POMERON CONTRIBUTION TO AZIMUTHAL CORRELATIONS IN

INCLUSIVE DISTRIBUTIONS

A. Bassetto
and
M. Toller
CERN - Geneva

ABSTRACT

We consider the contribution of the Pomeranchuk Lorentz pole to the two particle inclusive distribution in the pionization region. We compute the first non-dominant term, which depends on the relative azimuthal angle. We derive a general sum rule for azimuthal correlations and we show that the Pomeron term we have computed gives a relevant contribution.
Inclusive distribution functions \(^1\) have risen recently a considerable interest, both from the experimental and the theoretical points of view. Multiperipheral dynamics gives some general predictions \(^2\)–\(^3\) which seem to be in agreement with the existing data.

In a previous paper \(^3\) we have emphasized the importance of detecting two-particle correlations, in order to get a deeper insight in the dynamics. We have also pointed out that, if the total energy is sufficiently large, as the one attained at the ISR, one can exploit the simplifications occurring in the pionization region.

In this region the one and two particle inclusive distributions are expected to take the simplified form

\[
F^\alpha(P_\alpha, \lambda) = 2P^0_\alpha \frac{d^3\sigma^\alpha}{d^3P_\alpha} \sim \sigma_I N_\alpha(n_\alpha),
\]

\[
F^{\alpha\beta}(P_\alpha, P_\beta, \lambda) = 4P^0_\alpha P^0_\beta \frac{d^6\sigma^\alpha}{d^3P_\alpha d^3P_\beta} \sim \sigma_I N_\alpha(n_\alpha) N_\beta(n_\beta) + \sigma_I H^{\alpha\beta}(n_\alpha, n_\beta, \xi_{\beta \alpha}, \gamma_{\beta \alpha}).
\]

As in Ref. \(^3\) we have indicated by \(\alpha\) and \(\beta\) the observed particles, by \(\sigma_I\) the inelastic total cross-section, and by \(p^\alpha\) the transverse momentum of the particle \(\alpha\). The quantities \(\xi_{\beta \alpha}\) and \(\gamma_{\beta \alpha}\) are respectively the relative rapidity and the relative azimuthal angle of the particles \(\alpha\) and \(\beta\). The functions \(N\) and \(H\) should not depend on the total energy and on the properties of the initial particles.
For relatively large \( \xi_{\beta \alpha} \), the function \( H \) can be expanded in a sum of contributions of non-leading Lorentz poles plus the non-leading terms of the Pomeron contribution. For a Lorentz pole with intercept \( a' \) and a given quantum number \( M \), the asymptotic contribution is

\[
\exp \left[ |\xi_{\beta \alpha}| (a' - a) \right] \cos (M \phi_{\beta \alpha})
\]

\[
\cdot \nu^\alpha (p_{\alpha}) \nu^\beta (p_{\beta}),
\]

where \( a \sim 1 \) is the intercept of the Pomeron. We see that the dependence on the azimuthal angle \( \phi_{\beta \alpha} \) is related to the quantum number \( M \). The only pole with \( M = 1 \) and positive \( G \) parity which has hitherto been proposed \( ^{11},^{12} \) is the \( \phi' \), which is supposed to have an intercept \( a' \) near zero \( ^{13} \).

In these conditions, the first non-leading term of the Pomeron contribution, which is proportional to \( \cos \phi_{\beta \alpha} \), has to be taken into account. We remember that the leading term of the Pomeron contribution is just the first expression in the right-hand side of Eq. (2). Our aim is to compute explicitly the first non-leading term, starting from the functions \( N_\alpha (p_{\alpha}) \), of which we have a good experimental knowledge.

We are not taking into account the presence of cuts in the Lorentz complex plane. Very little can be said about their relevance in our problem. Cuts lying near the Pomeranchuk pole have been considered in Ref. 14), where it has been shown that, as these cuts can contain contributions with any value of the quantum number \( M \), they can induce azimuthal correlations between the fragmentation regions. A similar argument shows that cuts can induce azimuthal correlations also in the pionization region. These correlations can be distinguished from the ones we are considering because they should exhibit a much slower decrease for increasing \( \xi_{\beta \alpha} \). We
shall argue in the following that, as soon as we consider the pionization region, these cut contributions to the azimuthal correlations should be small.

The complete Pomeron contribution to the two particle inclusive distribution has the form

\[ \sigma^{-1} F_{KB}(p_x, p_y, s) \propto \left[ \tilde{\sigma}_A D^{0, a+1}_{00000} (a_z(\xi_{BA})) \tilde{\sigma}_B \right]^{-1} \]

\[ \times \sum_{i \bar{i} m' \bar{m}} \tilde{\sigma}_A D^{0, a+1}_{00 \bar{i} \bar{m}} (a_z(\xi_{dA}) a_x(\eta_d)) \tilde{\sigma}_B^d \]

\[ \times D^{0, a+1}_{\bar{i} m' \bar{m} 00} (a_x(-\eta_{d}) a_z(\xi_{dA}) a_x(\eta_d)) \tilde{\sigma}_B^d, \]

where the matrices \( D \) are the irreducible representations of the Lorentz group, \( a_z(\xi) \) is a boost of rapidity \( \xi \) along the \( z \) axis and \( u_z(\varphi) \) is a rotation of an angle \( \varphi \) around the \( z \) axis. The indices \( A \) and \( B \) refer to the incoming particles and we have put

\[ \frac{p_x}{M_d} = \sinh \eta_d. \]  

A non-trivial calculation, where we keep only the leading terms in \( \xi_{dA} \) and \( \xi_{B\beta} \), gives for the expression \( (4) \) the asymptotic approximation
\[ \sigma_{\pi^{-1}} F^{\alpha \beta}(P_\alpha, P_\beta, \delta) \simeq N_\alpha(P_{\alpha}) N_\beta(P_{\beta}) - T_\alpha(P_{\alpha}) T_\beta(P_{\beta}) \exp(-1/\delta_{\pi^{-1}}) \cos \varphi_{\pi^{-1}} + O[\exp(-2/\delta_{\pi^{-1}})], \]

where

\[ N_\alpha(P_{\alpha}) = (1 + \frac{P_{\alpha}^2}{M_{\alpha}^2})^{\alpha} \sum_j \rho_\alpha^j P_j \left( \frac{M_{\alpha}^2 - P_{\alpha}^2}{M_{\alpha}^2 + P_{\alpha}^2} \right), \]

\[ T_\alpha(P_{\alpha}) = \frac{P_{\alpha}}{M_{\alpha}} \left(1 + \frac{P_{\alpha}^2}{M_{\alpha}^2} \right)^{-\frac{1}{2}} \sum_j \rho_\alpha^j \left[-\sqrt{2\alpha} \cdot P_j \left( \frac{M_{\alpha}^2 - P_{\alpha}^2}{M_{\alpha}^2 + P_{\alpha}^2} \right) + 2\sqrt{\frac{2}{\alpha}} \left(1 + \frac{P_{\alpha}^2}{M_{\alpha}^2} \right)^{-1} \cdot P_j' \left( \frac{M_{\alpha}^2 - P_{\alpha}^2}{M_{\alpha}^2 + P_{\alpha}^2} \right) \right]. \]

The quantity \( \rho_\alpha^j \) differs from \( \tilde{\rho}_\alpha^j \) by a known constant.

Comparing Eqs. (7) and (8) we get the final result in the useful form

\[ T_\alpha(P_{\alpha}) = -\sqrt{\frac{2}{\alpha}} \cdot P_{\alpha} \left( \frac{M_{\alpha}^2}{M_{\alpha}^2 + P_{\alpha}^2} \right)^{\frac{1}{2}} \frac{d}{dP_{\alpha}^2} N_\alpha(P_{\alpha}). \]
If we use the simple parametrization

\[ N_d (r_d) = \frac{2}{\pi} h_d \Lambda^2 \exp (-\Lambda^2 r_d^2) , \]

\[ \Lambda^2 \approx 4 (\text{GeV}/c)^{-2} , \]

\[ \sum h_d \approx 1.5 , \]

we get (with \( a = 1 \))

\[ T_d (r_d) = \frac{2 \sqrt{2}}{\pi} h_d \Lambda^4 r_d (M_d^2 + r_d^2)^{\frac{3}{2}} \exp (-\Lambda^2 r_d^2) . \]

We see that even for \( |\xi_{\beta_d}| = 3 \) we have a correlation effect of the order of 10%. In any case, this effect has to be subtracted when one is looking for other contributions.

In the discussion of azimuthal correlations, it is useful to consider a sum rule, which is a direct consequence of the definition of inclusive distribution and of the vanishing of the total transverse momentum. The general form of this sum rule is

\[ \sum_d \int F_d (P_d , \beta) r_d^2 \frac{d^3 P_d}{2 P_d^0} = - \sum_{d, \beta} \int F_{d\beta} (P_d, P_{\beta}, \beta) \cdot r_d P_{\beta} \cos \phi_{d\beta} \frac{d^3 P_d}{2 P_d^0} \frac{d^3 P_{\beta}}{2 P_{\beta}^0} . \]
In the limit of large $s$, using Eqs. (1) and (2) and keeping only the leading term proportional to $\log s$, we get

$$\sum \int N_d(r_d) \frac{r_d^3}{d r_d} = -\sum \int H^B(r_d, r_B, \xi_{\beta d}, \psi_{\beta d}) \cdot \cos \psi_{\beta d} d \psi_{\beta d} \frac{1}{2} d \xi_{\beta d} \frac{1}{2} r_d^2 d r_d \frac{1}{2} r_B^2 d r_B. $$

(13)

A simple calculation using Eqs. (6), (10) and (11) shows that, if we assume that in the region $|\xi_{\beta d}| < 2$ the azimuthal correlation is given by Eq. (5), the sum rule is saturated up to 70%. This consideration shows that the contribution we have computed is important.

If $M = 1$ cuts are present near the Pomeranchuk pole their contribution to the sum rule is spread over a larger range of the variable $\xi_{\beta d}$. Then the sum rule suggests that this contribution is locally rather small.
NOTES AND REFERENCES


10) As for $a = 1$ some singularities arise which disappear in the final result we perform the calculation for a slightly smaller value of $a$.


