THE ALGEBRA OF CURRENT COMPONENTS AND COLLINEAR
PHOTOPRODUCTION OF MESONS

K. Dietz and W. Drechsler *)

CERN - Geneva

*) On leave from the Institut für Hochenergiephysik, Universität, Heidelberg.

65/1134/5
19 July 1965
I. INTRODUCTION

In two recent papers, Dashen and Gell-Mann \(^1\),\(^2\) studied the algebra of current components generated by the space integrals of the 144 Hermitian current densities \((\Psi^\dagger_q(x) \lambda_{i=1}^{16} \Psi_q(x))\); \(\lambda_k^\dagger = \lambda_k\), \(k = 1, \ldots, 16\) are the usual linear independent Dirac matrices, \(\lambda_i\), \(i = 0, \ldots, 8\) are the generators of \(U(3)\). The authors observed that the following subset of current components: \(\int d^3x (\Psi^\dagger \lambda_i \psi)\), \(\int d^3x (\Psi^\dagger \lambda_i \sigma_z \psi)\), \(\int d^3x (\Psi^\dagger \lambda_i \gamma_0 \vec{\sigma} \cdot \vec{\psi})\) and \(\int d^3x (\Psi^\dagger \lambda_i \gamma_0 \vec{\sigma} \cdot \vec{\psi})\) have the same matrix elements for particles at rest as for particles moving along the \(z\) direction. It was shown that the above 36 current components generate under equal time commutation a "hybrid" \(U(6)\) algebra, and that the particles belonging to a specified representation at rest continue to do so when they are moving along the \(z\) axis. This suggests an approximate "hybrid" \(U(6)\) symmetry for collinear processes. The same group was postulated for one-dimensional processes by Lipkin and Meshkov \(^3\) and called \(U(6)_W\). The generators of \(U(6)_W\) are determined by the \(W\) spin operators \(\vec{W}_z = \vec{\sigma} \cdot \vec{z}\), \(\vec{W}_x = \vec{\sigma} \cdot \vec{x}\), \(\vec{W}_y = \vec{\sigma} \cdot \vec{y}\) and the generators of \(U(3)\) and are identical with the operators sandwiched between the quark fields \(\Psi\) in the above-mentioned subset of current components.

In this paper we would like to study the consequences of \(U(6)_W\) invariance for collinear photoproduction of \(0^-\) and \(1^-\) mesons on nucleons. A similar investigation has been made by Johnson and Treiman for elastic scattering of pseudoscalar mesons on nucleons. However, as discussed by Harari \(^6\) the comparison of the Johnson-Treiman relations with experimental data on total elastic cross-sections at high energies provides no test of \(SU(3)\) and therefore also no test of the \(U(6)_W\) invariance. The reason is that high energy scattering is dominated by small momentum transfers where symmetry breaking effects are still expected to be important. The relations we obtain for the various photoproduction cross-sections are valid for both forward and backward direction. In the latter case sufficiently high momentum transfers may be obtained so that symmetry breaking effects no longer might obscure a significant check of \(SU(3)\) and \(U(6)_W\) symmetries. Experiments especially designed for detecting backward photoproduction have been performed in special cases \(^7\).
II. COUPLINGS AND W SPIN HELICITY AMPLITUDES

The following $U(6)_W$ invariant coupling between the incoming particles (photon $A$ and baryon $\psi$) and the outgoing particles (meson $\overline{\phi}$ and baryon $\overline{\psi}$) has been used:

$$
\gamma^\mu_{AEF} = \tilde{a}_1 \bar{\psi}^{ABC} \psi_{ABCD} \tilde{\phi}_E^C A_E^F + \tilde{a}_2 \bar{\psi}^{ABC} \psi_{ABD} \tilde{\phi}_E^C A_E^D + \tilde{a}_3 \bar{\psi}^{ABC} \psi_{ABD} \tilde{\phi}_E^B A_C^E + \tilde{a}_4 \bar{\psi}^{ABC} \psi_{AEF} \tilde{\phi}_B^E A_C^F
$$

(1)

The baryons belong to the representation $\bar{36}$ of $U(6)_W$, the octet part of which is given by the well-known expression:

$$
\psi_{ABC} = \frac{1}{3\sqrt{2}} \left[ \varepsilon_{\alpha\beta\gamma} \varepsilon_{\alpha\beta\gamma} N_{ABC}^\gamma + \ldots \right]
$$

(2)

The 36 mesons belonging to the adjoined representation $(35,1)$ of $U(6)_W$ fall into two groups: a $W$ spin triplet ($V_0^+, P_0^-, V_0^-$) with $X^0(959)$ included among the $P_0^-$, and a $W$ spin singlet $V_0^0$. $P$ and $V$ stand for 0⁻ and 1⁻ nonets, the subscripts 0,± refer to the $W$ components. Hence we represent the mesons by the tensor:

$$
\phi_{B}^{\alpha} = \phi_{j,\beta}^{\alpha} = \left[ \varepsilon_{\alpha\beta} P_0^0 + \frac{1}{2} V_0^0 + \sigma^+ V_+^0 + \sigma^- V_-^0 \right]_{\alpha}^{\beta} (1)
$$

(3)

The photon is regarded as a $U$ spin scalar with $W$ spin 1 and $W_z = \pm 1$:

*) In this work we disregard dectuplet production which will be considered separately.

65/1134/5
\[ A^b_c = A^b_{k\bar{g}} = \sum \theta^{\bar{g}} \lambda^{\alpha} \lambda^{\beta} \]

with \( \lambda = \frac{1}{2}(\lambda_3 + \frac{1}{\sqrt{3}} \lambda_8) \).

The vector \( \vec{e} \) can be identified with the usual transversal polarization vector of the photon. The Latin indices being W spin indices \((1,2)\), and the Greek indices being U(3) indices \((1,2,3)\). Inserting the expressions \((2), (3)\) and \((4)\) and performing the tensor contractions, Eq. \((1)\) leads to the following expression for \( \mathcal{M} \).

\[
\mathcal{M} = \sum \lambda \mathcal{M}_L
\]

\[
\mathcal{M}_L = -2\alpha_1 \vec{e}^\gamma \vec{e} \eta^\dagger \eta \{ (\vec{N}) (\vec{N}) \}
\]

\[
\mathcal{M}_2 = \frac{\alpha_2}{3} \left[ -\vec{e}^\gamma \vec{e} \eta^\dagger \eta \right] \left\{ (\vec{N}) (\vec{N}) \right\}_F + (\vec{M}) (\vec{N}) \}
\]

\[
- i \eta^\dagger \sigma_{\gamma} \vec{e}^\gamma \vec{e} \eta \left\{ (\vec{N}) (\vec{N}) \right\}_F + \frac{1}{3} (\vec{M}) (\vec{N}) \}
\]

\[
- i \eta^\dagger \sigma_{\gamma} \vec{e}^\gamma \eta \left\{ (\vec{N}) (\vec{N}) \right\}_D + \frac{1}{3} (\vec{M}) (\vec{N}) \}
\]

\[
\mathcal{M}_3 = \frac{\alpha_2}{3} \left[ -\vec{e}^\gamma \vec{e} \eta^\dagger \eta \right] \left\{ (\vec{N}) (\vec{N}) \right\}_F + (\vec{M}) (\vec{N}) \}
\]

\[
+ i \eta^\dagger \sigma_{\gamma} \vec{e}^\gamma \vec{e} \eta \left\{ (\vec{N}) (\vec{N}) \right\}_F + \frac{1}{3} (\vec{M}) (\vec{N}) \}
\]

\[
- i \eta^\dagger \sigma_{\gamma} \vec{e} \eta \left\{ (\vec{N}) (\vec{N}) \right\}_D + \frac{1}{3} (\vec{M}) (\vec{N}) \}
\]

\[
\mathcal{M}_4 = \frac{\alpha_4}{3} \left[ \vec{e}^\gamma \vec{e} \eta^\dagger \eta \right] \left\{ 4(\vec{N}) (\vec{N}) (\vec{N}) - 5 (\vec{N}) (\vec{N}) (\vec{N}) - 4(\vec{N}) (\vec{N}) (\vec{N}) + 5 (\vec{N}) (\vec{N}) (\vec{N}) - 4(\vec{N}) (\vec{N}) (\vec{N}) \}
\]

\[ + S(\vec{N}) (\vec{N}) + 5(\vec{N}) (\vec{N}) (\vec{N}) - 4(\vec{N}) (\vec{N}) (\vec{N}) \}
\]
The round brackets designate \( SU(3) \) traces. We used the abbreviations

\[
\begin{align*}
(\bar{N}(\bar{m}\lambda)N)_{D} &= T_{\chi}(\bar{N}\bar{m}\lambda N - \bar{N}N\bar{m}\lambda) \\
(\bar{N}(\bar{m}\lambda)N)_{F} &= T_{\chi}(\bar{N}\bar{m}\lambda N + \bar{N}N\bar{m}\lambda)
\end{align*}
\]

and analogously for \((\bar{N}(\bar{m}\lambda)N)_{D,F}\). \( N \) stands for the usual traceless baryon octet matrix and \( \bar{N} \) for its transpose \( ^{8} \). \( \bar{M} \) is the transposed matrix for a nonet of pseudoscalar or vector mesons depending on the state of polarization of the outgoing meson as described by the \( W \) spin polarization vector \( \vec{\xi}^{V} : \vec{\xi}^{V} = \vec{\xi}^{V}_{\chi} \) goes with \( P_{o} \) production, \( \vec{\xi}^{V} = \vec{\xi}^{V}_{\pm} \) with \( V_{\pm} \) production, and the terms independent of \( \vec{\xi}^{V} \) go with \( V_{0} \) production. The initial and final \( W \) spin or - what is the same thing in this case - usual helicity states of the baryons are described by \( \eta_{i}^{f} \) and \( \eta_{f}^{i} \), respectively. Our phase conventions are those of Jacob and Wick \( ^{9} \).

For the processes \( \chi^{+}p \rightarrow N_{8}^{+}W \) the expression (5) leads to the \( W \) spin helicity amplitudes of Table 1. A mixing angle of \( \tan \phi = 1/\sqrt{2} \) was assumed for \( \omega^{0} \) and \( \phi^{0} \) production, whereas for \( X^{0} \) production a possible mixing with the \( \eta^{0} \) was neglected because of the large mass difference between \( X^{0} \) and \( \eta^{0} \). \( ^{10} \). To convert the \( W \) spin helicity amplitudes of Table 1 into conventional helicity amplitudes, one has to introduce an additional phase factor \( (-1) \) between the two amplitudes for \( P_{o} \) and \( V_{0} \) production, respectively.
III. RESULTS AND DISCUSSION

From Table 1 we obtain two types of relations. Firstly, we rederive those relations which are consequences of SU(3) symmetry alone:

for pseudoscalar meson production:\footnote{11}:

\[ \sqrt{2} \, m_{\pi^+} + \sqrt{3} \, m_{\Lambda^0 K^+} = m_{\Sigma^+ K^+} \]  
\[ \sqrt{2} \, m_{\Sigma^+ K^0} + \sqrt{3} \, m_{\rho^0 \eta^0} = m_{\rho^0 \eta^0} \]  

for vector meson production:

\[ \sqrt{2} \, m_{\pi^+ \eta^0} + \sqrt{3} \, m_{\Lambda^0 K^0} = m_{\Sigma^0 K^0} \]  
\[ \sqrt{2} \, m_{\Sigma^0 K^0} + \sqrt{3} \, m_{\rho^0 \omega^0} = m_{\rho^0 \omega^0} + \sqrt{2} \, m_{\rho^0 \phi^0} \]  

Since we neglected $X^0 - \eta^0$ mixing, obviously, Eqs. (7) and (9) are different.

\textit{U}(6)$_W$ invariance leads to the following additional relations for the $0^-$ meson production amplitudes:

\[ m_{\Sigma^0 K^0} = -\frac{\sqrt{2}}{\gamma} m_{\rho^0 \omega^0} \]  
\[ m_{\rho^0 \eta^0} = \frac{3\sqrt{3}}{\gamma} m_{\rho^0 \eta^0} \]  
\[ m_{\rho^0 \phi^0} = \sqrt{2} \, m_{\rho^0 \eta^0} \]  

65/1134/5
Because \( U(6)_W \) connects spin and unitary spin the relations among the amplitudes for vector meson production depend on the helicities \((\lambda_H', \lambda_Y; \lambda_H, \lambda_Y)\) involved:

For \((-\frac{1}{2}, 0; \frac{1}{2}, 1)\) and \((\frac{1}{2}, 0; -\frac{1}{2}, -1)\):

\[
6 \, \mathcal{M}_{n+8'} + 5\sqrt{6} \, \mathcal{M}_{n' + \kappa'^*} = 7 \, \mathcal{M}_{\Sigma^0 + \kappa^*} + \sqrt{2} \, \mathcal{M}_{\rho + \pi^0} \tag{13}
\]

\[
\gamma_{\rho} f_+ f_0 = 0 \tag{14}
\]

For \((\frac{1}{2}, 1; \frac{3}{2}, 1)\) and \((-\frac{1}{2}, -1; -\frac{3}{2}, -1)\):

\[
6 \, \mathcal{M}_{n+8'} + 2\sqrt{6} \, \mathcal{M}_{n' + \kappa'^*} = \gamma_{\rho} f_+ f_0 + 2 \, \mathcal{M}_{\rho + \pi^0} + 3 \, \mathcal{M}_{\rho + \phi^0} \tag{15}
\]

For \((-\frac{1}{2}, 1; -\frac{1}{2}, 1)\) and \((\frac{1}{2}, -1; \frac{1}{2}, -1)\):

\[
3 \, \mathcal{M}_{n+8'} + 3\sqrt{6} \, \mathcal{M}_{n' + \kappa'^*} = - \frac{2}{7} \, \gamma_{\rho} f_+ f_0 + 2 \, \mathcal{M}_{\rho + \pi^0} + \frac{3}{7} \, \mathcal{M}_{\rho + \phi^0} \tag{16}
\]

The vanishing of the forward and backward production amplitude for longitudinally polarized \( \phi^0 \) provides a (experimental presumably difficult) test of \( \omega^0 - \phi^0 \) mixing in the \( U(6)_W \) model.

For the forward or backward production cross-section

\[
\sigma_{\chi + p \rightarrow N + \Lambda^0} = \frac{3}{k} \, \sigma_{N + \Lambda^0}
\]

we obtain the following relations:

\[
\sigma_{\chi' + p \rightarrow \Lambda^0} = \frac{2}{4^2} \, \sigma_{p + \Lambda^0} \tag{17}
\]

\[
\sigma_{\rho + \pi^0} = \frac{2}{4^2} \, \sigma_{p + \pi^0} \tag{18}
\]
\[
\bar{\sigma}_{p+r} = 2 \bar{\sigma}_{p+r} \eta^3
\]

\[
\frac{1}{3} \bar{\sigma}_{n+\mu^+} - \frac{1}{3} \bar{\sigma}_{A+\mu^+} \leq \bar{\sigma}_{\Sigma^+K^+} \leq 2 \bar{\sigma}_{\gamma^+K^+} + 3 \bar{\sigma}_{\Lambda^+K^+} + 2 \sqrt{\bar{\sigma}_{\eta^+n^+} \bar{\sigma}_{\Lambda^+K^+}}
\]

\[
\bar{\sigma}_{\eta^+n^+} + 0.25 \bar{\sigma}_{\gamma^+g} + 3.3 \bar{\sigma}_{\Sigma^+K^0} + 3 \bar{\sigma}_{\Sigma^+K^*} = 3.0 \bar{\sigma}_{\Sigma^+K^\mu^+} + 0.8 \bar{\sigma}_{\Lambda^+K^\mu^+} + 0.5 \eta^+K^+
\]

\[
\bar{\sigma}_{\eta^+n^+} + 0.12 \bar{\sigma}_{\gamma^+g} + 1.5 \bar{\sigma}_{\Sigma^+K^0} + 0.5 \bar{\sigma}_{\Sigma^+K^*} = 4.4 \bar{\sigma}_{\Sigma^+K^0} + 0.5 \bar{\sigma}_{\Lambda^+K^*} + 1,0 \bar{\sigma}_{\Lambda^+K^0}
\]

We have taken out a phase space factor \( q/k \) (\( k \) and \( q \) being the momenta in the initial and final states, respectively) to give a simple prescription to account for the mass differences. The right inequality of (20) follows from SU(3) symmetry alone. Whereas all other relations are consequences of U(6)\(_W\) invariance.

Next, we want to discuss one-particle exchange in the framework of U(6)\(_W\) symmetry. Slightly more general than the OME model, we consider in the t channel the exchange of two trajectories: one having parity \((-1)^J\) for each exchanged total angular momentum \( J \), the other having parity \((-1)^{J+1}\). For the production of transversally polarized vector mesons \( V_\perp^\pm \), only one trajectory contributes to each spin invariant in Eq. (5):

\[
P = (-\eta)^3 \quad \text{contributes only to} \quad \bar{\xi} V^* \xi
\]

\[
P = (-\eta)^{J+1} \quad \text{contributes only to} \quad \bar{\xi} V^* \times \xi
\]

If only one trajectory would contribute to a certain production process, the contribution of the other invariant has to vanish. This gives an additional equation between the functions \( a_1 \) to \( a_4 \). Especially, if five of the processes are determined by the exchange of one trajectory,
the only possible solution is $a_1 = a_2 = a_3 = a_4 = 0$, which means that all $0^-$ and $1^-$ photoproduction cross-sections should vanish. Therefore, we see that a combination of $U(6)_W$ invariance with a OME model gives in general further, presumably too many, restrictions. This statement is not confined to photoproduction. Experimental measurements of decay distributions of the produced resonances $^{12)}$ would help to clarify this point.

IV. CONCLUSION

Assuming $U(6)_W$ invariance as proposed by Lipkin and Meshkov and by Dashen and Gell-Mann, the relations (17) - (22) for forward and backward photoproduction cross-sections are derived. Especially in the backward direction these relations provide a test of the "hybrid" $U(6)$ subalgebra of current components. The consistency of this symmetry with one-meson exchange is discussed and considerable restrictions are found.

ACKNOWLEDGEMENTS

One of us (W.D.) would like to thank Professor L. Van Hove for the hospitality of the Theoretical Study Division during his stay at CERN.
REFERENCES

8) A collection of formulae can be found in L. Van Hove, "SU₃ and SU₅ Symmetry of Strong Interactions", Lectures at the CERN Easter School in Bed Kruschach, April 1965, CERN preprint TH.540, where further references are given.
10) R.H. Dalitz and D.G. Sutherland, Oxford preprint 1965, find a small $\nu^0 - X^0$ mixing angle $3\sigma \theta = 0.19$.
\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
\textbf{\( \gamma + p \)} & \( m_{N \rightarrow N \pi} (-\frac{1}{2}, 0; \frac{1}{2}, 1) \) & \( m_{N \rightarrow N \pi} (\frac{1}{2}, 0; -\frac{1}{2}, -1) \) & \( m_{N \rightarrow N \pi} (\frac{1}{2}, 1; \frac{1}{2}, 1) \) & \( m_{N \rightarrow N \pi} (-\frac{1}{2}, 1; -\frac{1}{2}, 1) \) \\
\hline
\textbf{\( n + \pi^+ \)} & \( \sqrt{2} \left( 10 a'_2 + 5 a'_3 + \frac{9}{2} a'_4 \right) \) & \( \sqrt{2} \left( 10 a'_2 - 5 a'_3 - \frac{1}{2} a'_4 \right) \) & \(-16 a'_2 - 2 a'_3 - 5 a'_4 \) & \( 4 a'_2 + 8 a'_3 + 4 a'_4 \) \\
\hline
\textbf{\( \Sigma^0 + K^+ \)} & \( 2 a'_2 + a'_3 + \frac{9}{2} a'_4 \) & \( 2 a'_2 - a'_3 - \frac{11}{2} a'_4 \) & \( \sqrt{2} \left( 2 a'_2 - 2 a'_3 - \frac{1}{2} a'_4 \right) \) & \( \sqrt{2} \left( 4 a'_2 - a'_3 + 4 a'_4 \right) \) \\
\textbf{\( \Lambda + K^+ \)} & \( \sqrt{3} \left( -6 a'_2 - 3 a'_3 - \frac{3}{2} a'_4 \right) \) & \( \Lambda + K^{**} \) & \( \sqrt{3} \left( -6 a'_2 + 3 a'_3 - \frac{3}{2} a'_4 \right) \) & \( 6 \sqrt{6} a'_2 + \frac{3}{2} \sqrt{6} a'_4 \) \\
\hline
\textbf{\( \Sigma^+ + K^0 \)} & \(-\sqrt{2} a'_2 + \sqrt{2} a'_3 \) & \( \Sigma^+ + K^{**} \) & \( \sqrt{2} \left( -a'_2 - a'_3 - a'_4 \right) \) & \(-2 a'_2 - 4 a'_3 - 4 a'_4 \) \\
\hline
\textbf{\( p + \pi^0 \)} & \( 7 a'_2 - 7 a'_3 \) & \( p + \rho^0 \) & \( 7 a'_2 + 7 a'_3 + a'_4 \) & \(-\sqrt{2} \left( 27 a'_1 + 11 a'_2 + 4 a'_3 + 4 a'_4 \right) \) \\
\hline
\textbf{\( p + \eta^0 \)} & \( 3 \sqrt{3} a'_2 - 3 \sqrt{3} a'_3 \) & \( p + \omega^0 \) & \( 9 a'_2 + 9 a'_3 + 9 a'_4 \) & \(-9 \sqrt{2} a'_1 - 9 \sqrt{2} a'_2 \) \\
\hline
\textbf{\( p + \rho^0 \)} & \( 3 \sqrt{6} a'_2 - 3 \sqrt{6} a'_3 \) & \( p + \phi^0 \) & \( 0 \) & \( 18 a'_1 \) \\
\hline
\end{tabular}
\caption{W-spin Helicity Amplitudes \( m_{N \rightarrow N \pi} (\lambda'_N, \lambda_M; \lambda_N, \lambda_Y) \)}
\end{table}