Transverse Coupling Impedance of a Simplified Ferrite Kicker Magnet Model

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Summary

We derive the horizontal and vertical transverse coupling impedance of a ferrite kicker magnet with rectangular aperture. The method is similar to the one developed in [1] to compute the longitudinal impedance of the same structure. One of the main features is the introduction of an infinite set of image current sources that satisfy the boundary conditions at the lateral conductors of the magnet. The impedance is computed with the help of field coefficients that are determined with the technique of field matching at the vacuum-ferrite boundary. Numerical results for the MKE kicker in the SPS are presented.

1 Introduction

In the previous report [1], the longitudinal coupling impedance of a simplified structure (Fig. 1) was derived. The formula was compared with the measurement for the SPS MKE kicker magnet and they were in good agreement at low frequencies up to 1 GHz [2].

This report gives the analytical formulae of the horizontal and the vertical transverse coupling impedances of the same model (Fig. 1). The speed of the beam is $c$ throughout this report.

2 Horizontal Transverse Coupling Impedance

The current density of a beam with horizontal dipole moment $I_0 \cdot \xi$ is

$$j_z(x, y, z, t) = \frac{I_0}{2} \{\delta(x - \xi) - \delta(x + \xi)\} \delta(y) \exp(j\omega(t - z/c)).$$

(1)

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Figure 1: A simplified ferrite kicker magnet model. The nominal beam is centered at \((x,y) = (0,0)\). Two ferrite blocks are placed at \((-a < x < a, b < |y| < d)\). They are surrounded by perfectly conducting material \((|x| > a \text{ or } |y| > d)\). The length in z-direction is infinity.

Since the variable \(\xi\) is assumed to be much smaller than \(a, b, d\), we put \(\xi^2 = 0\).

We add image current densities at \((x, y) = (2pa \pm \xi, 0)\), \(p = \pm 1, \pm 2, \cdots\), in order to have electric boundary conditions at \(|x| = a\). Thus, the source field becomes

\[
E^S_x = Z_0 I_0 \sum_{p=-\infty}^{\infty} \frac{\xi}{2\pi} \frac{\left( x - 2pa - \xi \right)^2 + y^2}{(x - 2pa - \xi)^2 + y^2},
\]

\[
E^S_y = -Z_0 H^S_x = \frac{Z_0 I_0 \xi}{2\pi} \sum_{p=-\infty}^{\infty} \frac{y}{(x - 2pa + \xi)^2 + y^2},
\]

where \(Z_0(= 377 \, \Omega)\) is the vacuum impedance.

The electromagnetic field in vacuum region \((|y| < b)\) is the sum of the source field and the waveguide modes:

\[
E_z = \sum_{n=0}^{\infty} \left(-A_n + B_n\right) \sin(k_{xn}x) \cosh(k_{xn}y),
\]

\[
E_x = E_x^S + \sum_{n=0}^{\infty} \frac{jk}{k_{xn}} A_n \cos(k_{xn}x) \cosh(k_{xn}y),
\]

\[
E_y = E_y^S + \sum_{n=0}^{\infty} \frac{jk}{k_{xn}} B_n \sin(k_{xn}x) \sinh(k_{xn}y),
\]

\[
Z_0 H_z = \sum_{n=0}^{\infty} (A_n - B_n) \cos(k_{xn}x) \sinh(k_{xn}y),
\]

where \(k = \omega/c, k_{xn} = 2(n + 1)\pi/(2a), n = 0, 1, 2, \cdots\). The transverse magnetic field component is not written here, because four boundary conditions are sufficient.
Also, the field in the ferrite region \((b < y < d)\) is expressed with the waveguide modes:

\[
E_z = \sum_{n=0}^{\infty} C_n \sin(k_{xn}x) \sin(k_{yn}(y - d)),
\]

\[
E_x = \frac{j}{\epsilon_x \mu_r - 1} \sum_{n=0}^{\infty} \left( -\frac{k_{xn}}{k} C_n + \mu_r \frac{k_{yn}}{k} D_n \right) \cos(k_{xn}x) \sin(k_{yn}(y - d)),
\]

\[
E_y = \frac{j}{\epsilon_y \mu_r - 1} \sum_{n=0}^{\infty} \left( -\frac{k_{xn}}{k} C_n - \mu_r \frac{k_{yn}}{k} D_n \right) \sin(k_{xn}x) \cos(k_{yn}(y - d)),
\]

\[
Z_0 H_z = \sum_{n=0}^{\infty} D_n \cos(k_{xn}x) \cos(k_{yn}(y - d)),
\]

where \(k_{yn} = \sqrt{\left(\epsilon_r \mu_r - 1\right) k^2 - k_{xn}^2}\).

Field matching at boundary \((y = b)\) gives

\[
(-A_n + B_n) \cosh(k_{xn}b) = C_n \sin(k_{yn}(b - d)),
\]

\[
E_{xn}^S + \frac{j k}{k_{xn}} A_n \cosh(k_{xn}b) = \frac{j}{\epsilon_x \mu_r - 1} \left( -\frac{k_{xn}}{k} C_n + \mu_r \frac{k_{yn}}{k} D_n \right) \sin(k_{yn}(b - d)),
\]

\[
E_{yn}^S + \frac{j k}{k_{xn}} B_n \sinh(k_{xn}b) = \frac{j \epsilon_r}{\epsilon_r \mu_r - 1} \left( -\frac{k_{yn}}{k} C_n - \mu_r \frac{k_{xn}}{k} D_n \right) \cos(k_{yn}(b - d)),
\]

\[
(A_n - B_n) \sinh(k_{xn}b) = D_n \cos(k_{yn}(b - d)).
\]

The Fourier coefficients of the source terms \((E_{xn}^S, E_{yn}^S)\) are

\[
E_{xn}^S = \frac{1}{a} \int_{-a}^{a} dx E_x^S(x, y = b) \cos(k_{xn}x)
= \frac{Z_0 I_0 \xi}{2a} \int_{-\infty}^{\infty} \frac{dx}{(x + b)^2} \cos(k_{xn}x)
= -\frac{Z_0 I_0 \xi}{2a} \frac{k_{xn} \exp(-k_{xn}b)},
\]

\[
E_{yn}^S = \frac{1}{a} \int_{-a}^{a} dx E_y^S(x, y = b) \sin(k_{xn}x)
= \frac{Z_0 I_0 \xi}{2a} \int_{-\infty}^{\infty} \frac{2xb}{(x + b)^2} \sin(k_{xn}x)
= \frac{Z_0 I_0 \xi}{2a} \frac{k_{xn} \exp(-k_{xn}b)}.
\]

Finally, the horizontal transverse impedance per unit length is

\[
\frac{Z_h}{L} = \frac{j}{I_0 \xi} (E_x - Z_0 H_y)
= \frac{1}{I_0 \xi} \sum_{n=0}^{\infty} \frac{k_{xn}}{k} (A_n - B_n)
= \frac{j}{2a} \sum_{n=0}^{\infty} \frac{k_{xn}^2}{k} \left[ \frac{k_{yn}(1 + \epsilon_r \mu_r) \text{sh} \ ch + k_{yn} \mu_r \epsilon_r \text{sh}^2 \ tn - \epsilon_r \text{ch}^2 \ ct}{\epsilon_r \mu_r - 1} - \frac{k}{k_{xn}} \text{sh} \ ch \right]^{-1},
\]

where \(\text{sh}, \text{ch}, \text{tn}, \text{ct}\) are \(\sinh(k_{xn}b), \cosh(k_{xn}b), \tan(k_{yn}(b - d)), \cot(k_{yn}(b - d))\), respectively.
3 Vertical Transverse Coupling Impedance

The current density of a beam with a vertical dipole moment $I_0 \cdot \xi$ is

$$j_z(x, y, z, t) = \frac{I_0}{2} \delta(x) \{\delta(y - \xi) - \delta(y + \xi)\} \exp(j \omega (t - z/c)).$$

Since the variable $\xi$ is assumed to be much smaller than $a, b, d$, we put $\xi^2 = 0$.

We add image current densities at $(x, y) = (2pa, \pm \xi), p = \pm 1, \pm 2, \cdots$, in order to have electric boundary conditions at $|x| = a$. Thus, the source field becomes

$$E^S_x = Z_0 H^S_y = \frac{Z_0 I_0}{4\pi} \sum_{p=-\infty}^{\infty} (-1)^p \left[ \frac{x - 2pa}{(x - 2pa)^2 + (y - \xi)^2} - \frac{x - 2pa}{(x - 2pa)^2 + (y + \xi)^2} \right]$$

$$E^S_y = - Z_0 H^S_x = \frac{Z_0 I_0}{4\pi} \sum_{p=-\infty}^{\infty} (-1)^p \left[ \frac{y - \xi}{(x - 2pa)^2 + (y - \xi)^2} - \frac{y + \xi}{(x - 2pa)^2 + (y + \xi)^2} \right]$$

The electromagnetic field in vacuum region ($|y| < b$) is the sum of the source field and the waveguide modes:

$$E_z = \sum_{n=0}^{\infty} (A_n + B_n) \cos(k_{xn}x) \sinh(k_{yn}y),$$

$$E_x = E^S_x + \sum_{n=0}^{\infty} \frac{j k}{k_{xn}} A_n \sin(k_{xn}x) \sinh(k_{yn}y),$$

$$E_y = E^S_y + \sum_{n=0}^{\infty} \frac{j k}{k_{xn}} B_n \cos(k_{xn}x) \cosh(k_{yn}y),$$

$$Z_0 H_z = \sum_{n=0}^{\infty} (A_n + B_n) \sin(k_{xn}x) \cosh(k_{yn}y),$$

where $k_{xn} = (2n + 1)\pi/(2a), n = 0, 1, 2, \cdots$.

Also, the field in the ferrite region ($b < y < d$) is expressed with the waveguide modes:

$$E_z = \sum_{n=0}^{\infty} C_n \cos(k_{xn}x) \sin(k_{yn}(y - d)),$$

$$E_x = \frac{j}{\epsilon_r \mu_r - 1} \sum_{n=0}^{\infty} \left( \frac{k_{xn}}{k} C_n + \mu_r \frac{k_{yn}}{k} D_n \right) \sin(k_{xn}x) \sin(k_{yn}(y - d)),$$

$$E_y = \frac{j}{\epsilon_r \mu_r - 1} \sum_{n=0}^{\infty} \left( \frac{k_{yn}}{k} C_n + \mu_r \frac{k_{xn}}{k} D_n \right) \cos(k_{xn}x) \cos(k_{yn}(y - d)),$$

$$Z_0 H_z = \sum_{n=0}^{\infty} D_n \sin(k_{xn}x) \cos(k_{yn}(y - d)).$$
where \( k_{yn} = \sqrt{(\epsilon_r \mu_r - 1)k^2 - k_{xn}^2} \).

Field matching at boundary \((y = b)\) gives
\[
(A_n + B_n) \sinh(k_{xn}b) = C_n \sin(k_{yn}(b - d)),
\]
\[
E_{xn}^S + \frac{jk}{k_{xn}} A_n \sinh(k_{xn}b) = \frac{j}{\epsilon_r \mu_r - 1} \left( \frac{k_{xn}}{k} C_n + \mu_r k_{yn} D_n \right) \sin(k_{yn}(b - d)),
\]
\[
E_{yn}^S + \frac{jk}{k_{xn}} B_n \cosh(k_{xn}b) = \frac{j \epsilon_r}{\epsilon_r \mu_r - 1} \left( -\frac{k_{yn}}{k} C_n + \mu_r k_{xn} D_n \right) \cos(k_{yn}(b - d)),
\]
\[
(A_n + B_n) \cosh(k_{xn}b) = D_n \cos(k_{yn}(b - d)).
\] (12)

The Fourier coefficients of the source terms \((E_{xn}^S, E_{yn}^S)\) are
\[
E_{xn}^S = \frac{1}{a} \int_{-a}^{a} dx E_x^S(x, y = b) \sin(k_{xn}x)
= \frac{Z_0 I_0 \xi}{2 \pi a} \int_{-\infty}^{\infty} dx \frac{2xb}{(x^2 + b^2)^2} \sin(k_{xn}x)
= \frac{Z_0 I_0 \xi}{2a} k_{xn} \exp(-k_{xn}b),
\]
\[
E_{yn}^S = \frac{1}{a} \int_{-a}^{a} dx E_y^S(x, y = b) \cos(k_{xn}x)
= \frac{Z_0 I_0 \xi}{2 \pi a} \int_{-\infty}^{\infty} dx \frac{b^2 - x^2}{(x^2 + b^2)^2} \cos(k_{xn}x)
= \frac{Z_0 I_0 \xi}{2a} k_{xn} \exp(-k_{xn}b).
\] (13)

Finally, the vertical transverse impedance per unit length is
\[
\frac{Z_v}{L} = \frac{j}{I_0 \xi} (E_y + Z_0 H_x)
= -\frac{1}{I_0 \xi} \sum_{n=0}^{\infty} \frac{k_{xn}}{k} (A_n + B_n)
= \frac{j Z_0}{2a} \sum_{n=0}^{\infty} \frac{k_{xn}^2}{k} \left[ \frac{k_{yn}}{k} (1 + \epsilon_r \mu_r) \sinh(ch) + \frac{k_{yn}}{k} (\mu_r ch^2 \tan(ch) - \epsilon_r \mu_r) \right]^{-1} \cot(k_{yn}(b - d)),
\] (14)

where \(sh, ch, tn, ct\) are \(\sinh(k_{xn}b), \cosh(k_{xn}b), \tan(k_{yn}(b - d)), \cot(k_{yn}(b - d))\), respectively.

4 Application of the Formulae to the SPS MKE Kicker

Next we put the parameters of the SPS MKE kicker magnet \((a = 67.5 \text{ mm}, b = 16 \text{ mm}, d = 76 \text{ mm}, L = 1.658 \text{ m}, \text{ Ferrite: 4A4 [3]})\) into the formulae. The result is shown in Figs. 2 and 3.

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Figure 2: Horizontal transverse coupling impedance. Solid and dotted lines show the real and the imaginary parts, respectively.

Figure 3: Vertical transverse coupling impedance. Solid and dotted lines show the real and the imaginary parts, respectively.

References

