KAON PHYSICS
WITH A HIGH-INTENSITY PROTON DRIVER

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Abstract

We study opportunities for future high-precision experiments in kaon physics using a high-intensity proton driver, which could be part of the front-end of a muon storage ring complex. We discuss in particular the rare decays \(K_L \rightarrow \pi^0 \nu \bar{\nu}\), \(K^+ \rightarrow \pi^0 \nu \bar{\nu}\), \(K_L \rightarrow \pi^0 e^+ e^-\), and lepton-flavour violating modes such as \(K_L \rightarrow \mu e\) and \(K \rightarrow \pi \mu e\). The outstanding physics potential and long-term interest of these modes is emphasized. We review status and prospects of current and planned experiments for the processes under consideration, and indicate possible improvements and strategies towards achieving the necessary higher sensitivity. Finally, we outline the machine requirements needed to perform these high-precision kaon experiments in the context of a muon storage ring facility.

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We study opportunities for future high-precision experiments in kaon physics using a high-intensity proton driver, which could be part of the front-end of a muon storage ring complex. We discuss in particular the rare decays $K_L \rightarrow \pi^0 \nu \bar{\nu}$, $K^+ \rightarrow \pi^0 \nu \bar{\nu}$, $K_L \rightarrow \pi^+ e^+ e^-$, and lepton-flavour violating modes such as $K_L \rightarrow \mu e$ and $K \rightarrow \pi \mu e$. The outstanding physics potential and long-term interest of these modes is emphasized. We review status and prospects of current and planned experiments for the processes under consideration, and indicate possible improvements and strategies towards achieving the necessary higher sensitivity. Finally, we outline the machine requirements needed to perform these high-precision kaon experiments in the context of a muon storage ring facility.

1 INTRODUCTION

G. Buchalla

1.1 Preliminary remarks
There are essentially two frontiers in particle physics: one striving for higher energies to access new degrees of freedom directly, the other aiming for higher precision in the study of rare processes. Each addresses the problem of achieving a better understanding of fundamental interactions from a different perspective and in a complementary way, and both are indispensable to obtain the complete picture.

The indirect method of high-precision measurements at low energies is exemplified by the detailed study of kaon decays, which has proved to be of the utmost importance for the development of particle physics. Among the landmark results in this field have been the concept of strangeness, leading to the quark model, which in turn provided the basis for QCD; the first hint of parity violation, pointing the way to the chiral nature of the weak gauge forces; the violation of CP, defining an absolute matter-antimatter asymmetry and a subtle connection to the three-generation structure of matter; and, last but not least, the characteristic suppression pattern for flavour-changing neutral currents (FCNC) in $K_L \rightarrow \mu^+ \mu^-$ or $K^0 - \bar{K}^0$ mixing, which suggested the charm quark and the GIM structure of flavour dynamics.

These examples illustrate how rare processes with kaons can provide indirect and yet crucial information on fundamental physics that is difficult to access in any other way. The fact that kaons can be copiously produced and that they have rather long lifetimes, with a large hierarchy between $K_L$ and $K_S$, are key elements in this respect.

Beyond the achievements of the past kaon physics offers clear perspectives for future progress. Prominent opportunities are the rare decays $K \rightarrow \pi \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$ or $K_L \rightarrow \mu e$. These processes
allow us to perform high-precision tests of Standard Model (SM) flavour physics, including the CKM mechanism for CP violation, and define very sensitive probes of new physics. The pursuit of these investigations is important because the physics of flavour represents the least understood and least tested sector of the SM. Kaon probes are necessary to complement both direct searches for new physics as well as the results on flavour dynamics from $b$ physics and similar fields.

The purpose of this document is to review particular possibilities in kaon physics with a long-term perspective. These require, on the one hand, a longer-term experimental effort, but will yield, on the other, results of the highest interest, which are competitive even on this longer time scale. Their interest may be an important element in the choice of the energy of the proton driver for muon storage rings. For this reason, in the first part of this document we highlight and motivate the cases of particular interest for the very high intensity kaon beams that could be obtained from such a proton driver. These have been the focus at this workshop, and we shall elaborate briefly on the studies performed with this perspective.

In the second part of this article, we discuss the experimental requirements for this type of precision kaon physics, including strategies for the detection of the rare modes under consideration, a comparison with related efforts at other laboratories, and accelerator aspects. We finally present a summary and our conclusions.

### 1.2 Overview

In Table 1 we list a selection of rare kaon processes that are of special interest for future studies of flavour physics. The chosen examples are intended to represent the typical categories one might distinguish for rare kaon decays with sensitivity to short-distance flavour dynamics: tests of the SM FCNC, the CKM mechanism and CP violation; searches for (charged-)lepton-flavour violation; probes of discrete symmetries by measuring the polarization of muons from $K$ decays. All three classes might reveal the presence of new physics, the first class by yielding results for SM parameters in conflict with other determinations, the latter two by merely giving a non-zero signal above the (exceedingly small) level expected in the SM. More examples have been discussed in the literature. For further information we refer to the review articles [1, 2, 3, 4, 5, 6]. Here we would like to emphasize some decays where the theoretical motivation is particularly strong, which have been the main focus of the present study:

- $K_L \rightarrow \pi^0 \nu \bar{\nu}$, (and also $K^+ \rightarrow \pi^+ \nu \bar{\nu}$)
- $K_L \rightarrow \pi^0 e^+ e^-$
- $K_L \rightarrow \mu e$

The significance of these modes is such that they are sufficient to motivate a dedicated kaon physics program. Further opportunities may then also be pursued later on, once a kaon facility is in place.

**Table 1: Overview of rare kaon decays.** (The SM expectation quoted for $B(K_L \rightarrow \pi^0 e^+ e^-)$ refers only to the contribution from direct CP violation, while the experimental limit refers to the total branching fraction.)

<table>
<thead>
<tr>
<th>Observable</th>
<th>Physics</th>
<th>SM</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(K_L \rightarrow \pi^0 \nu \bar{\nu})$</td>
<td>CPV, $\eta, J_{CP}$</td>
<td>$(2.8 \pm 1.1) \times 10^{-11}$ [7]</td>
<td>$&lt; 5.9 \times 10^{-1}$ [8]</td>
</tr>
<tr>
<td>$B(K^+ \rightarrow \pi^+ \nu \bar{\nu})$</td>
<td>$</td>
<td>V_{td}</td>
<td>$</td>
</tr>
<tr>
<td>$B(K_L \rightarrow \pi^+ \pi^- \nu \bar{\nu})$</td>
<td>CKM</td>
<td>$(2.8 \pm 0.8) \times 10^{-13}$ [10]</td>
<td>—</td>
</tr>
<tr>
<td>$B(K^+ \rightarrow \pi^+ \pi^0 \nu \bar{\nu})$</td>
<td>CKM</td>
<td>$(1 - 2) \times 10^{-14}$ [10]</td>
<td>$&lt; 4.3 \times 10^{-9}$ [11]</td>
</tr>
<tr>
<td>$B(K_L \rightarrow \pi^0 e^+ e^-)$</td>
<td>CPV, $\eta, J_{CP}$</td>
<td>$(4.6 \pm 1.8) \times 10^{-12}$ [7]</td>
<td>$&lt; 5.1 \times 10^{-10}$ [12]</td>
</tr>
<tr>
<td>$B(K_L \rightarrow \mu e)$</td>
<td>LFV, new physics</td>
<td>—</td>
<td>$&lt; 4.7 \times 10^{-12}$ [13]</td>
</tr>
<tr>
<td>$B(K^+ \rightarrow \pi^+ \mu^+ e^-)$</td>
<td>LFV, new physics</td>
<td>—</td>
<td>$&lt; 2.8 \times 10^{-11}$ [14]</td>
</tr>
<tr>
<td>$B(K_L \rightarrow \pi^0 \mu e)$</td>
<td>LFV, new physics</td>
<td>—</td>
<td>$&lt; 4.4 \times 10^{-10}$ [15]</td>
</tr>
<tr>
<td>$B(K^+ \rightarrow \pi^+ \mu^+ \mu^+)$</td>
<td>$\Delta L_\mu = 2$, new physics</td>
<td>—</td>
<td>$&lt; 3.0 \times 10^{-9}$ [16]</td>
</tr>
<tr>
<td>$P^+(K^+ \rightarrow \pi^0 \mu^+ \nu)$</td>
<td>new CPV scalar int.</td>
<td>$\sim 10^{-6}$</td>
<td>$&lt; 5 \times 10^{-3}$ [17]</td>
</tr>
</tbody>
</table>
2 THE GOLDEN MODES: $K_L \rightarrow \pi^0 \nu \bar{\nu}$ AND $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

G. Buchalla, T. Hurth

The decay modes $K \rightarrow \pi \nu \bar{\nu}$ are flavour-changing neutral current transitions, which are induced in the SM at one-loop order through the diagrams shown in Fig. 1.

The rare decay $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is one of the most attractive processes to study the physics of flavour. In particular, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is a manifestation of large direct CP violation in the SM. A small effect from indirect CP violation related to the kaon $\varepsilon$ parameter contributes less than $\sim 1\%$ to the branching ratio, and is therefore negligible. In addition, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ can be calculated as a function of fundamental SM parameters with exceptionally small theoretical error. The main reasons are the hard GIM suppression of long-distance contributions [18, 19], and the semileptonic character, which allows us to extract the hadronic matrix element $\langle \pi^0 | (\bar{s}d) | K^0 \rangle$ from $K^+ \rightarrow \pi^0 \nu \bar{\nu}$ decay using isospin symmetry. As a consequence, the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ amplitude is based on a purely short-distance-dominated flavour-changing neutral current matrix element, which is reliably calculable in perturbation theory. The CP properties help to improve further the theoretical accuracy, rendering even the charm contribution completely negligible, so that the clean top contribution fully dominates the decay. Next-to-leading QCD effects have been calculated and reduce the leading-order scale ambiguity of $\sim \pm 1\%$ to an essentially negligible $\sim \pm 1\%$ [20]. Isospin-breaking corrections in the extraction of the matrix element have also been evaluated, and lead to an overall reduction of the branching ratio by 5.6% [21]. Uncertainties from higher-order electroweak corrections are likewise at the level of a percent [22]. In total, the theoretical uncertainty in $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is below 3%. Consequently, on the order of a 1000 background-free events could still be used without being limited by theoretical uncertainties. The theoretically very clean relationship between the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ branching fraction and fundamental SM parameters reads

$$B(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 1.80 \times 10^{-10} \left( \frac{\text{Im} \lambda_1}{\lambda^5} X(x_t) \right)^2 = 4.16 \times 10^{-10} \eta^2 A^4 \frac{\left( \bar{m}_t(m_t) / 167\text{GeV} \right)^2}{10^3}$$

(1)

Here $\lambda_1 \equiv V_{ts}^* V_{td}$; $\lambda = 0.22$, $A \equiv V_{cb} / \lambda^2$ and $\eta$ are Wolfenstein parameters of the CKM matrix, and $X(x_t)$ is a function of the top-quark $\overline{MS}$-mass $\bar{m}_t(m_t)$; $x_t \equiv (\bar{m}_t(m_t) / M_W)^2$.

The CP-conserving mode $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is also of great interest, being sensitive to $|V_{td}|$. Compared to the neutral channel, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has a slightly larger theoretical uncertainty, which is due to the charm contribution that is non-negligible in this case. Explicit expressions can be found in the third reference of [20].

The study of $K \rightarrow \pi \nu \bar{\nu}$ can give crucial information for testing the CKM picture of flavour mixing. This information is complementary to the results expected from $B$ physics and is much needed to provide the overdetermination of the unitarity triangle necessary for a decisive test. Let us briefly illustrate some specific opportunities.

The quantity $B(K_L \rightarrow \pi^0 \nu \bar{\nu})$ offers probably the best accuracy in determining $|\text{Im} V_{ts}^* V_{td}|$ or, equivalently, the Jarlskog parameter $J_{CP} = \text{Im}(V_{ts}^* V_{td} V_{us} V_{ud}^*)$, the invariant measure of CP violation in the SM. The prospects here are even better than for $B$ physics at the LHC [23]. For example, a 10%
measurement $B(K_L \to \pi^0 \nu \bar{\nu}) = (3.0 \pm 0.3) \times 10^{-11}$ would directly give $\text{Im} \lambda_t = (1.38 \pm 0.07) \times 10^{-4}$, a remarkably precise result. The SM expectation for the branching ratio [7] is $(2.8 \pm 1.1) \times 10^{-11}$, where the uncertainty is due to our imprecise knowledge of CKM parameters. The current upper bound from direct searches [8] is $5.9 \times 10^{-7}$. An indirect upper bound, using the current limit on $B(K^+ \to \pi^+ \nu \bar{\nu})$ [9] and isospin symmetry, can be placed [24] at $2 \times 10^{-9}$.

A measurement of $B(K^+ \to \pi^+ \nu \bar{\nu})$ to 10% accuracy can be expected to determine $|V_{td}|$ with about the same precision. Combining 10% measurements of both $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ determines the unitarity triangle parameter $\sin 2\beta$, as seen in Fig. 2, with an uncertainty of about $\pm 0.07$,

$$\sin 2\beta = \frac{2r}{1 + r^2} \quad r = \frac{\sqrt{\sigma(B_+ - B_L) - P_0(K^+)}}{\sqrt{B_L}}$$

where $\sigma = (1 - \lambda^2/2)^{-2}$, $P_0(K^+) = 0.42 \pm 0.06$ is the internal charm contribution to $K^+ \to \pi^+ \nu \bar{\nu}$, which is known to next-to-leading order in QCD, and $B_+$ and $B_L$ represent the reduced branching ratios $B_+ = B(K^+ \to \pi^+ \nu \bar{\nu})/(4.11 \times 10^{-11})$ and $B_L = B(K_L \to \pi^0 \nu \bar{\nu})/(1.80 \times 10^{-10})$.

The time-integrated CP violating asymmetry in $B_d \to \Psi K_S$ is given by

$$A_{CP}(\Psi K_S) = -\sin(2\beta) \frac{x_d}{1 + x_d^2}$$

where $x_d = \Delta M_{B_d}/\Gamma_{B_d}$ gives the size of $B_d - \bar{B_d}$ mixing. Both determinations of $\sin 2\beta$, the one from (2) and the one from $B_d \to \Psi K_S$, have to coincide if the SM is valid. This implies the relation

$$\frac{2r(B_+, B_L)}{1 + r^2(B_+, B_L)} = -A_{CP}(\Psi K_S) \frac{1 + x_d^2}{x_d}$$

which represents a crucial test of the SM. As was stressed in [25], all quantities in the ‘golden relation’ (4) – except for the (calculable) $P_0(K^+)$ – can be directly measured by experiment, and the relation is practically independent of $m_t$ and $V_{cb}$.

A generic comparison between clean determinations of the unitarity triangle in $K$ and $B$ physics is illustrated in Fig. 3. Any discrepancy between the various observables would point to new physics. In this way these rare kaon decays provide us with an additional approach in our search for physics beyond the SM, complementary to the direct production of new particles. It is even possible that these rare processes lead to the first evidence of new physics. But also in the longer run, after new physics has already been discovered, these decays will play an important role in analyzing in greater detail the underlying new dynamics.

New physics contributions in $K_L \to \pi^0 \nu \bar{\nu}$ and $K^+ \to \pi^+ \nu \bar{\nu}$ can be parametrized in a model-independent way by two parameters [26], which quantify the violation of the golden relation (4). New
Fig. 3: Schematic determination of the unitarity triangle vertex ($\rho, \eta$) from the $B$ system (horizontally hatched) and from $K \to \pi \nu \bar{\nu}$ (vertically hatched). Both determinations can be performed with small theoretical uncertainty and any discrepancy between them would indicate new physics, as illustrated in this hypothetical example.

Fig. 4: Typical contributions to $K \to \pi \nu \bar{\nu}$ in supersymmetry with the exchange of squarks and gauginos.

effects in supersymmetric models [27] are induced through box and penguin diagrams with new internal particles such as charged Higgs or charginos and stops, as seen in Fig. 4, replacing the $W$ boson and the up-type quark of the SM shown in Fig. 1. In the so-called ‘constrained’ minimal supersymmetric standard model (MSSM), where all flavour-changing effects are induced by contributions proportional to the CKM mixing angles, the ‘golden relation’ (4) is valid. Therefore the measurement of $B(K_L \to \pi^0 \nu \bar{\nu})$ and $B(K^+ \to \pi^+ \nu \bar{\nu})$ still directly determines the angle $\beta$ and a significant violation of (4) would rule out this model (a recent general discussion of models with minimal flavour violation can be found in [28]).

Given the present experimental status of supersymmetry, however, a model-independent analysis including also flavour mixing via the squark mass matrices is more suitable. The new sources of flavour violation can then be parametrized by the so-called mass-insertion approximation, in an expansion of the squark mass matrices around their diagonals. It turns out that supersymmetric contributions in this more general setting, also called the ‘unconstrained’ MSSM, allow for a significant violation of, for example, relation (4). An enhancement of the branching ratios by an order of magnitude for $K_L \to \pi^0 \nu \bar{\nu}$ and by about a factor of three for $K^+ \to \pi^+ \nu \bar{\nu}$ relative to the SM expectations is possible, mostly due to the chargino-induced $Z$-penguin contribution [29]. Recent analyses [29, 30, 31] within the unconstrained MSSM focused on the correlation of rare decays and $\varepsilon'/\varepsilon$, the parameter of direct CP
violation in $K \to \pi \pi$ decays. This led to typical upper bounds of $B(K_L \to \pi^0 \nu \bar{\nu}) \leq 1.2 \times 10^{-10}$ and $B(K^+ \to \pi^+ \nu \bar{\nu}) \leq 1.7 \times 10^{-10}$.

3 PROBING CP VIOLATION WITH $K_L \to \pi^0 e^+ e^-$

G. D’Ambrosio, G. Isidori

The $K_L \to \pi^0 e^+ e^-$ mode is an ideal complement to $K_L \to \pi^0 \nu \bar{\nu}$ in clarifying the short-distance mechanism of CP violation and in searching for new physics. Within the SM both decays could provide precise information on the CKM factor Im($V_{ts}^* V_{td}$). However, the experimental difficulties in reaching this goal are rather different in the two cases, and the two channels are potentially affected by different new-physics effects.

3.1 Rate measurements

Similarly to $K_L \to \pi^0 \nu \bar{\nu}$, the direct CP-violating amplitude for $K_L \to \pi^0 e^+ e^-$ is dominated by short-distance dynamics, i.e., by top-quark loops, and is calculable with high accuracy in perturbation theory [32, 33]. Within the SM, this theoretically clean part of the amplitude leads to [33]

$$B(K_L \to \pi^0 e^+ e^-)_{CPV-dir}^{SM} = (2.5 \pm 0.2) \times 10^{-12} \left( \frac{\text{Im}(V_{ts}^* V_{td})}{10^{-4}} \right)^2,$$

setting to $10^{-12}$ the minimum level of sensitivity needed to access the short-distance information provided by this transition. However, contrary to $K_L \to \pi^0 \nu \bar{\nu}$, the direct CP-violating amplitude is not the only relevant contribution to $K_L \to \pi^0 e^+ e^-$. In this case it is necessary also to take into account the indirect CP-violating and the CP-conserving amplitudes, which are both dominated by long-distance dynamics.

The indirect CP-violating contribution alone can be written as

$$B(K_L \to \pi^0 e^+ e^-)_{CPV-ind} = \frac{\tau_L}{\tau_S} |\varepsilon|^2 \frac{\text{Im}(V_{ts}^* V_{td})}{10^{-4}} B(K_S \to \pi^0 e^+ e^-)$$

and can therefore be controlled precisely by means of the $K_S \to \pi^0 e^+ e^-$ branching ratio, which is expected to be in the $10^{-10} - 10^{-8}$ range [34, 35]. The two CP-violating amplitudes of $K_L \to \pi^0 e^+ e^-$, namely the direct and the indirect ones, will in general interfere, leading to a total CP-violating branching ratio that could easily reach the $10^{-11}$ level within the SM. Interestingly, the relative phase of the two amplitudes is determined by arg($\varepsilon$) and is known up to a sign. Thus from a measurement of both $B(K_S \to \pi^0 e^+ e^-)$ and $B(K_L \to \pi^0 e^+ e^-)_{CPV-tot}$ it is possible to determine Im($V_{ts}^* V_{td}$) up to a four-fold discrete ambiguity.

The last component of the $K_L \to \pi^0 e^+ e^-$ amplitude is the CP-conserving (CPC) term generated by the process $K_L \to \pi^0 \gamma^+ \gamma^- \to \pi^0 e^+ e^-$ [36, 37]. This contribution can be estimated theoretically using experimental information on the $K_L \to \pi^0 \gamma \gamma$ spectrum at small $M_{\gamma \gamma}$: the data presently available from KTeV [38] and NA48 [39] suggest that it is substantially smaller than the direct-CP-violating component, being around $(1 - 2) \times 10^{-12}$ [40]. Moreover, CP-conserving and CP-violating contributions to $K_L \to \pi^0 e^+ e^-$ do not interfere in the total rate and could be efficiently disentangled by a Dalitz-plot analysis [40]. In view of these arguments, the CP-conserving contamination should not represent a serious problem for the extraction of the interesting CP-violating component of the $K_L \to \pi^0 e^+ e^-$ rate.

The most serious problem in trying to measure $B(K_L \to \pi^0 e^+ e^-)_{CPV}$ is the large irreducible background generated by the process $K_L \to \gamma \gamma e^+ e^-$ [41]. Imposing the cut $|M_{\gamma \gamma} - M_{\gamma \gamma 0}| < 5$ MeV on the two-photon invariant mass spectrum of $K_L \to \gamma \gamma e^+ e^-$, the latter turns out to have a branching ratio $\sim 3 \times 10^{-8}$, more than $10^3$ times larger than the signal. Employing additional cuts on various kinematical...
variables, it is possible to reduce this background down to the 10^{-10} level \[41, 12\], but it is hard to reduce it below this figure without drastic reductions of the signal efficiency. We stress, however, that this does not imply that the signal is unmeasurable in a high-statistics experiment, where the physical background can be measured and modelled with high accuracy. For instance, assuming an effective background level of 10^{-10}, one could still determine \( \text{Im}(V_{ts}^*V_{td}) = 0, \pm 1.3 \times 10^{-4} \) (see text).

3.2 Time-dependent \( K_{L,S} \rightarrow \pi^0 e^+ e^- \) interference

Complementary information on the direct CP-violating component of the \( K_L \rightarrow \pi^0 e^+ e^- \) amplitude can be obtained by studying the time evolution of the \( K_{L,S} \rightarrow \pi^0 e^+ e^- \) decay \[37, 42\]. Although challenging from the experimental point of view, this method has two intrinsic advantages: i) the interference between \( K_S \) and \( K_L \) amplitudes is only due to the CP-violating part of the latter; and ii) the process \( K_S \rightarrow \gamma\gamma e^+ e^- \) is very suppressed with respect to \( K_S \rightarrow \pi^0 e^+ e^- \) [\( B(K_S \rightarrow \gamma\gamma e^+ e^-) \sim \text{few} \times 10^{-12} \)], so the background due to the \( |\gamma\gamma e^+ e^-\rangle \) final state is almost negligible at small times \( t \ll \tau_L \).

As an example, in Fig. 5 we show the time evolution of a pure \( |K^0\rangle \) beam at \( t = 0 \). In this case the probability distribution of decays into the final state \( |\pi^0 e^+ e^-\rangle \) or in the background channel \( |e^+ e^-\gamma\gamma\rangle \) (with \( M_{\gamma\gamma} \sim M_{\pi^0} \)), as a function of the proper time \( t \), can be written as

\[
I(t) = \frac{\tau_S}{2} \left\{ |A_S|^2 e^{-t/\tau_S} + 2 \text{Re} \left[ A_{CPV}^* A_S e^{-i(m_L-m_S)t} \right] e^{-t/(\tau_L+\tau_S)/2\tau_L \tau_S} \right. \\
+ \left[ |A_{CPV}|^2 + |A_{CP}|^2 + |A_{bkg}^L|^2 \right] e^{-t/\tau_L} \right\}.
\]

The three curves in Fig. 5 have been obtained assuming \( \tau_S |A_S|^2 = B(K_S \rightarrow \pi^0 e^+ e^-) = 10^{-8} \) and \( \tau_L |A_{bkg}^L|^2 = B(K_L \rightarrow \gamma\gamma e^+ e^-)_{\text{cuts}} = 10^{-10} \), and employing the following three values of \( \text{Im}(V_{ts}^*V_{td}) \): 0, \( \pm 1.3 \times 10^{-4} \). As can clearly be seen, the interference term is quite sensitive to the value of the direct CP-violating amplitude. On a purely statistical level, in this example one could reach a 10%
error on $\text{Im}(V_{ts}^*V_{td})$ with an initial flux of $2 \times 10^{15}/\epsilon_{\pi^0\text{ee}}$ $K_S$'s, where $\epsilon_{\pi^0\text{ee}}$ denotes the efficiency for decays occurring within the first 15 $K_S$ decay lengths.

It is worth stressing that the unknown value of $B(K_S \to \pi^0 e^+e^-)$ plays a major role in the time distribution. If $B(K_S \to \pi^0 e^+e^-)$ is found to be below $10^{-9}$, then it is probably too difficult to measure the interference term with high precision. On the other hand, if $B(K_S \to \pi^0 e^+e^-) > 10^{-9}$, then it is worthwhile to plan a measurement of the $K_{L,S} \to \pi^0 e^+e^-$ time interference, not only for its own interest, but also to obtain a precise determination of $B(K_S \to \pi^0 e^+e^-)$, which is needed anyway to disentangle the large indirect CP-violating component of $B(K_L \to \pi^0 e^+e^-)$. Interestingly, a heuristic argument based on vector-meson dominance favours a large value of $B(K_S \to \pi^0 e^+e^-)$ [40], a conjecture that will soon be tested by the NA48 collaboration [43].

### 3.3 Beyond the SM

The direct CP-violating amplitude for $K_L \to \pi^0 e^+e^-$ is very sensitive to possible extensions of the SM in the flavour sector and could eventually provide unambiguous evidence for new physics. For instance, within supersymmetric models with generic flavour structures, $B(K_L \to \pi^0 e^+e^-)_{\text{CPV}}$ could be enhanced with respect to its SM value up to one order of magnitude [29, 31]. This could happen either via a modification of the pure electroweak amplitudes ($Z$ penguin and $W$ box), present also in the $K_L \to \pi^0 \nu\bar{\nu}$ mode, or via nonstandard effects in the $\gamma$-penguin amplitude, absent in the $K_L \to \pi^0 \nu\bar{\nu}$ case [31]. Interestingly, the two types of effects could be disentangled by means of $K_{L,S} \to \pi^0 e^+e^-$ transitions only, measuring both $B(K_L \to \pi^0 e^+e^-)_{\text{CP-dir}}$ and the time-dependent interference [40]. Indeed, the latter is sensitive only to the vector component of the amplitude (dominated by the $\gamma$-penguin contribution), whereas the former is equally sensitive to the vector and axial components of the amplitude.

### 4 CHARGED-LEPTON-FLAVOUR VIOLATION IN KAON DECAYS IN SUPERSYMMETRIC THEORIES

A. Belyaev, M. Chizhov, A. Dorokhov, J. Ellis, M. E. Gómez, S. Lola

#### 4.1 Introductory remarks

In this section we discuss rare kaon decays that violate charged-lepton flavour conservation in supersymmetric theories with and without $R$ parity: for details and a complete set of references, see [44]. Recent data from the Super-Kamiokande [45] and other experiments have triggered an upsurge of interest in extensions of the SM with massive neutrinos and/or violation of the charged-lepton numbers in processes such as $\mu \to e\gamma$, $\mu \to 3e$, $\tau \to \mu\gamma$ and $\mu \to e$ conversion on heavy nuclei [46, 47, 48, 49, 50]. Here we discuss the prospects for muon-number violation in rare kaon decays, encouraged by hopes that the proton driver for a neutrino factory [51] could be used to improve the limits significantly. Any observable rate for processes like $K^0 \to \ell^+\ell^-$, $K^0 \to \pi^0\ell^+\ell^-$, $K^+ \to \pi^+\ell^+\ell^-$ ($\ell \neq \ell'$), or $K^+ \to \pi^-\ell^+\ell^+$ would constitute unambiguous evidence for new physics. The rates for such processes remain extremely suppressed if we extend the SM minimally to include right-handed neutrinos, but larger rates are possible in more ambitious extensions of the SM. Supersymmetry is one example of new physics that could amplify rates for some of the rare processes, either in the minimal supersymmetric extension of the standard model (MSSM) or in its modification to include the violation of $R$ parity.

#### 4.2 Kaon decays violating charged lepton number in the MSSM

In this paragraph, we evaluate the rates for lepton-number-violating rare kaon decays in the MSSM with massive neutrinos, assuming the seesaw mechanism [52], which we consider to be the most natural way to obtain neutrino masses in the sub-eV range. In particular, we assume Dirac neutrino masses $m_\nu$ of
the same order as the charged-lepton and quark masses, and heavy Majorana masses $M_{\nu R}$, leading to a light effective neutrino mass matrix of the form: $m_{\nu eff} = m_D^R \cdot (M_{\nu R})^{-1} \cdot m_D^P$

In the MSSM framework, rare kaon decays are generated by box diagrams involving the exchanges of charginos $\chi^\pm$ and neutralinos $\chi^0$. For instance, for $K^0 \rightarrow \mu^+ e^-$ we have the diagrams of Fig. 6.

![Fig. 6: MSSM box diagrams for $K^0 \rightarrow \mu^+ e^-$. There is another neutralino exchange diagram corresponding to the permutation of the $\mu$ and $e$. Since $\chi^0$ is a Majorana spinor, there are contributions from the neutralinos that differ in the number of mass insertions.](image)

Our procedure for evaluating these contributions is as follows: we find the maximal squark mixing that is allowed by the neutral-kaon mass difference $\Delta m_K$ [53], then we find the maximal slepton mixing allowed by $\mu \rightarrow e\gamma$ and $\mu-e$ conversion in nuclei in a model-independent way. Finally, having fixed these values, we calculate the rates for the rare kaon decays of interest here.

We parametrize the supersymmetric masses in terms of universal GUT-scale parameters $m_0$ and $m_{1/2}$, for sfermions and gauginos respectively, and use the renormalization-group equations of the MSSM to calculate the low-energy sparticle masses. Other relevant free parameters of the MSSM are the trilinear coupling $A$, for which we use the initial condition $A_0 = -m_{1/2}$, the sign of the Higgs mixing parameter $\mu$, and the value of $\tan \beta$. Models with different signs of $\mu$ give similar results: here we assume $\mu < 0$.

Typical results are shown in Fig. 7. As expected, the larger the value of $\tan \beta$ and the smaller the soft supersymmetric terms, the larger the branching ratios, apart from certain cancellations. In the case $\tan \beta = 10$ and $m_{1/2} = 250$ GeV, for the range $m_0 \geq 170$ GeV where $B(\mu \rightarrow e\gamma)$ and $R(\mu \rightarrow e)$ are consistent with the current experimental bounds, $B(K \rightarrow \mu e)$ is at most $2 \times 10^{-18}$. However, for the same value of $m_{1/2}$, when $\tan \beta = 20$ we find a significantly larger branching ratio at small values of $m_0 \sim 170$ GeV. Moreover, for smaller $m_{1/2} = 150$ GeV, we gain almost two orders of magnitude when we consider $m_0$ in the low-mass window between 100 and 150 GeV. We recall [49] that these lower values of $m_{1/2}, m_0$ are consistent with accelerator constraints and generically yield cold dark matter densities in the range preferred by cosmology [54].

This analysis demonstrates that, despite the limits from $\mu \rightarrow e\gamma$, $\mu-e$ conversion and $\Delta m_K$, the branching ratio of $K \rightarrow \mu e$ may be within the reach of the next generation of experiments, namely in the range $10^{-16} \rightarrow 10^{-18}$, at least if $\tan \beta$ is large and the soft supersymmetry-breaking terms are small.

### 4.3 Kaon decays violating charged lepton number in $R$-violating supersymmetry

We now discuss kaon decays violating charged-lepton flavour beyond the context of the MSSM. As is well known, the gauge symmetries of the MSSM allow additional dimension-four Yukawa couplings, of the form $\lambda L_i \bar{L}_j \bar{D}_k$, $\lambda' L_i Q_j \bar{D}_k$, $\lambda'' \bar{U}_i \bar{D}_j \bar{D}_k$ where the $L(Q)$ are the left-handed lepton (quark) superfields, and the $\bar{E}, \bar{D}, \bar{U}$ are the corresponding right-handed fields. If all these couplings were present simultaneously in the low-energy Lagrangian, they would generate unacceptably fast proton decay. In this study, we allow only lepton-number-violating processes and study the general case when several $R$-violating operators may be non-zero. Then we discuss the limits on their combinations that are obtainable from kaon decays.
Fig. 7: Illustrative predictions for $B(K^0 \to \mu e)$, $B(\mu \to e\gamma)$ and the $\mu \to e$ conversion rate, for $\tan\beta = 10$ (left column) and $\tan\beta = 20$ (right column), and $m_{1/2} = 150$ GeV (dashed lines) or 250 GeV (solid lines), as functions of $m_0$ (in GeV).
What is the connection of these limits with neutrino masses? We note that neutrino masses do not strictly constrain $K \to \mu e$ (and in certain cases the rest of the flavour-violating-processes), since neutrino masses may only constrain products of $LLE$ or $LQD$ operators, but not mixed $LLE$-$LQD$ products. Even for the diagrams with products of only $LQD$ operators, rare kaon decays involve quarks of the lightest and second-lightest generations. In this case the bounds from neutrino masses are significantly weaker, and the stricter limits come from the current measurements of the rare kaon decays themselves.

Fig. 8: Diagrams involving $R$-violating couplings that yield two-body $K^0 \to \ell^\pm \ell^\mp$ decays.

- Two-body $K^0$ decays to muons and electrons proceed via the diagrams shown in Fig. 8. Using the Feynman rules derived for the relevant effective kaon, pion and lepton interactions, we have recalculated the important kaon decay processes, and update the limits on the products of $R$-violating couplings using the present experimental limits. The diagrams of Fig. 8 lead to the following effective Lagrangian for $K^0\ell^+\ell^-$ interactions:

$$
\mathcal{L}_{K^0\ell^+\ell^-} = \frac{F_{K^0}}{2m_{\tilde{e}_i}} \left[ \lambda_{ijk}^s \lambda_{ij2}^{12} \left( \overline{\ell}_j L\ell_{k R} \right) - \lambda_{ikj}^s \lambda_{i21}^{12} \left( \overline{\ell}_j R\ell_{k L} \right) \right] K^0 (p_K) - \frac{f_K}{4m_{\tilde{u}_i}^2} \lambda_{ijk}^s \lambda_{ik2}^{12} \left( \overline{\ell}_j L\ell_{k L} \right) K^0 (p_K),
$$

where $F_{K^0} = m_{K^0} f_K / (m_s + m_d)$, $m_s + m_d \simeq 0.15$ GeV is the sum of the current masses of the $s$ and $d$ quarks, and $f_K = 0.1598$ GeV is the kaon decay constant. The value of $F_{K^0}$ is related to the pseudoscalar matrix element $\langle 0|\bar{s} \gamma^5 d|K^0 \rangle = -F_{K^0}$, and is obtained from $f_K$ by using the Dirac equations for quarks. All QCD corrections are included in this phenomenological approach. In the following, we assume that the $R$-violating couplings are real and that only one of their products is non-zero.

We have implemented the Feynman rules in the CompHEP package [63], using the effective Lagrangian, and have obtained analytical results for the kaon decay width as well as limits on the products of $\lambda \lambda'$ and $\lambda' \lambda'$ couplings from sneutrino and squark exchange, respectively. The results are shown in equations (16) and (17) of [44].

We now discuss the diagrams for 3-body kaon decays to pions and two charged leptons, of which there are two qualitatively different kinds.

- The kaon may decay into a pion of the same charge, in which case the leptons in the final state must have opposite signs: $K^\pm \to \pi^\pm \ell^\mp \ell^\pm$ and $K^0 \to \pi^0 \ell^+ \ell^-$. The corresponding diagram for the first process is shown in Fig. 9. The limit obtained from $K^0 \to \ell^+ \ell^-$ is typically 1-2 orders of magnitude better than that derived from $K^+ \to \pi^+ \ell^+ \ell^-$ decay, so we do not discuss further this class of three-body decays.
• The kaon may decay into a pion with the opposite charge, in which case the leptons in the final state must have the same signs: $K^{\pm} \to \pi^\mp \ell^\pm \ell^\pm$. This process involves two heavy virtual particles, the $W$ boson and a down squark. One should note that the decay width of this process is directly proportional to the mixing between the left- and right-handed squark states, denoted by $\tilde{b}_L$ and $\tilde{b}_R$, respectively. If there is no mixing, the same-sign-lepton process vanishes. The effective Lagrangian includes the following terms:

$$\mathcal{L}_{K^+\ell^-\nu_i} = V_{us}\sqrt{2}G_F f_K \gamma^\mu (\bar{\nu}_i \ell_{L\mu} \ell_{Li}) K^+ (p_K),$$  \hspace{1cm} (9)$$

$$\mathcal{L}_{\pi^+\ell^-\nu_i} = V_{ud}\sqrt{2}G_F f_\pi \gamma^\mu (\bar{\nu}_i \ell_{L\mu} \ell_{Li}) \pi^+ (p_\pi).$$  \hspace{1cm} (10)

Here $f_K$ and $f_\pi = 0.1307$ GeV are the kaon and pion decay constants, respectively, $G_F = 1.1663910^{-5}$ GeV$^{-2}$ is the Fermi constant and $V_{us}$, $V_{ud}$ are CKM matrix elements. We also have

$$\mathcal{L}_{K^+ (l^- j^-) C V_i} (\vec{d}_k) = \frac{\lambda'_{ik2} \lambda'_{j1k} V_{LR}}{4 m_d^2} F_{K^+} (\ell_{Lj} C \nu_i) K^+ (p_K),$$  \hspace{1cm} (11)$$

$$\mathcal{L}_{\pi^+ (l^- j^-) C V_i} (\vec{d}_k) = \frac{\lambda'_{ik1} \lambda'_{j1k} V_{LR}}{4 m_d^2} F_{\pi^+} (\ell_{Lj} C \nu_i) \pi^+ (p_\pi),$$  \hspace{1cm} (12)

where $F_{K^+} = m_{K^+}^2 f_K / (m_s + m_u)$, $F_{\pi^+} = m_{\pi^+}^2 f_\pi / (m_d + m_u)$, $m_{s+} + m_u \approx 0.15$ GeV, and $m_{d+} + m_u \approx 0.01$ GeV and $V_{LR}$ denotes the left-right squark mixing matrix element.

The Feynman diagrams for $K^+ \to \pi^+ \ell^+_i \ell^+_j$ decay are given in terms of these effective interactions, as shown in Fig. 10. As an example, one can obtain a constraint on the product of $\lambda'_{2k2} \lambda'_{11k}$ and $V_{LR}$ from $K^+ \to \pi^- e^+ \mu^+$ decay. In this case, we have the following numerical result $^1$: $V_{LR} (\lambda'_{2k2} \lambda'_{11k}) \times \left(\frac{100 \text{ GeV}}{m_d}\right)^2 \leq 10$. It is apparent that kaon decay into a pion and a like-sign lepton pair is too strongly suppressed to be useful at present.

### 4.4 Conclusions

We have discussed the flavour-violating decays of kaons into charged-lepton pairs in supersymmetric theories, in both the minimal supersymmetric standard model and $R$-violating models. In the first case, despite the limits from $\mu \to e\gamma$, $\mu-e$ conversion and $\Delta m_K$, the kaon decay branching ratios for large $\tan \beta$ and small soft supersymmetry-breaking terms may be accessible to a future generation of experiments using new intense proton sources. In the case of $R$-violating supersymmetry, we studied the expected rates for the decays $K \to \mu^+ e^\mp$ and $K \to \pi^+ \mu e$, for all two- and three-body processes, and obtained the bounds on products of $LLLL$ and $LQD$ operators summarized in (16) and (17) of [44]. We have also noted the possibility of like-sign lepton events in the presence of non-zero $\bar{b}_L - \bar{b}_R$ mixing, but for this to occur at a significant rate one would need large $R$-violating couplings. Our final conclusion is that lepton-flavour-violating rare kaon decays have the potential to provide important information on the issue of flavour physics. Any future observation would, in addition, help distinguish between different supersymmetric theories.

$^1$In [64] a similar constraint was found with a different choice of diagrams.
5 THE RARE DECAY $K^+ \rightarrow \pi^- l^+ l^+$

K. Zuber

The decays $K^+ \rightarrow \pi^- l^+ l^+$ can also be seen in the broader context of $\Delta L = 2$ lepton-number-violating processes. They are analogous to neutrinoless double-beta decay, and ways to determine three out of nine matrix elements of effective Majorana masses $\langle m_{\alpha\beta} \rangle$

$$\langle m_{\alpha\beta} \rangle = \left| \sum m_m \eta_{m\alpha}^\text{CP} U_{\alpha m} U_{\beta m} \right| \quad \text{with } \alpha, \beta = e, \mu, \tau.$$  \hfill (13)

with the relative CP-phases $\eta_{CP} = \pm 1$. For a general overview of $\Delta L = 2$ processes studied within this context, see [66]. The current limits on the possible processes involving kaons are given by the BNL E865 experiment [16]

$$B(K^+ \rightarrow \pi^- e^+ e^+) < 6.4 \times 10^{-10}$$  \hfill (14)

$$B(K^+ \rightarrow \pi^- \mu^+ e^+) < 5.0 \times 10^{-10}$$  \hfill (15)

$$B(K^+ \rightarrow \pi^- \mu^+ \mu^+) < 3.0 \times 10^{-9}$$  \hfill (16)

However, neutrinoless double beta decay and $\mu - e$ conversion on nuclei are already much more sensitive than the first two decays, and the main interest would be the decay $K^+ \rightarrow \pi^- \mu^+ \mu^+$ measuring

$$\langle m_{\mu\mu} \rangle = \left| \sum U_{\mu m}^2 m_m \eta_{m\mu}^\text{CP} \right| \quad \text{(17)}$$

Whilst the experimental limit (16) is not yet good enough to restrict $\langle m_{\mu\mu} \rangle$ and the combination $\lambda_{211}' \lambda_{212}'$ of $R$-violating supersymmetric couplings (see sec. 1.4.3 and [64]), it does lead to an improvement by about two orders of magnitude in the bound on the mixing of muon neutrinos with heavy neutrinos in the mass region of $249 \text{ MeV} < m_H < 385 \text{ MeV}$ due to resonance effects [67].

6 RARE KAON DECAY EXPERIMENTS

A. Ceccucci

6.1 Introduction

We focus on the rare kaon decays which can drive an experimental programme. Therefore the experimental status and the perspectives for future measurements of the following decay modes are reviewed:

- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$
- $K_L \rightarrow \pi^0 \nu \bar{\nu}$
- $K_L \rightarrow \pi^0 l^+ l^-$
- $K_L \rightarrow \mu e$

The Standard Model (SM) theoretical predictions and the current experimental limits for the above reactions are shown in Table 2. It is very likely that the availability of high-intensity kaon beams will trigger other experimental proposals, ranging from the search for CPT violation in kaon decays to the study of hypernuclei, but we consider the above reactions to be the most likely physics drivers for such a programme.
Table 2: Comparison between theory and experiment. Experimental upper limits are at 90% CL.

<table>
<thead>
<tr>
<th>Decay Mode</th>
<th>SM</th>
<th>Exp. Result</th>
<th>Experiment</th>
<th>Technique</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K^+ \to \pi^+ \nu\bar{\nu}$</td>
<td>$1 \times 10^{-10}$</td>
<td>$(1.5 \pm 0.2) \times 10^{-10}$</td>
<td>E787</td>
<td>kaon decay at rest</td>
</tr>
<tr>
<td>$K_L \to \pi^0 \nu\bar{\nu}$</td>
<td>$3 \times 10^{-11}$</td>
<td>$&lt; 5.9 \times 10^{-7}$</td>
<td>KTEV</td>
<td>$(\pi^0 \to e^+e^-\gamma)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$&lt; 1.6 \times 10^{-6}$</td>
<td>KTEV</td>
<td>$(\pi^0 \to \gamma\gamma)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$&lt; 2.6 \times 10^{-9}$</td>
<td>E787</td>
<td>Model indep. limit</td>
</tr>
<tr>
<td>$K_L \to \pi^0 e^+ e^-$</td>
<td>$5 \times 10^{-12}$</td>
<td>$&lt; 5.1 \times 10^{-10}$</td>
<td>KTEV</td>
<td>using $K^+ \to \pi^+ \nu\bar{\nu}$</td>
</tr>
<tr>
<td>$K_L \to \pi^0 \mu^+ \mu^-$</td>
<td>$1 \times 10^{-12}$</td>
<td>$&lt; 3.8 \times 10^{-10}$</td>
<td>KTEV</td>
<td></td>
</tr>
<tr>
<td>$K_L \to \mu e^- $</td>
<td>$-$</td>
<td>$&lt; 4.7 \times 10^{-12}$</td>
<td>E871</td>
<td>double spectrometer</td>
</tr>
</tbody>
</table>

6.2 $K^+ \to \pi^+ \nu\bar{\nu}$

The one event measured by AGS-E787 was observed in the 1995-1997 data sample, with an estimated background from all sources of $0.08 \pm 0.02$ events [9]. The event was obtained from data in the phase-space region above the $K^+ \to \pi^+ \pi^0$ ($K_{\pi2}$) peak. E787 also plans to access the region below the $K_{\pi2}$ peak, where a Signal-to-Noise (S/N) ratio of about unity is expected for the SM prediction. The two most significant backgrounds come from $K_{\pi2}$ and $K^+ \to \mu^+ \nu$ ($K_{\mu2}$). Other important backgrounds derive from the $K^+$ charge-exchange reaction (CEX) and from $\pi^+$ particles in the beam that scatter in the detector. The kaons are stopped in a scintillating-fibre target in the centre of the detector, and a low-mass central drift chamber measures the momenta of the decay products. The energy, range, and the decay sequence of charged particles are measured in the range-stack array of scintillators. Within the range stack, tracking is provided by two layers of straw tubes. The photon-veto system covers the entire solid angle.

Incremental improvement over E787 will be provided by AGS-E949 [68], which will be the primary user of the AGS in the years 2001-2003. About $3 \times 10^6 K^+$ at 700 MeV/c with a $K/\pi$ ratio of four are produced for $10^{13}$ incident protons. The improvement in sensitivity per year over E787 is a factor of 13. After two years of data taking, E949 should reach a sensitivity of $1.7 \times 10^{-11}$. Including the acceptance below the $K_{\pi2}$ peak, one would expect 7 – 13 events if the branching ratio is that predicted by the SM.

Further improvement in the measurement of $K^+ \to \pi^+ \nu\bar{\nu}$ is expected from the CKM experiment at the Fermilab Main Injector [69], which plans to exploit a decay-in-flight technique. The critical component is an intense (30 MHz) RF-separated 22 GeV/c $K^+$ beam based upon 3.9 GHz transverse mode super-conducting RF cavities. The experiment aims for a kaon-to-pion ratio of two for decays in the fiducial volume. The separator is formed by two RF cavities operating in a mode which deflects the beam laterally. The separation between the two cavities is chosen to deflect both $\pi^+$ and protons back onto the optical axis, where they are blocked by a beam stopper. The charged particles in the reaction are measured by a pair of Ring Imaging Cherenkov counters (RICH) and independently by upstream and downstream magnetic spectrometers. The veto system has a component surrounding the decay volume and a forward component after the $\pi^+$ RICH. The inefficiency to veto at least one photon from a $\pi^0$ should be $< 10^{-7}$. A muon veto follows the spectrometer to control the $K_{\mu2}$ and $K^+ \to \mu^+ \gamma\nu$ backgrounds, and the inefficiency to detect muons is required to be $< 10^{-5}$. The Fermilab Main Injector has now been in operation for over a year, and the beam line to transport 120 GeV slow-spill protons is under construction. This beam should be available for slow-spill tests in 2002. The CKM experiment requires $5 \times 10^{12}$ protons per second, corresponding to less than 20% of the total design intensity of the Main Injector. Full scientific approval of CKM is expected in June 2001.
6.3 $K_L \rightarrow \pi^0 \nu \bar{\nu}$

This decay mode is considered to be the “holy grail of the kaon system” [70]. Upper limits obtained so far lie 4 to 5 orders of magnitude above the SM prediction, as seen in Table 2. A first generation of dedicated experiments is now being proposed. Comparisons of the experimental techniques and of the overall expected sensitivities for currently proposed experiments are presented in Tables 3 and 4. The two main difficulties of the experiment are the incomplete knowledge of the initial state and the daunting background originating from other $K_L$ and $\Lambda$ decays with $\pi^0$s and neutron interactions.

Table 3: Experimental techniques for currently proposed $K_L \rightarrow \pi^0 \nu \bar{\nu}$ experiments.

<table>
<thead>
<tr>
<th></th>
<th>KEK-E391A</th>
<th>BNL-KOPIO</th>
<th>FNAL-KAMI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>far</td>
</tr>
<tr>
<td>Calorimetry</td>
<td>CsI,CeF$_3$</td>
<td>Shashlyk</td>
<td>pure CsI</td>
</tr>
<tr>
<td>$\sigma(E)/E(%)$</td>
<td>0.4/$\sqrt{E}$ + 0.5</td>
<td>3.5/$\sqrt{E}$</td>
<td>2/$\sqrt{E}$ + 0.5</td>
</tr>
<tr>
<td>$\sigma(t)$ (ps)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma(\theta)$ (photons)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constraint from pencil beam ($p_{t_{\max}}^0$) (MeV/c)</td>
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<td>6</td>
<td>11</td>
</tr>
<tr>
<td>Technique to measure $E_{K_L}$</td>
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<td>TOF</td>
<td>No</td>
</tr>
<tr>
<td>Required inefficiency to veto photons ($E_\gamma &gt; 100$ MeV)</td>
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<td>$10^{-4}$</td>
<td>$&lt;10^{-5}$</td>
</tr>
<tr>
<td>kaon decay rate (MHz)</td>
<td>0.5</td>
<td>25</td>
<td>2.8</td>
</tr>
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</table>

Table 4: Sensitivity for currently proposed $K_L \rightarrow \pi^0 \nu \bar{\nu}$ experiments.

<table>
<thead>
<tr>
<th></th>
<th>KEK-E391A</th>
<th>BNL-KOPIO</th>
<th>FNAL-KAMI</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td>far</td>
</tr>
<tr>
<td>Primary proton momentum (GeV/c)</td>
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<td>24</td>
<td>120</td>
</tr>
<tr>
<td>Production angle</td>
<td>6°</td>
<td>45°</td>
<td>24 mrad</td>
</tr>
<tr>
<td>Solid angle (\mu sr)</td>
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<td>500</td>
<td>0.36</td>
</tr>
<tr>
<td>Protons on target (3y, 10$^7$ s/y)</td>
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<td>$6.1 \times 10^{20}$</td>
<td>$3.3 \times 10^{20}$</td>
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<tr>
<td>Number of $K_L$ decays</td>
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<td>$1.5 \times 10^{14}$</td>
<td>$1.4 \times 10^{13}$</td>
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<td>Average $K_L$ momentum (GeV/c)</td>
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<td>0.7</td>
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<tr>
<td>Acceptance (%)</td>
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<td>7.1</td>
</tr>
<tr>
<td>Signal events (BR $\sim 3 \times 10^{-11}$)</td>
<td>3</td>
<td>65</td>
<td>30</td>
</tr>
<tr>
<td>Background events</td>
<td>$&lt;1.8$</td>
<td>35</td>
<td>17</td>
</tr>
</tbody>
</table>

A significant improvement on the background rejection could be achieved employing a beam of $K_L$ of known energy and direction, as could in principle be provided by one of the following two examples.

- A $\phi$ factory:
  At a $\phi$ factory, the $K_L$ is produced at fixed momentum ($\beta \simeq 0.21$) and its direction can be well determined using the reconstructed vertex for the accompanying $K_S \rightarrow \pi^+\pi^-$ decay and the interaction point [75]. Another advantage in the search for $K_L \rightarrow \pi^0 \nu \bar{\nu}$ at a $\phi$ factory is the absence of background generated by $\Lambda \rightarrow n\pi^0$ decays and neutrons.

- Exclusive formation by $\pi^- p \rightarrow K_L \Lambda$:
  This technique for producing $K_L$ of definite energy employing a 1 GeV/c $\pi^-$ beam via the reaction $\pi^- p \rightarrow K_L \Lambda$ has been used in the past [76].

Unfortunately, both methods appear to be orders of magnitude away from providing the necessary flux to reach SM sensitivities under reasonable running conditions.
The KEK-E391A [71] and the KAMI [72] proposals exploit a pencil beam design to limit the kaon transverse momentum and apply cuts on the minimum $\pi^0$ transverse momentum. A cut on $p_T > 150$ MeV/$c$ effectively rejects backgrounds coming from the reaction $\Lambda \rightarrow \pi^0 n$. To cope with the $K_L \rightarrow 2\pi^0$ background with two lost photons, the gamma veto inefficiency must be very small, $< 10^{-4}$ per photon.

A different approach is proposed by the KOPIO [73] experiment. Here, the intention is to use a very wide beam extracted at large angle with respect to the primary proton beam. The 24 GeV/$c$ primary proton beam can be delivered with 200 ps wide pulses spaced about 40 ns each [74]. This microstructure of the beam, coupled with the low kaon average momentum of 700 MeV/$c$, allows measurement of the kaon momentum via Time Of Flight (TOF). The kaon decay can therefore be transformed into the centre-of-mass frame, providing a constraint on the kinematics of the decay. In addition, the experiment plans to measure the photon direction by means of a preradiator placed in front of the electromagnetic calorimeter.

The performance of the calorimetry is stretched to the limits. Factors such as tails in the detector response should be considered in the evaluation of the response. As seen in Table 2, there are 4 to 5 orders of magnitude to bridge from the current upper limits to the SM prediction, and progress may be slower than expected.

The MSR front end appears suitable for a second-generation experiment. If the current proposals do not reach the SM sensitivity, a second-generation experiment should focus on improving the experimental technique. However, one should plan an experiment capable of collecting the 1000 events which would match the precision of the theoretical prediction.

### 6.4 $K_L \rightarrow \pi^0 e^+ e^-$

The experimental difficulty of this decay is twofold. On the one side one is forced to disentangle the short-distance direct CP violation from the indirect CP-violating and CP-conserving ones. On the other hand, a large background from $K_L \rightarrow \gamma \gamma e^+ e^-$ cannot be reduced below $10^{-10}$ without a significant efficiency loss. The best current limits are those presented by KTEV at Fermilab. Two candidates for the $K_L \rightarrow \pi^0 e^+ e^-$ decay remain in the signal box, for an expected background of $1.06 \pm 0.41$. The signal box is defined by a $\pm 2.65$ MeV/$c^2$ ($\pm 5$ MeV/$c^2$) cut on the $m_{\gamma\gamma} (m_{e^+e^-\gamma\gamma})$ variables. These cuts correspond to a $\pm 2\sigma$ cut on the signal distribution simulated by Monte Carlo. The limiting background is the radiative Dalitz decay $K_L \rightarrow e^+ e^- \gamma \gamma$ when $m_{\gamma\gamma} = m_{\pi^0}$. The most powerful variables for separating the signal from the background are the angle between the photons and the kaon in the $\pi^0$ rest frame and the minimum angle between any photon and any electron in the kaon rest frame. The result presented by KTEV [12] is based on data collected during the 1997 – 1999 Fermilab fixed target run and corresponds to an exposure of $2.6 \times 10^{11}$ $K_L$ decays. The background is inversely proportional to the $m_{\gamma\gamma}$ resolution, which is dominated by the energy resolution of the electromagnetic calorimeter. The energy resolution for photons achieved by the KTEV CsI detector (cf. Table 3) represents the state of the art, and a significant improvement over this figure is unlikely in the near future. Therefore the only hope to improve further on $K_L \rightarrow \pi^0 e^+ e^-$ is to measure an excess signal over the background. Progress will therefore only be made according to the square root of the available statistics. To address the SM prediction for direct CP violation in $K_L \rightarrow \pi^0 e^+ e^-$ about $10^{15}$ $K_L$ decays are needed. This corresponds to a flux which is about 20 times larger than the exposure accumulated by the AGS-E871 experiment, currently employing the most intense $K_L$ beam.

The $K_L \rightarrow \pi^0 \mu^+ \mu^-$ case is similar to the electron channel: a smaller experimental background is offset by smaller values for the predicted branching fraction. In the short term, progress is expected from analysis of the data collected by KTEV in 1999. In the future, the KAMI experiment at Fermilab should be able to improve the present upper limits significantly.
6.5 Lepton-flavour violation in kaon decays

The best limit to date on $K_L \to \mu e$ is that from the AGS-E871 experiment [13], namely $B < 4.7 \times 10^{-12}$ at 90% CL. The neutral kaon beam was produced by 24 GeV/c protons on a 1.4 interaction length $Pt$ target. It was selected at an angle 3.75° with respect to the direction of the incoming protons. The typical proton intensity was $1.5 \times 10^{13}$ per spill of 1.2 - 1.6 s duration. About $2 \times 10^8 K_L$ per spill with momentum between 2 and 16 GeV/c entered the decay volume, 7.5% of which decayed in the 11 m fiducial volume. The undecayed neutral beam was absorbed by a beam stopper placed downstream of the first two tracking stations. The tracking detectors consisted of six chamber stations and two consecutive dipole magnets, which had opposite polarities and provided net transverse momenta of 418 and 216 MeV/c. The chamber stations where the highest rates occurred, close to 1 MHz rate per wire, were 5 mm diameter straw tubes operated with a fast gas mixture [77]. Electron identification was provided by an atmospheric threshold Cherenkov counter and by a lead glass calorimeter. Muons were identified by scintillation counters and a range finder. The primary source of background was $K_L \to \pi e\nu$ decay in which a pion decayed upstream of the muon filter. The misidentification of the pion as a muon resulted in a maximum $m_{\mu e}$ of 489.3 MeV/c^2. The $m_{\mu e}$ resolution was 1.38 MeV/c^2 and therefore background from Gaussian tails is negligible. However, backgrounds from non-Gaussian tails could be important. There is probably room for improvement by up to a factor of ten over this result, but backgrounds from accidentals may become important for rates ten times larger than E871.

7 SOME CONSIDERATIONS ON USING THE PROTON DRIVER OF A MUON STORAGE RING (MSR) AS A KAON FACTORY

G. Kalmus

7.1 Introduction

The purpose of this section is to explore the feasibility of using the proton driver of a possible MSR facility to produce kaon beams of intensity and characteristics that are not only competitive with those available elsewhere, but potentially even better.

7.2 Assumptions

The qualitative behaviour of kaon production as a function of machine parameters (beam current and energy) is shown in Figs. 11 and 12. We see in Fig. 11 that the kaon yield rises rapidly as a function of energy: the $K_L$ production cross section shown is expected to be similar to the mean of the $K^+$ and $K^-$ yields. On the other hand, the absolute yield is not necessarily the most important consideration. One might also wish to optimize the yield for a given beam power, which is shown in Fig. 12. However, economic and other considerations might motivate a choice of proton driver energy somewhat below the optimal range between 30 and 100 GeV. For the purposes of this brief study, we make the following default assumptions.

a) We assume that the MSR proton driver achieves 24 GeV, and is a rapid-cycling synchroton with a 3µs beam pulse operating at 15 Hz.

b) The total beam power is 4MW, giving a beam current of about 160µA.

c) A stretcher ring is available and able to convert the very bad duty cycle ($< 10^{-4}$) to a good one ($\sim 100\%$).

d) Slow ejection from the stretcher ring at these high intensities is possible and efficient.

e) 10% of the protons from the MSR source can be used for kaon physics, i.e., 3 pulses every 2 seconds can be routed to the stretcher rings.

It is clear from Figs. 11 and 12 that a 2.2 GeV proton driver, being close to the kaon production threshold, would be vastly inferior as a kaon source, and not competitive. However, even in this case,
7.3 Machine parameters and secondary beams

Table 5 gives the parameters for proton machines that are existing, are under construction, or have been proposed. It should be noted that these are numbers for a full-intensity beam. In the case of the MSR proton driver, for the reasonable assumption of 3 pulses every 2 seconds, i.e., 10% of the beam, the cycle rate and protons/sec should be divided by 10.
Table 5: Comparison of proton intensities from existing, projected and proposed machines.

<table>
<thead>
<tr>
<th>Machine</th>
<th>Beam Energy (GeV)</th>
<th>Beam Current (µA)</th>
<th>Cycle Rate (Hz)</th>
<th>p/Pulse</th>
<th>p/sec</th>
</tr>
</thead>
<tbody>
<tr>
<td>CERN PS</td>
<td>26</td>
<td>1.6</td>
<td>0.5</td>
<td>$2 \times 10^{13}$</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>BNL AGS</td>
<td>24(30)</td>
<td>$\sim 5$</td>
<td>0.3</td>
<td>$3 \times 10^{13}$</td>
<td>$10^{14}$</td>
</tr>
<tr>
<td>FNAL MI (2002?)</td>
<td>120</td>
<td>1.6</td>
<td>0.33</td>
<td>$4 \times 10^{14}$</td>
<td>$6 \times 10^{13}$</td>
</tr>
<tr>
<td>JHF (2006/7?)</td>
<td>50</td>
<td>10</td>
<td>0.16</td>
<td>$4 \times 10^{12}$</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>KEK</td>
<td>12</td>
<td>0.16</td>
<td>0.25</td>
<td>$4 \times 10^{12}$</td>
<td>$10^{12}$</td>
</tr>
<tr>
<td>KAON (Defunct)</td>
<td>30</td>
<td>100</td>
<td>10</td>
<td>$6 \times 10^{13}$</td>
<td>$6 \times 10^{13}$</td>
</tr>
<tr>
<td>CERN MSR source (201N?)</td>
<td>24</td>
<td>160</td>
<td>15</td>
<td>$7 \times 10^{13}$</td>
<td>$10^{13}$</td>
</tr>
</tbody>
</table>

Table 6 gives the characteristics of beams available at the Brookhaven AGS operating at 24 GeV [79].

Table 6: Kaon beams available from the Brookhaven AGS.

<table>
<thead>
<tr>
<th>Beam</th>
<th>Kaon Mom. (GeV/c)</th>
<th>$\delta p/p$</th>
<th>FWHM (deg)</th>
<th>Prod. Angle (deg)</th>
<th>$\Delta \Omega$ (msr)</th>
<th>Flux per $10^{13}$ p</th>
<th>Purity</th>
<th>Rem.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C4</td>
<td>$\lesssim 0.83$</td>
<td>4</td>
<td>0</td>
<td>12</td>
<td>$4.6 \times 10^6$</td>
<td>$1.5 \times 10^6$</td>
<td>$\pi^+ / K^+$ = 0.4</td>
<td>$L=18m\sim 10^6 K^+$ stopped per $10^{13}$ p</td>
</tr>
<tr>
<td>D6</td>
<td>$\lesssim 1.9$</td>
<td>6</td>
<td>5</td>
<td>1.6</td>
<td>$5.5 \times 10^6$</td>
<td>$2.3 \times 10^6$</td>
<td>$\pi^- / K^-$ = 0.8</td>
<td>$L=31m$</td>
</tr>
<tr>
<td>B5</td>
<td>2–20</td>
<td>1–4.5</td>
<td>0.1</td>
<td>$K^0_L$ flux @ 3.75°C $\sim 1.3 \times 10^8$</td>
<td>$n / K^0_L = 20$</td>
<td>$L=10m$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the MSR proton driver that we assume has a similar energy to that of the BNL AGS, it is easy to estimate the intensities of analogous beams produced by the proton driver for the MSR, using $1/10$ of its intensity for a Kaon beam.

a) The C4 beam would deliver:

\[
\frac{4.6 \times 10^7 K^+}{\sec} \quad \text{at} \ 0.8 \ \text{GeV/c} \quad (18)
\]

\[
\frac{1.5 \times 10^7 K^-}{\sec} \quad \text{at rest} \quad (19)
\]

b) The D6 beam would deliver:

\[
\frac{5.5 \times 10^7 K^+}{\sec} \quad \text{at} \ 1.8 \ \text{GeV/c} \quad (20)
\]

\[
\frac{2.3 \times 10^7 K^-}{\sec} \quad \text{at} \ 1.8 \ \text{GeV/c} \quad (20)
\]

c) The B5 beam would deliver:

\[
1.3 \times 10^9 K_L /\text{sec in the range} \ 2 - 20 \ \text{GeV with} \ n / K^0_L \sim 20 \quad (21)
\]

7.4 Summary

On the basis of this brief survey, we consider the following points to be established.
a) A high-energy (> 10 GeV) MSR source is needed to produce competitive kaon beams.
b) For the MSR source assumed here, with an energy of 24 GeV and a beam power of 4MW, a 10% share of the protons will produce about 10 times the flux of kaons produced by 100% of the CERN PS or 3 times the flux produced by 100% of the Brookhaven AGS.
c) In order to be able to utilize these high fluxes, even in separated beams, the short, rapid cycling synchrotron pulses must be stretched so that the duty cycle is increased from $\sim 10^{-4}$ to $\sim 1$.
d) For neutral beams with $10^9 K_L/\text{sec}$ and $2 \times 10^{10}$ neutrons/sec, very high rates will be recorded in detectors.
e) The beam extraction, targeting and experiments will need to be very carefully designed.
f) If these technical problems can be overcome, then 10% of a 24-GeV MSR source could provide intense kaon beams at low energies, e.g., $\sim 5 \times 10^{13}$ stopping $K^+/\text{year}$ or $\sim 5 \times 10^{14} K_0^0/\text{year}$ decaying over a 15m length.

c) In order to stop 1% of $10^{15}$ energetic neutrons per year, the target size of the Brookhaven AGS.

g) Measuring $\frac{\text{Flux}}{\text{Total Flux}}$ of the target.

8 SUMMARY AND CONCLUSIONS

G. Buchalla, A. Ceccucci

We have studied opportunities for precision experiments in kaon physics using a high-intensity proton driver at a future muon storage ring (MSR) facility. Rare decays of kaons are excellent tools to test the flavour sector in great detail, to determine precisely the CKM matrix and to search with high sensitivity for signatures of new physics. We have stressed a number of highlights that combine outstanding physics motivation with the need of a longer term experimental effort towards the necessary measurements. As such they are suitable targets for a possible MSR front-end facility.

- $K_L \rightarrow \pi^0 \nu \bar{\nu}$: This rare decay mode is the benchmark process for determining the Jarlskog parameter $J_{CP} \sim \eta$, the invariant measure of CP violation in the standard model (SM). On the order of 1000 clean events could be used before being limited by the very small theoretical uncertainties. Simultaneously, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is an excellent new physics probe.
- $K^+ \rightarrow \pi^+ \nu \bar{\nu}$: This related CP-conserving process is sensitive to $|V_{td}|$ and has only slightly larger theoretical uncertainties than the neutral mode. It also represents a very important goal for future kaon experiments.
- $K_L \rightarrow \pi^0 e^+ e^-$: This process has a substantial contribution from direct CP violation, which is analogous to the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ amplitude and likewise has very small theoretical uncertainties. Moreover, the sensitivity to new physics is quite different for the two cases, so that a measurement of $K_L \rightarrow \pi^0 e^+ e^-$ is not redundant, but will provide additional information. However, in contrast to the neutrino mode, virtual photons can induce two more contributions in the case of $K_L \rightarrow \pi^0 e^+ e^-$, one from indirect CP violation proportional to $\varepsilon_K$ times the $K_S \rightarrow \pi^0 e^+ e^-$ amplitude, and also a CP-conserving piece adding incoherently to the rate. All three contributions have to be disentangled to extract the most interesting part from direct CP violation. For this reason the phenomenology of $K_L \rightarrow \pi^0 e^+ e^-$ is more complicated than it is in the case of $K_L \rightarrow \pi^0 \nu \bar{\nu}$. On the other hand, the prospects of kaon beams of very high intensity suggest the possibility of resolving these problems using various approaches that make use of sufficiently high statistics measurements. Building on previous proposals, we have therefore performed a new phenomenological analysis of $K_L \rightarrow \pi^0 e^+ e^-$, with a high-intensity kaon facility in mind and taking into account recent developments and results. New features in this study include in particular:

- The prospects of experimental results on $B(K_S \rightarrow \pi^0 e^+ e^-)$ in view of a heuristic theoretical estimate that this branching fraction may be large. This experimental input is crucial for analysing $K_L \rightarrow \pi^0 e^+ e^-$. 
- Methods to extract direct CP violation depending on the size of $B(K_S \rightarrow \pi^0 e^+ e^-)$, especially if it is large.
New experimental results on $K_L \to \pi^0\gamma\gamma$ and a re-analysis of the CP-conserving contribution, which appears to be well under control. Suitable cuts can further reduce the CP-conserving component in an efficient way.

Quantitative analysis of time-dependent $K_L - K_S$ interference in $K \to \pi^0e^+e^-$ decays, including the impact of a CP conserving contribution and the Greenlee background ($K_L \to e^+e^−\gamma\gamma$).

For the proposed measurements in $K \to \pi^0e^+e^−$ to be interesting, the typical number of required decaying $K_L$ is $\sim 10^{15}$ per year.

**Lepton-flavour violation in $K_L \to \mu e$, $K \to \pi\mu e$:** These processes are of special interest as they are absent in the SM, and thus serve as direct indicators of new physics. In general, these decays are often constrained in a given model both by bounds on $\mu \to e$ transitions as well as by flavour violation in the quark sector, particularly $\Delta m_{K}$ However, since decays such as $K_L \to \mu e$ involve flavour violation simultaneously in the quark and lepton sectors, direct measurements of these decays can in general give complementary information. An important scenario is provided by supersymmetric theories. In the MSSM the constraints from $\mu \to e$ transitions and $\Delta m_{K}$ are generally very tight, but $K_L \to \mu e$ branching ratios accessible to future high-intensity kaon experiments could still be allowed for certain regions of parameter space. Much larger effects just below current bounds could await discovery in the more general case of supersymmetry with $R$-parity violation. Additional interesting probes of LFV are decays of the type $K^+ \to \pi^-\mu^+\mu^+$. We have further reviewed the status and prospects of current and planned rare $K$ decay experiments that are relevant to the processes of special interest for this study. Possible improvements and strategies towards reaching the necessary higher sensitivity have been suggested.

We have also outlined machine requirements needed to realize the potential of kaon physics in the context of a MSR front end. The high-intensity proton source should have an energy of at least 10 GeV. As a benchmark, we have considered a proton machine with 24 GeV energy and 160 $\mu A$ beam current. A useful scenario for the purpose of comparison is given by the proposal of a European Hadron Facility (EHF) described in [80].

In conclusion, rare decays of kaons, such as $K_L \to \pi^0\nu\bar{\nu}$, $K_L \to \pi^0e^+e^-$ and $K_L \to \mu e$, present excellent opportunities to obtain competitive and complementary insight into flavour physics, one of the forefront areas in high-energy research. Current or planned near-future experiments will not fully exploit the physics potential of these processes. High-intensity kaon beamlines made available by a muon storage ring complex could allow us to perform the necessary second-generation kaon experiments.

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