MOLECULAR PUMPING PROPERTIES OF THE LHC ARC BEAM PIPE 
AND EFFECTIVE SECONDARY ELECTRON EMISSION FROM CU SURFACE 
WITH ARTIFICIAL ROUGHNESS 

A. Krasnov*

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An additional interesting application of the angular coefficients method is the estimation of an effective secondary electron emission from surface with artificial roughness. This method allows to take into account re-reflection of the electrons using experimental data of the secondary electron energy distributions and the surface reflectivity. The suppression affectivity of the secondary electron emission from Cu as a function of roughness parameter is presented. This result is a good input for designing of future accelerators with potential electron-cloud problems.

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An additional interesting application of the angular coefficients method is the estimation of an effective secondary electron emission from surface with artificial roughness. This method allows to take into account re-reflection of the electrons using experimental data of the secondary electron energy distributions and the surface reflectivity. The suppression affectivity of the secondary electron emission from Cu as a function of roughness parameter is presented. This result is a good input for designing of future accelerators with potential electron-cloud problems.

1 Introduction

Theoretically, the method of angular coefficients or Monte-Carlo method gives absolute accuracy in the calculation of residual gas density in a complicate vacuum system. But usually, badly known input parameters (desorption rates, sticking probabilities) significantly influence the results in practice. That is why the simple Knudsen diffusion model is widely used for rough estimation of main parameters of vacuum systems. Nevertheless, the accurate methods based on numerical calculations give useful possibility to optimize geometry of vacuum system under variable conditions and input parameters.

This paper describes the applications of the angular coefficients methods in the 2D case. The 2D solution makes it convenient for the geometry description and can be used in cases where geometrical parameters do not change or only slowly change along one direction.

2 Method of angle coefficients

2.1 General description.

A short description is presented below. Details about the method of angle coefficients can be found in ref.[1].

The angle coefficient $\phi_{i\leftarrow j}$ is the probability that a molecule leaving area $j$ reaches area $i$. If the total internal surface of a vacuum system is sub-divided into the small areas, the molecule flux balance between each other can be written as:

$$q_i = q_{0i} + \beta_i \sum_{j=1}^{N} \phi_{i\leftarrow j} q_j$$

(1)

where
\[ q_i , q_j \] are the total molecule fluxes leaving area with indexes \( i \) and \( j \) correspondingly;
\[ q_{0i} \] is the flux of desorbed molecules from the area with index \( i \);
\[ \beta_i = 1 - \alpha_i \] is the interaction parameter; \( \alpha_i \) is the sticking probability of the molecules to the area with index \( i \);
\( N \) is number of the areas.

There are \( N \) equations (1) with \( N \) unknown \( q_i \). The solution of this system is \( q_i \) as a linear function of the initially defined \( q_{0i} \).

The application of this method is correct under following conditions:
1. The mean free path of particles is much larger than the characteristic size of the vacuum volume;
2. The molecular flux is uniformly distributed along the corresponding area;
3. The description of \( N \) surfaces must represent a closed geometry.

The second condition is executed experimentally by choosing a large enough \( N \).
The third condition means that
\[
\sum_{i=1}^{N} \phi_{i \leftarrow j} = 1,
\]
(2)
i.e. the probability that a molecule leaving any element \( j \) reaches an element \( i \) must be equal to one. Note that equation (2) gives a simple way to estimate the error of the method in case there are geometrical difficulties with the accurate calculation of the angle coefficients.

2.2 Calculation of the angle coefficients.

It is generally accepted that the angular distribution of the desorbed or diffusely scattered molecules obeys a cosine distribution:
\[
dw = 2 \cos(\theta) d\Omega
\]
(3)
where \( \Omega \in [0,2\pi] \) is the solid angle and \( \theta \in \left[ 0, \frac{\pi}{2} \right] \) is the angle between direction of a molecule moving and normal to the surface. The transformation of (3) to 2D (angular distribution from a strip with infinitesimal width and infinite length) gives:
\[
dw = \frac{1}{2} \cos(\theta) d\theta \quad \theta \in \left[ -\frac{\pi}{2}, \frac{\pi}{2} \right]
\]
(4)
For two infinitesimal small elements $ds_i$ and $ds_j$ (Figure 1a) one obtains:

\[
\begin{align*}
\frac{d\theta_j}{ds_j} &= \frac{ds_i \cos(\theta_i)}{|\mathbf{R}_{ij}|} \\
dw &= \frac{\cos(\theta_j) \cos(\theta_i)}{2 \cdot |\mathbf{R}_{ij}|} ds_i
\end{align*}
\]

The probability that a molecule leaves element $ds_j$ of surface with total length $s_j$ is $dp = \frac{ds_j}{s_j}$. Note that second condition is used here. The probability that a molecule leaves $ds_j$ and reaches $ds_i$ is the differential angular coefficient $d\varphi_{i \leftarrow j}(ds_i, ds_j) = dw \cdot dp$. The integration of this equation along $s_i$ and $s_j$ gives:

\[
\varphi_{i \leftarrow j} = \frac{1}{s_j} \int_{s_j} \int_{s_i} \frac{\cos(\theta_j) \cos(\theta_i)}{2 \cdot |\mathbf{R}_{ij}|} ds_i ds_j
\]

If $s_j$ and $s_i$ are the simple line segments, as it is shown on Figure 1b, the integrals can be calculated analytically:

\[
\varphi_{i \leftarrow j} = \frac{([A_j B_i] + [B_j A_i]) - ([A_j A_i] + [B_j B_i])}{2 \cdot [A_j B_j]}
\]
Note, that the expression under the integrals in (5) is invariant to index exchange. It means that:
\[ \varphi_{i \leftarrow j}^{s} j = \varphi_{j \leftarrow i}^{s} i \]  \hspace{1cm} (7)

This equation is correct in the 3D case and under any angular distributions of molecules and can be used to simplify the calculation of the angular coefficients. Unfortunately, the equation (7) forbids the artificial normalisation of the angular coefficients in case when they do not satisfy the equation (2) \((\text{third condition})\).

2.3 Description of geometry.

The applied computer code uses the approximate description of a vacuum system as the multitude of the simple line segments. The coordinates of the lines must be entered in an anticlockwise direction for the general surface of the volume and clockwise for inside objects as it is shown on figure 2. The computer code uses this simple rule to automatically determine the \textit{working} side of the segments.

![Figure 2. Geometry description.](image)

2.4 Input data and calculation.

The desorbed molecule flux \( q_{0k} \), the sticking probability \( \alpha_{k} \) and sub-division factor \( n_{k} \) (see figure 2) must be defined for each \( k \) segment. The programme divides each segment on \( n_{k} \) equivalent elements, gives them new indexes and calculates \( \varphi_{i \leftarrow j} \)
using equations (6,7). The shadowed elements (figure 2) are found automatically. It is clear that for the shadowed elements the condition $\varphi_{i\leftrightarrow j} = 0$ should be satisfied. The system of equations (1) is solved using existing computer mathematic environment. The program code is based on the MATLAB computer library.

2.5 Output parameters.

2.5.1 Pumping speed and capture factor.

The pumping speed of a vacuum system can be written as:

$$S = Cf \cdot S_{\text{ideal}}$$

where $Cf$ is the capture factor; $S_{\text{ideal}} = \frac{1}{4} \hat{v} \cdot A$ is the ideal pumping speed of an inlet with total area $A$ for molecules with the mean velocity $\hat{v}$. $Cf$ can be defined using the relation between total output flux to total input flux:

$$Cf = 1 - \frac{\sum_{i \in E_{in}} \sum_{j=1}^{N} \varphi_{i\leftrightarrow j} q_{j}}{\sum_{i} q_{0i}}$$

(10)

here $E_{in}$ are input elements. The input elements should have $\alpha = 1$, $q_{0} > 0$ (usually $q_{0} = 1$ is assumed). Other elements should have $q_{0} = 0$. Note that if $\alpha = 0$ for all elements (except inputs elements), the capture factor should be equal to zero. This gives an additional way to estimate the calculation errors.

2.5.2 Conductivity

In the simple case, when the vacuum volume contains only one input surface with area $A$ and one output surface and for all inside surfaces $\alpha = 0$, the conductivity can be defined as:

$$U = C \cdot \frac{1}{4} \hat{v} \cdot A$$

here $C$ is the Clausing’s factor. $C$ is the probability that molecule entering through input surface leaves the volume through output surface.

$$C = \frac{\sum_{i \in E_{out}} \sum_{j=1}^{N} \varphi_{i\leftrightarrow j} q_{j}}{\sum_{i} q_{0i}}$$
The input elements should have $\alpha = 1, q_0 > 0$ ($q_0 = 1$ is usually assumed). Output elements should have $\alpha = 1, q_0 = 0$. For all other elements: $\alpha = 0, q_0 = 0$.

3 Application to LHC

The cross section of the proposed LHC arc beam pipe with beam screen is shown on figure 3. Most molecules desorbing due to synchrotron radiation, electron and ion exposure, reabsorb on inside surface of the beam screen except for hydrogen\[2,3,4\], which is pumped onto the cold bore through the slots in the beam screen. Electron shields are required to avoid additional heat load on cold bore due to electrons moving through slots in the dipole magnetic field [5,6,7].

Figure 3. Cross section of LHC beam pipe in the arc regions.

The thickness of the beam screen is $1.075\,mm$. The proposed slots have irregular length with an average value $L = 8 \cdot 10^{-3} \, m$ and width $d_1 = (1 \div 1.5) \cdot 10^{-3} \, m$. The average slot pitch is $L_p = 16 \cdot 10^{-3} \, m$. The total slot area per meter length of beam pipe is:

$$A = 8 \cdot \frac{L}{L_p} \cdot d_1 \quad (m^2/m).$$

The pumping speed per meter length of the beam pipe is:

$$S = Cf \cdot \frac{1}{4} \cdot \hat{v} \cdot A \quad (m^3/(s \cdot m)).$$
The longitudinal slot dimension $L$ is much larger than the transverse dimensions. It means that 2D angle coefficients method can be applied for $C_f$ calculation. The beam pipe is clearly separated in four equivalent pumping areas for $\text{H}_2$. This gives the possibility to take into account only one of these areas. Figure 4 shows the approximation used for one of the geometrical solution of the pumping area by line segments.

![Diagram](image)

**Figure 4.** Approximation of the pump area for hydrogen.

The capture factor has been obtained for following geometrical solutions:

1) ![Diagram](image) two slots in beam screen, without electron shield

2) ![Diagram](image) two slots in beam screen, electron shield without slot

3) ![Diagram](image) two slots in beam screen, electron shield with slot
only one slot in the beam screen.
electron shield without slot

In case 3) the width of the electron shield slot is defined as:

\[ d_2 = 3 \cdot 10^{-3} - d_1 \text{ (m)} \]

The calculated results for capture factor and pumping speed for H\textsubscript{2} at T=10K are presented in Table 1.

Table 1. The capture factor and pumping speed per meter length of LHC arc beam pipe.

<table>
<thead>
<tr>
<th>Case</th>
<th>( d_1 ) [\text{mm}]</th>
<th>( h ) [\text{mm}]</th>
<th>( C_f ) ( \alpha=0.1 )</th>
<th>( C_f ) ( \alpha=1 )</th>
<th>( S \text{ (m}^3/(s\cdot m)) ) for H\textsubscript{2} at T=10K ( \alpha=0.1 )</th>
<th>( S \text{ (m}^3/(s\cdot m)) ) for H\textsubscript{2} at T=10K ( \alpha=1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>–</td>
<td>0.48</td>
<td>0.68</td>
<td>0.155 0.220</td>
<td>0.155 0.220</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>–</td>
<td>0.44</td>
<td>0.75</td>
<td>0.215 0.365</td>
<td>0.215 0.365</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>0.26</td>
<td>0.36</td>
<td>0.085 0.115</td>
<td>0.085 0.115</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>2</td>
<td>0.23</td>
<td>0.31</td>
<td>0.110 0.150</td>
<td>0.110 0.150</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
<td>0.39</td>
<td>0.51</td>
<td>0.125 0.165</td>
<td>0.125 0.165</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>2</td>
<td>0.33</td>
<td>0.46</td>
<td>0.160 0.220</td>
<td>0.160 0.220</td>
</tr>
<tr>
<td>4</td>
<td>1.5</td>
<td>3</td>
<td>0.43</td>
<td>0.57</td>
<td>0.105 0.140</td>
<td>0.105 0.140</td>
</tr>
</tbody>
</table>

These calculations were done for the optimized value \( h \) (see Fig. 4). A variation of \( h \) in a range of technological tolerance \( \pm 0.5 \text{mm} \) does not significantly influence the results. The calculation error is less than 5\%.
4 Effective secondary electron emission from surface with artificial roughness.

The effect of secondary electron suppression by surface roughness is well known [8,9]. The surface with artificial roughness has a practical interest because it can be produced mechanically (rolling) very simply.

4.1.1 Geometrical effect.

Approximately, the angular distribution of the secondary electrons or the diffusely reflected electrons from surface with nonzero roughness obeys the cosine distribution. It means that equation (6) can be applied to calculate the angular coefficients for electrons in 2D case. Figure 5 shows a sample of the surface with additional (artificial) roughness. The roughness is determined by only one angular parameter $\alpha$.

Figure 5. Cross-section of the sample with artificial roughness (toothed surface).

The primary electrons hit the surface at normal incidence angle to the general plane. The secondary electron emission yield $\delta$ as a function of $\alpha$ can be estimated as [10]:

$$\delta(\alpha) = \delta_0 \exp \left( \frac{1}{2} \left( 1 - \sin \left( \frac{\alpha}{2} \right) \right) \right)$$

here, it is taken into account that $\frac{\pi}{2} - \frac{\alpha}{2}$ is the angle between primary electrons and a local normal vector to the surface. $\delta_0$ is the secondary electron emission yield at the normal incident primary electrons. A part of secondary should be captured by this kind of surface. Assuming that the secondary electrons are reflected elastically, the capture factor $Cf(\alpha, R(E))$ as a function of $\alpha$ and electron reflection $R(E)$ ($E$ - energy of the secondary electrons) can be calculated using the method of angular coefficients. The reflection coefficient of low energy electrons (90% of the secondary
electrons have an energy less than 20eV) for Cu surface is taken into account using the experimental data from ref.[10]. The energy distribution of the secondary electrons can be fitted as [10]:

\[
\rho(E) = \frac{1}{Z} \exp \left[ \frac{(\ln E / E_{\text{max}})^2}{2 \tau^2} \right]
\]

where \( E_{\text{max}} \approx 1.8eV, \tau \approx 1 \) are the fitting parameters; \( Z = \int_0^{20eV} \rho(E) dE \) is normalised coefficient.

Finally, the effective secondary electron emission yield can be estimated as:

\[
\delta_{\text{eff}}(\alpha) = \delta(\alpha) \int_0^{20eV} \rho(E) \cdot [1 - Cf(\alpha, R(E))] dE
\]

The result of numerical calculation of \( \delta_{\text{eff}}(\alpha) / \delta_0 \) is shown on Figure 6 (solid curve). This result is interesting to compare with a laboratory measurement of the secondary electron emission only. In a real situation the angular distribution of the primary electrons should be taken into account.

4.1.2 Effect of magnetic field. LHC conditions.

The presence of magnetic field, perpendicular to surface, simplifies calculation of capture factor for the secondary electrons by the regular roughness surface. If the Larmor’s radius (about 1.2mkm for 10eV electron in LHC arc magnetic field 8.3 Tesla) much smaller compare to the roughness sizes, the capture factor can be calculated as a sequence of collision with surface:

\[
Cf(\alpha, R(E)) = q_\alpha (1 - R(E)) + q_\alpha R(E) \cdot q_\alpha (1 - R(E)) + \\
+ q_\alpha^2 R^2(E) \cdot q_\alpha (1 - R(E)) + ... = \frac{q_\alpha (1 - R(E))}{1 - q_\alpha R(E)}
\]

where \( q_\alpha = 1 - \sin \frac{\alpha}{2} \) - part of electrons which return to the surface due to their “twist” motion in magnetic field. Here the cosine angular distribution (3) of secondary and reflected electrons is taken into account.

The result of numerical estimation of \( \delta_{\text{eff}}(\alpha) / \delta_0 \) in presence a magnetic field is shown on Figure 6 (dotted curve).
Figure 6. Secondary electron emission suppression by a surface with artificial roughness (toothed). Solid curve: without magnetic field. Dotted curve: with magnetic field.

5 Conclusions

The simplification of the method of angular coefficients for 2D case is done. The written computer program allows for a convenient geometry description. Applied to the calculation of the LHC arc beam pipe molecular pumping properties, it gives possibility to find out an optimum geometry and raises the accuracy of the residual gases density prediction for LHC operation.

An application of this method to estimate the effective secondary electron emission has shown a significant capture of electrons by a surface with artificial roughness. Both factors: the reduction of the secondary electron yield and the reduction of the effective surface reflectivity should significantly reduce the electron-cloud activity. This result is a good input for designing future accelerators with potential electron-cloud problems.

6 Acknowledgments

Many thanks to all CERN AT/VAC group for support of this work and their helpful discussions.
7 References.


