THE 200 MHZ TRAVELLING WAVE CAVITIES IN THE SPS

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Abstract
The RF power limits of the travelling wave cavities under heavy beam loading and their consequences will be discussed.

1 INTRODUCTION

The 200 MHz travelling wave cavities in the SPS are located in the tunnel at LSS3. They are bar loaded transmission lines consisting of four or five sections of eleven cells each [1]. The RF power amplifiers are located at the surface and are linked to the travelling wave cavities by coaxial lines (Z₀ = 50Ω). The series impedance of the travelling wave cavity structure is given by R₂ = 27.1 kΩ.

With respect to beam loading, the impedances Z₁ = V₉F / I₂ and Z₂ = Vb / I₀, are of interest. Here, V₉F is the axial voltage produced by the RF generator current, I₂, as it would be seen by the beam traversing the cavity not taking into account beam loading. Vb is the voltage induced by the beam current I₀.

The impedance Z₁ (adapted from [1]) is given by

\[ Z₁ = \sqrt{\frac{R₂ l² Z₀}{2}} \sin \frac{\omega l}{2} \]

where the group velocity, v₂, in the travelling wave cavity is given by v₂/c = 0.0946, and the cavity centre frequency \( ω_0 = 2π \times 200.222 \) MHz.

The impedance Z₁ is proportional to the interaction length of the cavity, l, and the transit time factor of the form \( \sin x/x \). The total phase slip between the proton bunches and the travelling wave, \( τ \), is in first approximation, as a function of the RF frequency \( ω \), given by

\[ τ = \frac{l}{v₂} (ω - ω₀) \]

The total voltage seen by a proton beam passing through a travelling wave cavity is then given by

\[ V = Z₁ I₂ + Z₂ I₀ \]  

This equation can be visualised in a vector diagram, Fig. 1, shown for a case above transition energy.

With respect to the beam current vector, \( \vec{I}_b \), the beam induced voltage \( \vec{V}_b \) points in a nearly opposing direction at an angle of about \(-τ/3\) (beam loading angle). The total voltage seen by the beam, \( \vec{V} \), is the vector sum of \( \vec{V}_RF \) (produced by the power amplifiers) and \( \vec{V}_b \).

The stable phase angle, \( φ_S \), is typically in the range of 45° (accelerating bucket) to 90° (stationary bucket) for most of the modes of operation. The beam loading angle is in the range of about \(-15°\) below transition to up to +15° above transition. The beam loading voltage is essentially a function of \( I₀ \) and \( l \) and typically in the range between zero and 3 MV. The total voltage V required per travelling wave cavity varies between 1 MV and 3 MV.

The interaction length of the cavity can be expressed in number of sections as \( l = (11n - 1)l₀ \) with \( l₀ = 0.374 \) m being the cell length. Fig. 2 shows which values of \( V_b \) are obtained with cavities of \( n = 3, 4 \) or 5 sections, as a function of beam current, \( I₀ \).

\[ V_b = \frac{1}{8} R₂ l² I₀ = \frac{1}{8} R₂ (11n - 1)² l₀² I₀ \]

The beam currents which are of particular interest are...
given in the following table:

<table>
<thead>
<tr>
<th>Beam</th>
<th>( N_{\text{RF}} )</th>
<th>( I_{b} )</th>
<th>( \eta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>LHC, ultimate</td>
<td>( 3 \times 80 \times 1.70 \times 10^{11} )</td>
<td>2.1 A</td>
<td>40%</td>
</tr>
<tr>
<td>LHC, nominal</td>
<td>( 3 \times 80 \times 1.05 \times 10^{11} )</td>
<td>1.3 A</td>
<td>40%</td>
</tr>
<tr>
<td>NGS</td>
<td>( 8 \times 10^{11} )</td>
<td>1.1 A</td>
<td>100%</td>
</tr>
</tbody>
</table>

Here the \( I_{b} \) values were established on the basis of a bunching factor of two. This table contains also one column with the RF power duty factor \( \eta \). As the LHC type beams do not occupy more than 40% of the SPS circumference, RF power will be needed only during that time. It is possible to take advantage of this because the filling time of the travelling wave cavities is much shorter than a revolution period. NGS type beams, as they are foreseen, will fill the SPS ring nearly completely (\( \eta = 100\% \)).

Fig. 2 shows how \( V_{b} \) (and hence also the power to compensate it) depends on \( I_{b} \) and the cavity length. The question arises which RF power is available and is there an optimum cavity length?

## 2 THE POWER LIMITS

At present there are two RF power limits. After the upgrade of the cavity power couplers and the RF amplifier power supplies, the amplifiers will be able to deliver up to 1.5 MW to a travelling wave cavity for \( \eta \leq 50\% \). For \( \eta = 100\% \) the coaxial lines limit this to 750 kW. These values determine the maximum available RF power, \( P_{\text{max}} \).

Due to several multipacting regions in the main power couplers there is also a minimum power limit of \( P_{\text{min}} = 100 \text{ kW} \).

For the case of \( I_{b} = 0 \), \( P_{\text{min}} = 100 \text{ kW} \) means that the cavity voltage \( V \) would be about 1 MV, for one travelling wave cavity or about 4 MV for all four together. To obtain voltages smaller than that, two cavities are counter phased with respect to each other as shown in the Fig. 3.

![Figure 3: Vector diagram in case of counter phasing.](image)

In the case of counter phasing, \( V_{1} = V_{2} = 1 \text{ MV} \), and the vectorial sum of \( \vec{V}_{1} \) and \( \vec{V}_{2} \) equals \( \vec{V} \). Varying the counter phasing angle (angle between \( \vec{V} \) and \( \vec{V}_{1} \) or \( \vec{V}_{2} \), respectively) total voltages of 0 to 2 MV will be obtained.

Having a closer look at \( V_{\text{RF,1}}^{2} \) and \( V_{\text{RF,2}}^{2} \) which are necessary to produce \( \vec{V}_{1} \) and \( \vec{V}_{2} \), respectively, one observes that \( V_{\text{RF,1}}^{2} \) due to the corresponding angles between \( \vec{V}_{1} \) and \( \vec{V}_{2} \) (or \( \vec{V}_{2} \)). This might lead to potentially dangerous situations. The first being that \( V_{\text{RF,1}}^{2} \) gets too small (hence \( P < P_{\text{min}} \)), the second being that \( V_{\text{RF,2}}^{2} \) gets too large (hence \( P > P_{\text{max}} \)).

In the worst case situation concerning \( P_{\text{max}} \) the demanded RF voltage \( V_{\text{RF,2}}^{2} \) will be largest in the case that \( \vec{V}_{2} \) and \( \vec{V}_{b} \) are exactly anti-parallel. For the travelling wave cavity providing \( V_{2} \) we then get \( V_{\text{RF}} = V + V_{b} \), \( V \) being 1 MV. Under these circumstances the total power \( P \) (see Fig. 4) is given by \( P = (\sqrt{P_{V}} + \sqrt{P_{b}})^{2} \), where \( P_{V} \) is the power necessary to produce \( V \) in the case of \( V_{b} = 0 \) and

\[
P_{b} = \frac{1}{64} R_{2} (2 I_{b})^{2} = \frac{1}{64} R_{2} (11 n - 1)^{2} l_{b}^{2} I_{b}^{2}
\]

the power necessary to compensate \( V_{b} \) in the case of \( V = 0 \).

![Figure 4: \( P \) vs. \( I_{b} \), \( n = 4 \).](image)

For a maximal available RF power \( P_{\text{max}} \), the power available to produce \( V \) is given by \( P_{V} = (\sqrt{P_{\text{max}}} - \sqrt{P_{b}})^{2} \). To give a numerical example let us consider \( P_{\text{max}} = 1.3 \text{ MW} \), \( P_{b} = 200 \text{ kW} \) (as in the case of \( I_{b} = 1.3 \text{ A} \), \( n = 4 \), see Fig. 4) then there will be only \( P_{V} = 480 \text{ kW} \) available to produce \( V \). The power limit \( P_{\text{max}} \) for this example was chosen lower than the 1.5 MW mentioned earlier. The reason has to do with a power reserve for transients.

The main beam loading compensation will be provided by the feed-forward system. At the moment the batch enters into the travelling wave cavity there will be an \( I_{b} \) transient of about 25%. Some power margin has to be taken into account for this purpose. This margin depends on \( I_{b} \) and \( \eta \). For the three beam types discussed earlier this leads to the following power limits, \( P_{\text{max}} \):

<table>
<thead>
<tr>
<th>( I_{b} )</th>
<th>( P_{b}, \text{RF} )</th>
<th>( \eta )</th>
<th>( P_{\text{max}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1 A</td>
<td>490 kW</td>
<td>40%</td>
<td>1.1 MW</td>
</tr>
<tr>
<td>1.3 A</td>
<td>200 kW</td>
<td>40%</td>
<td>1.3 MW</td>
</tr>
<tr>
<td>1.1 A</td>
<td>140 kW</td>
<td>100%</td>
<td>0.6 MW</td>
</tr>
</tbody>
</table>

As the counter phasing has some important consequences for the travelling wave cavity power requirements
optimum cavity length

Considering that in the case of \( \tau = 0 \) the necessary RF power is given by \([1]\)

\[
P = \frac{V^2}{R_2 l^2} + \frac{R_2 l^2 P_b^2}{64} + \frac{V I_b}{4} \cos \varphi_S = P(l)
\]

the question arises how to use the travelling wave cavities in the most efficient way. Differentiating \( P(l) \) with respect to \( l \) an optimum length \( l_{\text{opt}} \) is obtained

\[
l_{\text{opt}} = \sqrt{\frac{8V}{R_2 I_b}}
\]

and expressing this equation in number of sections

\[
n_{\text{opt}} = \frac{1}{11} \left( 1 + \sqrt{\frac{8V}{R_2 l^2 I_b}} \right). \tag{2}
\]

The optimum length is essentially a function of \( V \) and \( I_b \). For a particular set of values the necessary power as a function of cavity length is shown in Fig. 5.

The function \( P(n, \varphi_S) \) has a shallow minimum at around \( n = 4.2 \). Fig. 5 does not show what the optimum travelling wave cavity length would be for other values of \( V \) or \( I_b \). There are several ways to explore this issue. One is to calculate the optimum cavity length given by Eq. 2 for different beam currents and voltages. Fig. 6 shows the result. For \( 1 \text{ MV} \leq V \leq 2 \text{ MV} \) and \( I_b = 1.1 \text{ A} \) the optimum number of sections would be around 4.9, for \( I_b = 1.3 \text{ A} \) it would be about 4.5 and for \( I_b = 2.1 \text{ A} \) it would be about 3.6.

From the practical point of view there is still another approach possible. Calculate \( P \) as a function of \( V, I_b \) and \( \varphi_S \) (and \( n \)) and check for which set of parameters the relation \( P < P_{\text{max}} \) holds. The result is shown in Fig. 7.

As expected from the vector diagram (Fig. 3) the area \( P < P_{\text{max}} \) (green) is smallest for \( \varphi_S = 0^\circ \). Comparing the cases of \( n = 4 \) and \( n = 5 \) it becomes clear that for \( I_b < 1.5 \text{ A} \) and \( P_{\text{max}} = 1.3 \text{ MW} \) the case with \( n = 5 \) is more advantageous than the one with \( n = 4 \) in the sense that larger voltages are accessible. The border line between the region \( P < P_{\text{max}} \) and \( P \geq P_{\text{max}} \) is given by

\[
V = \sqrt{R_2 l^2 P_0} - \frac{1}{8} R_2 l^2 I_b \quad (\tau = 0, \varphi_S = 0)
\]

and shown in Fig. 8 for the case of \( P_{\text{max}} = 1.3 \text{ MW} \). The crossover current is then 1.5 A.

The most severe power limitation occurs for \( \varphi_S = 0 \) and a minimum required cavity voltage of 1 MV. Fig. 8 shows that, under this condition, the maximum allowed \( I_b \) will then be 2.0 A in the case of \( n = 5 \) and 2.3 A in the case of \( n = 4 \) (\( P_{\text{max}} = 1.3 \text{ MW} \) in both cases).

At present we have two travelling wave cavities of four sections each and two of five sections each. In the following it will be assumed that there will be four travelling wave
cavities of four sections each. There are reasons to consider also the option of one cavity of three sections and three of four sections each [3].

4 VOLTAGES AND BUCKET SIZES

In the previous sections the maximum available RF power and the travelling wave cavity length has been fixed. Now the question arises which accelerating voltages and bucket sizes are obtainable using two types of parameter sets, one which corresponds more to the technical design parameters (total accelerating voltage $V$, the time derivative of the dipole field $\dot{B}$, the particle momentum $pc$, and $I_b$) and one corresponding more to the beam physics aspects (bucket area $A$, $B$, $pc$, and $I_b$).

As a function of these parameter sets the necessary power $P$ per travelling wave cavity can be calculated. Regions in parameter space for which the relation $P_{\text{min}} \leq P < P_{\text{max}}$ does not hold, are excluded.

Fig. 9 shows in particular the consequences of the $P_{\text{min}}$ limit.

For $0 < V < 4$ MV and $I_b = 0$ the required power is constant because of counterphasing which demands 1 MV per travelling wave cavity for all demanded total voltages smaller than 4 MV. In the case of $I_b \neq 0$ the situation is different. For cavities where $V_{\text{RF}}$ lines up with $\dot{V}_b$ (see Fig. 3) the required power $P_1$ to produce 1 MV is smaller (lower branch in Fig. 9) than in the case where $V_{\text{RF}}$ and $\dot{V}_b$ point in opposite directions (requiring a larger $V_{\text{RF}}$, hence more power ($P_2$) to produce 1 MV, upper branch in Fig. 9). At 4 MV both branches merge.

Using the parameters of a specific LHC machine development cycle used in 1999, 26 GeV $\leq pc \leq 450$ GeV, $B \leq 0.35$ T/s and a voltage program with 500 kV $\leq V \leq 4.5$ MV, the required power as a function of time varies as shown in Fig. 10 for the case of four travelling wave cavities of 4 sections each, and assuming $I_b = 1.3$ A. At start of the cycle we observe that $P \leq P_{\text{min}}$, an undesirable situation.

After having considered a specific cycle, it might be interesting to see which parameter space is available. Firstly, the technical design parameters, as mentioned earlier, will be considered.

For the case of $I_b = 1.3$ A, $n = 4$, $P_{\text{min}} = 100$ kW, and $P_{\text{max}} = 1.3$ MW Fig. 11 shows contour plots for $P(V, pc, B)$. Here the blue regions are excluded, because of $P < P_{\text{min}}$ for one of the two cavities involved in counterphasing. The red regions are excluded because of $P_{\text{max}} \leq P$. As a function of $\dot{B}$ there is a minimum voltage necessary to sustain acceleration in the limit of a vanishing bucket size ($V_{\text{min}} = C \rho \dot{B}$, $C$ being the accelerator circumference and $\rho$ the bending radius). The regions below $V_{\text{min}}$ are left blank. One critical point in a cycle is at injection, where the RF voltage should provide a matched bucket for the incoming beam. Fig. 11 shows that the minimum possible voltage at injection is about 2.9 MV.

From the beam physics point of view it might be interesting to have a look at which bucket area is possible. For the LHC cycle used during machine developments it is programmed to be about 0.6 eVs to 0.7 eVs. The values of bucket area accessible are shown in Fig. 12. The colours are coded as previously. The minimum bucket area at injection is about 0.9 eVs. Under these conditions an injected bunch of 0.35 eVs longitudinal emittance and 4 ns bunch length would blow-up to about 0.75 eVs.
Figure 11: $P(V, pc, \dot{B})$ per travelling wave cavity. $I_b = 1.3$ A, $n = 4$, $P_{\text{min}} = 100$ kW, $P_{\text{max}} = 1.3$ MW.

Figure 12: $P(A, pc, \dot{B})$ per travelling wave cavity. $I_b = 1.3$ A, $n = 4$, $P_{\text{min}} = 100$ kW, $P_{\text{max}} = 1.3$ MW.

5 CONCLUSIONS

At present not only maximum power limits have to be considered but also minimum power requirements. Especially taking beam loading and counter phasing into account there are regions in the $\{A$ or $V, \dot{B}, pc, I_b, \ldots\}$ parameter space which are excluded. This might lead to a revision of the counterphasing scheme as used today. Another possibility to circumvent the $P_{\text{min}}$ problem might be a longer conditioning at low power levels and a re-conditioning from time to time.

As far as beam loading is concerned it was shown that there is an optimum travelling wave cavity length which is mainly a function of $I_b$. For the ultimate LHC beam four sections is the optimum and for the nominal four to five sections.

The principles and ways of analysis shown here apply also to other types of beam like the ultimate LHC type beam and the beam for the NGS project.

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7 REFERENCES

