Differences between Quark and Gluon jets
as seen at LEP

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Abstract

The differences between quark and gluon jets are studied using LEP results on jet widths, scale dependent multiplicities, ratios of multiplicities, slopes and curvatures and fragmentation functions. It is emphasized that the observed differences stem primarily from the different quark and gluon colour factors.

1 Introduction

The physics of the differences between quark and gluon jets continuously attracts an interest of both, theorists and experimentalists. Hadron production can be described by parton showers (successive gluon emissions and splittings) followed by formation of hadrons which cannot be described perturbatively. The gluon emission, being dominant process in the parton showers, is proportional to the colour factor associated with the coupling of the emitted gluon to the emitter. These colour factors are $C_A = 3$ when the emitter is a gluon and $C_F = 4/3$ when it is a quark. Consequently, the multiplicity from a gluon source is (asymptotically) $9/4$ higher than from a quark source.

In QCD calculations, the jet properties are usually defined inclusively, by the particles in hemispheres of quark-antiquark ($q\bar{q}$) or gluon-gluon ($gg$) systems in an overall colour singlet rather than by a jet algorithm. In contrast to the experimental results which often depend on a jet finder employed (biased jets), the inclusive jets do not depend on any jet finder (unbiased jets).
2 Results

2.1 Jet Widths

As a consequence of the greater radiation of soft gluons in a gluon jet compared to a quark jet, gluon jets are predicted to be broader. An experimental confirmation of this effect is shown in Fig. 1 where the fraction of a jet’s visible energy close to the jet axis is larger for quark jets than for gluon jets. All the QCD-based models describe the data very well.

![Figure 1: The differential energy profile of gluon and quark jets defined using a cone jet algorithm [1].](image)

2.2 Multiplicity Distributions and Ratios

The predicted larger soft gluon emission in gluon jets compared to quark jets has been confirmed by an observed difference between the hadron multiplicity in quark and gluon jets where the latter are found to be higher, as can be seen for example in Fig. 2 [2]. Only unbiased jets (here $g_{\text{incl.}}$) defined by particles found in the event hemispheres were used. The hemispheres are defined by the plane perpendicular to the principal event axis. There is a large theoretical interest in the ratio of the mean multiplicity of gluon and quark jets, $r = \langle N_g \rangle / \langle N_q \rangle$. This is predicted to be equal to the ratio $C_A/C_F = 2.25$ if the asymptotic condition $E_{\text{particle}} \ll E_{\text{jet}}$ is fulfilled. In real experimental conditions ($E_{\text{jet}}$ finite), the satisfaction of this condition is approached by taking
only soft particles into account [3]. In [4] soft particles in unbiased gluon and quark jets ($E_{\text{jet}} \sim 40$ GeV) were defined by momenta $p < 2.0$ GeV. In order to reduce the hadronization effects, transverse momenta of particles relative to the jet axes were required to be higher than 0.8 GeV, yielding $r = 2.32 \pm 0.18$ which agrees with the asymptotic value. The corresponding HERWIG results for $E_{\text{c.m.}} = 91$ GeV were found to be in a good agreement with the measurement. Moreover, for asymptotic $E_{\text{c.m.}} = 10$ TeV, HERWIG yielded $r = 2.25$, while JETSET set to have $C_A = C_F = 4/3$ gave $r = 1.00$.

Exploiting all the particles from finite energy jets leads to a reduced value of $r$ compared to the asymptotic one. The measured value $1.51 \pm 0.04$ from [2] is in excellent agreement with QCD calculations of this quantity [5, 6].

2.3 Scale Dependent Multiplicities and Ratios

Adopting a recently proposed method for obtaining the scale dependent unbiased gluon jet multiplicity, $N_g(Q)$ [6, 7], the ratios of multiplicities, $r$, of slopes, $r^{(1)}$, and of curvatures, $r^{(2)}$, defined as

$$r^{(1)} \equiv \frac{dN_g}{dN_q} \frac{dN_g}{dN_q}, \quad r^{(2)} \equiv \frac{d^2N_g}{d^2N_q} \frac{d^2N_g}{d^2N_q}, \quad y = \ln(Q/\Lambda), \quad Q = E_{\text{jet}}$$

were recently measured [8, 9] and compared to recent QCD calculations [5, 6, 10]. The method is based on a NLO expression for $N_{gg}$:

$$N_{gg}^{ch}(k_{\perp,Lu}) = 2[N_{qg}^{ch} - N_{qg}^{ch}(L, k_{\perp,Lu})]$$

$$N_{gg}^{ch}(k_{\perp,Le}) = 2[N_{qg}^{ch} - N_{qg}^{ch}(L_{q\bar{q}}, k_{\perp,Lu})]$$

where $N_{gg}$ is the inclusive multiplicity in 2-jet $gg$ system and $N_{qg}$ is the exclusive multiplicity in 2-jet $q\bar{q}$ events with no gluon radiation harder than $k_{\perp,Lu}$.
\( N_{q\bar{q}g} \) is the multiplicity of \( e^+e^- \) 3-jet events. The two expressions for \( N_{gg} \) reflect the ambiguity in the definition of the gluon jet \( p_\perp \) with respect to the \( q\bar{q} \) system when the gluon radiation is hard. The scales \( k_{\perp, Lu} \) and \( k_{\perp, Le} \) are proportional to

\[
p_{\perp, Lu} = \frac{s_{q\bar{q}g}}{s}, \quad p_{\perp, Le} = \frac{s_{q\bar{q}g}^2}{s_{q\bar{q}}}
\]

with \( s = E^2_{c.m.} \), \( s_{q\bar{q}} = p_qp_{\bar{q}}, \ s_{qg} = p_qp_g, \ s_{\bar{q}g} = p_{\bar{q}}p_g \) and \( p_q, p_{\bar{q}} \) and \( p_g \) the 4-momenta of the \( q, \bar{q} \) and \( g \). \( L \) specifies the \( e^+e^- \) c.m. energy (\( E_{c.m.} \)) and \( L_{q\bar{q}} \) the energy of the \( q\bar{q} \) system in the \( q\bar{q} \) rest frame. Note that the gluon jet terms depend on a single scale which corresponds to the unbiased jets, whereas the quark jet terms depend on two scales accounting for the bias in quark jet multiplicity due to the jet finder criteria used to select the \( q\bar{q}g \) events.

In order to obtain \( N_{gg} \) two event samples with jets found by Durham, Cambridge and Lucus jet finders were used. In the first sample, 3-jet light quark (uds) events\(^1\) from \( Z^0 \to q\bar{q} \) decays were kept. After energy ordering, the jet 3, having the lowest energy, is taken to be the gluon jet. This fact together with the condition \( \theta_2 \approx \theta_3 \) (the angles between the jet 1 and the other two are roughly the same, so called “Y events”) leads to a low sensitivity to gluon jet mis-identification. For Y events, the quantities depend only on \( E_{c.m.} \) and one inter-jet angle, which was conveniently chosen to be \( \theta_1 \). The measurement of \( N_{q\bar{q}g}^{ch}, L_{q\bar{q}}, k_{\perp, Lu} \) and \( k_{\perp, Le} \) is shown in Fig.3.

In the second sample, \( N_{q\bar{q}}^{ch}(L, k_{\perp, Lu}) \) from Eq.2 was directly measured as a

Figure 3: (a) The mean charged particle multiplicity of 3-jet uds flavour Y events from \( Z^0 \) decays, selected using the Durham, Cambridge and Lucus jet finders as a function of the opening angle \( \theta_1 \). (b) The corresponding scales defined in Eq.4.

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\(^1\)Theoretical expressions are based on massless quarks
mean multiplicity of 2-jet uds flavour events from $Z^0$ decays ($L = \ln(M_Z^2/\Lambda^2)$ fixed). Note that $N_{qq}^{\text{ch}}(L qq, k_{\perp, Lu})$ cannot be directly measured since $L_{qq}$, unlike $L$, is variable, so a direct measurement relevant for this analysis would require c.m. energies below the $Z^0$. Instead, the biased $N_{qq}^{\text{ch}}$ is determined from measurements of the unbiased $N_{qq}^{\text{ch}}$, using a NLO expression from [6]. Fig.4 shows that the results for $N_{gg}$ using the directly measured biased 2-quark jet multiplicity at Lund definition of $k_{\perp}$ are found consistent with the direct CLEO [11] and OPAL [4] measurements and with MC predictions (Fig.4(a)) as well as with the result using the calculated $N_{qq}^{\text{ch}}(L, k_{\perp, Lu})$ (Fig.4(b)). On the other hand, using the Leningrad definition of $k_{\perp}$, the results (Fig.4(b)) are inconsistent with MC predictions (found to have been accurate for $N_{qq}$ and $N_{gg}$ in many other studies) and also with direct CLEO (and possibly OPAL) measurements. These observations show a clear preference of the results based on Eq.2 using $k_{\perp, Lu}$ over those based on $k_{\perp, Le}$. In Fig.5 the data are compared to various QCD calculations [5, 6, 10]. For the predictions of [10] we observe that at 30 GeV, $r$ and $r^{(1)}$ exceed the data by about 20 and 6%, while $r^{(2)}$ agrees with the data. This suggests that higher order corrections are smaller for $r^{(2)}$ than for $r^{(1)}$ and for $r^{(1)}$ than for $r$. The data also confirm the prediction $r < r^{(1)} < r^{(2)} < C_A/C_F = 2.25$. For the predictions of [5] we observe a better agreement than [10] for $r$ and a similar agreement for $r^{(1)}$. The predictions of [6] are in good overall agreement with the data, however, it should be noted that these predictions are not entirely independent of the data.
Figure 5: The ratios of the mean multiplicity $r$, of slopes $r^{(1)}$ and of curvatures $r^{(2)}$ between unbiased gluon and uds quark jets as a function of scale.

### 2.4 Fragmentation Functions

The differences between quark and gluon jets manifest themselves also in the fragmentation functions, defined as

$$D(x_E, Q) \equiv \frac{1}{N_{\text{jet}}(Q)} \frac{dN_{\text{part}}(x_E, Q)}{dx_E}, \quad x_E = \frac{E_{\text{part}}}{E_{\text{jet}}}$$  \hspace{1cm} (5)

In [12] the quark and gluon fragmentation functions have been measured in udsc flavour general as well as in Y 3-jet events using Durham and Cambridge jet finders. The scale in these cases is not unambiguously defined but it should depend on $E_{\text{jet}}$ and the event topology. Studies of hadron production in events with a general topology have shown that the characteristics of the parton cascade depend mainly on the hardness of the process producing jets [13]

$$\kappa_H = E_{\text{jet}} \sin \theta / 2, \quad \theta = \text{angle to the closest jet}$$  \hspace{1cm} (6)

and accordingly, the scale in this analysis was put $Q = \kappa_H$. In Fig.6 the inclusive quark fragmentation function is compared to the gluon one. The latter is observed to be softer which can be explained by the fact that the radiation of soft gluons is larger for gluon jets and that gluon cannot be present as a valence parton inside a produced hadron (first splitting $g \rightarrow q\bar{q}$ has to occur). The scale dependence of the quark and gluon fragmentation functions
Figure 6: Quark and gluon jet fragmentation functions of $Y$ events, $\theta_2, \theta_3 \in [150^\circ \pm 15^\circ]$, compared to the predictions of various fragmentation models (Durham alg.).

Figure 7: Scale dependence of quark and gluon jet fragmentation function. Left: The data from lower energy experiments are multiplied by 0.5, since these refer to the multiplicities in $q\bar{q}$ events rather than in a single quark jet; $Q = E_{c.m.}/2$. 
is presented in Fig. 7. The figure on the left contains a summary of quark jet fragmentation function measurements. A good correspondence between the biased and unbiased measurements suggests that $\kappa_H$ is a meaningful choice of scale for a general 3-jet topology. The figure on the right shows the scaling violations of the biased gluon jets which are stronger than for quark jets. This is due to the fact that the scale dependence of the fragmentation functions for gluons is dominated by the splitting $P_{g\rightarrow gg} \sim C_A$, while that for quarks is dominated by the splitting $P_{q\rightarrow qg} \sim C_F$.

3 Conclusions

Shown examples of differences between quark and gluon jets underline the key role of the inequality $C_A > C_F$. Its consequences, namely larger widths and multiplicities as well as softer fragmentation function with stronger scaling violations of gluon jets with respect to quark jets have been confirmed experimentally.

A new method for the indirect measurement of unbiased $N_{gg}$ from biased $N_{q\bar{q}g}$ and $N_{q\bar{q}}$ was described and its usefulness proven. The results for $N_{gg}$ based on $k_{\perp,Lu}$ agree significantly better with previous measurements and MC predictions than the results based on $k_{\perp,Le}$. An overall conclusion is that the theory is in general agreement with the experimental results.

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References

[8] OPAL Coll., PN 488 (2001);