MUELLER-REGGE PHENOMENOLOGY IN THE CENTRAL REGION *)

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ABSTRACT

By performing a Mueller-Regge analysis of inclusive pion spectra in the central region (a) we verify approximately the factorization of secondary trajectories in the double-Regge limit; (b) we find exchange degenerate $P_\varphi$ and $P_f$ central couplings which (c) suggest the absence of short-range correlations between doubly charged pairs of pions. From the fitted central couplings (d) we estimate the magnitude of short-range $\pi^+\pi^-$ correlations and find rough agreement with preliminary data.

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Using the dual resonance model (DRM) as a guide, Chan et al. have discussed the implications of Mueller-Regge analysis for particle production in the central region at high energies. The DRM constraints forced them to introduce "non-dual" $J$ plane singularities in addition to the familiar Regge trajectories that are associated with the hadron spectrum, in order to account for rising single-particle spectra observed in ISR experiments. Furthermore, early-scaling criteria abstracted from the DRM have failed spectacularly*). For these reasons, Mueller-Regge phenomenology in the central region has failed to fulfil the promise it displayed in the fragmentation regions.

In spite of these embarrassments, circumstantial evidence mounts for the relevance of the Regge picture. There is a short-range correlation component present on the central plateau, and Ferbel's compilation of $x = 0$ cross-sections exhibits compatibility with the $s^{-1/4}$ approach to scaling expected in the Regge picture. It is therefore worth while to ask, in an exploratory spirit, whether by giving up DRM prejudices one can make a consistent picture out of the available information. It turns out that this can be done, and while elements of the exercise are questionable, the coherence of the results encourages us to report the analysis, if only to reopen discussion of the utility of a Mueller-Regge description of the central region. The reader should, however, bear in mind that even if the Mueller-Regge picture can serve to correlate many features of inclusive spectra, it is by no means a replacement for a microscopic theory of particle production.

According to the Mueller-Regge analysis, the single-particle inclusive cross-section for production of particle $c$ in the central region (finite centre-of-mass rapidity) is governed by the diagram in Fig. 1a. We denote this reaction as $(a|c|b)$. Similarly, the production of two particles $c$ and $d$, widely separated in rapidity but both in the central region (abbreviated $(a|c|d|b)$) is represented by Fig. 1b. Under the conventional assumption that the $J$ plane singularities which control

*) We have in mind the celebrated prediction of early scaling for $p + p \rightarrow \bar{p} +$ anything.
inclusive cross-sections are the ordinary, factorizable Regge poles familiar
in two-body scattering, the couplings \( \beta^x_1 \) to external particles (i.e.,
x = a and b) are known from total cross-section phenomenology. Therefore,
with the central coupling \( D_{ij}^{\pi} \) of Reggeons i and j to produced particle
c is fixed in fits to \( (a|c|b) \), it can be used - still assuming factoriza-
tion - to predict \( (a|c|d|b) \) and, by extension, the correlation function
for particles c and d. We now carry out this program, checking first
that factorization is not grossly violated.

Restricting Regge singularities to the usual set of (i) the
Pomeranchuk trajectory \( P \), and (ii) the natural-parity meson trajectories
\( f, A_2, \omega \), we have

\[
\frac{1}{\sigma_\infty(a|b)} \frac{d \sigma(a|\pi|b)}{d y} \Bigg|_{y=0} = D_{2\pi}^{\pi^-} + \nonumber
\]

\[
S^{-1/2} \left[ \frac{\beta_2^\pi}{\beta_2^\pi} - \frac{\beta_2^\pi}{\beta_2^\pi} \right] D_{2f}^{\pi^-} = \left[ \frac{\beta_2^\pi}{\beta_2^\pi} - \frac{\beta_2^\pi}{\beta_2^\pi} \right] D_{2\pi}^{\pi^-} + O(S^{-1/2}). \tag{1}
\]

The coupling \( D_{2\pi}^{\pi^-} \) is just the common asymptotic limit \(^*) on the Ferbel
plot \( \theta \) :

\[
D_{2\pi}^{\pi^-} \approx 0.84. \tag{2}
\]

The existence of this common limit is evidence for Pomeron factorization.
Next, using the two-body couplings of \( \theta \), we may determine \( D_{2\pi}^{\pi^-} \) and \( D_{2\pi}^{\pi^-} \)
by fitting the approach to the limit, assuming that the \( O(S^{-1/2}) \) term is
negligible, i.e., taking very literally Ferbel's straight lines. Fitting
\( (p|\pi|p) \), we obtain

\[
D_{2\pi}^{\pi^-} \approx -D_{2f}^{\pi^-} \approx 0.45, \tag{3}
\]

whereas fitting \( (\pi^+|\pi^-|p) \) we find

\[
D_{2\pi}^{\pi^-} \approx -D_{2f}^{\pi^-} \approx 0.36. \tag{4}
\]

\(^*)\) Ferbel's asymptotic values of total cross-sections \( \sigma_\infty \) (which do not
respect the Pomeranchuk theorem) differ by about 10% from those deduced
from Regge fits \(^3\). We use the latter.
The inequality of 0.36 and 0.45 suggests that (if the $s^{-1/2}$ term is in fact negligible) factorization is only approximate for the $\alpha(0) = \frac{1}{3}$ trajectories. However, let us take seriously the regularity *)

$$D_{\pi}^\pi \approx -D_{\pi}^\pi > 0$$

(5)

and not the numbers. So doing, we can write

$$\frac{1}{\sigma_\infty} \left. \frac{d\sigma}{dy} \right|_{y=0} \approx 0.84 + K s^{-1/4} D_{\pi}^\pi$$

(6)

for any pion production reaction. The coefficients $K$, computed from standard two-body couplings 9), are listed in Table I. Except for the qualitatively wrong prediction that

$$\frac{(p|\pi^-|p)}{\sigma_\infty (pp)} > \frac{(\pi^+|\pi^-|p)}{\sigma_\infty (\pi^+ p)}$$

(7)

the pattern of the Perbel plot is described satisfactorily. Indeed, if we choose a compromise numerical value of

$$D_{\pi}^\pi = -0.40$$

(8)

a fair over-all quantitative fit results 4). We regard this agreement as surprising, in view of the low energies at which most of the data were obtained.

Let us now apply what we have learned from the single-particle spectrum to two-body correlations. For the reaction $(a|c|d|b)$ we may define a normalized rapidity correlation function

$$R_{cd}(y_c, y_d) = \left( \frac{1}{\sigma_\infty (ab)} \frac{d\sigma}{dy_c dy_d} / \frac{1}{\sigma_\infty (ab)} \frac{d\sigma}{dy_c} \frac{1}{\sigma_\infty (ab)} \frac{d\sigma}{dy_d} \right) - 1.$$  

(9)

*) The conventional relation 1) is $D_{\pi}^\pi = D_{\pi}^\pi > 0$. 
In the scaling limit \((i = j = \text{Pomeron}, \ k \neq \text{Pomeron in Fig. 1})\), this correlation function is given by

\[
R_{cd}(y_c, y_d) \approx e^{-\frac{1}{2}(y_c - y_d)} \sum_{k = \omega, \sigma, A_2} \left( \frac{D_{\pi k}^c}{D_{\pi k}^d} \right) \tau_k \left( \frac{D_{\pi k}^d}{D_{\pi k}^d} \right)
\]

(10)

plus shorter range contributions of lower trajectories, where \(\tau_k\) is the signature of trajectory \(k\).

In the spirit of retaining only leading terms, we shall ignore the lower trajectories. Then (labelling pions by their charges) we have

\[
R_{-+} = R_{++} = e^{-\frac{1}{2}(y_c - y_d)} \left( D_{\pi \pi}^{-1} - (D_{\pi \pi}^{-1})^2 \right) / (D_{\pi \pi}^{-2})
\]

(11)

\[
R_{+-} = e^{-\frac{1}{2}(y_c - y_d)} \left( (D_{\pi \pi}^{+})^2 + (D_{\pi \pi}^{-})^2 \right) / (D_{\pi \pi}^{-2})
\]

(12)

\[
R_{00} = R_{00} = e^{-\frac{1}{2}(y_c - y_d)} (D_{\pi \pi}^{+})^2 / (D_{\pi \pi}^{-2})
\]

(13)

Immediately upon inserting (5) in (11)-(13) one finds

\[
R_{+-} : [R_{00} \text{ or } R_{-0} \text{ or } R_{00}] : [R_{++} \text{ or } R_{-+}] : 2 : 1 : 0.
\]

(14)

This means in particular that there should be no (correlation length 2) short-range correlations among like charged pions, in agreement with naive intuition. As the momentum of the \(\overline{c}d\) channel is timelike, one expects duality to relate an absorptive part of Fig. 1b - with \(k \neq P\) - to \((cd)\) resonances as in Fig. 1c. This connection has of course been appreciated for some time \([e.g., \text{Ref. 10}]\), but it has not been emphasized that the vanishing of \(R_{-+}\) is insensitive to the signs of \(D_{\pi P}^{\pm}\) and \(D_{\pi P}^{-}\); it is therefore in some sense independent of the unsuccessful early scaling predictions of the dual resonance model. Presently available data on \(R_{++}\) and \(R_{-+}\), obtained in the ANL/NAL 205 GeV/c pp bubble chamber exposure \(6\) show no hint of short-range correlations. It is clearly of great interest to improve statistics to subject the prediction to a more severe test.
We are now in a position to make a (very rough) numerical estimate of the magnitude of short-range correlations. Like the preceding discussion, this estimate is based on the extreme assumption that all particle production is due to a short-range correlation component, so success or failure is defined in terms of rough orders of magnitude. Using the values for central couplings given by (2) and (8), we calculate from (12) the energy-independent estimate

\[ R_+ \approx 0.45 \ e^{-y - y/2}. \]  

(15)

Even within our assumptions, the coefficient is to be believed only within a factor of two. However, as a rough estimate, it agrees well with existing data.

In fact the experimental situation is more complicated than a purely short-range correlation picture; the data are roughly represented by

\[ R_{\text{charged}} \approx 0.3 + 0.3 \ e^{-y - y/2}. \]  

(16)

with

\[ R_{\text{inel}} = \frac{1}{\sigma_{\text{inel}(ab)} \ dy_c \ dy_d} \ \frac{1}{\sigma_{\text{inel}(ab)} \ dy_d} \]  

(17)

Even in the simplest kind of two-component model \(^{11)} \), \(^{10)} \) one cannot consistently move between \( R \) and \( R_{\text{inel}} \) without giving up the Regge pole connection with total cross-sections. The strongest conclusion we may draw, therefore, is that our estimate indicates that a multiperipheral (here Regge) model can plausibly yield short-range correlations of the magnitude observed \(^*\).

\(^*\) This statement is consistent with the conclusions of Finkelstein \(^{12)} \), who estimated correlations in a simplified multiperipheral bootstrap equation.
It is well to emphasize again here the implications of Fig. 1c. In the Mueller-Regge picture, short-range correlations arise from (i.e., are dual to) resonance production. One would therefore have expected to see resonance wiggles in the correlation function, for small values of \( |y_c - y_d| \). Indications in the data \( 5), 6) \) of exponential behaviour down to \( |y_c - y_d| = 0 \) are in some sense an embarras de richesse, and somewhat puzzling \( 4) \).

Keeping always in mind our scheme's humble origins in Ferbel's data compilation, let us nevertheless notice its natural extension to kaon and proton production. The generalization of (5) is

\[
\sum_{k=A_2,\omega,f} D_{Pk}^c \tau_k \beta^i_k = 0, \tag{18}
\]

if (ic) is exotic, which leads to the "exchange degeneracy" requirements

\[
D_{Pf}^{K^+} = D_{Pf}^{K^+} = D_{Pf}^{K^+} = D_{Pf}^{K^+}, \tag{19}
\]

\[
D_{Pf}^{P} = D_{Pf}^{P} \quad D_{Pf}^{P} = D_{Pf}^{P} \tag{20}
\]

Using (19) and the two-body residues of Ref. 9, we may predict the relative magnitudes of the non-scaling \( s^{-1/4} \) terms for kaon production. The coefficients \( K \) in the analogue of (6)\( n \) These are given in Table II. Finally we notice that the systematics deduced here suggest early scaling (the absence of \( s^{-1/4} \) contributions) for reactions \( (a|c|b) \) in which \( (ac) \) and \( (bc) \) are exotic. While this criterion leads to no ridiculous predictions, it is difficult to see what possible relevance the \( (ac) \) and \( (bc) \) - as opposed to \( (a\bar{c}) \) and \( (b\bar{c}) \) - channels have for Mueller analysis.

Our investigation shows that the "circumstantial evidence" cited in the introductory paragraphs fits together in a systematic way that we, at least, find surprising. It underscores the desirability of careful measurements, species by species, of two-particle correlations.
ACKNOWLEDGEMENTS

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**TABLE I**: Relative magnitudes of leading non-scaling contribution to $(1/\sigma) d\sigma/dy|_{y=0}$ for pion production.

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**TABLE II**: Relative magnitudes of leading non-scaling contribution to $(1/\sigma) d\sigma/dy|_{y=0}$ for kaon production.
REFERENCES

5) P. Braccini - ibid.  
6) J. Whitmore - ibid.  
7) G. Neufhofer and F. Niebergall - ibid.  

FIGURE CAPTION

Figure 1a  Mueller-Regge diagram for the single-particle inclusive reaction $(a|c|b)$;  
Figure 1b  Mueller-Regge diagram for the two-particle inclusive reaction $(a|c|d|b)$;  
Figure 1c  The dual equivalent of Fig. 1b.