Fluctuations and NA49

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Abstract. A brief history of the study of fluctuations in high energy nuclear collisions at the CERN SPS performed by NA49 is presented. The ideas and the corresponding experimental data on fluctuations are discussed from the point of view of their sensitivity to the onset of deconfinement.

1. Introduction
What are the phases of strongly interacting matter? This question and in particular, the hypothesis that at high energy densities the matter is in the form of quark-gluon plasma (QGP) [1] rather than a gas of hadrons motivated the first stage of the broad experimental program of study of ultra-relativistic nucleus–nucleus collisions [2]. The results on the energy dependence of the mean hadron multiplicities and the shape of the transverse mass spectra obtained by the NA49 experiment at the CERN SPS indicate that the properties of the created matter change rapidly in the region of low SPS energies \( \sqrt{s_{NN}} \approx 6-12 \text{ GeV} \). The observed effects confirm predictions of the transition from hadron gas to QGP [4] and thus indicate that in fact a new form of strongly interacting matter exists in nature at sufficiently high energy densities.

Do results on energy dependence of event-by-event fluctuations confirm the conjecture of the onset of deconfinement at low SPS energies? This report summarizes the current status of the NA49 data on this subject.

The paper starts with a brief historical review of the event-by-event physics in NA49 (Section 2). The experimental status of the basic ideas relating the onset of deconfinement and the fluctuations in nuclear collisions are presented and discussed in Section 3. A summary given in Section 4 closes this report.

2. A brief history of fluctuations in NA49
The NA49 large acceptance spectrometer [5] was constructed with the aim to study event-by-event fluctuations. In a letter of intend [6] it was argued: "the focus on an analysis of macroscopic and microscopic observables, referring to the individual events, stems from our expectation that the phase transition, due to the statistical fluctuations in the collision dynamics, may not uniformly occur in the average central Pb+Pb collision". The first data on central Pb+Pb collisions at 158 A GeV were taken during the NA49 running period 1994-1996 and soon after
the pioneering analysis of event-by-event fluctuations started. It appeared to be more difficult
than originally expected. In particular the problems were caused by:

- a need to control the experimental biases on event-by-event basis,
- a difficulty to remove the influence of trivial fluctuations in collision geometry,
- the absence of quantitative models predicting event-by-event fluctuations in case of the
decomposition phase transition.

Numerous measures of fluctuations were proposed, see e.g. Refs. [7], and their advantages and
disadvantages were vividly discussed. Clearly a choice of the measure depends on the physics
goals. It should maximize the sensitivity to the effects which we are looking for and minimize
the sensitivity to the biasing contributions (experimental and physical).

Due to all these problems the first results on transverse momentum fluctuations [8] and the
fluctuations of the kaon to pion ratio [9] in central Pb+Pb collisions were published only in
1999 and 2000, respectively. The measured fluctuations (see Figs. 1 and 2) are approximately
reproduced by a "mixed event" model in which uncorrelated production of hadrons is assumed.
They clearly demonstrate that possible fluctuations of the early stage energy density within the
analyzed event sample, if existent, do not lead to significant changes of the event properties. All
collisions are "similar".

![Figure 1](image1)

**Figure 1.** Event-by-event distribution of the mean transverse momentum $M(p_T)$ of
accepted particles in the event (points) for central Pb+Pb collisions at 158A GeV. The
solid line shows the $M(p_T)$ distribution for "mixed events" (histogram).

![Figure 2](image2)

**Figure 2.** Distribution of the event-by-event kaon to pion ratio estimated using
a maximum likelihood method (points) for central Pb+Pb collisions at 158A GeV. As a
reference, the same procedure was applied to a "mixed event" sample (histogram).

In 1997 and 1998 the data on C+C and Si+Si collisions at 158A and 40A GeV were registered.
Together with the data on all inelastic Pb+Pb interactions at these energies, they allow to study
system size dependence of event-by-event fluctuations. In particular, the transverse momentum
[10] and multiplicity [11] fluctuations were analyzed. The data on mean $p_T$ fluctuations, see
Fig. 3 indicate an increase of fluctuations for C+C, Si+Si and semi-peripheral Pb+Pb collisions.
A similar effect is also observed at RHIC energies [12]. It is still lacking a well established
interpretation.

The hypothesis that the onset of deconfinement is located at low SPS energies [4] motivated
the energy scan program of the NA49 Collaboration [15]. Within this program data on central
Figure 3. The system size dependence of transverse momentum fluctuations at $158A$ GeV. The points correspond to the NA49 experimental data [10], whereas the solid line to the predictions of the string-hadronic model Hijing [26].

The predicted [4] rapid changes in the energy dependence of hadron properties averaged over central Pb+Pb collisions, like pion and kaon mean multiplicities and transverse mass spectra, were observed [3]. The search for the expected anomalous behavior of other quantities is in progress. Among them event-by-event fluctuations are of particular importance. This is because they are expected to be sensitive to the physics properties of the system which are not accessible in the study of the mean quantities. In the following section we review current status of the ideas and the experimental results on the energy dependence of electric charge, multiplicity and strangeness fluctuations in the domain of the onset of deconfinement.

3. Fluctuations and the onset of deconfinement

3.1. Net-electric charge fluctuations

A simple estimate of the net electric charge fluctuations show that they are much smaller in a Quark-Gluon Plasma than in a hadron gas [16]. This difference is caused by smaller charge units in the QGP (fractional charges of quarks) than in the hadron gas. Thus a decrease of the net-charge fluctuations is expected when the collision energy crosses the threshold for the deconfinement phase transition. A schematic sketch of this naive prediction is presented in Fig. 4.

The experimental results on central Pb+Pb collisions at $20A$, $30A$, $40A$, $80A$ and $158A$ GeV shown in Fig. 5 suggest only a very weak, if any, dependence on energy. They are close to those expected for a gas of pions correlated only by global charge conservation ($\Delta\Phi_q = 0$) and significantly above the naive prediction for the QGP $\Delta\Phi_Q \approx -0.5$ [17]. These results do not necessarily exclude reduced fluctuations in the QGP because these can be masked by contributions from resonance decays. In fact a simple model of the QGP hadronization and resonance decay can quantitatively explain the magnitude of the measured fluctuations, see Fig. 6.
Figure 4. A naive expectation for the energy dependence of the net-electric charge fluctuations in the region of the onset of deconfinement.

Figure 5. The energy dependence of the net-charge fluctuations measured by $\Delta \Phi Q$ [17] in central Pb+Pb collisions for two different rapidity intervals $\Delta y = 1.2$ (left) and $\Delta y = 3$ (right). For more details see the original publication [18].

The influence of resonance decays on charge fluctuations depends on the size of the rapidity interval, $\Delta y$, in which fluctuations are calculated. If $\Delta y$ is much bigger than the typical distance in rapidity of the daughter particles, $\Delta y_{SMEAR}$, the charge within the interval will not be changed by the decay and therefore the charge fluctuations should not be affected. On the other hand, if $\Delta y$ is small, a large fraction of daughter particles will leave the interval and the initial net-charge will be significantly changed. The mean rapidity difference of two pions originating from decays of $\rho(770)$ meson is approximately one unit of rapidity. Therefore in order to minimize the decay effect $\Delta y$ should be much larger than 1. However, this constraint is difficult to fulfill at SPS and lower energies because the width of the rapidity distribution of all produced particles, $\Delta y_{TOT}$ is not much broader than 1. Hence a rapidity interval which is large enough to be unaffected by the
resonance decays would contain almost all particles produced in a collision. The net-charge in this interval would then reflect the number of participant protons and the fluctuations would be determined by fluctuations of the collision centrality and not the particle production mechanism. A sketch which illustrates these considerations is shown in Fig. 7. At the SPS and AGS energies $\Delta y \approx \Delta_{Y_{SMEAR}} \approx \Delta_{Y_{TOT}} \approx 1$ and consequently the net-charge fluctuations are not sensitive to the initial QGP fluctuations. Hence their energy dependence may be not affected by the onset of deconfinement.

3.2. Multiplicity fluctuations

It was suggested that properly filtered multiplicity fluctuations should be sensitive to the equation of state at the early stage of the collision and thus to its changes in the deconfinement phase transition region [19]. This idea follows from the expectation that the early stage energy density changes from collision to collision and thus it causes fluctuations of the thermodynamical parameters of the matter like temperature, pressure, entropy and strangeness. Obviously the relation between energy density fluctuations and fluctuations of the other thermodynamical parameters depends on the equation of state. This opens a possibility to study equation of state via properly defined analysis of event-by-event fluctuations. This basic idea is sketched in Fig. 8 for the case of the temperature fluctuations in the phase transition domain.

As the entropy is closely related to the particle multiplicity, and it is expected to be approximately conserved during the evolution of the matter created at the early stage, the fluctuations of entropy are of primary interest. Based on the second principle of thermodynamics it can be shown [19] that the relative entropy fluctuations are given by:

$$\frac{(\delta S)^2}{S^2} = (1 + \frac{p}{\epsilon})^{-2}(\frac{\delta E}{E})^2,$$

(1)
Figure 7. A sketch illustrating different rapidity scales important in the study of the net-charge fluctuations as a possible signal of QGP. $\Delta y$ is the acceptance window in rapidity, $\Delta y_{\text{SMEAR}}$ is a typical distance in rapidity of decay products of resonances and $\Delta y_{\text{TOT}}$ is the width of the rapidity distribution of all particles. The observation of the predicted suppression of the net-charge fluctuations in QGP requires: $\Delta y_{\text{SMEAR}} < \Delta y < \Delta y_{\text{TOT}}$. This condition is not fulfilled at the SPS energies where $\Delta y_{\text{SMEAR}} \approx \Delta y \approx \Delta y_{\text{TOT}} \approx 1$.

Figure 8. A sketch illustrating how the early stage energy density fluctuations lead to temperature fluctuations which depend on the equation of state. The solid line shows equation of state with the 1st order phase transition. The width of the vertical bands indicates early stage energy density fluctuations for three different collision energies, whereas the width of the horizontal bands gives the corresponding temperature fluctuations.

where $S$ and $E$ denote entropy and energy, respectively, $p$ is pressure and $\epsilon$ energy density. For the experimental study the measure of multiplicity fluctuations, $R_e$ was proposed [19]:

$$R_e = \frac{(\delta \bar{n})^2/\bar{n}^2}{(\delta \bar{E})^2/\bar{E}^2},$$  \hspace{1cm} (2)
where $\delta\bar{n}$ and $\delta\bar{E}$ are dynamical fluctuations of particle multiplicity and energy, respectively. They are related to the variance of particle multiplicity $V(n)$ as:

$$V(n) = (\delta\bar{n})^2 + \langle(\delta n)^2\rangle,$$

where $\langle(\delta n)^2\rangle$ are dynamically averaged statistical fluctuations which can be measured using the sub-event method [20]. The measure $R_e$ is directly related to the early stage equation of state [19]:

$$R_e = (1 + \frac{p}{\epsilon})^{-2},$$

and therefore it should be sensitive to the onset of deconfinement. As it is defined as a ratio of relative multiplicity and energy fluctuations it is insensitive to the magnitude of the early stage energy density fluctuations. One should underline that the method assumes that the early stage entropy fluctuations are proportional to the dynamical multiplicity fluctuations, but it does not require knowledge of the value of the proportionality factor. This is a clear advantage in regard to the methods of studying the equation of state based on the analysis of the average quantities.

The energy dependence of $R_e$ calculated with the Statistical Model of the Early Stage (SMES) [4] is presented in Fig. 9 [19]. It shows a characteristic ‘shark fin’ structure caused by large fluctuations in the mixed phase region.

This simple idea motivated further theoretical and experimental effort directed toward understanding of multiplicity fluctuations in nucleus-nucleus collisions at high energies.

The effect of conservation laws on the multiplicity fluctuations were studied within canonical and micro-canonical ensembles. This led to an unexpected observation [21] that the scaled variance for positively (+) and negatively (-) charged particles calculated within canonical and grand canonical ensembles as a function of the mean multiplicity [21]. The results corresponds to a system with net charge equal to zero.
variance of the multiplicity distribution, \( w = V(n)/\langle n \rangle \), is different in various statistical ensembles (grand canonical, canonical and micro-canonical) even in the thermodynamical limit, \( V \to \infty \). The simplest example is presented in Fig. 10 where the scaled variance for positively and negatively charged particles calculated within grand canonical and canonical ensembles is plotted as a function of mean particle multiplicity [21]. A detailed study of multiplicities fluctuations in different statistical ensembles is in progress [22 24].

An important contribution to multiplicity fluctuations may come from the fluctuations in the collision geometry. Clearly an interaction with a small number of participating nucleons will result in much lower particle multiplicity than a central collision in which almost all nucleons are involved in the reaction process. Thus in order to extract the dynamical effects a precise control of the geometrical fluctuations is needed. In NA49 this is achieved by selecting events with fixed number of projectile participants, \( N_{\text{PROJ}}^{\text{Pb+Pb}} \), measured by the forward VETO calorimeter [25]. The presented results are corrected for the resolution of the VETO calorimeter and the finite bin size in the measured forward energy. In contrary to the naive expectations [26] it was found [11] that the scaled variance of the multiplicity distribution increases with a decreasing centrality of the Pb+Pb collisions at 158 A GeV. The preliminary results\(^1\) are shown in Fig. 11. The observed non-trivial centrality dependence focused the efforts on its tests and understanding and consequently the analysis of the energy dependence of multiplicity fluctuations was delayed.

3.3. Strangeness fluctuations
Similar to entropy (multiplicity) fluctuations, strangeness fluctuations are also sensitive to the early stage energy density fluctuations and via these to the equation of state at the early stage of the collisions [27]. The measure \( R_{s/e} \) proposed to study them is defined as:

\( R_{s/e} = \frac{\langle n_s \rangle}{\langle n_e \rangle} \)

\(^1\) Note that an error was found in the analysis of multiplicity fluctuations in C+C and Si+Si collisions at 158 A GeV, and thus the corresponding preliminary results shown in Refs. [11 14] should not be used any more.
Figure 12. The energy dependence ($F \equiv (\sqrt{s} - 2m)^{3/4} / s^{1/8}$) of the properly filtered strangeness fluctuations, $R_{s/e}$, calculated with SMES [4, 27]. The 'tooth' structure at the low SPS energies is caused by a change of fluctuations in the mixed phase region.

Figure 13. The energy dependence of the dynamical strangeness fluctuations in central Pb+Pb collisions at the SPS energies [28] and central Au+Au collisions at the top RHIC energy [29].

\[ R_{s/e} = \frac{(\delta \bar{n}_K)^2 / \bar{n}_K^2}{(\delta \bar{n})^2 / \bar{n}^2}, \]

where $\delta \bar{n}_K$ and $\delta \bar{n}$ are dynamical fluctuations of the multiplicity of kaons and pions, respectively.

The energy dependence of the measure $R_{s/e}$ calculated within the SMES model [4, 27] is shown in Fig. 12. The observed rapid change of the behavior is caused by the onset of deconfinement. The first results of NA49 [28] and STAR [29] concerning strangeness fluctuations in central Pb+Pb collisions at the SPS energies are presented in Fig. 13. The data seem to also suggest a change of energy dependence in SPS energies. However, a direct comparison between the data and the model predictions is not possible as the used experimental measure of fluctuations is different than $R_{s/e}$.

4. Summary

The basic ideas concerning fluctuation signals of the onset of deconfinement at the SPS energies were reviewed. The status of their experimental tests based on the NA49 data was presented. The naively expected significant suppression of the net-charge fluctuations is not observed. This may be due to dilution of the initial charge fluctuations by resonance decays.

The study of energy dependence of multiplicity fluctuations is in progress. It focuses on the search for the "shark fin" structure expected at the onset of deconfinement. In this respect two unexpected observations were made recently.

Theory: the scaled variance of multiplicity distribution calculated within various statistical ensembles (micro-canonical, canonical and grand canonical) was found to be different even in the thermodynamical limit. Experiment: the scaled variance of multiplicity distribution for Pb+Pb collisions at 158A GeV increases significantly when going from central to semi Peripheral collisions.
The preliminary results on the dynamical fluctuations of the kaon to pion ratio in central Pb+Pb collisions indicate a change of behavior at the SPS energies. The relation of this interesting effect to the onset of deconfinement is still unclear. This is because a proper comparison between the data and the model predictions is still missing.

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