Critical fluctuations of the quark density in nuclei

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We study the static scalar susceptibility of the nuclear medium, \textit{i.e.}, the change of the quark condensate for a small modification of the quark mass. In the linear sigma model it is linked to the in-medium sigma propagator. Near the critical density for the chiral phase transition it becomes large, reflecting the presence of large spontaneous fluctuations of the quark scalar density. A partial access to this phenomenon is provided by the nuclear response for probes which couple to the nucleon density fluctuations. We show that such probes experience a critical scattering, \textit{i.e.}, an anomalous scattering in the forward direction. The data on the longitudinal response in (e,e') scattering support the existence of this phenomenon at normal nuclear densities.

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I. INTRODUCTION

The problem of the quark condensate and chiral symmetry restoration in a dense medium has been extensively addressed. Little attention instead has been given to the question of the fluctuations and correlations of the quark scalar density. However for what concerns the chiral phase transition this quantity is as relevant as the evolution of the quark condensate. Indeed the latter is the order parameter of the broken symmetry, while the scalar quark density correlator may reveal the large spontaneous fluctuations which occur near the phase transition. Based on the analogy with the ferromagnetic case the following features can be expected. Above the critical density \( \rho_c \) (or the critical temperature for chirp symmetry restoration there is no long range order: the quark condensate vanishes. But in the vicinity of the critical density \( \rho_c \) there may exist, if the phase transition is of second order, a local ordering in the form of large fluctuations of the quark scalar density. At the critical density the range of the local ordering becomes infinite. At the same time the fluctuations become static, their decay time becomes infinite. A probe which couples to the scalar quark density should display these fluctuations in the form of a critical scattering phenomenon, concentrated around the zero momentum transfer, similar to the critical scattering of neutrons by ferromagnetic materials observed in the vicinity of the Curie point [1,2]. The closer the system is to the critical density the narrower the critical scattering peak. Below the critical density the problem of the fluctuations is more complex due to the existence of the long range order superimposed to the local ordering. The range and amplitude of the fluctuations differ according to their direction, parallel or perpendicular to the direction of the spontaneous magnetization.

Nuclear matter at normal density is well below the critical density. Nevertheless chiral symmetry is appreciably restored. The order parameter decreases, as compared to its vacuum value, by about 40\%. One may then wonder if there exist also large spontaneous fluctuations of the quark scalar density and if these fluctuations can be detected in some scattering experiments. The present work adresses these questions. Our investigation is performed in an effective theory, the linear sigma model. The two fields introduced in this model to insure chiral symmetry, the pion and the sigma, have a great relevance in nuclear physics, in such a way that we can use, as a source of information, the experience acquired in this field. We investigate in particular the correlations of the quark scalar density, showing how their range increases with density. We also discuss the possibility of detection of the fluctuations through the critical scattering phenomenon.

II. FORMALISM

For an infinite system which possesses translational invariance the quark density correlator only depends on the relative space-time separation \( x \). It can be defined as the retarded Green’s function \((-i\Theta(x^0)) \langle \bar{q}(x) q(0) \rangle\). Previous investigations [3,4] have adressed the question of the in-medium four-quark condensate, \textit{i.e.}, the quantity \( \langle \bar{q}(x) q(x) \rangle \) which is a correlator taken at the same space-time point.

In the linear sigma model the symmetry breaking piece of the Lagrangian is proportional to the sigma field:

\[ \mathcal{L}_{\chi SB} = c\sigma \] (1)

with \( c = f_\pi m_\pi^2 \). This quantity plays the role of the symmetry breaking Lagrangian of QCD.
\[ \chi_{SB}^{QCD} = -2 m_q \bar{q} q \]  

(2)

where \( \bar{q} q = (\bar{u} u + \bar{d} d)/2 \), \( m_q = (m_u + m_d)/2 \) and we have neglected isospin violation. Making use of the Gell-mann-Oakes-Renner relation we obtain the following correspondence between the QCD and effective theory correlators:

\[ \frac{\langle \bar{q} q(x) \bar{q} q(0) \rangle}{\langle \bar{q} q \rangle_{vac}^2} = \frac{\langle \sigma(x) \sigma(0) \rangle}{f_\pi^2} \]  

(3)

where \( \langle \bar{q} q \rangle_{vac} \) is the vacuum value of the condensate. The fluctuations of the quark density are thus carried by the sigma field, the chiral partner of the pion. The in-medium propagation of the sigma in the energy domain near the two-pion threshold, has been the object of several investigations (see e.g. [5–7]). Here we will focus on aspects which have been largely ignored, namely the low energy region, below the particle-hole excitation energies.

The increase in the range of the correlator is reflected in the increase of the static susceptibility which becomes divergent at the critical density, a consequence of the appearance of a soft scalar mode [8,9]. In QCD the conjugate variables are the quark scalar density, which is the order parameter, and the quark mass, which is the exciting field, analogous to the magnetic field. As an illustration of this analogy, in an effective theory such as the Nambu-Jona-Lasinio (NJL) model, the constituent quark mass differs from the current one by the effect of the interaction with the condensate:

\[ M_q = m_q - 2 G_1 \langle \bar{q} q \rangle \]  

(4)

This relation presents an analogy with that between the magnetic field inside the ferromagnet and the applied one in the Weiss theory of magnetism:

\[ H = H_0 + \lambda M \]  

(5)

These two quantities differ by the existence of an internal field, \( \lambda M \), proportional to the magnetization \( M \). In NJL, the action of the quark condensate on the quark mass plays the role of the internal field of the Weiss theory. There is an equivalence between the magnetic field and the mass on the one hand, and the condensate and the magnetization on the other hand. The existence of the internal field is responsible for the divergence of the magnetic susceptibility at the Curie temperature and it is natural to expect a similar divergence for the scalar susceptibility in the NJL or in the linear sigma models.

In QCD the scalar susceptibility represents the modification of the quark condensate to a small perturbation of the quark mass:

\[ \chi_S = \frac{\partial \langle \bar{q} q \rangle}{\partial m_q} = 2 \int dt' \, dr' \, G_R(r = 0, t = 0, r', t') \]  

(6)

where \( G_R \) is the retarded quark scalar correlator:

\[ G_R(r, t, r', t') = \Theta(t - t') \langle -i [\bar{q} q(r, t), \bar{q} q(r', t')] \rangle \]  

(7)

The susceptibility represents space and time integrated correlators. In the effective theory, the quark density fluctuations are carried by the sigma field and the corresponding scalar static susceptibility is given by:

\[ \chi_S = -2 \frac{\langle \bar{q} q \rangle_{vac}^2 \partial \langle \sigma \rangle}{f_\pi^2 \partial c} \]  

(8)

with:

\[ - \frac{\partial \langle \sigma \rangle}{\partial c} = \int_0^\infty d\omega \left( \frac{2}{\pi \omega} \right) Im D_{SS}(q = 0, \omega) = Re D_{SS}(q = 0, \omega = 0) \]  

(9)

where \( D_{SS}(q, \omega) \) is the Fourier transform of the scalar correlator:

\[ D_{SS}(q, \omega) = \int dt \, dr \, e^{i q \cdot r} e^{-i \omega} \{ -i T \left( \sigma(r, t) - \langle \sigma \rangle, \sigma(0) - \langle \sigma \rangle \right) \} \]  

(10)

We have replaced the retarded Green’s function by the time-ordered one which is identical for positive frequencies. One can also define a momentum dependent susceptibility according to:
\[
\chi_s(\mathbf{q}) = 2 \frac{(g \bar{q} q)^2}{f_\pi^2} \text{Re} D_{SS}(\mathbf{q}, \omega = 0)
\]

The range of the fluctuations of the quark density is thus given by the one of the sigma field. Notice that in the phase of broken symmetry, which is the case at ordinary densities, these fluctuations correspond to longitudinal ones, \textit{i.e.}, along the direction of the spontaneous ordering which is that of the scalar field. In a very simple picture where the sharp sigma mass is reduced in the nuclear medium, \textit{i.e.} \(m_\sigma\) replaced by some dropped value, \(m_\sigma^*\), as has been suggested by Hatsuda \textit{et al.} \[6\], the static correlator is \(\exp(-m_\sigma^* r)/r\). Hence, as \(m_\sigma^*\) goes to zero at full chiral symmetry restoration, the fluctuations acquire an infinite range as for fluids near the critical temperature. We will elaborate this concept and address the question of its experimental display. In this respect, we point out that the scalar propagator is not easily accessible directly but only through the scattering of probes which couple to the nucleon scalar density. In such a situation what is measured is the scalar polarization propagator which is not the same as the scalar field propagator. In the following we will show, using the standard \(\sigma - \omega\) model, how these two quantities are related. In addition, we also introduce the isoscalar vector polarization propagator accessible with an electromagnetic probe, showing that part of the relevant information can be obtained from this quantity.

When we deal with the nuclear system, the fact that the individual nucleons are not inert since they respond to the scalar field, \textit{i.e.}, to a change in the quark mass, is the crucial ingredient of the quark-meson coupling model (QMC). The corresponding scalar susceptibility of the nucleon is of the diamagnetic type which tends to lower the sigma field.

\[
\sigma, \omega
\]

The sharp sigma mass is reduced in the nuclear medium, \textit{i.e.} \(\Sigma\) limits. In order to simplify the formalism we ignore this distinction and omit the influence of \(\Sigma\) through a local interaction, into two pions. However, in the RPA ring approximation, the scalar meson entering the two-pion loop, \(\Sigma\), which affects the sigma propagator \[11\]. Indeed the sigma which enters the description of the nucleon meson, how these two quantities are related. In the formal discussion we will assume for simplicity that the nucleons are inert: only nuclear excitations enter. The individual response of the nucleons will be introduced later without difficulty.

Let us call \(g_S\) and \(g_V\) the \(\sigma\)NN and \(\omega\)NN coupling constants and \(D^0_{SV}\) the bare sigma and omega propagators. We define the bare scalar, vector and mixed polarization propagators, \(\Pi^0_{SS}, \Pi^0_{VV}\) and \(\Pi^0_{SV}\), which include the coupling constants. In the non relativistic limit they simply reduce to:

\[
\Pi^0_{SS} = g^2_S \Pi^0, \quad \Pi^0_{VV} = g^2_V \Pi^0, \quad \Pi^0_{SV} = \Pi^0_{VS} = gV \Pi^0
\]

where \(\Pi^0\) is the standard Lindhardt function which depends on \(q = (\omega, \mathbf{q})\). For completeness we should also introduce the two-pion loop, \(\Sigma_2\), which affects the sigma propagator \[11\]. Indeed the sigma which enters the description of the quark condensate is the chiral partner of the pion and is coupled to the squared pion field. It can thus transform, through a local interaction, into two pions. However, in the RPA ring approximation, the scalar meson entering the NN interaction has to be a chiral invariant \[12,13\] which is not coupled to the squared pion field (in the chiral limit). In order to simplify the formalism we ignore this distinction and omit the influence of \(\Sigma_2\) which becomes important at higher energies. We do not include either the vacuum polarization effects \[14\] keeping only the nucleon-hole excitations. In the RPA ring approximation we utilize for the particle-hole interaction in the isoscalar channel the \(\sigma - \omega\) model \[15,16\]. The dressed propagators (with the same notations as previously with the zero index suppressed) obey the following coupled equations:

\[
D_{SS} = D^0_S + D^0_S \Pi^0_{SS} D_{SS} + D^0_S \Pi^0_{SV} D_{VS}
\]

\[
D_{VS} = -D^0_V \Pi^0_{VS} D_{SS} - D^0_V \Pi^0_{VV} D_{VS}
\]

\[
\Pi_{SS} = \Pi^0_{SS} + \Pi^0_{SS} D^0_S \Pi_{SS} - \Pi^0_{SV} D^0_V \Pi_{VS}
\]

\[
\Pi_{VS} = \Pi^0_{VS} + \Pi^0_{VS} D^0_S \Pi_{SS} - \Pi^0_{VV} D^0_V \Pi_{VS}
\]

In this set of equations we have restricted our study to the space-like region, with \(|\mathbf{q}| \gg \omega\), and thus ignored terms in \(\omega^2/|\mathbf{q}|^2\) which appear in the \(\omega\) propagator. The solutions are:

\[
D_{SS} = D^0_S \frac{1 + D^0_V \Pi^0_{VV}}{1 - \Pi^0_{SS} D^0_S + \Pi^0_{VV} D^0_V + D^0_S D^0_V (\Pi^0_{SS} \Pi^0_{VS} - \Pi^0_{SS} \Pi^0_{VV})}
\]

and

\[
\Pi_{SS} = \frac{\Pi^0_{SS} - D^0_V (\Pi^0_{SV} \Pi^0_{VS} - \Pi^0_{SS} \Pi^0_{VV})}{1 - \Pi^0_{SS} D^0_S + \Pi^0_{VV} D^0_V + D^0_S D^0_V (\Pi^0_{SS} \Pi^0_{VS} - \Pi^0_{SS} \Pi^0_{VV})}
\]

They are related by:

\[
D_{SS} = D^0_S (1 + D^0_V \Pi_{SS})
\]
In the non-relativistic limit, they reduce to:

\[
D_{SS} = D_S^0 \frac{(1 + g_\Sigma^2 D_S^0 \Pi^0)}{1 - (g_\Sigma^2 D_S^0 - g_\Sigma^0 D_V^0) \Pi^0}
\]

(18)

\[
\Pi_{SS} = g_\Sigma^2 \frac{\Pi^0}{1 - (g_\Sigma^2 D_S^0 - g_\Sigma^0 D_V^0) \Pi^0}
\]

(19)

Similarly, the full vector and mixed propagators obey the following coupled equations:

\[
\Pi_{VV} = \Pi_{VV}^0 - \Pi_{VV}^0 D_V^0 \Pi_{VV} + \Pi_{VS}^0 D_S^0 \Pi_{SV}
\]

\[
\Pi_{SV} = \Pi_{SV}^0 + \Pi_{SV}^0 D_S^0 \Pi_{SV} - \Pi_{SV}^0 D_V^0 \Pi_{VV}
\]

(20)

The solution for \(\Pi_{VV}\) is obtained by interchanging the \(S\) and \(V\) symbol with however the interchange of \((-D_V^0)\) and \(D_S^0\), such that for instance:

\[
\Pi_{VV} \approx \frac{\Pi_{VV}^0 - D_S^0}{1 - \Pi_{SS}^0 D_S^0 + \Pi_{VV}^0 D_V^0 + D_S^0 D_V^0 (\Pi_{SV}^0 - \Pi_{SS}^0 \Pi_{VV}^0)} \Pi^0
\]

(21)

In the non-relativistic case, which applies if the density is not too large, \(\Pi_{VV}\) is identical, up to a coupling constant factor, to \(\Pi_{SS}\), hence opening the possibility of experimental information since the vector response is more accessible than the scalar one.

At this level, some comments are in order. The many-body effects are embodied in the RPA modification of \(D_{SS}\) and \(\Pi_{SS}\). We stress again that they are not the same for these two quantities. For instance, the first order expansion in the density of \(D_{SS}\) only involves the scalar field propagator, while for \(\Pi_{SS}\) both scalar and vector fields propagators enter. However the singularities are the same and given by the zeros of the common denominator. They are influenced by the scalar exchange but also by the vector one, owing to the in-medium \(\sigma - \omega\) mixing. Approaching the chiral phase transition, \(i.e.,\) a singularity of the RPA solution, the softening of the scalar-isoscalar modes should translate into an enhanced static susceptibility associated to large fluctuations.

We now comment the nature of these RPA effects. Although the above results are valid at any value of \(\omega\) and \(q\), we concentrate here on the static case \((\omega = 0)\) and for illustration, we use the non-relativistic expressions. In the space-like region the bare meson propagators are negative. As for \(\Pi^0\) which is purely real in the static limit, it is also negative:

\[
Re\Pi^0(q, 0) = \int_0^\infty d\omega \left( \frac{2}{\pi \omega} \right) Im\Pi^0(q, \omega) = -2 \rho \int_0^\infty d\omega \frac{1}{\omega} R^0(q, \omega) = -2 \rho S^{-1}(q)
\]

(22)

where \(\rho\) the nucleon density, \(R^0(q, \omega)\) the bare response function per nucleon and \(S^{-1}(q)\) is the inverse energy-weighted sum rule as defined by the last identity. For a Fermi gas with Fermi momentum \(k_F\), \(Re\Pi^0(q, 0)\) is:

\[
Re\Pi^0(q, \omega = 0) = \frac{-M_N k_F}{\pi^2} \left[ 1 + \frac{1}{t} \left( 1 - \frac{t^2}{4} \right) \ln \left( \frac{1 + t/2}{1 - t/2} \right) \right]
\]

(23)

At low momentum, it is a slowly varying function of \(t = |q|/k_F\):

\[
Re\Pi^0(q, \omega = 0) \simeq \frac{2 M_N k_F}{\pi^2} \left( 1 - \frac{t^2}{12} \right)
\]

(24)

Products of the type \(D^0(q, \omega = 0)\Pi^0(q, \omega = 0)\) are thus positive. As for the difference between the scalar and vector exchange which appears in the RPA denominator, it is linked, in the \(\sigma - \omega\) model, to the mean nuclear potential which is attractive. The RPA denominator thus leads to an enhancement effect. For \(D_{SS}\) the enhancement is further magnified by the numerator of the RPA solution of eq.18. In fact, in the first order expansion in density (or in \(\Pi^0\)) of \(D_{SS}\), the numerator cancels the repulsive effect from the vector exchange, leaving only the attractive scalar exchange which leads to large enhancement effects. While instead both scalar and vector exchanges enter in the expansion of \(\Pi_{SS}\), with a cancellation between the two: the enhancement is more moderate. At the critical density, \(\rho_c\), the RPA
denominator vanishes for \((q = 0, \omega = 0)\), the susceptibility diverges and the inelastic cross-section is singular in the forward direction. The increase in the vicinity of \(\rho_c\) of this cross-section in the near forward direction is the critical scattering phenomenon.

\[ \text{FIG. 1. From [23]: RPA charge longitudinal response (full line) in } ^{12}\text{C at } q = 200 \, \text{MeV/c. Long dash-short dash line is the independent particle response. The isoscalar and isovector partial contributions are also displayed. The experimental points are from [19].} \]

\[ \text{FIG. 2. Same as fig.1 but for } ^{40}\text{Ca at } q = 300 \, \text{MeV/c.} \]

III. DISCUSSION

We have seen that the RPA effects increase the magnitude of the quantity \(\text{Re} D_{SS}(q = 0, \omega = 0)\), i.e., of the static scalar susceptibility. We now introduce some numerical values, in order to get an idea of the expected magnitude of the enhancement factor. For this evaluation we ignore the modification of the sigma mass by the tadpole terms (\([6,13]\)), supposed to be incorporated in the empirical value of \(m_\sigma\) derived from the nuclear binding. We use the full relativistic expressions of eq. 21, discarding however the terms involving the small difference \(\Pi_{SV}^0 \Pi_{VS}^0 - \Pi_{SV}^0 \Pi_{VV}^0\).

We take for the bare polarization propagators at \(\omega = 0\), the following ansatz:

\[ \Pi_{SS}^0 = -2 g_S^2 \rho_S S^{-1}, \quad \Pi_{VV}^0 = -2 g_V^2 \rho S^{-1} \]

where \(\rho_S\) is the nucleon scalar density. The RPA denominator can thus be written in terms of the mean scalar and vector field, \(\langle \sigma \rangle\) and \(\langle \omega \rangle\), as:

\[ 1 - D^0_S \Pi_{SS}^0 + D^0_V \Pi_{VV}^0 \simeq 1 + 2 (g_V \langle \omega \rangle - g_S \langle \sigma \rangle) S^{-1}(q = 0) \]

\[ (26) \]
At this stage it is easy to incorporate the nucleon reaction to the sigma field. We take the value of the mean scalar field from the QMC model [17]. It grows more slowly with density than the mean vector field which has a linear dependence, owing to the scalar character of the density which enters in it but also to the reaction of the nucleons to the scalar field which is thus automatically incorporated in the value of $\langle \sigma \rangle_\rho$. The corresponding values from [17] at normal nuclear matter density $\rho_0$ are: $g_S \langle \sigma \rangle = 200$ MeV and $g_V \langle \omega \rangle = 152$ MeV. For the quantity $S^{-1}(q = 0)$ we take the free gas value defined by the eq. 22 and 24. Instead of the free nucleon mass we insert in it the effective value of $\chi$ model for the scalar susceptibility, which is an extra enhancement factor of $\simeq 40$ at $q = 300$ MeV/c, for instance, can be estimated to be $\simeq 2.5$. A theoretical evaluation of the corresponding enhancement factor arising from the RPA denominator, with the inputs from ref [17] and taking into account the momentum dependence of the $\sigma$ and $\omega$ exchanges gives a factor of $\simeq 3.6$ at $\rho_0$, roughly consistent with the “experimental” one. We remind that this number concerns the response. For the sigma propagator itself there is an extra enhancement factor of $\simeq 5$ from the RPA numerator. The total enhancement expected in the effective model for the scalar susceptibility, $\chi_S(q) \propto ReD_{SS}(q, \omega = 0)$, at a momentum of $300$ MeV/c, is then of $\simeq 15$. As $Re\Pi^0(q, \omega = 0)$ evolves slowly with density (as $k_F$), it should also be the case for the RPA enhancement.

The data also show that the collective character of the response is more pronounced at smaller momenta, hence the enhancement of the inverse energy weighted sum rule larger. The distortion gradually disappears at $\simeq 500MeV/c$. These features are qualitatively consistent with the existence of a critical scattering phenomenon peaked, as expected, at zero momentum transfer. Notice that the existence of critical fluctuations of the quark density does not imply a new type of three-body forces. The fluctuations arise from a mere rescattering of the sigma on the nucleons without intermediate excitation of the nucleon. The in-medium NN potential is thus unaffected. Moreover the mixing of $NN^{-1}$ excitations with the sigma has no influence either on the scalar spectral function at large energies, in the vicinity of the two-pion threshold, since both $Re\Pi^0(q, \omega)$ and $Im\Pi^0(q, \omega)$ vanish for $\omega$ larger than the particle-hole excitation energies.

In summary we have studied the scalar response of the nuclear medium, defined as the modification of the quark condensate in response to a small perturbation of the quark mass. In the linear sigma model it is linked to the sigma propagator. We have shown that at small momenta the nucleus responds collectively to the perturbation. A fluctuation
of the quark scalar density is carried by the sigma meson and relayed by the neighboring nucleons, so that it acquires a larger range than in free space. At the critical density for the chiral transition, which corresponds to the singularity of the RPA solution, the range becomes infinite and the susceptibility diverges. This phenomenon is associated with the existence of critical scattering for probes which couple to the quark density fluctuations. Since such are not readily available we can substitute for them probes which have a coupling to the nucleon scalar density fluctuations. They experience as well the critical scattering phenomenon. We have shown that, at small and moderate densities, the isoscalar part of the longitudinal response in (e, e') scattering, which couples to the nucleon scalar density fluctuations, is a good substitute to the former response, as long as the relativistic corrections are moderate. The data on the longitudinal response in (e, e') scattering appear to have a collective character, attributed to the isoscalar response. We have given a tentative interpretation of this collective nature as a critical scattering phenomenon linked to large fluctuations of the quark scalar density. However very small momentum data are lacking for the full exploration of the critical scattering. Nor can we follow the evolution at higher densities. It is possible that with increasing density, i.e., closer to the chiral phase transition, more resistance to the increase in the fluctuations develops (such as the nucleonic reaction to the scalar field). Nevertheless, if this interpretation, which is based on the effective model, is correct it naturally leads to conclude that the scalar susceptibility at small momenta is appreciably enhanced in the nuclear medium. It seems that critical fluctuations of the quark scalar density are already present at ordinary nuclear densities, even if the decrease of the order parameter, which is the quark condensate, is only moderate.

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