Supersymmetric corrections to Higgs decays and $b \to s\gamma$ for large $\tan \beta$\textsuperscript{a}

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If $\tan \beta$ is large, supersymmetric QCD corrections can become large, putting naive perturbation theory into doubt. I show how these $\tan \beta$-enhanced corrections can be controlled to all orders in $\alpha_s \tan \beta$. The result is shown for the decays $H^+ \to \bar{t}b$ and $b \to s\gamma$.

1 Large corrections to all orders

The Minimal Supersymmetric Standard Model (MSSM) contains two Higgs doublets $H_u$ and $H_d$. Their neutral components acquire the vacuum expectation values $v_u$ and $v_d$ with $v \equiv \sqrt{v_u^2 + v_d^2} = 174$ GeV. Recently scenarios with large $\tan \beta \equiv v_u/v_d$ (corresponding to $v_d \ll v_u \simeq 174$ GeV) have attracted increasing attention: this region of the parameter space is experimentally least constrained by the bounds from Higgs searches at LEP\textsuperscript{1}. A theoretical motivation for large $\tan \beta$ scenarios stems from GUT theories with bottom–top Yukawa unification, which require $\tan \beta = \mathcal{O}(50)^{2,3}$. At tree-level right-handed down-quark fields do not couple to $H_u$ and the bottom quark mass is related to the corresponding Yukawa coupling $h_b$ by $m_b = h_b v_d = h_b v \cos \beta$. For large $\tan \beta$ this has two important consequences: first $h_b$ is large, of order 1. Second, radiative corrections to the couplings of Higgs bosons to $b$ quarks proportional to $h_b \sin \beta$ can occur. These are enhanced by a factor of $\tan \beta$ compared to the tree-level result and stem from the supersymmetry breaking terms. The dominant $\tan \beta$-enhanced corrections are supersymmetric QCD (SQCD) contributions, i.e. loop diagrams with squarks and gluinos.

Observables can be affected by these large corrections in two ways:

I The leading order contribution is proportional to $\cot \beta$. This suppression is lifted in the loop corrections of the next-to-leading order (NLO).

II $\tan \beta$-enhanced corrections enter the counterterms, which appear in higher order corrections.

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An example for type-I corrections is the $H^+ T_{RSL}$ coupling $\propto m_t \cot \beta$ appearing in the one-loop matrix element for $b \to s\gamma$. SQCD vertex corrections are not suppressed by $\cot \beta$, so that the two-loop matrix element is $\tan \beta$-enhanced. Clearly, three- or more loops cannot produce more factors of $\tan \beta$, because the bare lagrangian contains $h_b$ and $\beta$ only in the combinations $h_b \cos \beta$ and $h_b \sin \beta$. The enhancement mechanism of type-II is related to the renormalization of $h_b$. SQCD corrections induce a counterterm to $h_b^{\text{tree}} = m_b/(v \cos \beta)$ which reads $\delta h_b = \delta m_b/(v \cos \beta)$ in terms of the mass counterterm $\delta m_b$. Also $\delta m_b$ contains terms proportional to $h_b \sin \beta$, so that $\delta h_b$ is $\tan \beta$-enhanced. Writing $\delta m_b = -m_b \Delta m_b$ one finds

$$h_b = h_b^{\text{ren}} + \delta h_b = h_b^{\text{tree}} (1 - \Delta m_b)$$

at the one-loop level. In this talk I consider the on-mass-shell renormalization scheme, in which $\delta m_b$ is adjusted to cancel the on-shell self-energy of the $b$ quark. For other schemes I refer to \(^4\). Then $\Delta m_b$ reads\(^3\):

$$\Delta m_b = \frac{2\alpha_s}{3\pi} M_{\tilde{g}} \mu \tan \beta I(m_{b_1}, m_{b_2}, M_{\tilde{g}}) \approx \frac{\alpha_s}{3\pi} \frac{\mu M_{\tilde{g}}}{M_{\tilde{g}, b_{1,2}}} \tan \beta.$$  

(2)

Here $m_{b_{1,2}}$ and $M_{\tilde{g}}$ are the $b$ squark and gluino masses and $\mu$ is the Higgsino mass parameter. The last formula in (2) is approximate with $M_{\tilde{g}, b_{1,2}}$ being the average of $m_{b_1}, m_{b_2}$ and $M_{\tilde{g}}$. The exact formula contains\(^3\)

$$I(a, b, c) = \frac{1}{(a^2 - b^2)(b^2 - c^2)(a^2 - c^2)} \left( a^2 b^2 \log \frac{a^2}{b^2} + b^2 c^2 \log \frac{b^2}{c^2} + c^2 a^2 \log \frac{c^2}{a^2} \right).$$  

(3)

By inspecting the one-loop SQCD corrections to Higgs and top decays calculated in\(^5\) one verifies that their $\tan \beta$-enhanced portion indeed stems from the counterterm diagram involving $\delta h_b$. The next-to-leading order result for $b \to s\gamma$\(^6\) in addition contains type-I corrections from the loop-corrected $H^+ T_{RSL}$ vertex.

In\(^3\) the type-II corrections have been derived in a different way, by investigating the loop-induced coupling of $b$ quarks to the other Higgs doublet $H_u$. This amounts to replacing $h_b$ by

$$h_b^{\text{eff}} = \frac{h_b^{\text{tree}}}{1 + \Delta m_b}.$$  

(4)

The scope of all analyses\(^3,5,6\) were one-loop corrections to $h_b$, and obviously (1) and (4) agree up to terms of order $(\Delta m_b)^2 = \mathcal{O}(\alpha_s^2)$. The two methods are related by a Ward identity. Since $\Delta m_b$ can be of order 1, there can be drastic numerical differences between (1) and (4). At first glance this raises doubts whether perturbation theory works if $\tan \beta$ is large.

In\(^4,7\) it has been investigated how higher orders are affected by these enhanced SQCD corrections: unlike type-I corrections the type-II corrections from $\delta h_b$ in (1) appear recursively in all orders of perturbation theory. $\delta h_b$ also enters the renormalized $\tilde{b}_L - \tilde{b}_R$ mixing in the $b$ squark mass matrix. As a consequence enhanced contributions to $\delta m_b$ at order $\alpha_s^n$ appear from one-loop self-energy diagrams in which a $\tilde{b}_L - \tilde{b}_R$ flip stems from the $\mathcal{O}(\alpha_s^{n-1})$ term of $\delta h_b$\(^4\). The enhanced higher order corrections simply sum to a geometric series\(^4\):

$$h_b = h_b^{\text{ren}} + \delta h_b = h_b^{\text{tree}} \left( 1 - \Delta m_b + (\Delta m_b)^2 - (\Delta m_b)^3 + \ldots \right) = \frac{h_b^{\text{tree}}}{1 + \Delta m_b}.$$  

(5)

The procedure of\(^3\) directly arrives at this result, as evidenced by (4). Indeed, $\tan \beta$-enhanced corrections are absent in the higher order contributions to the loop-induced $\tilde{b}bH_u$ vertex. The proof of this feature and the establishment of the geometric series in (5) involves power counting, the operator product expansion and standard infrared theorems\(^4\).
Next I present the effect of the resummation in (5) on two decay rates. The first plot shows the relative size of the SQCD corrections in the decay rate $\Gamma(H^+ \to t\bar{b})$ for two sets of parameters: $M_{H^+} = 350$ GeV, $M_{\tilde{b}} = 500$ GeV, $M_{\tilde{t}_2} = 200$ GeV (< $M_{\tilde{t}_1}$), $M_{\tilde{t}_2} = 180$ GeV (< $M_{\tilde{t}_1}$) (“light”) and $M_{\tilde{b}} = M_{\tilde{b}_2} = M_{\tilde{t}_2} = 1000$ GeV (“heavy”) 4:

The short-dashed line corresponds to the one-loop result of 5. The long-dashed curve is obtained by retaining only the $\tan \beta$-enhanced contribution from $\delta h_b$. The solid line supplements the result of 5 with the resummation of the large corrections.

SUSY constraints from $b \to s\gamma$ are highly model dependent. In the constrained MSSM with $M_{H^+} = 200$ GeV, $M_{\tilde{t}_2} = 250$ GeV, $M_2 = M_{\tilde{t}_1} = M_{\tilde{b}} = 800$ GeV and $\pm \mu = A_t = 500$ GeV we find 7:

Here the dashed curve is the next-to-leading order result of 6 and the solid curve represents the resummed prediction for the decay rate. The shaded area is the experimentally 9 allowed range $BR(b \to s\gamma) = (3.14 \pm 0.48) \times 10^{-4}$. Using the resummed result, we can determine the region of the $(M_{H^+}, M_{\tilde{\chi}_2^+})$-plane excluded by $BR(b \to s\gamma)$. In the following plot we have scanned over $m_{\tilde{t}_2} < m_{\tilde{t}_1} \leq 1$ TeV, $m_{\tilde{\chi}_2^+} < m_{\tilde{\chi}_1^+} \leq 1$ TeV and $|A_t| \leq 500$ GeV. The four lines correspond to
two values of \( \tan \beta \) and of the lighter stop mass as indicated in the plot. The region to the left of the corresponding line is excluded\(^7\):

![Graph showing the relationship between \( M_{\chi^2} \) and \( M_{H^+} \) with \( \tan \beta = 10 \) and stop masses of 100 GeV and 300 GeV.]

Similar results have been obtained in\(^8\).

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References

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