Fluxes, supersymmetry breaking and gauged supergravity

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Abstract

We report on the gauged supergravity interpretation of certain compactifications of superstring theories with p-form fluxes turned on.
We discuss in particular the interplay of duality symmetries in type IIB orientifolds and gauged isometries in the corresponding supergravity models.
Turning on fluxes is generally described by the gauging of some nilpotent Lie group whose generators correspond to axion symmetries of R-R and N-S scalars.

Contribution to the proceedings of “Sugra20” Conference, Department of Physics, Northeastern University, Boston (Ma) 02115 USA; March 2003
1 Introduction

Gauged supergravities in four dimensions have been the subject of a renewed interest in recent time, especially because of their connection with higher dimensional theories compactified on manifolds which allow fluxes of p-forms along the internal directions.

A particularly appealing class of such theories is given by type IIA and IIB compactifications on Ricci-flat manifolds which, in absence of fluxes, leave $N = 8, 4, 2$ unbroken supersymmetries in four dimensions. Turning on fluxes develops a scalar potential in these theories, which usually gives moduli stabilization, reduced supersymmetry and, in certain cases, leaves a vanishing cosmological constant [1]–[7]. In fact, theories with vanishing cosmological constant are generalized no-scale models, which were studied long ago in the pure supergravity framework [8, 9].

Another interesting class of models recently studied in the literature is given by compactifications on $T_6$ and $K^3 \times T_2$ of IIB orientifolds with three-form fluxes turned on [10, 11, 12]. These theories have $N = 4$ and $N = 2$ unbroken supersymmetries respectively, and the presence of fluxes may give rise to partial supersymmetry breaking $N = 4, 2 \rightarrow N = 1, 0$ with vanishing vacuum energy.

Due to the precise knowledge of the moduli space of these theories, an exact estimate of the (tree-level) scalar potential is possible in all these models. The flux-compactifications share the feature that their low energy description is given in terms of an extended supergravity with mass deformations and a scalar potential [13, 14, 15, 16].

In the supergravity framework this can be achieved through the gauging of some of the isometries of the non linear $\sigma$-model spanned by the scalar sector. Indeed, the relevant isometries which are gauged in presence of fluxes are those associated to axion symmetries of those scalars coming from R-R forms $C^{(p)}$ ($p = 0, 2, 4$) and from the N-S two-form $B$ field. In fact the latter only contributes in the generalized case of orientifolds of $T_6$ where the orientifold projection acts only on some directions of the $T_6$ torus [17].

A similar phenomenon occurs in the Scherk–Schwarz generalized compactification of M-theory [18]. Indeed, Scherk–Schwarz spontaneously broken supergravity can be shown to be completely equivalent to a gauged supergravity with a non semisimple gauge group given by the semi-direct product of a $U(1)$ factor, gauged by the Kaluza–Klein vector, times axionic symmetries corresponding to isometries in the scalar sector coming from five dimensional vectors [19, 20, 21] (see [20] also for the construction of the five–dimensional gauged supergravity which describes the spontaneously broken model deriving from $D = 6 \rightarrow D = 5$ Scherk–Schwarz dimensional reduction). The $U(1)$ symmetry is related to the central charge of the BPS massive representations of the spontaneously broken theory [19, 20, 21].

In all the above mentioned compactifications, the underlying supergravity relies on a gauge group which is a certain subgroup of the duality group having a linear (adjoint) action on the vector fields of the theory. Since the most general duality transformation in $D = 4$ is an element of $Sp(2n_V, \mathbb{R})$ (where $n_V$ denotes the number of vectors in the theory), the gauge group must be a lower block-triangular symplectic matrix, in order not to mix electric with magnetic fields.

The same ungauged $N$-extended supergravity can therefore have several inequivalent deformations, corresponding to different choices of the gauge group. The latter is chosen by selecting, among the duality symmetries, the ones which are realized linearly on the


## 2 Duality rotations and gauging

According to Gaillard and Zumino [22], the most general linear transformation between electric and magnetic field strengths $F^\Lambda, G_\Lambda$ ($\Lambda = 1, \cdots, n_V$), in a given four dimensional theory with vectors coupled to scalar fields, is a $Sp(2n_V, \mathbb{R})$ transformation, acting on the self-dual components $F^{\Lambda+}, G^+_{\Lambda}$ as

$$
\left( \begin{array}{c} F^\Lambda \\ G_\Lambda \end{array} \right)' = \left( \begin{array}{cc} A^\Lambda \Sigma & B^\Lambda \Sigma \\ C_{\Lambda \Sigma} & D^\Lambda \Sigma \end{array} \right) \left( \begin{array}{c} F^{\Sigma} \\ G^{\Sigma} \end{array} \right),
$$

where the matrix $S = \left( \begin{array}{cc} A & B \\ C & D \end{array} \right)$ is symplectic \(^b\).

If we define a complex symmetric matrix $N_{\Lambda \Sigma}$ through the equation

$$
G^{\Lambda+} = N_{\Lambda \Sigma} F^{\Sigma},
$$

then under a duality rotation (2.1) $N$ gets transformed to

$$
N' = (C + DN) \cdot (A + BN)^{-1}.
$$

The matrix $N$ appears in the vector Lagrangian as

$$
\mathcal{L} = 2\Im(N_{\Lambda \Sigma} F^{\Lambda+} F^{\Sigma}) = 2\Im(G^+_{\Lambda} F^{\Lambda}).
$$

For infinitesimal transformations we have

$$
\delta F^{\Lambda+} = a^\Lambda \Sigma F^{\Sigma+} + b^{\Lambda \Sigma} G^+_{\Sigma}, \quad b^{\Lambda \Sigma} = b^{\Sigma \Lambda};
$$

$$
\delta G^+_{\Lambda} = c_{\Lambda \Sigma} F^{\Sigma+} - a^\Sigma \Lambda G^+_{\Sigma}, \quad c_{\Lambda \Sigma} = c_{\Sigma \Lambda};
$$

$$
\delta N = c - a^T N - N a - N b N.
$$

The most general subgroup of $Sp(2n_V, \mathbb{R})$ leaving the Lagrangian invariant up to a total derivative is obtained by setting $B = b = 0$. In this case the vector fields $A^\Lambda$ (such that $F^\Lambda = dA^\Lambda$) transform as

$$
\delta A^\Lambda = a^\Lambda \Sigma A^{\Sigma}
$$

and

$$
\delta N = c - a^T N - N a.
$$

A duality transformation can be chosen as gauge symmetry if it can be found as a certain subalgebra of those duality symmetries which leave the action invariant, that is only if $B = 0$.

\(^a\)where a generic self-dual field strength $T^+$ is defined by $T^{\mu+}_{\nu \rho \sigma} = \frac{1}{2} (T_{\mu \nu} + \frac{1}{3} \epsilon_{\mu \nu \rho \sigma} T^{\rho \sigma})$.

\(^b\)that is $S^T \Omega S = \Omega$, which implies, by choosing as symplectic metric $\Omega = \left( \begin{array}{cc} 0 & \mathbb{I} \\ -\mathbb{I} & 0 \end{array} \right)$, that $(A^T C)$, $(B^T D)$ are symmetric and $A^T D - C^T B = \mathbb{I}$. Passing to the algebra, for $S = \mathbb{I} + s$, $s = \left( \begin{array}{cc} a & b \\ c & d \end{array} \right)$, this implies $b^T = b$, $c^T = c$, $d^T = -a$.
Axion symmetries have $C \neq 0$. In this case the gauge group can be non-abelian or abelian depending on whether the block $a$ is different from zero or not. Moreover, axion isometries are usually embedded in nilpotent groups, and this implies that the matrix $a$ itself is also lower (or upper) triangular.

$\sigma$-models coupled to vectors have a richer structure than ordinary $\sigma$-models. For coset spaces $G/H$, $G$ acts on the vector field strengths and their duals in a symplectic representation. In particular, in this representation the coset representatives are symplectic matrices $L(\phi) = \begin{pmatrix} A(\phi) & B(\phi) \\ C(\phi) & D(\phi) \end{pmatrix}$ through which, by defining

$$f(\phi) = \frac{1}{\sqrt{2}}(A - iB), \quad h(\phi) = \frac{1}{\sqrt{2}}(C - iD),$$

one gets $N = h \cdot f^{-1}$ as an explicit, scalar dependent, expression for the $N$ matrix. This formula allows the computation of the vector Lagrangian from first principles.

A simplification here occurs by choosing a solvable parametrization of the coset in the symplectic representation. In this case $B(\phi) = 0$ and $D(\phi) = (A^T)^{-1}$, so that

$$N = [C - i(A^T)^{-1}] \cdot A^{-1} = C \cdot A^{-1} - i(A \cdot A^T)^{-1}.$$  \hspace{1cm} (2.9)

Let us now suppose to gauge some subalgebra of the $(a, c)$ generators which has an adjoint action on the vectors (just trivial in the abelian case). Then, in such case

$$a^A_{\Sigma} = f^A_{\Sigma \Gamma} \xi^\Gamma; \quad c_{A\Sigma, \Gamma} = c_{A\Sigma, \Gamma} \xi^\Gamma$$  \hspace{1cm} (2.10)

where the constants $c_{A\Sigma, \Gamma}$ satisfy $c_{(A\Sigma, \Gamma)} = 0$ and an additional cocycle condition in the non abelian case, where $f^A_{\Sigma \Gamma} \neq 0:

$$\frac{1}{2} f^T_{A\Sigma} c_{\Delta \Omega, \Gamma} + c_{\Gamma \Omega, [\xi]A} f^T_{[\xi]A} + c_{\Gamma \Delta, [\xi]A} f^T_{[\xi]A} = 0$$  \hspace{1cm} (2.11)

These conditions appear because of the fact that, when $c$ generators (axions) are gauged, a gauge- and supersymmetry-invariant Lagrangian can be found only after adding to the Lagrangian a Chern-Simons type term, proportional to $c_{A\Sigma, \Gamma}$ \cite{23}.

The above is the general structure of the gauge transformations in extended supergravity when the axion symmetries are gauged.

This is a feature common to all orientifold models with fluxes turned on as well as to models obtained from Scherk–Schwarz generalized dimensional reduction.

### 3 T$_6$ orientifolds, nilpotent algebras and gaugings

In the present section we shall briefly review the construction, which was accomplished in \cite{17}, of some new four dimensional orientifold models (both in type IIB and IIA) where the orientifold projection involves an orbifold projection with respect to the space–inversion $I_{9-p}$ on $9-p$ coordinates, transverse to the $Dp$-brane world volume. Since in this construction the $Dp$–brane world volume fills the non-compact space-time, the torus $T_6$ is naturally split into Neumann directions $T_{p-3}$ and Dirichlet directions $T_{9-p}$. These models therefore will be denoted by $T_{p-3} \times T_{9-p}$. Their low-energy descriptions (in
absence of fluxes and $D$–branes) are all given in terms of a four dimensional ungauged $\mathcal{N} = 4$ supergravity coupled to six vector multiplets from the closed string sector. In the following discussion we shall always restrict ourselves to the bulk degrees of freedom. The scalar fields span a manifold with the following $G/H$ form:

$$\mathcal{M}_{\mathcal{N}=4} = \frac{SL(2, \mathbb{R})}{SO(2)} \times \frac{SO(6, 6)}{SO(6) \times SO(6)},$$

(3.12)

the duality group being $G = SL(2, \mathbb{R}) \times SO(6, 6)$. We summarize in Table 1 the bosonic field content of these models in the type IIB and IIA settings respectively (in our conventions the directions of $T_6$ are labeled by an index $n = 1, \ldots, 6$ whereas the directions of $T_{p-3}$ and of $T_{9-p}$ are labeled respectively by indices $i = 1, \ldots, p-3$ and $a = p-2, \ldots, 9$ and the non–compact space–time directions by Greek letters). The fluxes allowed by the orientifold projections are listed in Table 2.

### Table 1: Massless degrees of freedom for the IIB/IIA orientifolds

<table>
<thead>
<tr>
<th>$p$</th>
<th>scalars</th>
<th>vectors</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>$g_{ij}$, $\phi$, $C_{i\mu}$, $C^i_{\mu}$</td>
<td>$G^i_{\mu}$, $C^i_{\mu}$</td>
<td>IIB</td>
</tr>
<tr>
<td>7</td>
<td>$g_{ij}$, $g_{ab}$, $\phi$, $B_{ia}$, $C_{i\mu}$, $C_{i\mu}$, $C_{ijkl}$, $C_{ijab}$</td>
<td>$G^i_{\mu}$, $B_{ia}$, $C_{i\mu}$, $C_{ij\mu}$</td>
<td>IIA</td>
</tr>
<tr>
<td>5</td>
<td>$g_{ij}$, $g_{ab}$, $\phi$, $B_{ia}$, $C_{i\mu}$, $C_{ij}^i$, $C_{iab}$, $C_{iabc}$</td>
<td>$G^i_{\mu}$, $B_{ia}$, $C_{ij\mu}$, $C_{a\mu\nu}$</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$g_{ab}$, $\phi$, $C_{i\mu}$, $C_{i\mu}$</td>
<td>$B_{a\mu}$, $C_{a\mu}$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$g_{ij}$, $g_{ab}$, $\phi$, $B_{ia}$, $C_{i\mu}$, $C_{ij6}$, $C_{ij6}$</td>
<td>$G^i_{\mu}$, $C_{i\mu}$, $C_{i\mu}$, $B_{a\mu}$</td>
<td>II A</td>
</tr>
<tr>
<td>6</td>
<td>$g_{ij}$, $g_{ab}$, $\phi$, $B_{ia}$, $C_{i\mu}$, $C_{ij6}$, $C_{ij6}$</td>
<td>$G^i_{\mu}$, $B_{ia}$, $C_{ij\mu}$, $C_{a\mu}$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$g_{11}$, $g_{ab}$, $\phi$, $B_{1a}$, $C_{1\mu}$, $C_{1\mu}$, $C_{1ab}$</td>
<td>$G^i_{\mu}$, $B_{a\mu}$, $C_{1a\mu}$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Allowed fluxes for the IIB/IIA orientifolds. $F$, $H$ and $G$ fluxes are associated to the $B$, $C_2$ and $C_4$ fields

<table>
<thead>
<tr>
<th>$p$</th>
<th>fluxes</th>
<th>type</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>none</td>
<td>IIB</td>
</tr>
<tr>
<td>7</td>
<td>$H_{aij}$, $F_{aij}$, $G_{aijkl}$</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$H_{abc}$, $F_{iab}$, $H_{ija}$, $G_{ijabc}$</td>
<td>II A</td>
</tr>
<tr>
<td>3</td>
<td>$H_{abc}$, $F_{abc}$</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>$H_{ij6}$, $G_{ij6}$</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>$H_{ija}$, $H_{abc}$, $F_{ia}$, $G_{ijab}$</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>$H_{abc}$, $F_{ab}$, $G_{1abc}$</td>
<td></td>
</tr>
</tbody>
</table>

The $\mathcal{N} = 4$ orientifold models (in the absence of fluxes) can be consistently constructed as truncations of the unique four dimensional $\mathcal{N} = 8$ supergravity which describes the low-energy limit of toroidally compactified type II superstrings. In the scalar-field sector this
amounts to defining the embedding of the $\mathcal{N} = 4$ scalar manifold into the $\mathcal{N} = 8$ one. The duality group of the latter theory is $E_7(7)$ which acts non-linearly on the 70 scalar fields, and linearly, as a $Sp(56, \mathbb{R})$ symplectic transformation, on the 28 electric field strengths and their magnetic dual. Within this theory an intrinsic group-theoretical characterization of the ten dimensional origin of the scalar and vector fields can be achieved. In the so-called solvable Lie algebra representation of the scalar sector \cite{24, 25, 26}, the scalar manifold

$$\mathcal{M}_{\mathcal{N}=8} = \exp (\text{Solv}(e_{7(7)})), \quad (3.13)$$

is expressed as the group manifold generated by the solvable Lie algebra $\text{Solv}(e_{7(7)})$ defined by the Iwasawa decomposition of the $e_{7(7)}$ algebra:

$$e_{7(7)} = su(8) + \text{Solv}(e_{7(7)}). \quad (3.14)$$

In this framework, there is a natural one-to-one correspondence between the scalar fields and the generators of $\text{Solv}(e_{7(7)})$. The latter consist of the $e_{7(7)}$ Cartan generators $H_p$, parametrized by the $T_6$ radii $R_n = e^{\sigma_n}$ together with the dilaton $\phi$, and of the shift generators corresponding to the 63 positive roots $\alpha$ of $e_{7(7)}$, which are in one-to-one correspondence with the axionic scalars that parametrize them. This correspondence between Cartan generators and positive roots on one side and scalar fields on the other, can be pinpointed by decomposing $\text{Solv}(e_{7(7)})$ with respect to some relevant groups \cite{24}. The axions deriving from ten–dimensional tensor fields (i.e. the Kalb–Ramond form $B_{MN}$ and the R–R fields) transform in tensorial representations of the $SL(6, \mathbb{R})_g$ isometry group of the $T_6$ metric moduli and so do the corresponding solvable generators. If we express the $e_{7(7)}$ roots with respect to an orthonormal basis of Euclidean vectors $\{\epsilon_{r}\}$, $r = 1, \ldots, 7$, the precise correspondence between axions and $e_{7(7)}$ nilpotent generators reads:

$$C_{n_1n_2\ldots n_k} \leftrightarrow T^{m_1m_2\ldots m_{6-k}} = E^{a+\epsilon_{n_1}+\ldots+\epsilon_{n_k}}, \quad (3.15)$$

$$C_{n_1n_2\ldots n_k\mu\nu} \leftrightarrow T^{m_1m_2\ldots m_{6-k}} = E^{a+\epsilon_{m_1}+\ldots+\epsilon_{m_{6-k}}}, \quad (\epsilon^{n_1\ldots n_km_1\ldots m_{6-k}} \neq 0),$$

$$B_{nm} \leftrightarrow T_{B}^{nm} = E^{\epsilon_{n}+\epsilon_{m}},$$

$$B_{\mu\nu} \leftrightarrow T = E^{\sqrt{2}\epsilon_7},$$

$$G_{nm} \leftrightarrow T^{n} = E^{\epsilon_{n}-\epsilon_{m}}, \quad (n \neq m),$$

where

$$a = -\frac{1}{2} \sum_{i=1}^{6} \epsilon_i + \frac{1}{\sqrt{2}} \epsilon_7. \quad (3.16)$$

The torus radii $R_n = e^{\sigma_n}$ and the ten–dimensional dilaton $\phi$ enter this description in the following combination with the Cartan generators:

$$\vec{h} \cdot \vec{H} = \sum_{p=1}^{7} h^p H_{\epsilon_p} = \sum_{n=1}^{6} \sigma_n H_{(\epsilon_n+\frac{1}{\sqrt{2}} \epsilon_7)} - \frac{\phi}{2} H_a. \quad (3.16)$$

so that the kinetic term of the axion corresponding to the root $\alpha$ contains the correct exponential factor $e^{-2\vec{h} \cdot \vec{a}}$.

As far as the scalar sector is concerned, the embedding of the $\mathcal{N} = 4$ orientifold models $T_{p-3} \times T_{9-p}$ (in absence of fluxes) inside the $\mathcal{N} = 8$ theory (in its type IIA or
IIB Superstring

IIA Superstring

Figure 1: $SO(6, 6)$ Dynkin diagrams for the $T_{p-3} \times T_{9-p}$ models. The shaded sub-diagrams define the groups $SL(p-3, \mathbb{R}) \times SL(9-p, \mathbb{R})$ acting transitively on the metric moduli. The empty circles define simple roots corresponding to the metric moduli $g_{ij}, g_{ab}$, the gray circle denotes a simple root corresponding to a Kalb–Ramond field $B_{ia}$ and the black circle corresponds to a R–R axion.

IIB versions) is defined by the condition that the embedding of the $\mathcal{N} = 4$ duality group $SL(2, \mathbb{R}) \times SO(6, 6)$ inside the $E_{7(7)}$ fulfill the following condition:

$$SO(6, 6) \cap GL(6, \mathbb{R})_g = O(1, 1) \times SL(p-3, \mathbb{R}) \times SL(9-p, \mathbb{R}).$$

Condition (3.17) amounts to requiring that the $T_6$ metric moduli in our models are related either to the $T_{p-3}$ metric $g_{ij}$ or to the $T_{9-p}$ metric $g_{ab}$. It fixes the ten–dimensional interpretation of the fields in the ungauged $\mathcal{N} = 4$ models (except for the cases $p = 3$ and $p = 9$), and for a given $p$ it is consistent with the bosonic spectrum resulting from the corresponding orientifold reductions. In the $p = 3$ and $p = 9$ cases, the two embeddings are characterized by a different interpretation of the scalar fields, related to the $T_6/\mathbb{Z}_2$ orientifold reduction in the presence of D3 or D9 branes. The axions not deriving from the internal metric consist of $C_{i_1...i_{p-3}}$ in the external $SL(2, \mathbb{R})/SO(2)$ factor, $(p-3) (9-p)$ moduli $B_{ia}$ in the bi-fundamental of $SL(p-3, \mathbb{R}) \times SL(9-p, \mathbb{R})$ and 15 R–R moduli which we shall generically denote by $C_I$ and which span the maximal abelian ideal $\{T^I\}$ of $Solv(so(6, 6))$. The scalars $B_{ia}$ and $C_I$ parametrize a $15 + (p-3) (9-p)$ dimensional subalgebra $N_p$ of $Solv(so(6, 6))$ consisting of nilpotent generators only.

Switching on fluxes amounts in the four–dimensional theory to introducing a suitable gauge group contained inside $N_p$. Let us first list for the various models the generator content of $N_p$ and their algebraic structure.

$T_0 \times T_6$ model — In this case $N_3$ is abelian and its generic element is $C_{abcd} T^{abcd}$.

$T_2 \times T_4$ model — The algebra $N_5$ is 23–dimensional and has the following structure:

$$N_5 \equiv \{C_{\mu \nu} T + B_{ia} T_B^{ia} + C_{iabc} T^{iabc} + C_{ab} T^{ab}\},$$

$$[T_B^{ia}, T^{bc}] = T^{iabc}, \quad [T_B^{ia}, T^{jbcd}] = \epsilon^{ij} \epsilon^{abcd} T. \quad (3.18)$$
$T_4 \times T_2$ model — The algebra $N_7$ is 23–dimensional and has the following structure:

$$N_7 \equiv \left\{ C(0) T_0 + B_{ia} T^{ia}_B + C_{ia} T^{ia} + C_{ijab} T^{ijab} \right\},$$

$$[T^{ia}_B, T^{ib}_B] = T^{ia}, \quad [T^{ia}, T^{jb}] = T^{ijab}. \quad (3.19)$$

$T_6 \times T_0$ model — In this case $N_9$ is abelian and its generic element is $C_{ab} T^{ab}$.

$T_1 \times T_3$ model — The algebra $N_4$ is 20–dimensional and has the following structure:

$$N_4 \equiv \left\{ C_{a\mu} T_a + B_{ia} T^{ia}_B + C_{abc} T^{abc} \right\},$$

$$[T^{abc}, T^{ia}_B] = \epsilon^{abcde} T_e. \quad (3.20)$$

$T_3 \times T_3$ model — The algebra $N_6$ is 24–dimensional and has the following structure:

$$N_6 \equiv \left\{ C_a T^a + B_{ia} T^{ia}_B + C_{iab} T^{iab} + C_{ijab} T_{ij} \right\},$$

$$[T^a, T^{ib}_B] = T^{iab}, \quad [T^{ia}, T^{jbc}] = \epsilon^{ijk} \epsilon^{abc} T_k. \quad (3.21)$$

$T_5 \times T_1$ model — The algebra $N_8$ is 20–dimensional and has the following structure:

$$N_8 \equiv \left\{ C_i T^i + B_{ij} T^{ij}_B + C_{ij} T^{ij} \right\},$$

$$[T^i, T^{j6}_B] = T^{ij6}. \quad (3.22)$$

**Fluxes and gaugings: a preliminary analysis** — Let us consider the $T_2 \times T_4$ model in presence of the fluxes $H_{i\mu} = \epsilon_{ij} H_a X^a$ and $F_{iab}$. These fluxes appear as structure constants

$$[X_i, X_j] = \epsilon_{ij} H_a X^a, \quad [X_i, X^a] = F_{iab} X^b, \quad (3.23)$$

of a gauge algebra $G_g \equiv \{ X_i, X^a, X_\mu \}$ with connection $\Omega^a_\mu = G^a_i X_i + B_{a\mu} X^a + C^a_\mu X_\mu$, all other commutators vanishing.

The identification

$$X'_i = -F^{ab}_i T_{ab} + H_a T^a_i, \quad X'^a = F^{ab}_i T^a_b, \quad (3.24)$$

of the gauge generators with the isometries of the solvable algebra (in our conventions $F^{ab}_i = \frac{1}{2} \epsilon^{abcd} F_{i\mu} T^{a\mu}$ and $T^a_i = (1/3!) \epsilon_{abcd} T^{ibcd}$), reproduces only a contracted version of the algebra (3.23) in which three of the central charges $X_a$ vanish and we are left with $X'_a = -H_a T$. If we denote by $\{ X^\perp \} = \{ X_a \}/\{ X'_a \}$ these three central generators, we see that the subgroup $G'_g = \{ X'_i, X'^a, X'_\mu \}$ of the isometry group which is gauged coincides with the quotient:

$$G'_g = G_g/\{ X^\perp \}, \quad (3.25)$$

that amounts to imposing the vanishing of the central terms $X^\perp$ on all fields. One can verify that the vectors $G^a_\mu$, $B_{a\mu}$ and $C^a_\mu$ transform in the co-adjoint representation of $G_g$.

The non-abelian field strengths are:

$$H_{a\mu} = \partial_\mu B_{a\nu} - \partial_\nu B_{a\mu} - \epsilon_{ij} H_a G^i_\mu G^j_\nu,$$

$$F^{a\mu} = \partial_\mu C^a_\nu - \partial_\nu C^a_\mu + F^{iab}_i G^a_\mu B_{b\nu} - F^{iab}_i G^b_\nu B_{b\mu},$$

$$F^{i\mu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu. \quad (3.26)$$
and the covariant derivatives for the scalar fields read:

\[ D_\mu c = \partial_\mu c + H_a C^a_\mu - B_{a\mu} F^{ab}_i B_{b\mu}, \]
\[ D_\mu C^a_i = \partial_\mu C^a_i - B_{b\mu} F^{ba}_i - G^a_\mu F^{ab}_j B_{b\mu}, \]
\[ D_\mu B_{ia} = \partial_\mu B_{ia} + G_{i\mu} H_a, \]
\[ D_\mu C_{ab} = \partial_\mu C_{ab} + G^a_\mu F^{ab}_i. \]  

(3.27)

As far as the \( T_4 \times T_2 \) model is concerned let us consider the gauging which corresponds to turning on the fluxes \( F_ija, H_ija \). It is useful to describe the \( H \) and \( F \) forms as elements of an \( SL(2, \mathbb{R}) \) doublet labeled by an index \( \alpha = 1, 2 \), the couple of indices \((\alpha, a)\) span the representation \( 4 \) of \( SO(2, 2) \). We shall then denote the flux–forms by \( H^\lambda_{ij} \) where \( \lambda \) is the index of the \( 4 \) in which the invariant metric is diagonal \( \eta_{\lambda \lambda'} = \text{diag}(-1, -1, +1, +1) \). Similarly the generators \( T^\alpha a, T^\alpha j \) will be denoted by \( T^\lambda i \) and the vectors \( B_{a\mu}, C_{a\mu} \) by \( A^\lambda_\mu \). Inspection of the dimensionally reduced three–form kinetic term indicates for the four–dimensional theory a gauge group \( G \) with connection \( \Omega = X^i G^i_\mu + X^\lambda A^\lambda_\mu \) and the following structure:

\[ [X^i, X^j] = H^\lambda_{ij} X^\lambda. \]  

(3.28)

If we identify the gauge generators with isometries as follows:

\[ X^i = -H^\lambda_{ij} T^j_\lambda, \]
\[ X^\lambda = \frac{1}{2} H^\lambda_{ij} T^{ij}, \]  

(3.29)

then it can be shown that they close the algebra (3.28) only if \( H^\lambda_{ij} H^\lambda_{ij} = 0 \) which amounts to requiring that \( \int_{T^6} H^{(3)} \wedge F^{(3)} = 0 \) (this condition is consistent with a constraint found in [27] on the embedding matrix of a new gauge group in the \( N = 8 \) theory, which seems to yield an \( N = 8 \) “lifting” of the type IIB orientifold models \( T^{p-3} \times T^{9-p} \) discussed here).

The vectors transform indeed in the co–adjoint of \( G \) and the non–abelian field strengths are:

\[ F_\mu^\lambda = \partial_\mu A^\lambda_\mu - \partial_\nu A^\lambda_\nu - H^\lambda_{ij} C^i_\mu G^j_\nu, \]
\[ F_\mu^i = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu, \]
\[ F_\mu^i = \epsilon^{ijkl} (\partial_\mu C_{jkl\nu} - \partial_\nu C_{jkl\mu}). \]  

(3.30)

The relevant covariant derivatives for the axions read:

\[ D_\mu C_{ij} = \partial_\mu C_{ij} - \frac{1}{2} H_{ij \lambda} A^\lambda_\mu + \frac{1}{2} G^k_\mu H^\lambda_{ki} \Phi_{jk\lambda}, \]
\[ D_\mu \Phi^\lambda_i = \partial_\mu \Phi^\lambda_i - H^\lambda_{ij} G^j_\mu. \]  

(3.31)

4  Supersymmetry breaking in \( T^6/\mathbb{Z}_2 \) and \( K3 \times T^2/\mathbb{Z}_2 \) orientifolds

In this section we are going to describe the basic features of the supersymmetry breaking pattern induced by the presence of non trivial 3-form fluxes in two orientifold compactifications of IIB superstring. In particular, we will focus on the bulk sector of the theories.
which is the one responsible for the supersymmetry breaking and moduli stabilization, without considering here the D-brane degrees of freedom.

Before entering into the details, let us briefly summarize the conditions for the vacuum to preserve some unbroken supersymmetry. They are exhausted by the request that, in the vacuum, the supersymmetry transformation laws of all the fermions $f$ are zero for the corresponding supersymmetry parameter $\epsilon$, that is

\[
\begin{align*}
&<\delta f >= 0 \Rightarrow \left\{ \begin{array}{l}
\delta \psi_A = 0 : \quad <S_{AB}(\phi, g)> \epsilon^B = -\sqrt{<V(\phi)>_6} \epsilon_A \\
\delta \lambda^{IA} = 0 : \quad <N^{IA}(\phi, g)> \epsilon_A = 0
\end{array} \right.
\end{align*}
\]

where $\psi_A$ are the gravitini and $\lambda^{IA}$ generic spin 1/2 fields, while $S_{AB}$ and $N^{IA}$ denote the gravitino mass matrix and fermionic shifts respectively.

$V(\phi)$ is the scalar potential defined by

\[
\delta_B^A V(\phi) = -3 S^{AC} S_{BC} + N^{IA} N_{IB}
\]

Both the models considered in this section have a positive definite scalar potential, allowing vacuum configurations with partial super Higgs and zero vacuum energy. This is related to the fact that, in both cases, the gauging responsible for the presence of a scalar potential corresponds to switching on charges only for translational isometries of the scalar manifolds. It only depends, in fact, on universal properties leading to cancellation of positive and negative contributions in the scalar potential as it occurred in $N = 1$ no-scale models [8, 9].

### 4.1 The $T_6/Z_2$ orientifold

The bulk sector of the $T_6/Z_2$ orientifold model is described by $N = 4$ supergravity coupled to 6 vector multiplets, where, as discussed in section 3, the 38 scalars parametrize the coset manifold $SL(2, \mathbb{R})/U(1) \times SO(6, 6)/[SO(6) \times SO(6)]$ (for a derivation of this model from dimensional reduction of type IIB theory see [11] and references therein).

Its gauged supergravity description (first constructed in [13]) relies on an abelian gauge algebra which corresponds to a 12 dimensional subalgebra of the 15 shift symmetries of the scalars coming from the R-R 4-form $C_{\Gamma\Delta\Sigma\Omega} \equiv \epsilon_{\Lambda\Sigma\Gamma\Delta\Omega} B^{\Lambda\Sigma}; \; \Lambda, \Sigma, \cdots = 1, \cdots, 6$. The gauge vectors $A^\alpha_\mu_\Delta$ (with $\alpha = 1, 2$) come from the N-S and R-R two forms with one internal index. In terms of the $B^{\Lambda\Sigma}$ the covariant derivatives are

\[
D_\mu B^{\Lambda\Sigma} = \partial_\mu B^{\Lambda\Sigma} - f_{\alpha}^{\Lambda\Sigma\Delta} A^\alpha_\mu_\Delta
\]

where $f_{\alpha}^{\Lambda\Sigma\Delta}$ are the real N-S and R-R three-form fluxes. In presence of the fluxes $f_{\alpha}^{\Lambda\Sigma\Delta}$, the theory develops a scalar potential. In order for the potential to have a minimum with vanishing cosmological constant, it is required that the fluxes are subject to the constraint

\[
f_{-}^{\Lambda\Sigma\Delta} = i \alpha f_{2}^{\Lambda\Sigma\Delta}
\]

where $\alpha$ is a complex constant (the minus apex denotes the anti self-dual projection in the internal manifold). Condition (4.34) breaks the symmetry $SL(2, \mathbb{R}) \times GL(6) \subset SL(2, \mathbb{R}) \times SO(6, 6)$ to $U(4)$. 

In the vacuum, the $SL(2,\mathbb{R})/U(1)$ variable $S = C + ie^\phi$ is actually fixed at the value $S = -L_2/L_1 = i\alpha$, that is, in terms of the N-S and R-R IIB dilatons $(\phi, C)$

$$e^\phi = \Re\alpha, \quad C = -\Im\alpha \quad (\Re\alpha > 0), \quad (4.35)$$

where $L^\alpha$ is the complex component of the coset representative satisfying the $SL(2,\mathbb{R})$ condition $L^\alpha L^\beta - L^\alpha L^\beta = e^{i\alpha\beta}$. Equation (4.35) also implies that the N-S and R-R fluxes must both be present to have dilaton stabilization, so that the solution is non-perturbative at the string level.

The basic features of the model are encoded in the two matrices

$$F^{IJK} \equiv L^\alpha f_{IJK}^{\alpha}, \quad \bar{F}^{IJK} \equiv \bar{L}^\alpha f_{IJK}^{\alpha} \quad (4.36)$$

where $f_{IJK}^{\alpha} \equiv f_{\Lambda\Sigma\Delta}^{\alpha} E_I^{\Lambda} E_J^{\Sigma} E_K^{\Delta}$ and $E^I_\Lambda$ are the coset representatives of $GL(6)/SO(6) \subset SO(6,6)/[SO(6) \times SO(6)]$. Indeed, by using the $SU(4) \sim \text{spin}(SO(6))$ matrices one can define the two complex symmetric matrices

$$S_{AB} = -\frac{i}{48} \bar{F}^{-IJK}(\Gamma_{IJK})_{AB}$$
$$N^{AB} = \frac{1}{48} \bar{F}^{+IJK}(\Gamma_{IJK})^{AB} \quad (4.37)$$

where $S_{AB}$ is the gravitino mass matrix and $N^{AB}$ the fermionic shift in the dilatino supersymmetry transformation law. They have the same $U(1)$ charge, but contra-gradient $SU(4)$ representations $10, \overline{10}$ respectively. The scalar potential is actually given in terms of $N^{AB}$:

$$V \propto |N^{AB}|^2 \quad (4.38)$$

For generic values of the fluxes (still constrained by equation (4.34)) equation $N^{AB} = 0$ stabilizes the dilaton $e^\phi$ and axion $C$ together, while fixing $E^I_\Lambda$ to the diagonal form $\text{diag}(e^{\phi_1}, e^{\phi_2}, e^{\phi_3}, e^{\phi_1}, e^{\phi_2}, e^{\phi_3})$. There is no residual supersymmetry in this case.

Some residual supersymmetry is preserved when some extra constraints among the fluxes are satisfied. By adopting complex coordinates $I \rightarrow (i, \bar{i})$, $i, \bar{i} = 1, 2, 3$ (so that $F^{IJK} \rightarrow (f^{ijk}, f^{ijk}, f^{ijk}, f^{ijk})$), the four gravitino masses are given by

$$|f_{ijk}|, \quad |f_{ijk}|; \quad i \neq j \neq k \quad (4.39)$$

There are in fact four different values of this type. When any one of them vanishes, then one supersymmetry remains unbroken. In this case some of the $E^I_\Lambda$ moduli remain unfixed.

For instance, for preserving $N = 3$ supersymmetry it is needed that $f_{ijk} \neq 0$, while only $f_{ijk} = f_{ijk} \neq 0$. Then the $g_{ij}$ entries of the scalar metric are fixed to zero, while the $g_{ij}$ remain unfixed. Correspondingly, the six axions $B^{ij}$ are eaten by six of the vectors, while the remaining nine axions $B^{ij}$ remain massless. The resulting $N = 3$ moduli space is $U(3,3)/[U(3) \times U(3)]$, as predicted by $N = 3$ supergravity. There is a single massive spin 3/2 multiplet, which is long (not BPS saturated) and this is in agreement with the fact that there are not gauged central charges in this model. This is to be contrasted to what happen with the Scherk–Schwarz $N = 8$ spontaneously broken supergravity, where all the massive multiplets are 1/2 BPS saturated.
4.2 The $K3 \times T_2/Z_2$ orientifold

Let us now turn to discuss the $K3 \times T_2/Z_2$ orientifold model, which has been studied in the literature both as a compactification from ten dimensional IIB theory as well as a four dimensional gauged supergravity.

From a four dimensional point of view it is an $N = 2$ supergravity model coupled to 3 vector multiplets and 20 hypermultiplets. The scalars of the vector multiplets parametrize the special Kähler manifold

$$\mathcal{M}_V = \left[ \frac{SL(2, \mathbb{R})}{SO(2)} \right]^3 = \frac{SL(2, \mathbb{R})}{SO(2)} \times \frac{SO(2, 2)}{SO(2) \times SO(2)} \quad (4.40)$$

while the scalars in the hypermultiplets parametrize the quaternionic manifold

$$\mathcal{M}_H = \frac{SO(4, 20)}{SO(4) \times SO(20)} \quad (4.41)$$

The latter can be regarded as a fibration over

$$\frac{SO(4, 20)}{SO(4) \times SO(20)} = \frac{SO(3, 19)}{SO(3) \times SO(19)} \times \mathbb{R}^+ + 22 \quad (4.42)$$

where the presence on $\mathcal{M}_H$ of 22 translational isometries $(C^m, C^a)$, $(m = 1, 2, 3; \ a = 1, \cdots, 19)$ corresponding to the degrees of freedom of the 10 dimensional R-R 4-form, with two indices on the $K3$ and two on the torus, is put in evidence.

Switching on three-form fluxes corresponds to gauging some of the 22 translational isometries of $\mathcal{M}_H$ by some of the vectors $A^\Lambda_\mu$ ($\Lambda = 0, 1, 2, 3$) in the theory, where $\Lambda = (i, \alpha)$, $i \in SL(2, \mathbb{R})_{T2}$, $\alpha \in SL(2, \mathbb{R})_{IIB}$.

The covariant derivatives are

$$D_\mu C_m = \partial_\mu C^m + f^m_\Lambda A^\Lambda_\mu, \quad (4.43)$$

$$D_\mu C_a = \partial_\mu C^a + h^a_\Lambda A^\Lambda_\mu. \quad (4.44)$$

Here, the couplings $f^m_\Lambda, h^a_\Lambda$ correspond to the N-S and R-R three-form fluxes with one index on the torus and two on $K3$.

The presence of fluxes allows step-wise partial super-Higgs $N = 2 \to N = 1 \to 0$ with zero vacuum energy.

$N = 2$ and $N = 1, 0$ vacua correspond to two gaugings different in the choice of quaternionic isometries and of gauged vectors. In all cases many of the moduli are stabilized.

To obtain configurations with $N = 2$ supersymmetry one should switch on the fluxes $h_1^2 = g_2, h_3^2 = g_3$, taking as would be Goldstone bosons the scalars $C^a_{=1}, C^a_{=2}$. In this case the vectors which become massive are the vector partner of IIB dilaton and of $T^2$ complex structure moduli. In fact, two of the original massless hypermultiplets and the two vector multiplets of $A_2$ and $A_3$ combine into two long massive vector multiplets $[1, 4(\frac{1}{2}), 5(0)]$. We see that the $N = 2$ configurations are just an example of the Higgs phenomenon of two vector multiplets. The residual moduli space is

$$\frac{SO(4, 18)}{SO(4) \times SO(18)} \times \frac{SU(1, 1)}{U(1)} \quad (4.45)$$

\footnote{We only give here the local spectrum of the scalar manifolds, avoiding the discussion of discrete identifications.}
Let us note that to have $N = 2$ preserving vacua it is not possible to gauge more than two vectors, since it would be incompatible with the given symplectic frame, used to reproduce the orientifold configuration. It is easy to see that the given choice is the only one stabilizing the moduli in a way compatible with the domain of definition of the scalar fields. This same result is derived in Section 5 of [12], with topological arguments.

On the contrary, to obtain configurations with $N = 1,0$ supersymmetry one should switch on, at least, the couplings $f_0^1 = g_0, f_1^2 = g_1$ corresponding to the scalar isometries $C^{m=1}, C^{m=2}$. The massive vectors are in this case the graviphoton and vector partner of the $K3$ volume modulus. For general values of the fluxes, the supersymmetry is completely broken; the $N = 1$ case is obtained by imposing further $|g_0| = |g_1|$.

The massless spectrum of the $N = 1$ reduced theory is the following. From the 58 scalars of $SO(3,19)/(SO(3) \times SO(19)) \times \mathbb{R}^+$ there remain 20 scalars parametrizing $SO(1,19)/SO(19) \times \mathbb{R}^+$. From the 22 axions there remain 20. All together they complete the scalar content of 20 chiral multiplets. The spectrum includes two massless vector multiplets corresponding to $A_\mu^2$ and $A_\mu^3$ and an extra chiral multiplet whose scalar field comes from the $N = 2$ vector multiplet sector. There is then one long massive gravitino multiplet $[(3/2), 2(1), (1/2)]$, containing as vectors the graviphoton $A^0$ and the vector $A^1$.

Note that this $N = 2$ model allows partial Super Higgs, evading the no-go theorem for $N = 2$ supergravity [28]. This is possible because the symplectic frame chosen such as to reproduce the string model given uses a degenerate symplectic section for special geometry on $M_V$, where no prepotential function $F(X)$ exists.

More general vacua, preserving $N = 1,0$ supersymmetry, can be obtained by considering an arbitrary vector coupling $g_\Lambda$. In this more general case, $g_0$ and $g_1$ gauge two of the isometries $C^m$, while $g_2$ and $g_3$ gauge two of the isometries $C^n$. The $N = 1$ preserving vacua have 18 left over chiral multiplets.

Acknowledgments

This report is based on collaborations with C. Angelantonj, R. D’Auria, F. Gargiulo, M. A. Lledó and S. Vaulà, which we would like to thank. M.T. would like to thank H. Samtleben for useful discussions. M.T. would like to thank the Th. Division of CERN, where part of this work has been done, for their kind hospitality. The work of S.F. has been supported in part by European Community’s Human Potential Program under contract HPRN-CT-2000-00131 Quantum Space-Time, in association with INFN Frascati National Laboratories and by D.O.E. grant DE-FG03-91ER40662, Task C. The work of M.T. is supported by a European Community Marie Curie Fellowship under contract HPRN-CT-2001-01276.

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