INTRODUCTION (by J.M. Jauch)

(1) Classification of Particles.

(2) I-Spin and Strangeness.

(3) Classification of Interactions.

(4) Cross-sections and Lifetime of Elementary Processes.

(1) Classification of Particles.

Reference is made to the table of fundamental particles, distributed at the beginning of the lecture. The following general review is also useful:


We divide the various particles into certain groups which are listed below. The motivation for these divisions are certain general properties which the particles in each group have in common and which are listed under each group.
(a) Photons. Rest mass $m=0$, spin 1, and gauge invariance.

Photons interact through a universal constant $e = \frac{4\pi}{137}$ with charged particles. It is a remarkable feature that this interaction seems to be the only interaction which photons have with anything else. One may speak of the principle of "minimal interaction", formulated for instance tentatively in the following manner: "Every effect involving photons in interaction involves only the fundamental electromagnetic interaction". For instance the so-called anomalous magnetic moment of particles is not a new interaction, but is derivable from the fundamental electromagnetic interaction with great accuracy and complete agreement with experiment.

**Important Question:** What are basic and what are derived interactions. So far this question is only solved for the photons. For other fundamental particle interactions the answer is not known.

(b) Leptons. Relatively light, spin $\frac{1}{2}$, weakly interacting.

The most important property of the leptons is that they seem to satisfy a conservation law. Leptons, being particles of spin $\frac{1}{2}$ occur always in pairs of particles and antiparticles. The lepton conservation is true, if we associate with the antiparticles the opposite lepton number as to the particles. Thus for instance the following is a possible assignment of lepton numbers $\ell$ to the six known leptons:

<table>
<thead>
<tr>
<th>lepton</th>
<th>$\nu$</th>
<th>$\bar{\nu}$</th>
<th>$e^-$</th>
<th>$e^+$</th>
<th>$\mu^-$</th>
<th>$\mu^+$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell$</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
</tr>
</tbody>
</table>
The lepton conservation law, if true, would be stated as the selection rule

$$\Delta \ell = 0$$  \hspace{1cm} (1.1)

It has observable consequences, for instance in the two reactions

$$\begin{align*}
\nu^+ &\rightarrow \mu^+ + \nu \\
\nu^- &\rightarrow \mu^- + \bar{\nu}
\end{align*}$$  \hspace{1cm} (1.2)

the two neutrinos are different. Similarly

$$\mu^+ \rightarrow e^+ + \nu + \bar{\nu}$$  \hspace{1cm} (1.3)

leads to a pair of different neutrinos.

\textbf{(c) Mesons.} Intermediate mass, spin 0 (Bosons).

There are two subgroups: pions and K-mesons. The pions occur in a charge triplet with slightly different masses (see table).

$$\mathcal{M}_{\pi^+} = \mathcal{M}_{\pi^-} > \mathcal{M}_{\pi^0}$$

The K-mesons occur also with charges $^+$ and 0. But there are important differences from the pions. Within the experimental accuracy the masses of the K-mesons are all the same. Furthermore there exist two neutral pions which can be experimentally differentiated by different decay schemes with different lifetimes.

$$\begin{align*}
K^0_1 &\rightarrow \pi^+ + \pi^- \\
&\rightarrow \pi^0 + \pi^0 \quad \tau \sim 10^{-6} \text{ sec.}
\end{align*}$$  \hspace{1cm} (1.4)
(a) Nucleons, together with the antinucleons comprise four particles $n, p, \bar{n}, \bar{p}$. Strong interactions with the pion system! Spin $\frac{1}{2}$.

(b) Hyperons, $\Sigma^+, \Sigma^0, \Lambda^0, \Xi^-$. Masses $\gg$ nucleon masses. There are theoretical reasons for believing in the existence of a neutral $\Xi^0$. Furthermore all of them having spin $\frac{1}{2}$ (or half-integer at least) are expected to have antiparticles. None of these have been discovered yet.

The two last groups are conveniently placed into a larger group called the "baryons", because there seems to be a law, called the conservation of baryon numbers, which is similar to the law of lepton conservation.
(2) I-Spin and Strangeness.

(a) I-spin-space. We refer to the basic fact that nuclear forces are charge independent. Thus if we could switch off the electric charge and all other weak interactions, the neutron and the proton would be distinguishable.

This is similar to the spin in a magnetic field. As long as the magnetic field is not present the two spin states will be degenerate and they cannot be distinguished. This analogy is the basis of the I-spin formalism.

Two nucleons at rest are represented in this formalism by a column vector consisting of two components. For instance

\[
p = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad n = \begin{pmatrix} 0 \\ 1 \end{pmatrix}
\]  

(2.1)

When we have more than one particle we use the method of product spaces. For instance 2 particles are described in a four dimensional space. A system of orthogonal vectors in such a space is \( pp, pn, np, nn \). Explicitly we may write

\[
pp = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad pn = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad np = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix} \quad nn = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}
\]  

(2.2)

This can be generalized in an obvious manner to a system of \( N \) particles. The dimension of the space is then \( 2^N \).
(b) I-Spin Operators. In analogy to the ordinary spin we introduce the I-spin operators, for instance

\[
\tau_1 = \begin{pmatrix} 0^I & 0 \\ 0 & 1^I \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix}
\]

and

\[
\tau_+ = \frac{1}{2} (\tau_1 + i \tau_2)
\]

with the properties

\[
\tau_3 p = p, \quad \tau_3 n = n
\]

\[
\tau_+ n = p, \quad \tau_- n = n
\]

These operators satisfy the commutation rules

\[
\tau_1 \tau_2 = - \tau_2 \tau_1 = i \tau_3, \quad \text{etc.}
\]

In the case of many particles we introduce an I-spin operator for each particle

\[
\tau = \sum_{n=1}^{N} \tau^{(n)} \quad \tau = (\tau_1, \tau_2, \tau_3)
\]

and

\[
I = \frac{1}{2} \tau
\]

so that

\[
I^2 = I(I+1)
\]

for the substates with total I-spin equal to I.
Warning:

While the analogy of the I-spin formalism to the ordinary spin has always been stressed, it is perhaps not quite so well known, that the analogy is not complete. (see for instance L.L. Foldy, Phys.Rev. 93, 1395 (1954) on this point). The reason is that there exist general symmetry transformations which multiply every state vector in anyone of the subspaces of constant charge with a different phase factor. As a consequence any operator which does not commute with $T^2$ or $T_3$ cannot be an observable. In particular the I-spin operators $T_1$ and $T_2$ cannot be observables. This is the main reason why the physical significance of the I-spin formalism is much less transparent than that of the ordinary spin.

(c) Relation of I-Spin to Charge. In the case of one single nucleon the charge has the eigenvalues 0 and +1. (in units of e). We can therefore represent it by an operator

$$ Q = I_3 + \frac{1}{2} $$  \hspace{1cm} (2.9)

and we find

$$ Q \ p = p $$  \hspace{1cm} (2.10)

$$ Q \ n = 0 \ n $$

In the general case of N nucleon we find

$$ Q = \sum_{i=1}^{N} Q^{(i)} = I_3 + \frac{N}{2} $$  \hspace{1cm} (2.11)
If we decompose the states of the I-spin system in the usual manner in multiplets we expect the eigenvalues of $Q$ to occur in charge multiplets with values

$$q = -I + \frac{N}{2}, -I + \frac{N}{2} + 1, \ldots, +I + \frac{N}{2}$$  \hspace{1cm} (2.12)

All these multiplets have their centre of charge at the value $\frac{N}{2}$. Thus we conclude the important relation, valid for any system of nucleons:

$$\frac{\text{Baryon number}}{2} = \text{Centre of charge}$$  \hspace{1cm} (2.13)

(d) Extension of I-Spin to Pions. Pions occur in a triplet of charges $\pi^\pm, \pi^0$. We attribute to them a total I-spin = 1 and we can define the I-spin operators in complete analogy to angular momentum states

$$I = J = (J^1, J^2, J^3)$$  \hspace{1cm} (2.14)

$$J^1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad J^2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0-1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad J^3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$  \hspace{1cm} (2.15)

$$\pi^+ = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \pi^0 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad \pi^- = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$  \hspace{1cm} (2.16)
The following useful relations are immediately obtained from these definitions:

\[ \int_1^0 \mathcal{N}^+ = \frac{1}{\sqrt{2}} \mathcal{N}^0, \quad \int_1^0 \mathcal{N}^0 = \frac{1}{\sqrt{2}} (\mathcal{N}^+ + \mathcal{N}^-), \quad \int_1^0 \mathcal{N}^- = \frac{1}{\sqrt{2}} \mathcal{N}^0, \text{ etc.} \]

Notice:

The relation of the charge operator to the $I$-spin is now

\[ Q = I_3 \]

(2.17)

and differs from (2.9) by the term $+\frac{1}{2}$. But the Eq. (2.13) is still correct. The centre of charge is zero and so is the baryon number!

(e) Strangeness. For pions and nucleons we can write quite generally

\[ Q = I_3 + \frac{1}{2} \mathcal{N} \]

(2.11)

where $\mathcal{N}$ is the number of nucleons.

Question: Can this relation be generalized, so that it is valid for other charge multiplets of strange particles?
Let us see how it is with the $\Sigma$-particles, since there exist $\Sigma^+$ and $\Sigma^0$ one would naturally assign to it the I-spin 1, and since the centre of charge is zero we have in this case

$$Q = I_3$$

(2.18)

On the other hand, the $\Sigma$-meson is a baryon and therefore one would have expected $N=1$. Thus relation (2.11) is violated. But we can still write

$$Q = I_3 + \frac{1}{3} U$$

(2.19)

where $U$ is for the time-being an unknown integer. It is a strange fact that it differs from the baryon number $N$ and so we write

$$U = N + S$$

(2.20)

and call $S$ the strangeness number. Instead of (2.13) we have now

$$\frac{\text{Baryon number} + \text{Strangeness}}{2} = \text{Centre of charge}$$

(2.21)

Note:

The assignment of strangeness is not unique. It depends of course on the assignment of I-spin. When the charges occur in multiplets (as for instance the $\Sigma^+$ and $\Sigma^0$) we can try an enlightened guess.
There is, however, a much better guess possible if we combine the assignment with a selection rule. This selection rule was the starting point of the scheme and was invented first to explain the absence (or weakness) of certain reactions which otherwise would have to be observed.

These selection rules are

\[
|\Delta S| = 0 \quad \text{strong} \\
|\Delta S| = 1 \quad \text{weak} \\
|\Delta S| = 2 \quad \text{negligible}
\]  

(2.22)

We shall study below some of the consequences of these selection rules. But before doing it, let us complete the assignment of the various quantum numbers with the following table:

<table>
<thead>
<tr>
<th>S</th>
<th>N</th>
<th>I</th>
<th>U=S+N</th>
<th>Q</th>
<th>Particle</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>not known</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>±1,0</td>
<td>pion</td>
<td>0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>(\frac{1}{2})</td>
<td>1</td>
<td>0,1</td>
<td>nucleons</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>-1</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>-1</td>
<td>-1,0</td>
<td>(K^-, \overline{K}^0)</td>
<td>0</td>
</tr>
<tr>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>±1,0</td>
<td>(\Sigma^-, \Sigma^0)</td>
<td>(\frac{1}{2})</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
<td>(\frac{1}{2})</td>
<td>-1</td>
<td>-1,0</td>
<td>(\Xi^-, \Xi^0(?))</td>
<td>(\frac{1}{2})</td>
</tr>
</tbody>
</table>

**TABLE**

Assignment of I-spin and strangeness to the fundamental particles
The table must be completed for the antiparticles by adding the rule: strangeness of antiparticles = minus strangeness of particles.

This rule fits well into the already known rule that the pion is its own antiparticle (\( \pi^- = \pi^+ \)). On the other hand, it leads to the conclusion that there should exist two neutral K-mesons, viz.

\[
K^0 \quad \text{with } S = +1, \; N = 0, \; I = \frac{1}{2}, \; \text{and} \\
\overline{K}^0 \quad \text{with } S = -1, \; N = 0, \; I = \frac{1}{2}
\]

This is one of the predictions of the scheme which has been strikingly confirmed by experiments.

\(\text{(f) Consequences of Selection Rules.}\)

\(\text{(i)} \quad \text{Associated production and slow decay.}\)

Consider for example the reaction

\[
\overline{\pi}^- + p \rightarrow \Lambda^0 + K^0 \\
S = 0 \quad 0 \quad -1 \quad +1 \quad \Delta S = 0
\]

Thus this process is fast, while the decay

\[
\Lambda^0 \rightarrow p + \pi^- \\
S = -1 \quad 0 \quad 0 \quad \Delta S = 1
\]

must be slow. And this is just what is observed!
(B) Exclusion of unobserved processes.

The reaction

\[ n + n \rightarrow \Lambda^0 + \Lambda^0 \quad (|\Delta S| = 2) \]

is excluded but

\[ n + n \rightarrow \Lambda^0 + n + K^0 \quad (|\Delta S| = 0) \]

should occur. Similarly

\[ \pi^- + p \rightarrow \Sigma^+ + K^- \quad (|\Delta S| = 2) \]

is not observed but

\[ \pi^- + p \rightarrow \Sigma^- + K^+ \quad (|\Delta S| = 0) \]

occurs easily.

(X) Prediction of new particles.

(i) \( \Sigma^- \) was predicted as part of a triplet and was later discovered;

(ii) A \( \Lambda^- \) with \( S = N = I = 0 \) (first line in table) would fit very well into the scheme. It is not yet discovered.

(iii) \( \Xi^- \) would be part of a I-spin doublet. It is not yet discovered.

(C) The \( K^0 \)-doublet.

The occurrence of two neutral \( K \)-mesons with different decay scheme and lifetimes confirms this prediction. The analysis of this process will be discussed later.
(3) Cross-Sections and Lifetimes.

(a) Motivation. In Quantum Mechanics we describe the state of a system by a Schrödinger wave function $\Psi(t)$ which satisfies a Schrödinger wave equation

$$i \frac{\partial \Psi(t)}{\partial t} = H \Psi(t) \tag{3.1}$$

Here $H$ denotes the total Hamiltonian of the system. In principle if we knew the Hamiltonian and if we knew how to solve the Eq. (3.1) for certain initial conditions we could derive all observable results from it. Neither of these conditions is met and this is why the theory of fundamental particles is fragmentary in character.

In spite of this fragmentary character it is possible to derive certain results which are very useful in the study of fundamental processes and which lead to important clues. These results are obtained if we express the observable quantities in terms of the scattering operator and the clues to which they lead are the classification of the interactions into strong and weak interactions and various selection rules and branching ratios.

In order to express these relations we proceed now to express the observable quantities in terms of the scattering matrix.

(b) Definition of the Scattering Operator. Let us consider an example first, for instance the reaction

$$\overline{A} + N \rightarrow \Lambda + K$$

before \hspace{1cm} after

$t \rightarrow -\infty \hspace{1cm} t \rightarrow +\infty$
This process would be described by a wave function \( \psi(t) \) which in the remote past \( (t \to -\infty) \) would behave like two free particles \( \pi \) and \( N \), and in the distant future \( (t \to +\infty) \) it would behave like two free particles again, namely a \( \Lambda \) and a \( K \).

Let us denote by \( \psi_-(t) \) the free motion of a \( \pi \) and a \( N \) and by \( \psi_+(t) \) the free motion of a \( \Lambda \) and a \( K \). We also define \( \psi_+ = \psi_+(0) \), \( \psi_- = \psi_-(0) \). The connection between the two state vectors \( \psi_+ \) and \( \psi_- \) is then a linear transformation

\[
\psi_+ = S \psi_-
\]

(3.2)

One can prove that, whatever the total Hamiltonian for the whole system, provided the above-mentioned limiting property is true, the operator \( S \) must be a unitary operator, that is it satisfies

\[
S^*S = SS^* = I
\]

(3.3)

The operator \( S \) is called the scattering operator. The scattering matrix is the system of matrix elements

\[
(\beta | S | \alpha) = (\beta, S \alpha)
\]

(3.4)

The important property of the scattering matrix is the fact that the general transition probability for the transition from an "initial state" \( \alpha \) into a final state \( \beta \) is determined by the square of the matrix element

\[
P_{\beta \alpha} = \left| (\beta | S | \alpha) \right|^2
\]

(3.5)
A little care is needed in relating this expression to observable quantities since we are dealing in general with transitions from one state in a continuum of states to another state again in a continuum. We shall give the relations of the observable quantities to the matrix elements of $S$ in the following subsection:

(c) Lifetimes and Cross-Sections. We write for the general $S$-matrix

$$S = I + R \quad (3.6)$$

and express the general matrix element of $R$ in a representation in which the momenta of the incident and final states are diagonal. Let

\[ P_1', P_2', \ldots \text{ the momentum four vectors of the initial state } ^i \]
\[ P_1', P_2', \ldots \quad ^i \quad ^i \quad ^i \quad ^i \quad ^f \text{ final } \quad ^f \]

and

\[ P_i = P_1 + P_2 + \ldots \quad (3.7) \]
\[ P_f = P_1' + P_2' + \ldots \]

The general matrix element of $R$ has then the form

$$\langle f | R | i \rangle = \delta (P_f - P_i) \langle f | M | i \rangle \quad (3.8)$$
In (3. ) the occurrence of the (four-dimensional) 
\( \mathcal{J} \) -function expresses the conservation of momentum and energy. The matrix element \( \langle f | M | i \rangle \) is then in general free from \( \mathcal{J} \) -functions. It will be called the reduced matrix element.

Let \( n \) be the number of initial particles. There are two cases to consider:

\( n = 1 \) Decay of an unstable particle. The experimental quantity is the transition rate \( \Gamma = \frac{1}{\tau} \). \( \Gamma \) is the lifetime. One can derive the following relation

\[
\Gamma = \frac{1}{2\pi} \sum_s \sum_i \mathcal{J} (P_f - P_i) \left| \langle f | M | i \rangle \right|^2
\]  

\((GR_1)\)

\( n = 2 \) Collision of two particles. The experimentally observed quantity is the cross-section

\[
\sigma = (2\pi)^2 \sum_s \sum_i \mathcal{J} (P_f - P_i) \left| \langle f | M | i \rangle \right|^2
\]  

\((GR_2)\)

The symbols \( \sum_s \sum_i \) occurring in these formulae are the summation over the final and possible averaging over the initial states. These summations are to be interpreted as integrations if the continuous momentum variables are involved and summation over the discrete spin variables if any. The quantity \( I \) is the "total incident current". In the units used here it is simply the sum of the "dimensionless" velocities of the two incident particles. Only two cases are of practical importance, viz.

\*) c.f. J.M. Jauch and F. Rohrlich, Theory of Photons and Electrons, Section 8-6, p.163.
(i) Centre of mass system:

\[ I = q \left( \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} \right) \quad \varepsilon_{1,2} = \sqrt{\frac{m_1^2}{\varepsilon_1} + \frac{m_2^2}{\varepsilon_2} + q^2} \]

\( q \) = total momentum of particle 1 or 2

(ii) Laboratory system:

\[ I = \frac{p}{\varepsilon} \quad \varepsilon = \sqrt{m^2 + p^2} \]

\( p \) = momentum of incident particle.

The logical situation which shows the connection between Hamiltonian, Scattering operator and observable quantities can be summarized by the diagram

Note: \( GR_{1,2} \) denotes "golden rule" number 1 and 2, a terminology adapted from Fermi.