loose SMSKaonSelector criteria: out of the 1509 candidates, the results are:

\[
\begin{align*}
N^S_{K^0\pi} &= 0^{+2.8}_{-0} \text{ (stat)} \\
A^S_{K^0\pi} &= 0 \text{ (fixed)} \\
N^B_{K^0\pi} &= 1509.0^{+39.2}_{-38.5} \text{ (stat)} \\
A^B_{K^0\pi} &= -0.007 \pm 0.026 \text{ (stat)}
\end{align*}
\]

\(^3\)See Sec. 4.4.2

Measurement of Branching Fractions for \( B^\pm \rightarrow K^0 h^\pm \) decays
We find good agreement between the SMS selector and the global likelihood fit using the $DIRC$ PDF’s.

We have also checked that the asymmetries are compatible with zero in the on-resonance upper and lower side bands, in the off-resonance data and in continuum Monte Carlo.

In on-resonance lower side-band, we have 4529 events: fitting not using $A_1$ PDF and fixing the signal asymmetries, we get

\[
\begin{align*}
N^{S}_{K^0_{S}\pi} &= 9.3^{+8.5}_{-7.0} \text{ (stat)} \\
N^{S}_{K^0_{S}K} &= 0.3^{+0.9}_{-0.0} \text{ (stat)} \\
N^{B}_{K^0_{S}\pi} &= 2662.8^{+54.7}_{-54.0} \text{ (stat)} \\
N^{B}_{K^0_{S}K} &= 1856.9^{+46.2}_{-45.4} \text{ (stat)} \\
A^{B}_{K^0_{S}\pi} &= -0.001 \pm 0.020 \text{ (stat)} \\
A^{B}_{K^0_{S}K} &= 0.024 \pm 0.025 \text{ (stat)}
\end{align*}
\]

We see no asymmetry in the background and no statistically significant signal. To evaluate the cross-feed of background into signal events, we check on off-resonance data in the signal band, using $A_1$ PDF too: we get

\[
\begin{align*}
N^{S}_{K^0_{S}\pi} &= 0.1^{+1.2}_{-0.0} \text{ (stat)} \\
N^{S}_{K^0_{S}K} &= 0.1^{+1.2}_{-0.0} \text{ (stat)} \\
N^{B}_{K^0_{S}\pi} &= 2620.0^{+17.4}_{-16.7} \text{ (stat)} \\
N^{B}_{K^0_{S}K} &= 2020.0^{+15.6}_{-14.8} \text{ (stat)} \\
A^{B}_{K^0_{S}\pi} &= 0.023 \pm 0.065 \text{ (stat)} \\
A^{B}_{K^0_{S}K} &= 0.184 \pm 0.074 \text{ (stat)}
\end{align*}
\]

Finally, we have built samples of continuum Monte Carlo events with different amounts of signal and background and predefined asymmetries (including in the sample different amount of positive and negative charged candidates) and we have checked that the fit returns the correct yields and asymmetries.

Table 5-6 shows that the fit always returns the correct number of signal events within 1\sigma and that there is less than 1\% cross-feed to the other $K^0_{S}h$ channel or to background events. Table 5-7 shows tests on samples of continuum MC with fixed charge asymmetry: the fit returns the value of the asymmetry with very good agreement.

5.5.4 Systematic uncertainties

The amount of background and signal are both allowed to fluctuate within Poissonian statistics in the fit itself. Thus it is not necessary to estimate a systematic uncertainty from the background normalization. The
<table>
<thead>
<tr>
<th>Fit Variable</th>
<th>Input Value</th>
<th>Fitted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^{S}<em>{K</em>{S}^{0}\pi}$</td>
<td>6442</td>
<td>6387.0 ± 80.1</td>
</tr>
<tr>
<td>$N^{S}<em>{K</em>{S}^{0}K}$</td>
<td>0</td>
<td>18.5 ± 5.9</td>
</tr>
<tr>
<td>$A^{S}<em>{K</em>{S}^{0}\pi}$</td>
<td>−0.009</td>
<td>−0.009 ± 0.013</td>
</tr>
<tr>
<td>$A^{S}<em>{K</em>{S}^{0}K}$</td>
<td>0</td>
<td>0.56 ± 0.29</td>
</tr>
<tr>
<td>$N^{B}<em>{K</em>{S}^{0}\pi}$</td>
<td>0</td>
<td>29.2 ± 6.7</td>
</tr>
<tr>
<td>$N^{B}<em>{K</em>{S}^{0}K}$</td>
<td>0</td>
<td>7.4 ± 3.3</td>
</tr>
<tr>
<td>$A^{B}<em>{K</em>{S}^{0}\pi}$</td>
<td>0</td>
<td>−0.06 ± 0.23</td>
</tr>
<tr>
<td>$A^{B}<em>{K</em>{S}^{0}K}$</td>
<td>0</td>
<td>−0.06 ± 0.35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test 2: signal $K^{0}K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^{S}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$N^{S}<em>{K</em>{S}^{0}K}$</td>
</tr>
<tr>
<td>$A^{S}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$A^{S}<em>{K</em>{S}^{0}K}$</td>
</tr>
<tr>
<td>$N^{B}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$N^{B}<em>{K</em>{S}^{0}K}$</td>
</tr>
<tr>
<td>$A^{B}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$A^{B}<em>{K</em>{S}^{0}K}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Test 3: signal and continuum MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N^{S}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$N^{S}<em>{K</em>{S}^{0}K}$</td>
</tr>
<tr>
<td>$A^{S}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$A^{S}<em>{K</em>{S}^{0}K}$</td>
</tr>
<tr>
<td>$N^{B}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$N^{B}<em>{K</em>{S}^{0}K}$</td>
</tr>
<tr>
<td>$A^{B}<em>{K</em>{S}^{0}\pi}$</td>
</tr>
<tr>
<td>$A^{B}<em>{K</em>{S}^{0}K}$</td>
</tr>
</tbody>
</table>

**Table 5-6.** Summary of tests performed with Monte Carlo to validate the global likelihood fit using combinations of signal and continuum Monte Carlo.

**Measurement of Branching Fractions for $B^{\pm} \rightarrow K^{0}h^{\pm}$ decays**
<table>
<thead>
<tr>
<th>Fit Variable</th>
<th>Input Value</th>
<th>Fitted Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test 3: continuum MC with fixed asymmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{K_S^{0}\pi}^{S}$</td>
<td>0</td>
<td>0.0 ± 0.7</td>
</tr>
<tr>
<td>$N_{K_S^{0}K}^{S}$</td>
<td>0</td>
<td>0.0 ± 0.5</td>
</tr>
<tr>
<td>$N_{K_S^{0}\pi}^{B}$</td>
<td>295</td>
<td>291.6 ± 17.2</td>
</tr>
<tr>
<td>$N_{K_S^{0}K}^{B}$</td>
<td>0</td>
<td>3.4 ± 2.5</td>
</tr>
<tr>
<td>$A_{K_S^{0}\pi}^{B}$</td>
<td>−0.32</td>
<td>−0.314 ± 0.055</td>
</tr>
<tr>
<td>$A_{K_S^{0}K}^{B}$</td>
<td>0</td>
<td>−1 ± 1</td>
</tr>
<tr>
<td>Test 4: continuum MC with fixed asymmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{K_S^{0}\pi}^{S}$</td>
<td>0</td>
<td>0.0 ± 0.5</td>
</tr>
<tr>
<td>$N_{K_S^{0}K}^{S}$</td>
<td>0</td>
<td>0.9 ± 1.8</td>
</tr>
<tr>
<td>$N_{K_S^{0}\pi}^{B}$</td>
<td>0</td>
<td>0.2 ± 2.2</td>
</tr>
<tr>
<td>$N_{K_S^{0}K}^{B}$</td>
<td>220</td>
<td>218.8 ± 15.0</td>
</tr>
<tr>
<td>$A_{K_S^{0}\pi}^{B}$</td>
<td>0</td>
<td>1 ± 1</td>
</tr>
<tr>
<td>$A_{K_S^{0}K}^{B}$</td>
<td>0.27</td>
<td>0.271 ± 0.066</td>
</tr>
<tr>
<td>Test 5: continuum MC with fixed asymmetry</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$N_{K_S^{0}\pi}^{S}$</td>
<td>0</td>
<td>0.0 ± 0.7</td>
</tr>
<tr>
<td>$N_{K_S^{0}K}^{S}$</td>
<td>0</td>
<td>0.0 ± 0.7</td>
</tr>
<tr>
<td>$N_{K_S^{0}\pi}^{B}$</td>
<td>170</td>
<td>166.4 ± 13.5</td>
</tr>
<tr>
<td>$N_{K_S^{0}K}^{B}$</td>
<td>170</td>
<td>173.6 ± 13.8</td>
</tr>
<tr>
<td>$A_{K_S^{0}\pi}^{B}$</td>
<td>−0.18</td>
<td>−0.167 ± 0.080</td>
</tr>
<tr>
<td>$A_{K_S^{0}K}^{B}$</td>
<td>−0.18</td>
<td>−0.185 ± 0.078</td>
</tr>
</tbody>
</table>

Table 5-7. Summary of tests performed with continuum Monte Carlo with fixed charged asymmetry: in the fit the asymmetry of the signal is fixed to 0
systematic uncertainties in the unbinned likelihood analysis come primarily from the imperfect knowledge
of the correct parameterizations for each of the PDFs. Each parameter in each PDF was varied within ±1σ
and different samples of data were used to obtain alternative parameterizations.

The global fit was repeated changing every time one PDF parameter or using another parameterization for a
single PDF and the difference of the fit results from the central values of the nominal fit are taken to be the
estimated systematic uncertainty.

These studies are summarized below:

- \( m_{ES} \): The mean value of \( m_{ES} \) for signal decays was varied by ±0.5 MeV/\( c^2 \) and the width of
  ±0.2 MeV/\( c^2 \). The \( \xi \) parameter of the Argus function used to model the background shape, was
  allowed to vary within ±2σ. This uncertainty is estimated from the different values of \( \xi \) obtained by
  fitting on-resonance side-bands, off-resonance and continuum Monte Carlo data.

- \( \Delta E \): The mean for the \( \Delta E \) distribution for signal events are varied from −10 MeV to 0 MeV. The \( \sigma \)
  is varied of ±0.5 MeV, consistently with \( h^+h^- \) analysis (see Ref. [57]). Alternative parameterizations
  of background \( \Delta E \) distribution obtained fitting off-resonance and continuum Monte Carlo data are
  used.
Table 5-9. Systematics errors for $K^0\pi$ asymmetry

<table>
<thead>
<tr>
<th></th>
<th>$A^S_{K^0\pi}$</th>
<th>$A^S_{K^0\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{ES}$ (Signal)</td>
<td>Mean $m_{ES}$ (Signal)</td>
<td>+0.030 &lt;br&gt; -0.026</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{m_{ES}}$ (Signal)</td>
<td>+0.009 &lt;br&gt; -0.010</td>
</tr>
<tr>
<td>$m_{ES}$ (Bkg)</td>
<td>Mean $m_{ES}$ (Signal)</td>
<td>+0.011 &lt;br&gt; -0.009</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{\Delta E}$ (Signal)</td>
<td>+0.005 &lt;br&gt; -0.001</td>
</tr>
<tr>
<td>$\Delta E$ (Bkg)</td>
<td>Fisher (Signal)</td>
<td>±0.001</td>
</tr>
<tr>
<td></td>
<td>Fisher (Bkg)</td>
<td>±0.008</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>$\theta_c$: Frac Sat Peak</td>
<td>+0.002 &lt;br&gt; -0.004</td>
</tr>
<tr>
<td></td>
<td>$\theta_c$: Resolution</td>
<td>±0.001 &lt;br&gt; -0.001</td>
</tr>
<tr>
<td></td>
<td>$\theta_c$: Offset</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>$\theta_c$: charge dependent PDFs</td>
<td>±0.001</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>±0.035</td>
</tr>
</tbody>
</table>

- **Fisher Output**: The uncertainty due to the shape of $F$ in signal events is determined by using the shape obtained from $B^- \rightarrow D^0\pi^-$ decays in data. For background, alternative parameterizations are obtained from the fit done with off-resonance data and continuum Monte Carlo.

- **PID**: The $\theta_c$ parameterization offset and resolution have both been changed of $\pm 0.2$ mrad. The fraction of the satellite peak has been varied from 1% to 5% independently for $\pi$ and K. Another systematic cross-check we have performed is using separate DIRC PDFs for positive and negative charged kaons or pions: this gives no change in the $K^0\pi$ yield and a 0.5% difference in the asymmetry.

When just one systematic check is tried, the error is assumed to be symmetric. The systematic uncertainties in the fit results are summarized in Table 5-8 for the $K^0\pi$ yield and in Table 5-9 for the asymmetry in the $K^0\pi$ mode. The systematics due to this method for the $K^0\bar{K}$ mode is 0.

### 5.5.5 $\Delta E$ distribution from on-resonance data

We looked at the $\Delta E$ distribution the $m_{ES}$ signal band: figure 5-14 shows the fit of the signal with a fixed background shape taken from $m_{ES}$ side-band. Fitted $\Delta E$ mean and resolution are in good agreement with the ones estimated from $D^0\pi$ control sample and used in the global likelihood fit. The mean value is $-8.3 \pm 6.6$ MeV, the resolution is $22.2 \pm 5.4$ MeV and the area gives $66 \pm 15$ events: the $\chi^2/ndf$ of the fit is 61/57. Leaving the background floating the result is compatible with the quoted one.

*Marcella Bona*
5.5 Maximum likelihood analysis

Figure 5-14. $\Delta E$ distribution in the $m_{ES}$ signal region: in the fit only the parameters of the Gaussian are floating. The background shape is fixed accordingly with the on-resonance $m_{ES}$ side band ($m_{ES} < 5.27$ GeV) and rescaled properly.

5.5.6 ARGUS shape from on-resonance data

We include as a fit variable in the global likelihood the parameter $\xi$ of the ARGUS function which was used to parameterize $m_{ES}$ background shape. When we let all the parameters float, we get from the fit:

\[
N_{S_{\pi}}^{K^{0}_{S}} = 58.0^{+10.7}_{-9.9} \text{ (stat)}
\]
\[
N_{S_{K}\pi}^{K^{0}_{S}} = 0^{+2.7}_{-0} \text{ (stat)}
\]
\[
A_{S_{\pi}}^{K^{0}_{S}} = 0.21 \pm 0.18 \text{ (stat)}
\]
\[
A_{S_{K}\pi}^{K^{0}_{S}} = 0 \text{ (fixed)}
\]
\[
N_{B_{\pi}}^{K^{0}_{S}} = 1985.4^{+47.7}_{-47.0} \text{ (stat)}
\]
\[
N_{B_{K}\pi}^{K^{0}_{S}} = 1579.6^{+42.7}_{-42.0} \text{ (stat)}
\]
\[
A_{B_{\pi}}^{K^{0}_{S}} = 0.03 \pm 0.26 \text{ (stat)}
\]
\[
A_{B_{K}\pi}^{K^{0}_{S}} = -0.17 \pm 0.28 \text{ (stat)}
\]

Measurement of Branching Fractions for $B^{\pm} \to K^{0}h^{\pm}$ decays
\[ \xi = 24.1 \pm 1.9 \text{ (stat)} \]

In the nominal fit, we used 23.1 ± 1.3 and in the systematics we varied the \( \xi \) value used of 2\( \sigma \): the fitted value of 24.1 ± 1.9 is therefore taken into account through this systematic check. We get exactly the same result fixing the signal components to the values from the nominal fit.

### 5.6 Counting analysis

As a further cross-check, a simple cut-based analysis is performed to isolate samples of events that are consistent with the \( K^0 \pi \) and \( K^0 K \) hypotheses and signal yields are then obtained from an unbinned maximum likelihood fit to \( m_{ES} \) and \( \Delta E \) using the same parameterizations of the global likelihood method.

#### 5.6.1 Cuts

A cut on the Fisher discriminant output is chosen to optimize the statistical significance. The significance as a function of total efficiency is displayed in Fig. 5-15 for various cuts on the Fisher output. The maximum significance is achieved with \( F < -0.3 \). The efficiency of this cut on MC is 53.1% with respect to the likelihood selection.

To separate \( \pi \) and \( K \), the SMS loose selector is applied to the tracks passing the base PID cuts described in Sec. 5.4. Table 5-10 shows the cross-feed matrix computed from the \( D^0 \) control sample taking into account the proton cut and the SMS loose selection.
Table 5-10. Cross-feed matrix computed from the $D^0$ control sample (see Tab. 4-4).

<table>
<thead>
<tr>
<th></th>
<th>from real $K^0\pi$</th>
<th>from real $K^0_{s}K^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>to obs $K^0\pi$</td>
<td>0.937 ± 0.004</td>
<td>0.110 ± 0.003</td>
</tr>
<tr>
<td>to obs $K^0_{s}K^+$</td>
<td>0.063 ± 0.003</td>
<td>0.890 ± 0.004</td>
</tr>
</tbody>
</table>

**Figure 5-16.** $m_{ES}$ distribution in the $\Delta E$ signal region with superimposed the fitted function (sum of an ARGUS function, with $\xi = 23.1$, and a Gaussian with $m_{ES} = 5.280$ GeV/$c^2$ and $\sigma(m_{ES}) = 2.6$ MeV/$c^2$).

### 5.6.2 Results

First we estimate the event yields including in the fit the events passing all cuts except the base PID cuts. The efficiency of this selection is 25.4%.

Fig. 5-16 shows the $m_{ES}$ distribution with superimposed the fitted function. The result of the fit is:

\[
N^{S}_{K^0\pi} = 32.4^{+8.0}_{-7.3} \text{ (stat)}
\]

\[
N^{S}_{K^0_{s}K^+} = 4.5^{+5.7}_{-0.7} \text{ (stat)}
\]

\[
N^{B}_{K^0_{s}h^\pm} = 331.2^{+19.1}_{-18.4} \text{ (stat)}
\]

**MEASUREMENT OF BRANCHING FRACTIONS FOR $B^\pm \rightarrow K^0h^\pm$ DECAYS**
Table 5-11. Number of observed events and PID-unfolded events in each mode.

<table>
<thead>
<tr>
<th>mode</th>
<th>Observed</th>
<th>PID-unfolded</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi$</td>
<td>$26.9 \pm 5.9$</td>
<td>$26.9 \pm 7.3$</td>
</tr>
<tr>
<td>$K^0K$</td>
<td>$0.0 \pm 2.2$</td>
<td>$0.0 \pm 1.1$</td>
</tr>
</tbody>
</table>

Applying the base PID cuts, the Fisher cut and the loose SMS selection, we have selected a sample of 166 candidates in which the $h^\pm$ is compatible with the $\pi$ hypothesis and a sample of 143 candidates in which the $h^\pm$ is compatible with the $K$ hypothesis. Fitting the first sample ($K^0\pi$ candidates), we get:

$$N_{K^0\pi}^S = 26.9^{+6.2}_{-5.5} \text{ (stat)}$$
$$A_{K^0\pi}^S = 0.16^{+0.21}_{-0.22} \text{ (stat)}$$
$$N_{K^0\pi}^B = 139.1^{+12.5}_{-11.8} \text{ (stat)}$$
$$A_{K^0\pi}^B = 0.03 \pm 0.09 \text{ (stat)}$$

while fitting the second ($K^0_sK$ candidates):

$$N_{K^0_sK}^S = 0.0^{+2.2}_{-2.2} \text{ (stat)}$$
$$A_{K^0_sK}^S = 1.0 \pm 1.4 \text{ (stat)}$$
$$N_{K^0_sK}^B = 143.0^{+12.3}_{-11.6} \text{ (stat)}$$
$$A_{K^0_sK}^B = -0.07 \pm 0.08 \text{ (stat)}$$

Fig. 5-17 shows the $m_{ES}$ distribution with superimposed the fitted function for the candidates compatible with the $\pi$ hypothesis.

Applying the Cross-feed Table (Table 5-10) to the raw yields given by the fits determines the number of unfolded signal events in each mode. Table 5-11 summarizes the results. Taking into account the efficiency of the selection (22.2% and 21.0% for $\pi K^0_s$ and $K K^0_s$ respectively) the results are consistent with the finding of the global likelihood method.

## 5.7 Determination of branching fraction

We determine branching fractions for $\pi^+ K^0_s$ and an upper limit for the $K^+ K^0_s$ decay using the results of the global likelihood fit. The individual efficiencies are reported in previous sections. The total efficiencies are $41.7 \pm 1.6$ for $\pi^+ K^0_s$ and $40.3 \pm 1.5$ for $K^+ K^0_s$, where the error is combined statistical and systematic.
5.7 Determination of branching fraction

Branching fractions are calculated as

$$\text{BR}(K^0 h) = \frac{1}{\text{BR}(K^0 \rightarrow K^0) \cdot \text{BR}(K^0 \rightarrow \pi^+\pi^-)} \frac{N_S(K^0 h)}{\epsilon \cdot N_{B\bar{B}}},$$  

(5.4)

where $N_S$ is the central value from the fit, $\epsilon$ is the total efficiency, and $N_{B\bar{B}} = (22.57 \pm 0.36) \times 10^6$ is the total number of $B\bar{B}$ pairs in our data-set. $\text{BR}(K^0 \rightarrow K^0)$ is taken equal to 0.5 and $\text{BR}(K_s^0 \rightarrow \pi^+\pi^-)$ equal to 0.6861 [14]. Implicit in Eq. 5.4 is the assumption of equal branching fractions for $\Upsilon(4S) \rightarrow BB^0$ and $\Upsilon(4S) \rightarrow B^+B^-$. For the $K_s^0K$ mode we calculate the 90% confidence level upper limit yield and the result is increased by the total systematic error that, in this case, is reduced to the contributions from the efficiency and the number of $B\bar{B}$, since no contribution comes from varying the likelihood parameters. The results are summarized in Table. 5-12. The statistical significance of a given signal yield is determined by setting the yield to zero and maximizing the likelihood with respect to all other variables.

In summary, the branching fraction for $B^\pm \rightarrow K^0\pi^\pm$ is $18.2^{+3.3+1.6}_{-3.0-1.7} \cdot 10^{-6}$, while on the branching fraction of $B^\pm \rightarrow K^0K^\pm$ we set an upper limit of $2.5 \cdot 10^{-6}$.

**Figure 5-17.** $m_{ES}$ distribution in the $\Delta E$ signal region with superimposed the fitted function (sum of an ARGUS function, with $\xi = 23.1$, and a Gaussian with $m_{ES} = 5.280$ GeV/c$^2$ and $\sigma(m_{ES}) = 2.6$ MeV/c$^2$.) for candidates in which the $h^\pm$ is compatible with the $\pi$ hypothesis(left) and with the $K$ hypothesis(right).

**Measurement of Branching Fractions for $B^\pm \rightarrow K^0h^\pm$ decays**
Table 5-12. Summary of branching fraction results for the global likelihood fit. Shown are the central fit values $N_S$ and measured branching fractions $BR$. For the $K K^0$ mode we show the 90% confidence level upper limit signal yield and branching fraction. For $N_S$ and $BR$ the first error is statistical and the second is systematic.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$N_S(K^0 h)$</th>
<th>Stat. Sig. ($\sigma$)</th>
<th>$BR \times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^0\pi$</td>
<td>$58.7^{+10.7+1.6}_{-9.8-5.0}$</td>
<td>9.8</td>
<td>$18.9^{+3.3+1.6}_{-3.0-1.7}$</td>
</tr>
<tr>
<td>$K^0K$</td>
<td>0 (&lt; 7)</td>
<td>(0)</td>
<td>&lt; 2.5</td>
</tr>
</tbody>
</table>
Measurement of Branching Fractions for $B \rightarrow K_S^0 K_S^0$ Decays

This chapter describes the charmless two-body analyses for all-neutral final state containing $K_S^0$ mesons. The latest results from CLEO [65] on decays with all-neutral final state is $B(B^0 \rightarrow K^0\bar{K}^0) < 17 \times 10^{-6}$.

6.1 Data samples and event selection

The analyses presented in this chapter use the data samples described in Sec. 4.1 and the selection described in Sec. 4.2. Issues related to reconstruction of $K_S^0$ mesons have been discussed in Sec. 5.2. For the $K_S^0$ we require $t_{K_S^0}/\sigma_t > 5$, where $t_{K_S^0}$ is the measured (2-d) decay time and $\sigma_t$ is its error. The mass window is tightened to $|M(\pi^+\pi^-) - M(K_S^0)| < 11.2$ MeV/$c^2$ (3.5$\sigma$).

$B$ mesons are constructed by combining two $K_S^0$ candidates. To choose between multiple candidates in the same events, we use the variable $\delta \equiv (\Delta M_{K_{S,1}^0}^2 + \Delta M_{K_{S,2}^0}^2)^{1/2}$ and we keep the candidates with the smallest value of $\delta$.

We use the kinematic variables $m_{ES}$ and $\Delta E$. We require $5.20 < m_{ES} < 5.30$ GeV/$c^2$ and $|\Delta E| < 0.300$ GeV. From the conservative $\Delta E$ resolution value of 25 MeV, the signal region is defined as $|\Delta E| < 0.100$ GeV (i.e. 4 times the $\Delta E$ resolution). The regions $-0.3 < \Delta E < -0.1$ GeV and $0.1 < \Delta E < 0.3$ GeV are referred to as the lower and upper side-bands, respectively, and each of them has the same width of the signal region (0.2 GeV).

We also create a control sample collecting those events rejected by the $|\cos \theta_S|$ cut, in order to study the ARGUS function shape in different ranges of $|\Delta E|$ values.

6.2 Analysis strategy

Previous experiments have found no evidence of the decay $B^0 \rightarrow K_S K_S$: the theoretical expectation for the $B^0 \rightarrow K_S^0 K_S^0$ branching ratio is less than $10^{-6}$, but considering the efficiency, given that we reconstruct $K^0 \rightarrow K_S^0 \rightarrow \pi^+\pi^-$, we expect to see $K_S^0 K_S^0$ events with an effective branching ratio less than $10^{-7}$. Since we aim for setting the lowest upper limit possible on the branching ratio measurement, we have investigated two possible strategies to search for this channel in the B\bar{B}ar data sample: the usual global likelihood technique and a counting analysis optimization. With a toy Monte Carlo in the first case and with the on-
Table 6-1. Summary of $K^0_s K^0_s$ selection efficiency.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>reco + tag bit + $R_2$ + Sphericity</td>
<td>0.432 ± 0.004</td>
</tr>
<tr>
<td>$</td>
<td>\cos \theta_S</td>
</tr>
<tr>
<td>$t_{K^0_s}/\sigma_t &gt; 5$</td>
<td>0.366 ± 0.003</td>
</tr>
<tr>
<td>$</td>
<td>M(\pi^+\pi^-)-M(K^0_s)</td>
</tr>
</tbody>
</table>

resonance $m_{ES}$ side-band in the second case, we estimate the best upper limit we can obtain from each of them in order to establish the best method to use. Before being able to test these techniques, we performed the validation of the variables used in both the global likelihood fit and the counting analysis.

### 6.2.1 Efficiency

Table 6-1 summarizes the efficiency of the $K^0_s K^0_s$ selection which corresponds to the usual two-body one (see Sec. 4.2.1). Taking into account a minimal $|\cos \theta_S|$ cut at the value of 0.9 and no Fisher cut, the total efficiency is $(34.9 \pm 0.3)\%$. All efficiency estimates are derived in Monte Carlo except for the $K^0_s$ mass cut efficiency, which was estimated from the $D^+ \rightarrow K^0_s \pi^+$ control sample to be $(97 \pm 1)\%$.

Corrections to the efficiency value, accounting for the two $K^0_s$ reconstruction, are then included (see Sec. 3.3.4). These corrections amount to $1.05 \pm 0.12$ (stat.) $\pm 0.03$ (sys.) for the two $K^0_s$. Other systematic effects come from the $|\cos(\theta_S)|$ cut (1%) and the $K^0_s$ mass cut (1% per $K^0_s$ candidate). This contribution was estimated from the difference between the MC efficiency and that of the $D^+ \rightarrow \pi^+ K^0_s$ control sample. Another contribution to efficiency systematics is a 2.6% coming from the charged tracks reconstruction uncertainty in $K^0_s$ reconstruction.

The final number for efficiency is

$$36.6 \pm 4.6\%$$

### 6.3 The maximum likelihood analysis

We use the same unbinned maximum likelihood fit technique developed to determine from the data:

- $N^{S}_{K^0_s K^0_s}$, the number of $B \rightarrow K^0_s K^0_s$ decays;
- $N^{B}_{K^0_s K^0_s}$, the number of background $K^0_s K^0_s$ candidates;
The likelihood, $\mathcal{L}$, for the selected sample is given by the product of the probability density functions (PDFs) for each individual candidate and a Poisson factor. We use $m_{ES}$, $\Delta E$ and $F$ to separate signal and background.

To be used in the fit, a candidate must pass the preliminary selection, the cut on the $K_S^0$ invariant mass and the cut on $t_{K_S^0}/\sigma_{K_S^0}$ (see Sec. 5.2).

### 6.3.1 Definition of PDFs

The beam energy substituted mass of the $B^0$ candidate, $m_{ES}$, is parameterized as a Gaussian, with mean and width fixed to 5.280 GeV/$c^2$ and 2.6 MeV/$c^2$, respectively, for the signal (Fig. 6-1) and as an ARGUS function for the background. The value of the mean and the width of the signal $m_{ES}$ distribution come from the $B^- \rightarrow D^0\pi^-$ control sample.

The value of the $\xi$ parameter of the ARGUS function is determined to be $25.2 \pm 5.1$ from a fit to on-resonance $\Delta E$ side-band data (Fig. 6-2). A similar fit performed on off-resonance grand side-band region gives $\xi = 23.9 \pm 13.3$, and we find $\xi = 36.6 \pm 10.6$ in continuum Monte Carlo events (Fig. 6-3). All the values obtained from these samples are well compatible with each other and with the on-resonance fit. We use the value $\xi = 25.2 \pm 5.1$ in the rest of the likelihood analysis.

**Figure 6-1.** $m_{ES}$ distribution in the signal $K_S^0K_S^0$ Monte Carlo sample.
Figure 6-2. ARGUS fit to the $m_{ES}$ distribution in the on-resonance upper and lower side-band regions.

Figure 6-3. ARGUS fits to the $m_{ES}$ distribution for the grand side-band region in off-resonance (left) and continuum Monte Carlo (right).
The difference between the $B^0$ candidate’s energy and $\sqrt{s}/2$, $\Delta E$, is parameterized as a Gaussian for the signal (Fig. 6-4) and as a second order polynomial for the background (Fig. 6-5).

In the signal, the width of the Gaussian is 16.8 MeV for $K^{0}_s K^{0}_s$ Monte Carlo. A comparison of $B^- \rightarrow D^0 \pi^-$ decays in data and Monte Carlo indicates that the Monte Carlo resolution should be scaled by a factor $1.24 \pm 0.06$ to agree with data. As a consequence, in case of $K^{0}_s K^{0}_s$ decays, we have estimated the resolution on $\Delta E$ in real data to be $21.0 \pm 5$ MeV. To test the dependence of the fit from the $\Delta E$ resolution, we fitted 1000 toy MC experiments with a single Gaussian signal $\Delta E$ distribution having the same toy MC resolution or 1.24 times better or 1.24 times worse: the results of these fits are perfectly consistent with each others.

The distribution of the Fisher discriminant for the event, $\mathcal{F}$, is fitted with a double Gaussian for both background and signal. For the parameterization of the Fisher variable in signal events we have used signal $K^{0}_s K^{0}_s$ MC, while for the Fisher variable in the background events, we have used the on-resonance $m_{ES}$ side-band (Figure 6-6) defined as $5.20 < m_{ES} < 5.27$ GeV/$c^2$.

The Fisher distribution in on-resonance $m_{ES}$ side-band has been validated against continuum MC and off-resonance data (Figure 6-7).

Table 6-2 summarizes the functional form of PDFs and the samples used to obtain them.
### Table 1

<table>
<thead>
<tr>
<th>ID</th>
<th>ALLCHAN</th>
<th>72129</th>
<th>5588</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\chi^2$/ndf</td>
<td></td>
<td>18.06</td>
<td>55.68</td>
</tr>
<tr>
<td>P1</td>
<td></td>
<td>765.1</td>
<td>55.68</td>
</tr>
<tr>
<td>P2</td>
<td></td>
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<td>0.1280E-01</td>
</tr>
<tr>
<td>P3</td>
<td></td>
<td>0.3163</td>
<td>0.1166E-01</td>
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<tr>
<td>P4</td>
<td></td>
<td>114.3</td>
<td>53.22</td>
</tr>
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<td>P5</td>
<td></td>
<td>-0.4578E-01</td>
<td>0.1443</td>
</tr>
<tr>
<td>P6</td>
<td></td>
<td>0.4756</td>
<td>0.3835E-01</td>
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<table>
<thead>
<tr>
<th>ID</th>
<th>ALLCHAN</th>
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<th>210.0</th>
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<tbody>
<tr>
<td>$\chi^2$/ndf</td>
<td></td>
<td>13.66</td>
<td>12.61</td>
</tr>
<tr>
<td>P1</td>
<td></td>
<td>29.62</td>
<td>12.61</td>
</tr>
<tr>
<td>P2</td>
<td></td>
<td>0.2877E-01</td>
<td>0.1170</td>
</tr>
<tr>
<td>P3</td>
<td></td>
<td>0.2649</td>
<td>0.8093E-01</td>
</tr>
<tr>
<td>P4</td>
<td></td>
<td>10.64</td>
<td>10.67</td>
</tr>
<tr>
<td>P5</td>
<td></td>
<td>0.5876</td>
<td>0.4289</td>
</tr>
<tr>
<td>P6</td>
<td></td>
<td>0.2481</td>
<td>0.1344</td>
</tr>
</tbody>
</table>

### Figure 6-5
Background $\Delta E$ distribution comparison: off-resonance, on-resonance and continuum Monte Carlo data.

### Figure 6-6
$F$ distribution fits in signal $K^0_sK^0_s$ MC (left) and on-resonance $m_{ES}$ side-band (5.20 < $m_{ES}$ < 5.27 GeV) (right).
6.3 The maximum likelihood analysis

Table 6-2. Summary of functional form of PDFs used in the fit and of the sample used to obtain them. The samples in parentheses were used as a cross-check or to provide alternate parameterization to evaluate systematics.

<table>
<thead>
<tr>
<th>type of events and variable</th>
<th>shape</th>
<th>sample used (alternative)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal ( m_{ES} )</td>
<td>Gaussian</td>
<td>( K^0\bar{K}^0 ) MC (( B^- \rightarrow D^0\pi^- ))</td>
</tr>
<tr>
<td>Bkg ( m_{ES} )</td>
<td>ARGUS</td>
<td>on-res side-band (off-res, cont MC)</td>
</tr>
<tr>
<td>Signal ( \Delta E )</td>
<td>Gaussian</td>
<td>( K^0\bar{K}^0 ) MC with ( B^- \rightarrow D^0\pi^- ) scale factor</td>
</tr>
<tr>
<td>Bkg ( \Delta E )</td>
<td>Quadratic</td>
<td>on-res side-band (off-res, cont MC)</td>
</tr>
<tr>
<td>Signal Fisher</td>
<td>double Gaussian</td>
<td>( K^0\bar{K}^0 ) MC (( B^- \rightarrow D^0\pi^- ))</td>
</tr>
<tr>
<td>Bkg Fisher</td>
<td>double Gaussian</td>
<td>on-res ( m_{ES} ) side-band (off-res, cont MC)</td>
</tr>
</tbody>
</table>

Figure 6-7. Background \( F \) distribution comparison: off-resonance, on-resonance and continuum Monte Carlo data

Measurement of Branching Fractions for \( B \rightarrow K^0\bar{K}^0 \) Decays
Table 6-3. Results of several test fits using signal Monte Carlo and real data.

<table>
<thead>
<tr>
<th>Sample Description</th>
<th>$N_{sig}$</th>
<th>$N_{bkg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5583 K^0_S K^0_S$ MC</td>
<td>5532 ± 75</td>
<td>51 ± 9</td>
</tr>
<tr>
<td>57 continuum MC</td>
<td>0$^{+1}_{-0}$</td>
<td>57 ± 8</td>
</tr>
<tr>
<td>10 $K^0_S K^0_S$ MC and 57 continuum MC</td>
<td>8$^{+3}_{-3}$</td>
<td>59 ± 8</td>
</tr>
<tr>
<td>5 $K^0_S K^0_S$ MC and 57 continuum MC</td>
<td>4$^{+3}_{-2}$</td>
<td>58 ± 8</td>
</tr>
<tr>
<td>2 $K^0_S K^0_S$ MC and 57 continuum MC</td>
<td>1.0$^{+1.6}_{-0.9}$</td>
<td>58 ± 8</td>
</tr>
<tr>
<td>42 off-res</td>
<td>0$^{+1}_{-0}$</td>
<td>42 ± 6</td>
</tr>
<tr>
<td>360 on-res lower side-band</td>
<td>0$^{+1}_{-0}$</td>
<td>360 ± 19</td>
</tr>
<tr>
<td>198 on-res upper side-band</td>
<td>0$^{+1}_{-0}$</td>
<td>198 ± 14</td>
</tr>
</tbody>
</table>

6.3.2 Test on the maximum likelihood analysis

Several checks of the fitting technique were performed before testing the different maximum likelihood analysis hypotheses. Table 6-3 shows the results of fitting pure signal Monte Carlo, continuum Monte Carlo, off-resonance and on-resonance side-band data. No problems are observed.

We have used a toy Monte Carlo to estimate the 90% CL upper limit that can be obtained assuming 0 signal events. Background candidates are selected randomly from the PDFs for $m_{ES}$, $\Delta E$, and $F$. The mean number of background events to be generated is estimated from the on-resonance upper and lower side-bands and the off-resonance signal band, properly weighted.

The samples generated are then fitted and the result of the fit used to calculate the pulls of the variables to be extracted from the fit: the left plot in Fig. 6-8 shows the pull distribution for the number of background events. On these samples, we also calculate the 90% CL upper limit on the $K^0_S K^0_S$ yield: the right plot in Fig. 6-8 shows the upper limit distribution, whose mean value is 4.4 events that, taking into account the 34.9% efficiency, becomes an upper limit on the Branching Ratio:

$$B(B^0 \rightarrow K^0 \overline{K}^0) < 5.0 \times 10^{-6}$$

to be compared with CLEO result: $B(B^0 \rightarrow K^0 \overline{K}^0) < 17 \times 10^{-6}$.

Tab. 6-4 shows the optimization for the $|\cos \theta_S|$ cut and it shows that the upper limit on the achievable Branching Ratio is not improving while tightening the $|\cos \theta_S|$ cut. This test is done using the same $\Delta E$ and $m_{ES}$ parameterization (from the 0.9 $|\cos \theta_S|$ cut, see Section 6.3.1) and varying the Fisher one, since we assume no correlation between $|\cos \theta_S|$ and $\Delta E(m_{ES})$, while we expect correlation between $|\cos \theta_S|$ and the Fisher variable.
6.4 Counting analysis

Along with the maximum likelihood fit, a counting analysis has been optimized in order to estimate the best upper limit on $B(B^0 \rightarrow K^0 \bar{K}^0)$ we can extract with this technique from the Run1 data sample.

The counting analysis consists of cutting and counting the events in a 2-dimensional signal box within the $\Delta E-m_{ES}$ plane defined with $-0.1 < \Delta E < 0.1$ GeV and $5.2748 < m_{ES} < 5.2852$ GeV/c$^2$ (i.e. twice the $m_{ES}$ resolution). The region where $5.2748 < m_{ES} < 5.2852$ GeV/c$^2$ is called $m_{ES}$ signal band and the one where $5.20 < m_{ES} < 5.27$ GeV/c$^2$ is the already defined $m_{ES}$ side-band.

The optimization is done to choose the best cuts on the remaining discriminating variables used in two-body analysis: $|\cos \theta_S|$ cut and Fisher cut. We define the best cuts for this analysis the ones which give the lowest upper limit on $B^0 \rightarrow K^0 \bar{K}^0$ branching ratio.

**Figure 6-8.** The pull distribution for the number of background events (left) and the upper limit distribution (right) in 1000 $K_s^0 K_s^0$ toy MC experiments with the $|\cos \theta_S|$ cut value at 0.9.

**Table 6-4.** Results of several Toy Monte Carlo experiments with different cuts.

| $|\cos \theta_S|$ cut | mean value of # bkg events distribution (on the yield) | mean value of upper limit | eff. | upper limit on $B(10^{-6})$ |
|----------------------|------------------------------------------------------|---------------------------|------|----------------------------|
| $< 0.9$              | 282                                                  | 4.4                       | 34.9 | 5.0                        |
| $< 0.8$              | 147                                                  | 3.9                       | 30.8 | 5.0                        |
| $< 0.7$              | 80                                                   | 3.4                       | 26.9 | 5.0                        |
We estimate the number of background events in the signal box counting the number of events in the $m_{ES}$ side-band region and scaling it by the ratio area(signal box)/area($m_{ES}$ side-band), where the area is the integral of the ARGUS function. We assume that no signal events are observed in the signal box (i.e. no exceeding events are counted in that region). We estimate the ARGUS shape from events having $|\cos \theta_k| > 0.9$ to have a common parameterization for all the sets of cuts: from the fit in this sample, we get $\xi = 21.0 \pm 6.2$. With this $\xi$ value, the ratio area(signal box)/area($m_{ES}$ side-band) is 0.101.

Some results from this exercise are shown in Tab. 6-5: we quote the results with different cuts on $|\cos \theta_k|$ and with the Fisher cut giving the best upper limit. The yield upper limit is evaluated from the Feldman-Cousins tables [64] assuming that the number of events found is always equal to the estimated number of background events.

The results obtained reducing the signal box to $|\Delta E| < 50$ MeV are summarized in Tab. 6-6.

The best upper limit on the $K^0\bar{K}^0$ branching ratio is about $8 \cdot 10^{-6}$.
6.5 Analysis choice

From Monte Carlo and on-resonance data tests, performed in both the maximum likelihood fit case and the counting analysis case, we established that the lowest upper limit can be achieved using the maximum likelihood fit method. We choose to perform this maximum likelihood analysis with the $|\cos \theta_k|$ cut set at the value of 0.9: this is chosen since, in this case, we have reduced uncertainty on the background distributions.

6.6 Results on the Run1 data-set

The final fit sample contains 286 candidates and we find:

$$N_{\text{sig}} = 3.4^{+3.4}_{-2.4}$$
$$N_{\text{bkg}} = 283 \pm 17$$

The statistical significance of $N_{\text{sig}}$ is $1.5\sigma$, obtained by fixing the signal component to zero and recording the change in $-2 \log \mathcal{L}$.

In order to test the goodness of fit we have generated a set of 1000 pseudo-experiments with $N_{\text{sig}} = 3$ and $N_{\text{bkg}} = 283$: in Fig. 6-9 we plot $-2 \log \mathcal{L}$ for each pseudo-experiment. The arrow indicates the value obtained from the fit to the Run1 data-set. From the simulation we estimate the probability to find a greater value for $-2 \log \mathcal{L}$ to be 30%.

![Figure 6-9. The value of $-2 \log \mathcal{L}$ for 1000 pseudo-experiments made by Toy MC. The arrow indicates the result from the fit on the Run1 dataset.](image)

**Figure 6-9.** The value of $-2 \log \mathcal{L}$ for 1000 pseudo-experiments made by Toy MC. The arrow indicates the result from the fit on the Run1 dataset.
Figure 6-10. $m_{ES}$ distribution for events entering the maximum likelihood fit sample with superimposed the $m_{ES}$ PDFs used in the fit itself (Gaussian for the signal and ARGUS function for the background)

Figure 6-10 shows the $m_{ES}$ distribution with superimposed the PDF function used in the likelihood fit, while $\Delta E$ distribution for the same likelihood fit sample is shown in Fig. 6-11.

6.6.1 Systematics studies

Systematic errors on the results from the fit are determined using variations of the fit input parameters:

- Changing each parameter by $\pm 1\sigma$ for signal $\Delta E$ and $m_{ES}$ and for background $m_{ES}$.
- Using off-resonance and MC grand side-band parameterizations for background $\Delta E$.
- Using the $D^0\pi^-$ control sample fit parameters for signal $\mathcal{F}$.
- Using off-resonance and MC parameterizations for background $\mathcal{F}$.

In Tab. 6-7 there is a summary of the systematics errors on the signal yield. In some cases (first three entries in the table) we changed each parameter of $\pm 1\sigma$ and we estimated the error from the biggest positive and negative variations of the signal yield. In this way we can obtain an asymmetric interval. The other three entries are obtained using just one alternative parameterization in each case, giving only one possible variation (shown with its own sign in the table). We have then assumed a symmetric interval around the nominal value.

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Figure 6-11. $\Delta E$ distribution for events entering the maximum likelihood fit sample with superimposed the $\Delta E$ PDFs used in the fit itself (Gaussian for the signal and second order polynomial for the background).

Table 6-7. Systematics errors from likelihood fit parameters. The total refers to the absolute variations of the yield summed in quadrature. When just one systematics check is tried, the variation and thus the error are assumed to be symmetric.

<table>
<thead>
<tr>
<th>variable</th>
<th>systematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>signal $m_{ES}$</td>
<td>+0.3 - 0.4</td>
</tr>
<tr>
<td>background $m_{ES}$</td>
<td>+0.4 - 0.4</td>
</tr>
<tr>
<td>signal $\Delta E$</td>
<td>+0.1 - 0.4</td>
</tr>
<tr>
<td>background $\Delta E$</td>
<td>-</td>
</tr>
<tr>
<td>signal $\mathcal{F}$isher</td>
<td>+3.2</td>
</tr>
<tr>
<td>background $\mathcal{F}$isher</td>
<td>+1.2</td>
</tr>
<tr>
<td>total</td>
<td>+3.5 - 3.5</td>
</tr>
</tbody>
</table>

Measurement of Branching Fractions for $B \rightarrow K^0_sK^0_s$ Decays
Other sources of systematics in the branching fraction measurement come from the efficiency, (36.6±4.6%) contributing with a 13% systematic effect, and the number of $B^0$ decays (22574010 ± 18502(stat.) ± 361184(sys.)) adding a 1.6% systematic error.

6.6.2 Cross-check: the counting analysis

We also performed the counting analysis as a further cross check. A cut on the Fisher discriminant $F$ is used to suppress $q\bar{q}$ background.

The optimal set of cuts has been chosen with the 2-dimensional optimization described in Section 6.4: from table 6-5 we choose to use the first set of cuts which is giving one of the best upper limits and the $|\cos \theta|_c$ cut at the value of 0.9 is consistent with the one used in the maximum likelihood method analysis. The overall efficiency, including these cuts, is $17.5±0.3$, while the efficiency corrected with all the contributions described in Section 6.2.1 comes out to be $18.4±2.3\%$.

We estimate a number $N_B$ of background events of $1.7±0.5$ in the signal box with $|\Delta E|<0.1 GeV$: on Run1 dataset we find 3 events in the signal box. Using the Feldman-Cousins tables [64], we find the upper limit on the yield to be 6 events.

Systematics include the errors on the efficiency (evaluated in Section 6.2.1) and on $N_B$: the Fisher cut variation systematic has to be added in this case. We moved the Fisher cut from $-0.3$ down to $-0.4$ and up to $-0.2$ and recorded the branching ratio upper limit variations. We get $25\%$ systematic error from this Fisher cut variation.

Taking into account the relative efficiency and the errors, this result from the cut analysis is in good agreement with the nominal results (see Section 6.7).

6.7 Determination of the branching fraction

We have found good agreement between the counting analysis and the global likelihood fit signal yields. The branching fraction $B$ is defined as

$$ BR(B^0 \rightarrow K^0 K^0) = \frac{1}{BR(K^0 \overline{K}^0 \rightarrow K^0 K_s^0) \cdot BR(K_s^0 \rightarrow \pi^+ \pi^-)^2} \frac{N_S}{\epsilon \cdot N_B} , $$

where $N_S$ is the central value from the fit, $\epsilon$ is the total $K^0 K_s^0$ selection efficiency, $N_B$ = (22.57±0.36) $\times$ 10$^6$ is the total number of $B \overline{B}$ pairs in the dataset and $BR(K_s^0 \rightarrow \pi^+ \pi^-)$ = 0.6861 [14]. We assume the Standard Model prediction that $B^0 \rightarrow K^0 K_s^0$ proceeds through the $K^0 \overline{K}^0$ intermediate state (as opposed to $K^0 K_s^0$ or $K_s^0 K_s^0$) and use $BR(K^0 \overline{K}^0 \rightarrow K^0 K_s^0)$ = 0.5. Implicit in Eq. 6.1 is the assumption of equal branching fractions for $\Upsilon(4S) \rightarrow B^0 \overline{B}^0$ and $\Upsilon(4S) \rightarrow B^+ B^-$.  

1Since $CP$ violation affects in the neutral $K$ system have been measured to be so small ($\epsilon \sim 10^{-6}$) that they can be neglected, assuming conservation of angular momentum and $CPT$ invariance, the decay $B^0 \rightarrow K^0 \overline{K}^0 \rightarrow K^0 K_s^0 K_s^0$ is forbidden.
Since no significant signal is found in this mode we calculate the 90% confidence level upper limit yield and the result is increased by the total systematic error. We measure (with systematic uncertainties explained above)

\[ B(B^0 \to K^0 \overline{K}^0) = [1.8^{+1.8}_{-1.2}(\text{stat}) \pm 1.8(\text{syst})] \cdot 10^{-6} \]

The resulting upper limit at 90% confidence level is

\[ B(B^0 \to K^0 \overline{K}^0) < 7.3 \cdot 10^{-6} \]

This result is a significant improvement over the existing upper limit from the CLEO Collaboration [66], and is approaching the upper range of current theoretical estimates.

In the counting analysis case, as a cross check, we get

\[ B(B^0 \to K^0 \overline{K}^0) = [1.3 \pm 1.8(\text{stat}) \pm 1.7(\text{syst})] \cdot 10^{-6}. \]

The resulting upper limit at 90% confidence level is

\[ B(B^0 \to K^0 \overline{K}^0) < 8.5 \cdot 10^{-6}. \]

Both these results are compatible with the results from the likelihood fit.
Measurement of Branching Fractions for $B \rightarrow K_s^0 K_s^0$ Decays
Analysis of the time-dependent $CP$-violating asymmetry in $B \rightarrow \pi^+ \pi^-$ decays

This chapter describes the analysis of the time evolution in $B^0 \rightarrow h^+ h^-$ decays. In the Standard Model, the time-dependent $CP$-violating asymmetry in $B^0 \rightarrow \pi^+ \pi^-$ is related to the angle $\alpha$ of the Unitarity Triangle (Sec. 1.5.3.1). This decay mode has only recently been observed by CLEO [67] and confirmed by BaBar [6] and Belle [68]. Due to the small decay rate ($\frac{BR}{\pi}$), large continuum background, and significant cross-feed from $B^0 \rightarrow K^+ \pi^-$ decays, extraction of the $CP$ asymmetry in $\pi^+ \pi^-$ is intimately related to the branching fraction measurement. In addition, this measurement relies heavily on the infrastructure (tagging [69] and vertexing [71] in particular) developed for the $\sin 2\beta$ analysis [72].

7.1 $CP$ analysis requirements

The formalism has been already described in the Sec. 1.5. Defining $\Delta t = t_{CP} - t_{tag}$, where $t_{CP}$ and $t_{tag}$ are the proper decay times of the $CP$ and tagged $B$'s, respectively, the decay rate distribution $f_+ (f_-)$ (in Eq. 1.34) for $B_{CP} \rightarrow f$ when $B_{tag}$ is a $B^0$ ($\bar{B}^0$) can be rewritten as:

$$f_\pm(\Delta t) = \frac{e^{-|\Delta t|/\tau}}{4\tau} \left[ 1 \pm S_f \sin(\Delta m_{B_d}\Delta t) \mp C_f \cos(\Delta m_{B_d}\Delta t) \right],$$

(7.1)

where $\tau$ is the average $B^0$ lifetime, $\Delta m_{B_d}$ is the mixing frequency, and

$$S_f = \frac{2Im\lambda}{1 + |\lambda|^2} \quad \text{and} \quad C_f = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}.$$  

(7.2)

Interference effects are included in the physical quantity $\lambda$, defined in Eq. 1.22 or in 1.41: it can be rewritten as

$$\lambda \equiv \frac{q}{p} \frac{A_f}{A_f} = \eta_f e^{-2i\phi} \frac{A_f}{A_f},$$

(7.3)

where $e^{-2i\phi}$ is the weak mixing phase, $A_f (\overline{A}_f)$ is the amplitude for the decay $B^0 \rightarrow f$ ($\bar{B}^0 \rightarrow \overline{f}$), $\eta_f$ is the $CP$ eigenvalue of the final state, and the assumption of no $CP$ violation in mixing ($|\eta_f| = 1$) is implicit. In this analysis, observable $CP$ violation effects can arise from interference between different decay amplitudes ($|\overline{A}_f/A_f| \neq 1$) and interference between the mixing and decay weak phases (see Sec. 1.3.3).
In the case of imperfect tagging, Eq. 7.1 must be modified to include the mis-tag probabilities:

\[ f_{B^0 \text{ tag}} = (1 - w) f_+ + \overline{w} f_- \quad \quad f_{\overline{B}^0 \text{ tag}} = (1 - \overline{w}) f_- + w f_+ \]  

(7.4)

where \( w(\overline{w}) \) is the probability that a true \( B^0 (\overline{B}^0) \) meson is tagged as a \( \overline{B}^0 (B^0) \). Defining the dilutions, \( D = (1 - 2w) \) and \( \overline{D} = (1 - 2\overline{w}) \), the average dilution \( \langle D \rangle \), and the dilution difference \( \Delta D \)

\[ \langle D \rangle = \frac{D + \overline{D}}{2} \quad \quad \Delta D = D - \overline{D} \]  

(7.5)

the mistag probabilities can be written as

\[ w = \frac{1 - \langle D \rangle - \frac{\Delta D}{2}}{2} \quad \quad \overline{w} = \frac{1 - \langle D \rangle + \frac{\Delta D}{2}}{2}. \]  

(7.6)

The decay rate distributions, assuming perfect vertex resolution, are then

\[ f_{B^0 \text{ tag}} = e^{-|\Delta t|/\tau} \left[ 1 + \frac{\Delta D}{2} + \langle D \rangle \left(S_f \sin(\Delta m_{B_d} \Delta t) - C_f \cos(\Delta m_{B_d} \Delta t)\right) \right], \]

\[ f_{\overline{B}^0 \text{ tag}} = e^{-|\Delta t|/\tau} \left[ 1 - \frac{\Delta D}{2} - \langle D \rangle \left(S_f \sin(\Delta m_{B_d} \Delta t) - C_f \cos(\Delta m_{B_d} \Delta t)\right) \right]. \]  

(7.7)

The observed distribution \( F(\Delta t) \) is the convolution of \( f(\Delta t) \) with the signal vertex resolution function \( R_{\text{sig}}(\Delta t) \)

\[ F_{B^0 \text{ tag}} = f_{B^0 \text{ tag}} \otimes R_{\text{sig}} \quad \quad F_{\overline{B}^0 \text{ tag}} = f_{\overline{B}^0 \text{ tag}} \otimes R_{\text{sig}}. \]  

(7.8)

In neutral \( B \) meson decays to the \( K^\pm \pi^\mp \) final state the flavor of the parent meson is tagged by the charge of the kaon. For a \( B^0 \rightarrow K^+ \pi^- \) (\( \overline{B}^0 \rightarrow K^- \pi^+ \)) decay the event is therefore known to be a mixing event if the tag side is a \( B^0 \) (\( \overline{B}^0 \)). The maximum likelihood fit includes the effect of mixing in the \( K \pi \) term. The decay rate distributions for mixed and unmixed events when the tag side is a \( B^0 \) or \( \overline{B}^0 \) are (cf. Ref. [72]):

\[ F_{\text{mix, } B^0 \text{ tag}} = e^{-|\Delta t|/\tau} \left[ \left( 1 + \frac{\Delta D}{2} \right) - \langle D \rangle \cos(\Delta m_{B_d} \Delta t) \right] \otimes R_{\text{sig}}, \]

\[ F_{\text{unmix, } B^0 \text{ tag}} = e^{-|\Delta t|/\tau} \left[ \left( 1 - \frac{\Delta D}{2} \right) + \langle D \rangle \cos(\Delta m_{B_d} \Delta t) \right] \otimes R_{\text{sig}}, \]

\[ F_{\text{mix, } \overline{B}^0 \text{ tag}} = e^{-|\Delta t|/\tau} \left[ \left( 1 - \frac{\Delta D}{2} \right) - \langle D \rangle \cos(\Delta m_{B_d} \Delta t) \right] \otimes R_{\text{sig}}, \]

\[ F_{\text{unmix, } \overline{B}^0 \text{ tag}} = e^{-|\Delta t|/\tau} \left[ \left( 1 + \frac{\Delta D}{2} \right) + \langle D \rangle \cos(\Delta m_{B_d} \Delta t) \right] \otimes R_{\text{sig}}. \]  

(7.9)

### 7.2 Branching fraction \( B^0 \rightarrow h^+ h^- \) analysis results

Given data and selection described in Chapter 4, BaBar has published the results of the \( B^0 \rightarrow h^+ h^- \) branching fraction in Ref. [6]. The unbinned maximum likelihood fit and the parameterizations of the PDFs

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have been described in detail in Sec. 4.6.1. The fit is performed using the unbinned maximum likelihood fit including all events which pass the preselection and the PID cuts. The sample consists of 16032 candidates. The results from the fit are given in Table 7-1.

The statistical significances of the signals are determined to be $4.7\sigma$ for $\pi^+\pi^-$, $15.8\sigma$ for $K^+\pi^-$ and $1.3\sigma$ for $K^+K^-$. The systematic uncertainties in the fit results due to modelling of the PDFs are summarized in Tables 7-2 for the branching ratio.

For the decay modes $B \rightarrow \pi^+\pi^-$ and $B \rightarrow K^+\pi^-$ we measure the branching fractions to be $[4.1 \pm 1.0(\text{stat})\pm 0.7(\text{syst})] \cdot 10^{-6}$ and $[16.7\pm 1.6(\text{stat})^{+1.8}_{+1.2}(\text{syst})] \cdot 10^{-6}$, respectively. The statistical significance is $4.7 (15.8)$ standard deviations for the $\pi^+\pi^- (K^+\pi^-)$ decay. We do not find a significant signal yield in the mode $B \rightarrow K^+K^-$ and measure a 90% confidence level upper limit branching fraction of $2.5 \times 10^{-6}$. The asymmetry in the $B \rightarrow K^+\pi^-$ and $\bar{B} \rightarrow K^-\pi^+$ decay rates is measured to be $0.19 \pm 0.10 \pm 0.03$. These results have been published as Ref. [6] and they are referred to as the PRL results.

### 7.3 Analysis strategy

Applying the cuts described in Chapter 4 for the branching fraction analysis and considering the PRL results for the yields, the expected number of tagged $\pi\pi$ events is $1.5/\text{fb}^{-1}$, or $\approx 45$ in $30 \text{ fb}^{-1}$ (see Sec. 7.2). The

### Table 7-1. Results from the maximum likelihood fit.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\pi\pi}$</td>
<td>$41 \pm 10$</td>
</tr>
<tr>
<td>$N_{K\pi}$</td>
<td>$169 \pm 17$</td>
</tr>
<tr>
<td>$A_{K\pi}$</td>
<td>$0.19 \pm 0.10$</td>
</tr>
<tr>
<td>$N_{KK}$</td>
<td>$8.2^{+7.8}_{-6.4}$</td>
</tr>
<tr>
<td>$N_{b\pi\pi}$</td>
<td>$8152 \pm 95$</td>
</tr>
<tr>
<td>$N_{bK\pi}$</td>
<td>$4844 \pm 81$</td>
</tr>
<tr>
<td>$A_{bK\pi}$</td>
<td>$0.011 \pm 0.017$</td>
</tr>
<tr>
<td>$N_{bbK}$</td>
<td>$2817 \pm 58$</td>
</tr>
</tbody>
</table>

### Table 7-2. Systematic errors (%) on the branching ratio results from the maximum likelihood fit.

<table>
<thead>
<tr>
<th>Par.</th>
<th>bkg</th>
<th>$m_{ES}$</th>
<th>bkg $\Delta E$</th>
<th>bkg $\mathcal{F}$</th>
<th>$\langle m_{ES} \rangle$</th>
<th>$\sigma(m_{ES})$</th>
<th>$\langle \Delta E \rangle$</th>
<th>$\sigma(\Delta E)$</th>
<th>$\mathcal{F}(D^0\pi)$</th>
<th>$\theta_c$</th>
<th>Tot.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_{\pi\pi}$</td>
<td>$\pm 5.3$</td>
<td>$\pm 0.2$</td>
<td>$\pm 13$</td>
<td>$\pm 0.7$</td>
<td>$-0.2$</td>
<td>$\pm 4.2$</td>
<td>$\pm 5.9$</td>
<td>$\pm 3.7$</td>
<td>$+5.0$</td>
<td>$-5.5$</td>
<td>$\pm 17$</td>
</tr>
<tr>
<td>$N_{K\pi}$</td>
<td>$\pm 1.6$</td>
<td>$\pm 0.2$</td>
<td>$\pm 3.0$</td>
<td>$\pm 0.3$</td>
<td>$-1.4$</td>
<td>$\pm 0.5$</td>
<td>$+0.5$</td>
<td>$-1.4$</td>
<td>$-4.3$</td>
<td>$+7.3$</td>
<td>$\pm 7.0$</td>
</tr>
<tr>
<td>$N_{KK}$</td>
<td>$\pm 11$</td>
<td>$\pm 1.3$</td>
<td>$\pm 34$</td>
<td>$+10$</td>
<td>$-8.9$</td>
<td>$+5.1$</td>
<td>$+7.6$</td>
<td>$+4.6$</td>
<td>$\pm 3.8$</td>
<td>$\pm 17$</td>
<td>$\pm 43$</td>
</tr>
</tbody>
</table>
time-dependent measurement is therefore expected to be dominated by statistical uncertainty and, ignoring systematic uncertainties and assuming zero asymmetry in the background, the analysis that optimizes the effective $S/\sqrt{S+B}$ will also minimize the error on the $CP$ asymmetry. The $CP$ asymmetries, as well as the signal and background yields, are determined simultaneously from a global maximum likelihood fit using both tagged and untagged $h^+h^-$ events.

There are two $CP$ observables in the time-dependent analysis: $|\lambda|$ and $Im \lambda$: it would be most convenient to fit for $|\lambda|$ and $Im \lambda / |\lambda|$, since the latter provides a direct measurement of $\sin 2\alpha_{\text{eff}}$. However, it was found that this fit produces non-Gaussian pull distributions in detailed toy Monte Carlo studies. Moreover, from Eq. 7.2 it is seen that using $|\lambda|$ as a fit parameter implicitly constrains the coefficient of the cosine term to be in the physical region $(-1,1)$, while there is no such constraint in the data. So there are also conceptual problems associated with using $|\lambda|$ as a fit parameter. In contrast, fitting for $S_{\pi\pi}$ and $C_{\pi\pi}$ is found to be very robust and does not bias the fit. The current analysis therefore uses $S_{\pi\pi}$ and $C_{\pi\pi}$ as fit parameters.

### 7.4 Data samples and event selection

Results in this analysis are based on a greater luminosity with respect to the one described in Sec 4.1:

- Run 1 on-resonance data (20.4 fb$^{-1}$, 22.57 million $B\overline{B}$ pairs).
- Run 2 on-resonance data (9.95 fb$^{-1}$, ~11 million $B\overline{B}$ pairs).
- 19.1 million (9.47 fb$^{-1}$) $u\pi, d\overline{\pi}$ and $s\pi$ Monte Carlo.
- 12.3 million (9.46 fb$^{-1}$) $c\overline{\pi}$ Monte Carlo.
- 8.9 million (9.47 fb$^{-1}$) $\tau^+\tau^-$ Monte Carlo.
- 19k each of $\pi^+\pi^-$, $K^+\pi^-$, and $K^+K^-$ signal Monte Carlo.

Candidate $B^0$ mesons are reconstructed by combining pairs of charged tracks using four-vector addition, where we assume the pion mass for both tracks. The two-track vertex position is obtained with the standard B\Bar vertex algorithm, like the tag-side vertex is obtained with default B\Bar algorithm using the charged track and the loose $K^0_s$ candidate selection lists as input. For comparison, $\Delta t$ is calculated with and without the beam constraints. The B\Bar tagging algorithm is used to identify the opposite $B$ meson as a $B^0$ or $B^\pm$. We use all the four standard B\Bar tagging categories [4, 69]: lepton, kaon, neural network NT1 and NT2.

Except for a tighter background suppression cut and the addition of $\Delta t$ quality cuts, the selection criteria used in this analysis are identical to those in Chapter 4. The cut on $\cos \theta_k$ that was at 0.9 is tightened at $|\cos \theta_S| < 0.8$.

The $m_{ES}$ and $\Delta E$ 2-dimensional side-bands are defined as $|\Delta E| < 0.420$ GeV and $5.2 < m_{ES} < 5.3$ GeV/$c^2$. The signal band which is the fit region is defined with $|\Delta E| < 0.150$ GeV and $5.2 < m_{ES} < 5.2895$ GeV/$c^2$, while the $m_{ES}$ side-band is $|\Delta E| < 0.420$ GeV and $5.20 < m_{ES} < 5.26$ GeV/$c^2$. The

---

1This data-set corresponds to data taken in the first half of year 2001

MARCELLA BONA
7.4 Data samples and event selection

<table>
<thead>
<tr>
<th></th>
<th>$\epsilon(\pi^+\pi^-)$</th>
<th>$\epsilon(K^+\pi^-)$</th>
<th>$\epsilon(K^+K^-)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard efficiency</td>
<td>0.5464 ± 0.0036</td>
<td>0.5331 ± 0.0036</td>
<td>0.5189 ± 0.0036</td>
</tr>
<tr>
<td>PID efficiency</td>
<td>0.7527 ± 0.0042</td>
<td>0.7644 ± 0.0042</td>
<td>0.7549 ± 0.0043</td>
</tr>
<tr>
<td>$\Delta t$ Selection</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta t$</td>
<td>0.9853 ± 0.0014</td>
<td>0.9861 ± 0.0013</td>
<td>0.9887 ± 0.0012</td>
</tr>
<tr>
<td>$\sigma_{\Delta t}$</td>
<td>0.9562 ± 0.0023</td>
<td>0.9585 ± 0.0023</td>
<td>0.9598 ± 0.0023</td>
</tr>
<tr>
<td>$\Delta t$ efficiency</td>
<td>0.9422 ± 0.0026</td>
<td>0.9451 ± 0.0026</td>
<td>0.9489 ± 0.0026</td>
</tr>
<tr>
<td>nominal efficiency</td>
<td>0.3875 ± 0.0035</td>
<td>0.3851 ± 0.0035</td>
<td>0.3717 ± 0.0035</td>
</tr>
<tr>
<td>tracking correction</td>
<td>0.975</td>
<td>0.975</td>
<td>0.975</td>
</tr>
<tr>
<td>Total Efficiency</td>
<td>0.3778 ± 0.0034</td>
<td>0.3755 ± 0.0034</td>
<td>0.3624 ± 0.0034</td>
</tr>
</tbody>
</table>

Table 7-3. Summary of detection efficiencies for $\pi^+\pi^-$, $K^+\pi^-$, and $K^+K^-$ as determined in 8.8.0 signal Monte Carlo samples with 19k events. The Run 1 tracking efficiency correction factor is included in the total efficiency. The efficiency of each cut is relative to the previous one and the errors are statistical only.

The upper limit on $m_{ES}$ corresponds to our assumed end-point for the ARGUS function. Events in the fit region are used to extract yields and $CP$ parameters with an unbinned maximum likelihood fit, while events in the side-band region are used to extract various background parameters.

The $\Delta t$ selection using the beam spot constraints is:

- $|\Delta t| < 17\,\text{ps}$
- $0.3 < \sigma_{\Delta t} < 3.0\,\text{ps}$.

Table 7-3 summarizes the efficiency of the selection criteria as determined in signal $B^0 \rightarrow h^+h^-$ Monte Carlo samples. The efficiency of each cut is relative to the ones above it and the separate efficiencies for the standard, PID, and $\Delta t$ criteria are also shown. The tracking efficiency correction factor is the Run 1 estimate. Table 7-4 summarizes the tagging composition of the Run 1 and Run 2 events passing the selection criteria. Figure 7-1 shows the $m_{ES}$ distributions in each tagging category and for the the subset of untagged events. We use the same ARGUS shape parameter $\xi$ for all tag categories. The observed differences between the average $\xi$ and the values obtained in the Lepton and NT1 do not have a significant effect on the results (Sec. 7.9).

7.4.1 Optimization of the $|\cos \theta_S|$ cut

Toy Monte Carlo is used to optimize the cut on $|\cos \theta_S|$ relative to the branching ratio analysis cut ($< 0.9$). Given the large correlation between $\cos \theta_S$ and the Fisher discriminant, probability density functions (PDFs) for the latter variable need to be re-parameterized for each cut. In contrast to the 0.9 sample, the signal distribution is pure Gaussian for the 0.8 and 0.7 cuts. For background, the double-Gaussian PDF is a better representation of the data.

Analysis of the time-dependent $CP$-violating asymmetry in $B \rightarrow \pi^+\pi^-$ decays.
Analysis of the time-dependent $CP$-violating asymmetry in $B \rightarrow \pi^+\pi^-$ decays

<table>
<thead>
<tr>
<th>Category</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$B^0$</td>
<td>$\bar{B}^0$</td>
<td>Tot</td>
</tr>
<tr>
<td>Lepton</td>
<td>50</td>
<td>59</td>
<td>109</td>
</tr>
<tr>
<td>Kaon</td>
<td>920</td>
<td>877</td>
<td>1797</td>
</tr>
<tr>
<td>NT1</td>
<td>215</td>
<td>195</td>
<td>410</td>
</tr>
<tr>
<td>NT2</td>
<td>621</td>
<td>560</td>
<td>1181</td>
</tr>
<tr>
<td>untagged</td>
<td>–</td>
<td>–</td>
<td>3103</td>
</tr>
<tr>
<td>Total</td>
<td>1806</td>
<td>1691</td>
<td>6600</td>
</tr>
</tbody>
</table>

Table 7-4. Event yields in Run 1 and Run 2 separated by tag flavor and category.

**Figure 7-1.** Distributions of $m_{ES}$ in the Run 1 + 2 dataset for events in different tagging categories and for the subset of untagged events.
The separation between signal and background in the Fisher variable is reduced when cutting harder on $|\cos \theta_S|$, which partially compensates the increased signal-to-background ratio. To optimize this cut, 1000 toy experiments were generated corresponding to each of the three samples. Each experiment is fit with the likelihood function used in the branching ratio analysis with the usual PDFs for $\Delta E$, $\theta_c$ and the appropriate re-parameterized Fisher PDF based on the $|\cos \theta_S|$ cut. Events are generated with Poisson statistics corresponding to the published Run 1 result (see Sec. 7.2), scaled up to $30 \text{ fb}^{-1}$ and modified by the relative efficiency of the different $|\cos \theta_S|$ cuts. The signal efficiency is obtained assuming a flat distribution, while the background efficiency is obtained from the on-resonance $m_{ES}$ side-band data.

Figure 7-2 shows the distribution of fitted $\pi\pi$ yield divided by its error, which is an estimate of $S/\sqrt{S + B}$, for each of the toy samples. The 0.8 and 0.9 samples give the same significance, while the 0.7 sample is less optimal. The cut at 0.8 removes 60% of the background and leads to more Gaussian Fisher distributions. The reduction in background means that the time-dependent fits and toy Monte Carlo studies run twice as fast, which is not an insignificant advantage. We have therefore decided to use $|\cos \theta_S| < 0.8$ as our default cut.

**Figure 7-2.** The fitted $\pi\pi$ yield divided by its error for three samples of 1000 toy experiments corresponding to different cuts on $\cos \theta_S$.  

The separation between signal and background in the Fisher variable is reduced when cutting harder on $|\cos \theta_S|$, which partially compensates the increased signal-to-background ratio. To optimize this cut, 1000 toy experiments were generated corresponding to each of the three samples. Each experiment is fit with the likelihood function used in the branching ratio analysis with the usual PDFs for $m_{ES}$, $\Delta E$, and $\theta_c$ and the appropriate re-parameterized Fisher PDF based on the $|\cos \theta_S|$ cut. Events are generated with Poisson statistics corresponding to the published Run 1 result (see Sec. 7.2), scaled up to $30 \text{ fb}^{-1}$ and modified by the relative efficiency of the different $|\cos \theta_S|$ cuts. The signal efficiency is obtained assuming a flat distribution, while the background efficiency is obtained from the on-resonance $m_{ES}$ side-band data.

Figure 7-2 shows the distribution of fitted $\pi\pi$ yield divided by its error, which is an estimate of $S/\sqrt{S + B}$, for each of the toy samples. The 0.8 and 0.9 samples give the same significance, while the 0.7 sample is less optimal. The cut at 0.8 removes 60% of the background and leads to more Gaussian Fisher distributions. The reduction in background means that the time-dependent fits and toy Monte Carlo studies run twice as fast, which is not an insignificant advantage. We have therefore decided to use $|\cos \theta_S| < 0.8$ as our default cut.
Table 7-5. Results of a fit to the Run 1 data with the new analysis cuts. The PRL results, scaled by the relative efficiency, are shown for comparison.

### 7.4.2 Comparison with the branching fraction analysis result

Table 7-5 shows the new nominal fit results for the Run 1 dataset, including all of the cuts in Table 7-3. For comparison, the expected values, based on the PRL results in Tab. 7-1 and the relative efficiency of the new cuts, are also shown. The likelihood function for this fit is equivalent to the one used in the branching ratio analysis. The relative efficiency is 82% for signal events and 40% for background events.

### 7.5 Background characterization

#### 7.5.1 Composition

The sample selected with the cuts described in the previous section contains 97% continuum background. The $\pi^+\pi^-$ purity can be estimated from the fit result in Table 7-5 as $N_{\pi\pi}/\sigma_{\pi\pi}^2 \sim 40\%$. The background is again $q\bar{q}$ continuum and is made up of 72% $uds$, 25% charm, and 3% tau events.

The relative amount of $\pi\pi$, $K\pi$, and $KK$ events varies significantly over the different background samples. The charm sample has a much larger fraction of $K\pi$ and $KK$ decays than $uds$ events due to the dominance of $c \rightarrow s$ decays. The tau sample contains no kaons and is dominated by events where one or both of the tracks come from a $\tau \rightarrow \mu$ decay. These differences affect the relative fraction of $\pi\pi$, $K\pi$, and $KK$ background in each tagging category, which we take into account in the maximum likelihood fit.

Table 7-6 shows the percentage of events tagged in each category for the different species from fits using the Čerenkov angle to separate $\pi\pi$, $K\pi$, and $KK$ events. For Monte Carlo the results are cross-checked using truth information. The same trends are observed in data and Monte Carlo, with similar absolute tagging efficiencies. As a cross-check on the real data, we include the parameterization obtained from the fit region by floating the tag efficiencies. The fit region and side-band regions are in excellent agreement, giving
7.5.2 Parameterization of $\Delta t$

Continuum $q\bar{q}$ events typically produce $B \to h^+ h'^-$ candidates by combining one high momentum track from each jet. The resulting background $\Delta t$ distribution $R_{\text{bg}}$ is not expected to have a significant lifetime confidence that the higher statistics side-band sample can be used to determine the background tagging efficiencies. We therefore use the results from the combined Run 1 + 2 data in the maximum likelihood fit.

### Table 7-6

<table>
<thead>
<tr>
<th>Sample</th>
<th>Lepton</th>
<th>Kaon</th>
<th>NT1</th>
<th>NT2</th>
<th>Untagged</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC truth</td>
<td>0.8 ± 0.2</td>
<td>23.4 ± 1.1</td>
<td>5.2 ± 0.6</td>
<td>16.1 ± 1.0</td>
<td>54.5 ± 1.3</td>
</tr>
<tr>
<td>MC fit</td>
<td>0.7 ± 0.2</td>
<td>23.0 ± 1.1</td>
<td>5.2 ± 0.6</td>
<td>16.2 ± 1.0</td>
<td>54.8 ± 1.3</td>
</tr>
<tr>
<td>Run 1 $h^+ h'^-$ (FR)</td>
<td>1.2 ± 0.2</td>
<td>24.8 ± 0.9</td>
<td>6.7 ± 0.5</td>
<td>19.1 ± 0.8</td>
<td>48.2 ± 1.3</td>
</tr>
<tr>
<td>Run 1 $h^+ h'^-$ (SB)</td>
<td>1.0 ± 0.1</td>
<td>25.3 ± 0.5</td>
<td>6.5 ± 0.3</td>
<td>18.1 ± 0.5</td>
<td>49.2 ± 0.6</td>
</tr>
<tr>
<td>Run 1 + 2 $h^+ h'^-$ (SB)</td>
<td>1.0 ± 0.1</td>
<td>26.0 ± 0.4</td>
<td>6.6 ± 0.2</td>
<td>17.6 ± 0.4</td>
<td>48.9 ± 0.7</td>
</tr>
<tr>
<td>$K\pi$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC truth</td>
<td>1.2 ± 0.4</td>
<td>34.4 ± 1.7</td>
<td>3.5 ± 0.7</td>
<td>14.7 ± 1.3</td>
<td>46.2 ± 1.8</td>
</tr>
<tr>
<td>MC fit</td>
<td>1.1 ± 0.4</td>
<td>34.2 ± 1.7</td>
<td>4.1 ± 0.7</td>
<td>13.5 ± 1.2</td>
<td>47.0 ± 1.8</td>
</tr>
<tr>
<td>Run 1 $h^+ h'^-$ (FR)</td>
<td>1.1 ± 0.3</td>
<td>32.3 ± 1.3</td>
<td>5.7 ± 0.6</td>
<td>15.4 ± 1.0</td>
<td>45.5 ± 1.8</td>
</tr>
<tr>
<td>Run 1 $h^+ h'^-$ (SB)</td>
<td>1.0 ± 0.2</td>
<td>32.3 ± 0.7</td>
<td>5.6 ± 0.4</td>
<td>15.2 ± 0.6</td>
<td>45.8 ± 0.8</td>
</tr>
<tr>
<td>Run 1 + 2 $h^+ h'^-$ (SB)</td>
<td>1.0 ± 0.1</td>
<td>33.1 ± 0.6</td>
<td>5.4 ± 0.3</td>
<td>15.3 ± 0.5</td>
<td>45.2 ± 0.6</td>
</tr>
<tr>
<td>$KK$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC truth</td>
<td>2.2 ± 0.7</td>
<td>18.7 ± 1.8</td>
<td>6.8 ± 1.1</td>
<td>19.7 ± 1.8</td>
<td>52.5 ± 2.2</td>
</tr>
<tr>
<td>MC fit</td>
<td>2.5 ± 0.7</td>
<td>19.2 ± 1.7</td>
<td>6.5 ± 1.1</td>
<td>20.1 ± 1.8</td>
<td>51.8 ± 2.2</td>
</tr>
<tr>
<td>Run 1 $h^+ h'^-$ (FR)</td>
<td>1.9 ± 0.4</td>
<td>24.3 ± 1.3</td>
<td>5.9 ± 0.7</td>
<td>19.2 ± 1.2</td>
<td>48.7 ± 2.0</td>
</tr>
<tr>
<td>Run 1 $h^+ h'^-$ (SB)</td>
<td>1.5 ± 0.2</td>
<td>23.3 ± 0.8</td>
<td>6.6 ± 0.5</td>
<td>18.8 ± 0.7</td>
<td>49.8 ± 0.9</td>
</tr>
<tr>
<td>Run 1 + 2 $h^+ h'^-$ (SB)</td>
<td>1.5 ± 0.2</td>
<td>23.5 ± 0.7</td>
<td>6.9 ± 0.4</td>
<td>19.7 ± 0.6</td>
<td>48.3 ± 0.8</td>
</tr>
<tr>
<td>“Other”</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MC truth</td>
<td>0.7 ± 0.7</td>
<td>21.9 ± 3.4</td>
<td>7.5 ± 2.2</td>
<td>13.0 ± 2.8</td>
<td>60.3 ± 4.1</td>
</tr>
</tbody>
</table>

The “MC truth” results come from a direct count of events in each category/species using truth information. Events in the “other” category contain tracks not associated with either a pion or kaon. The Run 1 + 2 side-band results (highlighted) are used in the final fit.
component, even for $c\tau$ events. We therefore choose to parameterize the distribution as the sum of three Gaussians: core, tail, and outlier, with the same parameters for $\pi\pi$, $K\pi$, and $KK$ background. Attempting to incorporate event-by-event errors leads to unstable parameterizations, therefore, we do not make use of the $\Delta t$ error.

The Gaussian parameters are obtained from an unbinned maximum likelihood fit to the on-resonance side-band sample, with the outlier mean fixed to 0 ps. Figure 7-3 shows the fit result for Run 1 and Run 2 data. Table 7-7 lists the fitted parameters. The $\chi^2/ndof$ for the triple-Gaussian fit is 1.9 in Run 1 and 1.4 in Run 2. The Kolmogorov test returns a probability of 0.93 for both Run 1 and Run 2. There is a significant negative bias in $\mu_{\text{tail}}$ in Run 1 and a positive bias in Run 2. The origin of the shifts is not understood, but it is accounted for in the systematic error (Sec. 7.10).

Several cross-checks have been performed to validate the $\Delta t$ parameterization. Figure 7-4 shows the on-resonance side-band data compared to a parameterization obtained in the fit region. In this fit the signal and background yields are determined from $m_{ES}$, $\Delta E$, $F$, and $\theta_c$ simultaneously with the background $\Delta t$ parameters. For the signal, we use the same resolution function as the $\sin 2\beta$ analysis [70]. Good agreement is found between the parameters obtained from the fit region and side-band data.

In order to justify using an average of all species, we fit the side-band data with separate parameters for each species. We find no difference between the $\pi\pi$ and $K\pi$ parameters, and only small differences in the $KK$ sample. These differences are probably due to the significant correlation between Gaussian parameters and the fewer number of $KK$ events in the sample (cf. Table 7-5). The average fit results (Table 7-7) are used in the $CP$ fit.

Finally, we check the dependence on tagging by fitting the sub-samples corresponding to each category, as well as the untagged and all-tagged events. We find reasonable consistency across tagging categories, and between tagged and untagged events. The average background parameterization is used for all categories, and systematic errors are evaluated using the other parameterizations.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run 1</th>
<th>Run 2</th>
<th>MC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{core}}$</td>
<td>$-0.037 \pm 0.012$</td>
<td>$-0.005 \pm 0.016$</td>
<td>$-0.014 \pm 0.015$</td>
</tr>
<tr>
<td>$\sigma_{\text{core}}$</td>
<td>$0.637 \pm 0.022$</td>
<td>$0.611 \pm 0.027$</td>
<td>$0.673 \pm 0.022$</td>
</tr>
<tr>
<td>$f_{\text{tail}}$</td>
<td>$0.399 \pm 0.029$</td>
<td>$0.356 \pm 0.042$</td>
<td>$0.273 \pm 0.030$</td>
</tr>
<tr>
<td>$\mu_{\text{tail}}$</td>
<td>$-0.119 \pm 0.027$</td>
<td>$0.122 \pm 0.040$</td>
<td>$0.116 \pm 0.056$</td>
</tr>
<tr>
<td>$\sigma_{\text{tail}}$</td>
<td>$1.531 \pm 0.062$</td>
<td>$1.416 \pm 0.087$</td>
<td>$1.745 \pm 0.118$</td>
</tr>
<tr>
<td>$f_{\text{out}}$</td>
<td>$0.054 \pm 0.005$</td>
<td>$0.058 \pm 0.007$</td>
<td>$0.030 \pm 0.006$</td>
</tr>
<tr>
<td>$\mu_{\text{out}}$</td>
<td>(fixed)</td>
<td>(fixed)</td>
<td>(fixed)</td>
</tr>
<tr>
<td>$\sigma_{\text{out}}$</td>
<td>$5.486 \pm 0.231$</td>
<td>$5.272 \pm 0.288$</td>
<td>$6.294 \pm 0.564$</td>
</tr>
</tbody>
</table>

Table 7-7. Parameters for the triple-Gaussian background $\Delta t$ resolution function $R_{\text{bkg}}$ in on-resonance and continuum Monte Carlo samples. Separate parameterizations are used for Run 1 and Run 2. Means and resolutions are in ps.
7.5 Background characterization

Figure 7-3. Top: Distributions of $\Delta t$ in Run 1 (left) and Run 2 (right) on-resonance side-band samples. The result of a triple-Gaussian fit is overlayed. Bottom: The quantity $(N_i - F_i)/\sqrt{N_i}$ for each $\Delta t$ bin, where $F_i$ is the value of the function at the center of bin $i$.

Figure 7-4. The Run 1 $h^+ h^-$ on-resonance side-band sample compared to a parameterization obtained in the fit region. The parameterization is normalized to the number of events in the side-band sample.

Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+ \pi^-$ decays
Signal characterization

The tagging performance and vertex resolution of the tag side in a $B\overline{B}$ event do not depend on the decay channel of the fully reconstructed $B$ meson. With this assumption, the tagging efficiencies, dilutions, and $\Delta t$ resolution function ($R_{\text{sig}}$) for $\pi\pi$, $K\pi$, and $\bar{K}K$ decays are obtained from the high statistics sample of fully-reconstructed $B$'s (Breco sample).

Table 7-8 compares the tagging efficiency in signal Monte Carlo, the Run 1 + 2 Breco sample, and the $h^{+}h'^{-}$ fit region. The latter is obtained by floating the signal tag fractions in a $CP$-blind fit. No significant difference is observed between different species in signal Monte Carlo, and the on-resonance signal region gives consistent tagging efficiencies. Table 7-9 summarizes the average dilution and difference for each category as determined in the Run 1 + 2 Breco sample.

The signal $\Delta t$ resolution function is parameterized by the sum of three Gaussians (core, tail, outlier), where the core bias is a function of the tag category and the error on $\Delta t$. Table 7-10 lists the resolution function parameters obtained from a fit to the Run 1 + 2 Breco tagged + untagged sample.

\( \text{Table 7-8. Efficiency (\%)} \) for each tag category in signal $h^{+}h'^{-}$ Monte Carlo, the Run 1 and Run 2 $h^{+}h'^{-}$ fit region, and the Run 1 + 2 Breco sample. The latter (highlighted) is used in the fit.

\[
\begin{array}{cccccc}
\text{Sample} & \text{Lepton} & \text{Kaon} & \text{NT1} & \text{NT2} & \text{Untagged} \\
\pi^{+}\pi^{-} & 13.0 \pm 0.4 & 34.0 \pm 1.6 & 7.8 \pm 0.3 & 15.3 \pm 0.4 & 30.0 \pm 0.5 \\
K^{+}\pi^{-} & 12.4 \pm 0.4 & 33.7 \pm 0.6 & 8.4 \pm 0.3 & 14.7 \pm 0.4 & 31.0 \pm 0.5 \\
K^{+}K^{-} & 11.6 \pm 0.4 & 35.3 \pm 0.6 & 7.9 \pm 0.3 & 15.7 \pm 0.4 & 29.5 \pm 0.5 \\
\text{Run 1} h^{+}h'^{-} & 13.7 \pm 2.7 & 31.5 \pm 4.1 & 4.4 \pm 1.9 & 12.3 \pm 3.0 & 38.1 \pm 6.1 \\
\text{Run 2} h^{+}h'^{-} & 13.2 \pm 3.7 & 32.7 \pm 5.6 & 6.7 \pm 3.0 & 12.6 \pm 4.2 & 34.8 \pm 8.5 \\
\text{Run 1 + 2 Breco} & 11.0 \pm 0.3 & 35.8 \pm 0.5 & 8.0 \pm 0.3 & 13.9 \pm 0.4 & 31.3 \pm 0.5 \\
\end{array}
\]

\( \text{Table 7-9. Average dilution and difference determined from the Run 1 + 2 Breco sample. For comparison we show the parameters determined from Breco and } \pi\pi \text{ MC.} \)

\( \text{Table 7-10. Resolution function parameters obtained from a fit to the Run 1 + 2 Breco tagged + untagged sample.} \)

\( ^{2}\text{We blind the } CP \text{ parameters by adding a random offset between } \pm 5 \text{ and randomly flipping the sign of the asymmetries.} \)
### 7.7 The maximum likelihood analysis

An unbinned maximum likelihood fit is used to simultaneously extract yields and $CP$ asymmetry parameters from combined tagged and untagged $h^+h'^-$ sample. The only new feature of the fitting technique relative to the branching fraction analysis is the addition of a PDF describing the time dependence in each component and tagging category.

#### 7.7.1 Likelihood function

The overall construction of the likelihood function $L$ is similar to the branching fraction analysis. For this analysis, the sample consists of signal and background components for the three species ($\pi\pi$, $K\pi$, and $KK$), separated by the flavor and category of the tag side. In addition to the usual variables $m_{ES}$, $\Delta E$, the Fisher discriminant $F$ and the Čerenkov angle, the $\Delta t$ measurement facilitates provides additional background rejection.

The probability $P_{i,c}$ for a single event $i$ in tag category $c$ is the sum of probabilities over all components,

\[
P_{i,c} = \frac{N_{\pi\pi}}{N_c} f_{\pi\pi} P_{i,c}^{\pi\pi} + \frac{N_{b\pi\pi}}{N_c} f_{b\pi\pi} P_{i,c}^{b\pi\pi} \\
+ \frac{N_{KK}}{N_c} f_{KK} P_{i,c}^{KK} + \frac{N_{bKK}}{N_c} f_{bKK} P_{i,c}^{bKK} \\
+ \frac{N_{K\pi}}{2N_c} f_{K\pi} (1 + A_{K\pi}) P_{i,c}^{K-\pi^+} + \frac{N_{bK\pi}}{2N_c} f_{bK\pi} (1 + A_{bK\pi}) P_{i,c}^{bK-\pi^+}
\]

### Table 7-10. Parameters for $R_{sig}$ obtained from the Run 1 and Run 2 Breco tagged + untagged samples. The Breco MC results are shown for comparison.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run 1 Breco</th>
<th>Run 2 Breco</th>
<th>MC Breco</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scale (core)</td>
<td>1.411 ± 0.057</td>
<td>1.043 ± 0.104</td>
<td>1.151 ± 0.020</td>
</tr>
<tr>
<td>$\delta(\Delta t)$ lepton (core)</td>
<td>$-0.011 ± 0.119$</td>
<td>$-0.074 ± 0.163$</td>
<td>$-0.072 ± 0.021$</td>
</tr>
<tr>
<td>$\delta(\Delta t)$ kaon (core)</td>
<td>$-0.323 ± 0.068$</td>
<td>$-0.196 ± 0.095$</td>
<td>$-0.181 ± 0.014$</td>
</tr>
<tr>
<td>$\delta(\Delta t)$ NT1 (core)</td>
<td>$-0.186 ± 0.144$</td>
<td>$-0.357 ± 0.207$</td>
<td>$-0.109 ± 0.027$</td>
</tr>
<tr>
<td>$\delta(\Delta t)$ NT2 (core)</td>
<td>$-0.388 ± 0.104$</td>
<td>$-0.184 ± 0.153$</td>
<td>$-0.171 ± 0.022$</td>
</tr>
<tr>
<td>$\delta(\Delta t)$ notag (core)</td>
<td>$-0.361 ± 0.067$</td>
<td>$-0.155 ± 0.093$</td>
<td>$-0.212 ± 0.016$</td>
</tr>
<tr>
<td>Scale (tail)</td>
<td>3.0 ± 0.0</td>
<td>3.0 ± 0.0</td>
<td>4.358 ± 0.317</td>
</tr>
<tr>
<td>$\delta(\Delta t)$ (tail)</td>
<td>$-9.354 ± 2.844$</td>
<td>$-3.464 ± 3.341$</td>
<td>$-0.969 ± 0.143$</td>
</tr>
<tr>
<td>$f$ (tail)</td>
<td>0.006 ± 0.005</td>
<td>0.029 ± 0.039</td>
<td>0.061 ± 0.008</td>
</tr>
<tr>
<td>$f$ (outlier)</td>
<td>0.002 ± 0.003</td>
<td>0.000 ± 0.003</td>
<td>0.005 ± 0.001</td>
</tr>
</tbody>
</table>

**Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays**
\[ + \frac{N_{K^+\pi^-}}{2N_c} f_{c}^{K^+\pi^-} (1 - A_{K^+\pi^-}) P_{i,c}^{K^+\pi^-} + \frac{N_{bK^+\pi^-}}{2N_c} f_{c}^{bK^+\pi^-} (1 - A_{bK^+\pi^-}) P_{i,c}^{bK^+\pi^-} \]

where:

- \( N_k \) = fitted number of events of type \( k \) in the entire sample.
- \( f_{c}^{k} \) = fraction of events of type \( k \) that are tagged in category \( c \).
- \( N_c' = \sum N_k f_{c}^{k} \).
- \( A_k = (N_{K^+\pi^-} - N_{K^+\pi^-})/(N_{K^+\pi^-} + N_{K^+\pi^-}) \), the direct CP-violating asymmetry.
- \( P_{c}^{k} = P_{c}^{k}(m_{ES}) \cdot P_{c}^{k}(\Delta E) \cdot P_{c}^{k}(\mathcal{F}) \cdot P_{c}^{k}(\theta_{c}) \cdot P_{c}^{k}(\Delta t) \).

Untagged events are treated as a fifth category with \( D = \Delta D = 0 \). Due to the small number of signal events, we use the transformation of variables \( N_c^{k} \rightarrow N_k f_{c}^{k} \) in order to fit for the total yields \( N_k \) rather than the yield in each tagging category. Background tag fractions \( f_{c}^{k} \text{ come from Table 7-6. For signal we assume } f_{c}^{K^+\pi^-} = f_{c}^{bK^+\pi^-} = f_{c}^{bK^+\pi^-} \text{ and use the result from the Breco data sample (Table 7-8).} \)

The extended likelihood \( L_c \) for a single category is given by

\[
L_c = e^{-N_c} \prod_{i} N_c P_{i,c} \tag{7.11}
\]

where the Poisson term is the probability of observing \( N_c \) events in category \( c \) when \( N_c' \) are expected. Including this term allows for the direct fitting of yields rather than fractions. Finally, the total likelihood function is the product over all categories:

\[
\mathcal{L} = \prod_{c=1}^{5} L_c. \tag{7.12}
\]

The quantity \(-2 \ln(\mathcal{L}) = -2 \sum \ln(L_c)\) is minimized.

### 7.7.2 Probability Density Functions

The PDF parameterizations for \( m_{ES} \), \( \Delta E \), \( \mathcal{F} \), and \( \theta_{c} \) are described in detail in Sec. 4.6.1.

Top plots in Fig. 7-5 shows the \( m_{ES} \) distributions for signal events for Run 1 and Run 2. We use \( \mu_{m_{ES}} = 5.2800 \pm 0.0002 \text{ MeV}/c^2 \) and \( \sigma_{m_{ES}} = 2.6 \pm 0.2 \text{ MeV}/c^2 \) for both Run 1 and Run 2. The background \( m_{ES} \) shape is the usual ARGUS shape but the \( \xi \) parameter is left floating in the fit. Bottom plots in Fig. 7-5 shows the distribution of \( m_{ES} \) in the region \( 0.2 < |\Delta E| < 0.42 \text{ GeV} \) in the side-band region. We find similar shapes in Run 1 and Run 2, so we float common parameters for the entire dataset.

Figure 7-6 shows the \( \Delta E \) distribution for signal events in Run 2 and the combined Run 1 + 2 data. The Run 2 parameters are similar to the Run 1 results so we use a common mean, \( \mu_{\Delta E} = -5 \pm 5 \text{ MeV} \), and resolution, \( \sigma_{\Delta E} = 26^{+5}_{-2.5} \text{ MeV} \), for the entire dataset. The mean of \( K\pi \) (KK) events is shifted by approximately \(-45 \text{ MeV} \) \((-91 \text{ MeV}) \) relative to \( \pi\pi \), where the shift is momentum dependent due to the boost. The two parameters (three from a second order polynomial minus one of the normalization) of the background \( \Delta E \) shape are left floating in the fit. Again, we float common parameters for the entire dataset Run 1 and Run 2.
The signal Fisher discriminant distribution is obtained from signal $\pi^+\pi^-$ Monte Carlo and cross-checked with the $D^0\pi^-$ control sample. After tightening the cut on $\cos\theta_S$ relative to the branching fraction analysis we find that the signal Fisher shape is a pure Gaussian. Top plots in Fig. 7-7 shows the combined Run 1 + 2 sample and the signal Monte Carlo. We use the latter distribution for both Run 1 and Run 2, with the $D^0\pi^-$ sample used to estimate the systematic error. The background Fisher shape is the usual double Gaussian whose parameters are left floating in the fit. Bottom plots in Fig. 7-7 shows the Fisher distribution in the side-band region. For the background common parameters for the entire dataset are floated in the fit.

The Čerenkov angle pulls for pions and kaons are determined in a high-statistics data sample of $D^+ \to D^{0}\pi^+$, $D^0 \to K\pi$ decays, where the same PDFs are used for signal and background. We also use the same parameterization for positive and negative tracks. The pulls are defined as $(\theta_c - \theta_c(\text{exp}) + \text{offset})/\sigma_{\theta_c}$, where $\theta_c(\text{exp})$ is the expected angle for a pion or kaon with the given momentum (corrected for energy loss) and the offsets and resolutions depend on track polar angle. Left plots in Fig. 7-8 show the offset

### Figure 7-5

**Top plots:** distributions of $m_{ES}$ for $B^- \to D^0\pi^-$ decays in Run 1 (left) and Run 2 (right).
**Bottom plots:** distributions of $m_{ES}$ for Run 1 (left) and Run 2 (right) on-resonance data in the region $0.2 < |\Delta E| < 0.42 \text{GeV}$. 

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\[\Delta E_{\text{SMS1}} = 27.78 \pm 1.96\]

\[P_1 = -0.976 \pm 0.049, P_2 = 0.249 \pm 0.021\]

\[P_3 = 3.90 \pm 0.00, P_4 = 0.202 \pm 0.002, P_5 = 0.000, P_6 = 0.350 \pm 0.000\]

\[\Delta E_{\text{SMS1}} = 23.60 \pm 0.92, P_1 = 110.3 \pm 4.129, P_2 = -0.563 \pm 0.809, P_3 = 0.194 \pm 0.698, P_4 = 6.900, P_5 = 0.000, P_6 = 0.350 \pm 0.000\]

\[0 \leq \Delta E_{\text{SMS1}} \leq 0.2\]

\[0 \leq \Delta E_{\text{SMS1}} \leq 0.2\]

Figure 7-6. Distributions of \(\Delta E\) for \(B^- \rightarrow D^0\pi^-\) decays in Run 2 (left) and Run 1 + 2 (right).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Sig Func</th>
<th>Sig Params</th>
<th>Bkg Func</th>
<th>Bkg Params</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_{\text{ES}})</td>
<td>Gaussian</td>
<td>(\mu = 5.280 \text{ GeV}/c^2), (\sigma = 2.6 \text{ MeV}/c^2)</td>
<td>ARGUS</td>
<td>(\xi); free</td>
</tr>
<tr>
<td>(\Delta E)</td>
<td>Gaussian</td>
<td>(\mu = -5 \text{ MeV}), (\sigma = 26 \text{ MeV})</td>
<td>quadratic</td>
<td>(\Delta E_{p1}, \Delta E_{p2}); free</td>
</tr>
<tr>
<td>(F)</td>
<td>Gaussian</td>
<td>(\mu = -0.3547), (\sigma = 0.3098)</td>
<td>double Gaussian</td>
<td>(F_{\mu_1}, F_{\sigma_1}, F_{\mu_2}), (F_{\sigma_2}, F_{f_1}); free</td>
</tr>
<tr>
<td>(\theta_c) pulls</td>
<td>Gaussian</td>
<td>(\mu(\theta_c, \theta, p))</td>
<td>same as Sig</td>
<td>same as Sig</td>
</tr>
</tbody>
</table>

Table 7-11. Summary of PDFs for \(m_{\text{ES}}, \Delta E, F,\) and \(\theta_c\). The Čerenkov angle offsets and resolutions are functions of track polar angle \(\theta\).

and resolution parameterizations for Run 2 data (Run 1 parameterizations were given in Sec. 4.6.5). The global features are similar, with more centered offsets and somewhat better resolution in Run 2. We use separate parameterizations for the different datasets. Right plots in Fig. 7-8 show example pull distributions for Run 2. The “satellite” peaks observed in Run 1 appear to be absent in Run 2. Table 7-11 summarizes the parameterization of the non-\(\Delta t\) PDFs.

The signal \(\Delta t\) PDF depends on the flavor and category of the tag side. The functional form for the \(\pi\pi\) component is given by Eqs. 7.7 and 7.8, where the resolution function \(R_{\text{sig}}\) is defined in Table 7-10. We fit

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Figure 7-7. Top plots: distributions of $\mathcal{F}$ for $B^- \to D^0\pi^-$ decays in the combined Run 1 + 2 dataset (left) and signal $\pi^+\pi^-$ Monte Carlo (right). Bottom plots: distributions of $\mathcal{F}$ in Run 1 (left) and Run 2 (right) $m_{ES}$ side-band data.
for the coefficients $S_{\pi\pi}$ and $C_{\pi\pi}$ of the sine and cosine terms, respectively. Although the $K^+K^-$ final state is a $CP$ eigenstate, we treat this component as a pure lifetime. The functional form for the $K\pi$ component is given by Eq. 7.9. We fix the parameters $\tau$ and $\Delta m_{B_d}$ to their PDG values [14] and take the error into account as a systematic uncertainty. The background $\Delta t$ PDF is given by the triple-Gaussian resolution function $R_{\text{tag}}$, with the parameters defined in Table 7-7. The $\Delta t$ parameterization is summarized in Table 7-12, and the final set of free parameters in the maximum likelihood fit are listed in Table 7-13.

### 7.7.2.1 Correlations between PDF variables

An implicit assumption in the construction of the likelihood function is that the PDF dependent variables are uncorrelated. Table 7-14 summarizes the linear correlation coefficients for all pairs of $m_{ES}$, $\Delta E$, $F$, $\theta_c^+$, $\theta_c^-$, $\Delta t$, and $\sigma_{\Delta t}$. Correlations greater than 10% are highlighted.

The correlation between $m_{ES}$ and $\Delta E$ is not thought to have any significant impact on the fit (mostly because the MC correlation is inflated by the better $\Delta E$ resolution compared with data). The correlations between the Čerenkov angles, and between $\theta_c$ and $\Delta E$, in $K\pi$ and $KK$ events is due to the underlying momentum

**Figure 7-8.** Left plots: distributions of $\theta_c$ offsets (left) and resolutions (right) in the Run 2 $D^0$ control sample as functions of track polar angle. Top plots are for kaons, bottom plots are for pions. Right plots: distributions of $\theta_c - \theta_c(\exp) + \text{offset}$ for kaons (top) and pions (bottom) in the Run 2 $D^0$ control sample. The left (right) plots correspond to the momentum bin $1.75-1.875\,\text{GeV}/c$ ($1.875-2.0\,\text{GeV}/c$).
Table 7-12. Summary of PDFs for $\Delta t$.

<table>
<thead>
<tr>
<th>Component</th>
<th>Function</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal $\pi\pi$</td>
<td>$F_{B^0}$ or $F_{\overline{B}^0}$ tag</td>
<td>$S_{\pi\pi}$, $C_{\pi\pi}$; free $R_{\text{sig}}$; Table 7-10 $\langle D \rangle$, $\Delta D$; Table 7-9 $\tau = 1.548$ ps $\Delta m_{B_d} = 0.472 \hbar$ ps$^{-1}$</td>
</tr>
<tr>
<td>Signal $K^+\pi^-$</td>
<td>$F_{\text{mix}}$, $B^0$ tag or $F_{\text{unmix}}$, $\overline{B}^0$ tag</td>
<td>$R_{\text{sig}}$, $\langle D \rangle$, $\Delta D$, $\tau$, $\Delta m_{B_d}$</td>
</tr>
<tr>
<td>Signal $K^-\pi^+$</td>
<td>$F_{\text{mix}}$, $\overline{B}^0$ tag or $F_{\text{unmix}}$, $B^0$ tag</td>
<td>$R_{\text{sig}}$, $\langle D \rangle$, $\Delta D$, $\tau$, $\Delta m_{B_d}$</td>
</tr>
<tr>
<td>Signal $KK$</td>
<td>$\exp \otimes \text{res}$</td>
<td>$R_{\text{sig}}$; $\tau$</td>
</tr>
<tr>
<td>Background</td>
<td>triple Gaussian</td>
<td>$R_{\text{bg}}$; Table 7-7</td>
</tr>
</tbody>
</table>

Table 7-13. Summary of free parameters in the $CP$ fit.

- $N_{\pi\pi}$ Number of signal $\pi\pi$ events
- $N_{K\pi}$ Number of signal $K\pi$ events
- $A_{K\pi}$ charge asymmetry in signal $K^\pm\pi^\mp$ events
- $N_{KK}$ Number of signal $KK$ events
- $N_{b\pi\pi}$ Number of background $\pi\pi$ events
- $N_{bK\pi}$ Number of background $K\pi$ events
- $A_{bK\pi}$ charge asymmetry in background $K^\pm\pi^\mp$ events
- $N_{bKK}$ Number of background $KK$ events
- $\xi$ background $m_{ES}$ ARGUS shape parameter
- $\Delta E_{p1}$ background $\Delta E$ linear term
- $\Delta E_{p2}$ background $\Delta E$ quadratic term
- $\mathcal{F}_{\mu_1}$ background Fisher mean of first Gaussian
- $\mathcal{F}_{\sigma_1}$ background Fisher width of first Gaussian
- $\mathcal{F}_{\mu_2}$ background Fisher mean of second Gaussian
- $\mathcal{F}_{\sigma_2}$ background Fisher width of second Gaussian
- $\mathcal{F}_{f_0}$ background Fisher fraction of first Gaussian
- $S_{\pi\pi}$ coefficient of the sine oscillation in signal $\pi\pi$ events
- $C_{\pi\pi}$ coefficient of the cosine oscillation in signal $\pi\pi$ events

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dependence, which is properly taken into account in the PDF definition. Incorporating the correlation into the toy Monte Carlo generator, we have confirmed that there is no bias in the fit yields. The 14% correlation between the Fisher discriminant and the error on $\Delta t$ in the background samples is not yet understood. However, signal yields change only slightly between fits with and without $\Delta t$, which gives some confidence that the correlation does not significantly affect the yield estimate.

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7.8 Validation studies

Numerous studies using toy Monte Carlo, simulation, and real data have been performed to optimize and validate the analysis strategy. Figures 7-9 through 7-11 show pull plots for all free fit parameters in 695 toy experiments corresponding to the nominal $CP$ fit. Signal and background yields are generated according to a Poisson distribution with means equal to the PRL result scaled to $30 \, fb^{-1}$. The yield for signal $K\pi$ is generated with zero branching fraction and fit with the constraint $C_{K\pi} > 0$. Random values of $C_{K\pi}$ are generated between $\pm 1$ and random values of $S_{K\pi}$ are generated within the bounds given by the selected value of $C_{K\pi}$. The numbers of $B^0$ and $B^0$ tags are consistent with the generated value of $C_{K\pi}$. The most probable $CP$ asymmetry fit errors are $\sigma_{S_{\pi\pi}} \sim 0.6$ and $\sigma_{C_{\pi\pi}} \sim 0.45$ (Fig. 7-12).

7.8.1 Toy Monte Carlo

Due to the two-body nature of the decay, the daughter tracks in $B^0 \rightarrow h^+h'^-$ are essentially 100% anti-correlated in momentum and polar angle. First two plots in Fig. 7-13 show scatter plots of these two variables for tracks in signal $\pi\pi$ Monte Carlo. In addition, the assignment of the pion mass to all tracks leads to a systematic shift in the mean $\Delta E$, which is momentum dependent due to the boost (see right plot in Fig. 7-13). This underlying momentum dependence of $\theta_c$ and $\Delta E$ is the source of the large correlations between these variables reported in Table 7-14.

**Figure 7-9.** Left plots: pull plots for signal yields and the $K\pi$ charge asymmetry in 695 toy experiments generated with $30 \, fb^{-1}$ equivalent luminosity. Right plots: pull plots for background yields and the $K\pi$ charge asymmetry in 695 toy experiments generated with $30 \, fb^{-1}$ equivalent luminosity.
Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^\pm \pi^-$ decays

Figure 7-10. Left plots: pull plots for background $m_{ES}$, $\Delta E$, and Fisher parameters in 695 toy experiments generated with 30 fb$^{-1}$ equivalent luminosity. Right plots: pull plots for the background Fisher means and widths in 695 toy experiments generated with 30 fb$^{-1}$ equivalent luminosity.

Figure 7-11. Pull plots for $S_{\pi\pi}$ (left) and $C_{\pi\pi}$ in 695 toy experiments generated with 30 fb$^{-1}$ equivalent luminosity.

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7.8 Validation studies

![Histograms showing 
\(\sigma(S_{\pi\pi})\) and 
\(\sigma(C_{\pi\pi})\) distributions.
](image)

**Figure 7-12.** Error distributions for \(S_{\pi\pi}\) (left) and \(C_{\pi\pi}\) (right) in 695 toy experiments generated with 30 fb\(^{-1}\) equivalent luminosity.

![Correlation plots showing track momentum vs. polar angle (left), between polar angles of the two tracks (middle), and \(\Delta E\) vs. momentum for \(\pi\pi\) (upper band) and \(K\pi\) (lower band) decays (right).](image)

**Figure 7-13.** Correlation plots of track momentum vs. polar angle (left), between the polar angles of the two tracks (middle) in signal \(\pi\pi\) Monte Carlo and between \(\Delta E\) and momentum for \(\pi\pi\) (upper band) and \(K\pi\) (lower band) decays (right). The shift in \(K\pi\) is momentum dependent due to the boost.

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The kinematics of two-body decay are taken into account in the toy Monte Carlo generator:

- generate a $B$ meson in the $\Upsilon(4S)$ frame with momentum randomly selected from a Gaussian distribution ($\mu = 320$ MeV/$c$, $\sigma = 30$ MeV/$c$) and polar angle selected from a $1 - \cos^2(\theta)$ distribution,
- decay the $B$ candidate to two tracks isotropically,
- boost to the laboratory frame,
- use the resulting track momenta and polar angles to compute the expected value of $\theta_c$ and $\Delta E$.

This “generation” procedure faithfully reproduces the $(\theta_c, \Delta E, \Delta E_c)$ and $(\theta_c, \Delta E)$ correlations observed in Table 7-14. In the case the data $\Delta E$ resolution (26 MeV) is used instead of the Monte Carlo value (18 MeV) the correlation between $\Delta E$ and $\theta_c$ is reduced.

### 7.8.2 Effect of floating yields in the $CP$ fit

The rms of the $\Delta t$ distribution for $B\bar{B}$ events is greater than the corresponding rms for continuum $q\bar{q}$ events. It is therefore expected that adding this variable in the likelihood function will improve the statistical separation between signal and background. To see how much we gain in the branching fraction analysis by floating yields in the $CP$ fit, we generated 685 toy experiments and fit each one with and without the $\Delta t$ PDF in the likelihood. Figure 7-14 shows the difference in the fitted error on $N_{\pi\pi}$ and the two asymmetry parameters. The error on $N_{\pi\pi}$ improves by 8%, while the asymmetry errors increase only slightly. The conclusion is that fitting simultaneously for yields and asymmetries optimizes the branching fraction measurement and leads to a more accurate asymmetry measurement (since the uncertainty on the yield is included directly in the fit error).

### 7.8.3 Monte Carlo fits

Since we expect $\sim 250$ signal $\pi\pi$ and $K\pi$ events in 30 fb$^{-1}$, an important consistency check on the $\Delta t$ resolution function, and in the fit mechanism itself, is to fit for the $B^0$ lifetime and mixing frequency in the $h^+h^-$ sample. Table 7-15 shows the results of several test fits on Monte Carlo samples. Fitting for the lifetime and $\Delta m_{B_d}$ in pure signal, or in a mix of $\pi\pi$ and $K\pi$ events in the proper ratio, returns correct values for both parameters. We have also tried mixing the correct signal yield into the 9.5 fb$^{-1}$ continuum Monte Carlo sample and find consistent values of $\tau$ and $\Delta m_{B_d}$. Finally, fitting for $S_{\pi\pi}$ and $C_{\pi\pi}$ returns the correct values in high statistics signal Monte Carlo and consistent values in the sample with background.

To check that the fit errors on the $CP$ asymmetries in simulated Monte Carlo samples are consistent with what we estimate in toy Monte Carlo, we take the same sample of 9.5 fb$^{-1}$ continuum Monte Carlo and add in ten different sets of $(N_{\pi\pi}, N_{K\pi})$ with exact values determined by Poisson statistics. Table 7-16 summarizes the results of this test. The average error on $S_{\pi\pi}$ and $C_{\pi\pi}$ are 1.33 and 0.85, respectively. Scaled to 30 fb$^{-1}$ we would predict an expected error of 0.75 on $S_{\pi\pi}$ and 0.48, in excellent agreement with the toy Monte Carlo prediction (Fig. 7-12).
Figure 7-14. Difference in the fit error on $N_{\pi\pi}$ (left) and the two $CP$ parameters (right) with and without floating yields in the $CP$ fit. The average error on $N_{\pi\pi}$ is $\sim 10$, so the improvement in statistical error is $8\%$.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\tau$ (ps)</th>
<th>$\Delta m_{B_d}$ (fs ps$^{-1}$)</th>
<th>$S_{\pi\pi}$</th>
<th>$C_{\pi\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>7388 $\pi\pi$</td>
<td>1.589 ± 0.021</td>
<td>fixed</td>
<td>fixed</td>
<td>fixed</td>
</tr>
<tr>
<td>7388 $\pi\pi$</td>
<td>fixed</td>
<td>fixed</td>
<td>$-0.374 ± 0.040$</td>
<td>$0.013 ± 0.027$</td>
</tr>
<tr>
<td>7342 $K\pi$</td>
<td>fixed</td>
<td>0.472 ± 0.013</td>
<td>fixed</td>
<td>fixed</td>
</tr>
<tr>
<td>7342 $K\pi$</td>
<td>1.536 ± 0.021</td>
<td>fixed</td>
<td>fixed</td>
<td>fixed</td>
</tr>
<tr>
<td>1835 $\pi\pi$ + 7342 $K\pi$</td>
<td>1.513 ± 0.019</td>
<td>fixed</td>
<td>fixed</td>
<td>fixed</td>
</tr>
<tr>
<td>1835 $\pi\pi$ + 7342 $K\pi$</td>
<td>fixed</td>
<td>$-0.369 ± 0.085$</td>
<td>$0.002 ± 0.058$</td>
<td></td>
</tr>
<tr>
<td>9.5 fb$^{-1}$ equiv.</td>
<td>1.84 ± 0.22</td>
<td>fixed</td>
<td>fixed</td>
<td>fixed</td>
</tr>
<tr>
<td>9.5 fb$^{-1}$ equiv.</td>
<td>fixed</td>
<td>$-0.41 ± 0.78$</td>
<td>$-0.82 ± 0.80$</td>
<td></td>
</tr>
<tr>
<td>9.5 fb$^{-1}$ equiv.</td>
<td>fixed</td>
<td>0.460 ± 0.087</td>
<td>fixed</td>
<td>fixed</td>
</tr>
<tr>
<td>9.5 fb$^{-1}$ equiv.</td>
<td>fixed</td>
<td>0.438 ± 0.084</td>
<td>$-0.48 ± 0.83$</td>
<td>$-0.86 ± 0.78$</td>
</tr>
</tbody>
</table>

Table 7-15. Lifetime, $\Delta m_{B_d}$, and $CP$ fits to various signal and background Monte Carlo samples. The generated values are: $\tau = 1.548$ ps, $\Delta m_{B_d} = 0.472$ fs ps$^{-1}$, $S_{\pi\pi} = -0.4$, and $C_{\pi\pi} = 0$. When fixed, the fit parameters are set to: $\tau = 1.548$ ps, $\Delta m_{B_d} = 0.472$ fs ps$^{-1}$, $S_{\pi\pi} = C_{\pi\pi} = 0$.

Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays
Having validated the analysis on toy and fully simulated Monte Carlo events, we fit without \( \Delta t \) to validate the branching fraction portion of the likelihood. Table 7-17 summarizes fits to the Run 1, Run 2, and combined samples. The yields for both Run 1 and Run 2 are consistent with the PRL branching fractions, and the background parameters for \( m_{\text{res}} \), \( \Delta E \), and \( \mathcal{F} \) are consistent between the two datasets. To evaluate the effect of floating the background PDF parameters we also show a fit where the parameters are fixed to the values obtained from the floating fit. The contribution to the error on the \( \pi\pi \) yield can be estimated as \( \sqrt{\sigma_{\text{float}}^2 - \sigma_{\text{fix}}^2} = 1.6 \) events, which is significantly less than the systematic error derived from the conservative procedure used for the PRL analysis.

### 7.8.5 Lifetime and mixing fits

As a cross-check on the signal and background \( \Delta t \) parameterizations we perform \( CP \)-blind fits to the lifetime \( \tau \), mixing frequency \( \Delta m_{B_d} \), and \( CP \) asymmetries in Run 1 and Run 2 data. When fitting only \( \tau \) and \( \Delta m_{B_d} \) we fix \( S_{\pi\pi} = C_{\pi\pi} = 0 \) in order to be insensitive to \( CP \) asymmetry. For the \( CP \) fit we blind by adding a random offset between \( \pm 5 \) and randomly flipping the sign of the asymmetries.

Table 7-18 summarizes the fit results. We obtain values of \( \tau \) and \( \Delta m_{B_d} \) consistent with the PDG and fitting for these parameters does not significantly change the yields. Table 7-19 summarizes fits floating \( \tau \), \( \Delta m_{B_d} \), and the blinded \( CP \) asymmetries \( S_{\pi\pi} \) and \( C_{\pi\pi} \). Again, all results for \( \tau \) and \( \Delta m_{B_d} \) are consistent with the PDG.
7.8 Validation studies

Estimate the effect of floating background parameters for \( \pi \pi \) background parameters.

**Summary of data fits using only the branching fraction part of the likelihood function (no \( \Delta t \)).** To estimate the effect of floating background parameters for \( m_{ES} \), \( \Delta E \), and \( F \), we also show a fit with fixed background parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Run 1</th>
<th>Run 2</th>
<th>Run 1 + 2</th>
<th>Run 1 + 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( N_{\pi\pi} )</td>
<td>38.4 ± 9.9</td>
<td>26.6 ± 7.0</td>
<td>66.2 ± 12.4</td>
<td>66.2 ± 12.3</td>
</tr>
<tr>
<td>( N_{K\pi} )</td>
<td>144.4 ± 15.4</td>
<td>67.5 ± 10.3</td>
<td>211.8 ± 18.5</td>
<td>211.8 ± 18.4</td>
</tr>
<tr>
<td>( A_{K\pi} )</td>
<td>−0.191 ± 0.104</td>
<td>0.107 ± 0.150</td>
<td>−0.090 ± 0.087</td>
<td>−0.090 ± 0.087</td>
</tr>
<tr>
<td>( N_{bK\pi} )</td>
<td>5.9 ± 6.2</td>
<td>0.0 ± 1.5</td>
<td>2.0 ± 6.4</td>
<td>2.0 ± 6.4</td>
</tr>
<tr>
<td>( N_{b\pi} )</td>
<td>3081.9 ± 63.0</td>
<td>1453.8 ± 40.0</td>
<td>4533.2 ± 72.6</td>
<td>4533.2 ± 72.6</td>
</tr>
<tr>
<td>( N_{bK\pi} )</td>
<td>2010.5 ± 55.4</td>
<td>979.7 ± 35.2</td>
<td>2994.3 ± 65.4</td>
<td>2994.3 ± 65.3</td>
</tr>
<tr>
<td>( A_{bK\pi} )</td>
<td>−0.001 ± 0.027</td>
<td>−0.025 ± 0.036</td>
<td>−0.009 ± 0.021</td>
<td>−0.009 ± 0.021</td>
</tr>
<tr>
<td>( N_{bK} )</td>
<td>1318.9 ± 42.8</td>
<td>613.5 ± 26.3</td>
<td>1933.5 ± 48.9</td>
<td>1933.5 ± 48.9</td>
</tr>
<tr>
<td>( \xi )</td>
<td>20.6 ± 1.4</td>
<td>22.7 ± 2.1</td>
<td>21.3 ± 1.2</td>
<td>21.3 (fixed)</td>
</tr>
<tr>
<td>( \Delta E_{p1} )</td>
<td>−1.146 ± 0.147</td>
<td>−0.900 ± 0.212</td>
<td>−1.071 ± 0.121</td>
<td>−1.071 (fixed)</td>
</tr>
<tr>
<td>( \Delta E_{p2} )</td>
<td>0.649 ± 1.886</td>
<td>0.695 ± 2.697</td>
<td>0.691 ± 1.559</td>
<td>0.691 (fixed)</td>
</tr>
<tr>
<td>( F_{\mu_1} )</td>
<td>0.323 ± 0.067</td>
<td>0.516 ± 0.120</td>
<td>0.385 ± 0.061</td>
<td>0.385 (fixed)</td>
</tr>
<tr>
<td>( F_{\sigma_1} )</td>
<td>−0.134 ± 0.015</td>
<td>−0.099 ± 0.020</td>
<td>−0.122 ± 0.012</td>
<td>−0.122 (fixed)</td>
</tr>
<tr>
<td>( F_{\mu_2} )</td>
<td>0.187 ± 0.016</td>
<td>0.217 ± 0.020</td>
<td>0.198 ± 0.013</td>
<td>0.198 (fixed)</td>
</tr>
<tr>
<td>( F_{\sigma_2} )</td>
<td>0.018 ± 0.012</td>
<td>0.054 ± 0.025</td>
<td>0.029 ± 0.011</td>
<td>0.029 (fixed)</td>
</tr>
<tr>
<td>( F_{f_1} )</td>
<td>0.326 ± 0.007</td>
<td>0.356 ± 0.017</td>
<td>0.335 ± 0.007</td>
<td>0.335 (fixed)</td>
</tr>
</tbody>
</table>

**Table 7.17.** Summary of fits using only the branching fraction part of the likelihood function (no \( \Delta t \)).

<table>
<thead>
<tr>
<th>Sample</th>
<th>( \tau ) only</th>
<th>( \Delta m_{B_d} ) only</th>
<th>( \tau )</th>
<th>( \Delta m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>1.54 ± 0.15</td>
<td>0.553 ± 0.091</td>
<td>1.54 ± 0.16</td>
<td>0.553 ± 0.091</td>
</tr>
<tr>
<td>Run 2</td>
<td>1.49 ± 0.20</td>
<td>0.454 ± 0.221</td>
<td>1.49 ± 0.20</td>
<td>0.457 ± 0.220</td>
</tr>
<tr>
<td>Run 1 + 2</td>
<td>1.52 ± 0.12</td>
<td>0.539 ± 0.087</td>
<td>1.52 ± 0.12</td>
<td>0.540 ± 0.088</td>
</tr>
</tbody>
</table>

**Table 7.18.** Summary of data fits floating \( \tau \) only, \( \Delta m_{B_d} \) only, or both. These fits are performed with \( S_{\pi\pi} = C_{\pi\pi} = 0 \). Units are ps for \( \tau \) and \( 10^{-12} \) ps\(^{-1} \) for \( \Delta m_{B_d} \).

Analysis of the time-dependent \( CP \)-violating asymmetry in \( B \to \pi^+\pi^- \) decays
Table 7-19. Summary of fits floating $\tau$, $\Delta m_{B_d}$, and the blinded $CP$ asymmetries $S_{\pi\pi}$ and $C_{\pi\pi}$. Units are $\text{s}$ for $\tau$ and $\text{ps}^{-1}$ for $\Delta m_{B_d}$.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\tau$</th>
<th>$\Delta m_{B_d}$</th>
<th>$S_{\pi\pi}$ (blind)</th>
<th>$C_{\pi\pi}$ (blind)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run1</td>
<td>1.53 ± 0.16</td>
<td>0.540 ± 0.096</td>
<td>2.73 ± 0.84</td>
<td>4.42 ± 0.56</td>
</tr>
<tr>
<td>Run2</td>
<td>1.49 ± 0.20</td>
<td>0.634 ± 0.114</td>
<td>−0.65 ± 1.14</td>
<td>3.50 ± 0.63</td>
</tr>
<tr>
<td>Run1 + Run2</td>
<td>1.52 ± 0.12</td>
<td>0.542 ± 0.087</td>
<td>1.45 ± 0.56</td>
<td>4.21 ± 0.49</td>
</tr>
</tbody>
</table>

Figure 7-15. Scans of the likelihood function vs. $S_{\pi\pi}$, $C_{\pi\pi}$, and $A_{K\pi}$.

7.9 Results

The unblinded fit results are shown in Tab. 7-20. Figure 7-15 shows scans of the likelihood function with respect to the $CP$ parameters. To estimate how likely the error obtained on the full dataset is, we generated 1279 toy experiments with yields given by the data fit (no Poisson fluctuations). Figure 7-16 shows the pull distributions for $S_{\pi\pi}$ and $C_{\pi\pi}$. Figure 7-17 shows the error distribution from the ensemble of toy experiments, with the data results indicated by the arrows.

Figure 7-18 shows distributions of $\eta_{ES}$ and $\Delta E$ for events enhanced in signal $\pi\pi$ and $K\pi$ decays using likelihood ratio cuts. The curves represent projections of the fit result scaled by the efficiency of the additional cuts. Figure 7-19 shows the $\Delta t$ distribution for $\pi\pi$-selected events, with a looser selection than the one applied in Fig. 7-18. We find that the background resolution function describes the tails of the $\Delta t$ distribution well, and the core is consistent with $B$ decay.

7.9.1 Cross-checks

Table 7-21 summarizes several tests to cross-check the stability of the result. We fit separately the Run 1 and Run 2 datasets and find the weighted averages are consistent with the results for the entire dataset. We also fit the $B^0$ and $\bar{B}^0$ tag samples separately, again with consistent results. To test the stability of the fit...
Table 7-20. Summary of the unblinded fit results on the Run 1 + 2 dataset. The last row gives the correlation between the time-dependent $CP$ asymmetry parameters.
Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays

Figure 7-16. Pull distributions for $S_{\pi\pi}$ and $C_{\pi\pi}$ from 1279 toy experiments with fixed yields generated according to the Run 1 + 2 result.

Figure 7-17. Error distributions for $S_{\pi\pi}$ and $C_{\pi\pi}$ from 1279 toy experiments with fixed yields generated according to the Run 1 + 2 result. The data result is indicated by the arrows.
Figure 7-18. Distributions of $m_{ES}$ and $\Delta E$ for samples enhanced in signal $\pi\pi$ and $K\pi$ decays using likelihood ratio cuts. The solid curves represent projections of the maximum likelihood fit result.

Table 7-21. Separate fits to Run 1 and Run 2, $B^0$ and $\bar{B}^0$ tags, and the subsample of Lepton and Kaon tagged events.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$A_{K\pi}$</th>
<th>$S_{\pi\pi}$</th>
<th>$C_{\pi\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td>$-0.16 \pm 0.10$</td>
<td>$+1.34 \pm 0.82$</td>
<td>$-0.03 \pm 0.52$</td>
</tr>
<tr>
<td>Run 2</td>
<td>$-1.19 \pm 0.76$</td>
<td>$-1.09 \pm 0.10$</td>
<td>$+0.11 \pm 0.14$</td>
</tr>
<tr>
<td>$B^0$ tags</td>
<td>$+0.24 \pm 0.15$</td>
<td>$-0.44 \pm 0.77$</td>
<td>$-0.06 \pm 1.28$</td>
</tr>
<tr>
<td>$\bar{B}^0$ tags</td>
<td>$-0.36 \pm 0.14$</td>
<td>$+0.42 \pm 0.65$</td>
<td>$-0.08 \pm 1.4$</td>
</tr>
<tr>
<td>no NT tags</td>
<td>$-0.07 \pm 0.08$</td>
<td>$+0.09 \pm 0.58$</td>
<td>$-0.54 \pm 0.53$</td>
</tr>
</tbody>
</table>

against tag category, we fit only the Lepton and Kaon categories and find consistent results, with slightly larger errors for $S_{\pi\pi}$ and $C_{\pi\pi}$ (as expected).

Figure 7-1 indicates some discrepancy in ARGUS shape between different tag categories. Since we assume one shape in the nominal fit, we have to investigate the possible systematic effect on the $CP$ parameters by refitting the data with different (floating) values of $\xi$ for each category. Table 7-22 summarizes the results. The Lepton and NT1 categories are somewhat different than the nominal value, but the $\pi\pi$ yield changes by only 0.2 events, and the values of $S_{\pi\pi}$ and $C_{\pi\pi}$ change by only $-0.005$ and $+0.012$, respectively. There appears to be no bias in using an average value of $\xi$.

Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays
Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays

Figure 7-19. Distribution of $\Delta t$ for a sample enhanced in $\pi\pi$ events using likelihood ratio cuts. The solid histogram represents the expected distribution for signal and background, while the dashed histogram shows the expected background shape.

<table>
<thead>
<tr>
<th>Category</th>
<th>Fit result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>34.6 ± 10.6</td>
</tr>
<tr>
<td>Kaon</td>
<td>20.2 ± 2.2</td>
</tr>
<tr>
<td>NT1</td>
<td>13.7 ± 4.7</td>
</tr>
<tr>
<td>NT2</td>
<td>22.0 ± 2.7</td>
</tr>
<tr>
<td>Untagged</td>
<td>22.1 ± 1.7</td>
</tr>
</tbody>
</table>

Table 7-22. Results of a fit floating separate values of $\xi$ for each tagging category. The average $\xi$ from the nominal fit is 21.2 ± 1.2.
### Table 7-23. Results of a fit floating signal tagging efficiencies in Run 1 and Run 2, along with yields, background parameters, and CP parameters.

<table>
<thead>
<tr>
<th>Category</th>
<th>Floating</th>
<th>Nominal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{\text{Lepton}}$(Run1)</td>
<td>$0.137 \pm 0.027$</td>
<td>$0.110$ (fixed)</td>
</tr>
<tr>
<td>$f_{\text{Kaon}}$(Run1)</td>
<td>$0.315 \pm 0.041$</td>
<td>$0.358$ (fixed)</td>
</tr>
<tr>
<td>$f_{\text{NT1}}$(Run1)</td>
<td>$0.044 \pm 0.019$</td>
<td>$0.080$ (fixed)</td>
</tr>
<tr>
<td>$f_{\text{NT2}}$(Run1)</td>
<td>$0.123 \pm 0.030$</td>
<td>$0.139$ (fixed)</td>
</tr>
<tr>
<td>$f_{\text{Lepton}}$(Run2)</td>
<td>$0.132 \pm 0.037$</td>
<td>$0.110$ (fixed)</td>
</tr>
<tr>
<td>$f_{\text{Kaon}}$(Run2)</td>
<td>$0.327 \pm 0.056$</td>
<td>$0.358$ (fixed)</td>
</tr>
<tr>
<td>$f_{\text{NT1}}$(Run2)</td>
<td>$0.067 \pm 0.030$</td>
<td>$0.080$ (fixed)</td>
</tr>
<tr>
<td>$f_{\text{NT2}}$(Run2)</td>
<td>$0.126 \pm 0.042$</td>
<td>$0.139$ (fixed)</td>
</tr>
</tbody>
</table>

As a check on the consistency of the tagging efficiencies in signal events, we performed a fit floating the efficiencies for each category separately for Run 1 and Run 2. Table 7-23 summarizes the result. The values are consistent between Run 1 and Run 2, and they agree with the nominal values obtained from the Breco sample (Table 7-8).

To study the effect of this possibility of the Fisher discriminant shape varying across tag categories, we generated toy MC with different Fisher shapes for each category, based on fits to on-resonance side-band data, and fit each pseudo-experiment with the average Fisher shape. All parameters show no bias.

### 7.10 Systematic studies

Tables 7-24–7-30 summarize the absolute variations in $A_{K\pi}$, $S_{\pi\pi}$, and $C_{\pi\pi}$ arising from uncertainties in various parameters, determined by fluctuating each parameter up and down by 1σ. Table 7-31 summarizes systematic uncertainties determined by substituting different parameter sets for signal and background $\Delta t$, and the background tagging efficiencies determined from the fit region (Table 7-6). Table 7-32 summarizes the systematic errors coming from all sources, and the total systematic error calculated as the quadrature sum of the individual uncertainties. Although the nominal branching fraction results should still be considered the PRL ones, we have also calculated the total systematic error on $N_{\pi\pi}$. We find an uncertainty of ±4.6 events, which is a fractional error of 7%.

### 7.11 Summary

This analysis has produced a measurement of the time-dependent CP violating asymmetry in $B^0 \to \pi^+\pi^-$ decays, and an updated measurement of the charge asymmetry in $B^0 \to K^+\pi^-$ decays. In 30.4 fb$^{-1}$ we
We calculate the parameterizations.

\[
\mathcal{A}_{K\pi} = -0.07 \pm 0.08 \pm 0.02,
\]

where the first error is statistical and the second is systematic. The systematic error on \(\mathcal{A}_{K\pi}\) is the quadrature sum of the total from Table 7-32 and an uncertainty of \(+0.01\) from possible charge bias in track reconstruction and particle identification [6]. We calculate the 90% confidence limit on \(\mathcal{A}_{K\pi}\), \([-0.21, 0.07]\), including the statistical and systematic errors and assuming Gaussian errors.

To conclude, even if this result is still statistically limited, the analysis method is demonstrated to be robust and promising as soon as a higher statistics is available.

Marcella Bona
### Table 7-26.

Systematic errors due to uncertainties on background tagging efficiencies.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A_{K\pi}$</th>
<th>$S_{\pi\pi}$</th>
<th>$C_{\pi\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$+\sigma$</td>
<td>$-\sigma$</td>
<td>$+\sigma$</td>
</tr>
<tr>
<td><strong>Run 1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lepton ($\pi\pi$)</td>
<td>$-0.00002$</td>
<td>$0.00001$</td>
<td>$-0.00428$</td>
</tr>
<tr>
<td>Lepton ($K\pi$)</td>
<td>$-0.00043$</td>
<td>$0.00045$</td>
<td>$0.00060$</td>
</tr>
<tr>
<td>Lepton ($KK$)</td>
<td>$0.00003$</td>
<td>$-0.00003$</td>
<td>$-0.00006$</td>
</tr>
<tr>
<td>Kaon ($\pi\pi$)</td>
<td>$-0.00003$</td>
<td>$0.00002$</td>
<td>$-0.00078$</td>
</tr>
<tr>
<td>Kaon ($K\pi$)</td>
<td>$-0.00043$</td>
<td>$0.00042$</td>
<td>$0.00004$</td>
</tr>
<tr>
<td>Kaon ($KK$)</td>
<td>$-0.00011$</td>
<td>$0.00010$</td>
<td>$0.00001$</td>
</tr>
<tr>
<td>NT1 ($\pi\pi$)</td>
<td>$0.00005$</td>
<td>$-0.00006$</td>
<td>$-0.0062$</td>
</tr>
<tr>
<td>NT1 ($K\pi$)</td>
<td>$0.00020$</td>
<td>$-0.00024$</td>
<td>$0.0013$</td>
</tr>
<tr>
<td>NT1 ($KK$)</td>
<td>$-0.00002$</td>
<td>$0.00001$</td>
<td>$0.00004$</td>
</tr>
<tr>
<td>NT2 ($\pi\pi$)</td>
<td>$0.00001$</td>
<td>$-0.00002$</td>
<td>$-0.00023$</td>
</tr>
<tr>
<td>NT2 ($K\pi$)</td>
<td>$-0.00009$</td>
<td>$0.00009$</td>
<td>$0.00002$</td>
</tr>
<tr>
<td>NT2 ($KK$)</td>
<td>$-0.00004$</td>
<td>$0.00004$</td>
<td>$0.00000$</td>
</tr>
</tbody>
</table>

**Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays**
### Table 7-27. Systematic errors due to uncertainties on tagging dilution and dilution differences.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( \mathcal{A}_{K\pi} )</th>
<th>( S_{\pi\pi} )</th>
<th>( C_{\pi\pi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton ( \langle D \rangle )</td>
<td>0.00050, -0.00042</td>
<td>0.01837, -0.01885</td>
<td>0.00043, -0.00167</td>
</tr>
<tr>
<td>Lepton ( \Delta D )</td>
<td>0.00062, -0.00057</td>
<td>-0.00366, 0.00445</td>
<td>-0.01653, 0.01621</td>
</tr>
<tr>
<td>Kaon ( \langle D \rangle )</td>
<td>-0.00014, 0.00016</td>
<td>-0.01417, 0.01513</td>
<td>-0.00088, 0.00058</td>
</tr>
<tr>
<td>Kaon ( \Delta D )</td>
<td>-0.00010, 0.00010</td>
<td>0.00308, -0.00294</td>
<td>-0.01633, 0.01637</td>
</tr>
<tr>
<td>NT1 ( \langle D \rangle )</td>
<td>0.00007, -0.00005</td>
<td>-0.00572, 0.00566</td>
<td>-0.01005, 0.01039</td>
</tr>
<tr>
<td>NT1 ( \Delta D )</td>
<td>0.00006, -0.00004</td>
<td>0.00426, -0.00444</td>
<td>-0.01087, 0.01073</td>
</tr>
<tr>
<td>NT2 ( \langle D \rangle )</td>
<td>0.00062, -0.00057</td>
<td>-0.00366, 0.00445</td>
<td>-0.01653, 0.01621</td>
</tr>
<tr>
<td>NT2 ( \Delta D )</td>
<td>0.00046, -0.00045</td>
<td>0.00028, -0.00034</td>
<td>-0.00730, 0.00720</td>
</tr>
</tbody>
</table>
### Table 7-28

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\mathcal{A}_{K\pi}$</th>
<th>$S_{\pi\pi}$</th>
<th>$C_{\pi\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma$</td>
<td>$-\sigma$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>Run 1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$ Outlier</td>
<td>0.00020</td>
<td>-0.00023</td>
<td>-0.00101</td>
</tr>
<tr>
<td>$f$ Tail</td>
<td>-0.00004</td>
<td>0.00005</td>
<td>-0.00312</td>
</tr>
<tr>
<td>$\mu$ Kaon</td>
<td>0.00002</td>
<td>-0.00004</td>
<td>0.01156</td>
</tr>
<tr>
<td>$\mu$ Lepton</td>
<td>0.00004</td>
<td>-0.00005</td>
<td>-0.00338</td>
</tr>
<tr>
<td>$\mu$ NoTag</td>
<td>0.00006</td>
<td>-0.00006</td>
<td>0.00011</td>
</tr>
<tr>
<td>$\mu$ NT1</td>
<td>-0.00005</td>
<td>0.00007</td>
<td>0.00005</td>
</tr>
<tr>
<td>$\mu$ NT2</td>
<td>0.00017</td>
<td>-0.00016</td>
<td>0.00415</td>
</tr>
<tr>
<td>$\mu$ Tail</td>
<td>0.00009</td>
<td>0.00003</td>
<td>-0.00004</td>
</tr>
<tr>
<td>$\sigma$ Core</td>
<td>0.00036</td>
<td>-0.00036</td>
<td>0.00152</td>
</tr>
<tr>
<td>$\sigma$ Tail</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>Run 2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f$ Outlier</td>
<td>-0.00005</td>
<td>0.00003</td>
<td>0.00079</td>
</tr>
<tr>
<td>$f$ Tail</td>
<td>-0.00029</td>
<td>-0.00028</td>
<td>0.02767</td>
</tr>
<tr>
<td>$\mu$ Kaon</td>
<td>-0.00007</td>
<td>0.00007</td>
<td>-0.00890</td>
</tr>
<tr>
<td>$\mu$ Lepton</td>
<td>0.00002</td>
<td>-0.00005</td>
<td>-0.02081</td>
</tr>
<tr>
<td>$\mu$ NoTag</td>
<td>0.00017</td>
<td>-0.00019</td>
<td>0.00029</td>
</tr>
<tr>
<td>$\mu$ NT1</td>
<td>0.00013</td>
<td>-0.00015</td>
<td>0.00768</td>
</tr>
<tr>
<td>$\mu$ NT2</td>
<td>-0.00003</td>
<td>0.00000</td>
<td>0.00031</td>
</tr>
<tr>
<td>$\mu$ Tail</td>
<td>0.00030</td>
<td>-0.00050</td>
<td>-0.01058</td>
</tr>
<tr>
<td>$\sigma$ Core</td>
<td>-0.00003</td>
<td>0.00003</td>
<td>0.01705</td>
</tr>
<tr>
<td>$\sigma$ Tail</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
</tbody>
</table>

*Table 7-28. Systematic errors due to uncertainties on the signal $\Delta t$ parameterization.*

Analysis of the time-dependent $CP$-violating asymmetry in $B \rightarrow \pi^+\pi^-$ decays.
### Table 7-29. Systematics errors due to uncertainty on the background $\Delta t$ parameterization.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A_{K\pi}$ $+\sigma$</th>
<th>$A_{K\pi} -\sigma$</th>
<th>$S_{\pi\pi}$ $+\sigma$</th>
<th>$S_{\pi\pi} -\sigma$</th>
<th>$C_{\pi\pi}$ $+\sigma$</th>
<th>$C_{\pi\pi} -\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core $\mu$</td>
<td>$-0.00017$</td>
<td>$0.00016$</td>
<td>$-0.00015$</td>
<td>$0.00022$</td>
<td>$0.00083$</td>
<td>$-0.00079$</td>
</tr>
<tr>
<td>Core $\sigma$</td>
<td>$-0.00004$</td>
<td>$0.00003$</td>
<td>$-0.00122$</td>
<td>$0.00117$</td>
<td>$-0.00005$</td>
<td>$0.00025$</td>
</tr>
<tr>
<td>Tail $f$</td>
<td>$-0.00019$</td>
<td>$0.00020$</td>
<td>$-0.00551$</td>
<td>$0.00547$</td>
<td>$-0.00174$</td>
<td>$0.00175$</td>
</tr>
<tr>
<td>Tail $\mu$</td>
<td>$-0.00017$</td>
<td>$0.00017$</td>
<td>$0.00096$</td>
<td>$-0.00099$</td>
<td>$0.00083$</td>
<td>$-0.00088$</td>
</tr>
<tr>
<td>Tail $\sigma$</td>
<td>$-0.00048$</td>
<td>$0.00050$</td>
<td>$-0.00496$</td>
<td>$0.00469$</td>
<td>$-0.00315$</td>
<td>$0.00335$</td>
</tr>
<tr>
<td>Outlier $f$</td>
<td>$-0.00029$</td>
<td>$0.00033$</td>
<td>$-0.00250$</td>
<td>$0.00255$</td>
<td>$-0.00151$</td>
<td>$0.00160$</td>
</tr>
<tr>
<td>Outlier $\sigma$</td>
<td>$-0.00020$</td>
<td>$0.00024$</td>
<td>$0.00035$</td>
<td>$-0.00026$</td>
<td>$-0.00019$</td>
<td>$0.00033$</td>
</tr>
<tr>
<td>Run 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Core $\mu$</td>
<td>$-0.00014$</td>
<td>$0.00013$</td>
<td>$-0.00050$</td>
<td>$0.00056$</td>
<td>$0.00004$</td>
<td>$0.00001$</td>
</tr>
<tr>
<td>Core $\sigma$</td>
<td>$0.00010$</td>
<td>$-0.00011$</td>
<td>$-0.00056$</td>
<td>$0.00075$</td>
<td>$0.00195$</td>
<td>$-0.00218$</td>
</tr>
<tr>
<td>Tail $f$</td>
<td>$-0.00030$</td>
<td>$0.00035$</td>
<td>$0.00266$</td>
<td>$-0.00293$</td>
<td>$-0.00063$</td>
<td>$0.00081$</td>
</tr>
<tr>
<td>Tail $\mu$</td>
<td>$-0.00007$</td>
<td>$0.00007$</td>
<td>$-0.00138$</td>
<td>$0.00151$</td>
<td>$-0.00020$</td>
<td>$0.00022$</td>
</tr>
<tr>
<td>Tail $\sigma$</td>
<td>$-0.00064$</td>
<td>$0.00068$</td>
<td>$0.00799$</td>
<td>$-0.00798$</td>
<td>$-0.00156$</td>
<td>$0.00163$</td>
</tr>
<tr>
<td>Outlier $f$</td>
<td>$-0.00014$</td>
<td>$0.00014$</td>
<td>$0.00126$</td>
<td>$-0.00124$</td>
<td>$0.00014$</td>
<td>$-0.00018$</td>
</tr>
<tr>
<td>Outlier $\sigma$</td>
<td>$0.00003$</td>
<td>$-0.00004$</td>
<td>$-0.00066$</td>
<td>$0.00084$</td>
<td>$-0.00024$</td>
<td>$0.00034$</td>
</tr>
</tbody>
</table>

### Table 7-30. Systematic errors due to uncertainty on $\tau$ and $\Delta m_{B_s}$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A_{K\pi}$ $+\sigma$</th>
<th>$A_{K\pi} -\sigma$</th>
<th>$S_{\pi\pi}$ $+\sigma$</th>
<th>$S_{\pi\pi} -\sigma$</th>
<th>$C_{\pi\pi}$ $+\sigma$</th>
<th>$C_{\pi\pi} -\sigma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$</td>
<td>$0.00019$</td>
<td>$-0.00020$</td>
<td>$-0.00193$</td>
<td>$0.00194$</td>
<td>$-0.00091$</td>
<td>$0.00085$</td>
</tr>
<tr>
<td>$\Delta m_{B_s}$</td>
<td>$0.00017$</td>
<td>$-0.00017$</td>
<td>$-0.01092$</td>
<td>$0.01122$</td>
<td>$-0.00658$</td>
<td>$0.00657$</td>
</tr>
</tbody>
</table>

**Marcella Bona**
7.11 Summary

These errors are symmetrized when calculating the total systematic errors. The total systematic error is calculated as the quadrature sum of the individual uncertainties.

<table>
<thead>
<tr>
<th>Variation</th>
<th>$A_{K\pi}$</th>
<th>$S_{\pi\pi}$</th>
<th>$C_{\pi\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Fisher $D^0_{\pi^-}$</td>
<td>−0.00334</td>
<td>−0.019</td>
<td>0.007</td>
</tr>
<tr>
<td>Dbl Gauss ↔ Bif Gauss $F$</td>
<td>−0.00675</td>
<td>−0.014</td>
<td>0.045</td>
</tr>
<tr>
<td>$R_{s_{\text{sig}}}$ Run 1 ↔ Run 2</td>
<td>−0.00107</td>
<td>0.053</td>
<td>0.012</td>
</tr>
<tr>
<td>$R_{b_{\text{kg}}}$ Run 1 ↔ Run 2</td>
<td>−0.00118</td>
<td>0.047</td>
<td>0.016</td>
</tr>
<tr>
<td>$R_{b_{\text{kg}}}$ params from fit region (Run1)</td>
<td>0.00055</td>
<td>0.006</td>
<td>0.007</td>
</tr>
<tr>
<td>$R_{b_{\text{kg}}}$ params from fit region (Run2)</td>
<td>−0.00039</td>
<td>0.004</td>
<td>−0.001</td>
</tr>
<tr>
<td>$R_{b_{\text{kg}}}$ params from tagged events</td>
<td>−0.00064</td>
<td>0.010</td>
<td>0.005</td>
</tr>
<tr>
<td>$R_{b_{\text{kg}}}$ params from untagged events</td>
<td>−0.00052</td>
<td>−0.014</td>
<td>−0.005</td>
</tr>
<tr>
<td>Average $R_{b_{\text{kg}}}$ for Run1 and Run2</td>
<td>−0.00051</td>
<td>0.025</td>
<td>0.004</td>
</tr>
<tr>
<td>Signal $\pi\pi$ ↔ B-reco MC Tagging (divide by 2)</td>
<td>0.00022</td>
<td>0.040</td>
<td>0.0080</td>
</tr>
<tr>
<td>$\epsilon_{\text{tag}}$(bkg) from fit region</td>
<td>−0.00008</td>
<td>0.002</td>
<td>−0.007</td>
</tr>
</tbody>
</table>

Table 7-31. Additional systematic errors evaluated from variation of signal and background $\Delta t$ parameterizations, and from using the background tagging efficiencies determined from background events in the fit region. These errors are symmetrized when calculating the total systematic errors.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$A_{K\pi}$</th>
<th>$S_{\pi\pi}$</th>
<th>$C_{\pi\pi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{\text{ES}}$</td>
<td>0.00259</td>
<td>0.00224</td>
<td>0.00733</td>
</tr>
<tr>
<td>$\Delta E$</td>
<td>0.01427</td>
<td>0.01346</td>
<td>0.00931</td>
</tr>
<tr>
<td>$F$</td>
<td>0.00676</td>
<td>0.00676</td>
<td>0.02360</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>0.00390</td>
<td>0.00400</td>
<td>0.02083</td>
</tr>
<tr>
<td>Sig Tagging</td>
<td>0.00123</td>
<td>0.00113</td>
<td>0.04761</td>
</tr>
<tr>
<td>Bkg Tagging</td>
<td>0.00088</td>
<td>0.00088</td>
<td>0.00651</td>
</tr>
<tr>
<td>Sig $\Delta t$</td>
<td>0.00123</td>
<td>0.00136</td>
<td>0.06833</td>
</tr>
<tr>
<td>Bkg $\Delta t$</td>
<td>0.00192</td>
<td>0.00188</td>
<td>0.05191</td>
</tr>
<tr>
<td>$\tau$ and $\Delta m_{B_d}$</td>
<td>0.00025</td>
<td>0.00026</td>
<td>0.01139</td>
</tr>
<tr>
<td>Total</td>
<td>0.01670</td>
<td>0.01598</td>
<td>0.10456</td>
</tr>
</tbody>
</table>

Table 7-32. Summary of systematic errors from all sources. The total systematic error is calculated as the quadrature sum of the individual uncertainties.

Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays
Analysis of the time-dependent $CP$-violating asymmetry in $B \to \pi^+\pi^-$ decays
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Chapter 7


