Lepton-flavor-violating decays $L \rightarrow l\gamma\gamma$
as a new probe of supersymmetry with broken R-parity

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Lepton-flavor-violating decays of the type $L \rightarrow l\gamma\gamma$ ($\mu \rightarrow e\gamma\gamma$, $\tau \rightarrow e\gamma\gamma$, and $\tau \rightarrow \mu\gamma\gamma$) are proposed as new probes of R-parity-violating supersymmetry. Non-penguin diagrams with a sneutrino that decays into two photons via a triangle graph might trigger such decays even in the absence of the corresponding radiative decays into one photon only, e.g. $\mu \rightarrow e\gamma$. Thus, processes of the type $L \rightarrow l\gamma\gamma$ may provide an independent probe of new flavor physics.

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Supersymmetry (SUSY) is one of the most promising candidates for new physics, curing some of the shortcomings of the Standard Model (SM), for instance, the emergence of quadratic divergences in the Higgs sector \[1\]. Despite its appealing theoretical features, however, there is so far no experimental evidence for SUSY up to the electroweak scale. Apart from direct evidence, i.e. direct production and decays of SUSY particles, indirect probes could be employed to search for SUSY, i.e. processes in which the SUSY particles emerge virtually either in loops or as tree-level mediators. In this respect Flavor-Changing Neutral Current (FCNC) or Lepton-Flavor-Violating (LFV) processes are a natural searching ground for SUSY, since the SM background to such processes is either loop-suppressed (FCNC) or does not exist (LFV).

In this paper the effects of new physics on a class of LFV processes, $L \rightarrow l\gamma\gamma$, are investigated, in which a lepton decays radiatively into a lighter one plus two photons. As an example, R-Parity-Violating (RPV) SUSY is discussed. In the conventional RPV framework, the SUSY superpotential is supplemented with new RPV interaction terms $\hat{W}_{\text{RPV}} \supset \frac{1}{2} \lambda_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{L}_j^b \hat{E}_k^c + \lambda'_{ijk} \epsilon_{ab} \hat{L}_i^a \hat{\bar{Q}}_j^b \hat{\bar{D}}_k^c$, \[1\] where $\hat{Q}$ and $\hat{L}$ are SU(2) doublet quark and lepton supermultiplets, respectively, and $\hat{D}^c$ and $\hat{E}^c$ denote the SU(2) singlet down-type quark and lepton supermultiplet; $i, j, k = 1, 2, 3$ label the generations and $a, b$ are the SU(2) indices that are responsible for the antisymmetric property $\lambda_{ijk} = -\lambda_{jik}$.

Within the framework of RPV SUSY, $\mu \rightarrow e\gamma$ (and $\tau \rightarrow e\gamma$, $\tau \rightarrow \mu\gamma$) can be mediated by penguin-like diagrams of the type depicted in Fig. \[1\]. Depending on the RPV couplings involved, the particles running in the loop are either quark–squark, or lepton–slepton. The process $L \rightarrow l\gamma$, where $L$ and $l$ are leptons of generations $i$ and $j$, respectively, is then proportional to either $\lambda_{inn} \lambda_{jmn}$ or $\lambda_{inn} \lambda_{mijn}$ or $\lambda'_{inn} \lambda'_{jmn}$ \[2\].

So far, very little attention has been devoted to $L \rightarrow l\gamma\gamma$, under the assumption that it is always suppressed with respect to $L \rightarrow l\gamma$. An exception to this rule is discussed in \[3\], where a leptoquark model in which $L \rightarrow l\gamma\gamma$ is much larger than $L \rightarrow l\gamma$ is presented. Note that their model yields $\text{BR}(\mu \rightarrow e\gamma\gamma) < 10^{-18}$; see also \[4\]. In general, an additional photon can easily be accommodated - one just has to add a photon to the diagram in Fig. \[1\] and obtain the $\lambda$-reducible diagrams shown in Fig. \[5\] (they are reducible in the sense that, after removing a photon line from $L \rightarrow l\gamma\gamma$, a legitimate RPV diagram for $L \rightarrow l\gamma$ is obtained). Obviously processes mediated through such diagrams do not provide any further information on the corresponding RPV couplings, since they differ only by kinematic factors and by an extra electromagnetic coupling.

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FIG. 1: A penguin diagram responsible for the decays $L \rightarrow l\gamma$ in RPV SUSY. The photon can couple to any line provided it is charged. RPV couplings are symbolized by a blob.

(a)

(b)

FIG. 2: The topology of (a) $\lambda$-reducible and (b) $\lambda$-irreducible diagrams for $L \rightarrow l\gamma\gamma$.

However, it is quite possible that the decay $L \rightarrow l\gamma$ does not exist, or its rate is too small to be observed, whereas the decay $L \rightarrow l\gamma\gamma$ still has an appreciable rate. This could be the case if, for instance, all products of the type $\lambda\lambda$ and $\lambda\lambda'$ are much smaller than products of the type $\lambda\lambda'$, in which case the $\lambda$-irreducible topology of Fig. 2b may give rise to a much larger rate than the $\lambda$-reducible diagrams of Fig. 2a.

For example, if $\lambda_{122}$ and $\lambda'_{233}$ are the only non-vanishing lambdas, then $\Gamma(\mu \rightarrow e\gamma) = 0$ while $\Gamma(\mu \rightarrow e\gamma\gamma) \neq 0$. Accordingly, such RPV coupling product combinations can be detected or constrained by the measured upper limit on the branching ratio of $\mu \rightarrow e\gamma\gamma$, which is at present $7.2 \times 10^{-11}$ [7, 8]. No experimental limits exist so far for the analogous decays of the $\tau$-lepton.

There are proposals to measure (or constrain) the branching ratios of such SM-forbidden decays of the $\mu$ down to branching ratios of the order of $10^{-14}$ [9], $10^{-15}$ [10] or even down to $10^{-18}$ [11]. This highlights the phenomenological impact of LFV muon decays.

A general basis of effective operators describing the decays $L \rightarrow l\gamma\gamma$ with the topology of Fig. 2, i.e. $L \rightarrow lS_k \rightarrow l\gamma\gamma$, will be constructed first. Here $S_k$ is a scalar particle of type $k$ and $i, j$ are generation indices of $L, l$, respectively. The momentum notation $L(p) \rightarrow l(p')\gamma(k)\gamma(k')$ will be used.

The amplitude can be defined as

$$\mathcal{M}_{L \rightarrow l\gamma\gamma} = \sum_{n=1,2} \sum_{P=L,R} C_{n(ijkm)}^P \mathcal{O}_n^P,$$

where $m$ is the family index of the fermion that runs in the loop, and the index $k$ corresponds to the intermediate scalar. Also, $C_{n(ijkm)}^L, C_{n(ijkm)}^R$ are Wilson-like coefficients with $L(R) = (1 - (+)\gamma_5)/2$, and

$$\mathcal{O}_1^L = [\bar{u}_l(p')L u_L(p)] \left( k \cdot k' g^{\lambda\nu} - k' \lambda k^\nu \right) \epsilon^*_\lambda \epsilon^*_\nu.$$
\[ \mathcal{O}_L^I = [\bar{u}_i(p') L u_L(p)] i \varepsilon^{LpRq} k_{jk} k_{r} \bar{c}_i c^r \]  

(3)

are effective operators. If the incoming lepton is right-handed, then \(L \rightarrow R\) in (3).

In the framework of RPV SUSY with a sneutrino (\(\tilde{\nu}_k\)) exchange, the Wilson coefficients are given by:

\[ \begin{align*}
C_{L/(i j km)}^L; C_{R/(i j km)}^L (\ell_m \text{ in the loop}) & = 0; \\
& \frac{\alpha}{4 \pi} \frac{i \lambda^*_{kji} \lambda_{kmn}}{m_{\tilde{\nu}_k} M_{\tilde{\nu}_k}^2} \cdot [f_{1/2}(x); g_{1/2}(x)], \\
C_{L/(i j km)}^L; C_{R/(i j km)}^L (d_m \text{ in the loop}) & = 0; \\
& N_c Q^2 \frac{\alpha}{4 \pi} \frac{i \lambda^*_{kji} \lambda_{kmn}}{m_{d_m} M_{\tilde{\nu}_k}^2} \cdot [f_{1/2}(x); g_{1/2}(x)],
\end{align*} \]  

(4)

where \(M_{\tilde{\nu}_k}\) is the mass of the \(k\)th generation sneutrino and \(m_f\), \(f = \ell\) or \(d\), is the mass of the fermion of generation \(m\) that runs in the loop. Furthermore \(N_c = 3\), \(Q_d = 1/3\) and the abbreviation \(x = 2m_f^2 / k \cdot k'\) is being used. For diagrams with a charge-conjugate sneutrino (\(\tilde{\nu}^*\)) exchange, in (4) \(C_{L/(i j km)}^L \leftrightarrow C_{R/(i j km)}^R\) (i.e. with \(\lambda_{kji}^* \rightarrow \lambda_{kij}^*\)) and \(M_{\tilde{\nu}_k} \rightarrow M_{\tilde{\nu}_k^*}\). The functions \(f_{1/2}\) and \(g_{1/2}\) (originating from the loop integral of the triangle) are given by

\[ \begin{align*}
f_{1/2}(x) & = -2x \left[ 1 + (1 - x) \arcsin^2 \left( \frac{1}{\sqrt{x}} \right) \right], \\
g_{1/2}(x) & = 2x \arcsin^2 \left( \frac{1}{\sqrt{x}} \right).
\end{align*} \]  

(5)

The emergence of these functions reflects the different structure of the integral related to the parity of the couplings of the fermion in the loop. The subscripts refer to the spin of the particle running in the triangle. Note that when \(x < 1\), the arcsin \(1 / (\sqrt{x})\) develops an imaginary part, which corresponds to the loop particle residing on its mass shell.

In what follows, results will be given for both constituent and current light quark masses: \(m_d = 300\) MeV, \(m_s = 450\) MeV, \(m_t = 4.5\) GeV, and \(m_d = 10\) MeV, \(m_s = 120\) MeV, \(m_b = 4.2\) GeV, respectively. We believe that using constituent quark masses makes more sense, since for our case here, it would allow the decay \(\mu \rightarrow e\ell d\ell\), for current \(d\) quarks [12].

For the case of the muon decay, \(\mu \rightarrow e\gamma\gamma\), and for either down quarks with constituent masses or for the \(\tau\)-lepton running in the loop, the relation \(m_\ell^2 > m_\nu^2 > k \cdot k'\) holds. Then, the limit \(m_\ell^\mu > k \cdot k'\) (\(m_\ell = m_d, m_s, m_b\) or \(m_\tau\)) provides a good approximation. Therefore, the loop integral functions can be replaced with their \(x \rightarrow \infty\) limits, \(f_{1/2}(x \rightarrow \infty) = -\frac{1}{4}\) and \(g_{1/2}(x \rightarrow \infty) = 2\). Using this approximation, decay widths are given by

\[ \Gamma(\mu \rightarrow e\gamma\gamma) \big|_{m_\ell > k \cdot k'} \simeq A_{RPV}^{\mu} \left( \frac{\alpha}{\pi} \right)^2 \frac{1}{256\pi^2} \frac{13m_\mu^2}{34320m_\ell^2 M_{\tilde{\nu}_k}^2}, \]  

(6)

where \(f = d_i\) (\(i = 1, 2, 3\) for \(d, s\) or \(b\), respectively) or \(f = \tau\) is the fermion in the loop and \(S = \tilde{\nu}_k\) or \(\tilde{\nu}_k^*\) (\(k = 1, 2\) or \(3\)), is the scalar responsible for the flavor changing transition \(\mu \rightarrow e\). Also

\[ \begin{align*}
A_{RPV}^{\mu} & = |\lambda_{k12}\lambda_{k33}|^2, \\
A_{RPV}^{\tau} & = |\lambda_{k21}\lambda_{k33}|^2, \\
A_{RPV}^{d, \tilde{\nu}_k} & = |\lambda_{k12}\lambda_{k3i}|^2 / 9, \\
A_{RPV}^{d, \tilde{\nu}_k^*} & = |\lambda_{k21}\lambda_{k3i}|^2 / 9.
\end{align*} \]  

(7)

The branching ratios scaled by the relevant RPV couplings for single particles running in the triangle are given in Table I in the spirit of this paper, only such down quarks and leptons running in the triangle are considered that would not give rise to the decay \(L \rightarrow l^\ell\), i.e., only those leptons that cannot generate the \(\lambda\)-reducible-type diagrams.

Comparing the current limit \(BR(\mu \rightarrow e\gamma\gamma) < 7.2 \times 10^{-11}\) [7] with the estimate for the maximal possible branching ratio for this decay mode in Table I in conjunction with the existing bounds on the relevant RPV coupling products as given in Table II, it can be argued that, at present, \(\mu \rightarrow e\gamma\gamma\) does not impose any new constraints on the RPV SUSY parameter space. However, the situation will improve with the much more stringent experimental constraints that can be anticipated: on the third row of Table II, the expected sensitivity of \(\mu \rightarrow e\gamma\gamma\) to the relevant RPV coupling products is exhibited, assuming that future experiments will be sensitive to \(BR(\mu \rightarrow e\gamma\gamma) \lesssim 10^{-14}\), see e.g. [8].

For the case of \(\tau\)-decays, taking the existing bounds on the relevant RPV coupling products into account, the largest branching ratio is obtained for \(\tau \rightarrow \mu\gamma\gamma\) with an \(s\)-quark in the loop. In particular, for \(|\lambda_{k33}\lambda_{k22}| = 0.01\) (its upper bound, see [12]), \(m_s = 450\) MeV and \(M_{\tilde{\nu}} = 100\) GeV, \(BR(\tau \rightarrow \mu\gamma\gamma) \sim 10^{-10}\). This result is by far smaller than the typical present limits on \(\tau\) decay branching ratios, which are of the order of \(10^{-7}\), see e.g. [12]. Moreover, planned experiments [13], which will produce about \(5 \times 10^6\) taus per year, will also be insensitive to the decays \(\tau \rightarrow l\gamma\gamma\).

A few remarks are in order to conclude the discussion of the branching ratios:
From \([2, 20]\)

\[ \tau (m = 1) \quad \mu \rightarrow e\gamma \text{ possible} \quad \tau \rightarrow e\gamma \text{ possible} \quad 8.9 \times 10^{-9} \]

\[ \tau (m = 2) \quad \mu \rightarrow e\gamma \text{ possible} \quad 5.0 \times 10^{-9} \quad \tau \rightarrow \mu\gamma \text{ possible} \]

\[ \tau (m = 3) \quad 3.2 \times 10^{-9} \quad \tau \rightarrow e\gamma \text{ possible} \quad \tau \rightarrow \mu\gamma \text{ possible} \]

TABLE II: Current limits and expected sensitivity to RPV coupling products from future experiments. Limits are given for sparticle masses of 100 GeV. The limits shown on the fourth column scale as \(m_\nu/100 \text{ [GeV]}\), see text.

- From \(\mu \rightarrow e\gamma\) conversion in nuclei.
- This limit depends strongly on the strange-quark content in the nucleon and is, therefore, model-dependent. In particular, if the strange quark content in the nucleon is consistent with zero (see e.g. [21]), then this limit does not apply.
- From charged current universality.
- From \(\Gamma(\tau \rightarrow e\nu\bar{\nu})/\Gamma(\tau \rightarrow \mu\nu\bar{\nu})\).
- From \(\mu \rightarrow e\gamma\) conversion in nuclei [22].
- From \(BR(D^+ \rightarrow K^o\mu^+\nu_\mu)/BR(D^+ \rightarrow K^o\mu^+\nu_\mu)\).
- From \(D^0 \rightarrow D^0\) mixing. Model-dependent.
- From \(\Gamma_{had}(Z^0)/\Gamma_{lep}(Z^0)\).

There is also a second possibility for \(L \rightarrow l\gamma\) of the \(\lambda\)-irreducible type to proceed: through the soft SUSY-breaking RPV terms \(10 \ C_{ijk} \hat{f}_L \hat{f}_L \hat{E}_k^c\) or \(C_{ijk} \hat{L}_j \hat{Q}_i \hat{E}_k^c\) and slepton or squark in the loop. In this case the sneutrino couples to a pair of sleptons or down squarks of different helicity index, i.e. \(\hat{f}_L \hat{f}_R\), where \(\hat{f}\) is a slepton or down squark, while the photons couple to \(\hat{f}_L \hat{f}_L\) or to \(\hat{f}_R \hat{f}_R\). Assuming for simplicity that the \(\hat{f}_L \hat{f}_R\) mixing is of \(O(1)\) and setting \(C_{ijk} \equiv M_{SUSY} \hat{C}_{ijk}\), \(C'_{ijk} \equiv M_{SUSY} \hat{C}'_{ijk}\), the width for \(L \rightarrow l\gamma\) is (for \(m_\hat{f}^2/m_L^2 \gg 1\) and \(m_\hat{f}^2/m_L^2 \ll 1\)):

\[
\Gamma(L \rightarrow l\gamma) \simeq \tilde{A}_{iij} \tilde{f}^S \left(\frac{\alpha}{\pi}\right)^2 \frac{1}{256\pi^2} \frac{Q^4_{\tilde{f}} m_\tilde{f}^2 M_{SUSY}^2}{8640 m_L^4 m_S^4},
\]  

(8)
where \( \tilde{f} = \tilde{\ell} \) with \( Q_{\tilde{f}} = -1 \) or \( \tilde{f} = \tilde{d} \) with \( Q_{\tilde{f}} = -1/3 \) if the sparticle in the loop is a slepton or down squark, respectively, and \( S = \tilde{\nu} \) or \( \tilde{\nu}^* \). Also,

\[
\tilde{A}^{ij\tilde{f}_m\tilde{\nu}_k}_{RPV} = |\tilde{C}_{kmm}\lambda_{kij}|^2, \quad \tilde{A}^{ij\tilde{f}_m\tilde{\nu}^*_k}_{RPV} = |\tilde{C}_{kmm}\lambda_{kji}|^2, \\
\tilde{A}^{ij\tilde{d}_n\tilde{\nu}_k}_{RPV} = 9 |\tilde{C}'_{kmm}\lambda_{kij}|^2, \quad \tilde{A}^{ij\tilde{d}_n\tilde{\nu}^*_k}_{RPV} = 9 |\tilde{C}'_{kmm}\lambda_{kji}|^2. \tag{9}
\]

Here \( m, k = 1, 2, 3 \) denote the family of the slepton (\( \tilde{\ell} \)) or down squark (\( \tilde{d} \)) and sneutrino or antineutrino, respectively.

Plugging in numbers (e.g., setting \( M_{SU3} = m_{\tilde{e}_n} = m_{\tilde{\nu}_k} = 100 \text{ GeV} \)), \( BR(\mu \rightarrow e\gamma\gamma) \sim |\tilde{C}_{kmm}\lambda_{k12}|^2 \sim 3.7 \times 10^{-14} \) and \( BR(\tau \rightarrow e\gamma\gamma) \sim |\tilde{C}_{kmm}\lambda_{k13}|^2 \sim 1.9 \times 10^{-12} \). Both values are much smaller than the ones obtained with fermions in the loop.

- If an RPV bilinear term exists, i.e. \( B\tilde{L}\tilde{H}_u \) [16], then \( L \rightarrow l\gamma\gamma \) can also proceed via the \( \lambda \)-irreducible topology with one trilinear (\( \lambda \)) and one bilinear (\( B \)) RPV insertion. Therefore, the decay would be \( \propto \lambda \times B \). In this case the off-shell sneutrino emitted from the decaying lepton mixes with the down-type Higgs boson, which then goes into the two photons with the well known (see e.g. [17]) one-loop SUSY amplitude for \( H \rightarrow \gamma\gamma \) (with the replacement \( m_H^2 \rightarrow 2k \cdot k' \)). This possibility will be investigated in detail elsewhere [18].

- Another set of related processes that may be generated through the same \( \lambda \)-irreducible diagrams are \( \tau \rightarrow (e, \mu)gg \). Work on these processes is in progress [18].

- The \( \tau \)-decays \( \tau \rightarrow (\mu, e)\gamma\gamma \) of the \( \lambda \)-irreducible type that are considered in this paper, with the lepton loop, have tree-level “analogs” that probe identical combinations of lambdas: \( \tau \rightarrow \mu ee \) and \( \tau \rightarrow \mu ee \) [22]. However, the contribution from lepton loops is not necessarily the largest one: it was found above that the dominant contribution comes from the \( s \)-quark loop, for which the branching ratio can amount to up to \( \mathcal{O}(10^{-10}) \) (for \( \tau \rightarrow \mu\gamma\gamma \)), taking into account the current constraints [12], which are obtained from processes of the type \( \tau \rightarrow l + \text{hadrons} \).

- As it can be seen from the Table, \( \mu - e \) conversion in nuclei is a strong rival to our processes. However, couplings that involve \( b \) quark and \( \tau \) lepton (and perhaps \( s \) quark) are beyond the reach of \( \mu - e \) conversion experiments. Although our bounds on such processes are weak, they will strengthen if PRISM experiment will come into action.

In this paper a new class of lepton flavor-violating-processes, \( L \rightarrow l\gamma\gamma \) has been presented and discussed that might appear even if the corresponding decay into one photon only is suppressed or not existing at all. The underlying mechanism for such decays is the decay of the heavy lepton into the lighter one and a virtual scalar particle that goes into the two photons via a triangle diagram. Hence, such processes might probe the flavor structure of leptons coupling to these scalar particles that are a common feature of many models for new physics. Branching ratios for such decays have been calculated within the framework of R-parity-violating supersymmetry with sneutrinos as the scalar particles. Imposing RPV bounds emerging from other processes, the resulting branching ratios are well below current observation thresholds. However, for a planned new round of experiments, especially for \( \mu \) decays, these processes - if they exist - might shed light on potential new physics or in turn might help to set more stringent exclusion bounds.

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