THE INVISIBLE HIGGS DECAY WIDTH
IN THE ADD MODEL AT THE LHC

M. Battaglia¹, D. Dominici²†, J.F. Gunion³ and J.D. Wells⁴
¹Department of Physics, University of California at Berkeley, and Lawrence Berkeley National Laboratory, Berkeley, CA 94720; ²Department of Physics, CERN, Theory Division, CH-1211 Geneva 23, Switzerland; ³Department of Physics, University of California at Davis, Davis, CA 95616; ⁴Department of Physics, University of Michigan, Ann Arbor, MI 48109-1120

Abstract
Assuming flat universal extra dimensions, we demonstrate that for a light Higgs boson the process \( pp \rightarrow W^*W^* + X \rightarrow \text{Higgs, graviscalars} + X \rightarrow \text{invisible} + X \) will be observable at the 5 \( \sigma \) level at the LHC for the portion of the Higgs-graviscalar mixing (\( \xi \)) and effective Planck mass (\( M_D \)) parameter space where channels relying on visible Higgs decays fail to achieve a 5 \( \sigma \) signal. Further, we show that even for very modest values of \( \xi \) the invisible decay signal probes to higher \( M_D \) than does the (\( \xi \)-independent) jets/\( \gamma + \text{missing energy} \) signal from graviton radiation. We also discuss various effects, such as Higgs decay to two graviscalars, that could become important when \( m_h/M_D \) is of order 1.

1. INTRODUCTION
In several extensions of the Standard Model (SM) there exist mechanisms which modify the Higgs production/decay rates in channels that are observable at the LHC. One example is the Randall Sundrum model where the Higgs-radion mixing not only gives detectable reductions (or enhancements) in Higgs yields, but also allows the possibility of direct observation of radion production and decay [1,2]. It is also possible for the Higgs rate in visible channels to be reduced as a result of a substantial invisible width. For example, this occurs in supersymmetric models when the Higgs has a large branching ratio into the lightest gravitinos or neutralinos. Invisible decay of the Higgs is also predicted in models with large extra dimensions felt by gravity (ADD) [3, 4]. In ADD models the presence of an interaction between the Higgs \( H \) and the Ricci scalar curvature of the induced 4-dimensional metric \( g_{ind} \) generates, after the usual shift \( H = (\frac{v + h}{\sqrt{2}}, 0) \), the following mixing term [5]

\[
L_{mix} = \epsilon h \sum_{\vec{n} > 0} s_{\vec{n}}
\]

with

\[
\epsilon = \frac{-2\sqrt{2}}{M_P} \xi v m_{\vec{n}}^2 \sqrt{\frac{3(\delta - 1)}{\delta + 2}}.
\]

Above, \( M_P = (8\pi G_N)^{-1/2} \) is the Planck mass, \( \delta \) is the number of extra dimensions, \( \xi \) is a dimensionless parameter and \( s_{\vec{n}} \) is a graviscalar KK excitation with mass \( m_{\vec{n}}^2 = 4\pi^2 \vec{n}^2/L^2 \), \( L \) being the size of each of the extra dimensions. (Note that with respect to [5] our normalization is such that we have taken only the real part of the \( \phi_{\vec{n}}^G \) fields, writing \( \phi_{\vec{n}}^G = \frac{1}{\sqrt{2}}(s_{\vec{n}} + ia_{\vec{n}}) \) and using \( \phi_{\vec{n}}^G = [\phi_{\vec{n}}^G]^* \) to restrict sums to \( \vec{n} > 0 \), by which we mean the first non-zero entry of \( \vec{n} \) is positive.) After diagonalization of the full

‡ On leave from Dipartimento di Fisica, Univ. Firenze, Sesto F. (FI) 50019, Italy
mass-squared matrix one finds that the physical eigenstate, \( h' \), acquires admixtures of the graviscalar states and vice versa. Dropping \( \mathcal{O}(\varepsilon^2) \) terms and higher,

\[
h' \sim \left[ h - \sum_{\tilde{m} > 0} \frac{\varepsilon}{m_h' - \im \tilde{m} \Gamma_{h'} - m_{\tilde{m}}^2} \right], \quad s_{\tilde{m}}' \sim \left[ \tilde{s}_{\tilde{m}} + \frac{\varepsilon}{m_h' - \im \tilde{m} \Gamma_{h'} - m_{\tilde{m}}^2} \right].
\]

In computing a process such as \( WW \rightarrow h' + \sum_{\tilde{m} > 0} s_{\tilde{m}}' \rightarrow F \), normalization and admixture corrections of order \( \varepsilon^2 \) that are present must be taken into account and the full coherent sum over physical states must be performed. The result at the amplitude level is

\[
A(WW \rightarrow F)(p^2) \sim \frac{g_{WW} h F}{p^2 - m_h^2 + \im m_h' \Gamma_{h'} + \im g(p^2) + F(p^2)}
\]

where \( F(p^2) \equiv -\varepsilon^2 \text{Re} \left[ \sum_{\tilde{m} > 0} \frac{1}{p^2 - m_{\tilde{m}}^2} \right] \) and \( G(p^2) \equiv -\varepsilon^2 \text{Im} \left[ \sum_{\tilde{m} > 0} \frac{1}{p^2 - m_{\tilde{m}}^2} \right] \). Taking the amplitude squared and integrating over \( dp^2 \) in the narrow width approximation gives the result

\[
\sigma(WW \rightarrow h' + \sum_{\tilde{m} > 0} s_{\tilde{m}}' \rightarrow F) = \sigma_{SM}(WW \rightarrow h \rightarrow F) \left[ \frac{1}{1 + F'(m_{h,\text{ren}}^2)} \right] \frac{\Gamma_h}{\Gamma_h + \Gamma_{h \rightarrow \text{graviscalar}}}.
\]

where \( m_{h,\text{ren}}^2 - m_h^2 + F(m_{h,\text{ren}}^2) = 0 \) and we have defined \( m_h \Gamma_{h \rightarrow \text{graviscalar}} \equiv G(m_{h,\text{ren}}^2) \). We will argue that for a light Higgs boson both the wave function renormalization and the mass renormalization effects will be small. In this case, the coherently summed amplitude gives the Standard Model cross section suppressed by the ratio of the SM Higgs width to the sum of the SM Higgs width and the Higgs width arising from mixing with the graviscalars.

2. INVISIBLE WIDTH

As described, there is a decay of the Higgs arising from the mixing (or oscillation) of the Higgs itself into the closest KK graviscalar levels. These graviscalars are invisible since they are weakly interacting and mainly reside in the extra dimensions whereas the Higgs resides on the brane. The mixing width \( \Gamma_{h \rightarrow \text{graviscalar}} \sim G(m_h^2)/m_h \) thus corresponds to an invisible decay width. The equation for \( G(m_h^2) \) below eq. (4) shows that it is calculated by extracting the imaginary part of the mixing contribution to the Higgs self energy. The result is \[5, 6\]

\[
\Gamma(h \rightarrow \text{graviscalar}) \equiv \Gamma(h \rightarrow \sum_{\tilde{n} > 0} s_{\tilde{n}}) = 2\pi \xi^2 \nu^2 \frac{3\delta - 1}{\delta + 2} \frac{m_h^{1+\delta}}{M_D^{2+\delta}} S_{\delta - 1}
\]

\[
\sim (16 \, \text{MeV}) 20^{\delta/2} \xi^2 \nu S_{\delta - 1} \frac{3\delta - 1}{\delta + 2} \left( \frac{m_h}{150 \, \text{GeV}} \right)^{1+\delta} \left( \frac{3 \, \text{TeV}}{M_D} \right)^{2+\delta}
\]

where \( S_{\delta - 1} = 2\pi^{\delta/2}/\Gamma(\delta/2) \) denotes the surface of a unit radius sphere in \( \delta \) dimensions while \( M_D \) is related to the \( D \) dimensional reduced Planck constant \( \sqrt{M_D} \) by \( M_D = (2\pi)^{\delta/(2+\delta)} \sqrt{M_D} \). Our eqs. (6) are a factor of 2 larger than those presented in refs. [5, 6].

2.1 The wave function renormalization factor and mass renormalization

A simple estimate of the quantity \( F'(m_{h,\text{ren}}^2) \), appearing in the wave function renormalization factor found in eq. [5], suggests that it is of order \( \xi^2 m_h^3 \), where \( \Lambda \) is an unknown ultraviolet cutoff energy presumably of order \( \Lambda \sim M_D [7] \). Assuming this to be the case, \( F' \) will provide a correction to coherently computed LHC production cross sections that is very probably quite small for the \( m_h \ll M_D \) cases that we are about to explore. However, one must keep in mind that a precise calculation of \( F' \) is
not possible. Similarly, the mass renormalization from $F(m_{h_{\text{ren}}}^2)$ should be of order $\xi^2 m_h^6 / M_D^2$ and, therefore, small for $m_h \ll M_D$. There are other incomputable sources of $v^4 / M_D^2$ corrections lurking in the theory beyond these sources, and the results presented here are computed using the first, and perhaps only, calculable terms in the perturbation series.

2.2 Contribution to the invisible width from direct two graviscalar decay

In addition to decay by mixing, one expects also a contribution to the invisible width of the Higgs from its decays into two graviscalars. This can be evaluated by using the transformation of eq. (3) between the physical eigenstate $h'$ and the unmixed $h$ to derive the relevant trilinear $h's_k s_l$ vertices. These are used to compute the corresponding matrix element. The final expression for $\Gamma(h' \to \text{graviscalar pairs})$ can be written as

$$\Gamma(h' \to \text{graviscalar pairs}) = \frac{18}{\pi} \frac{m_h^{3+2\delta}}{M_D^{4+2\delta}} \xi^2 \left( \frac{\delta - 1}{\delta + 2} \right)^2 \left( \frac{\pi^{\delta/2}}{\Gamma(\delta/2)} \right)^2 I, \quad (7)$$

where $I$ is an integral coming from the sum over all the possible kinematically allowed $h' \to s_k s_l$ decays. The integral $I$ decreases rapidly as $\delta$ increases. As a result, $\Gamma(h' \to \text{graviscalar pairs})$ is only significant compared to $\Gamma(h \to \text{graviscalar})$ if $\delta \leq 4$. The ratio of the two widths is given by:

$$\frac{\Gamma(h' \to \text{graviscalar pairs})}{\Gamma(h \to \text{graviscalar})} = \frac{3(\delta - 1)}{2\pi^{2(\delta + 2)}} \xi^2 \left( \frac{m_h}{M_D} \right)^{2+\delta} \frac{\pi^{\delta/2}}{\Gamma(\delta/2)} I. \quad (8)$$

From this result, we immediately see that even for small $\delta$ the pair invisible width will be smaller than the mixing invisible width unless $m_h$ is comparable to $M_D$.

To lowest order in $\xi^2 (m_h / M_D)^{2+\delta}$, decays of other states nearly degenerate with the $h'$ can be neglected in the computation of a cross section obtained by coherently summing over the $h'$ and the nearly degenerate $s'_m$ states. Thus, to this same order of approximation, $\Gamma(h' \to \text{graviscalar pairs})$ should simply be added to $\Gamma(h \to \text{graviscalar})$ in the expression for the narrow-width cross section of eq. (5).

![Figure 1](image.png)

**Fig. 1:** In the left-hand plot, we display the total invisible width of a 1 TeV Higgs boson into one and two graviscalars as a function of $M_D$ for various values of $\xi$ ($\xi = 1$ solid, $\xi = 2$ dashed, $\xi = 3$ dotted). For this plot we have fixed $\delta = 2$. The plot on the right shows the ratio of the two-graviscalars decay width to the one-graviscalar decay width for the same choices of parameters.

In Figure 1 we show an extreme case corresponding to $\delta = 2$ and $m_h = 1000$ GeV. Depending on the values of the parameters $\xi$ and $M_D$, the pair invisible width can be a significant correction to the invisible width from direct mixing. More generally, for $m_h > M_D$ the graviscalar-pair invisible width...
can provide a 3% to 20% correction to the direct-graviscalar-mixing invisible width. However, if \( m_h \) is substantially smaller than \( M_D \), then the graviscalar pair width is not important. For example, for \( \delta = 2 \), \( m_h = 120 \text{ GeV} \) and \( M_D = 500 \text{ GeV} \), \( \Gamma(h \rightarrow \text{graviscalar pairs})/\Gamma(h \rightarrow \text{graviscalar}) < 0.0015 \) for \( \xi < 2 \). Therefore, in the following analysis, where we will assume a light Higgs, we can safely neglect the contribution to the invisible width from the decay into two graviscalars and use the expression given by eq. (6).

3. MEASUREMENTS AT LHC

For a Higgs boson with \( m_h \) below the \( WW \) threshold, the invisible width causes a significant suppression of the LHC Higgs rate in the standard visible channels. For example, for \( M_D = 500 \text{ GeV} \) and \( m_h = 120 \text{ GeV} \), \( \Gamma(h \rightarrow \text{graviscalar}) \) is of order 25 GeV already by \( \xi \sim 1 \), i.e. far larger than the SM prediction of 3.6 MeV. Even when \( m_h \) is greater than the \( WW \) threshold, Fig. 1 shows that the partial width into invisible states can be substantial even for \( M_D \) values of several TeV; therefore, for any given value of the Higgs boson mass, there is a considerable parameter space where the invisible decay width of the Higgs boson could be the first measured phenomenological effect from extra dimensions.

Detailed studies of the Higgs boson signal significance, with inclusive production, have been carried out by the ATLAS [8,9] and CMS [10] experiments. If \( 115 \text{ GeV} < m_h < 130 \text{ GeV} \), the \( h \rightarrow \gamma \gamma \) channel appears to be instrumental for obtaining a \( \geq 5\sigma \) signal at low luminosity. The \( t\bar{t}h \) and \( h \rightarrow ZZ^* \rightarrow 4\ell \) channels also contribute, with lower statistics but a more favorable signal-to-background ratio. Preliminary results indicate that Higgs boson production in association with forward jets may also be considered as a discovery mode. However, here the background reduction strongly relies on the detailed detector response.

In the ADD model, these results are modified by the appearance of an invisible decay width suppressing the Higgs signal in the standard visible channels. Here, we fix \( m_h = 120 \text{ GeV} \) and perform a full scan of the ADD parameter space by varying \( M_D \) and \( \xi \) for different values of the number of extra dimensions \( \delta \) and demonstrate that there are regions at high \( \xi \) where the significance of the Higgs boson signal in the canonical channels drops below the \( 5\sigma \) threshold. However, the LHC experiments will also be sensitive to an invisibly decaying Higgs boson through \( W W \)-fusion production, with tagged forward jets. A detailed CMS study has shown that, with only 10 fb\(^{-1} \), an invisible channel rate of \( \Gamma_{\text{inv}}/\Gamma = 0.12-0.20 \) times the SM \( WW \rightarrow \text{Higgs} \) production rate gives a signal exceeding the \( 5\sigma \) significance for \( 120 \text{ GeV} < m_h < 400 \text{ GeV} \) [11,10]. Given that the effective (from the sum over the \( h \) state and nearby degenerate states) \( WWh \) coupling is of SM strength, this defines the region in the ADD parameter space where the Higgs boson signal can be recovered through its invisible decay.

Figure 2 summarizes the results for specific choices of parameters. In the green (light grey) region, the Higgs signal in standard channels drops below the \( 5\sigma \) threshold with 30 fb\(^{-1} \) of LHC data. But in the area above the bold blue line the LHC search for invisible decays in the fusion channel yields a signal with an estimated significance exceeding \( 5\sigma \). It is important to observe that, whenever the Higgs boson sensitivity is lost due to the suppression of the canonical decay modes, the invisible rate is large enough to still ensure detection through a dedicated analysis.

The analysis of \( \text{Jet}/\gamma^+ \) missing energy is also sensitive to the ADD model over a range of the \( M_D \) and \( \delta \) parameters [12]. The invisible Higgs decay width appears to probe a parameter space up to, and beyond, that accessible to these signatures (see Figure 2). Further, the sensitivity of these channels decreases significantly faster with \( \delta \) compared to that of the invisible Higgs width if \( \xi \sim 1 \). Finally, it is interesting that, in the region where both signatures can be probed at the LHC, a combined analysis will provide a constraint on the fundamental theory parameters.

A TeV-class \( e^+e^- \) linear collider will be able to further improve the determination of the Higgs invisible width. Extracting the branching fraction into invisible final states from the Higgsstrahlung cross section and the sum of visible decay modes affords an accuracy of order 0.2-0.03% for values of
Fig. 2: Invisible decay width effects in the $\xi - M_D$ plane for $M_H = 120$ GeV. The green (grey) regions indicate where the Higgs signal at the LHC drops below the 5 $\sigma$ threshold for 30 $fb^{-1}$ of data. The regions above the blue (bold) line are the parts of the parameter space where the invisible Higgs signal in the $WW$-fusion channel exceeds 5 $\sigma$ significance. The vertical lines show the upper limit on $M_D$ which can be probed by the analysis of jets/\gamma with missing energy at the LHC. The plots are for different values of $\delta$: 2 (upper left), 3 (upper right) 4 (lower left), 5 (lower right).

the invisible branching fraction in the range 0.1-0.5. But the ultimate accuracy can be obtained with a dedicated analysis looking for an invisible system recoiling against a $Z$ boson in the $e^+e^- \rightarrow hZ$ process. A dedicated analysis has shown that an accuracy $0.04 < \delta BR/BR < 0.025$ can be obtained for $0.1 < BR < 0.5$ [13]. This accuracy would establish an independent constraint on the $M_D$, $\xi$ and $\delta$ parameters.

ACKNOWLEDGEMENTS
JFG and JDW are supported by the U.S. Department of Energy.

References


