Review of Coupled Bunch Instabilities in the LHC

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Abstract

In order to reach the required luminosity, the LHC will have a large number of high intensity bunches. Coupled bunch instabilities can therefore be excited by the higher order modes (HOMs) of the RF cavities, by parasitic cavities and by the transverse resistive wall effect. This report summarises the growth times of the coupled bunch instabilities taking into account the HOMs (damped or undamped) relevant for the 200 MHz normal conducting cavities, the 400 MHz superconducting cavities, as well as other parasitic cavities. It is shown that, with the damped HOMs of the RF cavities, the coupled bunch instabilities remain within control for the LHC operation. As far as the transverse resistive wall effect at injection is concerned, it is demonstrated that the corresponding growth times can be safely compensated by the proposed transverse feedback system [1].
1 Introduction

The LHC will be operated with a total of 2808 bunches per beam and with a bunch separation of 25 nsec. With the proposed bunch intensities ($1.1 \times 10^{11}$ for nominal operation and $1.7 \times 10^{11}$ for the ultimate case), the wake fields generated by a bunch in narrow band structures such as RF cavities, other parasitic cavities and the transverse resistive wall effect will last long enough to affect consecutive bunches and thus potentially lead to coupled bunch instabilities.

The LHC will have two (four cells) superconducting cavities per beam operating at 400 MHz and four normal conducting cavities operating at 200 MHz for efficient injection capture. The interaction of the bunches with these cavities can lead to coupled bunch instabilities (CBI) in the longitudinal as well as in the transverse planes in case the wake fields are strong enough. This problem is potentially more severe in the longitudinal plane as there will be no longitudinal feedback in the LHC [2]. In the transverse planes, there is a feedback system and it is necessary to confirm that the growth times of the CBI are slow enough to stay within the power and gain limits of the feedback system. It is worth underlining that an active feedback operating up to 20 MHz will be able to treat each bunch separately on its fundamental dipole mode, while the higher order bunch oscillation modes will have to be stabilised by Landau damping. Similarly, the feedback system will also be required to ensure a sufficient margin for the control of the transverse emittances.

In a first step, the growth times for each individual type of cavities are evaluated both for the damped and undamped higher order modes. This stresses the importance of the 200 MHz cavities in terms of CBI, but also confirms that the situation remains under control, provided the cavities’ HOMs are damped. As far as the present evaluation is concerned, a few additional comments concerning the scenarios included in the study are as follows:

- Although the 200 MHz cavities will be parked after injection, they physically remain in the machine so that their impedance has to be included both for the injection and physics conditions. The same argument applies for the 400 MHz cavities at injection.
- Presently there exists a possibility that the 200 MHz cavities will not be initially installed in the machine (injection being carried out with the 400 MHz cavities at 8 MV). This option has also been included in the study, in order to check that this operation scenario is also safe in terms of CBI.
- Although, during the design phase, a special effort was made to avoid any unnecessary parasitic cavities, there are inevitably some elements in the machine with undesired trapped modes. This is the case for the transverse dampers and for the CMS experimental chamber. Consequently, these elements have been included.
- A lot of work has been done in the past to study the CBI in the LHC [3–7]. The published results illustrate the evolution of the design for many components. A special effort has been made to use the most recent available data for this study. This is particularly true for the compilation of the HOMs of the different elements.
- The LHC filling pattern is not symmetric but is composed by more or less symmetric batches. Unfortunately, most of the standard computational codes available for CBI calculations require a symmetric filling of the buckets. It is therefore assumed that the bunches are symmetrically placed and the calculations are done for the ultimate intensity. Since it has been reported earlier [6,8,9] that the maximum growth rate of a non-symmetrically filled ring is always smaller than that of the corresponding
symmetric filling, the estimates using symmetric filling and the ultimate intensity guarantee a safe prediction of an upper bound for the growth rates.

- The transverse resistive wall effect can also couple the motion of different bunches. The transverse resistive wall estimates are rather complicated in the LHC due to different designs of the superconducting, warm, injection and interaction regions. It will be shown that the proposed transverse feedback system can cope with the estimated growth times.

2 Longitudinal Symmetric Coupled Bunch Instabilities

Two mode numbers 's' and 'a' describe a longitudinal coupled bunch mode. For 'k' bunches in the machine, there are 'k' coupled bunch modes characterised by a longitudinal mode number 's' which takes the values; s=0,1,2,...,(k-1), and defines the phase shift

\[ \Delta \phi = \frac{2\pi s}{k} \]

between the bunches. An index 'a' describes the individual bunch motion for each coupled bunch oscillation mode 's'. Thus a=1 is the dipole mode where the bunches move rigidly as they execute longitudinal synchrotron oscillations, a=2 is the quadrupole mode, where the head and tail of the bunch oscillate longitudinally out of phase etc.

The unperturbed modes have frequencies given by

\[ \omega_p^\parallel = (pk + s + aQ_s)\omega_0 \]

where p=0,±1,±2..., \(Q_s\) is the synchrotron tune and \(\omega_0\) is the angular revolution frequency.

In the presence of the machine impedance, there is a coherent frequency shift. The coherent frequency shift in the Sacherer-Zotter formalism is given by [10,11]:

\[ \Delta \omega_{s,a}^\parallel = i \left( \frac{a}{a + 1} \right) \frac{I_b \omega_0^2 \eta}{3(L/2\pi R)^32\pi \beta^2 (E_T/e)\omega_s} \left[ \frac{Z_n^\parallel}{n} \right]_{s,a}^{eff} \]

where \(L\) is the total bunch length equal to \(2\sqrt{2}\sigma_l\) (\(\sigma_l = \text{rms bunch length}\)) for a parabolic bunch and \(2\sqrt{\pi}\sigma_l\) for a Gaussian bunch. \(R\) is the average machine radius, \(I_b\) is the average bunch current, \(\eta\) is the phase slip factor, \(E_T\) is the total beam energy, \(\beta\) is the relativistic beta factor and \(\omega_s\) is the angular synchrotron frequency=\(Q_s\omega_0\). The effective longitudinal impedance is defined by:

\[ \left[ \frac{Z_n^\parallel}{n} \right]_{s,a}^{eff} = \sum_{p=-\infty}^{p=+\infty} \frac{Z_n^\parallel(\omega_p^\parallel)}{(\omega_p^\parallel/\omega_0)} h_a(\omega_p^\parallel) \sum_{p=-\infty}^{p=+\infty} h_a(\omega_p^\parallel), \]

Where \(h_a(\omega_p^\parallel)\) denotes the bunch mode spectrum characteristic of the synchrotron mode 'a'.

The real part of the coherent frequency shift gives the real coherent mode frequency shift and the imaginary part yields the instability growth rate.

The growth times of the longitudinal CBI have been calculated considering the LHC parameters presented in Table 1. In case the 200 MHz cavities would not be available at injection, the corresponding parameters for injecting into the 400 MHz system are indicated in parantheses.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ring Circumference [m], C</td>
<td>26658.883</td>
</tr>
<tr>
<td>Number of protons per bunch, (N_p)</td>
<td>(1.1\times10^{11}) (nominal)</td>
</tr>
<tr>
<td>Circulating beam current [A], (I_0)</td>
<td>(0.706) (nominal)</td>
</tr>
<tr>
<td></td>
<td>(1.7\times10^{11}) (ultimate)</td>
</tr>
<tr>
<td>Momentum compaction, (\alpha)</td>
<td>0.000347</td>
</tr>
<tr>
<td>Betatron tunes (H/V), (Q_T)</td>
<td>63.28/63.31</td>
</tr>
<tr>
<td>Energy [GeV], (E_T)</td>
<td>450</td>
</tr>
<tr>
<td></td>
<td>7000</td>
</tr>
<tr>
<td>RF frequency [MHz], (f_{RF})</td>
<td>200.35 (400.789)</td>
</tr>
<tr>
<td></td>
<td>400.79</td>
</tr>
<tr>
<td>Harmonic number, (h)</td>
<td>17820 (35640)</td>
</tr>
<tr>
<td>Number of symmetric bunches, (k)</td>
<td>3564</td>
</tr>
<tr>
<td></td>
<td>3564</td>
</tr>
<tr>
<td>RF voltage [MV], (V_{RF})</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>(8)</td>
</tr>
<tr>
<td></td>
<td>16</td>
</tr>
<tr>
<td>rms bunch length [cm], (\sigma_l)</td>
<td>17.5 (11.6)</td>
</tr>
<tr>
<td></td>
<td>7.73</td>
</tr>
<tr>
<td>rms energy spread, (\sigma_E/E)</td>
<td>(3.06\times10^{-4}) (4.68\times10^{-4})</td>
</tr>
<tr>
<td></td>
<td>(1.11\times10^{-4})</td>
</tr>
<tr>
<td>Synchrotron tune, (Q_s)</td>
<td>0.002546 (0.005878)</td>
</tr>
<tr>
<td></td>
<td>0.00212</td>
</tr>
<tr>
<td>Synchrotron frequency [Hz], (f_s)</td>
<td>28.64 (66.08)</td>
</tr>
<tr>
<td></td>
<td>23.86</td>
</tr>
</tbody>
</table>

Table 1: Parameters of LHC used for CBI estimates

### 2.1 Longitudinal CBI estimates for HOMs of the 200 MHz cavities

The undamped monopole modes of the 200 MHz capture cavities are given in Table 2 while the damped monopole modes data are given in Table 3 (with 2 HOM couplers and with 4 HOM couplers) [12]. The corresponding shunt impedances are illustrated in Figures 1 and 2. The quoted HOM values apply for one cavity and the shunt impedances are multiplied by the number of cavities (i.e. 4) for growth time estimates. Using these modes, the growth times of the coupled bunch instabilities have been estimated mainly using the computer program ZAP [11] (the comparison of ZAP with the other codes is given in Ref. [13]). Though the experimental observations on SPS fit quite well with a Gaussian distribution, both parabolic and Gaussian distributions have been considered. It is recalled that the growth times are evaluated for the ultimate LHC intensity.

The CBI are counteracted by Landau damping from the synchrotron frequency spread within the bunches. A spread in synchrotron frequency arises from the non-linearity of the RF bucket. The CBI mode is Landau damped if the shifted mode frequency lies within the effective spread of the bunch. In the different Tables, the flag ‘Damped’ indicates that the mode is actually ‘Landau damped’.

As can be seen from Table 4, with the undamped HOMs of the 200 MHz cavities, the growth times are very fast. The dangerous HOMs are mainly at 245.7, 487.5, 631.1,
<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Rs (MΩ)</th>
<th>Q</th>
<th>Frequency (MHz)</th>
<th>Rs (MΩ)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>245.0</td>
<td>0.103</td>
<td>33400</td>
<td>1556.0</td>
<td>0.002</td>
<td>118200</td>
</tr>
<tr>
<td>487.5</td>
<td>1.047</td>
<td>57470</td>
<td>1596.0</td>
<td>0.029</td>
<td>70580</td>
</tr>
<tr>
<td>631.1</td>
<td>0.132</td>
<td>49490</td>
<td>1653.0</td>
<td>0.009</td>
<td>102900</td>
</tr>
<tr>
<td>716.2</td>
<td>0.731</td>
<td>60000</td>
<td>1672.0</td>
<td>0.221</td>
<td>67680</td>
</tr>
<tr>
<td>748.9</td>
<td>0.367</td>
<td>53260</td>
<td>1678.0</td>
<td>0.003</td>
<td>151200</td>
</tr>
<tr>
<td>943.6</td>
<td>0.685</td>
<td>88480</td>
<td>1788.0</td>
<td>0.007</td>
<td>100100</td>
</tr>
<tr>
<td>1015.0</td>
<td>0.023</td>
<td>79100</td>
<td>1810.0</td>
<td>0.050</td>
<td>62940</td>
</tr>
<tr>
<td>1101.0</td>
<td>0.736</td>
<td>72080</td>
<td>1836.0</td>
<td>0.102</td>
<td>76180</td>
</tr>
<tr>
<td>1111.0</td>
<td>0.001</td>
<td>62080</td>
<td>1986.0</td>
<td>0.018</td>
<td>81090</td>
</tr>
<tr>
<td>1279.0</td>
<td>0.021</td>
<td>61940</td>
<td>2027.0</td>
<td>0.167</td>
<td>71670</td>
</tr>
<tr>
<td>1283.0</td>
<td>0.155</td>
<td>98680</td>
<td>2086.0</td>
<td>0.085</td>
<td>66110</td>
</tr>
<tr>
<td>1336.0</td>
<td>0.116</td>
<td>83310</td>
<td>2238.0</td>
<td>0.064</td>
<td>81580</td>
</tr>
<tr>
<td>1380.0</td>
<td>0.003</td>
<td>84860</td>
<td>2299.0</td>
<td>0.495</td>
<td>37990</td>
</tr>
<tr>
<td>1382.0</td>
<td>0.010</td>
<td>71680</td>
<td>2317.0</td>
<td>0.162</td>
<td>60440</td>
</tr>
<tr>
<td>1438.0</td>
<td>0.106</td>
<td>66310</td>
<td>2321.0</td>
<td>0.169</td>
<td>60800</td>
</tr>
<tr>
<td>1485.0</td>
<td>0.010</td>
<td>82730</td>
<td>2535.0</td>
<td>0.044</td>
<td>62390</td>
</tr>
<tr>
<td>1489.0</td>
<td>0.340</td>
<td>75160</td>
<td>2617.0</td>
<td>0.005</td>
<td>55560</td>
</tr>
<tr>
<td>1531.0</td>
<td>0.001</td>
<td>108800</td>
<td>2623.0</td>
<td>0.563</td>
<td>56040</td>
</tr>
<tr>
<td>1528.0</td>
<td>0.010</td>
<td>86690</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Undamped monopole modes of the 200 MHz cavity [12]

Figure 1: Undamped monopole modes of the 200 MHz cavity [12]
Table 3: Damped monopole modes of the 200 MHz cavity [12]
Figure 2: Damped monopole modes of the 200 MHz cavity [12]

<table>
<thead>
<tr>
<th>Bunch mode</th>
<th>Parabolic Bunch</th>
<th>Gaussian Bunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection</td>
<td>Top</td>
<td>Injection</td>
</tr>
<tr>
<td>a=1</td>
<td>16 msec</td>
<td>86 msec</td>
</tr>
<tr>
<td>a=2</td>
<td>24 msec</td>
<td>267 msec</td>
</tr>
</tbody>
</table>

With Damped HOMs
2 Couplers
| a=1        | Damped          | Damped         | Damped         | Damped         |
| a=2        | 238 msec        | Damped         | Damped         | Damped         |

With Damped HOMs
4 Couplers
| a=1        | Damped          | Damped         | Damped         | Damped         |
| a=2        | Damped          | Damped         | Damped         | Damped         |

Table 4: Effect of undamped and damped monopole modes of the 200 MHz cavities
716.2, 943.6 and 1100.99 MHz. As shown in Table 4, the growth time for the bunch mode \( a=1 \) at injection energy using parabolic bunch shape is less than that for \( a=2 \), but the situation is reversed when a Gaussian bunch shape is used. The growth times are therefore evaluated for both bunch shapes in all cases, to avoid missing the fastest growth time. With 2 couplers on these cavities, as shown in Table 4, all the bunch modes are damped except for \( a=2 \) at injection with a parabolic bunch shape. With 4 couplers, all coupled bunch modes are suppressed at injection as well as at top energy.

### 2.2 Longitudinal CBI estimates for HOMs of the 400 MHz cavities

The undamped and damped monopole modes data for the 400 MHz superconducting cavities [14] are given in Table 5. This data is for one cavity (consisting of 4 cells) and the shunt impedances should be multiplied by the number of cavities per beam (=2). This factor has thus been included in the growth times calculations.

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Undamped HOM</th>
<th>Damped HOM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \text{R}_{\text{sh}} )</td>
<td>( \text{Q} )</td>
</tr>
<tr>
<td>779.0</td>
<td>0.301</td>
<td>50000</td>
</tr>
<tr>
<td>1184.0</td>
<td>0.068</td>
<td>50000</td>
</tr>
<tr>
<td>1238.0</td>
<td>0.076</td>
<td>50000</td>
</tr>
</tbody>
</table>

Table 5: Undamped and damped monopole modes for the 400 MHz superconducting cavity [14]

Table 6 illustrates the results obtained for the HOMs of the 400 MHz cavities (damped and undamped). The numbers in parantheses correspond to the case where the 400 MHz cavities are used at injection rather than the 200 MHz cavities. With undamped HOMs, the bunch excitation is not suppressed and thus, it is necessary to damp the HOMs with dedicated couplers. With the damped HOMs, the CBI are neither excited at injection nor at top energy.

### 2.3 Longitudinal CBI estimates for HOMs of Transverse Damper and trapped modes of CMS chamber

The transverse feedback system is composed of 4 dampers per beam. The monopole modes for these dampers have been calculated and measured [15]. The monopole modes for one damper system are given in Table 7. As shown in Table 8, with the undamped monopole modes of the transverse dampers, the bunch motion is unstable (though not very fast) at both energies. However, with the damped monopole modes, all the coupled bunch modes are suppressed at injection as well as top energy.

The detailed design of the CMS experimental chamber is available and the trapped monopole modes have been estimated by Yun Luo \(^1\) using the MAFIA code [16].

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\(^1\) On leave of absence from IHEP, China
Table 6: Effect of undamped & damped monopole modes of the 400 MHz cavities

<table>
<thead>
<tr>
<th>Bunch mode</th>
<th>Parabolic Bunch</th>
<th>Gaussian Bunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Undamped HOMs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a=1</td>
<td>0.267 sec</td>
<td>0.403 sec</td>
</tr>
<tr>
<td>(Damped)</td>
<td>(Damped)</td>
<td></td>
</tr>
<tr>
<td>a=2</td>
<td>0.116 sec</td>
<td>Damped</td>
</tr>
<tr>
<td>(Damped)</td>
<td>(Damped)</td>
<td></td>
</tr>
<tr>
<td>With Damped HOMs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a=1</td>
<td>Damped</td>
<td>Damped</td>
</tr>
<tr>
<td>(Damped)</td>
<td>(Damped)</td>
<td></td>
</tr>
<tr>
<td>a=2</td>
<td>Damped</td>
<td>Damped</td>
</tr>
<tr>
<td>(Damped)</td>
<td>(Damped)</td>
<td></td>
</tr>
</tbody>
</table>

Table 7: Undamped and damped monopole modes of the Transverse Damper [15]

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Undamped HOMs</th>
<th>Damped HOMs</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rsh (KΩ)</td>
<td>Q (KΩ)</td>
</tr>
<tr>
<td>82.8</td>
<td>2.875</td>
<td>820</td>
</tr>
<tr>
<td>419.3</td>
<td>26.67</td>
<td>1270</td>
</tr>
<tr>
<td>641.2</td>
<td>2.184</td>
<td>1680</td>
</tr>
<tr>
<td>880.6</td>
<td>0.816</td>
<td>1020</td>
</tr>
</tbody>
</table>

trapped monopole modes are listed in Table 9. These modes are for half of the chamber, so that the values of the shunt impedances have been multiplied by two for the calculation of the growth times. The monopole mode spectrum for the full chamber is shown in Figure 3. Despite of the numerous modes of this chamber, all the coupled bunch modes remain stable at injection as well as at top energy.

2.4 Summary for the longitudinal plane

The CBI growth times with undamped HOMs of both RF cavities put together along with the undamped modes of transverse dampers and the CMS chamber show that the CBI would be excited and thus would not be acceptable for the LHC. However, with the damped HOMs of RF cavities and transverse dampers, the CBI are Landau damped and thus will not be a problem for the LHC operation.

3 Transverse Symmetric Coupled Bunch Instabilities

As described in section 2, the two mode numbers 's' and 'a' are again required to describe a transverse coupled bunch instabilities. The main difference as compared to the longitudinal case is that the index 'a' can take a value equal to zero, meaning that the bunches move rigidly as they execute the transverse oscillations. Thus, a=0, describes the rigid
| Bunch mode | Parabolic Bunch | | | Gaussian Bunch | | | |
| --- | --- | --- | --- | --- | --- | --- |
| | Injection | Top | Injection | Top | | | |
| With Undamped HOMs | | | | | | | |
| a=1 | Damped | Damped | Damped | 1.32 sec | | | |
|  | (Damped) | (Damped) | | | | | |
| a=2 | Damped | Damped | Damped | Damped | | | |
|  | (Damped) | (Damped) | | | | | |
| With Damped HOMs | | | | | | | |
| a=1 | Damped | Damped | Damped | Damped | | | |
|  | (Damped) | (Damped) | | | | | |
| a=2 | Damped | Damped | Damped | Damped | | | |
|  | (Damped) | (Damped) | | | | | |

Table 8: Effect of monopole modes of the Transverse Dampers

![Graph showing Shunt Impedance](image)

Figure 3: Trapped monopole modes in the CMS chamber
<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Rs (KΩ)</th>
<th>Q</th>
<th>Frequency (MHz)</th>
<th>Rs (KΩ)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>726.6983</td>
<td>0.0288</td>
<td></td>
<td>970.0500</td>
<td>1.0564</td>
<td></td>
</tr>
<tr>
<td>728.1463</td>
<td>0.0949</td>
<td></td>
<td>980.0851</td>
<td>1.0514</td>
<td></td>
</tr>
<tr>
<td>730.5508</td>
<td>0.2248</td>
<td></td>
<td>990.0418</td>
<td>1.0302</td>
<td></td>
</tr>
<tr>
<td>733.8926</td>
<td>0.4158</td>
<td></td>
<td>1000.0659</td>
<td>1.0833</td>
<td></td>
</tr>
<tr>
<td>738.1447</td>
<td>0.4754</td>
<td></td>
<td>1009.9896</td>
<td>1.0884</td>
<td></td>
</tr>
<tr>
<td>743.2677</td>
<td>0.9347</td>
<td></td>
<td>1020.0383</td>
<td>1.0421</td>
<td></td>
</tr>
<tr>
<td>749.2051</td>
<td>0.8326</td>
<td></td>
<td>1029.9429</td>
<td>1.1326</td>
<td></td>
</tr>
<tr>
<td>755.8727</td>
<td>1.2044</td>
<td></td>
<td>1039.9324</td>
<td>1.0715</td>
<td></td>
</tr>
<tr>
<td>763.1447</td>
<td>1.4392</td>
<td></td>
<td>1049.8354</td>
<td>1.0948</td>
<td></td>
</tr>
<tr>
<td>770.8464</td>
<td>1.3979</td>
<td></td>
<td>1059.7559</td>
<td>0.9486</td>
<td></td>
</tr>
<tr>
<td>778.8123</td>
<td>1.4366</td>
<td></td>
<td>1069.6556</td>
<td>1.0365</td>
<td></td>
</tr>
<tr>
<td>787.0434</td>
<td>1.6605</td>
<td></td>
<td>1079.5973</td>
<td>1.0086</td>
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<td>1256.1288</td>
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</table>

Table 9: Monopole modes in half of the CMS chamber
dipole mode and the mode a=1 applies to the case where the head and the tail of the bunch oscillate transversely out of phase. The frequency of the unperturbed modes in this case is described by,

\[ \omega_p^T = (pk + s + QT + aQ_s)\omega_0 \]

where p=0,±1,±2...and QT is the betatron tune. The effective bunch spectrum is modified for the finite chromaticity. The coherent frequency shift in the Sacherer-Zotter formalism is given by [10,11]:

\[ \Delta\omega_{s,a}^T = -i\left(\frac{1}{a + 1}\right)\frac{I_b\beta c^2}{2L(E_T/e)\omega_\beta[Z_T]_{s,a}^{s,a}} \]

The effective bunch spectrum is modified for the finite chromaticity as:

\[ [Z_T]_{s,a}^{s,a} = \sum_{p=-\infty}^{p=+\infty} h_a(\omega_p^T - \omega_\xi) \]

where \( \omega_\beta = QT\omega_0 \) and \( \omega_\xi \) is the chromatic frequency given as \( \omega_\xi = \xi\omega_0/\eta \) with \( \xi \) as the chromaticity.

A complex coherent frequency shift is estimated for the given narrow band impedances in the machine. The real part of the frequency shift gives the frequency shift while the imaginary part gives the instability growth rate.

The transverse rigid dipole mode, a=0, requires in addition to synchrotron spread also a betatron tune spread for Landau damping. However, for modes a>0, synchrotron frequency spread is sufficient to obtain Landau damping. In the Zotter formalism [10], a guess for the betatron tune spread is required as an input for the a=0 mode. The Landau damping condition for a=0 transverse rigid dipole mode with nonlinear betatron tune spread in the Zotter formalism is however not included in ZAP. It has therefore been evaluated with the BBI program.

### 3.1 Transverse CBI estimates for the HOMs of the 200 MHz cavities

The growth times of transverse CBI have been calculated considering the LHC parameters given in Table 1. The undamped dipole modes of the 200 MHz capture cavities [17] are given in Table 10 and Figure 4. The damped dipole modes are presently not available for these cavities.

The growth times are again evaluated for the ultimate intensity and a corrected chromaticity equal to zero. The growth times in the presence of the undamped dipole modes of the 200 MHz cavities are given in Table 11. The a=0 and a=1 modes show undamped motion. At injection energy, the growth time of the a=0 mode is long enough and can be handled by the transverse feedback. The remaining a=1 mode can be cured by positive chromaticity and by space charge tune spread. At top energy, the space charge tune spread is two orders of magnitude smaller than at injection and will not be sufficient anymore. However, the tune spread arising from beam-beam effect will Landau damp all the coupled bunch instabilities. To guarantee the stability of the beams from injection to top energy, the Landau damping octupoles will be used to provide some tune spread [18,19] and help to stabilise the beam, possibly in conjunction with the feedback system.
<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Rs (MΩ/m)</th>
<th>Q</th>
<th>Frequency (MHz)</th>
<th>Rs (MΩ/m)</th>
<th>Q</th>
</tr>
</thead>
<tbody>
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<td>3.43</td>
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</tr>
<tr>
<td>796.2</td>
<td>1.15</td>
<td>44824</td>
<td>801.2</td>
<td>3.40</td>
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</tr>
<tr>
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</table>

Table 10: Undamped dipole modes of the 200 MHz cavity [17]

<table>
<thead>
<tr>
<th>Growth Times</th>
<th>Parabolic Bunch</th>
<th>Gaussian Bunch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bunch mode</td>
<td>Injection</td>
<td>Top</td>
</tr>
<tr>
<td>a=0</td>
<td>121 msec</td>
<td>522 msec</td>
</tr>
<tr>
<td>a=1</td>
<td>97 msec</td>
<td>Damped</td>
</tr>
</tbody>
</table>

Table 11: Effect of undamped dipole modes of the 200 MHz cavities

### 3.2 Transverse CBI estimates for the HOMs of the 400 MHz cavities

The undamped and damped dipole modes for 400 MHz superconducting cavities [14] are given in Table 12. In order to convert the $R/Q$ value from the HOM spectrum to the transverse impedance, a tube radius of 15 cm was considered [17]. The estimated growth times in the presence of the undamped and damped HOM of the 400 MHz cavities are shown in Table 13. The CBI growth times are not very fast (a=0 can be handled by the transverse feedback and a=1 is Landau damped) even in the case of the undamped HOMs. The situation is a fortiori even safer with the damped HOMs, since the a=0 mode has very slow growth times and the a=1 mode is Landau damped.
Figure 4: Undamped dipole modes of the 200 MHz cavity [17]

<table>
<thead>
<tr>
<th>Frequency (MHz)</th>
<th>Undamped HOM Rsh (MΩ)</th>
<th>Q (KΩ)</th>
<th>Damped HOM Rsh (MΩ)</th>
<th>Q (KΩ)</th>
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</thead>
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<td>534.0</td>
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<td>50000</td>
<td>14.0</td>
<td>93</td>
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</tbody>
</table>

Table 12: Undamped and damped dipoles for the 400 MHz superconducting cavity [14]
### Table 13: Effect of undamped and damped dipoles of the 400 MHz cavities

<table>
<thead>
<tr>
<th>Bunch mode</th>
<th>Parabolic Bunch</th>
<th>Gaussian Bunch</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Injection</td>
<td>Top</td>
</tr>
<tr>
<td>With Undamped HOMs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a=0</td>
<td>0.24 sec</td>
<td>2.38 sec</td>
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<tr>
<td></td>
<td>(0.65 sec)</td>
<td>(4.72 sec)</td>
</tr>
<tr>
<td>a=1</td>
<td>Damped</td>
<td>Damped</td>
</tr>
<tr>
<td></td>
<td>(Damped)</td>
<td>(Damped)</td>
</tr>
<tr>
<td>With Damped HOMs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a=0</td>
<td>101 sec</td>
<td>1070 sec</td>
</tr>
<tr>
<td></td>
<td>(267 sec)</td>
<td>(1945 sec)</td>
</tr>
<tr>
<td>a=1</td>
<td>Damped</td>
<td>Damped</td>
</tr>
<tr>
<td></td>
<td>(Damped)</td>
<td>(Damped)</td>
</tr>
</tbody>
</table>

3.3 Transverse Resistive Wall Instabilities

The resistive wall contribution in the LHC is mainly related to the beam screen (the beam screen and the coated copper chambers in the BPMs occupy about 90% of the machine’s circumference). The cryogenic part of the beam screen is made of copper cladded stainless steel to keep the resistance as low as possible both for instability and ohmic heating considerations. The resistivity of cold copper is a function of the residual resistance ratio (RRR) and of the magnetic field $B$. The magnetic field increases the path length of the conduction electrons which leads to a substantial resistance increase at cryogenic temperatures. As for the frequencies around 10 kHz, only 0.3% of the beam image current flows through the stainless steel outside the copper coating, it follows that the surface resistance is almost entirely defined by the thin copper layer. Thus, instead of considering a double layered wall formulation, the impedance of a thin wall is considered [20].

Previous experience with co-laminating stainless steel with copper (thickness 50 micron, with RRR=100) showed that the copper close to the steel gets contaminated (much lower local RRR) during the fabrication process such that the surface impedance is increased. To counteract this effect, it has been decided to increase the thickness of the copper layer from 50 to 75 micron. At injection energy, this design is equivalent to a thickness of 50 microns with an RRR of 100, corresponding to the model used in the present evaluation. However, at top energy, due to the effect of magneto-resistance, the RRR reduces to 30. The transverse resistive wall impedance is calculated as:

$$R(\omega) = \frac{cpl}{\pi tl^2b^3}$$

where, $t=$ thickness of the copper layer, $\rho =$ resistivity of copper at room temperature/RRR, $l=$ total length of the copper coated chamber and $b=$ beam pipe radius.

The relevant parameters used for the calculations of the resistive wall impedances and the growth times are listed in Table 14. The frequency of the lowest dangerous mode is 8 KHz (fractional part of the tune = 0.3). The beta value at the beam screen locations is larger by a factor of 1.3 than the ring average beta values and thus the impedances are
scaled up by this factor. The impedance of the warm part of the machine is taken as 20 MΩ/m [21]. Furthermore, the horizontal impedance is taken as 1.4 times smaller than the vertical impedance as dictated by the beam screen geometry [22]. The ring average beta values are taken as the ratio of ring radius to the tune value (=67m). For these estimates, the total intensity is that of 2808 bunches (unlike the number of 3564 symmetric bunches used in the previous sections). As shown in Table 14, the growth time in the vertical plane

| Length of the beam screen [Km] including Cu-plated chambers | 23.9 |
| Cu thickness [µm] | 75 |
| Effective Cu thickness [µm] | 50 |
| Vertical radius [m] | 0.018 |
| Frequency of the lowest mode [kHz] | 8 |
| \(\beta_{\text{beamscreen}}/\beta_{\text{ringaverage}}\) | 1.3 |

| Vertical/Horizontal Impedance | 1.4 |
| RRR | Injection | Collision |
| 100 | 30 |
| Vertical Impedance due to screen [MΩ/m] | 36 | 120 |
| Horizontal Impedance due to screen [MΩ/m] | 26 | 86 |
| Total vertical Impedance [MΩ/m] | 56 | 140 |
| Total Horizontal Impedance [MΩ/m] | 46 | 106 |

| Vertical Growth time [msec] | Nominal Intensity [0.56 A] | 38.6 | 238.6 |
| Ultimate Intensity [0.86 A] | 24.8 | 154.4 |

| Horizontal Growth time [msec] | Nominal Intensity [0.56 A] | 47.0 | 316 |
| Ultimate Intensity [0.86 A] | 30.4 | 205 |

Table 14: Effect of Transverse Resistive Wall

at injection energy with the present resistive wall impedance is ~25 msec for the ultimate intensity. This is within the acceptable limits of the transverse feedback (specified to cope up with a growth time of 14 msec [1] for the resistive wall instability). The corresponding growth time at top energy is ~155 msec. Thus, in the absence of beam-beam tune spread, the transverse oscillations will grow. If the transverse feedback system will be kept on since injection until beams are put into collision, the beam stability will be ensured.

3.4 Summary for the transverse planes

The CBI growth times in the presence of undamped dipole modes of the 200 MHz and damped dipole modes of the 400 MHz cavities can be handled by the transverse dampers. The transverse resistive wall instability growth time at injection energy for the ultimate intensity is within the specifications of the transverse feedback system. However, to ensure the stability of the beam from injection to top energy, the transverse feedback should be kept on until collisions take place.
4 Conclusions

In the presence of the undamped higher order modes of the normal conducting capture cavities (200 MHz) and the superconducting cavities (400 MHz), coupled bunch instabilities are excited in the LHC and the growth times are fast enough to blow up the bunch dimensions and/or cause loss of particles, therefore this would not be acceptable for LHC. Once these HOMs are damped by means of dedicated couplers, the instabilities are Landau damped in the longitudinal plane and the growth times are long enough in the transverse plane so that the excitations can be suppressed by the transverse feedback system. All the higher order coupled bunch modes other than the dipole coupled bunch mode will be Landau damped at injection as well as at top energy. In addition to this, even in the case where the trapped modes of the CMS chamber and the damped modes of the transverse dampers are included, the instabilities thresholds are not exceeded. The coupled bunch instabilities growth times for the initial operation of the LHC with 400 MHz cavities alone (no 200 MHz cavities) also show that the instabilities would be within control.

With the present knowledge of the machine impedance, the resistive wall effect is within control. In the vertical plane, the resistive wall instability growth time at injection is ~25 msec for the ultimate intensity. Nevertheless, it remains true that the impedance of any new component to be installed in the machine has still to be carefully optimized such that the total transverse impedance of the machine remains below 100 MΩ/m at injection. Indeed, this value would correspond to an instability growth time of ~14 msec which happens to be at the limit of what the transverse feedback system could compensate. With the present resistive wall budget, the instability growth time at top energy is ~155 msec. Consequently, in the absence of beam-beam tune spread, the transverse oscillations will grow. It is therefore recommended to keep the transverse feedback system on from injection until beams are put into collision.

5 Acknowledgements

I thank D. Brandt and L. Vos for many useful discussions, for providing all the necessary input for the estimates and critically going through this manuscript to give their valuable comments. I am thankful to F. Ruggiero for his useful suggestions on this report. This work would not have been possible without the help from SL/RF group, special thanks to J. Tuckmantel for providing all the necessary data.

References