Studies on the Gluon Self Coupling with OPAL

The general (i.e. theoretical) studies described in this note will be mentioned at the Moriond conference (March 11-17; OPAL QCD presentations on Tuesday, March 13) and eventually at the meeting of the German Physical Society (March 19-??). The sensitivity and current statistics of the data will only be indicated, without showing the data themselves. Please send your comments to Siggi Bethke (SIGGI at CERNVM).

Abstract

Four-jet final states of hadronic $Z^0$ decays are analysed in terms of observables which test the existence of the gluon self coupling, as postulated by QCD. Angular correlations between the jet axes are studied. It is shown that the experimental method of reconstructing jets is sensitive to the underlying parton kinematics of the events. The data are corrected for detector resolution and fragmentation effects, and are then compared to the predictions of second order perturbative QCD calculations and of an abelian vector gluon model ("QED"). The data are compatible with QCD and do not reproduce the predictions of the abelian vector gluon model.

Contents

1 Introduction.  
2 TGV Observables and Theoretical Predictions.  
3 Experimental Reconstruction of TGV Observables.  
4 Experimental Results.  
5 Summary and Discussion.
1 Introduction.

The basic ingredients of Quantum Chromodynamics (QCD), the gauge theory of the interactions of quarks and gluons, are the principle of "asymptotic freedom" and the process of gluon self coupling. Asymptotic freedom determines that the QCD coupling strength, $\alpha_s$, decreases with increasing energy, an expectation which has been confirmed in recent studies of multijet production rates in hadronic final states of the $e^+ e^-$ annihilation [1]. An independent and important verification of the validity of QCD is to find direct signatures of the gluon self coupling, a process which is not possible in an alternative "QED" like abelian vector theory. Up to date only one experimental study is published about the possible observation of the gluon self coupling [2], which however suffered from low statistical significance and which used, as it now turns out, on incorrect theoretical predictions of the abelian vector theory.

In this note, the OPAL hadronic event sample is analysed in terms of observables which are sensitive to the gluon self coupling. In Section 2 the observables and the respective theoretical predictions of QCD and "QED" are described. The experimental method and the resolution to reconstruct partonic properties of 4-jet events from hadronic final states are described in Section 3. The data results and a comparison with the predictions of QCD and QED are presented in Section 4.

2 TGV Observables and Theoretical Predictions.

The gauge structure of QCD is only visible in second or higher perturbative order, where the gluon self coupling contributes through the existence of the triple gluon vertex (TGV). In $e^+ e^-$ annihilation, the TGV as illustrated in Fig. 1a, is predicted to be the dominant source of 4-jet events. Other processes which lead to 4-parton final states are double gluon bremsstrahlung (Fig. 1b,c) and gluon splitting into a quark-antiquark pair (Fig. 1d). In case of the abelian vector model ("QED"), where gluons cannot couple to gluons, diagram (a) is no longer possible and 4-quark final states (diagram (d)) are predicted to be produced more often than in the case of QCD. The relative contributions of the TGV, of double gluon bremsstrahlung and of the 4-quark final states, predicted by second order perturbation theory for QCD and for "QED", are given in Table 1. The numbers listed were calculated using the second order matrix element implementations of the Lund Monte Carlo program [3]. While 4-quark final states contribute only about 3.8% of all 4-parton final states in QCD, for "QED" this number is about 24%.

Observables which are sensitive to the different spin structures of events of the process (a), namely a gluon of spin 1 goes to two spin 1 particles, and process (d), spin 1 goes to two spin 1/2 particles, have been proposed to test the existence of the TGV. In this analysis, the following observables, which do not require gluon- or quark-identification within individual events, are studied:

- The angle $\theta_{BZ}$, proposed by Bengtsson and Zerwas [4], which is defined as the angle between the two planes spanned by the parton 3-vectors $\vec{p}_1$ and $\vec{p}_2$ and by the vectors $\vec{p}_3$ and $\vec{p}_4$, where the convention is such that the 4 partons within a 4-jet event are ordered according to their energies, $E_1 \geq \ldots \geq E_4$. The angle $\theta_{BZ}$ is illustrated in Fig. 2a, for a typical configuration of a 4-parton event.

- The angle $\theta_{NR}$, originally proposed by Nachtmann and Reiter [5] and as modified by Bengtsson [6] is defined by the angle between the two vectors $\vec{p}_1 - \vec{p}_2$ and $\vec{p}_3 - \vec{p}_4$, where the partons are again ordered according to their energies, as indicated above. $\theta_{BZ}$ is illustrated in Fig. 2b.
• The angle $\theta_{KSW}$, proposed by Körner, Schierholz and Wilrodt [7] is defined as the angle between the two planes spanned by the two parton momenta in each hemisphere of the event, whereby the two hemispheres are given by the plane perpendicular to the event thrust axis. For the event configuration shown in Fig. 2, $\theta_{KSW}$ would be defined as the angle between the cross product vectors $\vec{p}_1 \times \vec{p}_3$ and $\vec{p}_2 \times \vec{p}_4$.

The expected QCD distributions for these observables have been calculated using the 4-parton event generator of the Lund Monte Carlo program [3]. This generator offers the possibility to generate 4-parton events also according to "QED", where the colour factors of the original QCD generator are modified to the respective abelian vector case. In the course of this analysis, it was discovered that for the $\theta_{BZ}$ and $\theta_{NR}$ distributions the difference between QCD and "QED" like 4-parton events was about a factor of 3 smaller than originally predicted in the theoretical publications [4,6]; also AMY presented a larger difference between QCD and "QED". It could be clarified that in some of the previous works [4,6] the utilized abelian (QED) generator was incorrect, predicting a larger relative contribution of 4-quark events (51%) for the "QED" case [8]. The correct number is, however, only 24%; see Table 1. The false prediction of 51% was up to now used in a number of further publications and summary articles as [2,9], thus overestimating the predicted difference and signature between QCD and "QED" by about a factor of three. The correct distribution of these observables will be discussed and shown in Section 4, together with the results of the OPAL data.

3 Experimental Reconstruction of TGV Observables.

To analyse experimental distributions of the TGV observables, 4-jet events must be defined and reconstructed from the measured data. For this purpose the so called JADE jet finder [10] is used, because it was shown in one of our previous works [1] and other references quoted therein that this jet finder defines and reconstructs jets in close agreement to methods used in theoretical calculations. For different values of the jet resolution parameter $y_{cut}$, the minimum invariant jet mass allowed between resolvable jets, each event is classified as $n$-jet event, and the respective four-momenta of the jet axes are calculated from the sum of the particle four-momenta associated with each jet. Both the measured momenta of charged and neutral energies are used in this analysis, as described in further detail in [1]. For identified 4-jet events, the TGV observables are then calculated from the reconstructed momenta of jets. The data distributions are then corrected, bin by bin, for detector resolution and acceptance. These corrections are obtained from a sample of MC generated events [3], which passed a simulation of the OPAL detector hardware and which underwent the same selection criteria as the real data.

The data distributions must, at some point, be compared to the theoretical expectations of QCD and of "QED". Since nothing is known about a hypothetical abelian, "QED"-like hadronisation process and therefore no model simulations exist for that case, the data cannot directly be compared to the pure theoretical predictions. There are two possibilities to overcome this difficulty:

1. It is assumed that hadronisation within "QED" is identical to the hadronisation as described within the typical QCD models (e.g. the Lund model). In this case, 4-parton final states will be generated according to second order "QED", with a subsequent "standard" fragmentation into hadrons. These hadronised events can than be compared with the data, if in addition the possible background from misidentified 4-jet events is taken into account.

2. The data are corrected for fragmentation effects using the QCD model that describes global properties of the data best. In this way, one obtains data distributions "on the parton level".
These data distributions are then compared to the analytical QCD and "QED" calculations.

In this analysis, we follow the procedure described as item 2. The data will be corrected for fragmentation effects in bin-by-bin corrections as given by the Lund QCD shower model calculations. This correction automatically includes the correction for background coming from non-4-jet events which after hadronisation are misreconstructed as 4-jet events. For this procedure, the binsize of the experimental distributions must be chosen according to the experimental resolution for reconstructing the TGV observables.

From the point of view of partonic final states calculated in second order perturbation theory, two processes will influence and alter the expected parton distributions:

(a) The (soft) parton shower as described in the QCD shower models, which may be equivalent to higher order QCD effects. These are not analytically calculable but may distort, increase or destroy the clear picture from second order calculations.

(b) Hadronisation effects from the process $\text{partons} \rightarrow \text{hadrons}$.

Both these effects are studied with the Lund QCD shower program. The influence of the soft parton shower is evaluated by plotting the TGV observables, calculated from the final partons of each generated event (after identifying it as 4-jet event with the same jet finder as above), against the corresponding result obtained from the partons at an intermediate step within the shower. This intermediate step is characterized by internally requiring invariant parton masses to exceed a certain threshold (higher than the 1 GeV cutoff used for the complete, final shower) in order to be counted as a new, semi-final parton. This way the observables, calculated for each event at different stages of the parton shower, can be displayed in a scatter plot. It is expected (or at least hoped) that most events will populate the main diagonal in such a plot. The mean and the width of a distribution of the difference between both these measures directly indicates the distortion and finite resolution for these observables imposed by the (soft) parton shower.

The mean parton multiplicity after the full, standard parton shower at $Z^0$ energies is about 9.1, while in second order QCD only up to 4 partons can be generated. If in the Lund model the parton shower is stopped at an invariant mass cutoff of 5 GeV, the mean parton multiplicity is 3.8, which is comparable to the case of second order QCD. A typical scatter plot of $\theta_{BZ}$ calculated for a shower cutoff of 5 GeV, and of $\theta_{BZ}$ calculated for the same events but after the full parton shower (evolved down to 1 GeV), is shown in Fig. 3a. A clear correlation between the two calculations is visible around the main diagonal of that plot. Figure 3b shows the distribution of $(\theta_{BZ} - \theta'_{BZ})$, where the mean of -1.7 degrees indicates that the soft parton shower increases $\theta_{BZ}$ on average by a few percent (the mean $\theta_{BZ}$ is about 40 degrees). The r.m.s. width of that distribution is 13.6 degrees, indicating the approximate resolution on that observable, caused by the soft parton shower or, equivalently, higher order effects. For the analysis of $|\cos\theta_N R|$ and $\theta_{KSW}$ ($0 \leq \theta_{KSW} \leq 180$), the reconstruction correlations are of similar quality, with r.m.s. resolutions of better than 0.14 in $|\cos\theta_N R|$ and of 32 degrees in $\theta_{KSW}$. For these studies, the events were required to have 4 jets for a jet definition parameter $y_{\text{cut}} = 0.01$; better resolutions are obtained for larger $y_{\text{cut}}$ values, as also used in the data analysis described later.

Similar studies as for the influence of the soft parton shower have been done to analyse the effects of hadronisation. In Fig. 4 correlation plots for all three observables are shown, where for each event the angle calculated from the final hadrons ($\theta_i$) is plotted against the result as calculated from the final partons ($\theta'_i$), before hadronisation started. Also shown are the distributions for $\theta'_i - \theta_i$. For a 4-jet resolution of $y_{\text{cut}} = 0.010$, the value for which Fig. 4 was generated, the resolution for reconstructing
the TGV angles after hadronisation, compared to the partonic final states, is 17 degrees for \(\theta_{BZ}\), 0.18 for \(|\cos \theta_{NR}\)| and 37 degrees for \(\theta_{KSW}\). The resolutions improve slightly for larger values of \(y_{cut}\).

It is therefore concluded that the analyzing power of the experimental method is well suited to study effects of the TGV in hadronic decays of the \(Z_0\) boson.

4 Experimental Results.

This analysis is based on the OPAL hadronic data sample recorded in 1989. The selection criteria are the same as those described in [1], with the exception of the requirement that \(|\cos \theta_T|\), the angle between the event thrust axis and the beam line, must be less than 0.9. This requirement is replaced by the demand that all reconstructed jet axes must satisfy \(|\cos \theta_{jet}| < 0.9\), since the thrust axis is not well defined in spherical, 4-jet like events. 4-jet events are reconstructed for different jet resolution criteria (\(y_{cut} = 0.010, 0.015, 0.020, 0.030\) and \(0.040\)), and for each of these samples the distributions of the TGV observables \(\theta_{BZ}, \cos \theta_{NR}\) and \(\theta_{KSW}\) are calculated from the reconstructed event jet axes. For the calculation of each of these observables, additional event selection criteria are imposed:

- \(\theta_{BZ}\): For this observable, only events are accepted for which the angle between the jet axes \(\vec{p}_1\) and \(\vec{p}_2\) is less than 160 degrees and the angle between \(\vec{p}_3\) and \(\vec{p}_4\) less than 130 degrees. This requirement ensures that only events are included where the two planes, between which \(\theta_{BZ}\) is measured, are both reliably defined.

- \(\theta_{NR}\): This observable is only calculated for events where the energy of the two lowest energetic jets is smaller than those of the two highest energetic jets, which is ensured by requiring that \(E_3/E_2 < 0.5\). This cut increases the expected difference between QCD- and "QED"-like 4-jet events by about a factor of two for this observable; however, it also decreases the available data statistics by a factor of 2 to 5, depending on the value of \(y_{cut}\) at which 4-jets are defined.

- \(\theta_{KSW}\): In order to ensure that the two planes between which this angle is calculated are well defined, the angles between the jet momenta of each hemisphere are required to be less than 150 degrees and larger than 30 degrees.

After these requirements and for \(y_{cut} = 0.01\), 1056 events remain for the analysis of \(\theta_{BZ}\), 921 events for \(\theta_{NR}\) and 1505 events for \(\theta_{KSW}\). The corresponding numbers for \(y_{cut} = 0.02\) are 540, 234 and 747, respectively, decreasing to 140, 26 and 209 for \(y_{cut} = 0.04\).

The distributions for these three observables, after corrections for detector acceptance and for hadronisation effects as predicted by the Lund shower Monte Carlo program, are shown in Fig. 5 for \(y_{cut} = 0.010\). The data are compared to the theoretical expectations of QCD and "QED" as well as for a pure (abelian) sample of \(q\bar{q}q\bar{q}\) events, calculated with the second order Lund parton generator. The bin size of the distributions was chosen according to the respective experimental resolution for these observables, as described in Section 3. The errors of the data points include the statistical errors of the data and the uncertainty of the correction procedure.

In the case of \(\theta_{BZ}\) and \(\theta_{NR}\), the data are compatible with the QCD expectation, while the agreement with "QED" is significantly worse. The expectations for a pure \(q\bar{q}q\bar{q}\) sample are considerably different from the QCD results (which are essentially \(qgqg\) events, see Table 1), demonstrating the potential power of both these observables to discriminate between QCD and QED, especially if quark- and gluons jets could be tagged in each individual event. The fact that the data points are more
QCD-like than the second order QCD prediction itself (i.e. the data are systematically beyond the QCD prediction), can be explained - within the QCD shower model - by the influence of the (soft) gluon shower, which enhances the expected effects as mentioned in Section 3.

The experimental signature for a preference of QCD in the $\theta_{KSW}$ distribution is not as pronounced as in the other cases. This can be explained by the model calculations of hadronisation effects presented in Section 3, where it was observed that some events are reconstructed with $\theta_{KSW}(\text{hadrons}) \approx 180 - \theta_{KSW}(\text{partons})$, caused by a reversion of the energy order of two jets in one hemisphere. Apparently, the difference between QCD and “QED” is just due to this relative energy ordering of jets in each hemisphere, such that folding $\theta_{KSW}$ to the region of 0 to 90 degrees virtually destroys the sensitivity of this observable. Thus, for the time being, $\theta_{KSW}$ does not add much significance to the experimental study of the gluon self coupling, but is still an important consistency check for the analysis of 4-jet events.

In order to check that the significance of the results discussed so far does not depend on the specific choice of the jet resolution parameter $y_{\text{cut}} = 0.010$ to 0.040. Also shown are again the corresponding expectations from QCD and “QED”, calculated in second order perturbation theory. In general, the data are always compatible with QCD, while “QED” does not describe the data well. This is especially true for the lower values of $y_{\text{cut}}$, since the event statistics drop very fast for increasing $y_{\text{cut}}$. It is again obvious that the data are systematically somewhat beyond the QCD prediction; a fact that can be attributed to a “soft” gluon shower or equivalently to higher order effects, which are not taken care of in the second order (i.e. Born term) calculations shown in the figures. This explanation is supported by studies done with the Lund shower generator, which predicts such an effect for $\theta_{BZ}$ in the whole region of $y_{\text{cut}}$, and for $\theta_{NR}$ at small $y_{\text{cut}}$ (below 0.02) only.

5 Summary and Discussion.

The OPAL hadronic event sample was analysed in terms of 4-jet event observables which are sensitive to the existence of the triple gluon vertex. Such a study is an important test of the validity of QCD as the theory of the strong interactions, and is complementary to testing the characteristic running of the coupling strength, $\alpha_s$. Starting with an event sample of more than 2600 identified and reconstructed 4-jet events and after some additional kinematic cuts, the observables are calculated from the reconstructed jet axes. The differential distributions are corrected for detector resolution and acceptance as well as for hadronisation effects as predicted by the Lund shower model.

The same model was used to study the experimental resolution for reconstructing the observables. Compared to the stage and the predictions of second order QCD and “QED” calculations, the resolution is influenced by the further, soft parton shower as well as by the process of hadronisation. As a result of this study, the resolutions of reconstructing the “hard” parton dynamics by analysing the observables from the hadronic final states are about or better than 20 degrees for $\theta_{BZ}$; 0.2 for $|\cos\theta_{NR}|$ and 40 degrees for $\theta_{KSW}$. These resolutions, the close correlations between the observables when calculated from the partons and from the hadronic final states and the fact that hadronisation and soft parton shower affect the shapes and mean values of the analysed distributions only by a few per cent, undoubtedly show that experimental studies on the existence of the gluon self coupling are meaningful with hadronic $Z^0$ decays.

The differential shapes of the corrected data distributions as well as their means are in agreement.
with the expectations predicted by second order QCD calculations. Predictions of a second order abelian vector gluon model ("QED") do not agree with the data. This disagreement, however, can presently not be interpreted as a hard evidence against this type of model due to the following reasons:

- Theoretical predictions for the analysed distributions can only be obtained from second order QCD and "QED" calculations. These include only the Born term (i.e. the leading order term) for 4-jet production; in both cases it is not known how large higher order contributions might be and how they would influence the theoretical predictions. QCD shower models are used for trend studies about the influence of the "soft", i.e. higher order parton shower, but they are not useful to repeat this check for the case of "QED".

- The process of hadronisation is not really understood in the case of QCD, and nothing is known about a "QED"-like hadronisation (should there be any expected). Therefore this study relies on assumptions made about the transition from partons to hadrons. In the case of QCD, our present "knowledge" about hadronisation, due to the long experience with and the overall success of these models, may be sufficient for this analysis; for "QED" it certainly is not.

- Given these theoretical uncertainties and the fact that the expected difference between QCD and "QED" is almost a factor of three smaller than incorrectly stated in previous analyses, the currently available statistics of about 30000 hadronic $Z^0$ decays is not yet sufficient for an unambiguous evidence for or against one of the two theories. In the special case of OPAL, the need for more Monte Carlo events on the detector level is even more dominant.

In summary, it is nevertheless important to see that the data are compatible with and show a preference for the QCD predictions. There is justified hope that in the near future the significance of this analysis can be substantially increased.

These and further studies on the TGV are (and will be) done at CERN and at Montreal.

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References


List of Tables

1 Relative contributions to 4-parton final states of TGV events (a), of double gluon bremsstrahlung (b),(c) and of 4-quark final states (d), calculated in second perturbative order of QCD and “QED”. ........................................... 9

List of Figures

1 Basic Feynman diagrams for the process $e^+e^- \rightarrow 4\text{jets}$. ......................... 9

2 Definitions of $\vartheta_{BZ}$ (a) and of $\vartheta_{NR}$ (b) for a typical 4-parton event configuration. ........................................... 9

3 (a),(c): Correlations of $\vartheta_{BZ}$ ($|\cos\vartheta_{NR}|$), calculated from final state partons at the end of the full QCD parton shower, versus $\vartheta'_{BZ}$ ($|\cos\vartheta'_{NR}|$), calculated from the partons at an intermediate state of the shower ($Q_0 = 5 \text{ GeV}$). (b),(d): distributions of $\vartheta'_{BZ} - \vartheta_{BZ}$ and $|\cos\vartheta'_{NR}| - |\cos\vartheta_{NR}|$. ........................................... 10

4 (a), (c), (e): Correlations of the observables calculated from final state hadrons ($\theta_i$) versus those calculated from the partons at the end of the QCD shower ($\theta'_i$). (b), (d), (f): distributions of $\theta'_i - \theta_i$ etc. ........................................... 11

5 Measured distributions of $\vartheta_{BZ}$ (a), $\cos\vartheta_{NR}$ (b) and $\vartheta_{KSW}$ (c), corrected for detector resolution and hadronisation, compared to second order perturbative calculations of QCD and “QED”. ........................................... 12

6 Mean values of the TGV observables as a function of the jet resolution parameter $y_{cut}$: the data are corrected for detector and hadronisation effects. ........................................... 13
Table 1: Relative contributions to 4-parton final states of TGV events (a), of double gluon bremsstrahlung (b),(c) and of 4-quark final states (d), calculated in second perturbative order of QCD and "QED".

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<td>75.8%</td>
</tr>
<tr>
<td>(d)</td>
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<td>24.2%</td>
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Figure 1: Basic Feynman diagrams for the process $e^+e^- \rightarrow 4$ jets.

Figure 2: Definitions of $\theta_{BZ}$ (a) and of $\theta_{NR}$ (b) for a typical 4-parton event configuration.
Figure 3: (a),(c): Correlations of $\theta_{BZ}$ ($|\cos \theta_{NR}|$), calculated from final state partons at the end of the full QCD parton shower, versus $\theta'_{BZ}$ ($|\cos \theta'_{NR}|$), calculated from the partons at an intermediate state of the shower ($Q_0 = 5$ GeV). (b),(d): distributions of $\theta'_{BZ} - \theta_{BZ}$ and $|\cos \theta'_{NR}| - |\cos \theta_{NR}|$. 
Figure 4: (a), (c), (e): Correlations of the observables calculated from final state hadrons ($\theta_i$) versus those calculated from the partons at the end of the QCD shower ($\theta'_i$) (b), (d), (f): distributions of $\theta'_i - \theta_i$ etc.
Figure 5: Measured distributions of $\theta_{BZ}$ (a), $\cos\theta_{NR}$ (b) and $\theta_{KSW}$ (c), corrected for detector resolution and hadronisation, compared to second order perturbative calculations of QCD and "QED".
Figure 6: Mean values of the TGV observables as a function of the jet resolution parameter $y_{\text{cut}}$; the data are corrected for detector and hadronisation effects.