QCD Studies at LEP

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Abstract
Studies of hadronic final states of $e^+e^-$ annihilations at LEP are reviewed. The topics included cover hadronic event shapes, measurements of $\alpha_s$, determinations of QCD colour factors and tests of the non-Abelian gauge structure of QCD, differences between quark and gluon jets, QCD with heavy quarks and selected results of two-photon scattering processes.

1 Introduction
The LEP experiments ALEPH [1], DELPHI [2], L3 [3] and OPAL [4] have contributed more than 240 publications on hadronic physics and tests of Quantum Chromodynamics (QCD), the theory of the Strong Interaction between quarks and gluons (see e.g. [5]). On the occasion of the 50th anniversary of the CERN laboratory in October 2004, four years after the close-down of the LEP collider, this article gives an overview of some of the main QCD results at LEP.

The emphasis of this review is concentrated on studies which, based on perturbation theory, test key features of QCD. For earlier reviews of hadronic physics at LEP, the reader is referred to [6, 7, 8, 9, 10].

2 Hadronic Events at LEP
According to the current understanding of high energy particle collisions and reactions in the framework of the Standard Model, see e.g. [11, 12, 13], hadronic final states in $e^+e^-$ annihilations are produced through an intermediate virtual photon or a $Z^0$ boson, which decays into a quark-antiquark pair. The development of a quark-gluon cascade from the initial quark-antiquark system is calculated in fixed order QCD perturbation theory, so far in full next-to-leading order (NLO, equivalent to $O(\alpha_s^3)$) [14, 15, 16], or in the (next-to-)leading logarithmic approximation ((N)LLA) [17]. The nonperturbative process of hadronisation into visible particles is described by phenomenological string- [19] or cluster- [20] fragmentation models or, alternatively, by applying analytical power corrections [21].

At c.m. energies above the threshold of $W$- or $Z^0$-boson pair production, hadronic final states are also generated through the decays of these bosons to 4 fermions, if at least one of these pairs is a quark-antiquark system. The physics of 4-fermion final states is not included in this review but is discussed elsewhere [12].

During its time of physics operation, from August 1989 to November 2000, the LEP collider delivered an integrated luminosity of about $1 fb^{-1}$ to each of the four experiments. Of this, about 200 $pb^{-1}$ were collected during the “LEP-I” phase of operation, from 1989 to 1995, at or around the $Z^0$ mass resonance, i.e. at $E_{cm} \sim M_{Z^0} = 91.1875 \pm 0.0021$ GeV [22]. This, together with the large resonant $e^+e^-$ annihilation cross section at the $Z^0$ mass, resulted in data samples of about 4 million hadronic events for each experiment. A typical example of an event $e^+e^- \rightarrow 4$ jets is shown in Figure[1].
The “LEP-II” phase, from 1996 to 2000 at c.m. energies at and above the pair-production of W bosons, up to a maximum of 209 GeV, resulted in integrated luminosities of about 750 pb$^{-1}$. The approximate total numbers of hadronic events, obtained by each LEP experiment, are summarised in Table 1.

Due to the large event statistics, the clean and precise environment of $e^+e^-$ annihilations, the high c.m. energies, the improved detector technology and advanced theoretical calculations, significant achievements were achieved at LEP, compared to the time before, see e.g. [23, 24, 10].

3 Hadronic Event Shapes and Jet Production

At the time of LEP operation, measurements of jet production rates and of hadronic event shape parameters developed into precision tools to determine $\alpha_s$, to probe details of perturbative QCD predictions, to study hadronisation properties and to optimise and test hadronisation models. The development was largely influenced by the introduction of new jet algorithms [25], most notably the Durham (D-) scheme algorithm [26], of new event shape measures [27] and of improved theoretical predictions [16, 17, 18]. Overviews of jet and event shape observables can be found e.g. in [15, 25, 28].

The precision of data description by QCD model calculations is exemplified in Figure 2, where the measured relative production rates of multijet events are compared to the predictions of QCD shower models, at $E_{cm} = 91.2$ GeV [29], and in Figure 3, where the distributions of the shape observable Thrust (T) [30], measured at different c.m. energies, are compared with analytical predictions of QCD [31]. QCD shower models as well as QCD analytical predictions, with their parameters optimised to provide an overall good description of the data, are able to reproduce even subtle dynamic features of the data, over the entire LEP energy range. Hadronisation effects are, for many observables, small and well under control.

4 Determinations of $\alpha_s$

The coupling parameter of the Strong Interactions, $\alpha_s$, is - similar to the fine structure constant $\alpha_{em}$, the Weinberg angle $\sin^2\theta_w$ and the mass of the electron $m_e$ - one of the basic constants of nature, whose values, however, are not given by theoretical predictions but must be measured by experiment. Precise measurements of $\alpha_s$ and the experimental verification of the energy dependence of $\alpha_s$, specifically as predicted by QCD (see e.g. [5, 32, 33]), therefore were (and still are) one of the key research issues at LEP.

4.1 $\alpha_s$ from Electroweak Precision Measurements

Determinations of $\alpha_s$ from electroweak precision measurements crucially depend on the strict validity of the predictions of the Standard Model. QCD corrections affect almost all electroweak precision observables and measurements at LEP. In particular, the hadronic partial decay width of the $Z^0$, $\Gamma_{had}$, obtains QCD corrections of the form \( (1 + \Sigma_n(C_n \alpha_s^n)) \), \( n = 1, 2, 3, \ldots \). These corrections are known up to next-next-to-leading order (NNLO), i.e. to $O(\alpha_s^3)$ or $n = 3$ [34], see also [35] and references quoted therein.

In the most recent combination of the LEP-I and LEP-II measurements of all four experiments, the LEP Electroweak Working Group (LEP EWWG) [12, 22], see also [13], obtained

\[
\alpha_s(M_{Z^0}) = 0.1226 \pm 0.0038 \ (\text{exp.}) +0.0033 \ -0.0009 \ (M_H) +0.0028 \ -0.0005 \ (QCD)
\]

from $R_Z = \Gamma_{had}/\Gamma_Z = 20.767 \pm 0.025$, whereby the second error accounts for variations of the unknown Higgs boson mass between 100 and 900 GeV/c$^2$. The third error comes from a parametrisation of the unknown higher order QCD corrections, i.e. from variations of the QCD renormalisation scale and renormalisation scheme [22].
In the same analysis \[22\], the fitted leptonic pole cross section, \( \sigma_\ell^0 = (2.0003 \pm 0.0027) \) pb, resulted in

\[
\alpha_s(M_{Z^0}) = 0.1183 \pm 0.0030 \text{ (exp.)} +^{0.0026}_{-0.0000} \text{ (MH)}.
\]

Since \( \sigma_\ell^0 = \frac{12\pi \alpha_s^2}{M_Z^2} \Gamma_\ell^2 \) and \( \Gamma_Z \sim \Gamma_{had} \), \( \sigma_\ell \) has a steeper dependence on \( \alpha_s \) than \( \Gamma_{had} \). In next-to-leading order, the QCD coefficient \( C_1 \) for \( \Gamma_{had} \) turns to 2\( C_1 \) for \( \sigma_\ell \), \( C_2 \) turns to \( (2C_2 + C_1^2) \), etc. The experimental error of \( \alpha_s \) from \( \sigma_\ell \) is thus smaller than that from \( \Gamma_{had} \). However, with increased QCD-coefficients \( C_i \), the renormalisation scale uncertainty also increases, c.f. equation 13 of \[32\], such that the QCD uncertainty on \( \alpha_s \) from \( \sigma_\ell \) is expected to roughly double w.r.t. \( \alpha_s \) from \( \Gamma_Z \).

A global fit of all LEP data to determine \( \alpha_s \) together with the masses of the \( Z^0 \) boson, of the top-quark and of the Higgs boson, gives \[22\]

\[
\alpha_s(M_{Z^0}) = 0.1200^{+0.0031}_{-0.0029} \text{ (exp.)}.
\]

The latter result is the most precise available from combined electroweak fits of the LEP data. There is no additional uncertainty due to the unknown Higgs mass. The QCD uncertainties for this particular result of \( \alpha_s \), however, were never determined, and prove to be difficult to be guessed due to the unknown size of the effective QCD coefficients that enter the overall fit. Similar as argued in the case of \( \sigma_\ell^0 \), the QCD uncertainty on \( \Gamma_{had} \) cannot simply be applied to other observables.

### 4.2 \( \alpha_s \) from \( \tau \) lepton decays

The most significant determination of \( \alpha_s \) at small energy scales is obtained from the normalised hadronic branching fraction of \( \tau \) leptons, \( \mathcal{R}_\tau = \frac{\Gamma(\tau \to hadrons \nu_\tau)}{\Gamma(\tau \to e \nu_\tau, \mu \nu_\mu)} \), which is predicted as \[30\]

\[
\mathcal{R}_\tau = 3.058(1.001 + \delta_{\text{pert}} + \delta_{\text{nonpert}}).
\]

Here, \( \delta_{\text{pert}} \) and \( \delta_{\text{nonpert}} \) are perturbative and nonperturbative QCD corrections; \( \delta_{\text{pert}} \) was calculated to complete \( \mathcal{O}(\alpha_s^2) \) \[30\] \[37\] and is similar to the perturbative prediction for \( \mathcal{R}_Z \).

L3 \[38\] determined \( \alpha_s \) from measured branching fractions of tau leptons into electrons and muons. ALEPH \[39\] and OPAL \[40\] also presented measurements of the vector and the axial-vector contributions to the differential hadronic mass distributions of \( \tau \) decays, which allow simultaneous determination of \( \alpha_s \) and of the nonperturbative corrections. The latter were parametrised in terms of the operator product expansion (OPE) \[41\]. They were found to be small and to largely cancel in the total sum of \( \mathcal{R}_\tau \), as predicted by theory \[30\]. \( \alpha_s(M_\tau) \) is obtained for different variants of the NNLO QCD predictions \[38\] \[42\] \[43\]. The combined result of \( \alpha_s \) from \( \mathcal{R}_\tau \) (c.f. \[32\]) is

\[
\alpha_s(M_\tau) = 0.322 \pm 0.005(\text{exp.}) \pm 0.030(\text{theo.}).
\]

When extrapolated to the energy scale \( M_{Z^0} \), this results in \( \alpha_s(M_{Z^0}) = 0.1180 \pm 0.0005(\text{exp.}) \pm 0.0030(\text{theo.}) \).

### 4.3 \( \alpha_s \) from event shape observables

Determinations of \( \alpha_s \) from hadronic event shape observables, from jet production rates and related observables are based on pure QCD predictions. They do not depend on the assumption of strict validity of the Standard Model, however they require assumptions on or parametrisations of nonperturbative hadronisation effects.

QCD predictions for distributions and for mean values of hadronic event shapes, of jet production rates and of energy correlations are available in complete NLO \[14\] \[15\] \[16\]. In addition, for many observables, resummation of the leading and next-to-leading logarithms (NLLA) is available \[17\] which can be matched to the NLO expressions (resummed NLO).

All LEP experiments have contributed studies which are based on hadronic event shape observables, at all major LEP energies, see \[32\] \[35\] and references quoted therein. The LEP QCD working group has recently provided an overall combination of all respective LEP results which is based...
on applying common experimental procedures, consistent theoretical predictions and definitions of the theoretical uncertainties [44, 45]. For each observable and each energy a combined value of $\alpha_s$ is obtained. The results for different observables are displayed in Figure 4, demonstrating the necessity for a careful treatment and application of theoretical uncertainties to obtain a consistent and compatible situation. The results of $\alpha_s$ combined for all major LEP c.m. energies are given in Table 2. The overall combination of all these results finally gives

$$\alpha_s(M_{Z^0}) = 0.1202 \pm 0.0003 \, \text{(stat.)} \pm 0.0049 \, \text{(syst.)}.$$  

Analytical approaches to approximate nonperturbative hadronisation effects lead to "power corrections" which are proportional to powers of $1/Q$ [21]. These include, in addition to $\alpha_s$, only one further parameter $\alpha_0$ which stands for the unknown behaviour of $\alpha_s$ below an infrared matching scale $\mu_I$. Both the energy dependence of mean values as well as differential distributions of hadronic event shapes, without applying corrections for hadronisation effects, are well described by analytic predictions based on NLO QCD plus power corrections, see Figures 5 and 6 [46].

A summary of fit results of $\alpha_s$ and of $\alpha_0$ [46] is given in Figure 7. The combined results on $\alpha_s$ from power correction fits are

$$\alpha_s(M_{Z^0}) = 0.1187 \pm 0.0014 \, \text{(fit)} \pm 0.0001 \, \text{(stat.)} \pm 0.0001 \, \text{(theo.)}$$

from mean values, and

$$\alpha_s(M_{Z^0}) = 0.1111 \pm 0.0004 \, \text{(fit)} \pm 0.0020 \, \text{(sys.)} \pm 0.0044 \, \text{(theo.)}$$

from distributions [46]. The large systematic difference between these two results indicates the presence of large but yet unknown corrections which are a matter of further studies.

### 4.4 Other $\alpha_s$ results from LEP

There are further studies of $\alpha_s$ from LEP, which however have not yet reached the same experimental maturity, in terms of multiple verification by all experiments, of the range of different systematic checks and of verifications of the limited overall uncertainties. These are e.g. determinations of $\alpha_s$ from studies of scaling violations of fragmentation functions from ALEPH [47] and DELPHI [48], which can be combined to [32]

$$\alpha_s(M_{Z^0}) = 0.125 \pm 0.007 \, \text{(exp.)} \pm 0.009 \, \text{(theo.)}.$$  

Another notable result is the determination of $\alpha_s$ from 4-jet event production rates [49], which is based on a NLO, i.e. $O(\alpha_s^2)$ QCD prediction [18]. The 4-jet event production rate is proportional to $\alpha_s^2$ in LO QCD, compared to $\alpha_s$ for 3-jet like shape observables, and thus is more sensitive to $\alpha_s$. ALEPH obtains, with a rather rigorous definition of errors,

$$\alpha_s(M_{Z^0}) = 0.1170 \pm 0.0001 \, \text{(stat.)} \pm 0.0013 \, \text{(sys.)}.$$  

Further results on $\alpha_s$ are obtained in fits of the QCD group constants and studies of the nonabelian nature of QCD, which are reviewed in section 6.

### 4.5 LEP summary of $\alpha_s$

The LEP measurements of $\alpha_s$, in the energy from $M_T = 1.78$ GeV to $<E_{\text{cm}}>= 206$ GeV, are summarised in Figure 8 together with earlier results from the TRISTAN collider (see [32]) and with recent results from a "LEP-style" re-analysis of PETRA data at lower c.m. energies [50, 51]. The data are compared to the QCD prediction of the running coupling constant, calculated in $4^{th}$ order perturbation theory [52] with 3-loop matching at the heavy quark pole masses [53], for the
The current world average value of $\alpha_s(M_{Z^0}) = 0.1183 \pm 0.0027^{[32, 33]}$. The specific energy dependence of $\alpha_s$ and the concept of Asymptotic Freedom are strigently testified by the LEP results.

A combined value of $\alpha_s(M_{Z^0})$ from LEP data alone is calculated using the three most significant results from $\tau$ decays, from $R_Z$, both in complete NNLO QCD, and from the combined results from event shapes and jet production, using resummed NLO QCD predictions:

\[
\begin{align*}
\tau \text{ decays} : & \quad \alpha_s(M_{Z^0}) = 0.1180 \pm 0.0030, \\
R_Z : & \quad \alpha_s(M_{Z^0}) = 0.1226^{+0.0058}_{-0.0038}, \text{ and} \\
\text{shapes} : & \quad \alpha_s(M_{Z^0}) = 0.1202 \pm 0.0050.
\end{align*}
\]

Since the errors are dominated by theoretical uncertainties which are largely correlated with each other, a combined value of $\alpha_s(M_{Z^0})$ is calculated assuming an overall correlation factor between the three results which is adjusted such that the total $\chi^2$ is unity per degree of freedom, giving

$$\alpha_s(M_{Z^0}) = 0.1195 \pm 0.0034$$

for an overall correlation factor of 0.67, as the final combined result from LEP.

## 5 Colour Factors and nonabelian gauge structure of QCD

The central element giving rise to asymptotic freedom is the gluon self-coupling in QCD which was studied in angular correlations and energy distributions of 4-jet events. The significance of such a measurement after one year of data taking at LEP is displayed in Figure 9 [54]. Here, the distribution of the Bengtson-Zerwas angle [55] between the energy-ordered jet axes of reconstructed 4-jet events is compared with the predictions of QCD and with an Abelian theory where the gluon self-coupling does not exist.

The current state-of-the-art of such studies, which involve the analysis of several 4-jet angular correlations or fits to hadronic event shapes, is summarised [56] in Figure 10. The data, with combined values of

\[
\begin{align*}
C_A & = 2.89 \pm 0.01 \text{ (stat.)} \pm 0.21 \text{ (syst.)} \\
C_F & = 1.30 \pm 0.01 \text{ (stat.)} \pm 0.09 \text{ (syst.)}
\end{align*}
\]

are in excellent agreement with the gauge structure constants of QCD ($C_A \equiv N_C = 3$, $C_F = 4/3$ and $T_R = 1/2$), and rule out an Abelian vector gluon model ($C_A = 0$, $C_F = 1$ and $T_R = 6$). The existence of light colour-charged spin-1/2 supersymmetric partners of the gluon, the gluinos, is strongly disfavoured.

## 6 Differences between q- and g-jets

QCD predicts that quarks and gluons - due to their different colour charges - fragment differently: gluon initiated jets are expected to be broader than quark jets, the multiplicity of hadrons in gluon jets, $N_{had}$, should be larger than in quark jets, and particles in gluon jets are expected to be less energetic.

At LEP, corresponding studies at earlier $e^+e^-$ colliders were further refined, e.g. by anti-tagging gluon jets through the help of high resolution silicon vertex detectors [57], by analysing gluon-inclusive jets recoiling against two other jets which are double-tagged to be a $b$-quark-antiquark system [58], or by extracting the charged particle multiplicity of hypothetical gluon-gluon jet events from measurements of symmetric 3-jet events at LEP and from average hadronic (quark-antiquark-) events in $e^+e^-$ annihilation [59].

\[\footnote{Note that this world average included previous results of $\alpha_s$ from $R_Z$ and from $M_{\tau}$.} \]
One result of the latter type is displayed in Figure 11, where the average charged particle multiplicities of gluon-gluon and of quark-antiquark configurations are compared to the QCD predictions \[60, 61\]. These data, which confirm the QCD prediction of a higher colour charge of gluons compared to quarks, also provided a fit of the ratio \(C_A/C_F = 2.22 \pm 0.11\) \[59\], in perfect agreement with the QCD expectation of 2.25.

### 7 QCD with Heavy Quarks

#### 7.1 Gluon splitting into \(c\bar{c}\) and \(b\bar{b}\) quark pairs

The fraction of \(e^+e^- \rightarrow \text{hadrons}\) events in which a gluon splits into a pair of heavy quarks, \(c\bar{c}\) or \(b\bar{b}\), is commonly referred to as \(g_{c\bar{c}}\) and \(g_{b\bar{b}}\), respectively. These quantities are infrared safe, due to the cutoff by finite quark masses, and can therefore be calculated by means of perturbative QCD. Such predictions, however, depend on the value of \(\alpha_s\) as well as on the values of the quark masses. From leading and next-to-leading logarithmic approximations \[62\], \(g_{c\bar{c}}\) is expected to be in the range of 1 percent and \(g_{b\bar{b}}\) to be about 1 permille.

Measurements of \(g_{c\bar{c}}\) and \(g_{b\bar{b}}\) are available by all LEP experiments as well as from the SLD experiment at the SLAC Linear Collider. They are based on selections of 3-jet events with active tagging of two b-quarks, of two charmed mesons and/or of two leptons in the gluon jet. These measurements are summarised in Table 3. Combining them results in

\[
g_{c\bar{c}} = (3.05 \pm 0.14 \text{ (exp.)} \pm 0.34 \text{ (sys.)})10^{-2} \text{ and} \tag{2}
g_{b\bar{b}} = (2.74 \pm 0.28 \text{ (exp.)} \pm 0.72 \text{ (sys.)})10^{-3} \tag{3}
\]

where the experimental errors were combined in quadrature, the total errors where determined by introducing a common correlation factor between all measurements such that the overall \(\chi^2\) per degree of freedom adjusts to unity, and the systematic error is the quadratic difference of the latter two. Without the result from SLD, the LEP results average to \(g_{b\bar{b}} = (2.94 \pm 0.31 \pm 0.83)10^{-3}\).

#### 7.2 Flavour independence of \(\alpha_s\) and Measurements of the running b-quark mass

Studies of the flavour dependence of \(\alpha_s\) revealed a difference in jet rates and event shapes between b quark and light quark events, of the order of a few percent (see ref. 4 in \[70\]). These differences can be explained, in terms of NLO QCD calculations for massive quarks \[71\], by effects of the large b-quark mass. With proper account of these effects, the flavour independence of \(\alpha_s\), which is a fundamental property of QCD, could be established within about 1 % accuracy for b-quarks, 4 % for c-quarks and 5 to 10 % for the light u-, d- and s-quarks, see e.g. \[9\].

Taking the flavour independence of \(\alpha_s\) for granted, the NLO QCD predictions for massive quarks can also be used to determine the b-quark mass at the energy scale of the Z boson. QCD predicts that the quark masses depend on \(\alpha_s(Q^2)\) and thus are energy dependent, see e.g. \[12\]. A summary of the measurements of the b-quark mass from LEP experiments \[73, 74, 70\] is given in Figure 12. Also shown is the QCD prediction for the running b-quark mass, normalised to its value at the production threshold, \(m_b(m_b) = (4.2 \pm 0.2)\text{ GeV} \[73\], and using the world average value of \(\alpha_s(M_{Z^0}) = 0.1184 \pm 0.0031\) \[32\].

Combining the LEP measurements with the same treatment of (correlated) errors as described in the previous subsection, results in

\[
m_b(M_{Z^0}) = (2.82 \pm 0.02 \text{ (stat.)} \pm 0.37 \text{ (sys.)}) \text{ GeV} ,
\]

which excludes a constant b-quark mass with a significance of 3.3 standard deviations.
8 Two photon physics

Extensive studies of two-photon scattering processes leading to hadronic final states have been performed at LEP; for summary reports on this particular topic see e.g. [76, 77, 78]. Scaling violations are seen in a compilation of measurements of the photon hadronic structure function $F_2^\gamma(x, Q^2)$ from LEP and from previous $e^+e^-$ experiments [76], see Figure 13. The LEP data, especially those obtained at LEP-II, extend the range of measurements of $F_2^\gamma$ to $<Q^2>$ up to 780 GeV$^2$, the largest scale of photon structure probed in $e^+e^-$ collisions.

LEP measurements also extend the range of data at very small $x$, down to $x \sim 10^{-3}$, as seen in Figure 14. The data are compatible with a rise of $F_2^\gamma$ as predicted by leading (LO) and higher order (HO) perturbative QCD [79], while the simple quark-parton model (QPM) is naturally inadequate to describe data in this regime.

9 Summary and conclusions

The successful running of LEP has led to a significant increase of knowledge about hadron production and the dynamics of quarks and gluons at high energies. Precise determinations of $\alpha_s$ at the smallest and the largest c.m. energies available to date, superior treatment and evaluation of experimental and theoretical uncertainties, experimental confirmation of asymptotic freedom and of the gluon self coupling, detailed studies of differences between quark and gluon jets, verification of the running b-quark mass and of the flavour independence of $\alpha_s$, deeper understanding of power corrections and of hadronisation models to describe the nonperturbative hadronisation domain, and detailed studies of hadronic systems in two-photon scattering processes were summarised in this report, proving QCD as a consistent theory which accurately describes the phenomenology of the Strong Interaction.

Future developments in this field are within reach: NNLO QCD calculations and predictions for jet and event shape observables will soon be available; they will initiate further analyses of the LEP data which will provide even more accurate and more detailed determinations of $\alpha_s$.

Acknowledgements. The scientific results summarised in this review are achieved by the coherent work of a huge number of scientists, engineers and technicians, at CERN as well as at all the institutes who participated in the LEP program worldwide. Special thanks go to CERN and the groups running the LEP collider so efficiently, and also to the large number of funding agencies who had the breath to support the project through all these years of planning, of constructing and of running LEP. I am indebted to S. Kluth, to R. Nisius, to P. Zerwas and to the LEPQCDWG for their inspiring inputs, and for allowing to use their material in this review.

References


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Table 1: Typical numbers of hadronic events obtained by each of the four LEP experiments, at and around the principal c.m. energies. Numbers for $E_{cm} \geq 161$ GeV are corrected for and do not include 4-fermion final states.

<table>
<thead>
<tr>
<th>$E_{cm}$ [GeV]</th>
<th>91.2</th>
<th>133</th>
<th>161</th>
<th>172</th>
<th>183</th>
<th>189</th>
<th>200</th>
<th>206</th>
</tr>
</thead>
<tbody>
<tr>
<td># of events</td>
<td>$4 \times 10^6$</td>
<td>800</td>
<td>300</td>
<td>200</td>
<td>1200</td>
<td>3000</td>
<td>3000</td>
<td>3000</td>
</tr>
</tbody>
</table>
Table 2: Combined results of $\alpha_s(Q)$ for major LEP c.m. energies $Q$ [15].

<table>
<thead>
<tr>
<th>$Q$ (GeV)</th>
<th>$\alpha_s(Q)$</th>
<th>stat. error</th>
<th>exp. error</th>
<th>hadr. error</th>
<th>theory error</th>
<th>total error</th>
</tr>
</thead>
<tbody>
<tr>
<td>91.2</td>
<td>0.1199 ± 0.0002</td>
<td>±0.0008 ±0.0017</td>
<td>+0.0048</td>
<td>-0.0047</td>
<td>±0.0052</td>
<td>±0.0051</td>
</tr>
<tr>
<td>133.0</td>
<td>0.1135 ± 0.0016</td>
<td>±0.0012 ±0.0013</td>
<td>+0.0045</td>
<td>-0.0044</td>
<td>±0.0051</td>
<td>±0.0050</td>
</tr>
<tr>
<td>161.0</td>
<td>0.1081 ± 0.0025</td>
<td>±0.0015 ±0.0011</td>
<td>+0.0041</td>
<td>-0.0040</td>
<td>±0.0051</td>
<td>±0.0050</td>
</tr>
<tr>
<td>172.0</td>
<td>0.1049 ± 0.0029</td>
<td>±0.0017 ±0.0009</td>
<td>+0.0040</td>
<td>±0.0040</td>
<td>±0.0053</td>
<td>±0.0053</td>
</tr>
<tr>
<td>183.0</td>
<td>0.1077 ± 0.0013</td>
<td>±0.0009 ±0.0008</td>
<td>+0.0037</td>
<td>±0.0041</td>
<td>±0.0040</td>
<td>±0.0040</td>
</tr>
<tr>
<td>189.0</td>
<td>0.1092 ± 0.0008</td>
<td>±0.0009 ±0.0008</td>
<td>+0.0037</td>
<td>±0.0038</td>
<td>±0.0040</td>
<td>±0.0040</td>
</tr>
<tr>
<td>200.0</td>
<td>0.1080 ± 0.0009</td>
<td>±0.0010 ±0.0007</td>
<td>+0.0036</td>
<td>±0.0039</td>
<td>±0.0040</td>
<td>±0.0040</td>
</tr>
<tr>
<td>206.0</td>
<td>0.1078 ± 0.0009</td>
<td>±0.0008 ±0.0007</td>
<td>+0.0033</td>
<td>±0.0036</td>
<td>±0.0035</td>
<td>±0.0038</td>
</tr>
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</table>

Table 3: Compilation of results on fractions of gluons splitting into $c\bar{c}$ and $b\bar{b}$.

<table>
<thead>
<tr>
<th>exp.</th>
<th>$g_{c\bar{c}} \times 10^{-2}$</th>
<th>$g_{b\bar{b}} \times 10^{-3}$</th>
<th>ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>3.26 ± 0.23 ± 0.42</td>
<td>2.77 ± 0.42 ± 0.57</td>
<td>[63, 64]</td>
</tr>
<tr>
<td>DELPHI</td>
<td>—</td>
<td>3.3 ± 1.0 ± 0.8</td>
<td>[65]</td>
</tr>
<tr>
<td>L3</td>
<td>2.45 ± 0.29 ± 0.53</td>
<td>—</td>
<td>[66]</td>
</tr>
<tr>
<td>OPAL</td>
<td>3.26 ± 0.21 ± 0.38</td>
<td>3.07 ± 0.53 ± 0.97</td>
<td>[67, 68]</td>
</tr>
<tr>
<td>SLD</td>
<td>—</td>
<td>2.44 ± 0.59 ± 0.34</td>
<td>[69]</td>
</tr>
</tbody>
</table>
Figure 1: Hadronic event of the type $e^+e^- \rightarrow 4$ jets recorded with the ALEPH detector at LEP-I.
Figure 2: Relative production rates of n-jet events (n = 2 to 5) for different values of the jet resolution parameter $y_{\text{cut}}$, measured at the $Z^0$ resonance at LEP [29]. The data are compared to predictions of the JETSET QCD shower and hadronisation model (hadrons). The predictions for partons, before hadronisation, are also given in order to illustrate the size of the hadronisation effect.
Figure 3: Measured distributions of Thrust [31], after corrections for backgrounds and detector effects, together with fitted QCD predictions.
Figure 4: The combined $\alpha_s(M_Z)$ obtained from different observables at LEP [45]. The shaded band represents the overall combined fit for all observables. The inner error bars and the dashed band represent the statistical uncertainties.
Figure 5: Mean values of Thrust $T$ as a function of the c.m. energy $\sqrt{s}$. The full line shows the QCD fit including power corrections, the perturbative part of which is indicated by the dashed line [46].
Figure 6: Differential distributions of the wide jet broadening $B_w$ at different c.m. energies. The dotted lines show a common QCD fit including power corrections. Full lines indicate the fit ranges used to adjust $\alpha_s$ and $\alpha_0$ [10].
Figure 7: Combined results of $\alpha_s$ and $\alpha_0$ from fits to the mean values and to the differential distributions of event shape observables, measured at LEP and at lower c.m. energies [46].

Figure 8: Summary of measurements of $\alpha_s(Q^2)$ from LEP. Results from $e^+e^-$ annihilations at PETRA [50, 51] and TRISTAN (see [32]) are also included. Open symbols are from event shapes in resummed NLO, filled symbols from $\tau$ and $Z^0$ hadronic decay widths, in full NNLO QCD. The curves represent the QCD predictions of the running running coupling for the current world average of $\alpha_s$ [33].
Figure 9: Distribution of the azimuthal angle between the planes spanned by the two highest and the two lowest energetic jets in 4-jet events measured at LEP [54], together with predictions by QCD and by an abelian “QED like” theory which does not include gluon self-coupling.
Figure 10: Measurements and combination of the QCD colour factors $C_A$ and $C_F$ [56].
Figure 11: Charged particle multiplicities for $gg$ and for $q\bar{q}$ final states as a function of the energy scale [59].
Figure 12: Measurements of the b-quark mass at LEP, compared with the value of $m_b$ at the bottom quark production threshold and the QCD expectation of the running quark mass.
Figure 13: Compilation of measurements of the hadronic photon structure function $F_2^\gamma$ in $e^+e^-$ collisions [76].

Figure 14: Measurements of $F_2^\gamma$ at small $Q^2$ and small $x$ [76].