Neutrino oscillations and signals in $\beta$ and $0\nu2\beta$ experiments

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Abstract

Assuming Majorana neutrinos, we infer from oscillation data the expected values of the parameters $m_{\nu_e}$ and $m_{ee}$ probed by $\beta$ and $0\nu2\beta$-decay experiments. If neutrinos have a ‘normal hierarchy’ we get the 90% CL ranges $|m_{ee}| = (0.5 \div 5)$ meV, and discuss in which cases future experiments can test this possibility. For ‘inverse hierarchy’, we get $|m_{ee}| = (10 \div 57)$ meV and $m_{\nu_e} = (40 \div 57)$ meV. The $0\nu2\beta$ data imply that almost degenerate neutrinos are lighter than 0.95 $\text{heV}$ at 90% CL, competitive with the $\beta$-decay bound. We critically reanalyse the data that were recently used to claim an evidence for $0\nu2\beta$, and discuss their implications. Finally, we review the predictions of flavour models for $m_{ee}$ and $\theta_{13}$.

The most successful extensions of the SM to date suggest that neutrinos are Majorana particles, with masses around $v^2/M_{\text{GUT}} \sim$ meV, where $v = 174$ GeV (the electroweak scale) and $M_{\text{GUT}} \sim 2 \cdot 10^{16}$ GeV (the grand unification scale). If this indication is correct, the goal of future neutrino experiments will be the reconstruction of the neutrino Majorana mass matrix, namely the measurement of 3 masses, 3 mixing angles and 3 CP-violating phases. Within this framework, data on solar and atmospheric neutrinos [1, 2, 3] are interpreted as pieces of evidence for effects due to 2 mixing angles and 2 squared mass differences, and the LSND anomaly [4] cannot be explained. We are far from knowing all 9 neutrino parameters. Furthermore, only 6 of them might be measured by oscillations; thus, we will eventually need “non-oscillation” experiments to access the remaining ones. In this work, we discuss how future study of neutrinoless double beta decay ($0\nu2\beta$) and tritium end-point spectrum ($\beta$) could contribute to this goal. In particular, we carefully study the expected signals for these experiments on the basis of our present knowledge. We follow two different strategies of investigation: we first pursue a purely phenomenological approach, and next we add speculative ingredients from models of quark and lepton masses. In our view, neither of these approaches is entirely satisfactory: the first one allows us to derive safe but not extremely strong restrictions; the second one can give stronger but unsafe restrictions. Therefore these two approaches are “complementary”.

In section 1 we obtain the ranges of parameters measured by oscillation experiments, and summarize how future experiments are expected to improve on these quantities. In section 2 we work out the connections between oscillation and non-oscillation ($\beta$ and $0\nu2\beta$-decay) experiments. We improve on

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earlier similar works, by combining these considerations with the existing oscillation data by means of appropriate statistical techniques. In this way, we determine the precise ranges of \( \beta \) and \( 0\nu2\beta \)-decay signals expected on the basis of present oscillation data, for allowed neutrino mass spectra. In the case of almost degenerate neutrinos, we also compute the upper bound on the common Majorana neutrino mass implied by present \( 0\nu2\beta \) experiments. An evidence for \( 0\nu2\beta \) has been claimed in a recent paper [5]. In the appendix we critically reanalyse the data that should contain such evidence and discuss their implications. In section 3 we review the expected values of \( m_{\nu e} \) and \( \theta_{13} \) (the two still unknown parameters with brighter future experimental prospects) in large families of flavour models discussed in the literature. Our results are summarized in section 4.

1 Oscillation experiments

The three-flavour Majorana neutrino mass matrix \( m_{\ell\ell'} \) (where \( \ell, \ell' = \{e, \mu, \tau\} \)) is described by 9 real parameters. It is convenient to choose them to be the 3 real positive masses \( m_i \) and parameterize the neutrino mixing matrix \( V \) (that relates the fields with given flavour to those with given mass, \( \nu = V_{\ell}\nu_i \)) as

\[
V = R_{23}(\theta_{23}) \cdot \text{diag}(1, 1, e^{i\phi}) \cdot R_{13}(\theta_{13}) \cdot \text{diag}(1, 1, e^{-i\phi}) \cdot R_{12}(\theta_{12}) \cdot \text{diag}(1, e^{i\alpha}, e^{i\beta})
\]

where \( R_{ij}(\theta_{ij}) \) represents a rotation by \( \theta_{ij} \) in the \( ij \) plane and \( i, j = \{1, 2, 3\} \). We order the neutrino masses \( m_i \) such that \( m_3 \) is the most splitted state and \( m_2 > m_1 \), and define \( \Delta m_{ij}^2 = m_j^2 - m_i^2 \). With this choice, \( \Delta m_{23}^2 \) and \( \theta_{23} \) are the ‘atmospheric parameters’ and \( \Delta m_{12}^2 > 0 \) and \( \theta_{12} \) are the ‘solar parameters’, whatever the spectrum of neutrinos (‘normal hierarchy’ i.e. \( m_1 \ll m_2 \ll m_3 \) so that \( \Delta m_{23}^2 > 0 \); ‘inverted hierarchy’ i.e. \( m_3 \ll m_1 \ll m_2 \), so that \( \Delta m_{23}^2 < 0 \) or ‘almost degenerate’).

1.1 Present situation

Almost all present experimental information on the 9 neutrino parameters comes from oscillation experiments [1, 2, 3], and can be summarized as

\[
|\Delta m_{23}^2|^{1/2} \sim 50 \text{ meV}, \quad (\Delta m_{21}^2)^{1/2} \sim (0.03 \div 25) \text{ meV}, \quad \theta_{12} \sim \theta_{23} \sim 1, \quad \theta_{13} \lesssim 1/4, \quad \phi \sim (0 \div 2\pi)
\]

and more precisely in fig. 1, obtained from our up-to-date fits of solar, atmospheric and reactor data. Details on how our fits have been performed can be found in [6, 7]. The atmospheric mixing angle \( \theta_{23} \) and the solar mixing angle \( \theta_{12} \) have to be large (indeed, it looks as if the first is almost maximal, while the second may be just ‘large’); the CHOOZ experiment implies that the third mixing \( \theta_{13} \) cannot be that large. The solar mass splitting, \( \Delta m_{12}^2 \), could be slightly or much smaller than the atmospheric one, \( |\Delta m_{23}^2| \). Nothing is known on \( \phi \).

A brief comment about statistics could help in better interpreting our results. In fig. 1 and in the rest of the paper we plot \( \Delta \chi^2(p) \equiv \chi^2(p) - \chi^2_{\text{best fit}} \), where \( p \) is the quantity in which we are interested; \( \chi^2(p) \) is usually obtained from a multi-parameter fit, as \( \chi^2(p) = \min_{p'} \chi^2(p, p') \), where \( p' \) are other parameters in which we are not interested. For example, a fit of atmospheric data gives \( \chi^2(\Delta m_{23}^2, \theta_{23}, \theta_{13}) \), from which we obtain the functions \( \Delta \chi^2(\Delta m_{23}^2) \) and \( \Delta \chi^2(\theta_{23}) \), plotted in figs. 1a and 1b respectively. It is important to appreciate that, when a quantity is extracted in this way from a multi-parameter fit, \( \text{there is no loss of statistical power due to the presence of other parameters} \). In fact, with some technical assumptions (e.g. the Gaussian approximation\(^1\)) one can derive the following two equivalent results:

\(^1\)When extracting the oscillation parameters from present data, only in one case this assumption is not sufficiently well realized in practice. For simple physical reasons the Beryllium contribution to solar neutrino rates depends in very strong oscillatory way on \( \Delta m_{12}^2 \), for \( \Delta m_{12}^2 \sim 10^{-9} - 10^{-10} \text{ eV}^2 \) (see fig. 1a). Therefore the \( \Delta \chi^2(\theta_{12}) \) shown in fig. 1b has been obtained from a full Bayesian analysis, performed assuming a flat prior in \( \ln \Delta m_{12}^2 \) along the lines of [6, 8]. Only in this case, and only around \( \theta_{12} \sim 1 \), there is some difference between a Bayesian result and the Gaussian approximation, and we use the Bayesian result.
In the frequentistic framework $\Delta \chi^2(p)$ is distributed as a $\chi^2$ with one degree of freedom. (2) In the Bayesian framework $\exp[-\Delta \chi^2(p)/2]$ is the probability of different $p$ values, up to a normalization factor. Our inferences on $m_{ee}$ also depend on unknown parameters ($\theta_{13}$ and the CP-violating phases): using the Gaussian approximation we obtain more simple and conservative results, as explained in section 2.2.

In fig. 1 and in the rest of the paper we do not include the significant but controversial information from SN1987A, that would disfavour $\theta_{13} \gtrsim 1^\circ$ (if $\Delta m^2_{23} < 0$) and solar solutions with large mixing angle [9, 10]. However, we recall here the origin of these bounds. The average $\bar{\nu}_e$ energy deduced from Kamiokande II and IMB data is $E_{\bar{\nu}_e} \sim 11$ MeV, assuming the overall flux suggested by supernova simulations (experimental data alone do not allow to extract both quantities accurately). This is somehow smaller than the value suggested by supernova simulations in absence of oscillations, $E_{\bar{\nu}_e} \sim 15$ MeV. For both figures it is difficult to properly assign errors; but oscillations that convert $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu,\tau$ increase the disagreement, since supernova simulations suggest $E_{\bar{\nu}_\mu,\tau} \sim 25$ MeV. With an inverted hierarchy, $\theta_{13} \gtrsim 1^\circ$ gives rise to adiabatic MSW conversion, swapping $\bar{\nu}_e \leftrightarrow \bar{\nu}_\mu,\tau$ completely. This is why this case is ‘disfavoured’ if the predictions of supernova models on neutrino energy and flux are correct. The same argument applies to large solar mixing angles: $\theta_{12} \sim 1$ induces a partial swap of the $\bar{\nu}_e$ into $\bar{\nu}_\mu,\tau$, whatever the mass spectrum of neutrinos. LMA oscillations have a smaller $\Delta m^2_{12}$ than LOW and (Q)VO, and are therefore less ‘disfavoured’. SMA gives almost no $\bar{\nu}_e$ oscillations, but is strongly disfavoured by solar data. For a full analysis, see [10].

1.2 Perspectives of improvement

Future oscillation experiments can significantly improve the situation. Concerning the ‘solar’ parameters, SNO, KamLAND and Borexino can reduce the error on $\sin^2 2\theta_{12}$ down to around 5%, and measure $\Delta m^2_{12}$ to few per-mille (if it lies in the VO or QVO regions), or few per-cent (in the LMA region), or around 10% (in the LOW region) [11].\textsuperscript{2} Concerning the ‘atmospheric’ parameters, K2K, Minos or CNGS can reduce the error on $|\Delta m^2_{23}|$ and $\theta_{23}$ down to about 10% and discover $\theta_{13}$ if larger than few degrees [13, 14] (the precise value depends on $|\Delta m^2_{23}|$). Far future long-baseline experiments can reduce the error on $|\Delta m^2_{23}|$ and $\theta_{23}$ down to few % (with a conventional beam [15]) and maybe 1% (with a neutrino factory beam [16]). These experiments could also discover a $\theta_{13}$ larger than 0.5$^\circ$ and tell something about $\phi$, if LMA is the true solution of the solar neutrino problem. Future reactor experiments [17] can be sensitive to a $\theta_{13} \gtrsim 3^\circ$.

\textsuperscript{2}If $\Delta m^2_{12} \lesssim 2 \times 10^{-3}$ a new reactor experiment with a shorter baseline than KamLAND would be necessary [11, 12]. If $\Delta m^2_{12} \approx 10^{-6}$ eV$^2$ Borexino and KamLAND will not see an unequivocal signal.
If a non zero $\theta_{13}$ will be discovered, earth matter corrections to $\nu_{\mu} \rightarrow \nu_e$ and $\bar{\nu}_{\mu} \rightarrow \bar{\nu}_e$ will tell the sign of $\Delta m_{23}^2$ [18] (i.e. if the atmospheric anomaly is due to the lightest or heaviest neutrinos). The sign of $\theta_{23} - 45^\circ$ (which tells whether the neutrino state with mass $m_3$ contains more $\nu_\tau$ or more $\nu_\mu$) can be measured by comparing

$$P(\nu_e \rightarrow \nu_e) = 1 - \sin^2 2\theta_{13} \sin^2 \frac{\Delta m_{23}^2 L}{4E_\nu} \quad \text{with} \quad P(\nu_\mu \rightarrow \nu_e) = \sin^2 \theta_{23} \cdot [1 - P(\nu_e \rightarrow \nu_e)]$$

(even including matter effects in $P(\nu_\mu \rightarrow \nu_e)$, the issue on $\theta_{23}$ remains). Note that muon disappearance experiments alone cannot distinguish $\theta_{23}$ from $90^\circ - \theta_{23}$, and that the present bound $\sin^2 2\theta_{23} \gtrsim 0.95$ [1] allows the relatively loose range $1/3 \lesssim \sin^2 \theta_{23} \lesssim 2/3$. If $\Delta m_{12}^2$ is in the upper part of the LMA region, so that it affects long-baseline experiments, the sign of $\theta_{23} - 45^\circ$ can be measured even if $\theta_{13} = 0$.

We can summarize the content of this section by saying that we can confidently include among the known parameters $|\Delta m_{23}^2|$, $\theta_{23}$, $\theta_{12}$, and we have reasonably good perspectives for $\Delta m_{12}^2$, $\theta_{13}$, sign($\Delta m_{23}^2$), $\theta_{23} - 45^\circ$, $\phi$ (we express our expectations by the order of quotation; however, effects due to $\theta_{13}$ or $\phi$ could be too small to be ever observed).

### 2 Non-oscillation experiments

Oscillation experiments cannot access to 3 of the 9 parameters of the Majorana mass matrix: the overall neutrino scale, and the two CP-violating “Majorana phases” denoted as $\alpha$ and $\beta$ in eq. (1) [19]. Indeed, the neutrino mass matrix enters as $m^\dagger m$ in the Hamiltonian of propagation of ultrarelativistic neutrinos. Non oscillation experiments are needed to probe the full neutrino mass matrix [20]. We focus the discussion on $\beta$-decay tritium experiments [21] and $0\nu2\beta$-decay germanium experiments [22], that seem to have the best chances of reaching the necessary sensitivity. These techniques are among the oldest and better established. We will not discuss different techniques that at present are not competitive, though they have certain advantages. For instance calorimetric measurements [23] are not limited to the end-point of $\beta$ spectrum. Search of $0\nu2\beta$ with other nuclear species could allow to reduce nuclear theoretical uncertainties [24].

Before continuing, we recall other possibilities to investigate the mass of neutrinos offered by the astrophysics and the cosmology:

- **At the next gravitational collapse of a supernova**, the general strategy [25] will consist in identifying structures in the time and/or energy distributions of neutrinos sensitive to neutrino masses, as the neutronization peak, the rising (or falling) ramp of the cooling phase, a hypothetical sharp cutoff due to black hole formation. The sensitivity of these approaches has been quantified in several works, assuming the capabilities of present detectors (SuperKamiokande, SNO, LVD,. . .). The difference in time of flight between neutrinos and gravitons will only be sensitive to neutrinos heavier than about 1 eV [26], comparable to present $\beta$-decay bounds. The difference in time of flight between different neutrino species will only be sensitive to neutrino mass differences larger than few 10 eV [27]. If neutrino emission were suddenly terminated by black hole formation, a measurement of the difference in time of flight between neutrinos of different energy will only be sensitive to neutrino masses larger than few eV [28].

- **Information on neutrino masses might also be obtained from data on large scale structures in the universe**, together with very accurate measurements of anisotropies in the temperature of the cosmic background radiation. It is thought that it is possible to improve on the present limit $\sum_\nu m_\nu \lesssim (5 \div 10) \text{eV}$ down to about $\sim 0.3 \text{eV}$ [29] (the precise value depends on uncertain cosmological parameters). However, even if an effect due to neutrino masses will be detected, it will be difficult to ascertain that it is really due to neutrino masses, rather than to other mechanisms that could produce similar effects. Cosmology has better sensitivity than other experiments to heavy sterile neutrinos.
Figure 2: We parameterize by \( h \) the theoretical uncertainty on the inferences of \( |m_{ee}| \) from the data due to the disagreement among different calculations of nuclear matrix elements (see eq. (3) and discussion therein). In the horizontal scale, the year of the calculation. Compiled from [32].

- The ‘Z burst’ [30] is a possible source of the ultra-high energy cosmic ray events observed above the Greisen-Zatsepin-Kuzmin (GZK) cutoff [31]. A cosmic ray neutrino that scatters with a nonrelativistic cosmic microwave background neutrino encounters the Z resonance at the energy \( E^\text{res}_\nu = M^2_Z/2m_\nu \), that is slightly above the optimal value even if neutrinos are as heavy as possible: \( m_\nu = 1 \text{ eV} \) gives \( E^\text{res}_\nu = 4 \times 10^{21} \text{ eV} \). It seems difficult to imagine a cosmological source that produces enough energetic neutrinos without producing, at the same time, too many photons [30].

2.1 \( \beta \)-decay and 0\( \nu \)2\( \beta \) experiments

Present \( \beta \)-decay experiments (‘direct mass search’) and neutrinoless double \( \beta \)-decay (0\( \nu \)2\( \beta \)) experiments give bounds on combinations of entries of the neutrino Majorana mass matrix \( m_{\ell\ell}' \). Such bounds are somewhat above the mass scale suggested by oscillations: at 95% CL

\[
 m_{\nu e} \equiv (m^\dagger m)_{ee}^{1/2} < 2.2 \text{ eV} \quad \text{from } \beta\text{-decay [33]} \quad \text{and} \quad |m_{ee}| < 0.38h \text{ eV} \quad \text{from } 0\nu2\beta \text{ [34].} \quad (3)
\]

Here \( h \equiv M_0/M \) is the inverse of the nuclear matrix element \( \mathcal{M} \) for 0\( \nu \)2\( \beta \) of \( ^{76}\text{Ge} \), normalized to a reference matrix element \( \mathcal{M}_0 \), chosen to be the one of ref. [35]. 0\( \nu \)2\( \beta \) experiments try to measure (or bound) \( |m_{ee}| \) by measuring the rate of the transition \( \Gamma(^{76}\text{Ge} \rightarrow ^{76}\text{Se} 2\beta) = \Phi \cdot \mathcal{M}^2 \cdot |m^2_{ee}| \), where \( \Phi \) is a phase space factor. Any uncertainty on the nuclear matrix element \( \mathcal{M} \) reflects directly on the measurement of \( |m_{ee}| \). Indeed, different calculations find rather different nuclear matrix elements, as shown in fig. 2, that suggests the range \( h = 0.6 \div 2.8 \). We will not attempt to assign an error on \( \mathcal{M} \). Rather, we always include an explicit factor \( h \) whenever we quote an experimental result on 0\( \nu \)2\( \beta \), as we do in eq. (3).

The result of the Heidelberg–Moscow experiment at Gran Sasso (HM) quoted above [34] was obtained by selecting a certain search window, and comparing the \( n = 21 \) events occurring there with the \( b = 20.4 \pm 1.6 \) expected background events. Knowing that \( m_{ee} = 350 \text{ meV} \) would yield \( s = 9.3/h^2 \) signal events, from the Poissonian probability of measuring \( s + b \) events one obtains the likelihood for \( m_{ee} \), \( \mathcal{L} \propto e^{-s}(1 + s/b)^n \).

The result of the MAINZ collaboration [33] quoted above is in agreement with TROITSK results [36], though the \( \beta \) spectrum of this last experiment presents some unexpected features. Anyhow, this limit is rather close to the sensitivity of these setups, \( \sim 2 \text{ eV} \), so that new experiments are certainly needed to progress. Indeed, there are plans to extend the sensitivities of \( \beta \) and especially 0\( \nu \)2\( \beta \) experiments to mass scales closer to those suggested by oscillations, shown in eq. (2):

\[
 m_{\nu e} \rightarrow 300 \text{ meV}, \quad |m_{ee}| \rightarrow 10h \text{ meV.} \quad (4)
\]
We refer here to the proposals of the collaborations Katrin [37] for \(0\nu2\beta\)-decay and Genius [38] for \(0\nu2\beta\)-decay, that evolve from the experiments mentioned above. There is also another proposal (named Majorana) [39] to improve on \(0\nu2\beta\) with \(^{76}\text{Ge}\), which evolves from the second most powerful experiment, IGEX [40]. The estimated sensitivity is several times weaker than what the Genius experiment would like to achieve. There is also another interesting proposal named GEM [41].

One should note that in eqs. (3) and (4) we suggest that \(\beta\)-decay experiments will at best extract a single parameter, \(m_{\nu_e}\). This is the case of the three neutrino scenario considered here, for the mass splittings suggested by oscillations are too little to be resolved by any experiment till now proposed.\(^3\) For example the energy resolution of Katrin will be more than one order of magnitude larger than the scales in eq. (2) [44]. Only if neutrinos have an almost degenerate spectrum the \(\beta\)-decay experiments mentioned here could see neutrino masses. We will consider this case closely in section 2.5 (for a discussion of related matter, and more general possibilities, see [42, 43]).

2.2 How to use the inputs from oscillations?

The connection of oscillations with \(\beta\) and \(0\nu2\beta\)-decay has been explored in a number of works [45, 46, 47, 42, 43].\(^4\) The quantities probed by \(\beta\) and \(0\nu2\beta\) experiments can be written in terms of the masses \(m_i\), of the mixing angles \(\theta_{ij}\) and of the CP-violating phases \(\alpha\) and \(\beta\) as

\[
m_{\nu_e} = \left( \sum_i |V_{ei}|^2 m_i^2 \right)^{1/2} = \left( \cos^2 \theta_{13} (m_1^2 \cos^2 \theta_{12} + m_2^2 \sin^2 \theta_{12}) + m_3^2 \sin^2 \theta_{13} \right)^{1/2} \quad (5)
\]

\[
|m_{ee}| = \sum_i |V_{ei}|^2 m_i = |\cos^2 \theta_{13} (m_1 \cos^2 \theta_{12} + m_2 e^{2i\alpha} \sin^2 \theta_{12}) + m_3 e^{2i\beta} \sin^2 \theta_{13}|. \quad (6)
\]

In both formulæ, we can identify the 3 individual contributions associated with the 3 masses \(m_i\). It is useful to make few general remarks:

1. \(m_{ee}\) depends on oscillation parameters and on the overall neutrino mass scale, while \(m_{ee}\) is also sensitive to the Majorana phases.

2. The \(m_2\)-contributions to \(m_{\nu_e}\) and \(m_{ee}\) are guaranteed to be non-zero (for inverted spectrum the same is true for the \(m_1\)-contribution).

3. Since \(|V_{ei}| \leq 1\), the mixing factors suppress more strongly the \(m_i\)-contributions to \(m_{ee}\) than those to \(m_{\nu_e}\). For example, at the best-fit LMA solution \(V_{e2}^2 \approx 1/3\) we have a \(m_2\)-contribution to \(m_{\nu_e}\) 70\% larger than the one to \(m_{ee}\). This is even more evident in the hierarchical case, when the \(m_3\)-contributions are suppressed by the small angle \(\theta_{13}\).

\(^3\)We approximate the exact formula for the \(\beta\)-decay spectrum close to end-point in presence of mixed neutrinos in terms of a single effective neutrino mass parameter [33] \(m_{\nu_e}^2 \equiv |V_{ei}|^2 m_i^2\) as

\[
\frac{dN}{dE_{\nu}} \propto \sum_i |V_{ei}|^2 \Gamma(m_i) \approx \Gamma(m_{\nu_e}) \quad \text{where} \quad \Gamma(m) \equiv E_{\nu} \sqrt{E_{\nu}^2 - m^2}.
\]

(the measured electron energy in the \(^3\text{H} \rightarrow ^3\text{He} \, e^+ \nu_e\), decay is related to \(E_{\nu}\) by kinematics). This approximation is trivially a good one if neutrinos are almost degenerate. However, its usefulness is more general [42], since: (1) the difference between the approximated and exact spectrum, integrated around the end-point, vanishes at order \(O(m_i^2)\) (this is interesting for end-point search of neutrino mass with limited energy resolution); (2) far from the end-point, the difference is zero at order \(O(m_i^2)\) (this is interesting for calorimetric search of neutrino mass). These properties do not hold for other definitions of the ‘effective mass’, say \(m_{\nu_e} = |V_{ei}|^2 \cdot m_i\) [43]. However, if a future \(\beta\) decay experiment will attain a very high sensitivity, and at the same time will be able to resolve the separation between the mass levels, it will be necessary to introduce more parameters to describe the measured \(\beta\) spectrum, and it will be possible to extract more interesting information.

\(^4\)It is commonly assumed that the dominant contribution to \(0\nu2\beta\) comes from massive neutrinos. We remark that, even with this assumption there might be surprises: e.g. if CPT is violated, the rate of double \(\beta\)-transition could be different from that of double \(\beta^+\), or the \(\beta^-\) absorption (EC) followed by \(\beta^+\) emission. However, we will not consider these possibilities.
4. Let us denote as \( m_{\text{min}} \) the lightest neutrino mass: \( m_{\text{min}} = m_1 \) \((m_{\text{min}} = m_3)\) in the case of normal (inverted) hierarchy. Increasing \( m_{\text{min}}^2 \) increases \( m_{\nu_e}^2 \) by the same amount, and \( m_{\nu_e} \) is always larger than \( m_{\text{min}} \). Instead, the behaviour of \( m_{ee} \) as a function of \( m_{\text{min}} \) is less simple, especially when the individual contributions become comparable in size.

5. Most importantly, while the contributions to \( m_{ee} \) are all positive, the individual contributions to \( m_{ee} \) may compensate each other for certain values of the Majorana phases.

Let us elaborate on the last point, enlarging the theoretical perspective. One should realize that we cannot even invoke ‘naturalness’ to argue that \( m_{ee} \) should not be much smaller than its individual \( m_i \)-contributions \([48]\). In fact, from a top/down point of view the \( ee \) element of the neutrino mass matrix is a very simple object: some flavour symmetry could easily force a small \( m_{ee} \), giving rise to apparently unnatural cancellations when \( m_{ee} \) is written in terms of the low energy parameters \( m_i \) and \( \theta_{ij} \), as in eq. (6)\(^5\). A small \( m_{ee} \) is stable under quantum corrections, that renormalize it multiplicatively. For these reasons, bottom/up oscillation analyses that try to determine the range of \( m_{ee} \) consistent with oscillation data must consider seriously the most pessimistic case. As discussed in \([49]\) naturalness considerations would be automatically incorporated in a Bayesian analysis (i.e. no need to invent ‘fine-tuning parameters’) if we assumed any almost flat prior probability distribution for the phenomenological parameters (the neutrino masses, mixing angles, and CP-violating phases). This is why in this work we stick to the Gaussian approximation.

The expression of \(|m_{ee}|\) minimized in the Majorana phases \( \alpha \) and \( \beta \) was given in \([46]\). In this work, we will use it, together with the inputs from oscillations, to infer the signals expected in 0ν2β and \( \beta \)-decay experiments in a number of interesting limiting cases, namely: normal hierarchy, inverted hierarchy, almost degenerate neutrinos. In the first two cases (sections 2.3 and 2.4), we will further show the expression of \(|m_{ee}|\) which results from further minimization in the lightest neutrino mass \( m_{\text{min}} \); this is useful to discuss why (with present data) we cannot exclude the possibility \( m_{ee} = 0 \), even if neutrinos have Majorana masses and oscillate.

### 2.3 Normal hierarchy

If neutrino masses have a partial hierarchy \( m_1 \ll m_2 \approx (\Delta m^2_{12})^{1/2} \ll m_3 \approx (\Delta m^2_{23})^{1/2} \) we could be sure that \( m_{ee} \) is non-zero:

\[
|m_{ee}| > (-\Delta m^2_{12} \cos 2\theta_{12})^{1/2} - \theta_{13}^2 (\Delta m^2_{23})^{1/2}
\]

(7)

(up to higher orders in \( \theta_{13} \) and \( \Delta m^2_{23} / \Delta m^2_{12} \), and barring other possibilities already excluded by oscillation data) only if the following conditions hold

\[
\theta_{12} > \frac{\pi}{4} \quad \text{and} \quad \theta_{13} < \left[ -\frac{\Delta m^2_{12}}{\Delta m^2_{23}} \cos 2\theta_{12} \right]^{1/4}.
\]

(8)

Thus, there is no warranty to have a detectable value of \( m_{ee} \), since solar data allow \( \theta_{12} > \pi/4 \) at a reasonable confidence level only in the LOW and (Q)VO solutions, where \( \Delta m^2_{12} \) is too small to give a detectable contribution to \( m_{ee} \). The minimal guaranteed \( m_{ee} \) is always smaller than the ‘solar’ mass scale \((\Delta m^2_{12})^{1/2}\).

We now specialize our discussion to the case of ‘normal hierarchy’ \( m_1 \ll m_2 \ll m_3 \). As discussed in section 3.1 this case naturally arises in see-saw models, even in presence of large mixing angles. A negligible \( m_1 \) is a prediction of those see-saw models where only 2 right-handed neutrinos give a significant contribution to \( m_{\ell\ell'} \).

\(^5\)An analogous remark holds in supersymmetric see-saw models, that generate \( \mu \to e\gamma \) (and other) decay amplitudes approximately proportional to the \( e\mu \) element of the squared neutrino Yukawa matrix. Even in this case a flavour symmetry that demands a small \( e\mu \) element would give rise to apparently unnatural cancellations, when \( \mu \to e\gamma \) is written in terms of low energy parameters.
The crucial question is: can this prediction be practically tested? Non-oscillation $0\nu 2\beta$ experiments can do that, provided that the 'solar' and the 'atmospheric' contributions to $m_{ee}$ are not comparable [46]. In fact, we have

$$|m_{ee}| = |\sqrt{\Delta m^2_{12}} \sin^2 \theta_{12} \cos^2 \theta_{13} e^{2i\alpha} + \sqrt{\Delta m^2_{23}} \sin^2 \theta_{13} e^{2i\beta}| \equiv |m_{ee}^{\text{sun}} + e^{2i(\beta-\alpha)} m_{ee}^{\text{atm}}|$$

CP-violating phases enter through the combination $\beta - \alpha$. Therefore, if oscillation experiments will tell that $m_{ee}^{\text{sun}} \gg m_{ee}^{\text{atm}}$, this sub-class of see-saw models will imply that $|m_{ee}|$ is in the narrow range

$$m_{ee}^{\text{sun}} - m_{ee}^{\text{atm}} \leq |m_{ee}| \leq m_{ee}^{\text{sun}} + m_{ee}^{\text{atm}}$$

An analogous prediction is possible in the opposite eventuality, $m_{ee}^{\text{atm}} \gg m_{ee}^{\text{sun}}$: in this case $|m_{ee}|$ would be approximately equal to $m_{ee}^{\text{atm}}$.

What present oscillation data tell us on these eventualities? The answer is illustrated in fig. 3, where we show the values of these two contributions in the regions of parameters compatible with solar and atmospheric neutrino oscillations. From these results (and applying the statistical procedure described in section 1) we extract the $\chi^2$ distributions of the solar and atmospheric contributions to $m_{ee}$ plotted in fig. 4a. We see that the largest contribution allowed by data is the solar one: if LMA is the solution of the solar anomaly, solar data suggest a $m_{ee}^{\text{sun}}$ in the 99% CL range 1 ÷ 10 meV. The CHOOZ bound on $\theta_{13}$, together with the SuperKamiokande measurement of $\Delta m^2_{23}$, give the 99% CL upper bound $m_{ee}^{\text{atm}} \leq 3$ meV. Therefore the atmospheric contribution alone is too small for giving a signal in planned experiments. The dominant upper bound on both parameters $m_{ee}^{\text{atm}}$ and $m_{ee}^{\text{sun}}$ (rightmost lines in fig. 4a) is provided by CHOOZ, that excludes large $\theta_{13}$ and large $\Delta m^2_{12}$.

In fig. 4b we show the probability distribution of $|m_{ee}|$ (thick line), taking into account our ignorance on the CP-violating phase $\alpha - \beta$. The dashed lines also show the probability distribution of $|m_{ee}|$ in the special cases of maximal ($\alpha - \beta = 0$) and minimal ($\alpha - \beta = \pi/2$) interference between the atmospheric and solar contributions.

Assuming Majorana neutrinos with normal hierarchy, $|m_{ee}|$ is well predicted and large enough to be measured in forthcoming $0\nu 2\beta$ experiments in two eventualities.
1. $\theta_{13}$ is somewhat below the Chooz bound and $\Delta m_{12}^2$ is in the upper part of the LMA region;

2. $\theta_{13}$ is around the Chooz bound, and $\Delta m_{12}^2$ is not in LMA region.

In the first case it will be possible to test, and eventually exclude, the sub-class of models that predict $m_1 \ll m_2$ (the accuracy of this test will be limited by the theoretical uncertainty on the nuclear matrix element relevant for the $0\nu2\beta$ decay). In the second case the actual value of $m_1$ is irrelevant even if $m_1 \sim m_2 \sim (\Delta m_{12}^2)^{1/2}$; however, the case favored by the data is the first one.

According to present data, ‘solar’ and ‘atmospheric’ contributions to $m_{ee}$ can be comparable and sizable. In this case, what we will learn from a future very sensitive $0\nu2\beta$ experiment?

- A measurement of $|m_{ee}|$ would be equivalent to a measurement of the Majorana CP-violating phase $\alpha - \beta$, if oscillation parameters are known.

- A strong upper bound on $|m_{ee}|$ would suggest $\alpha - \beta \approx \pi/2$ and a correlation between oscillation parameters: $\tan^2 \theta_{13} \approx \sin^2 \theta_{12} \sqrt{\Delta m_{12}^2 / \Delta m_{23}^2}$ (or that both $m_{ee}^{\text{sun}}$ and $m_{ee}^{\text{atm}}$ are negligibly small).

Concerning $\beta$-decay, the range for $m_{\nu_e}$ is $(3 \div 10)$ meV at 90% CL and $(0 \div 16)$ meV at 99% CL (a negligible $m_{\nu_e}$ corresponds to the LOW solution and small $\theta_{13}$). The largest contribution to $m_{\nu_e}$ could be the one associated with atmospheric oscillations without contradicting present data (see remark 3 at page 6), but would remain too small to give any detectable effects in planned $\beta$-decay experiments.

### 2.4 Inverted hierarchy

In our notation ‘inverted hierarchy’ means $m_3 \ll m_1 \approx m_2 \approx (\Delta m_{32}^2)^{1/2}$, with $m_1$ and $m_2$ separated by the ‘solar’ mass splitting. The general expressions (5) and (6) simplify to

$$m_{\nu_e} \approx (\Delta m_{23}^2)^{1/2} \cos \theta_{13}, \quad |m_{ee}| \approx (\Delta m_{23}^2)^{1/2} |\cos^2 \theta_{12} + e^{2i\alpha} \sin^2 \theta_{12} - \cos^2 \theta_{13}|$$

so that $m_{\nu_e}$, and the value of $m_{ee}$ maximized with respect to Majorana phases, is practically equal to the atmospheric mass splitting, $m_{ee}^{\text{max}} = m_{\nu_e} = (\Delta m_{32}^2)^{1/2}$. The case of inverted hierarchy seems more favourable, even from a pessimistic point of view. Indeed, a non zero value

$$|m_{ee}| > [\Delta m_{32}^2 (\cos^2 2\theta_{12} - \theta_{13}^4)]^{1/2} \cos \theta_{13} + \frac{\Delta m_{12}^2}{2(\Delta m_{32}^2)^{1/2}} \sin^2 \theta_{12}$$

(9)
up to higher orders in $\theta_{13}$ and $\Delta m_{21}^2/\Delta m_{32}^2$. If $\theta_{13}$ and $\Delta m_{12}^2$ can be neglected, eq. (9) reduces to the well-known result $|m_{ee}| > \Delta m_{32}^2 \cdot \cos^2 2\theta_{12}$. The condition on $\theta_{12}$ is satisfied by best fit LMA oscillations. Furthermore, as discussed in section 1, SN1987A data could prefer a small $\theta_{13}$ $\lesssim 1^\circ$ for inverted hierarchy.

Assuming a negligible $\theta_{13}$ and $\Delta m_{12}^2 \ll |\Delta m_{23}^2|$, the situation is illustrated in fig. 5, where we plot contour lines of the minimal value of $m_{ee}$ (obtained for $\alpha = \pi/2$) and of the maximal value of $m_{ee}$ (obtained for $\alpha = 0$ and equal to $m_{\nu e}$) superimposed to the 68,90,99% CL confidence regions for the solar mixing angle and for the atmospheric mass difference. A cancellation in $m_{ee}$ is possible for $\tan^2 \theta_{12} \approx 1$; such values of $\theta_{12}$ would be strongly disfavoured if we could restrict our analysis to the LMA solution of the solar neutrino anomaly.

Applying the statistical procedure described in section 1, we extract the $\chi^2$ distributions of $m_{ee}$ and $m_{\nu e}$. The result is shown in fig. 6: oscillation data suggest a $m_{\nu e}$ in the 90% CL range $(40 \div 57)$ meV and a $|m_{ee}|$ in the 90% CL range $(10 \div 57)$ meV. However a much smaller $|m_{ee}|$ can be obtained at slightly higher CL from patterns of solar oscillations (mainly LOW or (Q)VO with maximal mixing) somehow disfavoured by data, but favoured by certain theoretical considerations. As discussed in section 3, inverted hierarchy is naturally obtained from a Majorana mass matrix of pseudo-Dirac form, that predicts all the conditions that give a small or zero $m_{ee}$: $\theta_{12} \approx \pi/4$, $\theta_{13} \approx 0$ and $\alpha \approx \pi/2$.

In general, if neutrinos have Majorana masses with inverted spectrum

- an accurate measurement of $|m_{ee}|$ would be equivalent to a measurement of the Majorana CP-violating phase $\alpha$, if oscillation parameters are known;
- a strong upper bound on $m_{ee}$ would imply $\alpha \approx \pi/2$ and a correlation between oscillation parameters, that probably would be practically indistinguishable from $\theta_{12} \approx \pi/4$.

### 2.5 Almost degenerate neutrinos

In this case, the common neutrino mass is essentially equal to the parameter $m_{\nu e}$ probed by $\beta$-decay experiments. This is presumably the only case that can be tested by near future $\beta$-decay experiments,
Figure 6: *In the case of inverted hierarchy, we plot the probability distribution of the minimal and maximal values of $m_{ee}$ (fig. 6a), and the probability distribution of $m_{\nu_e}$ (fig. 6b).*

and certainly the only case that could affect present experiments.

We discuss in the appendix why we do not consider the recently claimed “evidence for 0$\nu$2$\beta$” [5] convincing. Therefore, in this section we will limit to consider the existing bound on 0$\nu$2$\beta$ [34], and we will compute the bound on $m_{\nu_e}$ from 0$\nu$2$\beta$ and oscillation data, that can be compared with the one from $\beta$ decay. The argument goes as follows: letting aside the region where cancellations are possible [50, 46],

$$\theta_{12} \neq \frac{\pi}{4} \quad \text{and} \quad \tan \theta_{13} < |\cos 2\theta_{12}|^{1/2}$$

(almost degenerate neutrinos lead to a non-zero value of $m_{ee}$):

$$|m_{ee}| \geq m_{\nu_e} \cdot (\cos^2 \theta_{13} |\cos 2\theta_{12}| - \sin^2 \theta_{13}).$$

Thence, the question is whether we know $\theta_{12}$ and $\theta_{13}$ sufficiently well to exclude a cancellation.

The answer is a ‘conditional yes’, as can be seen from fig. 7. There we show the $\Delta \chi^2$ obtained combining all existing 0$\nu$2$\beta$ and oscillation data, as function of $m_{\nu_e}$. At 90% CL, this procedure yields the interesting bound $m_{\nu_e} < 0.95$ MeV, which is better than the limit from $\beta$-decay if $h$ is not too large. However, the distribution is strongly non Gaussian, so that at 99% CL there is no bound. If we knew that LMA is the true solution of the solar neutrino problem, we would have the 99% CL bound $m_{\nu_e} < 1.5$ MeV.

These considerations illustrate the importance of obtaining a precise measurements of the ‘solar’ mixing angle $\theta_{12}$, and of reducing nuclear uncertainties to get further information on massive neutrinos. It will be important to reconsider the inference on neutrino mass scale $m_{\nu_e}$ from 0$\nu$2$\beta$, as soon as we will get more precise information on the solar mixing angle $\theta_{12}$ from SNO, KamLAND, Borexino. The outcome would be particularly interesting if KamLAND will confirm the LMA solution with less-than-maximal mixing.

We summarize the possible rôle of future $\beta$-decay and 0$\nu$2$\beta$-decay experiments as follows

- If neutrinos have Majorana masses and if future oscillation data will tell us that eq. (11) is satisfied at a high CL, 0$\nu$2$\beta$ experiments alone will be sufficient to rule out (or confirm) the possibility that neutrinos have a large common mass $m_{\nu_e}$.

- If neutrinos have a large common Majorana mass $m_{\nu_e}$, $\beta$-decay experiments will be necessary to measure it, and 0$\nu$2$\beta$ experiments will permit to investigate the Majorana phases. Since $V_{e3}^2$ is small, $\alpha$ is the only relevant phase, to a good approximation.

6The dominant bound on $\theta_{13}$ comes from the Chooz experiment (when combined with the SK atmospheric constraints on $|\Delta m_{23}^2|$). The replacement $\theta_{13} = 0$, however, approximates reasonably well the accurate treatment of the Chooz bound.
Predictions, expectations, guesses and prejudices

Perhaps the simplest and more conservative theoretical explanation of the lightness of neutrinos is given by the idea that neutrino masses are suppressed by the scale of lepton number violation [51]. This scenario is well compatible with unification of electroweak and strong interactions at a very high energy scale and provides an appealing mechanism for the generation of the observed baryon asymmetry. Such considerations single out, among all possible models of neutrino masses and mixings, those characterized, at low energies, by three light Majorana neutrinos. Given the present experimental knowledge, that favors $\Delta m_{23}^2$ as the leading oscillation frequency, it is also natural to define a zeroth order approximation of the theory, where $\Delta m_{12}^2$ and $\theta_{13}$ vanish (which allows us to neglect the CP-breaking parameter $\phi$) whereas $\theta_{23}$ and $\theta_{12}$ are maximal. This approximation is of course not realistic and should be regarded only as a limiting case, possibly arising from an underlying symmetry. Many effects can perturb this limit, such as small symmetry breaking terms, radiative corrections, effects coming from residual rotations needed to diagonalize the charged lepton mass matrix, or to render canonical the leptonic kinetic terms. A detailed analysis of these effects would require a separate discussion for each conceivable model, with no guarantee of being able to really cover all theoretical possibilities. Here we will adopt a less ambitious approach which consists in dealing only with the simplest perturbations of the leading textures, with the hope that the results will be sufficiently representative of the many existing models. This point of view is only partially supported by present data. If $\Delta m_{12}^2$ and $\theta_{13}$ were as large as experimentally allowed, neutrino data would not clearly point to any simple pattern of the type considered here.

Of course, there are many other promising theoretical ideas on neutrino masses, that maintain their interest (and survive) even after the recent experimental developments. Some of these recent approaches insist on (certain minimal version of) grand unification models [52], and, in particular aim at a link with proton decay rate [53]. There are also attempts to find stricter relations with leptogenesis [54]; or with low energy lepton flavor violating processes in supersymmetric models [55]; etc. We do not want to underestimate the validity of these ideas, and we instead hope that some or all of them will find an important role in a future theory of massive neutrinos. Rather, it should be clear that our approach to the theory of neutrino masses is guided by simple and phenomenological considerations, to the aims of exploiting regularities in the observable parameters of massive neutrinos, and of making a guess about $m_{ee}$ and $\theta_{13}$.
3.1 Normal hierarchy

We begin with the hierarchical neutrino spectrum, whose leading texture, up to redefinitions of the field phases, in the mass-eigenstate bases of charged leptons, is given by

\[
m_\nu = \begin{pmatrix}
0 & 0 & 0 \\
0 & 1 & 1 \\
0 & 1 & 1
\end{pmatrix}.
\]  

(13)

This structure might be due to a peculiar structure of the mass matrix of the right-handed neutrinos, of the (Dirac) Yukawa couplings of the neutrinos, or even of the charged leptons in a certain flavor basis [56]; this may arise, for instance, within a U(1) flavor symmetry [57] under which \(\nu_\mu\) and \(\nu_\tau\) are neutral and \(\nu_e\) possesses a non-vanishing charge [58, 59, 60]. The determinant of the block (2, 3) can be forced to vanish (so that one naturally obtains \(\Delta m^2_{12} \ll \Delta m^2_{23}\)) by making use of the see-saw mechanism, if one of the heavy right-handed neutrinos has larger Yukawa couplings and/or is lighter than the other ones [61].

The resulting neutrino mass matrix has approximately rank one. This setup can be implemented within the framework of a U(1) symmetry [59, 60], which, however, only predicts a generically large, not necessarily maximal \(\theta_{23}\). Another possibility to generate the leading texture in eq. (13) is at 1 loop [63] or at tree level [64] in supersymmetric models that violate both lepton number and R-parity. An important feature of (13) is that \(\theta_{12}\) is undetermined at leading order.

To get a realistic mass matrix, we consider deviations from the symmetric limit in (13), parametrized by:

\[
m_\nu = \begin{pmatrix}
\delta & \epsilon & \epsilon \\
\epsilon & 1 + \eta & 1 + \eta \\
\epsilon & 1 + \eta & 1 + \eta
\end{pmatrix},
\]  

(14)

where \(\delta, \epsilon\) and \(\eta\) denote small \((\ll 1)\) real parameters, defined up to coefficients of order one that can differ in the various matrix elements. The mass matrix in (14) does not describe the most general perturbation of the zeroth order texture (13). We have implicitly assumed a symmetry between \(\nu_\mu\) and \(\nu_\tau\) which is preserved by the perturbations, at least at the level of the order of magnitudes. It is difficult to understand the precise origin of these small deviations. However, it is possible to construct models based on a spontaneously broken U(1) flavor symmetry, where \(\delta, \epsilon\) and \(\eta\) are given by positive powers of one or more symmetry breaking parameters. Moreover, by playing with the U(1) charges, we can adjust, to certain extent, the relative hierarchy between \(\eta, \epsilon\) and \(\delta\) [65].

Another example is given by those models where the neutrino mass matrix elements are dominated, via the see-saw mechanism, by the exchange of two right-handed neutrinos [66]. Since the exchange of a single right-handed neutrino gives a successful zeroth order texture, we are encouraged to continue along this line. Thus, we add a sub-dominant contribution of a second right-handed neutrino, assuming that the third one gives a negligible contribution to the neutrino mass matrix, because it has much smaller Yukawa couplings or is much heavier than the first two. The Lagrangian that describes this plausible sub-set of see-saw models, written in the mass eigenstate basis of right-handed neutrinos and charged leptons, is

\[
\mathcal{L} = \lambda_\ell L_\ell N h + \lambda_{\ell'} L_{\ell'} N' h + \frac{M}{2} N^2 + \frac{M'}{2} N'^2 \quad \Rightarrow \quad m_{\ell\ell'} \propto \frac{\lambda_\ell \lambda_{\ell'}}{M} + \frac{\lambda_{\ell'} \lambda_{\ell'}}{M'}
\]

where \(\ell, \ell' = \{e, \mu, \tau\}\). The pattern of perturbations in eq. (14) can be reproduced if \(\lambda_e \ll \lambda_\mu \approx \lambda_\tau\) and \(\lambda_{\ell'} \approx \lambda_{\ell'}\). Even though the number of see-saw parameters is larger than the number of low-energy observables, there is one neat prediction:

\[
\text{det } m = 0 \quad \text{i.e. } \quad m_1 = 0.
\]

7The suggestion to interpret the data on massive neutrinos by assuming a number of right handed neutrinos as small as possible is quite old, see [62]. Note however that in this work it was missed the fact that the model with 2 right-handed neutrinos is predictive, as we discuss in the following.
(We recall that the possibilities to test this case were discussed at length in section 2.3).

Let us come back to the mass matrix $m_\nu$ of eq. (14). After a first rotation by an angle $\theta_{23}$ close to $\pi/4$ and a second rotation with $\theta_{13} \approx \epsilon$, we get

$$m_\nu \approx m \begin{pmatrix} \delta + \epsilon^2 & \epsilon & 0 \\ \epsilon & \eta & 0 \\ 0 & 0 & 2 \end{pmatrix},$$

up to order one coefficients in the small entries. To obtain a large solar mixing angle, we need $|\eta - \delta| \lesssim \epsilon$. In realistic models there is no reason for a cancellation between independent perturbations and thus we assume $\delta \lesssim \epsilon$ and $\eta \lesssim \epsilon$, separately.

Consider first the case $\delta \approx \epsilon$ and $\eta \lesssim \epsilon$. The solar mixing angle $\theta_{12}$ is large but not maximal, as preferred by the large angle MSW solution. We also have $\Delta m^2_{23} \approx 4 m^2$, $\Delta m^2_{12} \approx m^2 \epsilon^2$ and

$$m_{ee} \approx \sqrt{\Delta m^2_{12}},$$

the largest possible prediction, in the case of normal hierarchy, in agreement with the results in section 2.3.

If $\eta \approx \epsilon$ and $\delta \ll \epsilon$, we still have a large solar mixing angle and $\epsilon^2 \approx \Delta m^2_{12}/\Delta m^2_{23}$, as before. However $m_{ee}$ will be much smaller than the estimate in (16). Unfortunately, this is the case of the models based on the above mentioned U(1) flavor symmetry that, at least in its simplest realization, tends to predict $\delta \approx \epsilon^2$. In this class of models we find

$$m_{ee} \approx \sqrt{\Delta m^2_{12}} \left( \frac{\Delta m^2_{12}}{\Delta m^2_{23}} \right)^{\frac{1}{2}},$$

below the sensitivity of the next generation of planned experiments. It is worth to mention that in both cases discussed above, we have

$$\theta_{13} \approx \left( \frac{\Delta m^2_{12}}{\Delta m^2_{23}} \right)^{\frac{1}{2}},$$

which might be very close to the present experimental limit.

If both $\delta$ and $\eta$ are much smaller than $\epsilon$, the (1, 2) block of $m_\nu$ has an approximate pseudo-Dirac structure and the angle $\theta_{12}$ becomes maximal. This situation is typical of models where leptons have U(1) charges of both signs whereas the order parameters of U(1) breaking have all charges of the same sign [60]. We have two eigenvalues approximately given by $\pm m \epsilon$. As an example, we consider the case where $\eta = 0$ and $\delta \approx \epsilon^2$. We find $\sin^2 2\theta_{12} \approx 1 - \epsilon^2/4$, $\Delta m^2_{12} \approx m^2 \epsilon^3$ and

$$m_{ee} \approx \sqrt{\Delta m^2_{12}} \left( \frac{\Delta m^2_{12}}{\Delta m^2_{23}} \right)^{\frac{1}{6}},$$

In order to recover the large angle MSW solution we would need a relatively large value of $\epsilon$. This is in general not acceptable because, on the one hand the presence of a large perturbation raises doubts about the consistency of the whole approach and, on the other hand, in existing models where all fermion sectors are related to each other, $\epsilon$ is never larger than the Cabibbo angle. We are then forced to embed the case under discussion within the LOW solution, where the solar frequency is much smaller and, as a consequence, $m_{ee}$ is beyond the reach of the next generation of experiments.

### 3.2 Inverted hierarchy

If the neutrino spectrum has an inverted hierarchy, the leading texture depends on the relative phase $2\alpha$ between the two non-vanishing eigenvalues. In this case theory can give significant restrictions, since
one has to explain why two neutrinos are degenerate. This degeneracy can be more easily explained for special values of the relative phases. In particular, when $\alpha = \pi/2$, the resulting texture has an exact symmetry under the combination $L_e - L_\mu - L_\tau$ [67, 68] and reads:

$$m_\nu = m \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}. \quad (20)$$

The Abelian symmetry $L_e - L_\mu - L_\tau$ would allow different $(1,2)$ and $(1,3)$ entries, that are chosen equal in (20) to recover a maximal $\theta_{23}$. Independently from the relative size of these entries, we have $\theta_{12} = \pi/4, \theta_{13} = 0, \Delta m_{23}^2 = 2m^2, \Delta m_{12}^2 = 0$. Notice that an exact $L_e - L_\mu - L_\tau$ symmetry implies $m_{ee} = 0$. If it were possible to find a symmetry realizing the inverted hierarchy with $\alpha \neq \pi/2$, then we could avoid $m_{ee} = 0$. We add small perturbations according to:

$$m_\nu = m \begin{pmatrix} \delta & 1 & 1 \\ 1 & \eta & \eta \\ 1 & \eta & \eta \end{pmatrix}. \quad (21)$$

The perturbations leave $\Delta m_{23}^2$ and $\theta_{23}$ unchanged, in first approximation. We obtain $\theta_{13} \approx \eta, \tan^2 \theta_{12} \approx 1 + \delta + \eta$ and $\Delta m_{12}^2 / \Delta m_{23}^2 \approx \eta + \delta$, where coefficients of order one have been neglected.

There is a well-known difficulty of this scenario to reproduce the large angle MSW solution [67, 69]. Indeed, barring cancellation between the perturbations, in order to obtain a $\Delta m_{23}^2$ close to the best fit LMA value, $\eta$ and $\delta$ should be smaller than about 0.1 and this keeps the value of $\sin^2 2\theta_{12}$ very close to 1, somewhat in disagreement with global fits of solar data [6]. Even by allowing for a $\Delta m_{12}^2$ in the upper range of the LMA solution, or some fine-tuning between $\eta$ and $\delta$, we would need large values of the perturbations to fit the LMA solution. On the contrary, the LOW solution can be accommodated, but, even in the optimistic case $\delta \gg \eta$, we obtain:

$$m_{ee} = \frac{1}{2} \sqrt{\Delta m_{12}^2} \left( \frac{\Delta m_{12}^2}{\Delta m_{23}^2} \right)^{\frac{1}{2}}, \quad (22)$$

too small to be detected by planned experiments. This is an example where the underlying symmetry forces a cancellation between potentially large contributions to $m_{ee}$, which persists also after the inclusion of the perturbations. In fact the combination $m_1 \cos^2 \theta_{12} + m_2 e^{2i\alpha} \cos^2 \theta_{12}$ is not of order $m$, as we could naively expect, but of order $m \delta$. The largest allowed value for $\theta_{13}$ is

$$\theta_{13} \approx \frac{\Delta m_{12}^2}{\Delta m_{23}^2}, \quad (23)$$

which, for the LOW solution, is practically unobservable.

### 3.3 Almost degenerate neutrinos

Also in the case of degenerate spectrum, the zeroth order texture depends on the relative phases between the eigenvalues and this dependence leads to widely different expectation for $m_{ee}$. We have two limiting cases: one for $\alpha = 0$, and another one with $\alpha = \pi/2$. In the first case $m_{ee}$ will be comparable to the average neutrino mass, while in the second case the large mixing in the solar sector tends to deplete the $m_{ee}$ entry [70]. There are examples of both these possibilities among the theoretically motivated models and here we will discuss two representative scenarios. It should be said that it is more difficult to accommodate a degenerate neutrino spectrum in a model of fermion masses, than a spectrum with normal or inverted hierarchy. The neutrino degeneracy should be made compatible with the observed hierarchy in the charged fermion sectors, and this is not an easy task. Once the degeneracy is achieved, at leading
order, it should be preserved by renormalization group evolution from the energy scale where neutrino masses originate down to the low energy scales where observations take place. The stability under these corrections may impose severe restrictions on the theory, especially if a very small mass splitting is required by the solar neutrino oscillations [71, 72]. There is also a generic difficulty in embedding such a model in a grand unified context where the particle content often includes right-handed neutrinos. Indeed, it is difficult to see how a presumably hierarchical Dirac mass matrix combines with a Majorana mass matrix for the right-handed neutrinos to give rise, via the see-saw mechanism, to a degenerate neutrino spectrum. One idea that partially overcomes some of the above-mentioned problems is that of flavor democracy [73], where an approximate $S_{3L} \otimes S_{3R}$ symmetry leads to the following pattern of charged fermion mass matrices:

$$m_f = \hat{m}_f \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \delta m_f \ , \quad f = \{e,u,d\} \quad (24)$$

where $\delta m_f$ denotes small symmetry breaking terms. The corresponding spectrum is hierarchical and the smallness of quark mixing angles can be explained by an almost complete cancellation between the two unitary left transformations needed to diagonalize $m_u$ and $m_d$. The neutrino mass matrix has the general structure:

$$m_\nu = m \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + r \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} + \delta m_\nu \ , \quad (25)$$

where $r$ is an arbitrary parameter and $\delta m_\nu$ represents a small correction to the symmetric limit. The freedom associated to $r$, $\delta m_\nu$ and $\delta m_e$ allows us to select an almost degenerate and diagonal $m_\nu$ and to extract the large atmospheric and solar mixing angles from the charged lepton sector. However, a non-vanishing $\Delta m_{21}^2$ and maximal $\theta_{12}, \theta_{23}$ angles are not determined by the symmetric limit, but only by a specific choice of the parameter $r$ and of the perturbations, that cannot be easily justified on theoretical grounds. If, for example, we choose $\delta m_\nu = \text{diag}(0, \epsilon, \eta)$ with $\epsilon < \eta \ll 1$ and $r \ll \epsilon$, the solar and the atmospheric oscillation frequencies are determined by $\epsilon$ and $\eta$, respectively. The mixing angles are entirely due to the charged lepton sector. A diagonal $\delta m_e$ will give rise to an almost maximal $\theta_{12}, \tan^2 \theta_{23} \approx 1/2$ and $\theta_{13} \approx \sqrt{m_e/m_\nu}$. By going to the basis where the charged leptons are diagonal, we can see that $m_{ee}$ is close to $m$ and independent from the parameters that characterize the oscillation phenomena. Indeed $m$ is only limited by the $0\nu2\beta$ decay. The parameter $r$ receives radiative corrections [74] that, at leading order, are logarithmic and proportional to the square of the $\tau$ lepton Yukawa coupling. It is important to guarantee that this correction does not spoil the relation $r \ll \epsilon$, whose violation would lead to a completely different mixing pattern. This raises a ‘naturalness’ problem for the LOW solution. It would be desirable to provide a more sound basis for the choice of small perturbations in this scenario that is quite favorable to signals both in the $0\nu2\beta$ decay and in sub-leading oscillations controlled by $\theta_{13}$.

If $\alpha = \pi/2$, we could have $m_{ee}$ well below the average neutrino mass or even below the solar oscillation frequency. This is exemplified by the following leading texture

$$m_\nu = m \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2}(1 + \eta) & -\frac{1}{2}(1 + \eta) \\ \frac{1}{\sqrt{2}} & -\frac{1}{2}(1 + \eta) & \frac{1}{2}(1 + \eta) \end{pmatrix} \ , \quad (26)$$

where $\eta \ll 1$, corresponding to an exact bimaximal mixing, with eigenvalues $m_1 = m$, $m_2 = -m$ and $m_3 = (1 + \eta)/m$. This texture has been proposed in the context of a spontaneously broken $SO(3)$ flavor symmetry and it has been studied to analyze the stability of the degenerate spectrum against radiative
the error on the atmospheric parameters, and probe values of the solar anomaly. MiniBoone will test the LSND anomaly. K2K, Minos and CNGS should reduce oscillation parameters in the near future. KamLand and Borexino should finally identify the true solution promising. We reviewed how it should be possible to access and reliably measure at least 4 of the 6 As recalled in section 1, the experimental program on neutrino oscillations is well under way and looks
4 Conclusions

corrections [72]. We add small perturbations to (26) in the form:

\[ m_\nu = m \begin{pmatrix} \frac{\delta}{\sqrt{2}} & \frac{1}{\sqrt{2}}(1 + \eta) & \frac{1}{\sqrt{2}}(1 - \epsilon) \\ \frac{1}{\sqrt{2}}(1 - \epsilon) & -\frac{1}{2}(1 + \eta - \epsilon) & \frac{1}{2}(1 + \eta - 2\epsilon) \end{pmatrix} \]

(27)

where \( \epsilon \) parametrizes the leading flavor-dependent radiative corrections (mainly induced by the \( \tau \) Yukawa coupling) and \( \delta \) controls \( m_{ee} \). We first discuss the case \( \delta \ll \epsilon \). To first approximation \( \theta_{12} \) remains maximal, disfavouring an interpretation in the framework of the LMA solution. We get \( \Delta m^2_{12} \approx m^2 \frac{e^2}{\eta} \) and

\[ \theta_{13} \approx \left( \frac{\Delta m^2_{12}}{\Delta m^2_{23}} \right)^{1/2}, \quad m_{ee} \ll m \left( \frac{\Delta m^2_{23} \Delta m^2_{12}}{m^4} \right)^{1/2} \] (28)

If we assume \( \delta \gg \epsilon \), we find \( \Delta m^2_{12} \approx 2m^2 \delta \), \( \theta_{23} = \pi/4 \), \( \sin^2 2\theta_{12} = 1 - \delta^2/8 \) and \( \theta_{13} = 0 \). Also in this case the solar mixing angle is too close to \( \pi/4 \) to fit the large angle MSW solution. We get:

\[ m_{ee} \approx \frac{\Delta m^2_{12}}{2m} \] (29)

too small for detection if the average neutrino mass \( m \) is around the eV scale. This rather extreme example shows that there is no guarantee for \( m_{ee} \) to be close to the range of experimental interest, even with degenerate neutrinos where the involved masses are much larger than the oscillation frequencies.

We conclude this section with a comment on a relevant issue raised in [75]: would a measurement \( m_{ee} \sim \text{eV} \) imply the LMA solution, since in such a case the other LOW, (Q)VO solutions are unstable under radiative corrections? The answer is no. Ref. [72] showed examples of degenerate neutrino spectra with \( m_{ee} = 0 \), \( \theta_{13} = 0 \) and \( \theta_{12} = \pi/4 \) where \( \Delta m^2_{12} \) is radiatively generated only at order \( \lambda^4 \), rather than at order \( \lambda^2 \) as naively expected. It is easy to find examples with \( m_{ee} \neq 0 \). The basic observation is that the results in [72] continue to hold if the neutrino mass matrix has the from in eq. (5) of [72] in a flavour basis where the charged lepton mass matrix is not fully diagonal because has a non vanishing 12 entry. The flavour rotation in the 12 plane that leads to the usual basis of \( e, \mu, \tau \) mass eigenstates, generates non-vanishing \( m_{ee} \) and \( \theta_{13} \) and shifts \( \theta_{12} \) from \( \pi/4 \). Proceeding along the lines of [72] one concludes that in models where

\[ \alpha = \frac{\pi}{2}, \quad \sin \theta_{13} = \tan \theta_{23} \tan(\theta_{12} - \pi/4) \quad \text{so that} \quad |m_{ee}| \approx \frac{2\theta_{13}}{\tan \theta_{23}} m + O(\theta_{13}^2) \]

radiative corrections generate a solar mass splitting compatible with the LOW and (Q)VO solutions of the solar anomaly; at the same time, \( m_{ee} \) can be relatively large without violating the CHOOZ bound.

4 Conclusions

As recalled in section 1, the experimental program on neutrino oscillations is well under way and looks promising. We reviewed how it should be possible to access and reliably measure at least 4 of the 6 oscillation parameters in the near future. KamLand and Borexino should finally identify the true solution of the solar anomaly. MiniBoone will test the LSND anomaly. K2K, Minos and CNGS should reduce the error on the atmospheric parameters, and probe values of \( \theta_{13} \) below its present bound. Further experimental progresses and improvements (maybe culminating in a neutrino factory) seem feasible in longer terms, after the first generation of long-baseline experiments, although we cannot exclude that effects due to \( \theta_{13} \) or CP violation are too small to ever be observed.

The fact that oscillations are becoming an established fact can only reinforce the motivation for other approaches to massive neutrinos, and primarily \( \beta \) and neutrino-less double \( \beta (0\nu2\beta) \) decay experiments,
In fig. 8a we show the 90% and 99% CL ranges of \( m_{ee} \) in the cases of normal hierarchy (i.e. \( 0 = m_1 \ll m_2 \ll m_3 \), so that \( \Delta m_{23}^2 > 0 \)) inverted hierarchy (i.e. \( 0 = m_3 \ll m_1 \approx m_2 \), so that \( \Delta m_{23}^2 < 0 \)) and almost degenerate neutrinos at 1 eV (a value chosen for illustration). LOW collectively denotes large mixing angle solutions other than LMA. In fig. 8b, without distinguishing LMA or LOW, we plot the 90% CL range for \( m_{ee} \) as function of the lightest neutrino mass, thereby covering all spectra.

These results are summarized in fig. 8a, where we also show the ranges of \( m_{ee} \), restricted to the LMA and to the other large mixing angle solutions of the solar anomaly (collectively denoted as ‘LOW’). The SMA solution is only allowed at higher CL than the ones we consider.

Beyond these two special cases (where the lightest neutrino has a little or negligible mass), there is a family of more generic spectra (sometimes named ‘partial hierarchy’, ‘partial degeneracy’, . . . ) conveniently parametrised by the lightest neutrino mass, \( m_{\nu_{\min}} \). In fig. 8b we show how the 90% CL range for \( m_{ee} \) varies as a function of \( m_{\nu_{\min}} \) (note that this figure improves on traditional plots in \( m_{\nu_{\min}} - m_{ee} \) plane [46], which are done at fixed values of the oscillation parameters). When \( m_{\nu_{\min}} \) is larger than the oscillation scales (so that neutrinos are almost degenerate at a common mass \( m_{\nu_e} \)) we get the 90% range \( m_{ee} = (0.17 \div 1) \ m_{\nu_e} \) (see fig. 8a for more detailed results). We also considered the implications of present data:
In the case of almost degenerate neutrinos (section 2.5) we converted the present $0\nu2\beta$ bound ($m_{ee} < 0.38 \text{ eV}$ at 95% CL; $h \sim 1$ parameterizes nuclear uncertainties, see section 2.1) into a 90% CL bound on their common mass, $m_{\nu_e} < 0.95 h \text{ eV}$, competitive with the direct $\beta$-decay bound.

This bound could become stronger with future oscillation data. In particular, if KamLAND will confirm the LMA solution of the solar neutrino anomaly, it will be important to update the inferences on $m_{ee}$ performed in the present work. Within the three Majorana neutrino context we assumed, existing and planned $\beta$ decay experiments can see a signal only if neutrinos are almost degenerate. However, future $0\nu2\beta$ experiments alone could rule out this possibility, if future oscillation data will safely tell us that $|\cos \theta_{12}| > \tan^2 \theta_{13}$. If neutrinos indeed have a large common Majorana mass, only with both $\beta$ and $0\nu2\beta$ experiments will it be possible to measure it, and learn on relevant Majorana phase.

A recent paper [5] claims a $0\nu2\beta$ evidence for almost degenerate neutrinos. In the appendix we reanalysed the data on which such claim is based and proposed a statistically fair way of extracting the signal from the background. Assuming that the relevant sources of background are the ones discussed in [5], we get a 1.5$\sigma$ hint (or less, depending on the data-set used) for $0\nu2\beta$ from the published data. In section 3.3 we provide a counterexample to the statement that measuring $m_{ee} \sim \text{eV}$ would imply the LMA solution: the other LOW, (Q)VO solutions would not necessarily be unstable under radiative corrections.

All $m_{ee}$ ranges obtained above have been derived allowing for cancellations between the different contributions to $m_{ee}$. One could be tempted to obtain more optimistic results by invoking ‘naturalness’ arguments to disfavour large cancellations. As discussed in section 2.2, since $m_{ee}$ is a theoretically clean quantity (the $ee$ element of the neutrino mass matrix) some flavour symmetry could force it to be small, giving rise to apparently unnatural cancellations when $m_{ee}$ is written in terms of the phenomenological parameters. Phenomenology alone implies only the relatively weak results summarized above.

For this reason in section 3 we have estimated $m_{ee}$ and $\theta_{13}$ in models of neutrino masses. We considered few theoretically well-motivated textures, that existing models can reproduce in some appropriate limit, and we added small perturbations to mimic the realistic case. Even though this procedure cannot of course cover all possibilities, it illustrates, for each type of spectrum, the typical theoretical expectations for $m_{ee}$ and $\theta_{13}$ and provides examples where flavour symmetries force a small $m_{ee}$. Qualitatively we can summarize our analysis by saying that values of $m_{ee}$ above 10 meV are certainly possible for models of quasi-degenerate neutrinos, but they would come as unexpected in models characterized by normal or inverted hierarchy. In models realizing the LMA solutions, independently from the type of spectrum, the angle $\theta_{13}$ is usually close to its present experimental limit.

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A Evidence for neutrino-less double beta decay?

A recent paper [5], which reanalysed data of the Heidelberg–Moscow experiment at Gran Sasso (HM), claimed an evidence for $0\nu2\beta$ at the 2.2$\sigma$ or 3.1$\sigma$ level, when Bayesian or frequentistic techniques are adopted. The measured value would be $m_{ee}/h = (0.39 \pm 0.11) \text{ eV}$, implying almost degenerate neutrinos with mass $m_{ee} \geq |m_{ee}|$, partly testable at KATRIN [37]. Such a result would be of the highest importance. However, we believe that the published data only contain a $\sim 1.5\sigma$ hint (or less, depending on the data-set used) and we would like to explain why in this appendix.

The data in fig.s 2 and 3 of [5] are reported in our fig.s 9a and b, where we show the two histograms with the energy distribution of the second data-set (from detectors 1, 2, 3, 5) and of the third data-set (from detectors 2, 3, 5, with pulse-shape discrimination applied in order to reduce the background). A
0ν2β signal would show up as a peak centered at the energy \( E = Q_0 = 2039.0 \text{ keV} \) and width equal to the energy resolution, \( \sigma_E = 1.59 \pm 0.19 \text{ keV} \). A quick look at the data shows no clear peak; so one wonders what is known a priori on the spectrum, and which is the procedure of analysis to be applied.

Our first step is to repeat the exercise done in [5]: we compute the likelihood of having one peak of width \( \sigma_E \) centered at a generic energy \( E \), on the top of a constant background of unknown intensity \( b \) in the energy range \( (2000 \div 2080) \text{ keV} \). The continuous blue lines in fig.s 9a,b (the right ordinates show their scale) show our results.\(^8\) They reasonably agree with the corresponding fig.s 5 and 6 of [5], although we find a somewhat lower probability that a peak is present at \( E \approx Q_0 \) than in [5] (maybe because [5] fits data with thinner binning than in the published data). In both cases more pronounced peaks seem present at other energies.

The evidence of [5] is obtained by restricting the search window to the energy range \( E = Q_0 \pm \text{few}\cdot \sigma_E \), such that it does not include the other peaks, and by fitting the data under the hypothesis of a peak at \( Q_0 \) plus a constant background. An obvious criticism to this procedure is that the data look like a peak over a constant background because one has chosen the specific window where this happens. This procedure would be justified if the size of the search window were not a significant arbitrary choice, e.g. if one could clearly see in the data a peak at \( Q_0 \), surrounded by the level of constant background estimated by the statistical procedure adopted. However, this structure is not clearly visible in the data-sets. In order to state quantitatively our concern, we show in fig. 9c how the evidence for a 0ν2β signal (quantified as \( \Delta \chi^2 = \chi^2_{\text{best}} - \chi^2_{\text{no 0ν2β}} \)) fluctuates when one chooses windows of different sizes around \( Q_0 \). A ‘too small’ window does not allow to discriminate the signal from the background, since it can only be distinguished from the signal exploiting their different energy dependence. With an appropriate window the evidence

\(^8\)What we precisely plot is \( 1 - \mathcal{L}(\text{background})/\mathcal{L}(\text{background + peak at } E) \), where \( \mathcal{L}(\text{background}) \) is the likelihood of having a constant background, while \( \mathcal{L}(\text{background + peak at } E) \) is the likelihood of having a constant background plus a peak at an energy \( E \). The level of the background and the height of the peak are treated as free parameters. The likelihoods \( \mathcal{L} \equiv e^{-\chi^2/2} \) are computed as \( \chi^2 = (\sigma_E - 1.59 \text{ keV})^2/(0.19 \text{ keV})^2 + \sum_i \chi_i^2 \), where the \( \chi_i^2 \) relative to the \( i \)th bin (which has \( n_i \) observed and \( \mu_i \) expected events) is given by Poissionan statistics as \( \chi_i^2 = 2(\mu_i - n_i \ln \mu_i) \). Finally, the \( \chi^2 \) is minimized with respect to the nuisance parameters \( b \) and \( \sigma_E \). In Gaussian approximation, the Bayesian and frequentistic techniques employed in [5] both reduce to this procedure. We repeated all analyses described in this appendix using a Bayesian procedure, finding very little differences with respect to its Gaussian limit.
Figure 10: Fig. 10a: the $\chi^2(m_{ee}/h)$ extracted in different ways from the Heidelberg–Moscow data without pulse-shape discrimination. Fig. 10b: bounds on the degenerate neutrino mass, using our reanalysis of these data.

for a peak at $Q_0$ can reach the $3\sigma$ level. Both the evidence and the central value of the signal change when the size of the window is varied. There is almost no evidence when a large window is chosen; but if the background were constant, a large window would be the fairest way to estimate its level.

In summary: the data contain no evidence for a $0\nu2\beta$ signal under the assumption of an energy-independent background in the plotted range. Evidence could only be claimed if the background had a favourable energy spectrum (smaller around $Q_0$ and larger far from $Q_0$) and if one could understand it, allowing to disentangle the signal from the background. Therefore the crucial question is: what is the composition of the background, and in particular what are the peaks in the rest of the spectrum, which seem to emerge from a constant background? The authors of [5] observe that some peaks lie around the known energies of $\gamma$-peaks of $^{214}$Bi:

$$E_a = \{2010.7\text{ keV},\ 2016.7\text{ keV},\ 2021.8\text{ keV},\ 2052.9\text{ keV}\};$$

this is how they motivate the choice of a small search window that excludes them.

If this is the real composition of the backgrounds, one should analyse all data assuming that they are composed of: a constant component, plus the $^{214}$Bi peaks, plus a possible signal $s$. Therefore we write the number of events as

$$\frac{dn}{dE} = b + \sum_a p_a \rho(E - E_a) + s \rho(E - Q_0)$$

where $\rho(E) = \frac{e^{-E^2/2\sigma_E^2}}{\sqrt{2\pi}\sigma_E}$.

The relative intensity of the $^{214}$Bi $\gamma$-lines is known [76]. We marginalize the $\chi^2$ with respect to the nuisance parameters that describe the backgrounds and the energy resolution, obtaining the $\chi^2$ distribution of the signal, $\chi^2(s)$ (1 degree of freedom). In this way we find a weak evidence of a peak at $Q_0$: $\chi^2_{\text{best}}(s) - \chi^2(0) \approx 2 (0.5)$ using the second (third) data-set; namely, a $1.5\sigma$ evidence at most. The best fit is shown in fig.s 9 (dotted lines). The constant component of the background is only slightly lower than under the hypothesis that all events are due to a constant background (dashed lines). In both cases at $Q_0$ there is no peak significantly above the background level, so that we obtain little evidence for a $0\nu2\beta$ signal. In fig. 10a we show the $\chi^2(m_{ee}/h)$, as obtained from three different analyses of the HM data without pulse-shape discrimination: by choosing a small search window (as in [5]), by modeling the background (as proposed here), and assuming a constant background (as in the HM paper [34], where a slightly different data-set was analysed). In fig. 10b we show the bounds on the degenerate neutrino mass, using our reanalysis of these data. Note that the upper bound on $m_{\nu_e}$ in fig. 7 is stronger, because we used the HM analysis of data with pulse-shape discrimination. The connection of massive neutrinos with cosmology is well known (e.g. [29], for earlier works about neutrinos as warm dark matter see [77]).

With actual data and simulations of the apparatus, it could be possible to do better than what we can do, by calculating also the absolute intensity of the faint $^{214}$Bi lines in the $(2000 \div 2080)\text{ keV}$ region.
based on other, much more intense lines of this isotope. Using the line at $E \sim 1764.5$ keV and the data
in fig. 2 of [34], we estimate that the faint $^{214}$Bi lines should be a few times weaker than in our best-fit.
A significant $0\nu2\beta$ signal could finally result if most peaks are not statistical fluctuations, and could be
identified.

However a comparison between the two data-sets discoursages such hopes. We expect that the pulse-
shape discrimination should suppress the background from $\gamma$-lines (since the annihilation energy of the
positron may be deposited in a relatively wider region) more than the hypothetical $0\nu2\beta$ signal (since the
two $e^-$ deposit their energy in a narrow region). A detailed simulation of the detector is needed to
estimate this issue quantitatively. The problem is that fig.s 9a,b and our best-fit values indicate that pulse-shape
discrimination should suppress the background from $\gamma$-lines (since the annihilation energy of the
positron may be deposited in a relatively wider region) more than the hypothetical $0\nu2\beta$ signal (since the
two $e^-$ deposit their energy in a narrow region). A detailed simulation of the detector is needed to
estimate this issue quantitatively. The problem is that fig.s 9a,b and our best-fit values indicate
rather that pulse-shape discrimination gives an almost equal suppression: the constant background gets
underestimated.

In conclusion, we do not see a really significant evidence for $0\nu2\beta$ in the published data. A better
understanding and control of the background is necessary to learn more from present data (and maybe
recognize a signal) and also in view of future search of $0\nu2\beta$ with $^{76}$Ge detectors.

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Note that the three non-zero parameters of the Zee model, $A$, $Z$, and $B$ can be adjusted to reproduce this texture (though one could have guessed that $m_{\mu}$ was the smallest element, rather than $m_{\tau}$).


Data available at the internet address nuclear-data.nuclear.lu.se/nucleardata/toi.