operating at Orsay\(^1\)\); single bremsstrahlung events, \(e^+ e^- \rightarrow e^+ e^- \gamma\), of low momentum transfer were detected with a counting rate of a few counts per minute. While several hundred large angle \(e^- e^-\) scattering events at a total energy \(E^* = 2 \times 300\) MeV were reported in 1965 by the Princeton-Stanford group\(^2\), a few \(e^+ e^-\) scattering events and annihilations into \(\pi^+ \pi^-\) pairs at \(E^* = 2 \times 380\) MeV were observed the following year at Novosibirsk \(^3\) with VEPP2. Two years later, several thousand \(e^+ e^-\) large angle scattering events and annihilations into either pion pairs, kaon pairs or \(\pi^+ \pi^- \pi^0\) states were measured at Orsay\(^4\text{-}^6\) with ACO.

These steps could be made because of the following achievements: positron beams of higher intensity for injection, higher vacuum giving longer beam storing time, improved beam control and the change from weak focusing magnetic rings to strong focusing ones with separate functions. It must be pointed out that a large fraction of the time of operation of all existing rings has been devoted to machine studies which were being done alternately with high energy physics experiments. Important results have been obtained especially on beam instabilities and on the limitations of beam-beam interaction rates; one may thus be confident in the prediction that the "luminosities" which will be achieved in the rings presently in the process of being designed will be much higher than those of the first generation rings.

2. MAIN CHARACTERISTICS OF ELECTRON-POSITRON COLLIDING BEAM RINGS

2.1 Magnetic guide field and beam structure

High energy electrons and positrons produced in a linear or circular accelerator can be stored for many hours in a more or less circular vacuum chamber, the trajectories of the particles being closed by a series of bending magnets. The orbits of the stored particles are confined within a volume of small transverse dimensions by magnetic quadrupoles which focus the two beams alternately in the horizontal plane and the vertical plane. As an example, Figure 1 shows the lay out of the magnetic elements of AONE\(^7\), a 1.5 GeV \(e^+ e^-\) ring built at Frascati. The mean radius of this machine is 16 meters.
Fig. 1 General lay-out of the Frascati $e^+ - e^-$ colliding beam ring ADONE.
The number of electrons and positrons stored is of the order of $10^{10} \div 10^{11}$ in the rings presently in operation; such values correspond to circulating currents $I_z$ of $10 \div 50$ mA. The area $S$ of a transverse cross section of the beams at an interaction point is typically a few mm$^2$.

Since the electrons have their trajectories deflected in the bending magnets, they emit photons; this radiation, called "synchrotron radiation", plays an essential role in the structure of the beams. The energy losses which it causes must be compensated by an acceleration provided by one or several radio-frequency cavities. It follows that the stored particles are longitudinally bunched by the periodic accelerating field. In the existing rings, the number of bunches is less than or equal to 3; the length of each bunch is of the order of $10 \div 50$ cm.

2.2 Injection, particle spill out and beam lifetime

Particles being injected in the ring at a certain distance from the ideal closed orbit, the injection process generates oscillations of a large amplitude; nevertheless these oscillations are progressively damped by the synchrotron radiation. Such damping does not conserve phase space and so permits multiple injection into the same orbit. One can thus build up high circulating currents from relatively weak sources of positrons. Typical values of injection repetition rates and injection times are $0.5 \div 50$ Hz and $1 \div 30$ minutes respectively.

Losses of stored particles are due to their collisions either with other stored particles which belong to the same bunch ("Touschek effect"), or with stored particles of the opposite beam (single bremsstrahlung: $e^+e^-\rightarrow e^+e^-\gamma$), or with the residual gas of the vacuum chamber (elastic or inelastic scattering). In order to achieve beam lifetimes of the order of 10 hours, ultra-high vacuum must be maintained in the chamber despite the high power of synchrotron radiation which hits the walls.

It is important to note that when the total intensity of stored particles is 50 mA with a lifetime of 10 hours, the average number of lost particles per second and per meter along the orbit is $3 \times 10^4$. This spill out of high energy particles turns out to be in general an important source of background since it can generate large showers which trigger the

\textit{*)} Except in the BY-PASS where $\sim$ 150 bunches are filled.
experimental set up used to detect beam-beam collisions.

2.3 Beam energy

The mean energy $E$ of the beams is entirely determined by the magnetic field $B$ in the bending magnets of the ring. By varying $B$, one changes $E$ at will between two limits which usually correspond to a drop of the interaction rate or, at high energies, to the saturation of the magnetic elements.

The RMS $\sigma_E$ of the energies of the stored particles does not critically depend upon the magnetic structure of the ring and is approximately equal to $E\gamma (x_0/2p)^{1/2}$ where $\gamma = E/m_0c^2$, $x_0 = R/m_0c$ and $\rho$ is the magnets' radius of curvature. This energy spread is therefore very small; for example, in the Orsay storage ring ACO, one has $\sigma_E/E \leq 10^{-4}$. This is a valuable characteristic common to all $e^+ - e^-$ storage rings.

Absolute energy calibration of a ring with a precision better than 0.5% is difficult to achieve with magnetic measurements. Nevertheless, by such measurements one can tune a ring so that its energy is reproduced to a few parts in $10^4$ over periods of several weeks, so that one can take full advantage of the very good energy definition of the stored beams.

2.4 Luminosity and its limitations

The rate $\Lambda$ of collisions $e^+e^- \rightarrow f$ per beam crossing region is given by:

$$\Lambda = \sigma f_0 B \int \rho_+ \rho_- dS = \mathcal{L} \sigma \quad \text{(events/s),} \quad (1)$$

where $\sigma$ is the cross section of the events considered, $f_0$ the frequency of revolution of the particles in the ring, $B$ the number of bunches per beam, $\rho_+$ and $\rho_-$ the transverse bunch densities; the quantity $\mathcal{L}$ measured in cm$^{-2}$ s$^{-1}$ is called the "Luminosity" of the ring: it includes all the other parameters just mentionned and is the prime parameter of a ring since it characterizes the collision rate between the 2 beams.

The variation of $\mathcal{L}$ as a function of the energy $E$ may be sketched very roughly in the following way $^\ast$). In storage rings whose energies are limited to about 1.5 GeV (such as VEPP2, ACO and ADONE) the peak

$^\ast$) For more details, the reader may consult the lectures given by Sands$^3$) on "Storage Ring Luminosity".
luminosity occurs at the maximum energy at which the ring can be operated. In higher energy rings, the peak luminosity is reached at an energy which lies somewhere between 1.5 GeV and 2.5 GeV. Below the energy \( E_T \) corresponding to \( \mathcal{L}_{\text{peak}} \), \( \mathcal{L} \) is limited by the space charge effect of one beam acting on the other one, the so-called "Amman-Ritson" effect \(^{10}\). This effect sets a limit on the transverse density \( \rho \) of each bunch which is proportional to the energy \( E \) of the beams. Since the natural transverse area \( S \) of the beams varies like \( E^2 \), it follows directly from Eq. (1) that

\[
\mathcal{L} = \left( \frac{E}{E_T} \right)^4 \mathcal{L}_{\text{peak}} \quad (E \leq E_T) \tag{2}
\]

when the beams collide head on. This \( E^4 \) dependence is the one observed in ACO. If one makes the beam cross at a variable angle and if one operates the ring at constant currents, then one expects:

\[
\mathcal{L} = \left( \frac{E}{E_T} \right) \mathcal{L}_{\text{peak}} \quad (E \leq E_T). \tag{3}
\]

The latter energy dependence is of course more favorable than Eq. (2) but it has not been achieved yet in \( e^+ - e^- \) rings.

At energies larger than \( E_T \), the power \( P \) of the synchrotron radiation becomes so high that the limitation on \( \mathcal{L} \) comes from the maximum current which can be accommodated by the RF cavity (or cavities). Since \( P \propto E^4 \), one has \( I_4 \propto E^{-4} \). Just above \( E_T \), there is a domain of energy which extends up to some energy \( E_c \) in which one may slow down the drop in luminosity due to the decrease in stored intensities by reducing \( B \) and/or the coupling between vertical and horizontal oscillations of the particles. In this energy range, \( \mathcal{L} \) varies in the following way:

\[
\mathcal{L} = \left( \frac{E_T}{E} \right)^3 \mathcal{L}_{\text{peak}} \quad (E_T \leq E \leq E_c). \tag{4}
\]

Above \( E_c \), \( \mathcal{L} \propto E^{-10} \) and therefore drops very rapidly.

2.5 Data on existing rings and rings being designed or proposed

Table 1 gives some of the main characteristics of the rings which are operated presently and of those being designed or proposed for construction.
<table>
<thead>
<tr>
<th>Name</th>
<th>Location</th>
<th>Energy (GeV)</th>
<th>Mean radius $\sqrt{S/\sigma}$ (meters)</th>
<th>Focusing (type)</th>
<th>Maximum number of bunches</th>
<th>Number of interaction regions free for experiments</th>
<th>Luminosity ($cm^{-2} s^{-1}$)</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>AdA</td>
<td>Frascati (Italy)</td>
<td>.25</td>
<td>0.64</td>
<td>Weak</td>
<td>2</td>
<td>1</td>
<td>$2.10^{25}$</td>
<td>Shut down in 1965</td>
</tr>
<tr>
<td>VEPP2</td>
<td>Novosibirsk (USSR)</td>
<td>.7</td>
<td>1.6</td>
<td>Weak</td>
<td>3</td>
<td>1</td>
<td>$3.10^{28}$ at $E=0.5 \text{ GeV}$</td>
<td>First high energy physics experiments started in 1966</td>
</tr>
<tr>
<td>ACO</td>
<td>Orsay (France)</td>
<td>.55</td>
<td>3.4</td>
<td>AG separate functions</td>
<td>2</td>
<td>1</td>
<td>$3.10^{28}$ at $E=0.5 \text{ GeV}$</td>
<td>First high energy physics experiments started in 1967</td>
</tr>
<tr>
<td>AONE</td>
<td>Frascati (Italy)</td>
<td>1.5</td>
<td>16</td>
<td>AG separate functions</td>
<td>3</td>
<td>4</td>
<td>$10^{29}$ achieved at 1 GeV with head-on collisions, 2 $10^{29}$ design value at 1.1 GeV with beams crossing at an angle.</td>
<td>Final tests on two beam operation. Crossing at an angle still to be achieved,</td>
</tr>
<tr>
<td>NY-PASS</td>
<td>Cambridge (USA)</td>
<td>3.5 to 7</td>
<td>27</td>
<td>AG one low $\pi$ insertion</td>
<td>300</td>
<td>1</td>
<td>Design value $2 \times 10^{31}$ at 3 GeV</td>
<td>Multicycle injection achieved. First tests on $e^+ e^-$ injection by the end of 1969.</td>
</tr>
<tr>
<td>VEPP3</td>
<td>Novosibirsk (USSR)</td>
<td>3.5</td>
<td>12</td>
<td>AG</td>
<td>19</td>
<td>2</td>
<td>Design value : $10^{30}$</td>
<td>First $e^+ e^-$ injection in 1969</td>
</tr>
<tr>
<td>DORIS</td>
<td>Hamburg (Germany)</td>
<td>3 to 4.5</td>
<td>46</td>
<td>AG separate functions, 2 low $\pi$ insertions</td>
<td>480</td>
<td>2</td>
<td>Design values : $10^{32}$ in the first phase, $10^{33}$ in later phases.</td>
<td>Final design parameters are presently chosen. First injection: 1974.</td>
</tr>
<tr>
<td>SPEAR</td>
<td>Stanford (USA)</td>
<td>2 to 3</td>
<td>34</td>
<td>AG separate functions, low $\pi$ insertions</td>
<td>36</td>
<td>2</td>
<td>$10^{32}$ in the first phase, $10^{33}$ in later phases.</td>
<td>Proposal</td>
</tr>
<tr>
<td>COPPELIA</td>
<td>Orsay (France)</td>
<td>2 to 3</td>
<td>46</td>
<td>AG separate functions, space charge compensated in the interaction region</td>
<td>20</td>
<td>2</td>
<td>$10^{32}$ in the first phase, $10^{33}$ in later phases.</td>
<td>Proposal</td>
</tr>
</tbody>
</table>
3. DETECTION

3.1 Generalities

A detector to be used with an e^+e^- colliding beam ring must satisfy various requirements which put severe restrictions on its structure.

i) Electron-positron reaction products have in general a broad angular distribution over the total 4π steradians of solid angle (see Fig. 2). Since the counting rates of many of the interesting events are usually low (see Table 2), the detector must sustain as large a solid angle as possible. The same condition follows from the fact that many of the particles produced are unstable; a good efficiency of detection of the decay products also requires a large solid angle detector.

ii) Collisions occur at random times (except for the effect of the bunching of the stored particles); this implies that the detector must be triggerable.

iii) Spill out of stored particles is in general an intense source of background, especially since condition i) above implies that the detector be close to the vacuum chamber and surround most or all of it. The most efficient way of rejecting this background is based on the fact that true e^+ e^- collisions must originate from the very well defined beam-beam interaction region whose transverse section has an area of only a few mm^2; furthermore this interaction region is also limited in length. The probability that a lost particle generate tracks which cross within this small volume is very small. To take full advantage of this selection criterium, one has to be able to make a precise geometric reconstruction of the origin of the events detected.

iv) Several channels are open at the same time at moderate energies (E < 1 GeV), plenty of them are open at higher energies. Since the counting rates of many reactions are expected to be low, it is desirable that the same detector record several types of events and provide means of discriminating among them. At total C of M energies below 2 M_p, the particles to be detected are: γ, e^+, μ^+, π^+, K^+, η or their decay products. The identification of these particles can be based on electric charge detection, shower detection, range measurement and on the evidence of nuclear interactions. Kinematic constraints are also useful in
Fig. 2 Angular distributions of some electron-positron reactions. (This figure is taken from the SLAC proposal for a high energy \( e^+e^- \) colliding-beam storage ring, Stanford, 1965).
### TABLE 2
Predicted rates of some e⁺e⁻ reactions with an assumed luminosity of $10^{33}$ cm⁻² s⁻¹.

<table>
<thead>
<tr>
<th>Beam energy $E$ (GeV)</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q^2$ (annihilation) (GeV/c)²</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>36</td>
</tr>
<tr>
<td>final state</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>e⁺ e⁻</td>
<td>1.0(+6)</td>
<td>4.7(+5)</td>
<td>2.7(+5)</td>
<td>1.1(+4)</td>
</tr>
<tr>
<td>γγ</td>
<td>1.3(+5)</td>
<td>5.8(+4)</td>
<td>3.2(+4)</td>
<td>1.4(+4)</td>
</tr>
<tr>
<td>μ⁺ μ⁻</td>
<td>4.9(+4)</td>
<td>2.1(+4)</td>
<td>1.2(+4)</td>
<td>5.4(+3)</td>
</tr>
<tr>
<td>π⁺ π⁻</td>
<td>6.1(+1)</td>
<td>1.1</td>
<td>6.1(-2)</td>
<td>1.1(-3)</td>
</tr>
<tr>
<td>π⁰ γ</td>
<td>9.3</td>
<td>3.6(-1)</td>
<td>3.6(-2)</td>
<td>1.4(-3)</td>
</tr>
<tr>
<td>ρ⁺ ρ⁻</td>
<td>2.7(+3)</td>
<td>4.7(+2)</td>
<td>1.0(+2)</td>
<td>9.7</td>
</tr>
<tr>
<td>p ¯p</td>
<td>2.9(+2)</td>
<td>6.5</td>
<td>3.6(-1)</td>
<td>6.2(-3)</td>
</tr>
<tr>
<td>Λ ¯Λ</td>
<td>6.5(-1)</td>
<td>3.8(-2)</td>
<td>6.5(-4)</td>
<td></td>
</tr>
<tr>
<td>Σ⁺ Σ⁻</td>
<td>4.8</td>
<td>3.1(-1)</td>
<td>5.4(-3)</td>
<td></td>
</tr>
</tbody>
</table>

The rates are expressed in number of counts per hour. Powers of tens are written in parentheses, e.g. $1.7(-2) = 1.7 \times 10^{-2}$. The detector is assumed to sustain a large solid angle defined by $45^\circ < \theta < 135^\circ$, $0 < \phi < 2\pi$, where $\theta$ is the angle between the beam line and the final particle momenta, $\phi$ the azimuth angle.

One has assumed that the form factors of all hadrons decrease as $q^{-4}$ and one has neglected the contribution of the electric form factor to the baryon-antibaryon production rates.
discriminating among the various possible final products. In the multigev energy range the identification of a particular reaction among the scores of those which are possible requires a kinematic analysis based on accurate momentum and angle measurements. However, such an analysis might sometimes be difficult to carry out because of the following point.

v) Many of the reactions lead, either directly or after the decay of unstable particles, to the production of \( \gamma \) rays (for example: \( e^+ e^- \rightarrow \pi^0 \gamma \), \( e^+ e^- \rightarrow \pi^+ \pi^- \pi^0 \), \( e^+ e^- \rightarrow \rho^+ \rho^- \rightarrow \pi^+ \pi^- \pi^0 \pi^0 \), \( e^+ e^- \rightarrow \bar{\rho}^0 \pi^0 \), \( e^+ e^- \rightarrow \bar{\pi}^0 \pi^0 \), \( e^+ e^- \rightarrow pN \rightarrow p\bar{\pi}^0 \) etc...). It is quite important that these \( \gamma \) rays be detected with high efficiency \(^*\). It follows that shower detectors must be so located as to be separated from the interaction region by little absorbing material. This condition implies that these detectors be inside the solenoid which produces the field necessary for momentum analysis, or at least inside the iron used for flux return.

3.2 First generation of detectors

A set of spark chambers triggered by multifold coincidences between scintillation counters provides the easiest way to build a detector which meets all the above requirements in a range of \( \mathcal{M} \) of energy limited to about 2 GeV. Figure 3 gives an example of such an experimental set up, namely the first one used at Novosibirsk to detect \( e^+ e^- \), \( \mu^+ \mu^- \) and \( \pi^+ \pi^- \) pairs.\(^{11}\) It consists of a symmetrical arrangement of optical spark chambers triggered by four-fold coincidences. The spark chambers located closest to the ring vacuum chamber have thin plates in order to make accurate geometrical measurements. Further away, thick plate spark-chambers develop electromagnetic cascade showers and stop pions or muons; the latter spark chambers allow range measurements and can show interactions of pions with nuclei (stars, for instance). The whole set up is covered by a layer of scintillation counters which veto cosmic rays.

A similar arrangement was built at Orsay, except for the fact that the spark chambers being somewhat wider and closer to the beams sustained a larger solid angle. The corresponding increase of the solid angle of detection allowed the observation of three-body final states \( (e^+ e^- \rightarrow \omega \rightarrow \pi^+ \pi^- \pi^0) \). Figure 4 shows some typical spark chamber pictures

\(^{*}\) Note that these \( \gamma \) rays do not necessarily have a high energy.
Fig. 3 First experimental set-up used with VEPP 2 at Novosibirsk.

1) Anticoincidence scintillation counter; 2) Lead absorber 20 cm thick; 3) "Range" spark chamber; 4) "Shower" spark chamber; 5) Duraluminium absorber 2 cm thick; 6) Thin-plate spark chambers; 7) Window of outer vacuum chamber; 8) Interaction region; 9) Inner vacuum chamber; 10) Scintillation counters; 11) Storage ring magnet.
Fig. 4 Some typical spark-chamber pictures of events registered at $E^* = 2 \times 382$ MeV with ACO (Orsay).
events registered at Orsay at a total C of M energy $E^* = 2 \times 382$ MeV, that is, on the lower tail of the $\omega$ resonance.

Other detectors of similar structure have been built or proposed\textsuperscript{12}) for ACO, ADONE and higher energy rings; most of them can detect $\gamma$ rays as well as charged particles and sustain a very large solid angle since they surround almost completely the vacuum chamber. The present trend is to replace part or all of the optical spark chambers with wire spark chambers operated with a computer. The main advantages of this change are well known: direct digitization of the information, possibility of higher triggering rates, and absence of light collection problems.

3.3 Future detectors

Non-magnetic detectors of the type described in the preceding paragraph may also be used at higher energies ($E > 1$ GeV) to study QED reactions ($e^+e^- \rightarrow e^+e^-$, $\gamma\gamma$, $\mu^+\mu^-$), to search new intermediate vector mesons, and to get a first picture of strong interaction reactions. Nevertheless, the untangling of all the possible channels which are open at high energy requires precise momentum measurements with a magnetic field.

The kinematic analysis should take into account the possibility that some energy and momentum may be taken away by undetected $\gamma$ rays, since the probability that the incoming electrons radiate in the direction of the beam line becomes high at high energy. In particular, there is a non negligible probability that the reactions which have the highest rates (viz. $e^+e^-$ scattering and $\mu^+\mu^-$ pair production) simulate\textsuperscript{*}) rarer ones (for example: $e^+e^- \rightarrow pp$) when accompanied with the emission of two photons of approximately opposite momenta.

It seems necessary to achieve momentum measurements within 1% or so. Large solenoids must therefore be used; typical parameter values are: diameter and length $\approx 3$ m, field $\approx 1.5$ Tesla. Superconducting coils are desirable in order to avoid power dissipations of several megawatts.

Since it is difficult to collect the light from many optical spark

\textsuperscript{*}) As far as the kinematics are concerned.
chambers and from scintillation counters and/or Čerenkov counters located inside a solenoid, efforts are being made to design wire proportional chambers of various shapes and to operate them in strong magnetic fields. It is hoped that it will be possible to use such chambers not only to locate tracks but also as triggering devices. A complete set up will require an order of magnitude of $10^5$ wires, and therefore will need an on-line computer to handle the storing and the checking of the information.

One then realizes that a detector to be used with a multigev $e^+ - e^-$ ring will be a very heavy and complicated apparatus whose size and complexity of operation will be somewhat similar to those of a bubble chamber. As is the case for a bubble chamber, the same detector will be used for a long period of time, without modification, to record a large amount of data to be analyzed later. This analysis might be done by groups of physicists working in different laboratories than the one where the ring is operated.

4. PURELY ELECTROMAGNETIC CHANNELS

4.1 Generalities

Among the various electromagnetic channels, the most interesting reactions are the three following one:

\[ e^+ + e^- \rightarrow e^+ + e^- \] (Bhabha scattering), \hspace{1cm} (a)
\[ e^+ + e^- \rightarrow \gamma + \gamma \] (Two-quantum annihilation), \hspace{1cm} (b)
\[ e^+ + e^- \rightarrow \mu^+ + \mu^- \] (Muon production). \hspace{1cm} (c)

The cross sections of these reactions can be exactly calculated by QED and the radiative corrections have also been evaluated \(^{13-16}\). The latter are quite large for high resolution experiments.

The lowest order Feynman diagrams for these three processes are shown in Figure 5.
Fig. 5 Feynman diagrams for processes (a), (b) and (c).

The corresponding cross sections in the $C$ of $\mathcal{M}$ are found to be:

(a) Bhabha scattering

\[
\frac{d\sigma}{d\Omega} = \frac{F_0^2}{8\gamma^2}\left[ \begin{array}{c}
\frac{q_{14}^4 + s_{14}^4}{q^4} |F(q^2)|^2 + 2 \frac{q_{14}^4}{q^2 s^2} \text{Re} F(q^2) \text{Re} \left( F(s^2) \right) \\
+ \frac{q_{14}^4 + s_{14}^4}{s^4} |F(s^2)|^2
\end{array} \right], \quad (5)
\]
(b) **Two-quantum annihilation** (γ >> 1 and θ >> 1/γ)

\[
\frac{d\sigma}{d\Omega} = \frac{r_0^2}{8\gamma^2} \left[ \frac{q^4}{q^2} |F(q^2)|^2 + \frac{q^2}{q'^2} |F(q'^2)| \right]
\]

\[\text{(6)}\]

(c) **Muon production**

\[
\frac{d\sigma}{d\Omega} = \frac{r_0^2}{16\gamma^2} \beta_\mu \left[ (1 + \cos^2 \theta) + (1 - \beta_\mu^2) \sin^2 \theta \right] |F(s^2)|^2
\]

\[\text{(7)}\]

where \(q^2 = -4E^2 \sin^2 \theta/2\), \(q'^2 = -4E^2 \cos^2 \theta/2\), \(s^2 = 4E^2\) and \(θ\) is the angle between the final particle momenta and the beam line.

Form factors, \(F(q^2) = 1 / (1 + q^2/\Lambda^2)\), have been introduced in these formulas according to the usual procedure in order to parameterize possible deviations from pure QED predictions.

The Feynman diagrams of Fig. 5 show that reaction (a) can be used to test the \(e^-\gamma\) vertex function. It also tests the photon propagator both for space-like and time-like transfers. However, in an experiment which does not distinguish the charges of the final particles, the main contribution to the cross section is by far the one associated with time-like transfers. Reaction (b) tests the electron vertex and the electron propagator, while reaction (c) probes the muon structure with extremely time-like photons and provides also a very clean way to look for vacuum polarization effects in the photon propagator.

To illustrate the importance of the steps which can be made in this field by \(e^+e^-\) colliding beam experiments, let us consider the measurement of \(μ\) pair production with ADONE as proposed by a Roma-Frascati team\textsuperscript{17}. The goal of this proposal is to measure the cross section of reaction (c) within a few per-cent in the GeV region. To make it more precise, let us assume that \(E_+ = E_- = 1\) GeV and that the overall accuracy of the measurement is 5%. If this experiment leads to \(\sigma_{\text{measured}} = \sigma_{\text{theory}}\), the lower limit of the cut-off parameter \(\Lambda\) of Eq. (7) would be set at 13 GeV/c (68% confidence level). The distance which corresponds to such
a cut-off is 0.015 fermi.

A similar experiment carried out with the BY-PASS at E = 3 GeV would bring this limit down to 5 \times 10^{-3} fermi.

4.2 Experimental results

4.2.1 Bhabha scattering

The absolute cross section of $e^+ - e^-$ elastic scattering has been measured with ACO at a total C of $\phi$ energies close to the mass of the $\phi$ resonance. The identification of these two-body events was based on the colinearity of the tracks in the transverse view* and on the recognition of showers in thick plate spark chambers (see Fig. 4). The detector could not distinguish the electric charges carried by the final particles.

The $e^+ - e^-$ pairs detected were those emitted in a cone whose axis was perpendicular to the beam line, and which had a total angular aperture of 74°. The total number of accepted events was 443.

Simultaneously with the observation of these large angle scattering events, the luminosity of the ring was measured by recording the double bremsstrahlung reaction

$$e^+ + e^- \rightarrow e^+ + e^- + \gamma + \gamma'$$

in which the two $\gamma$ rays are emitted back-to-back along the beam line.

The time integral of the luminosity was then used to determine the absolute Bhabha cross section according to Eq. (1). The comparison of the number thus obtained with the one deduced from the theoretical differential cross section integrated over the solid angle of detection leads to the following value† of the cut-off $1/\Lambda^2$:

$$\Lambda^{-2} = (+0.002 \pm 0.061 \text{ (statistical)} \pm 0.030 \text{ (systematic)}) \text{ (GeV/c)}^{-2}$$

For comparison we give the results of Barber et al.19 based on the

* In the "transverse view", the tracks are seen as projected on a plane perpendicular to the beam line.
† I am very grateful to Dr. Buon for letting me quote this result prior to its publication.
study of $e^- e^-$ scattering at 556 MeV/particle (C of M). This experiment also tested the space-like photon propagator and the electron vertex function. Barber et al. found:

$$\Lambda^2 = (-0.06 \pm 0.06 \text{ (statistical)}) \quad (\text{GeV/c})^{-2},$$

with a systematic error smaller than the statistical one.

4.2.2 Mu Pair Production

There has not been, up to now, any $e^+ - e^-$ colliding beam experiment which has had as the unique goal the detection of $\mu^+ \mu^-$ pairs: the latter were obtained as by-products of an experiment carried out with ACO on $\pi^+ \pi^-$ production. Muon tracks are easily differentiated from those of the other two-body events ($e^+ e^-$ pairs and $\pi^+ \pi^-$ pairs) by their regular aspect and by their longer range (see Fig. 4).

A total of 62 $\mu^+ \mu^-$ events were recorded at an average energy (total C of M) of about 650 MeV. This number of events differs by less than 3% from the one computed by using Eq. (7) and the integral of the luminosity during the data taking time. One can then set the following limit to the cut-off parameter $\Lambda$ of Eq. (7):

$$|\Lambda^2| < 0.16 \quad (\text{GeV/c})^{-2} \quad (68\% \text{ confidence level}).$$

5. STRONG INTERACTIONS

5.1 Generalities

To the lowest order in the fine structure constant $\alpha$, hadron production in $e^+ - e^-$ collisions proceeds according to the graph of Fig. 6.

![Fig. 6 Graph of the one photon channel for hadron production. a, b, c... represent strong interacting particles.](image)
The electron and the positron annihilate into a virtual time-like photon which is then coupled to the strongly interacting particles. By such processes, one can probe the structure of stable and unstable hadrons with the electromagnetic current, and thus learn about strong interactions although the beam particles are not strongly-interacting particles themselves.

Of particular note is the multiplicity of open hadronic channels when the beam energy reaches $1.5 \pm 1.7$ GeV. Here is a non-exhaustive series of them:

$$e^+e^- \rightarrow \pi^+\pi^-, K^+K^-, \pi^0, \rho^+\rho^-, K^+K^-$$
$$e^+e^- \rightarrow p\bar{p}, n\bar{n}, \Lambda\bar{\Lambda}, \Sigma^+\Sigma^-, \Omega^0$$
$$e^+e^- \rightarrow N^+N^+, \Omega^+\Omega^-$$

In contrast with the experimental difficulties that one might encounter when dealing with such a variety of final states, the interpretation of the experiments concerning each channel is particularly simple because of the structure of the relevant Feynman diagram (see Fig. 6). Since hadron production passes mostly through one intermediate virtual photon $*$, the final particles must be in a state with $J^{PC} = 1^{--}$. One particular consequence of this constraint is the following. When the final state is made of $n$ pions, one has necessarily $T$ (isospin) = 1 for $n$ even and $T = 0$ for $n$ odd. Therefore the isovector resonances can be investigated by examining the $2\pi$ state, while isoscalar resonances are studied by examining the $3\pi$ state.

Theoretical predictions on hadron production cross sections are quite uncertain. While several models$^{20-21}$ can accommodate the experimental results on the nucleon form factors as measured in the space-like region and described by the well known "dipole fit"

$$G_{Ep}(q^2) = \frac{G_{mp}(q^2)}{\mu_p} = (1 - \frac{q^2}{0.71 (\text{GeV}/c)^2})^{-2}, \quad (8)$$

these models sometimes differ by orders of magnitude in the time-like

$*$ Putzolu$^{15}$ has shown that the contribution to the cross section of the interference term between the one-photon diagrams and the two-photon diagrams cancels when the measurement does not distinguish the electric charges of the final particles.
region, which will be explored with $e^+e^-$ colliding beam experiments. Such experiments appear as essential in clarifying the situation in this field.

Although absolute cross sections are difficult to estimate, symmetry schemes can relate in a precise way the form factors of particles which belong to the same multiplets. To illustrate this point, we write the $e^+e^-$ annihilation cross sections in the form

$$\sigma(e^+e^- \rightarrow A\bar{B}) = \sigma_{MAB} (1 + \cos^2 \theta) + \sigma_{EAB} \sin^2 \theta$$

which is valid in the lowest order of the fine structure constant $\alpha$. The quantities $\sigma_{MAB}$ and $\sigma_{EAB}$ thus introduced are simply related to the magnetic and electric form factors of the $\gamma AB$ vertex. SU(3), for example, predicts relations among these quantities $\sigma_M$ and $\sigma_E$ such as:

$$\sigma_{Ep\bar{p}} = \sigma_{E+p\bar{p}^+} = \sigma_{E-Z-Z^-} \text{ etc...},$$

$$\sigma_{M_{p\bar{p}}} = \frac{9}{4} \sigma_{M_{n\bar{n}}} = 9 \sigma_{M_{A\bar{A}}^{0-0}} = 3 \sigma_{M_{A\bar{A}}^{2-0}} \text{ etc...},$$

$$\sigma_{EN-N^-} = \sigma_{EY-Y^+} = \sigma_{E\Omega-\bar{\Omega}} \text{ etc...},$$

$$\sigma_{Ep+} = \sigma_{E+K^+} \text{ etc...}$$

The measurement of the relative yields of various hadron pairs at a given beam energy will thus allow very direct tests of these symmetries.

5.2 Vector meson production

5.2.1. The vector meson dominance model (VDM)

The vector meson dominance model\textsuperscript{22-26} is one of the first which has been proposed in view of understanding the hadron electromagnetic form factors, and since then it has played an essential role in this field.

This model is based on the existence of three neutral vector mesons ($\rho^0, \omega$ and $\phi$) which have the same quantum numbers ($J, P$ and $C$) as those of the photon. The vector dominance hypothesis assumes that, to a very
good approximation, the entire electromagnetic current of the hadrons is a linear combination of the fields which describe the three mesons. The basic equation of the VDM is therefore the following identity:

\[ J_{\mu}^{em} (x) = - \left[ \frac{m_\rho^2}{g_\rho} \rho_\mu^0 (x) + \frac{m_\omega^2}{g_\omega} \omega_\mu (x) + \frac{m_\phi^2}{g_\phi} \phi_\mu (x) \right] \cdot \]  

(9)

The coupling constants have been written in the form \( m_\nu^2/g_\nu \), (\( \nu = \rho^0, \omega, \phi \)) for convenience.

Kroll, Lee and Zumino\(^2\) have constructed a Lagrangian field theory in which the identification implied by Eq. (9) can be made exactly\(^\ast\). They have further shown that this identity is completely consistent with gauge invariance, provided that the vector mesons are coupled only to conserved currents. The \( \rho \) is then assumed to be coupled to the isospin current with the coupling constant \( g_\rho \), while the currents which are the sources of the \( \omega \) and \( \phi \) fields are taken as linear combinations of the baryon number current \( B_\mu \) and the hypercharge current \( Y_\mu \). Two angles, \( \theta_Y \) and \( \theta_N \), and two coupling constants, \( g_Y \) and \( g_N \), are in general necessary to describe the \( \omega-\phi \) mixing in this formalism and it can be shown that

\[ g_\phi = 2g_Y/\cos \theta_Y, \]  

(10)

and

\[ g_\omega = 2g_Y/\sin \theta_Y, \]  

(11)

where \( g_Y \) is the coupling constant of the octet central member \( \phi_8 \) to the hypercharge current.

All these coupling constants but \( g_N \) and the two mixing angles can be deduced from the measurement of the cross sections of the reactions which follow:

\[ e^+ + e^- \rightarrow \rho \rightarrow \pi^+ + \pi^-, \]  

(d)

\[ e^+ + e^- \rightarrow \omega \rightarrow \pi^+ + \pi^- + \pi^0, \]  

(e)

\[ e^+ + e^- \rightarrow \phi \rightarrow \pi^+ + \pi^- + \pi^0, \]  

(f)

\[ e^+ + e^- \rightarrow \phi \rightarrow \pi^0_S + K^0, \]  

(g)

\[ e^+ + e^- \rightarrow \phi \rightarrow K^+ + K^-. \]  

(h)

\(^\ast\) Kroll, Lee and Zumino have identified the hadronic electromagnetic current operator with a linear combination of the renormalized vector meson fields.
The simplest analysis\(^*\) of these reactions is carried out in the following way. Assuming vector dominance, and treating the vector mesons as essentially stable particles, the production of any hadronic final state \( f \) proceeds (in the lowest order in \( a \)) according to the graph of Fig. 7. This graph shows that if \( f \) represents a possible decay mode of a vector meson \( V \), the \( e^+e^- \rightarrow f \) reaction cross section will have a resonant behaviour with a peak occurring at a total \( C \) of \( M \) energy \( E^* = 2E = m_v \). In the vicinity of this maximum, one can assume a Breit-Wigner description for the cross section,

\[
\sigma(E) = \frac{3\pi}{4} \frac{1}{E^2} \frac{\Gamma(V \rightarrow f) \Gamma(V \rightarrow e^+e^-)}{[(2E - m_V)^2 + \Gamma_V^2/4]} ,
\]

(12)

where \( \Gamma(V \rightarrow f) \) (resp. \( \Gamma(V \rightarrow e^+e^-) \)) is the partial width of the \( V \) meson decay into the \( f \) state (resp. the \( e^+e^- \) state) and \( \Gamma_V \) is the total width of the \( V \) resonance. It follows that the peak cross section is given by

\[
\sigma_{\text{peak}} = \frac{12\pi}{m_V^2} \frac{\Gamma(V \rightarrow e^+e^-) \Gamma(V \rightarrow f)}{\Gamma_V^2} .
\]

(13)

The excitation curve \( \sigma(E) \) which can be obtained by studying the reaction \( e^+e^- \rightarrow f \) leads to the measurement of the three quantities\(^**\).

\(^*\) A thorough study may be found in Goudin's lectures on "Electron-positron annihilation into hadrons".

\(^**\) The masses of the \( \omega \) and of the \( \phi \) mesons were accurately known prior to colliding beam experiments.
$m_V$, $\Gamma_V$ and $\sigma_{\text{peak}}$. One can therefore compute the product $B_f \times \Gamma(V \rightarrow e^+e^-)$ where $B_f$ represents the branching ratio $\Gamma(V \rightarrow f)/\Gamma_V$. This branching ratio is accurately known for the $\rho \rightarrow \pi^+\pi^-$ decay and the $\omega \rightarrow \pi^+\pi^-\pi^0$ decay. So far, the information on the $\phi$ meson partial decay modes is not very precise but the storage ring experiments have themselves provided a measurement of the branching ratios $\Gamma(\phi \rightarrow \pi^+\pi^-\pi^0)/\Gamma(K^0_S K^0_L)$ and $\Gamma(\phi \rightarrow K^+K^-)/\Gamma(\pi^+_S K^0_L)$, since the three processes (f), (g) and (h) have been studied with $e^+e^-$ colliding beams.

Finally, from the knowledge of $\Gamma(V \rightarrow e^+e^-)$ one can deduce the value of the coupling constant $g_V$ which appears in Eq. (9). This is achieved by using a pole model for the $V \rightarrow e^+e^-$ decay as illustrated by Fig. 8.

![Fig. 8 Graph of the $V \rightarrow e^+e^-$ decay.](image)

A straightforward calculation gives the following partial width:

$$\Gamma(V \rightarrow e^+e^-) = \frac{4\pi\alpha^2}{3} \frac{m_V}{g_V^2}.$$  \hspace{1cm} (14)

Eqs. (10) and (11) are then used to compute $g_Y$ and $\theta_V$. The other mixing angle, $\theta_N$, can be computed $^{27}$ from the partial decay width of the $\phi$ going into $K^+K^-$ or $K^0_S K^0_L$ without having to make any assumption on the SU(3) symmetry-breaking interactions:

$$\Gamma(\phi \rightarrow K^+K^- \text{ or } K^0_S K^0_L) = \frac{1}{12} \frac{g_Y^2}{4\pi} \frac{\cos^2 \theta_V}{\cos^2(\theta_Y - \theta_N)} \frac{g_K^3}{m_\phi}.$$  \hspace{1cm} (15)

In the preceding analysis, the vector mesons were treated as infinitely narrow resonances. Finite width corrections must be made which
bring in some additional factors in the formulas derived above. Details on these corrections will be found in References 30 to 34.

5.2.2 Experimental results

The yield of $\pi^+\pi^-$ pairs at the $\rho$ resonance has been measured both at Novosibirsk$^{11}$ and at Orsay$^4$. Roughly speaking, these measurements cover the following range of energies (total C of M): 580 MeV to 1030 MeV by 30 MeV steps. In the Orsay experiment, special attention has been paid to the $\omega$ resonance region in order to look for a possible $\rho-\omega$ interference effect$^{35)}$. A total of about 2500 $\pi^+\pi^-$ pairs have been detected in this set of experiments.

The yield of $\pi^+\pi^-\pi^0$ states has been measured at Orsay in a 60 MeV energy range covering the $\omega$ resonance$^5$ ($\sim 200$ events) and in a 15 MeV range covering the $\phi$ resonance$^5$ ($\sim 50$ events).

Finally, the yields of $K^0_S K^0_L$ and $K^+K^-$ pairs have also been measured at the $\phi$ resonance energy ($\sim 150$ $K^0_S K^0_L$ pairs and $\sim 2000$ $K^+K^-$ pairs).

All these measurements lead to the excitation curves of reactions (d) to (g) as shown in Figs. 9 to 11. In Fig. 9, it is the square of the modulus of the pion electromagnetic form factor $|F_{\pi}|^2$ which has been plotted versus the C of M energy, rather than the cross section of reaction (d) itself. $F_{\pi}$ is a scalar function defined by

$$<\pi^+\pi^-|J_{\mu}^{em}(0)|\gamma> = i(q_+ - q_-)_\mu F_{\pi}(s),$$

where $q_+$ (resp. $q_-$) is the four-momentum of the ingoing positron (resp. electron) and $s = (q_+ + q_-)^2 = 4E^2$. The cross section of reaction (d) is then easily found to be

$$\sigma_{total}(e^+e^- \rightarrow \pi^+\pi^-) = \frac{\pi a^2}{3} \frac{1}{s} \left(1 - \frac{4m_\pi^2}{s}\right)^{3/2} |F_{\pi}(s)|^2.$$

While the curves of Figs. 10 and 11 were obtained by using the standard Breit-Wigner expression multiplied by a phase space factor, the curve shown in Fig. 9 is a four parameter fit obtained by Parkinson$^{33}$.)
Fig. 9 Pion electromagnetic form factor $|F_\pi|^2$ versus energy. The curve shows a four parameter fit to the experimental points which has been obtained by Parkinson$^{33}$.
Fig. 10 The $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section at the energy of the $\omega$ resonance. The experimental results have been fitted with a Breit-Wigner expression in which the width of the resonance was taken as a free parameter.
Fig. 11 Excitation curves of the $\phi$ resonance.
using a formula derived from a relativistic generalization of the Breit-Wigner formula. Other expressions of the pion form factor have been proposed by Gounaris and Sakurai\textsuperscript{30}, Vaughn and Wali\textsuperscript{31}, and Roos and Pišút\textsuperscript{32}.

The extraction of the coupling constants and of the mixing angles from these data can be found in Refs. 36 and 37, as well as a comparison of these experimental results to theoretical predictions based on the VDM. We shall merely quote here the values obtained of the coupling constants and the mixing angles :

<table>
<thead>
<tr>
<th>$\frac{g_0^2}{4\pi}$</th>
<th>$\frac{g_\omega^2}{4\pi}$</th>
<th>$\frac{g_\rho^2}{4\pi}$</th>
<th>$\frac{g_\nu^2}{4\pi}$</th>
<th>$\theta_Y$</th>
<th>$\theta_N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.17±0.12</td>
<td>14.8±2.3</td>
<td>11.0±1.6</td>
<td>1.58±0.18</td>
<td>40.8°±3.5°</td>
<td>21.2°±9°</td>
</tr>
</tbody>
</table>

Finally, let us mention some evidence of a $\rho$–$\omega$ interference which comes out of the pion form factor measurements carried out at Orsay\textsuperscript{35}). The data on pion pairs production (reaction (d)) obtained with ACO at the energy of the $\omega$ resonance have been left out of the analysis given above. The overall set of measurements including those made at the $\omega$ resonance are shown in Fig. 12. This figure also shows two curves which were fitted to these experimental results. The confidence level corresponding to fit (1) which does not assume any $\rho$–$\omega$ interference is 8 %, while the confidence level corresponding to fit (2) which assumes an interference effect is 47 %. When taken at their face values, the best fit parameters of fit (2) lead to a relatively high figure of the $\omega \to \pi^+\pi^-$ transition rate : $\Gamma^{1/2}(\omega \to \pi^+\pi^-) = (0.63 \pm 0.23)$ MeV$^{1/2}$.

\textsuperscript{\textdagger} The value of the coupling constant $g_\rho$ was deduced from Gounaris\textsuperscript{30b}) analysis of the pion form factor.
Fig. 12 Pion electromagnetic form factor. The complete experimental Orsay results have been fitted (1) without assuming any $\rho-\omega$ interference effect, (2) assuming a $\rho-\omega$ interference, the amplitude and phase of the $\omega\to\rho$ transition being taken as free parameters.
6. **ASYMPTOTIC BEHAVIOUR**

Last but not least, let us briefly mention the very important problem of the asymptotic behaviour of hadron production cross sections in $e^+e^-$ collisions. Particular emphasis was recently given to it by Gatto\textsuperscript{39}).

Conjectures have been made on (i) the asymptotic behaviour of the cross section of each hadronic channel, (ii) the asymptotic behaviour of the total cross section of a set of reactions such as $e^+e^-\rightarrow p+"anything"$, where "anything" indicates the sum over all possible hadron states, (iii) the asymptotic behaviour of the cross section summed over all hadronic channels\textsuperscript{*}).

Many of these predictions are based upon crossing from the $e^+e^-$ annihilation channels to the elastic or deep inelastic electron-positron scattering. Of particular note are the results of Drell, Levy and Yan\textsuperscript{39}) which are based on Bjorken's\textsuperscript{40}) work on the structure functions of the nucleon. Drell et al. have found with their "parton" model that the cross section of the $e^+e^-\rightarrow p+"anything"$ reaction should asymptotically be comparable in magnitude to that of lepton pair creation and very much larger than the "elastic" annihilation process which leads to a $p\bar{p}$ pair.

A similar conclusion on the total cross section of the $e^+e^-\rightarrow$ hadrons process had been previously found by Gribov, Ioffe and Pomeranchuk\textsuperscript{41}) by studying the commutation relations between the components $j_0$ and $j_1$ of the hadron electromagnetic current and the properties of spectral functions of the Lehmann-Källen representation.

Gatto\textsuperscript{38}) has given a general classification of the field algebra in terms of asymptotic behaviours of the $e^+e^-$ annihilation cross-section, and has pointed out that the algebra which predicts the slower decrease of the annihilation cross section as a function of energy ($\sigma(E) \sim \frac{1}{E^2}$) is the only one which has the virtue of leading to finite E.M. mass differences.

7. **CONCLUSION**

Luminosities intense enough to allow high energy $e^+e^-$ experiments were achieved three years ago, first with VEPP 2 at Novisibirsk, and soon after with ACO at Orsay.

Important results have been obtained since then, which bear mostly

\textsuperscript{*} The analysis can be carried out separately for the $T = 0$ channels and the $T = 1$ channels.
on the properties of the three neutral vector mesons. These experiments represent the first step of a systematic investigation of the hadron structure carried out with time-like photons. They have confirmed that $e^+ e^-$ rings are by far the best suited machines in this field. The recent successful two-beam operation of ADONE makes one confident that this investigation will be extended to higher energies in the near future, while the BY-PASS, VEPP 3 and DORIS promise further progress in the pursuit of still higher energies and luminosities.

* * *

- 92 -
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