FREE ELECTRON LASERS: A SHORT REVIEW OF THE THEORY AND EXPERIMENTS

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ABSTRACT
This note is devoted to a short review of the theoretical and experimental aspects of Free Electron Lasers (FEL). We will discuss both recirculated and single-passage FELs and their relevant design problems.

1. INTRODUCTION

Since laser sources have been experimentally demonstrated the concept of a "universal" or "radio-like" coherent light source has been recognized as a powerful tool for a large number of potential applications.

The concept of fully tunable lasers is therefore as old as laser Physics.

The research activity in this field, developed through the years, is summarized in Fig. 1 where we have plotted the power against the wavelength (and wavelength range) of the commonly considered tunable conventional sources. Many of the light sources of Fig. 1 are far from being real tunable lasers. It is self-evident that the ultimate tunable laser has not been developed, but its desired performance can be easily outlined:

a) stability
b) long life
c) easily manageable
d) high power
e) easily tunable via external settings to any selected frequency.

These and many other "science-fiction" performances will be the characteristics of a real tunable laser. The areas of application of these kinds of sources are as wide as their versatility and include such different fields as spectroscopy, remote detection, photochemistry etc.1).

To give an example, conventional tunable lasers like excimers, dyes and harmonic generation have been used for UV spectroscopy, while color center lasers have been exploited in the middle IR spectroscopy and have provided substantial advances. The use of real tunable sources in connec-

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Fig. 1 Comparative chart between FEL and conventional coherent sources. Average power vs \( \lambda \). Curves FEL (SP); single passage FEL average power vs \( \lambda \) 1st and 3rd harmonic respectively, maximum electron beam (e.b.) power 20 MW, \( K=1 \), \( \lambda_u=5 \) cm, \( N=50 \), \( L_C=6m \), \( \tau_M=12\mu s \), \( \delta=5\% \). Straight curves: FEL storage ring average power vs \( \lambda \) without (continuous) and with (dashed) Touschek effect respectively. \( I=100 \) A and operating parameter of LEDA-F, see Ref. 2. [\( L_C \) = cavity length, \( \delta \) = duty cycle].

In the field of photochemistry the tunable sources will allow more reliable analytic techniques and laser-based chemical processing ranging from controlled thermal chemistry to laser initiated radical reactions.

The above list of potential applications of tunable sources may be complemented with more technological subjects such as mechanical processing, information transfer and communication. The noticeable interest in the field is therefore fully justified. The state of the art of the tunable sources has been discussed in Ref. 2 where a comprehensive review of the experimental results and of the literature has been given. As a comment to Fig. 1 we notice that among the solid state lasers the most reliable is the Alexandrite, with a tuning bandwidth of about 1000 \( \AA \), operating at room temperature. The color center lasers have a more significant tuning range, but require improvements in terms of stability and life-time. The most limiting factor of dye lasers is represented by the stability, even though their tunability over the years has been improved, to cover the range between the visible and the near IR. However, the power limitation beyond 10 \( \mu m \) is evident. In this region and in the short wavelength region (VUV, X-rays) tunable non-conventional sources are provided by the Free Electron Lasers (FEL) 3).
In Fig. 2 the tunability curve of FELs is shown, the continuous line ranges from VUV to the microwaves. Although the basic mechanism of FEL allows a wide tunable range, this device, as it stands, does not provide the universal laser we are talking about. A fully tunable FEL requires, indeed, a "universal" accelerating electron machine able to provide an electron beam with continuous varying energies from MeV to GeV region. Even this kind of machine does not exist, but its characteristics for FEL application can be easily listed:

a) **easy energy tunability**
b) **modest size**
c) **high beam power (average and peak)**
d) **good beam qualities (small energy spread and emittances)**.

An overview of the design characteristics of an accelerating electron device dedicated to FEL has been presented in Ref. 4. In that paper the performances of storage rings, diode machines, induction linacs, electrostatic devices and R.F. accelerating machines have been discussed within the framework of their relevance to FEL. In Fig. 3 we have summarized the relative range of energy and current of accelerating devices. These are impressive, from MeV and kA to GeV and hundreds of mA.

![Graph](image)

**Fig. 2** FEL scenario
Fig. 3 Current vs. Energy for existing accelerators

Such a flexible accelerator is very far from the present technological capabilities. Nevertheless, exploiting such different tools as relative energy tunability, higher-harmonics emission, undulator gap variations etc. an FEL can provide a range of tunability much larger than that of the conventional sources.

In the next sections we will briefly summarize the main features of the FEL theory with particular emphasis on the design criteria of both recirculated and linear devices. The final section is devoted to concluding remarks where we complete the comparison with the conventional lasers.

2. FEL: THEORY AND DESIGN CRITERIA

In an FEL a beam of ultrarelativistic electrons interacts with an undulator magnet (UM) where it undergoes transverse oscillations and emits radiation at a fixed wavelength; the radiation is stored in an optical cavity, it reinteracts with the copropagating e-beam and is amplified.

An undulator magnet is a spatial array of magnets arranged as in
Fig. 4, with spatial period $\lambda_u$ and it was originally proposed as a tool to enhance the brightness of the synchrotron light\(^5\).

We will give a simple heuristic explanation of the central emission frequency in an undulator. Madey\(^6\) has shown that for ultrarelativistic e-beam the undulator field can be treated as a radiation field with wavelength

$$\lambda^* = 2\lambda_u \hspace{1cm} (1)$$

and density number of pseudophotons\(^7\)

$$n = \frac{e}{4} \frac{k^2}{\lambda_u^2 c^2}, \quad k = \frac{e B}{2\pi m c^2} \hspace{1cm} (2)$$

where $\alpha$ is the fine structure constant, $r_o$ is the classical electron radius, $k$ is the undulator parameter and $B (= B_o$ for helical undulator, $B_o/\sqrt{2}$ for linear undulator) is the average on axis field ($B_o$ is the on axis field).

The relation (1) can be understood as follows. The vector potential of the undulator field can be written as

$$A = \{ \Re e \left[ B_o \lambda_u \exp(iz/\lambda_u) \right] \} \hat{y} \hspace{1cm} (3)$$

As a consequence the magnetic field is given by

$$B = \nabla \times A = \Re e \left[ -iB_o \exp(iz/\lambda_u) \right] \hat{x}$$

$$\hat{x}, \hat{y}, \hat{z} \equiv \text{unit vectors} \hspace{1cm} (4)$$

From (4) we get also

$$\nabla \times B = \nabla \times \nabla \times A = -\nabla^2 A = 1/\lambda_u^2 A \hspace{1cm} (5)$$

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**Fig. 4** Undulator magnet geometry
which clearly contradicts the Maxwell equation. The last term on the right hand side of (5) can however be reinterpreted, according to Ref. 3, as a kind of "photon mass". This is indeed the price to be paid when one treats from the very beginning the undulator vector potential in the transverse gauge. Furthermore, transforming the vector potential to that seen by a relativistic electron moving through the UM we have

$$A' = \Re e \left[ B_0 \kappa_u \exp[i \gamma/\kappa_u (z' + \beta ct')] \right] \hat{y} \quad (6)$$

$$\beta = v_z/c, \quad \gamma^2 = (1 - \beta^2)^{-1}$$

thus yielding for the magnetic and electric field the expressions

$$B' = \Re e \left[ -i \gamma B_0 \exp[i \gamma/\kappa_u (z' + \beta ct')] \right] \hat{x} \quad (7)$$

$$E' = \Re e \left[ -i \gamma \beta B_0 \exp[i \gamma/\kappa_u (z' + \beta ct')] \right] \hat{y}$$

and finally obtaining the "Maxwell equation"

$$v' \times B' = \frac{1}{c} \frac{\partial E'}{\partial t} + \frac{1}{\kappa_u} A' \quad v' \times E' = - \frac{1}{c} \frac{\partial B'}{\partial t} \quad (8)$$

From the first of equations (8) it follows that the "mass term" is neglected once $\gamma >> 1$. Furthermore, since $\beta \sim 1$ the fields (7) approximate closely to those of a radiation field. Since a light pulse must be a radiation field in all the reference frames, the field defined by (7) must remain a "radiation" field in the laboratory frame too. Transforming back its wavelength to that frame one finds

$$\lambda^* = (1 + \beta) \lambda_u \quad (9)$$

thus getting for $\beta \sim 1$ the relation (1). Finally the equation (2) is a straightforward consequence of the definition of the field energy density.

Since the undulator field can be treated as an ordinary electromagnetic wave, its interaction with the electron can be viewed as a scattering. According therefore to the well known formula of the Compton scattering, the wavelength of the scattered light at an angle $\theta$ is given by (see Fig.5)

Fig. 5 Compton scattering diagram
\[ \lambda = 2\lambda_0 \frac{1 - \beta \cos \theta}{1 + \beta} \]  \hspace{1cm} (10) 

Expanding for small angles and ultrarelativistic energies one finds

\[ \lambda = \frac{\lambda_0}{2\gamma^2} (1 + k^2 + \gamma^2 s^2) \]  \hspace{1cm} (11)

The corrective term \( k^2 \) is due to the transverse electron motion and it is analogous to the effect suggested by Brown and Kibble in the analysis of the electron motion in an intense laser wave, where an intensity dependent contribution to the Compton wavelength shift was found (see Ref. 3 for further comments).

One of the most peculiar characteristics of the light emitted by a charged particle running in a UM is the bandwidth. This quantity can be easily evaluated according to the following simple argument:

1) The duration of the emitted light pulse is linked to the difference between electron and photon flight times (see Fig. 4)

\[ \Delta t = \frac{N\lambda_0 u}{c} (1 - \beta) \equiv \frac{N\lambda_0}{c} \]  \hspace{1cm} (12)

\((N\) is the number of undulator periods\).

2) According to the indetermination principle the bandwidth can be easily evaluated from

\[ \Delta \omega \Delta t \equiv \hbar, \; \Delta \omega \equiv \frac{\hbar c}{N\lambda_0} \]  \hspace{1cm} (13)

3) Combining both (11) and (13) we get the relative homogeneous bandwidth

\[ \left( \frac{\Delta \omega}{\omega} \right)_0 = \frac{1}{2N} \]  \hspace{1cm} (14)

The denomination "homogeneous" and inhomogeneous derives (with the same meaning) directly from the standard theory of the photon emission by atoms or molecules. As is well known, the spectrum of an \( N \)-period light pulse has a width given by (14), while its shape is

\[ f(\omega) \propto \left( \frac{\sin v/2}{v/2} \right)^2, \; v = 2\pi N \frac{\omega_o - \omega}{\omega_o} \left( \omega_o = \frac{2\pi c}{\lambda} \right) \]  \hspace{1cm} (15)

The spectrum given by Eq. (15) plays a fundamental role in the FEL theory and
it has been plotted in Fig. 6 together with two experimental spectra (Stanford 8) and Orsay 9). The above brief comments are relevant to the spontaneous emission. For a recent and detailed analysis the reader is referred to Ref. 4. We are now interested in stimulated emission and gain. By the former we mean emission in the presence of other e.m. modes and variation of the intensities of the those modes. A rigorous analysis of the gain can be found in Ref. 3, but here we will evaluate the gain function in a rather direct way. The gain mechanism can be understood as a balance between an absorption and an emission photon process. The gain function will be therefore given by the difference between the probabilities of emitting and absorbing a photon. The functional form of emitting or absorbing a photon is the same as Eq. [15], the only distinguishing feature is the

![Diagram](image_url)

Fig. 6 Undulator magnet forward emission spectrum: (a) Theoretical spectrum; (b) Experimental spectrum (Stanford, Ref. 8); (c) Experimental spectrum (Orsay, Ref. 9).
electron recoil, so that

\[ g(\omega) \approx f(\omega - \varepsilon) - f(\omega + \varepsilon) \approx \frac{\sin v/2}{v/2} \left( \frac{\sin v/2}{v/2} \right)^2 \]

\[ \sqrt{v} = 2nN \frac{\omega_0 (1 - (\omega_0 \lambda_e / c) \gamma)}{\omega_0}, \lambda_e = \gamma / mc . \]

At the lowest order in the small quantity \((\omega_0 \lambda_e / c) \gamma\) we get

\[ g(\omega) = -\frac{4\pi^2}{\gamma} \frac{\lambda L}{\Sigma E} \frac{I}{I_0} \mathcal{F} \frac{k^2}{1 + k^2} \frac{(\omega - \omega_0)^2}{\omega_0^2} \frac{d}{dv} \left( \frac{\sin v/2}{v/2} \right)^2 \]

where \(\Sigma E\) is the e-beam cross section, \(\mathcal{F}\) is the filling factor

\[ \mathcal{F} = \begin{cases} 1 & \text{if } \Sigma E > \Sigma L, \\ \frac{\Sigma E}{\Sigma L} & \text{if } \Sigma E < \Sigma L \end{cases} \]

\(\Sigma L\) is the laser-beam cross section, \(L\) is the undulator length, \(I\) the e-beam peak current and \(I_0 = eC/r_0(=1.7 \times 10^4 A)\) is the Alfvén current. Since emission at higher harmonics occurs in undulator magnets on and off axis, an analogous gain formula for higher harmonics can be derived (see Ref. 10 for a general formulation).

In particular, linear undulators allow odd-harmonic emission on axis and the relevant gain can be written as

\[ g_n(\omega) = -g_n^0 \frac{n}{n^2} \frac{d}{dv} \left[ \frac{\sin n v/2}{n v/2} \right]^2, \quad n = 1, 2, \ldots \]

\[ v_n = 2nN \frac{n \omega_0 - \omega}{n \omega_0} \]

\[ g_n^0 = \frac{4\pi}{\gamma} \frac{\lambda L}{\Sigma E} \frac{I}{I_0} \mathcal{F} \frac{\lambda}{\lambda_n} \frac{n \omega_0}{n \omega_0} \frac{k^2}{1 + k^2} \frac{(\omega - \omega_0)^2}{\omega_0^2}, \quad \lambda_n = \lambda / n \]

\[ \mathcal{F}_n = n\text{-th harmonic filling factor} \]

\[ F_n(\xi) = \xi \left[ n \left( \frac{J_n[(n+1)(\xi^2)] - J_{n-1}(n \xi)]}{2} \right)^2 \right], \xi = 1/2 \frac{k^2}{1 + k^2} \]

\((J_n(\cdot)) \equiv n\text{-th cylindrical Bessel function})
The analysis we have developed so far is relevant to a small-signal, single-mode, homogeneously-broadened FEL operation. The strong signal and multimode behaviour will be treated below. By homogeneous broadening we mean an FEL operating with an electron beam whose energy spread and emittances produce negligibly small effects.

It is well known that those beam qualities produce both a broadening of the emission line and a reduction of the gain \(^3\). The value of the inhomogeneous linewidth in terms of the beam emittances and energy spread is \(^3\)

\[
(\Delta \omega) = (\Delta \omega)^0 \sqrt{1 + \mu_x^2 + \mu_y^2 + \mu_z^2}.
\]

(20)

The \(\mu\)-coefficients are the ratio between the inhomogeneous and homogeneous widths and have played a crucial role in the design and optimisation of an FEL device \(^1\).

In particular,

\[
\mu_x = 4N\sigma_e, \quad \sigma_e = \text{r.m.s. energy spread}
\]

\[
\mu_{x,y} = \frac{4Ny^2}{1 + k^2} \left\{ \frac{1}{2\sigma_{x,y}} \left( \frac{x_{x,y}}{2\pi} \right)^4 + 2(k\lambda_y - \delta_{x,y})^2 h_{x,y} \sigma_{x,y}^2 \right\}^{1/2}
\]

\[
\mu_{x,y} = \frac{4Ny^2}{1 + k^2} \left\{ \frac{1}{2\sigma_{x,y}} \left( \frac{x_{x,y}}{2\pi} \right)^4 + 2(k\lambda_y - \delta_{x,y})^2 h_{x,y} \sigma_{x,y}^2 \right\}^{1/2}
\]

(21)

where \(\varepsilon_{x,y}\) are the radial and vertical emittances, \(\sigma_{x,y}\) the transverse e-beam dimensions, \(h_{x,y}\) are coefficients depending on the undulator geometry, namely \(h_x = h_y = 1\) for helical undulators and \(h_x = 1, h_y = 2\) for the linear case, with polarization along the \(y\)-axis. Physically \(\delta\) is the magnitude of the sextupolar term along the \(x\)-direction \(^3\).

It is worth noting that the inhomogeneous broadening due to the emittances consists of two distinct contributions; the first due to the angular divergence, the second to the finite beam size which explores regions of different magnetic field strength.

The expression Eq.(21) suggests that one can choose an optimum \(\sigma_{x,y}\) to minimize the effect of the inhomogeneous broadening \(^3,^*)\_1\_e.

\[
\sigma_{x,y} = \left\{ \frac{1}{2|h_{x,y}|} \right\}^{1/4} \left( \frac{\lambda_x \varepsilon_{x,y}}{2k} \right)^{1/2}.
\]

(22)

Therefore one finally gets

\[*)\text{ Since, } \sigma_{x,y} = \sqrt{(\varepsilon_{x,y} - \beta_{x,y})/2\pi} \text{ where } \beta_{x,y} \text{ is the beta amplitude function, Eq.(22) amounts to } \beta_{x,y} = \left\{ (1/2|h_{x,y}|) \right\}^{1/2} (y\lambda_y)/k\pi.\]
\[
\mu_{x,y} = 2N \sqrt{2} |h_{x,y}| \frac{k}{1+k^2} \lambda_u^y \varepsilon_{x,y} \quad .
\]

(23)

We must remark, however, that while the choice of Eq. (22) minimizes the effect of the inhomogeneous broadening it may, at the same time, create difficulty with the filling factor. Therefore, inserting the auxiliary condition that the e.b. cross section is of the order of the laser mode waist, we get

\[
\varepsilon_{x,y} \equiv \sqrt{2} |h_{x,y}| \pi \left( \frac{k}{y} \right) \Lambda^y \lambda_u \quad .
\]

(24)

A further condition on the emittance can be obtained requiring that \( \mu_{x,y} \leq 1 \) and thus

\[
\varepsilon_{x,y} \leq \frac{\lambda}{\sqrt{2} |h_{x,y}|} \frac{y}{Nk} \quad .
\]

(25)

Combining Eq. (23) and (24) one can also get a condition on the sextupolar terms \(|h_{x,y}| \) (11).

When a very small beam section is required we can neglect in Eq. (21) the inhomogeneous broadening induced by the undulator field inhomogeneities. With the requirement that the beam cross section is of the order of the laser waist and that (23) be less than unity we find the conditions (11)

\[
\varepsilon_{x,y} \leq \lambda, \quad \beta_{x,y} \sim \Lambda \lambda_u \quad .
\]

(26)

To give an idea of the \( \mu \) parameters on the spontaneous emission and the gain, we have plotted in Fig. 7 those functions against \( v \) for different values of \( \mu \). It is evident that with increasing values of the inhomogeneous parameters the curves are both widened and reduced.

Longitudinal mode locking arises for FEL operation with bunched e-beams. It has indeed been shown that in this situation a natural phase locking is induced by the FEL interaction, and the strength of the coupling between the modes is given by the further parameter

\[
\mu_c = \frac{\Lambda}{\sigma_z} \quad .
\]

(27)

where \( \sigma_z \) is the electron bunch r.m.s. longitudinal length. The larger is \( \mu_c \), the greater is the number of coupled modes. The bunched e-beam structure is also responsible for the so called FEL lethargic behaviour, i.e. the slow down of the light pulse due to the interaction and the necessity to shorten the cavity length with respect to the nominal round-trip period to keep the synchronization between light and electron bunches (3). The im-
Fig. 7 Inhomogeneous broadened gain: (a) $\mu_\varepsilon = \mu_\chi = \mu_\gamma = 0$; (b) $\mu_\varepsilon = 1$, $\mu_\chi = \mu_\gamma = 0$; (c) $\mu_\varepsilon = 0$, $\mu_\chi = 1$, $\mu_\gamma = 0$; (d) $\mu_\varepsilon = \mu_\chi = \mu_\gamma = 1$.

The importance of this effect for short-pulse operating FEL devices will be discussed below.

The above notions are the minimal theoretical background to understand the FEL operation; in the next two subsections we will describe in some detail FEL devices in storage rings and single-passage devices.

3. FEL STORAGE RING OPERATION

We have seen that the practical realization of a FEL requires an e-beam with good qualities, namely large peak current and relatively low energy spread and emittance. A storage ring (SR) provides a very good e-beam for the FEL operation.

In these devices the e-beam is continuously recirculated through the interaction region and as a consequence the energy spread and the emittances increase. Therefore, according to the arguments presented so far, the increase of the inhomogeneous broadening reduces the FEL amplification. This dynamical behaviour is peculiar to storage ring FELs. A correct description of the storage ring FEL operation requires indeed the self-consistent analysis, turn by turn, of both the laser and the electron beams. Storage ring FELs have been suggested for laser operation in the short wavelength region from visible down to VUV and x-ray (see Fig. 2). Just to start these introductory remarks we show in Fig. 9 the layout of an SR designed for FEL operation \cite{12}. The machine has a twofold symmetry (problems arise for non-symmetrical structures). The symmetry is provided by the insertion of two, long, low-field undulators (for the FEL operation) and two, short, high-field undulators to enhance the synchrotron radiation.
Fig. 8 Storage ring layout (design study LEDA-F2): B = bending magnet; F, D = focusing and defocusing (horizontal) quadrupole magnet; S = sextupole magnet; WM (FEL) = undulator magnet for FEL operation; WM = high-field undulator magnet for enhancing synchrotron radiation emission.

emission (this fact will be clarified below).

The focalization is provided by an alternate distribution of horizontal focusing and defocusing quadrupole magnets (F and D in Fig. 8). Also inserted in each quarter of the machine are two bending magnets and two sextupoles to minimize the dependence of the transverse oscillation frequency on the particle energy. The free space between the quadrupoles of the long straight section is utilized for the injection of the electrons into the machine and to insert the RF accelerating system which accelerates the electron to higher energies than the injection one and supplies the energy lost by synchrotron emission in the bending magnets.

Since the particles emit synchrotron radiation the off-energy particles tend to reduce, turn by turn, the energy shift from the synchronous ones with a damping time

$$\tau_s \sim \frac{TE}{U_o}, \quad U_o = \frac{4\pi r_0^2}{3} \frac{\gamma^4}{\rho} m_0 c^2$$

(28)

where $E_o$ is the machine nominal energy, $T$ the revolution period, $U_o$ the energy radiated per turn and $\rho$ the bending magnet radius (assumed identical for all the magnets).
The betatron motion too is damped, with damping times

$$t_x = t_y = 2t_z .$$

(29)

The above expressions are only approximate. The correct ones involve the so-called damping partition numbers $J_i$, namely

$$\frac{T_i}{t_i} = J_i \frac{U_0}{2E_0} \quad (i = x, y, z) .$$

(30)

According to the Robinson Theorem 13) the $J$ numbers obey the following identity

$$\sum_{i} J_i = 4 .$$

(31)

In any case Eq. (29) and (30) are good approximations for a typical plain machine $(J_x \approx 2, J_x \approx 1, J_y = 1$. The exact expression should contain a small correction to take into account the (eventual) radial gradient in the bending magnets).

After these few remarks on SR physics let us briefly discuss what are the achievable e-beam qualities.

3.1. Emittances

The smallest emittances in an S.R. are achieved with the magnetic lattice suggested by Chasman, Green and Rowe 14). Such a magnetic structure consists of $M$ achromatic bends and, according to Krinsky 15), Sommer 16) and Potaux 17), the minimum horizontal emittance can be written as

$$\epsilon_x^{\text{min}} = (7.7 \times 10^{-13} \text{ m.rad}) \frac{\gamma^2}{J_x M^3}$$

(32)

which is very accurate for $M \geq 4$ 18). In actual storage ring design it is difficult to achieve the minimum value given by (32) and a more realistic estimate is 15)

$$\epsilon_x = 2\epsilon_x^{\text{min}} = (1.5 \times 10^{-12} \text{ m.rad}) \frac{\gamma^2}{J_x M^3} .$$

(33)

3.2 Energy spread

According to the explanations given so far, it may be thought that, due to damping, the e-beam becomes point like. This is not the case. The synchrotron radiation is, indeed, emitted in quanta of discrete energy
which generate a kind of noise. As a consequence the electrons undergo a diffusion mechanism, counteracting the damping, the resulting energy spread is \(3\)

\[
\sigma_e = \sqrt{\frac{C}{q J_{sp}}} \gamma \sim \sqrt{\frac{C}{2\rho}} \gamma \tag{34}
\]

where

\[
C_q = \frac{55}{32\sqrt{3}} \frac{m}{m_o c} = 3.84 \times 10^{-13} \text{ m} . \tag{35}
\]

Beside the quantum excitation two other effects may cause beam heating, namely the "Touschek effect" and the "anomalous bunch lengthening". We will discuss them within the framework of the current-limiting factors.

3.3 \(e\)-beam current

Current limitation in SR's is due to such reasons as beam-gas interaction, ion trapping, intra-beam scattering, anomalous bunch lengthening etc. For a more complete description of these effects the interested reader may like to see the paper by Le Duff in Ref. 19. In this note we will briefly discuss the Touschek and bunch lengthening effects.

When two particles performing transverse oscillations collide, a part of their transverse momentum is transformed into a longitudinal momentum change. As a consequence, one particle gains and the other loses momentum. If the momentum variation is larger than the momentum acceptance of the SR both particles are lost\(^*\). This is the Touschek effect and its consequence is a reduction of the beam life-time. The probability of intrabeam scattering increases with the beam density. Therefore, the effect is also a limiting factor of both emittance and current density. Calculations of the maximum achievable current and beam life-time have been given and can be found in Refs. 20.

Let us now discuss a little more quantitatively, the so called anomalous bunch lengthening. The phenomenological model \(^{21}\) predicts that when the bunch current exceeds a certain threshold value the energy spread and the bunch length will both increase with the stored current in the bunch.

\(^*\) Strictly speaking this is a single-particle Touschek effect. However, another effect takes place namely the multiple Touschek effect, in which the energy transfer between particles does not lead to particle losses but appears as a noise source for the particle motion. The obvious consequence is an increase of the energy spread in the beam.
It can be shown that, under specific conditions the following two equalities hold \(15,20\):

\[
\frac{\sigma_z}{R} = \frac{\sigma}{v_s} \frac{\sigma_E}{R}, \quad R = \text{machine radius}
\]

\[
\sigma^3_{\varepsilon} = \frac{1}{\sqrt{2\pi}} \frac{e v_s}{\sigma E_0} \frac{I}{Z_n/n} |Z_n/n|
\]

where \(\sigma_z\) is the longitudinal bunch length, \(\sigma\) the momentum compaction, \(v_s\) the machine tune, \(I\) the average current and \(|Z_n/n|\) the characteristic impedance. The peak current is given to

\[
\hat{I} = \frac{\sqrt{2\pi} R}{\sigma_z} \frac{I}{\varepsilon}
\]

Combining both (37) and (36) yields \(15\)

\[
\varepsilon \frac{I}{|Z_n/n|} = 2\pi E_0 \sigma^3_{\varepsilon}
\]

We now have all the most important parameters to write down the gain for an SR FEL. We must underline that Eq.(38) has a particularly interesting meaning. It shows that the energy spread is not only an undesirable feature in the sense that it reduces the gain but, since larger energy spread allows larger peak currents, a suitable balance between the two competitive effects may give rise to an "optimum" energy spread for FEL operation. The explicit expression of the gain function is \((\xi_L \sim L \lambda)\)

\[
g(\omega) = 2 \frac{\sigma^3}{c} \frac{\sigma F_1(\xi)}{|Z_n/n|} \mu^2_{\varepsilon} f(\nu; \mu_c; \mu_X; \mu_{\varepsilon})
\]

where \(f(\ldots)\) is the inhomogeneous gain function and reduces to the ordinary gain function when all the \(\mu\)'s are zero. The presence of the coupling coefficient \(\mu_c\) in Eq.(39) is due to the bunched beam operation and, therefore, to the longitudinal phase locking. Within this framework it is not an extra independent variable but, according to Eqs.(27) and(36) may be written as

\[
\mu_c = \frac{4N^2 \varepsilon}{\sigma R} \frac{v_s}{\mu_{\varepsilon}}
\]

Once the emittance is fixed by Eqs.(32-37) and by the further condition of Eq.(26) one can find the optimum \(\sigma_{\varepsilon}\) by looking at the maximum of the function \(\mu^2_{\varepsilon} f(\ldots)\) against \(\mu_{\varepsilon}\).

Analogous optimization criteria can be found for the current limita-
tion due to the Touschek effect, but we will not discuss this case since the optimisation procedure closely follows that developed above. Let us now quickly discuss the power achievable with an SR FEL (for a complete analysis the interested reader is referred to Refs. 19-20).

At the beginning of this subsection we have briefly outlined the SR FEL dynamics and saw that the FEL interaction acts as a kind of noise which, in the chosen hypothesis, induces a diffusion counteracted only by the damping due to the synchrotron emission in the bending and undulator magnets. We can, therefore, expect that the average laser power \( P_L \) will be related to the synchrotron emission one. The relationship can be stated more quantitatively as follows. The laser process itself degrades the e-beam qualities, then the gain decreases and the laser is switched off. We must wait a time of the order of the damping time to have a new laser pulse. The average laser power is therefore approximately given by

\[
P_L \approx \frac{N_e \Delta E}{\tau_z} = \frac{N_e}{\tau_z} E \Delta E \leq \frac{1}{2N} P_S
\]

(41)

where \( \Delta E \) is the maximum energy variation, \( N \) is the number of particles in the beam and \( P_S \) is the synchrotron radiation power given by (see Eq. 28)

\[
P_S = \frac{N_e U_0}{T} = \frac{N_e}{\tau_z} E
\]

(42)

Using the above scaling law and the design parameters of the LEDA-F machine we have plotted in Fig. 1 the FEL-SR power levels against the wavelength and have also included the current limitation due to the Touschek effect.

4. SINGLE-PASSAGE FEL OPERATION

The first FEL operation was accomplished with the Stanford superconducting Linac. This electron source was characterized by extremely good beam qualities which made it an almost unique tool for the first experimental attempts.

Even though an ideal machine for FEL operation, the superconducting Linac has long been considered an impracticable solution for FELs owing to its technological complexity and large operational costs. However, recent progress in superconducting cavity technology made the machine operation less critical, in principle, and reduced considerably the costs. It is therefore desirable that, with its special characteristics, this
accelerating device be carefully reconsidered for FEL operation. Single-passage FEL's using more conventional sources have been proposed and up to now have operated with an induction Linac \(^{23}\), an RF Linac \(^{24}\), a Van der Graaf machine \(^{25}\) and a microtron \(^{26}\).

The chart of existing experiments is shown in Fig. 2. A rather detailed review of the low energy accelerators dedicated to FEL operation has been made in Ref. 4. Here we will briefly discuss a few of the characterizing features of each e-beam source.

4.1 Linacs

The most comprehensive review on Linacs has been given in Ref. 26. There are essentially two types, namely the RF and the induction. The first can provide currents of the order of hundreds of mA and (for FEL operation) an energy of hundreds of MeV. Induction Linacs can furnish e-beams of tens of kA and tens of MeV \(^{27}\); the advanced test accelerator is indeed designed to provide a beam of 10 kA at 50 MeV \(^{27}\).

It is clear that conventional RF Linacs can be dedicated to Compton regime FELs, whilst induction Linacs FEL operate in the so called high gain collective regime \(^{3}\). RF Linacs have been operated around two frequencies, 3 GHz (S-band) and 1.3 GHz (L-band). The main limitation of these machines is the large energy spread whose main sources are the variation in RF accelerating field along the length of the bunch and single-bunch beam loading \(^{28}\). A typical energy spread is 1% at 50 MeV which can result in a too large inhomogeneous broadening to ensure laser action. A typical measure to overcome this limitation is to compress the energy by an order of magnitude.

The typical longitudinal microbunch length varies from 3-4 ps for superconducting Linacs, to 6 and 15 ps for S and L band normal Linacs respectively. According to Eq. (28) S-band Linacs (as well as superconducting ones) may create problems for FEL operation with long undulators at FIR wavelengths. The use of the energy compression mechanism may solve this problem too, but inevitably creates difficulties with the peak current. A criterion to optimize bunch length and peak current has been discussed in Ref. 11. Large peak currents can, in principle, be obtained with a Linac, but limitations arise for the average current. Using a subharmonic bunching technique 100 A peak current in 30 ps has been obtained at Los Alamos \(^{24}\). At Osaka University \(^{30}\) the same method is used to obtain very high single bunch current (3kA in 16 ps).

Let us now briefly discuss the problem of emittance in Linacs. In Fig. 9 we have plotted the product of the normalized emittances against the average currents of the most representative sample of existing low-energy accelerators \(^{11}\). These two quantities are roughly correlated by an empirical relationship \(^{11,\ast}\).
Fig. 9  Current vs emittances for existing accelerating devices without radiative damping

\[
\frac{\bar{T} [A]}{\beta^2 y^2 x_y} \approx 1.1 \times 10^4
\]  \hspace{1cm} (43)

which has been exploited to get scaling relationships for the FEL gain and power \(^{11}\). However, since no fundamental limit other than the cathode emission exists, one may expect that with careful research the limit of Eq. 43 can be improved by an order of magnitude or more \(^{31}\).

Induction Linacs, as already mentioned, can provide very high beam currents, but are, at the moment, limited in energy (tens of MeV) and pulse duration (tens of ns). As to the energy spread it can be derived from the following relationship \(^{28}\)

\[
\sigma_\varepsilon \sim 10^{-3} \sqrt{1 + 3.6 \times 10^{-3} \left( \frac{[A]}{\beta y} \right)^2}
\]  \hspace{1cm} (44)

Typical values are around a few \(^\circ\). As far as the emittance is concerned, the above considerations relevant to RF Linacs also hold for induction ones.

\(^*)\) The emittances are expressed in cm.rad.
4.2 Microtrons

A review of FEL experiments in progress with microtrons and their relevant technology has been made in Ref. 4. The microtrons offer with respect to the Linacs the important advantage of a much lower energy spread while the pulse length is longer than that of the S-band Linacs. These two effects can be explained by the energy compression mechanism which is automatically set up by the microtron operating principle 32). Typical values of energy spread are

\[ \sigma_E \sim 1.5 \times 10^{-2} \frac{E_R}{E_0} \]  \hspace{1cm} (45)

where \( E_R \) is the resonant energy gain per orbit and \( E_0 \) is the nominal machine energy.

From Eq. (11) and the above relationship, we find a condition on the resonant energy gain to avoid energy spread problems, namely

\[ E_R \leq \frac{10^2}{6} \frac{E_0}{N} \]  \hspace{1cm} (46)

Taking into account typical operating FEL microtron parameters and the fact that \( E_R \leq 1 \) MeV, the condition of Eq. (46) is largely satisfied. Typical values of the bunch length are around 20-30 ps. Limitations can arise for FEL operation at long wavelengths (FIR) or with long undulators *). For the microtron emittances the conclusions arrived at for the Linacs also hold.

The most serious disadvantage of a microtron is the peak current which can reach only a few Amperes. The intrinsic limits are the amount of power which can be pumped into the cavity, and the cathode geometry. The race-track microtrons having a separate injector section can be used to overcome this difficulty and enjoy both the advantages of linacs and conventional microtrons 4).

4.3 Van der Graaf accelerators

Finally we mention the Van der Graaf accelerators. A machine of this type has been already exploited as an e-beam source for the UCSB FEL experiment 25). The beam of a Van der Graaf accelerator is characterized

*) We must however underline that the FEL optimization is a rather complicated process, which should be carried out with regard to the various effects contributing to the gain. Taking these effects into account separately may lead to misleading conclusions.
by extremely good qualities. For example, the UCSB accelerator has furnished a beam with an emittance (normalized) at 2.5 MeV of about 7.5x10^{-6} m.rad. Furthermore the beam has a continuous structure and can reach average currents of a few Amperes with maximum energy of tens of MeV \ref{33}.

4.4 General Features of Single-Pass FELs

We have discussed so far the accelerator performances rather than those relevant to the laser. We will now briefly discuss the main characteristics of the single-pass FEL gain and saturation.

Assuming that the main limitation on the beam current is the RF power, we can write FEL single passage gain as follows \ref{3}:

\[
\begin{align*}
g_h &= g_h^{\theta} \mathcal{R} \mathcal{E}_q [\theta; \mu_C, \mu_x, \mu_y, \mu_e] & h & \equiv \text{helical} \\
g_{\xi, n} &= g_{\xi, n}^{\theta} \mathcal{R} \mathcal{E}_q [\theta; \mu_C, \mu_x, \mu_y, \mu_e] & \xi & \equiv \text{linear}
\end{align*}
\]

\[
g_h^{\theta} = \frac{88 \times 10^{-4}}{N^2} \frac{k}{1+k^2} \frac{P[\text{MW}]}{\delta} \frac{\lambda}{\lambda_u},
\]

\[
g_{\xi, n}^{\theta} = \frac{88 \times 10^{-4}}{n^2 N^2} \frac{k^2}{1+k^2} \frac{P[\text{MW}]}{\delta} \frac{\lambda}{\lambda_u} \left[ J(n-1)/2(n^\xi)-J(n+1)/2(n^\xi) \right]^2,
\]

where \( \delta \) is the machine duty cycle, \( P[\text{MW}] \) is the e-beam power in megawatts and \( \theta \) and \( \theta_n \) are the "delay-parameters" given by

\[
\begin{align*}
\theta &= \frac{4N}{\pi} \frac{\omega_0 \delta T}{g_h^{\theta}} \\
\theta_n &= \frac{4N}{\pi} \frac{\omega_0 \delta T}{g_{\xi, n}^{\theta}}
\end{align*}
\]

where \( \delta T = T_C - T \), \( T_C \), the cavity round trip period and \( T \) the bunch-bunch time (see Fig. 10).

The quantity \( \mathcal{R} \mathcal{E}_q \) represents the maximum value of the multimode gain function and contains also the dependence on the different parameters entering the process.

The typical behaviour of \( \mathcal{R} \mathcal{E}_q \) against \( \theta \) is shown in Fig. 11, together with the dimensionless laser power \( \chi \ref{3} \). It should be noticed that the maximum gain and the maximum output laser power do not correspond to the same value of \( \theta \). Therefore optimization of the gain does not result in the maximum output laser power.

The average laser power can be evaluated according to the following formula \ref{34}:

\[
P_L[\text{MW}] = P[\text{MW}] f[\text{Hz}] \chi(\theta) (\tau_M[\mu s] - \tau_R[\mu s])
\]

\[
(49)
\]
Fig. 10 e-beam structure from an RF machine: $t_b$ = microbunch time duration; $t_M$ = macropulse time duration; $T$ = microbunch time separation; $f$ = repetition frequency.

Fig. 11 Gain function and dimensionless laser power vs $\theta$

where $f$ is the machine repetition frequency, $t_M$ is the e-beam macropulse duration (see Fig. 10) and $t_R$ is the pulse rise-time linked to the gain by the following expression

$$t_R[\mu s] \approx 0.14 \frac{L_c[m]}{g - \gamma_T} \quad (50)$$

where $g$ is the gain as a function of the above parameters, $\gamma_T$ is the cavity loss and $L_c$ is the length of the cavity. In Fig. 1 the curves represent the average output power of an FEL operating at the 1st and 3rd harmonic respectively, with an e-beam power of 20 MW. It is evident that in the region $10 < \lambda(\mu m) < 100$ the FEL, in principle, may generate larger power than the conventional sources.
5. CONCLUSIONS

In this note we have presented a review of both the storage rings and single-pass FELs. We have emphasized the problems relevant to the electron sources and laser light output but no mention has been made of the cavity and undulator technology which are dealt with more completely in Ref. 4. We have also stressed that the future development of FELs as a "workhorse" for tunable applications strongly depends on the reliability of the electron source.

The use of the FEL for industrial applications will depend on its cost being relatively modest. In Ref. 34 a comparative cost analysis of FEL with other lasers was carried out and the results are summarized in Fig. 12. It is clear that the FEL is competitive when it is operating with a high efficiency extraction system (in the figure with an efficiency of 10%)\(^*\).

As a concluding remark we would like to stress that the goal of this paper has been twofold, namely to give a review of the basic ideas and problems underlying the FEL physics, and to indicate how this new laser device must be realistically considered within the framework of tunable sources.

![Fig. 12 FEL cost/watt vs λ with efficiencies of 1% and 10%](image)

\(^*\) The analysis has been limited to single-pass operating FELs where a market analysis for the electron source can be carried out. For the SR the technology is so specific that a market analysis makes no sense.
REFERENCES


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