A Nonlinear Collimation System For CLIC

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A Nonlinear Collimation System for CLIC

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Abstract
We describe a nonlinear collimation system for CLIC at 3 TeV centre-of-mass energy [9]. This nonlinear system employs 3 skew sextupoles, two of which are at dispersive locations. The skew sextupoles guarantee the collimator survival in case of a full beam impact. We discuss the optics of this system, and outline the plan for further work.

1 INTRODUCTION

A collimation system for a future linear collider must fulfill three functions, namely it should (1) reduce the background in the particle-physics detector by removing particles at large betatron amplitudes or energy offsets, which otherwise would be lost downstream generating muons near the collision point, or emit synchrotron radiation photons in the final quadrupoles that could strike sensitive detector elements; (2) withstand the impact of a full bunch train in case of a machine failure; and (3) not produce intolerable wake fields that might degrade the orbit stability or dilute the beam emittance.

Various types of linear or nonlinear collimation systems have been described in the past [2, 3, 4, 5, 6]. Some of these have used pairs of skew quadrupoles and octupoles either in nondispersive or dispersive regions in order to increase the spot size at the energy collimation [3, 5]. A characteristic feature of all these systems is that they are separated between energy and betatron collimation, and typically employed the skew sextupoles only in one or the other half. At beam energies of 250–500 GeV, these systems were not significantly shorter than a conventional linear system.

The system for 1.5–TeV beam energy presented here uses a skew sextupole at a dispersive location, whose purpose is to increase the vertical beam size at the spoiler. A single vertical spoiler is placed about 90° in betatron phase advance behind the sextupole and collimates in all three degrees of freedom simultaneously. This reduces the total length of the system, and minimizes the wake-field effects. The skew sextupole also serves to amplify the beam centroid amplitude in case of a momentum error or an incoming horizontal betatron oscillation, thus allowing the positioning of the spoiler further away from the center of the beam-pipe. A second skew sextupole downstream of the spoiler, and 180° from the first sextupole, cancels all aberrations induced by the former.

The collimation for the orthogonal betatron phase, for which we assume much looser requirements, i.e., larger collimation amplitudes, is accomplished by placing a third much weaker skew sextupole π/2 upstream of the first, in a region without dispersion.

2 SCHEME

The Hamiltonian of a skew sextupole at a location with horizontal dispersion \( D \) is

\[
H_s = \frac{1}{6} K_s (y^3 - 3(x + D\delta)^2 y),
\]

where \( x \) and \( y \) are the transverse betatron amplitudes at the sextupole, and \( \delta \) the relative momentum offset. The integrated sextupole strength \( K_s \) can be expressed in terms of the sextupole length \( l_s \), the pole-tip field \( B_T \), the magnetic rigidity \( (B\rho) \), and sextupole aperture \( a_s \) as

\[
K_s = \frac{2B_T l_s}{(B\rho)a_s^2}.
\]

At the skew sextupole a particle suffers deflections \( \Delta x' = -\partial H_s/\partial x \), \( \Delta y' = -\partial H_s/\partial y \) or

\[
\Delta x' = K_s(D_{\text{sext}} + x)y \quad \text{(3)}
\]

\[
\Delta y' = -\frac{1}{2}K_s(y^2 - x^2 - D_{\text{sext}}^2\delta^2 - 2D_{\text{sext}}\delta x). \quad \text{(4)}
\]

The position at a downstream spoiler is obtained from

\[
x_{\text{sp}} = x_{0,\text{sp}} + R_{12}\Delta x',
\]

\[
y_{\text{sp}} = y_{0,\text{sp}} + R_{34}\Delta y',
\]

where the subindex 0 indicates the position in the absence of the skew sextupole and \( R_{12}, R_{34} \) are the optical transport matrix elements between the sextupole and the spoiler.

The rms beam size at the spoiler is computed by squaring the expressions for \( x_{\text{sp}} \) and \( y_{\text{sp}} \), and averaging over the transverse and energy distribution. We assume that, in the above expressions, the vertical beam size and emittance are negligibly small compared with the horizontal beam size and, especially, with the product of energy spread and dispersion.

Retaining the dominant terms only, the vertical beam size is determined by the part of the skew-sextupole deflection which is quadratic in energy, or

\[
\sigma_y \approx C_\delta \frac{1}{2} |R_{34}| K_s |D_{\text{sext}}^2\delta_{\text{rms}}^2.
\]

The factor \( C_\delta \) is obtained from averaging the term \( \delta^4 \) over the energy distribution and dividing the result by the square of the rms energy spread. In case of a Gaussian distribution one finds \( C_\delta = \sqrt{\pi} \), and in case of a flat distribution with sharp cutoff, which is more representative for a linear collider like CLIC, \( C_\delta = \sqrt{9/\pi} \).

Although we do not constrain it, the linear dispersion tends to be small at the spoiler, \( D_{\text{sp}} \approx 0 \), which avoids the
coupling of energy errors into betatron motion via wake fields. (However, since in the proposed scheme only a vertical aperture is required, this condition is not really necessary; indeed, a nonzero dispersion would enlarge the beam size at the spoiler which is calculated below.) The horizontal beam size at the spoiler is determined from

$$\sigma_x \approx \left( R_{12} K^2_{s} D_{\text{sext}}^2 \sigma_{\text{rms}} \beta_{y, \text{sext}} \epsilon_y + \beta_{x, \text{sp}} \epsilon_x \right)^{1/2}. $$

(8)

We will assume that the second term under the square root is dominant. For spoiler survival, a minimum beam size $$\sigma_{x, \text{min}}$$ required so that $$\sigma_y \sigma_x \geq \sigma_{x, \text{min}}^2$$, which we can rewrite as

$$C_\delta \frac{1}{2} |R_{34} K_x| D_{\text{sext}}^2 \sigma_{\text{rms}} \sqrt{\beta_{x, \text{sp}} \epsilon_x} \geq \sigma_{x, \text{min}}^2. $$

(9)

At a given value of $$D_{\text{sext}}$$, Eq. (9), determines the minimum value of the product $$|K_x R_{34}|$$ required at the sextupole.

We denote the collimation amplitude for the horizontal and vertical betatron motion as $$\pm n_x \sigma_x$$ and $$\pm n_y \sigma_y$$, respectively, and the energy collimation depth in units of $$\delta$$ by $$\pm \Delta$$. A single vertical spoiler is employed to collimate in all three degrees of freedom. It is natural to produce a large horizontal beta function at the sextupole, since here the dispersion is large as well, and a large vertical beta function at the spoiler. The collimation of the horizontal motion and in energy then occurs via the nonlinear vertical deflection, Eq. (4), received at the skew sextupole. The vertical collimation is obtained from the linear optics in the usual way.

The beta functions follow from the required collimation amplitudes as

$$\beta_{x, \text{sext}} = \frac{D_{\text{sext}}^2 \Delta^2}{\epsilon_x n_x^2}, $$

(10)

$$\beta_{y, \text{spoiler}} = \frac{K^2_{s} R_{34}^4 D_{\text{sext}}^4 \Delta^4}{4 \epsilon_y n_y^2}. $$

(11)

The three equations (9), (10), and (11) contain the product $$|K_x R_{34}|$$. Choosing $$K_x$$ as large as possible and maintaining a reasonable pole-tip radius and $$B_T \leq 1.4$$ T, the minimum value of $$R_{34}$$ is determined from $$D_{\text{sext}}$$. The achievable value of the dispersion $$D_{\text{sext}}$$ is limited by the emittance growth $$\Delta(\epsilon_x)$$ due to synchrotron radiation in the dipole magnets. The latter restricts the value

$$\Delta(\epsilon_x) \approx (4 \times 10^{-8} \text{ m}^2 \text{ GeV}^{-1}) E^5 B_5 < f \epsilon_x $$

(12)

to a fraction $$f$$ of the initial emittance. Here $$B_5$$ is the radiation integral [7], $$I_5 = \sum_i L_i < \mathcal{H} > / \rho_i$$, the sum runs over all bending magnets, with bending radius $$\rho_i$$, length $$L_i$$, and ‘curly $$\mathcal{H}$$’ function defined by Sands [8].

A solution to Eqs. (9), (10), (11), and (12) can be found, e.g., by adjusting the length of the collimation system and the locations of sextupoles and spoiler.

Absorbers must intercept the particles that are scattered by the spoiler. One absorber can be located half a FODO cell behind the second skew sextupole, following the spoiler. This arrangement has the advantage that the scattered particles are further deflected by the strong skew sextupole before they impinge on the absorber. Although the location of the absorber is then more than 90° behind the spoiler, the $$R_{12}$$ and $$R_{34}$$ matrix elements between spoiler and absorber are still significant. A second absorber is placed 90° after the first one — which coincides with locations upstream and downstream of the bending magnets — such that both inwards and outwards scattered particles can be caught.

The collimation as described so far acts only on one phase of the betatron motion. The phase advance to the collision point should be adjusted such that this phase is the phase of the final-doublet (FD) quadrupoles, 90° away from the phase at the collision point. Betatron motion in this phase is most critical, since the corresponding particles traverse the final quadrupoles at large amplitudes.

Nevertheless some collimation and machine protection are required also in the orthogonal phase. To provide this, we place a further nonlinear element, e.g., another, weaker, skew sextupole upstream of the first strong skew sextupole and of the bending section. This magnet should not be a multiple of $$\pi$$ apart from the main skew sextupoles, so that it affects the orthogonal component of betatron motion.

Particles arriving with large amplitudes at this additional skew sextupole get nonlinearly deflected so that they acquire a substantial offset at the first main skew sextupole, which further amplifies their deflection. Therefore, they hit the same spoiler as the particles which are collimated in the other betatron phase.

Denoting the $$(3, 4)$$ transport matrix elements between the additional (pre-) skew sextupole and the first main skew sextupole by $$R_{34}^{\text{pre}}$$, the collimation depths in units of $$\sigma$$ for the IP betatron phase by $$n_{y}^{IP}$$ and $$n_{x}^{IP}$$, and the spoiler half gap by $$a_{y, \text{sp}}$$, the minimum integrated strength of the additional skew sextupole is

$$K_{x}^{\text{pre}} = \frac{2}{n_{y}^{IP} \beta_{y}^{\text{pre}}} \left( \frac{2 a_{y, \text{sp}}}{K_x R_{34}} \right)^{1/2} \left( \frac{\epsilon_y}{\epsilon_{\text{rms}}} \right)^{1/2}, $$

(13)

which ensures that particles vertically offset by more than $$n_{y}^{IP} \text{rms}$$ beam sizes will hit the spoiler. The horizontal collimation depth in the IP phase, $$a_{x}^{IP}$$ is then fixed by an equation analogous to (13). We assume that $$n_{x}^{IP}$$ is so large and, hence, $$K_{x}^{\text{pre}}$$ sufficiently small, that the geometric aberrations induced by this first skew sextupole need not be corrected. The residual relative blow up can be estimated as

$$\Delta \sigma_{y}/\sigma_{y} \approx \sqrt{3 K_{x}^{\text{pre}} \beta_{y}^{\text{pre}}} \beta_{y} \epsilon_{x}/\sqrt{\epsilon_{y}}, $$

to be added in quadrature, where the beta functions are those at the pre-s Sextupole.

The advantage of the described arrangement for orthogonal collimation is that it makes maximum use of the already existing strong skew sextupoles, and no additional spoilers are necessary.

### 3 Optics

Table 1 lists some beam parameters of CLIC at 3 TeV [9], and the collimation amplitudes required [10]. Figure 1
Table 1: Beam and collimation parameters.

<table>
<thead>
<tr>
<th>variable</th>
<th>symbol</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>beam energy</td>
<td>$E$</td>
<td>1.5 TeV</td>
</tr>
<tr>
<td>rms momentum spread</td>
<td>$\delta_{\text{rms}}$</td>
<td>2.8 $\times$ 10^{-3}</td>
</tr>
<tr>
<td>hor. geom. emittance</td>
<td>$\epsilon_x$</td>
<td>0.23 pm</td>
</tr>
<tr>
<td>vert. geom. emittance</td>
<td>$\epsilon_y$</td>
<td>6.8 fm</td>
</tr>
<tr>
<td>hor. betatron coll. depth</td>
<td>$n_x$</td>
<td>10</td>
</tr>
<tr>
<td>vert. betatron coll. depth</td>
<td>$n_y$</td>
<td>80</td>
</tr>
<tr>
<td>energy collimation</td>
<td>$\Delta$</td>
<td>0.013</td>
</tr>
<tr>
<td>hor. IP betatron coll. depth</td>
<td>$n_{x,\text{IP}}$</td>
<td>460</td>
</tr>
<tr>
<td>vert. IP betatron coll. depth</td>
<td>$n_{y,\text{IP}}$</td>
<td>2000</td>
</tr>
</tbody>
</table>

Table 2: Optics parameters.

<table>
<thead>
<tr>
<th>variable</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>2.07 km</td>
</tr>
<tr>
<td>beta functions ($x$, $y$) at skew sext.</td>
<td>175, 82 km</td>
</tr>
<tr>
<td>dispersion at skew sext.</td>
<td>61 mm</td>
</tr>
<tr>
<td>skew sextupole pole tip field</td>
<td>1.4 T</td>
</tr>
<tr>
<td>skew sextupole pole tip radius</td>
<td>4 mm</td>
</tr>
<tr>
<td>skew sextupole length</td>
<td>3 m</td>
</tr>
<tr>
<td>skew sextupole strength $K_s$</td>
<td>104 m^{-2}</td>
</tr>
<tr>
<td>$R_{12}$, $R_{34}$ from sext. to spoiler</td>
<td>110, 307 m</td>
</tr>
<tr>
<td>beta functions ($x$, $y$) at spoiler</td>
<td>20.5, 586 km</td>
</tr>
<tr>
<td>dispersion at spoiler</td>
<td>$\sim$ 0 m</td>
</tr>
<tr>
<td>rms spot size ($x$, $y$) at spoiler</td>
<td>69, 209 $\mu$m</td>
</tr>
<tr>
<td>vertical spoiler half gap</td>
<td>$a_{y,\text{sp}}$</td>
</tr>
<tr>
<td>hor. beta function at pre skew sext.</td>
<td>5.4 km</td>
</tr>
<tr>
<td>vert. beta function at pre skew sext.</td>
<td>19.5 km</td>
</tr>
<tr>
<td>dispersion at pre skew sextupole</td>
<td>0 mm</td>
</tr>
<tr>
<td>pre-skew sextupole pole tip field</td>
<td>23 mT</td>
</tr>
<tr>
<td>pre-skew sextupole pole tip radius</td>
<td>20 mm</td>
</tr>
<tr>
<td>pre-skew sextupole length</td>
<td>3 m</td>
</tr>
<tr>
<td>pre-skew sextupole strength $K_{s,\text{pre}}$</td>
<td>0.068 m^{-2}</td>
</tr>
<tr>
<td>$R_{12}$ from pre-sext. to sext.</td>
<td>290 m</td>
</tr>
<tr>
<td>$R_{34}$ from pre-sext. to sext.</td>
<td>113 m</td>
</tr>
</tbody>
</table>

We plan to compare the performance and collimation efficiency of this nonlinear system with those of alternative linear designs [10] using the code BDSIM [12].

5 REFERENCES