SOME REMARKS ON HIGHER MESONS *)

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ABSTRACT

A model based on exchange degeneracy and linear trajectories is given to correlate presently known meson resonances with $I = 1$. A critical discussion is given of some features of this model. Some remarks are made on the problem of the widths of the higher mesons in this scheme and on some related problems in higher meson spectroscopy.

*) This represents a revised version of an earlier CERN internal report TH.730 (1966) (unpublished).

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It is the purpose of this note to attempt a fit of all known \( I=1 \)
mesons *) on straight line Regge trajectories. Several papers 3,4,5,6 have appeared in recent months in which partial attempts to use linear
Regge trajectories are made, but none of them have reproduced the splitting
in the \( R \) region seen in the missing mass experiment 7. Since they also
neglected peaks such as \( \sigma \) and \( A_1 \) which have been reported at
lower masses, one may feel that this could be remedied if they were in-
cluded. The second purpose is, within the context of the model, to dis-
cuss, in what we hope is a semi-quantitative way, the problem of narrow
widths, which has aroused some interest recently.

To some extent we feel that, lacking a detailed dynamical model,
our calculations are very trivial. Our apology is only that similar
approaches show promising results in the baryon spectrum 8. One is thus
encouraged to hope that trying it for mesons may either produce good
results, or by failing badly, lead one either to some dynamical under-
standing 9, if success continues for the baryons, or to look more suspi-
ciously at the theory for baryons, which so far is completely checked in
only rather few cases. A further hope is that the proposed trajectories
may be of interest to Regge theoreticians, particularly for consideration
of unnatural parity exchange. Finally the considerations on the widths
may, with a little additional experimental information, lead one to new
selection rules of strong interaction dynamics, in the way that the failure of
\( \phi \) to decay to \( \phi \to \pi \) has done.

*) Only for the dipion resonance at 1650 MeV is there evidence 1 that
\( I=1 \), not 2, for the charged resonances above 1.5 GeV. It should
be noted that 1.7 GeV is a very natural point at which higher \( I \)
spin resonances might occur as can be seen from the following piece
of numerological argument. In the baryon system, evidence for a 10
resonance has recently been given by Cool et al. at 1860 MeV 2. If
one conjectures that the mass \( m_M \) at which new "non quark-anti-
quark" mesons appear is related to the mass \( m_B \) at which new
"non three-quark" baryons appear by \( m_M^2 - m_N^2 = m_B^2 - m_P^2 \), we find
\( m_M = 1620 \text{ MeV} \). We shall assume \( I=1 \) for all higher bumps, however.
Our theoretical input will be the concept of "exchange degeneracy", introduced by Arnold \(^9\) and supplemented by straight line Regge trajectories \(^8\). Arnold's idea is that resonances should occur at intervals of \(\Delta J = 1\), not of \(\Delta J = 2\) as in the usual scheme. This is very natural on any Fermi-Yang model in which the mesons are bound states of baryons and antibaryons, since then meson exchange in the \(t\) channel should dominate over double baryon exchange in the \(u\) channel, giving a predominantly direct potential only. We are aware that this is only a rather rough approximation \(^10\), and we shall discuss the errors involved for the case of the \(O-A_2\) trajectory for illustration.

Our experimental input will consist of the low-lying resonances \(\overline{\Pi}(139)0^-,\ \rho(768)1^{-+},\ A_2(1286)2^{++},\ A_1(1080)1^{++},\ B(1208)1^{++}\) and \(\delta(962)0^{++}\), where our notation is (mass in MeV)\(^{\text{exp}}\). We take our parameters from the results presented by Focacci et al. \(^11\), when they have observed the peak, since their results for the higher peaks are the ones we shall be concerned with most, and from the Rosenfeld tables \(^12\) in other cases. We wish to emphasize that for only \(\Xi,\ \rho,\ \text{and} \ A_2\) are these spin-parity assignments reasonably well confirmed \(^\ast\), and in the case of \(\delta\) we have to rely entirely on theoretical conjecture \(^17\)-\(^19\) \(^\ast\ast\).

For the trajectories consisting of \((\rho,A_2)\) and of \((\overline{\Xi},B)\) treatments have been given previously \(^4\),\(^5\),\(^6\),\(^11\). For the \(A_1\) and \(\delta\) trajectories, we have no partners from which to derive the trajectories. We then need the following information: Deutschmann et al. \(^20\) have presented evidence of a \(\overline{\Xi}^+\Xi^0\) resonance at 1910 MeV and, while

\(^\ast\) Even for the \(A_2\) there have recently been some worries about the spin-parity assignment \(^13\) and about the possibility of two peaks in the \(3\overline{\Xi}\) system \(^14\)-\(^16\).

\(^\ast\ast\) The argument given in Ref. \(^19\) on the small rate depends on the existence of only Regge poles. Regge cuts of negative parity could spoil it [see V.N. Gribov, International Conference on High Energy Physics, Berkeley, Calif. (1966)]: measurement of \(\delta\) production, if it is really \(0^{+-}\), could be interesting as a test for Regge cuts.
Goldhaber \(^{21}\) apparently feels this resonance to be as yet unconfirmed, we feel that its statistical significance of five standard deviations, should, for consistency with the criteria of Ref. \(^{11}\), lead us to take it seriously. The most natural choice is then \(3^{-+}\) if it is to be associated with the \(\Sigma\) in a Regge trajectory.

As seen from our presentation of the results in the Table, we still have not a candidate for \(R_{3}(1748)\) \(^{11}\). For this we propose that it should be a \(2^{++}\) state, associated with the \(A_{1}\). This we describe as trajectory \((A_{1}, R_{3})_1\); an alternative proposal is that \(R_{3}\) is a \(5^{++}\) state \(^{6}\); this we describe as trajectory \((A_{1}, R_{3})_2\). Since this predicts a \(2^{++}\) state at 1450 MeV and, more seriously, a \(0^{++}\) state at 460 MeV, we are inclined to favour the first assignment.

Let us now make some general remarks about our scheme. First of all, let us mention that the value of 0.47 found for \(\alpha(p, A_{2}) \langle 0 \rangle\) agrees neither with the value of 0.58 for the \(p\) trajectory, nor with that of 0.35 found for the \(A_{2}\) trajectory \(^{10}\). If we were to choose \(\alpha(p) = 0.58\) and \(\alpha(p, 0.58) = 1\), we would find a \(3^{-+}\) state at 1820 MeV, not at 1650 MeV. Hence one must allow some curvature in the trajectory. The baryon trajectories, on the other hand, seem to fit rather well with linear trajectories, at least for \(\kappa^2 > 1\) GeV\(^2\). A further remark is that the positions of the higher resonances are very sensitive to those of the input. Thus if \(m_{A_{2}}\) is chosen as 1305 MeV \(^{12}\) instead of 1286 MeV \(^{11}\), then we find \(m_{\Xi} = 2260\), not 2206, MeV.

A further feature of our scheme which we find somewhat disquieting is the fact that the values of \(\alpha'\) which we find for the unnatural parity meson trajectories differs substantially from those of the natural parity meson trajectories, whereas for the baryon trajectories the values of \(\alpha'\) agree within 10% for all three established trajectories. A further point is that \(\alpha(A_{1} - R_{3}) \langle 0 \rangle\) is 0.39 for our first solution. This may be interesting for explaining reactions involving unnatural parity exchange, but contradicts, on the other hand, a suggestion by Durand \(^{22}\) that np charge exchange may be explained by having \(\alpha_{A_{1}}(0) < 0\).
From the Table it will be seen that we have resonances close to the observed peaks at S, T, U and all three peaks in the R region. In addition we have a fourth peak in the R region at 1652 MeV with $J^{PC} = 2^{+}$. This is rather gratifying in view of the reported existence of a $\pi^{+}\pi^{-}$ resonance at 1640 MeV $^{21}$. Of course, we also predict other resonances, at 2090 MeV and at 1355 MeV which have not yet been observed. Although one can argue that the former may have escaped detection because of a large width, the failure to detect the second in the experiment of Wehmann et al. $^{23}$ is rather disturbing. We should also point out that two resonances are predicted in the vicinity of S, three in the vicinity of T, and three in the vicinity of U. This may be relevant for the observation in $\bar{p}p$ (total cross-sections) of resonances near the T and U of Ref. $^{11}$, but with much larger widths of about 30 MeV $^{24}$. 

Clearly, our scheme owes something to the quark model $^{17}$. Though the idea of exchange degeneracy depends only on a baryon-antibaryon model for the mesons, the restriction to only singlet and octet representations and to the spin values of the trajectory we employ are harder to justify if nucleons and nucleon resonances and their antiparticles are the relevant baryons. Our reason for trying straight line trajectories, rather than the more concrete model of Ref. $^{17}$, is simply that the observation by Deutschmann et al. $^{20}$ of a 1910 MeV resonance decaying to $\pi^{+}\pi^{-}$ is incompatible with that scheme, so that some modification, at least, is required, and the only guide available is the success of the baryon trajectory scheme.

We now discuss the problem of the widths of these mesons. This has seemed rather hard to understand since the widths of all the resonances seen in the missing mass spectrometer above 1.5 GeV are $\leq 38$ MeV. This contradicts a general feeling that the widths of resonances should increase with their mass. Indeed the baryons do seem to show some slow increase with mass, at least as far as present analysis is concerned. We shall try to estimate how serious this problem is within our framework. It should be borne in mind that bubble chamber data $^{21},^{1}$ on peaks in the R region suggest much larger widths. If these resonances are the same as those seen by Focacci et al. $^{11}$, we feel that the higher statistics and better resolution of the latter experiment should lead us to take their narrow width seriously.
Our discussion will be based on a very simple expression for the partial width \( \Gamma_\ell \) for the decay of a meson of mass \( M \) into a quasi-two-body state with relative orbital angular momentum \( \ell \),

\[
\Gamma_\ell = \gamma \frac{q}{m} \frac{(qR)^{2\ell}}{[1 \cdot 3 \cdot 5 \cdots (2\ell - 1)]^2}
\]

where \( q \) is the momentum available in the decay, \( R \) is some "radius of interaction" and \( \gamma \) is the reduced width, which is a priori unlikely to vary widely from one resonance to the next. We shall show that for the higher meson resonances this formula leads, on normalizing \( \gamma \) to the width of the \( \rho \), to a reasonable explanation of the widths. For \( R \) we take \( 1/550 \text{ MeV}^{-1} \), since this is the mass appearing in the electromagnetic form factors. Such a choice seems reasonable, as it gives a width of 100 MeV for \( f_0 \).

If we take the \( U \) to be \( 5^+ \), for instance, we find that a reasonable estimate of its width is 60 MeV, if we keep all quasi-two-body states implied by our model. One may, of course, anticipate that there will be some further contribution to the width from many particle states, but presumably it should be of the same order as that from the quasi-two-body states. Most of the contribution to our estimate comes from the decay into \( \rho + f_0 \) or \( \omega + A_2 \) in d wave, and if we were to decrease the value of \( \gamma \) for these modes by 2, agreement with Ref. \(^{11}\) would be obtained. Since \( \gamma \) can certainly be expected to vary by 10 from one resonance to another without implying any particular selection rule (cf., the \( A_2 \to \bar{K}K \) with \( f_0 \to \pi \pi \)), it is clear that we cannot conclude that anything very surprising is happening.

\text{*) We are unable to give any really satisfactory justification for this parametrization, which is essentially a "relativized" version of the formula given in Chapter VIII of Blatt and Weisskopf \(^{25}\). A similar expression was suggested by Layson \(^{26}\) for baryon widths. This formula employs nothing much more than the idea that strong forces are of short range, which is almost the only thing we know. Strictly speaking, the formula given here is only correct if \( qR \) is small, but the correction terms tend to decrease \( \Gamma_\ell \) and for our purposes are not important.}
The reason why the width does not increase very rapidly in this scheme is that the spacing between resonances is decreasing rapidly while the spin is increasing rapidly. There is thus rather little phase space available for decay into nearby states with low orbital angular momentum while the states with large phase space have high angular momentum barriers. It is very interesting here to remark the recent observation by Bartke et al. 27 of a resonance at 3690 MeV with $\Gamma = 50 \pm 30$ MeV in $N^+\bar{\pi}$, which lends support to this picture for baryons also.

In the region of $S$ similar estimates show that $S(1950)^{4-}$ should have a narrow width compatible with the limit of 35 MeV $^{11}$. For $S(1910)^{3-}$, however, an appreciably larger width of about 150 MeV, with $\pi\bar{\pi}$, $\pi\Delta$, and $\phi\phi$ each having comparable probability is expected, which may well imply that the resonance observed by Focacci et al. is the $4^{-}$ state.

So far our results are in fair accord with experiment even if we do not change $\gamma$ appreciably from the value found for the $\phi$. Reasonable results also ensue for the $R_1(1650)^{3-}$ for which we anticipate a width of 15 MeV for $\pi\pi\pi$, although it appears that $\phi+\phi$ should have a width of 40 MeV since it occurs in a $p$ wave. For the $2^{+}$ state proposed at 1650 MeV we anticipate comparable widths of about 60 MeV each for $\pi\pi\phi$, $\pi\phi$ and $\pi\pi$. This is in fair accord with the total width of 100 MeV quoted in Ref. $^{21}$, but disagrees somewhat with the preliminary evidence given there that $\pi\pi\phi$ dominates over $\pi\phi$. However, if we consider the $2^{-}$ and $2^{+}$ states in the $R$ region, we find that, keeping $\gamma$ at its value deduced from $\phi$, we would expect that each resonance should have a width of about 200 MeV for each of the modes $0^{-1}$ in a $p$ wave and $0^{-2}$ in an $s$ wave. This indicates that $\gamma$ should be $\lesssim 1/10$ of the value of $\gamma$ deduced from the $\phi$ if we identify these resonances with the $R_2$ and $R_3$ of Ref. $^{11}$.

Let us compare our results on widths with that of previous treatments. Freund $^{5}$ suggests that asymptotically the partial width to two pseudoscalars, for instance, should tend to infinity as $2^{2J}(J-\frac{3}{2})$; this result appears to disagree with our simple model, but it should be noted
that Freund's widths are proportional to $\beta(t)^2$, where $\beta(t)$ is the reduced residue function, which he, somewhat arbitrarily in our view, sets constant above $t=m^2$. It seems that no reliable estimates of $\beta(t)$ at high $t$ values are available but in view of its rapid variation at lower $t$ values we feel it may well decrease rapidly at large $t$. This would be reflected in a Lagrangian theory of the decay of these higher mesons by the fact that the coupling constant should decrease rapidly with increasing resonance mass, so as to incorporate the centrifugal barrier effect found in a boundary condition model of these mesons.

A proposal closely related to ours was made independently by Dalitz at the Berkeley Conference. He proposed using a smaller radius than that used here, but did not make numerical comparisons in the $R$ region. It is clear from our discussion that no very strong overlap function effect between resonances with widely different spin is necessary, but that the largest suppression of $\psi$ seems necessary for the decays of $2-^-$ states into $2^+0^-$ and $0^-1^-$ states. This effect will only be made worse by decreasing the radius of interaction and, since no strong effect is observed in the decay of $2^+0^-0^-$ such as $f_0 \to \pi \pi$, the jump in spin is not enough to make us feel that this should be a really large effect. A factor of 10 suppression is not yet enough, however, to make us feel that anything really remarkable is happening, such as occurred in the suppression of $\psi \to \rho \pi$ where a factor of 100 suppression was involved *). It seems that an improved bound must be set on the partial widths of these resonances, either by improvement of the total width measurement, or, perhaps more probably, by finding that the relevant modes are only a small fraction of the total width, before anything really surprising and significant can be concluded.

We make some brief remarks on two other problems. The first concerns the $\bar{p}p$ data of Giacomelli et al. For peaks which they observe in the $T$ and $U$ region: Giacomelli et al. determine $(J^{1/2})^R_{NN} \approx 20$ MeV, where $J$ is the spin of the resonance. Taking for $U$, for instance, the most conservative spin and orbital angular momentum available in our scheme, namely $J=4$, $\ell=4$, we deduce $J_{NN}^{10} = 10 J_5$. This may lend some support to a Fermi-Yang model of the mesons.

*) It is clear that we are not committing the first basic error of strong coupling theory discussed by P.E. Low [Brandeis Lecture Notes, 1965 - "Particle Symmetries", Gordon and Breach publ., p.587 (1966)] and we feel that to take factors of 10 too seriously would be to commit the second.
Our second remark is on the $\Lambda$ meson \textsuperscript{28}). Zalewski \textsuperscript{29}) informs us that an attempt to fit the branching ratios of this resonance leads to strong disagreement with SU(3). This may mean that, just as in the case of the $R$ region, which first estimates from the missing mass experiment gave as a broad peak of width $\sim$100 MeV, several narrower resonances, including the $3^- K^*$ suggested by Ahmadzedah \textsuperscript{6}), will be found in this region. Analysis of spin and parity assuming only one resonance will then give misleading results.

Finally, let us state that we do not believe that this model will give a complete answer to the meson spectrum. We present it only as a model to show what may be learnt from current experimental evidence and to show simple effects such as the ease with which several resonances having masses very close together can arise, and that there is not yet anything decisively disturbing about the narrow widths observed. Its success or, in the case of its failure, the implications for the baryons, we leave to the reader to judge \textsuperscript{*}).

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\textsuperscript{*}) In the interval between the completion of the manuscript and the author's return from vacation the pessimism of this last paragraph has received support from the appearance of a paper by Vanderhagen et al. \textsuperscript{30}), in which a $0^- \eta^{-}$ resonance is reported at 1320 MeV. (The flexibility of numerology is well illustrated by changing $m$ to the average mass of the pseudoscalars, $\sim$400 MeV, and using a linear instead of a quadratic relation in the first footnote above.)

It should also be pointed out that we have taken a rather naive approach to Regge recurrences and have not worried about possible conspiracy trajectories, which have led Halpern \textsuperscript{31}) to propose a $0^-$ assignment for the $\delta(962)$, or daughter trajectories \textsuperscript{32}). For completeness we should perhaps also mention other suggestions on the $\delta$, of $0^-$ by Wu and Tuan \textsuperscript{33}), from a group theoretical viewpoint, and by Logan et al. \textsuperscript{34}) who suggest it may be the $\rho'$ introduced by Fischer and Høgsaasen \textsuperscript{35}).
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