THE RELATIVE HADRONIC WIDTHS OF THE PSI MESONS AND

$\psi \rightarrow D\bar{D} \rightarrow \text{HADRONS}$

N.A. Törnqvist *)
CERN -- Geneva
and
Research Institute for Theoretical Physics +)
University of Helsinki

ABSTRACT

The large ratios between the widths of the $\psi$ mesons can be understood if the decays proceed through the process $\psi \rightarrow D\bar{D} \rightarrow \text{hadrons}$. Comparison with the $\phi$ width agrees favourably with this hypothesis.

*) Supported by the Academy of Finland.
+)
Present address.
We shall, in this note, discuss a natural mechanism, within the charm model, by which one can understand the rapidly growing widths of the $\psi$ particles with increasing mass \(^{1)}\).

If one assumes the phenomenological rule of Zweig \(^{2)}\) to hold exactly for hadronic vertices, the $\psi$ mesons can decay to normal hadrons in two ways:

(i) directly through the non $\bar{c}c$ components of the $\psi$, or
(ii) from the $\bar{c}c$ component through higher order diagrams, in particular $\psi \to D\bar{D} \to$ hadrons.

The remarkable narrowness of the $\psi(3.1)$ and $\psi(3.7)$ requires in this scheme that these are almost pure $\bar{c}c$ states with non $\bar{c}c$ impurities less than 1%. Mass matrix analyses \(^{3),4)}\) also predict small non $\bar{c}c$ impurities, consistent with zero, but unfortunately with large errors. The mechanism (ii) must be present due to unitarity, but again because of the narrow widths it cannot be very strong.

An appealing possibility is, of course, that the $\psi$'s actually are pure $\bar{c}c$ states, due to some dynamical principle of strong interactions, and that they decay as described by (ii) above. In analogy (although not necessarily) the $\phi$ would be a pure $s\bar{s}$ state (as a mass matrix analysis with linear masses in fact predicts). The decay $\phi \to 3\pi$ would go through $\phi \to K\bar{K} \to 3\pi$. Estimates of the latter process assuming the $K\bar{K}$ amplitude to be dominated by the near-lying poles ($u, K^*, \rho$) and that the imaginary part is dominant give the right order of magnitude \(^{5)}\). As we shall see in more detail below if the $\phi$ mass would be below the $K\bar{K}$ threshold its $3\pi$ width would be much smaller because then only the real part contributes. In accordance with this, the estimate of Gaillard, Lee and Rosner \(^{6)}\)

\[
\Gamma(\phi_c) \approx \frac{m(\phi_c)}{m(\phi)} \Gamma(\phi \to 3\pi) \tag{1}
\]

should be done for the $\psi(4.1)$ rather than for $\psi(3.1)$ (both of which would lie above the threshold). For the $\psi(3.1)$, Eq. (1) gives a value almost two orders of magnitude too large, which has been taken as an indication against the charm picture. However, for $\psi(4.1)$ one predicts a
width $\Gamma_{\text{n.h.}} = 2.7$ MeV directly into normal hadrons [i.e., final states not containing $D$, $\bar{D}$, $\psi(3.1)$, $\psi(3.7)$, etc]. This is not an unreasonable value considering the pessimistic estimate involved on effects due to opening of new channels and phase space. In fact we shall obtain below a similar estimate from comparison of the relative widths of the three $\psi$'s.

For the $\psi$ particles the simplest diagram describing a transition from a pure $c\bar{c}$ $\psi$ to a "$\omega$" or "$\phi$" type of state involves an intermediate charmed meson pair (e.g., $D\bar{D}$), Fig. 1a. In a quark diagram picture this corresponds to a double twist diagram, Fig. 1b. A more general diagram is that of Fig. 2a, where the intermediate $D\bar{D}$ annihilates in a final state interaction. By duality such diagrams can often be reduced to an intermediate "$\omega$" or "$\phi$" like Fig. 1b. However, $D\bar{D}$ annihilations of the $u\bar{t}$ type are different and would account for, e.g., the $\psi(3.7) \rightarrow \psi(3.1)\mu^+\mu^-$ decay, Fig. 2b.

It is easy to understand physically that the formation of the (virtual) $D\bar{D}$ pair is difficult if one is far below threshold and therefore the processes described by Figs. 1 and 2 should be strongly dependent on the mass of the $\psi$ meson. Above threshold the imaginary part of the diagram may become large and make the process go much faster.

This picture is quite different from the three gluon annihilation hypothesis with asymptotic freedom 7). The latter predicts a decreasing width to normal hadrons ($\Gamma_{\text{n.h.}}$) with increasing mass, which would require that the increasing total width have to be accounted for by other decay modes [$\psi(3.1)+\text{hadrons}$, $\eta_c+\text{hadrons}$, radiative decays, etc]. This provides a good test of our hypothesis against that of Ref. 7).

It is of course not possible to calculate reliably the diagram of Fig. 1 but we can obtain some rough estimate of its dependence on the $\psi$ mass. If we assume for simplicity the $\psi$'s to be identical except for mass *) their widths to normal hadrons will be proportional to the absolute square of the diagram evaluated at the three masses in question. The absolute normalization will be more difficult to calculate; it will depend

*) This may of course turn out not to be a good approximation. E.g., the $\psi(3.1)$ and $\psi(3.7)$ may in analogy with $\rho$ and $\rho'$ have very different couplings to $D\bar{D}$. However, it may still be possible that in the sum over intermediate charmed pairs ($D\bar{D}$, $\Upsilon$, $D\bar{D}^*$, etc.), whose thresholds presumably lie closely spaced, such differences average out.
sensitively on the magnitude of the $\Psi \bar{D} \bar{D}$ coupling constant, number of intermediate charmed meson states, phase space, etc. We shall not attempt to estimate it, only the ratios $\Gamma_{n.h.} (3.1) : \Gamma_{n.h.} (3.7) : \Gamma_{n.h.} (4.1)$.

The best way to estimate the diagram is to assume a reasonable imaginary part (which is through unitarity related to $D\bar{D}$ production) and calculate the real part from a dispersion relation. Using $\pi_{D\bar{D}}(0)=0$ this can be written in the subtracted form:

$$\text{Re} \left( \pi_{D\bar{D}} (s) \right) = \frac{s}{2\pi} \int_{4m_d^2}^{\infty} \frac{\text{Im} \left( \pi_{D\bar{D}} (s') \right)}{s' (s'-s)} \, ds' \quad .$$  \tag{2}

It converges if the $D\bar{D}$ production cross-section (proportional to $\frac{1}{s} \text{Im} \pi_{D\bar{D}}$) falls with energy.

If we assume an imaginary part growing like $1 - (4m_d^2/s)^{\frac{3}{2}}$ corresponding asymptotically to a constant contribution to the ratio $R = \sigma_{\text{hadrons}}/\sigma_{\mu^+\mu^-}$ from $D\bar{D}$ production, one obtains a $\pi_{D\bar{D}}(s)$ proportional to the QED vacuum polarization due to scalar mesons:

$$\text{Re} \left( \pi_{D\bar{D}} (s) \right) = \frac{2}{3\pi} \left\{ 1 - 3 \left( \frac{4m_d^2}{s} \right) \left( 1 - \frac{\sqrt{\frac{s}{4m_d^2}}} \text{arctg} \sqrt{\frac{s}{4m_d^2}} \right) \right\} ;
\quad 5 \leq 4m_d^2 \quad .$$ \tag{3}

In Fig. 3a, the function $\pi_{D\bar{D}}$ is displayed and one observes the fast increasing real part below threshold. This behaviour is quite model-independent, any imaginary part will give similar behaviour. Thus qualitatively the diagram of Fig. 1 always predicts an increasing rate with $\Psi$ mass (apart from phase space factors, etc.).

A better estimate of the diagram can be obtained if we use the experimental knowledge of the enhancement in the total cross-section above threshold, which presumably is due to a resonance $\Psi (4.1)$. Then it is reasonable to assume that also the $D\bar{D}$ production cross-section has a similar enhancement, which corresponds to a peaked $\text{Im} \pi_{D\bar{D}}$ just above threshold. This means that one could imagine the decay to proceed dominantly through the chain.
\[
\psi \rightarrow \psi(4.1) \rightarrow D\bar{D} \rightarrow \text{hadrons}
\]

(Fig. 2c) *).

If one assumes a F wave Breit-Wigner behaviour \((M = 4.15 \text{ GeV}, \Gamma = 0.25 \text{ GeV})\) for the imaginary part one finds through (2) a real part which falls much steeper than Fig. 3a below threshold (Fig. 3b). The behaviour is not very sensitive to the exact position of the threshold once it is between 3.7 and 4.1. In the figure the value \(2m_D = 3.8 \text{ GeV}\) is used. The tail of \(\text{Re}(\pi_{D\bar{D}})\) below threshold goes as \(1/(\pi^2 - s)\); above threshold the imaginary part is dominant.

To put in only one resonance for the imaginary part, without "background" terms, clearly underestimates the high energy contribution. A better guess would be to add a high energy background to \(\text{Im}(\pi_{D\bar{D}})\) as suggested by the total cross-section data and a finite asymptotic value of \(R\). We can achieve this by simply adding (properly weighted) the \(\pi_{D\bar{D}}\) as calculated in the two cases discussed above (Figs. 3a and 3b). We weight the two contributions such that the contribution to \(R\) from \(D\bar{D}\) production is proportional to \(R - 2\).

In the Table, we give the predicted ratios \(\Gamma_{n.h.}(\psi(4.1)) : \Gamma_{n.h.}(\psi(3.7)) : \Gamma_{n.h.}(\psi(3.1))\) [normalized to \(\Gamma_{n.h.}(\psi(3.1))\)] for the three different imaginary parts discussed above:

<table>
<thead>
<tr>
<th>(\text{Im}(\pi_{D\bar{D}}(s)))</th>
<th>(\frac{s-4m_D^2}{s})</th>
<th>(\text{P wave resonance} + \text{Resonance + background})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma_{n.h.}(\psi(4.1)))</td>
<td>10</td>
<td>530</td>
</tr>
<tr>
<td>(\Gamma_{n.h.}(\psi(3.7)))</td>
<td>3</td>
<td>12</td>
</tr>
<tr>
<td>(\Gamma_{n.h.}(\psi(3.1)))</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table: The predicted ratios for the widths to normal hadrons normalized to the widths of \(\psi(3.1)\).

* One can argue that due to orthogonality of the wave functions of the \(\psi\) mesons the transitions \(\psi(3.1) \rightarrow \psi(4.1)\) and \(\psi(3.7) \rightarrow \psi(4.1)\) are small and cause additional suppression of the \(\psi(3.1)\) and \(\psi(3.7)\) widths compared to \(\psi(4.1)\). I thank G. Preparata for this point.
We have conservatively assumed that the rate is proportional to $m_\psi |\pi_D(s)|^2$. This should give a low estimate since one could well argue that the phase space grows faster and that new channels make the widths larger with increasing mass.

Taking the direct hadronic width of the $\psi(3.1)$ to be 50 keV (the virtual photon contribution being subtracted from the current value of 77 MeV) the last two columns correspond to a $\Gamma_{n.h.}(\psi(3.7))$ of 350 resp. 600 keV. The experimental current estimate for $\Gamma_{tot}$ is between 200 and 800 keV. For the $\psi(3.7)$ one finds $\Gamma_{n.h.} = 85$ and 27 MeV, respectively. These numbers should, of course, be considered as crude orders of magnitude estimates. But it is interesting that this value is not in bad disagreement with the value 2.7 MeV obtained through (1) in the comparison with the $\phi$ width.

The estimates given above are, as we have pointed out, likely to be too small. If one adds decay channels like $\psi(4.1) \rightarrow \psi(3.7) + \text{hadrons}$, $\psi(3.1) + \text{hadrons}$, $\eta_c + \text{hadrons}$, etc., which are not included in our estimates, final states not involving charmed pairs may even correspond to a large part of the total 250 MeV width of $\psi(4.1)$. If our estimate of $\Gamma_{n.h.}(\psi(4.1))$ is too low by a factor 5-10 the branching ratio into $D\bar{D}^*\pi\pi$ could be very small making charmed mesons hard to detect.

ACKNOWLEDGEMENTS

We thank in particular Professors J. Ellis, M. Jacob, G. Preparata and C. Schmid for valuable discussions.
REFERENCES

1) J. Aubert et al. - Phys.Rev.Letters 33, 1404 (1974);
   J. Augustin et al. - Phys.Rev.Letters 33, 1406 (1974);
   C. Bacci et al. - Phys.Rev.Letters 33, 1408 (1974);
   G.S. Abrams et al. - Phys.Rev.Letters 33, 1453 (1974);
   J. Augustin et al. - "Total Cross-Section for Hadron Production by

2) G. Zweig - CERN Preprints TH. 401, 412 (1964);


4) K. Kajantie, C. Montonen, M. Roos and N. Törnqvist - Helsinki Preprint
   3-75;
   C. Montonen, M. Roos and N. Törnqvist - Helsinki Preprint 30-74, to be

5) J. Patasupathy - "Unitarity and Partial Widths for $\phi\to 3\pi$", Bombay
   Preprint (1975).

6) M.K. Gaillard, B.W. Lee and J.L. Rosner - "Search for Charm", Fermilab

7) A. de Rujula and S.L. Glashow - Phys.Rev.Letters 34, 46 (1975);

FIGURE CAPTIONS

Figure 1  a) Feynman diagram for $\psi\to D\bar{D} \to \omega$;
          b) The corresponding doubly twisted quark line diagram.

Figure 2  a) The process $\psi\to D\bar{D}\to$ hadrons;
          b) Quark line diagram for $\psi\to D\bar{D}\to \gamma\pi$;
          c) The process $\psi\to \psi(4.1)\to D\bar{D}\to$ hadrons.

Figure 3  a) The function $\tau_{D\bar{D}}(s)$ for a pointlike coupling to
          $D\bar{D}$;
          b) The function $\tau_{D\bar{D}}(s)$ computed from Eq. (2) for a resonance
          in Im($\tau_{D\bar{D}}$) just above the thresholds. The parameters are
          $m_D = 4.15$ GeV, $\Gamma = 0.25$ GeV, $2m_D = 3.8$ GeV.