A simulation of Gauge Mediated Supersymmetry Breaking with \( \tilde{\tau}_1 \) as the NLSP in the ATLAS detector

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Abstract. The feasibility for mass measurements of SUSY particles from minimal GMSB models with \( \tilde{\tau}_1 \) as the NLSP in the ATLAS detector is studied using parameterized simulations. The measured particles are selected such that model parameters can be estimated.

1 Introduction

The Higgs field added to the Standard Model (SM) to render it renormalizable has a very famous hierarchy problem associated with it. The plausible models available today that can explain this hierarchy all imply new exotic physics. Probably the most favored alternative is that the world is supersymmetric (SUSY) at high energies. And since all current low energy observations can be explained without SUSY, there must exist a mechanism that breaks SUSY at lower energy into the Standard Model (SM) if SUSY shall have any room to play in the domain of TeV physics. An interesting model from an experimental point of view is Gauge Mediated SUSY Breaking (GMSB) [1]. Much of the GMSB phenomenology applicable for the ATLAS detector has already been explored in a previous study [2]. However, a very interesting part of the GMSB parameter space was not covered. That is when the \( \tilde{\tau}_1 \) slepton is the only next-to-lightest sparticle (NLSP). Depending on the lifetime of the NLSP which is one of the free parameters in GMSB, two distinct cases emerge. Either the \( \tilde{\tau}_1 \) decays close to the production vertex with \( \tau \) leptons in the final state which make the experimental situation quite difficult, or the \( \tilde{\tau}_1 \) is quasi-stable and decays outside of the detector. In the latter case the experimental signal is very clean due to the NLSPs that appear as spectacular heavy charged particles in the muon system. Actually one of the most important handles on the model as is pointed out in [3] is the NLSP lifetime. However, lifetime measurements require detailed full simulation which is outside the scope of this work. A detailed discussion can be found in [4].

2 The model

The effective models of GMSB is a sub class of models within the framework of the Minimal Supersymmetric extension of the Standard Model (MSSM). Due to general arguments e.g. soft SUSY breaking (SSB) the number of parameters in MSSM are more than 120 in addition to the 19 in the SM. Much of the MSSM parameter space is highly unphysical including e.g. rapid proton decay and lepton number violation and hence a so called R parity conservation is added ad hoc. In order to understand the model some assumptions must be made concerning the origin of the SSB terms. One such assumption is GMSB. In GMSB the observable sector \( O \) containing the Standard Model fields communicates with a messenger sector \( M \) usually built up by GUT preserving gauge fields. \( M \) then mediates the SSB by overlapping with a rather unspecified secluded sector \( S \)

\[
O \xrightarrow{SU(5)} M \xleftarrow{X} S
\]

This automatically generates small flavor violations and preserves Grand Unification. In the minimal version of GMSB (mGMSB) the model is highly predictive and completely determined by only six parameters:

- \( M_m \), the messenger scale. The messenger scale it set to 250 TeV.
- \( \Lambda_m = F_m/M_m \), the effective SUSY breaking scale. Here chosen to be 30 TeV. \( F_m \) is the fundamental SUSY breaking scale felt by the messenger fields.
- \( N_5 \), the parameterization of the \( SU(5) \) fields. Here \( N_5 = 3 \).
- \( \tan \beta \), the ratio of the Higgs vacuum expectation values at the electroweak scale.
- \( \text{sgn} \mu \), the sign of the Higgsino mass term. Here \( \text{sgn} \mu = +1 \)
- \( C_{\text{grav}} \), the gravitino mass scaling factor.

Throughout this analysis the GMSB events are generated by the GMSB implementation in ISAJET 7.48 [5]. The major theoretical arguments at the moment against GMSB are the origin of the Higgs parameters \( \mu \) and \( B \) see [1], which in mGMSB are assumed to be determined at the EW-scale, and the somewhat ad hoc use of gauge fields.
3 Phenomenology

3.1 The NLSP and the mass spectrum

The dominant SUSY production in a high energy hadron collider is through glue and quark interactions into gluino or squark pairs [6]. However, there is also an interesting contribution from prompt neutralino, chargino and slepton production [7]. Due to the imposed R parity the particles will be pair produced and decay down to the LSP. The LSP in mGMSB models is a very light gravitino for any relevant value of the fundamental SSB scale $F \sim C_{\text{grav}} M_P$:

$$m_{\tilde{g}} = \frac{F}{\sqrt{3} M_P} \leq 1 \text{keV},$$

where $M_P$ is the reduced Planck mass. This is expected since the model is constructed to keep flavor violations small and hence gravitational effects must be suppressed. The upper bound on $m_{\tilde{g}}$ actually comes from cosmology [1]. For most of the parameter space the NLSP is either a $\tilde{\chi}_1^0$, co-NLSP $\tilde{t}_R$, a $\tilde{\tau}_1$ or $\tilde{\chi}_1^0$ and $\tilde{\tau}_1$ co-NLSP. In a very small corner of parameter space $\tilde{\nu}$ NLSP is also possible. An inverted case with the $\tilde{g}$ as NLSP happens when $M_m$ is very large. For the parameters in this analysis (G3) the $\tilde{\tau}_1$ is the NLSP, see Table 1.

The lifetime of the NLSP can be controlled by the $C_{\text{grav}}$ parameter. For $C_{\text{grav}} = 1$ the NLSP will decay close to the interaction vertex while for $C_{\text{grav}} = 5000$ the decay will most likely take place outside of the detector. As $\tan \beta$ is increased the $\tilde{\tau}_1$ becomes the NLSP due to the mixing effects proportional to $m_\tau$. The other SUSY mass differences due to the shift in $\tan \beta$ appears to be of less phenomenological importance. The ISAJET sparticle mass spectrum is shown in Table 2.

The connection between the model parameters and the physical mass spectrum is according to [1] at leading order proportional to $N_\eta A_m$ for the gaugino masses, and $\sqrt{N_\eta A_m}$ for the slepton masses. There is also an overall logarithmic dependence on $M_m$ from to the boundary conditions for the renormalization group equations.

<table>
<thead>
<tr>
<th>Particle</th>
<th>G2</th>
<th>G3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{g}$</td>
<td>699</td>
<td>699</td>
</tr>
<tr>
<td>$\tilde{u}_L$</td>
<td>658</td>
<td>658</td>
</tr>
<tr>
<td>$\tilde{d}_L$</td>
<td>662</td>
<td>662</td>
</tr>
<tr>
<td>$\tilde{t}_2$</td>
<td>674</td>
<td>672</td>
</tr>
<tr>
<td>$\tilde{b}_2$</td>
<td>640</td>
<td>644</td>
</tr>
<tr>
<td>$\tilde{e}_L$</td>
<td>204.1</td>
<td>204.5</td>
</tr>
<tr>
<td>$\tilde{\tau}_2$</td>
<td>204</td>
<td>207</td>
</tr>
<tr>
<td>$\tilde{\nu}$</td>
<td>189</td>
<td>188</td>
</tr>
<tr>
<td>$h^0$</td>
<td>106</td>
<td>111</td>
</tr>
<tr>
<td>$A^0$</td>
<td>334</td>
<td>311</td>
</tr>
<tr>
<td>$\tilde{\chi}_1^0$</td>
<td>113.9</td>
<td>115.5</td>
</tr>
<tr>
<td>$\tilde{\chi}_2^0$</td>
<td>277</td>
<td>265</td>
</tr>
<tr>
<td>$\tilde{\chi}_1^{\pm}$</td>
<td>191.8</td>
<td>193.0</td>
</tr>
</tbody>
</table>

Table 2. Masses in GeV as $\tan \beta$ is increased from 5 to 12. Note that $\tilde{e}$ and $\tilde{\mu}$ are degenerate. The model G2 was investigated in [2]. The sleptons are co-NLSP in this case.

3.2 Topology

The typical signature at the end of the decay chain for the G3 model is

$$\tilde{\chi}_1^0 \rightarrow \tilde{\nu}_R \tilde{\mu}^\mp \tilde{\tau}_1 \mu^\pm (\tau) \mu^\pm \rightarrow \tilde{G} \tau \mu^\pm (\tau) \mu^\pm$$

Due to the gravitino ($\tilde{G}$) a lot of missing transverse energy ($E_T$) is produced. The only exception is when the NLSP is quasi-stable and charged, then its mass and momentum can be completely reconstructed in the muon system.

The signature for the G3 models unconditionally involves $\tau$ which makes the detection challenging. The decay mode at the end of the decay chain is either of the type

$$\tilde{\nu}_R (\tilde{\chi}_1^0) \rightarrow \tilde{\tau}_1 + \mu^\pm (e^\pm) + (\tau)$$

when the $\tilde{\tau}_1$ decays outside the detector, or if the $\tilde{\tau}_1$ lifetime is very short

$$\tilde{\tau}_1^\pm \rightarrow \tau^\pm + \tilde{G}$$

The $\tau$ lepton in the decay (1) is most likely undetectable due to the limited available phase-space.

A very large fraction of the events contains the decay

$$\tilde{\chi}_1^0 \rightarrow \tilde{\nu}_R (\tilde{\chi}_1^0) + \mu^\pm (e^\pm)$$

which provides a very clear same flavor opposite charge (SOC) dilepton signal. The SOC results in sharp mass edges (see appendix A) and also removes most of the light QCD background. By forming the flavor combination

$$\mu^+ \mu^- + e^+ e^- + \mu^{\pm} e^\mp$$

a lot of the combinatorial background can be subtracted.

A standard handle on the SUSY signals is $M_{eff}$. $M_{eff}$ is defined as

$$M_{eff} = \max_{i} \sum_{j=1}^{4} |p_{T,j}^\tau| + E_T,$$

i.e. it is defined as the scalar sum of the four leading jets plus missing transverse energy. This is very useful since squarks and gluinos generate a lot of hard jets. For prompt $\tilde{\chi}_1^0$, $\tilde{\chi}_1^0$ and $\tilde{l}$ production this is not the case, and one has to rely on $E_T$ and e.g. a lepton signature.
3.3 From mass measurements to model parameters

In the same way as parameters in the SM can be extracted from various precision observables, parameters in an underlying SUSY model can be estimated. Following the approach outlined in [2], sparticle masses are selected such that they can be used in a fit for the most probable model. If $\sigma$ and $C_{\text{tree}}$ are held fixed there are four independent parameters. Hence, in absence of degeneracy four independent masses should be enough to pin point the model. The light higgs mass $m_h$ is assumed to be measured independently within $\pm 3$ GeV. The minimum set of masses is a squark or gluino mass, the $\chi_i^0$ and $\chi_i^2$ masses\(^1\) and a slepton mass. Conceptually the situation looks like:

\[
p_j \xrightarrow{M_j} m_i \xrightarrow{D} m_i^*\tag{5}
\]

where $m_i = M_i(p_j)$ are the masses of the model parameters $p_j$, $m_i^* = D(m_i)$ represents the smearing of the detector. To find the inverse mapping back to $p_j$ a fit can be made using the estimated errors from the mass measurements.

\[
\chi^2 = \sum_i \frac{(M(p_j) - m_i^*)^2}{\sigma_i^2}\tag{6}
\]

Since the mass resolutions in these models in the end turn out to be comparable with those in [2], the actual fit to the model parameter space is not evaluated in this analysis.

4 Experimental setup

4.1 Production

4.1.1 SUSY Signal and integrated luminosity

The total cross-section for SUSY production in the G3 models is 17 pb. Assuming the LHC low luminosity of $10^{33}$ cm$^{-2}$s$^{-1}$, one year yields a total integrated luminosity of 30 fb$^{-1}$. Let us say that 10 fb$^{-1}$ will be available, then 170000 SUSY events should be simulated for correct statistical fluctuations. Here all SUSY models use 170000 signal events to represent one year of low luminosity.

Of the total signal 10% is due to prompt gaugino and slepton production. Since the jet multiplicity for prompt production is much lower than for the strong production, a jet veto enables very effective selection of this type of events. Prompt production can e.g. be interesting in an exclusive cross-section analysis.

4.1.2 Standard model background

For model G3b the SM background is strongly suppressed by the presence of heavy charged particles. These can be experimentally identified through time of flight and/or ionisation measurements. However, since the signature in G3a contains SM particles only, the SM background is significant. The Monte Carlo cross-sections for the main SM contributions are listed in Table 3. Only backgrounds with potentially large $E_T$ and dilepton signals have been evaluated. All background events are simulated by PYTHIA [8]. Examples of background compared to signal are shown in Figure 1. The number of plotted events is always rescaled to 10 fb$^{-1}$ and hence the statistical fluctuations are equal or larger than expected.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\sigma$(pb)</th>
<th>Ev. prod.</th>
<th>Ev. simul.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t\bar{t}$ (833 pb)</td>
<td>580</td>
<td>5.8 M</td>
<td>400 k</td>
</tr>
<tr>
<td>$W\ell(\not{p}_T &gt; 20 \text{ GeV})$</td>
<td>35000</td>
<td>350 M</td>
<td>1200 k</td>
</tr>
<tr>
<td>$Z\ell(\not{p}_T &gt; 20 \text{ GeV})$</td>
<td>580</td>
<td>5.8 M</td>
<td>1.0 k</td>
</tr>
<tr>
<td>WW, WZ, ZZ</td>
<td>65</td>
<td>650 k</td>
<td>1200 k</td>
</tr>
<tr>
<td>QCD($\not{p}_T &gt; 50 \text{ GeV}$)</td>
<td>$23 \cdot 10^6$</td>
<td>2.3 x 10$^{11}$</td>
<td>300 k</td>
</tr>
<tr>
<td>$b\bar{b}(\not{p}_T &gt; 50 \text{ GeV})$</td>
<td>880000</td>
<td>880 M</td>
<td>300 k</td>
</tr>
</tbody>
</table>

Table 3. SM backgrounds used in the simulations. All Z are $Z \rightarrow \tau^+\tau^-$. A separate sample of $b\bar{b}$ is included since it produces potentially dangerous di-muon signals at the 1.2% level.

4.2 Trigger

The signal has a fairly high probability to pass the trigger due to the presence of very high transverse momentum ($p_T$) jets, see e.g. the leading jet variable in Figure 1, and large amount of missing energy. The results given by ATLFAST for the standard trigger menu options XE50+J50 and J180 are shown in Table 4. XE50 represents $E_T$ above 50 GeV, and J180 represents jets with $p_T$ above 180 GeV. To extract the relevant efficiencies for the XE trigger which only uses the calorimeter information, muons were removed at the generator output. The numbers within parenthesis includes an optional refinement from the muon system, i.e. standard ATLFAST behaviour. For prompt production in G3a, the trigger criteria $\tau 20 + \text{XE30}$ combined with a jet veto was investigated. The acceptance for prompt production with this trigger was found to be 9%.

Since the trigger efficiencies are so high for both models, a 100% trigger efficiency is assumed in the rest of the analysis. It also gives a conservative estimate of the background.

4.3 Detection

4.3.1 Ordinary particles

The response of the ATLAS detector is simulated by fast parameterizations using the ATLFAST package [10]. The default settings are used. That is, the acceptance is limited to $|y| < 2.5$, and reliable results should be expected.
for ordinary particles since the $p_T$ thresholds are conservative: $\gamma > 5$ GeV, $\mu^+ > 5$ GeV and $\mu^- > 6$ GeV. Jets are reconstructed with an R=0.4 cone after binning and smearing the fragmented generator particles according to the calorimeter performance. The jet threshold is $p_T > 15$ GeV. The $\mu$ and $e$ reconstruction efficiency is assumed to be 100%. Note that in the ATLFAST muon trigger simulation, parameterized muon efficiencies are included but not used since 100% trigger efficiency is assumed. The (mass)$^2$ is always assumed to be much smaller than the (momentum)$^2$ except for the quasi-stable NLSP, hence $p_T$ equals $E_T$.

### 4.3.2 Tau particles

The tau particles are reconstructed by the hadronic decay modes using standard ATLFAST tau jet isolation criteria. The efficiency and jet rejection are parameterized using the result from a separate tau study [11] where full simulation of both calorimeter isolation and inner tracker constraints are used. The parameterization is $p_T$ dependent. E.g. for $15 < p_T < 30$ GeV a 60% efficiency gives a jet rejection of 10. Here the tau efficiency is fixed to 60%. The ATLFAST results for $M_{\tau\tau}$ and $\delta M_{\tau\tau}$ from $Z$ decays were
Table 4. Fraction of the events that pass the trigger criteria. The results are from ATLFAST with information only from the calorimeters as is the case for the first level trigger, an optional trigger object refinement from the muon system is shown within parentheses. The equivalent trigger rates are indicated in a few cases. No detector efficiencies are included. Note that the true trigger rates are higher since no pile-up is included in the simulation, at least a factor of five is expected e.g. for the XE50 trigger. The estimates for the high $p_T$ trigger should however be more reliable.

<table>
<thead>
<tr>
<th>Channel</th>
<th>XE50+J50[%%]</th>
<th>J180[%%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>G3a</td>
<td>92</td>
<td>83</td>
</tr>
<tr>
<td>G3b</td>
<td>92 (62)</td>
<td>87, 0.005 Hz</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>56 (31)</td>
<td>5 (7), 0.01 Hz</td>
</tr>
<tr>
<td>WJ ($p_T &gt; 20$ GeV)</td>
<td>4</td>
<td>0.7</td>
</tr>
<tr>
<td>ZJ ($p_T &gt; 20$ GeV)</td>
<td>6</td>
<td>0.6</td>
</tr>
<tr>
<td>WW, WZ, ZZ</td>
<td>10</td>
<td>1.3</td>
</tr>
<tr>
<td>QCD ($p_T &gt; 50$ GeV)</td>
<td>0.05</td>
<td>0.5, 100 Hz</td>
</tr>
<tr>
<td>$bb$ ($p_T &gt; 50$ GeV)</td>
<td>1.3 (0.7)</td>
<td>0.4 (0.6)</td>
</tr>
</tbody>
</table>

4.3.3 Quasi-stable charged sleptons

The detection of the quasi-stable $\tilde{\tau}_1$ is parameterized using the results in [14] and is implemented into ATLFAST. Effects of the sagitta measurement, multiple scattering and energy loss fluctuations are taken into account. For a mass measurement $(m_{\tau_2} = p/(\beta \gamma))$ $\beta$ is limited to $0.8 < \beta < 0.91$, see Figure 2. At $\beta = 0.85$ the $\beta$ measurement contribution to the resolution is 10% and the momentum contribution 3%, the two terms are independent. The lower bound on $\beta$ comes from a very conservative estimate of the time window in the trigger chambers needed for the second coordinate [13], and the upper bound from the cut on time of flight needed to reject the muons at the three sigma level [14]. Thus $m_{\tau_2}$ can be measured down to the level of the estimated 0.1% systematic effects due to the high statistics given by 60k of $\tilde{\tau}_1$ for 10 fb$^{-1}$. Once $m_{\tau_2}$ is constrained the upper bound on $\beta$ can be relaxed. 90% reconstruction efficiency is assumed within the acceptance.

5 Model G3b

In this model the $\tilde{\tau}_1$ is a quasi-stable NLSP and it decays outside of the detector. The golden event signature is a very heavy charged particle in the muon system. Virtually no standard model background is present when an additional lepton is required in the event. As described in the previous section, the $\tilde{\tau}_1$ mass is measured independently in the muon system. This enables the momentum resolution to be vastly improved by a mass constraint. Since everything in the decay chain $\chi_{1,2}^0 \rightarrow l^+ l^- l^-\tilde{\tau}_1 (\tau)$ is well measured except for the very soft $\tau$, the reconstruction is almost trivial.

5.1 Slepton reconstruction

Only the lowest mass combination is kept in order to reduce the combinatorial background. No charge information is useful here due to the unmeasurable $\tau$ lepton. The raw invariant mass spectrum for $M_{\tau_2 l}$ is shown in Figure 3. After adding the missing $\tau$ mass to $M_{\tau_2 l}$ it should equal the $M_\tau$ mass. The edge in the effective mass spectrum ($M$) is extracted by a fit with the function

$$f(x) = \frac{a + b(x - M)}{1 + e^{c(x - M)}} + d.$$
The fit yields

\[ M_{\tilde{t}_1} = M_{\tilde{t}_2} + m_t = 103.3 \text{ GeV (103.3 ± 0.05 GeV)}. \tag{8} \]

The masses within parenthesis are from the fit, and the error is the statistical error only. Within the validity of the fitted function, the result is robust with respect to the mass window.

### 5.2 Neutralino reconstruction

The \( \chi_{1,2}^0 \) masses can either be found directly by adding the next particle in the decay chain as is shown in Figure 4, or indirectly from \( M_{\tilde{e}_{1,2}} \) as is shown in Figure 5. The \( M_{\chi_2^0} \) is well determined from \( M_{\tilde{e}_{1,2}} \) with the candidate selected from Figure 3 within the mass window 99-102 GeV. The result from a fit to the same function as in Figure 3 gives

\[ M_{\chi_2^0} = M_{\tilde{e}_{1,2}} + m_e = 115.3 \text{ GeV (115.3 GeV ± 0.1 GeV)}. \tag{9} \]

The result in insensitive to the selected mass window. \( M_{\chi_2^0} \) seems to be better determined from the dilepton spectrum. For an introduction to effective invariant masses see appendix A. Edges in the dilepton spectrum are expected at

\[ M_{ll}^{\text{max}} = M_{\chi_2^0} \sqrt{1 - \frac{M_{\tilde{e}_1}^2}{M_{\chi_2^0}^2} \left( 1 - \frac{(M_{\tilde{e}_1} + M_{\tilde{e}_2})^2}{M_{ll}^2} \right)} = 14.5 \text{ GeV (14.3±0.05 GeV)}, \tag{10} \]

and at

\[ M_{ll}^{\text{max}} = M_{\chi_2^0} \sqrt{1 - \frac{M_{\tilde{e}_2}^2}{M_{\chi_2^0}^2} \left( 1 - \frac{(M_{\tilde{e}_1} + M_{\tilde{e}_2})^2}{M_{ll}^2} \right)} = 46.8 \text{ GeV (46.2±0.3 GeV)}. \tag{11} \]

The fitted function consists of two first order polynomials. The statistical error is very small, hence the dominating error is due to the uncertainty in the shape of the background subtracted signal. Please note that all figures containing dilepton pairs are improved by the flavor subtraction \( \mu^+\mu^- + e^+e^- - \mu^\pm e^\mp \).

### 5.3 Squark reconstruction

The \( \tilde{q}_{L,R} \) masses can be extracted from edges in the invariant mass spectrum \( M_{llj} \). The dominant decay is \( \tilde{q}_R \rightarrow \chi_0^0 q \), see the upper plot in Figure 6. The events are selected by requiring \( M_{ll} < 15 \text{ GeV} \), \( p_T > 65 \text{ GeV} \), 2 jets with \( p_T > 25 \text{ GeV} \) and 2 jets with \( p_T > 50 \text{ GeV} \). The lower plot in Figure 6 also shows the spectrum \( M_{llj} \), but with

\[ M_{llj}^{\text{max}} = M_{llj} \sqrt{1 - \frac{M_{\chi_0^0}^2}{M_{llj}^2} \left( 1 - \frac{(M_{\tilde{q}_L} + M_{\tilde{q}_R})^2}{M_{llj}^2} \right)} = 320 \text{ GeV (316±3 GeV)}, \tag{12} \]

15 GeV < \( M_{llj} < 46 \text{ GeV} \). Then the dominant decay is \( \tilde{q}_L \rightarrow \chi_0^0 q \). The expected edges are
Fig. 5. The dilepton invariant mass spectrum $M_{ll}$. Fits to the edges yield the result: $M_{ll} = 14.3 \pm 0.05$ GeV, and $M_{ll} = 46.2 \pm 0.3$ GeV.

and

$$M_{ll}^{max} = M_{ll} \sqrt{1 - \frac{M_{\tilde{\chi}_0^0}^2}{M_{ll}^2}} \sqrt{1 - \frac{(M_{\tilde{\chi}_0^0} + M_{l})^2}{M_{ll}^2}} = 546 \text{ GeV} \ (539 \pm 5 \text{ GeV}). \quad (13)$$

The errors are errors from the fit generously rounded upwards. The estimation is based on values taken from several fits within different regions and different sets of data. Errors from absolute energy scale calibrations are not included. Given the six mass measurements, it is easy to solve for the six masses.

5.4 Proof of the NLSP flavor

In the G3b model one cannot prove the flavor of the NLSP unless the soft tau is found. Since the $p_T$ of the $\tau$ is so low it will be very hard to find it the calorimeter. However, apart from the soft tau everything else is measurable. Thus, after three sparticle masses have been determined in the decay chain, the tau momentum can be solved by imposing the mass constraints, see appendix B for details.

Three masses and four 4-vectors are needed in order to fix the soft $\tau$ momentum. A powerful candidate for this type of the decay chain is $\tilde{\ell}_L \rightarrow \chi_0^0 l \rightarrow \tilde{\ell}_R l^+ l^- \rightarrow \tilde{\tau}_1 (\tau)^- l^+ l^-$ since leptons are easy to measure. A very simple test of this technique has been done assuming perfect knowledge of the reconstructed sparticle masses. This is not a bad approximation judging from the results in the previous sections if the systematic affect are under control, but the robustness against mass errors remains to be tested. The solution in appendix B has been implemented and applied on the events from ATLFAST. The particles are reconstructed in the same way as is described in the previous sections by successive bottom-up reconstruction of the decay chain. The $\tilde{\ell}_L$ was selected within a mass window of $\pm 6$ GeV. To get a hint of the performance, the opening angle between the true $\tau$ vector and the mass constrained estimation $\tau^*$ is shown in Figure 7. The vector $\tau$ could e.g. be replaced with tau vertex candidates from the inner tracker and a correlation of the type seen in Figure 7 would constitute a definite proof of the NLSP flavor.

6 Model G3a

In the G3a model the NLSP is assumed to decay close to the vertex. As a result, no SUSY particle is directly de-
when a soft lepton is selected, see the upper plot in Figure 8. When selecting hard leptons with $p_T > 15$ GeV an edge is expected at
\[ M_{\mu_l}^{\text{max}} = \sqrt{M_{\tilde{\mu}_l}^2 - M_{\tilde{\nu}_l}^2} = 51.7 \text{ GeV (51.1±1 GeV)}, \quad (16) \]
see the lower plot in Figure 8. The plots are surprisingly free of SM background. Very powerful are the dilepton and hard tau requirement. Please note that the statistics is too low, e.g. a factor 10 for $t\bar{t}$. Hence the fluctuations are too large to represent the real background. Proper statistics should only improve the situation. The edges for $M_{\mu_l}$ and $M_{\mu_l\mu_l}$ are the same as in G3b. However for the SOC dilepton spectrum the background is more complicated. The background under the signal is not flat and causes a systematic shift of the second edge. In order to reduce the background an extra requirement of at least one tau with $p_T > 65$ GeV is applied. The results from the dilepton and dilepton plus jet fits are

\[ M_{\mu_l}^{\text{max}} = 14.5 \text{ GeV (14.3±0.1 GeV)}, \]
\[ M_{\mu_l}^{\text{max}} = 46.8 \text{ GeV (42.3±5 GeV)}, \]
and
\[ M_{\mu_l\mu_l}^{\text{max}} = 320 \text{ GeV (313±6 GeV)}, \]
\[ M_{\mu_l\mu_l}^{\text{max}} = 546 \text{ GeV (530±5 GeV)}, \]

The measured endpoint is systematically lower in many cases. This is expected to some extent since taus and jets leak out of the cone. The tau also has an unmeasured neutrino.

At this point, just as in model G3b, we now have enough mass relations to solve for six unknown masses which enables us to do a fit for the most probable model parameters.

7 Conclusions

The signals from GMSB models with $\tilde{\tau}_1$ as the NLSP have been investigated. The response of the ATLAS detector has been simulated in a simple parameterized way and with no pile-up. Both the case with a quasi-stable NLSP (G3b), and the case with a NLSP decaying close to the vertex (G3b). The G3b model can be fully reconstructed with good mass resolution similar to related models studied previously [2]. The G3a model is harder than G3b from an experimental point of view due to the fact that the decay chain reconstruction is based on tau leptons in the final state. However, also in this case can masses of sleptons, neutralinos and squarks be determined with a precision of the order of GeV, and few GeV for squarks. As in the case [2] a reasonable estimation of the model parameters should be possible from the measured masses.

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**A Effective mass kinematics in decay chains**

One of the basic tools in SUSY measurements are edges in the effective mass spectrum. These masses can be formed from squares of arbitrary 4-vectors at different stages in the decay chain. As an illustration I here derive the upper edge for the simplest possible non trivial effective mass, i.e.:

\[ m^2_{BD} = (p_B + p_D)^2 \]  

in the decay:

\[ A \rightarrow B + C, C \rightarrow D + E \]  

The derivation is general, but since it is used in the dilepton effective mass analysis, the leptons (B and D) are assumed to be massless. The equality sign indicates when this approximation is made. The first step is to find the momentum in the CM of the decaying particle:

\[ (E_B, p_B) \leftarrow A \rightarrow (E_C, -p_B) \]  

\[ (E_D', p_D') \leftarrow C' \rightarrow (E_E', -p_D') \]
The boost between the frames is then simply
\begin{equation}
\beta = \frac{m_A^2 - m_C^2}{m_A^2 + m_C^2},
\end{equation}
\begin{equation}
\gamma = \frac{m_A^2 - m_C^2}{2 m_A m_C}.
\end{equation}

By boosting to the unprimed system one finds
\begin{equation}
E_D = \gamma E'_D + \gamma \beta E'_D = \frac{m_A^2 - m_C^2}{2 m_C} m_A
\end{equation}

The upper edge in the effective mass can now be found when \( B \) and \( D \) are back-to-back
\begin{equation}
\max m_{BD}^2 = 4E_B E_D = m_B^2 \left(1 - \frac{m_C^2}{m_A}ight) \left(1 - \frac{m_C^2}{m_B}ight)
\end{equation}

The same technique can be used to construct more complex effective masses.

### B Mass constraints

Given three masses in a decay chain it is possible to reconstruct the momentum of one of the decay products. This solution is due to [2] but is given here explicitly for the generic case. The reconstructed momentum is solved for \( \mathbf{p}_1 \). The equation system is:
\begin{align}
(p_0 + p_1 + p_2)^2 &= M_A^2 \\
(p_0 + p_1 + p_2 + p_3)^2 &= M_B^2 \\
(p_0 + p_1 + p_2 + p_3 + p_4)^2 &= M_C^2
\end{align}

Expand the squared 4-vectors and collect \( \mathbf{p}_1 \) on the left hand side by subtracting the equations \( E_a \equiv E_0 + E_2 \), \( \mathbf{p}_a \equiv \mathbf{p}_0 + \mathbf{p}_2 \):
\begin{align}
2E_1 E_a - 2 \mathbf{p}_1 \cdot \mathbf{p}_a &= M_A^2 - m_0^2 - m_1^2 + m_2^2 + m_A^2 + m_B^2 \\
-2E_0 E_2 + 2 \mathbf{p}_0 \cdot \mathbf{p}_2 &= C_1
\end{align}
\begin{align}
2E_1 E_3 - 2 \mathbf{p}_1 \cdot \mathbf{p}_3 &= M_2^2 - M_1^2 - m_0^2 + m_2^2 + m_3^2 + m_A^2 + m_B^2 \\
-2E_0 E_3 - 2E_2 E_3 + 2 \mathbf{p}_0 \cdot \mathbf{p}_3 + 2 \mathbf{p}_2 \cdot \mathbf{p}_3 &= C_2
\end{align}
\begin{align}
2E_1 E_4 - 2 \mathbf{p}_1 \cdot \mathbf{p}_4 &= M_3^2 - M_2^2 - m_0^2 + m_3^2 + m_4^2 + m_A^2 + m_B^2 \\
-2E_0 E_4 - 2E_2 E_4 - 2E_3 E_4 + 2 \mathbf{p}_0 \cdot \mathbf{p}_4 + 2 \mathbf{p}_2 \cdot \mathbf{p}_4 + 2 \mathbf{p}_3 \cdot \mathbf{p}_4 &= C_3
\end{align}

We want \( \mathbf{p}_1 \) alone on the left hand side. \( E_i ([\mathbf{p}_1]) \) is eliminated by subtraction:
\begin{align}
(31) \cdot E_3 - (32) \cdot E_a \Rightarrow \\
(2 \mathbf{p}_3 E_a - 2 \mathbf{p}_a E_3) \cdot \mathbf{p}_1 &= E_3 C_1 - E_a C_2 \\&\Rightarrow \\
D_1 \cdot \mathbf{p}_1 &= F_1
\end{align}
\begin{align}
(32) \cdot E_4 - (33) \cdot E_3 \Rightarrow \\
(2 \mathbf{p}_4 E_3 - 2 \mathbf{p}_3 E_4) \cdot \mathbf{p}_1 &= E_4 C_2 - E_3 C_3 \\&\Rightarrow \\
D_2 \cdot \mathbf{p}_1 &= F_2
\end{align}

Fig. 10. The same \( M_{ij} \) distributions as in G3b, but for G3a and with SM background included. The background is negligible under the fitted region.

After expanding Equation (17) and solving for \( \mathbf{p}^2 \) one finds
\begin{equation}
\mathbf{p} = \frac{m_A^2 - m_C^2}{2 m_A}. 
\end{equation}

Thus
\begin{equation}
E_B \simeq \frac{m_A^2 - m_C^2}{2 m_A}, 
\end{equation}
\begin{equation}
E_C \simeq \frac{m_A^2 + m_C^2}{2 m_A}.
\end{equation}
Given two linear equations we can now find a solution for e.g. \( p_{1x} \) and \( p_{1y} \)

\[
\begin{align*}
D_{1x}p_{1x} + D_{1y}p_{1y} + D_{1z}p_{1z} &= F_1' \\
D_{2x}p_{1x} + D_{2y}p_{1y} + D_{2z}p_{1z} &= F_2' \\
\Rightarrow \quad \left( \frac{D_{1x}}{D_{2x}}, \frac{D_{1y}}{D_{2y}} \right) \left( p_{1x}, p_{1y} \right) &= \left( \frac{F_1' - D_{1z}}{F_2' - D_{2z}} \right) \left( p_{1z} \right)
\end{align*}
\]

(36)

\[
\begin{align*}
p_{1x} &= A_{11} + A_{12}p_{1z} \\
p_{1y} &= A_{21} + A_{22}p_{1z}
\end{align*}
\]

(37)

These solutions are then plugged into equation (31) and squared to avoid the square root in the expression for the energy

\[
E_1^2 E_a = C_1 + 2p_1 \cdot p_a
\]

\[
E_1^2 E_a' = C_1^2/4 + (p_1 \cdot p_a)^2 + C_1 p_1 \cdot p_a
\]

\[
E_1^2 E_a^2 = C_1^2/4 + (p_1 \cdot p_a)^2 + C_1 p_1 \cdot p_a
\]

\[
(p_{1x}^2 + p_{1y}^2 + p_{1z}^2 + m_1^2)^2 E_a = C_1^2/4 + (p_{1x} p_{ax} + p_{1y} p_{ay} + p_{1z} p_{az})^2 + C_1 (p_{1x} p_{ax} + p_{1y} p_{ay} + p_{1z} p_{az})
\]

(39)

Once the solution for \( p_{1x} \) and \( p_{1y} \) is plugged in it gets kind of messy, but it is just an ordinary quadratic equation in \( p_{1z} \):

\[
-(A_{11}^2 + A_{12}^2 p_{1z}^2 + 2A_{11} A_{12} p_{1z} + A_{21}^2 + A_{22}^2 p_{1z}^2 + +2A_{21} A_{22} p_{1z} + p_{1z}^2 + m_1^2) E_a^2 + C_1^2/4 + (A_{11}^2 + +A_{12}^2 p_{1z}^2 + 2A_{11} A_{12} p_{1z} + A_{21}^2 + A_{22}^2 p_{1z}^2 + +2A_{21} A_{22} p_{1z} + p_{1z}^2 + m_1^2) E_a = 0
\]

(40)

After coefficient identification the final solution reads

\[
p_{1z} = \frac{-B_1 \pm \sqrt{B_1^2 - 4B_0 B_2}}{2B_2}
\]

(41)

where

\[
B_0 = -(A_{11}^2 - A_{12}^2 - m_1^2) E_a^2 + C_1^2/4 + A_{11}^2 p_{ax}^2 + A_{21}^2 p_{ay}^2 + 2A_{11} A_{21} p_{ax} p_{ay} + +C_1 A_{11} p_{ax} + A_{21} p_{ay}
\]

(42)

\[
B_1 = 2(-A_{11} A_{12} - A_{11} A_{22}) E_a^2 + 2A_{11} A_{12}^2 p_{ax}^2 + +2A_{21} A_{22} p_{ay}^2 + 2 p_{ax} p_{ay} (A_{11} A_{22} + A_{12} A_{21}) + +2A_{11} p_{ax} p_{ax} + 2A_{21} p_{ay} p_{az} + +C_1 (A_{12} p_{ax} + A_{22} p_{ay} + p_{az})
\]

(43)

\[
B_2 = -(A_{12}^2 - A_{22}^2 - 1) E_a^2 + A_{12}^2 p_{ax}^2 + A_{22}^2 p_{ay}^2 + +p_{ax}^2 + 2A_{12} A_{22} p_{ax} p_{ay} + 2A_{12} p_{ax} p_{az} + +2A_{22} p_{ay} p_{az}
\]

(44)