

Aspects of Collapsing Cycles

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Abstract

Much has been learned about string theory over the last few years by studying properties of cycles and branes in a given background geometry. Here we discuss three situations (quantum volume, attractor flows/D-brane stability, and dynamical topology change) in which cycles in a Calabi-Yau background evolve and/or degenerate in some manner, yielding various insights into aspects of quantum geometry.

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1 Introduction

Over the last few years we have learned much by pushing string theory into extreme geometrical realms. Here we would like to discuss three recent works that continue this type of exploration. In the first, we will discuss aspects of quantum volume, focusing on some of the unusual features of volume as probed by wrapped branes [1, 2]. In the second, we will discuss aspects of attractor flows, seeking to find qualitative differences between the spectra of large and small volume Calabi-Yau compactifications [3]. In the third, we will discuss collapsing cycles in the context of topology changing transitions, seeking to find situations in which fairly generic boundary conditions force the background Calabi-Yau space to undergo topology change as a function of the eleventh dimension in M-theory [4].

2 Quantum Volume

We will work in the context of type II string theory compactified on a Calabi-Yau manifold M . If the real Kähler form on M is denoted by J , the classical area and volume of the nontrivial 2, 4, and 6-cycles of M are determined by the usual expression: $\int_{C_r} J^r$ where r is the complex dimensions of the cycle C_r . The question that naturally arises, though, is: How do these expressions change when quantum effects are taken into account? Now, in any situation in which a classical concept is extended into the quantum domain, without further information, some ambiguity creeps in as many quantum expressions have the same classical limit. To sharpen our choice of what quantum volume means, therefore, we should introduce some physical guide. In perturbative string theory, the contributions of world sheet instantons provide one such guide. Namely, consider a correlation function between three states corresponding to elements of $H^{1,1}(M)$. As is well known, the lowest order α' contribution involves the classical intersection form on M , but there are nonperturbative corrections in α' coming from string world sheets that wrap around holomorphic curves in M . These contributions are sensitive to the volume of such curves as they are weighted by the exponential of the worldsheet action, which takes the form $\exp(2\pi i \int_{C_2} B + iJ)$. If one uses mirror symmetry to determine the *exact* value of such correlation functions [5] (by doing a variation of Hodge structure calculation on the mirror manifold W), one can then adjust the value of $\int_{C_2} B + iJ$ to ensure agreement with the calculation on M , thereby giving one definition of the quantum volume of C_2 . As found in [6], this approach yields

$$\int_{C_2} B + iJ = \frac{\int_{\gamma} \Omega}{\int_{\gamma_0} \Omega} \quad (1)$$

where Ω is the holomorphic 3-form on W and γ, γ_0 are suitably chosen 3-cycles on W .

It is worth noting a few features of this definition of the quantum volume of 2-cycles on M . First, we are naturally led to a notion of complex volumes. Second, in the large radius limit on M , if one sets the B -field to zero, we recover the usual classical values for the volumes of 2-cycles. Third, in the small radius regime, there are departures from classical expectations. For instance, as in [6, 7], on the quintic three-fold (and other one-parameter Calabi-Yau examples), there is a *lower* bound on the quantum volume of 2-cycles—explicitly,

on the quintic, 2-cycle volumes satisfy

$$\text{Vol}_{\text{quintic}}(2 - \text{cycles}) \geq .6881. \quad (2)$$

while classically the only constraint on such volumes is that they are non-negative.

The fourth point is simply that this approach, since it relies on worldsheet instantons in perturbative string theory, is only applicable to 2-cycles. To go beyond this limitation it is natural to turn to developments in nonperturbative string theory.

In particular, if we are interested in the quantum volume of a p-cycle, we can wrap a D-p-brane around it in a BPS manner, and then note that the mass of this state is a physical probe of the volume of the cycle. Hence, we can declare that the quantum volume of the p-cycle is the mass of the state obtained by such a BPS wrapping of a p-brane, suitably normalized. Of course, to make this definition useful, one needs to be able to reliably calculate these masses. For 3-cycles this is immediate: in Type IIB string theory, we wrap a 3-brane on a supersymmetric representative γ of given 3-cycle homology class, and the resulting mass is

$$\text{Mass}(3 - \text{brane}) = \frac{|\int_{\gamma} \Omega|}{|\int_M \Omega \wedge \overline{\Omega}|^{1/2}}. \quad (3)$$

For even dimensional cycles on M , we can write down a similar exact formula by invoking mirror symmetry. Namely, mirror symmetry maps $\oplus H^{2p}(M, \mathbb{Z}) \rightarrow H^3(W, \mathbb{Z})$ and hence any even cycle C on M is mapped to some 3-cycle γ on W . The mass of the corresponding BPS wrapped brane is then given by the formula above evaluated for γ on W . In turn, this gives us the corresponding quantum volume of C on M . Notice that in the case that C is a 2-cycle, the magnitude of the resulting quantum volume agrees with that of perturbative string theory, up to an overall normalization factor.

We see, therefore, that everything boils down to (a) being able to calculate the period integrals of Ω over arbitrary 3-cycles at arbitrary locations in the moduli space of a given Calabi-Yau manifold (and its mirror) and (b) explicitly realizing the map $\oplus H^{2p}(M, \mathbb{Z}) \rightarrow H^3(W, \mathbb{Z})$. The subtlety here is that the calculations in (a) quickly become involved and the map in (b) is not known in general. In practice, we deal with this in the following way.

First, we are often interested in understanding where in moduli space the quantum volume of a given cycle vanishes. If this cycle is an odd-dimensional cycle, then there is no need for the map in (b). If the cycle is even-dimensional (which we shall henceforth assume), we can get by with a weaker form of the map in (b). Namely, if we know the map in (b) up to an overall scale factor, then this is enough to determine ratios of quantum volumes, as well as the precise locations of the zeroes of the quantum volumes. So, if we find a basis γ_j of $H^3(W)$ which are *proportional* to integral cycles (with the same, generally unknown, overall proportionality factor) then by this reasoning we are free to use these cycles in the range of the map in (b). Such cycles, it turns out, are much easier to identify than are integral cycles. As for the map itself, we make use of the general insights of [6, 8] which relates 3-cycles with $\log^j(z)$ monodromy about a large complex structure point $z = 0$ on W (assuming for simplicity a one-dimensional moduli space) to cycles of complex dimension j on M . Now the subtlety here is that this identification is true up to admixtures of cycles of lower dimension, and it is generally a challenge to work out these lower dimensional contributions explicitly. Hence, when we describe cycles of complex dimension j this implicitly means up

to such undetermined admixtures. Finally, as for (a), the difficulty in carrying out the period calculations at arbitrary points in the moduli space, we make use of the classical theory of Meijer functions which naturally encode all the required analytic continuations of solutions of Picard-Fuchs equations which these computations require (for details see [2]).

Let us turn to some examples. For simplicity, let's begin with the quintic hypersurface whose Kähler form we parameterize as $J = se$ where s is a real number and e is an integral generator of $H^2(\text{Quintic})$. As the quintic hypersurface has $h^{1,1} = 1$, we can think of s as parameterizing a real slice through the one-complex dimensional Kähler moduli space. Classically, the volumes of the single homologically nontrivially 2, 4, and 6 cycles are just: 2-cycle volume: s , 4-cycle volume: $5s^2/2$; 6-cycle volume: $5s^3/6$. In particular, note that only at $s = 0$ does any cycle vanish, and at that point *every* cycle vanishes.

Quantum mechanically the story is different. By carrying out the procedure as above, we find that the quantum volumes of the 2, 4, and 6 cycles as a function of s are as given in Figure 2 (where in Figure 1, for comparison, we plot the classical result). Notice that for the quantum volumes there is only one location where a cycle vanishes, and it is not at $s = 0$. Moreover, the cycle that vanishes is the 6-cycle [1, 9], not the 2-cycle as one would have naturally speculated. This is a bit odd, since although the entire Calabi-Yau has zero quantum volume at this point, the 2 and 4 cycle have nonzero volumes, as they are bounded below by positive numbers throughout the moduli space. In [2, 11] this phenomenon has been found to hold in a range of other one parameter examples, so appears to be generic.

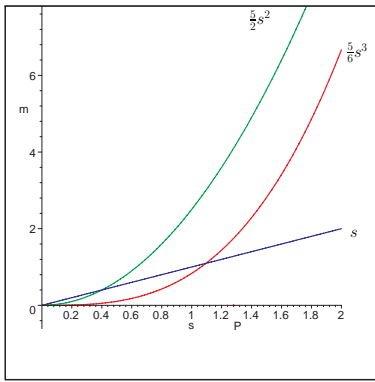


Figure 1. Classical 2, 4, 6 cycle volumes on the Quintic.

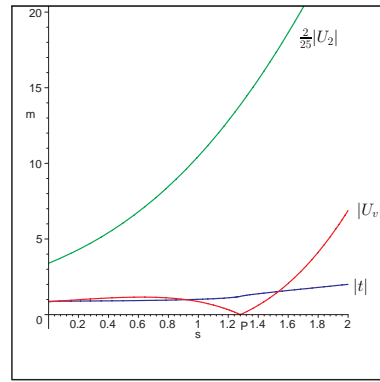


Figure 2. Quantum 2, 4, 6 cycle volumes on the Quintic.

As a second example, let's consider two-dimensional orbifolds. In the case of a \mathbb{Z}_2 orbifold, it was shown in [7] — using the perturbative string theory approach to quantum volume — that there is a point in the moduli space where a 2-cycle collapses to zero quantum volume (the orbifold point itself). But for the case of \mathbb{Z}_3 orbifolds, there was a bit of a puzzle in [7] because calculations showed that the 2-cycle does not vanish anywhere. Yet by virtue of there being a singularity in the moduli space of the associated physical model, we expect that some state has gone to zero mass at that point. But which state? Well, with our

nonperturbative extension of quantum volume and our experience with the quintic above, we can answer that question and, moreover, not be too surprised by the result. As shown in Figure 3, we see that the 4-cycle in this example goes to zero volume at a point in the moduli space, even though the 2-cycle never does.

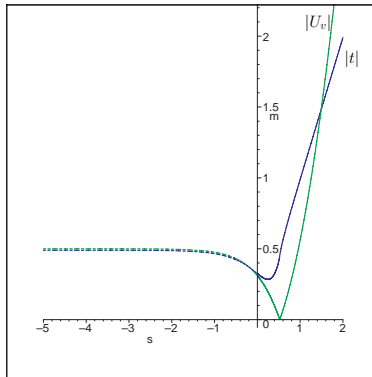


Figure 3. Quantum 2 and 4 cycles volumes for a \mathbb{Z}_3 orbifold.

Finally, in [2] these calculations have been extended to one dimensional loci in two-dimensional moduli spaces, with a range of similar violations of classical geometric intuition making themselves apparent, and in [11] the whole series of one parameter examples has been studied with special attention paid to an interesting manifestation of T -duality in this context.

3 Attractor Flows

In the discussion of the last section, we implicitly assumed that in any given homology class in $H_3(M)$ or $H_3(W)$ there is a supersymmetric representative. As a matter of fact, however, the existence of supersymmetric cycles is a difficult and as yet unsettled question. So, an interesting and important question to ask is (a) under what conditions are we assured that the relevant supersymmetric cycles exist thereby implying that the associated BPS state exists, and (b) are there examples in which the answer to (a) can be used to qualitatively distinguish large radius and small radius compactifications? That is, might it be that the existence of a supersymmetric cycle in a given homology class might have a Kähler form dependent answer, existing, say, when the radius is large but not when it is small?

A full answer to these questions is beyond current understanding, but, nevertheless, much can be gleaned through a variety of techniques. One such method makes use of *attractor flows*.

Recall that in [12], spherically symmetric black hole solutions to the equations of $\mathcal{N} = 2$ supergravity were studied and shown to obey an attractor mechanism: the value of the fields at the horizon of the hole are largely insensitive to the boundary values of the fields at spatial

infinity. Explicitly, these equations take the form:

$$\partial_\tau U = -e^U |Z| \quad (4)$$

$$\partial_\tau z^a = -2e^U g^{a\bar{b}} \partial_{\bar{b}} |Z| \quad (5)$$

where $\tau = 1/r$, r being the radial direction in space, U arises from the spacetime metric in the form $ds^2 = -e^{2U} dt^2 + e^{-2U} dx^i dx^i$, z^a are coordinates on the Kähler moduli space, $g^{a\bar{b}}$ is the inverse metric on the Kähler moduli space in these coordinates, and $Z = Z(\gamma) = \frac{\int_\gamma \Omega}{\int_M (\Omega \wedge \bar{\Omega})^{\frac{1}{2}}}$.

As one can see, these equations relate flows in moduli space to the spatial profile of the fields in a black hole background. As such, they provide an interesting and novel link between moduli space and spacetime physics.

One expects that these BPS solutions are the supergravity description of the BPS D-brane states in the full string theory. In the beautiful work of [13], these equations were studied for both their physical and mathematical content and one observation was this. If we study an attractor flow solution to these equations which has the property that $|Z|$ vanishes at a *regular* point in the moduli space, we don't expect the corresponding BPS state to exist. The reason is that $|Z|$ corresponds to the mass of a BPS brane wrapped on γ . If it vanishes along a flow, at that point we expect to have a new massless degree of freedom, and this is generally accompanied by a singularity in the moduli space. If there is no such singularity, we expect that the corresponding BPS state simply does not exist, i.e. the corresponding supersymmetric cycle in the homology class γ does not exist. Roughly, we associate $|Z| = 0$ with a collapsing cycle, and the latter give rise to singularities in the moduli space. If there is no singularity, the existence of a supersymmetric cycle in the given class is thrown into question.

This is a natural conjecture, but in [14, 15] it was suggested that there are circumstances in which the physics is more rich. Namely, there might be a would-be attractor solution in which there is a zero of $|Z|$ at a regular point (a “regular zero”) but for which the flow crosses a curve of marginal stability before that point is encountered. At the crossing point, the state decays and the attractor flow splits. As for the physical realization of this state, as shown in [14, 15], if γ splits into $\gamma_1 + \gamma_2$, the spacetime configuration can be realized by a spherical shell carrying the charge associated with γ_2 which surrounds the charge of γ_1 sitting at the origin.

An interesting aspect of these solutions, then, is that they provide a means of identifying candidate states that exist in one region of the moduli space but not at another. For instance, in [14, 15] it was shown how the state identified by [16] — which exists at the Gepner point on the quintic (which is at small radius) but apparently decays before getting to large radius — can be realized as a splitted attractor flow starting from the Gepner point. Naturally, then, one wonders about examples in which the reverse would be true: Start at large radius where classical geometry is a good guide to physics and identify states in the string spectrum. Then, go to ever smaller radius where non-classical geometrical effects become increasingly pronounced. Are there states for which these effects cause the state to decay, giving us a qualitative impact of short distance geometry on large distance expectations. Indeed in [3] such states have been found, and in Figures 4, 5, and 6 we give an example.

Namely, in Figure 4 we show a family of attractor flows on the Kähler moduli space of the quintic hypersurface, all associated with the charge vector $(Q_6, Q_4, Q_2, Q_0) = (0, 1, 3, -5)$.

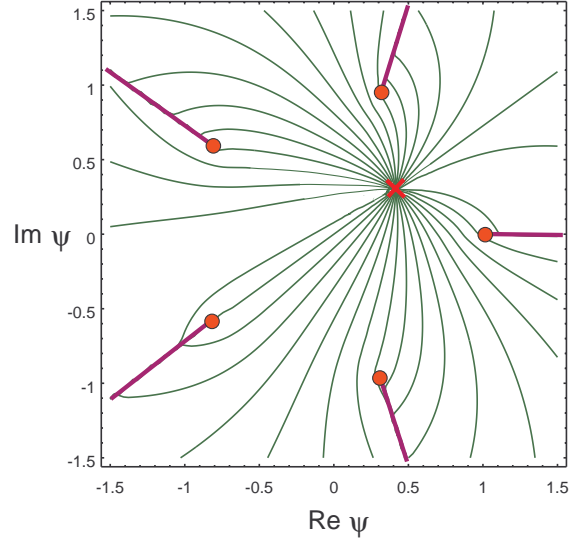


Figure 4: Attractor flows in the quintic moduli space.

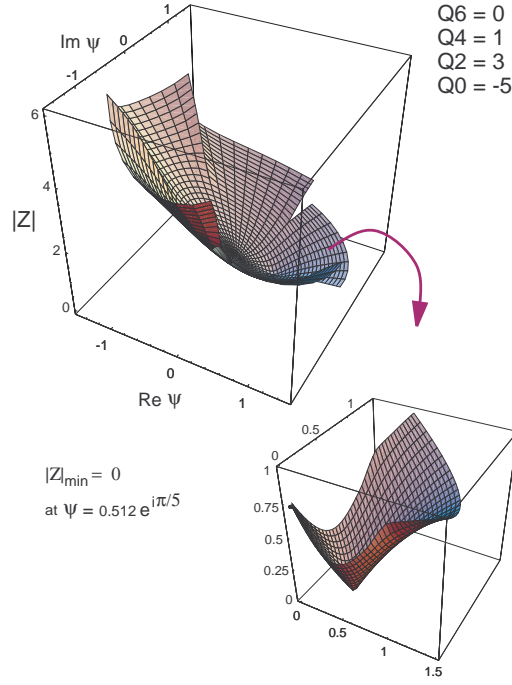


Figure 5: Moduli dependence of the mass of a BPS wrapped brane.

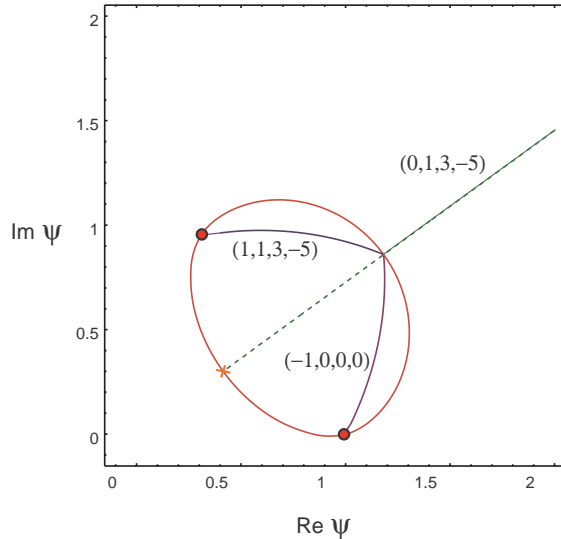


Figure 6: A splitted attractor flow crossing marginal stability line.

The flows terminate at the attractor point $\psi = .4146 + .3009i$. As we see in Figure 5, at this point the state has $|Z| = 0$, but this is a regular point (since the only singular point is the conifold point, indicated by the red circle(s) in Figure 4). In Figure 6, we show how this regular zero is physically interpreted in terms of split flow, crossing a curve of marginal stability. This is but one of many such examples, as discussed in greater detail in [3], thereby giving a nice qualitative distinction between long and short distance geometry.

4 Dynamical Topology Change

The final arena of collapsing cycles we will discuss has to do with topology change. Namely, over the years, several works [17, 18] have established definitively that there are physically smooth processes in string theory which result in a change in the topology of spacetime. In these studies, as well as studies of topology change in M theory [19], one considers a one parameter family of vacuum solutions — a one parameter family of spacetimes — that passes from one Calabi-Yau manifold to another which is topologically distinct. The referenced works succeeded in showing that there is no obstruction to such topology change, but no dynamics was ascribed to motion through the family. In this section, we discuss [4] a variation on this theme of topology change in which dynamics does drive the evolution from one topology to another. Specifically, (a) the topology change occurs within a single (not a family of) spacetime background and (b) for generic choices of initial conditions, the dynamics (*i.e.*, the field equations) drive us through a topology change.

To be concrete, we focus our attention on Calabi-Yau compactifications of M theory to five dimensions in the presence of G flux (the four form field strength). As discussed in [21, 22, 23, 24], the effective five dimensional theory does not admit a flat space vacuum solution. Rather, the spacetime metric is warped and the solution is of the domain wall type with one of the five dimensions singled out as the transverse direction. In addition to the

effective five dimensional spacetime metric, the moduli of the Calabi-Yau will generically vary along the transverse direction. In [4], we show that there exist Calabi-Yau compactifications in which the field equations force the Kähler moduli to pass from one Kähler cone into an adjacent cone, while the overall volume of the Calabi-Yau manifold remains large. This implies that the Calabi-Yau manifold undergoes a flop transition and continues on to a topologically distinct Calabi-Yau manifold as we move along the transverse dimension.

One may think of this work as being complementary to that of [25, 26, 27] in which it was shown that in the presence of certain dyonic black holes, a Calabi-Yau with particular moduli at spatial infinity can be driven by the attractor equations through a flop transition on the way to the black hole’s horizon. Here we briefly describe vacuum solutions whose structure requires topology change, referring the reader to [4] for details.

Following the pioneering work of [22], the supersymmetric domain-wall or three-brane solution to the effective five dimensional field equations is given by

$$\begin{aligned} ds_5^2 &= e^{2A} dx_4^2 + e^{8A} dy^2 \\ V &= \left(\frac{1}{3!} d_{ijk} f^i f^j f^k \right)^2, \\ e^A &= V^{1/6}, \\ b^i &= V^{-1/6} f^i, \\ F_{11,\mu\nu\rho\sigma}^i &= i\sqrt{2}\epsilon_{\mu\nu\rho\sigma}\partial_{11}V^{-1/2}f^i \end{aligned} \tag{6}$$

where the f^i ’s are defined in terms of one-dimensional harmonic functions

$$d_{ijk}f^j f^k = H_i, \quad H_i = \sum_n \alpha_i^{(n)} |y - y_n| + c_i \tag{7}$$

$$= \sum_{n=0}^k 2\alpha_i^{(n)} y + k_i, \quad y_k < y < y_{k+1} \tag{8}$$

and the k_i are arbitrary constants of integration. In these expressions, V is the volume of the Calabi-Yau manifold, the b^i are the Kähler moduli, and y is the coordinate in the “eleventh” dimension (the coordinate transverse to the end of the world branes in strongly coupled heterotic string theory).

We see explicitly that the moduli are y dependent, giving rise to the possibility that in a given example they might pass from one Kähler cone to another as y varies from one wall in spacetime to another. Indeed this possibility is borne out by studying a simple example of a pair of Calabi-Yau manifolds connected by a flop transition—namely, the well studied $(h^{1,1}, h^{2,1}) = (3, 243)$ Calabi-Yau manifolds considered in [29, 28, 25, 26, 30]. Figure 7 shows a topology changing solution to these equations for this example, giving rise to the configuration schematically illustrated in Figure 8.

One particularly interesting issue regarding these solutions arises from examining the Bianchi identity for the G-field. Namely, in the eleven-dimensional theory on S_1/\mathbb{Z}_2 , the Bianchi identity for G is modified by boundary sources [20] to the form

$$dG = \left(\left[\text{tr} F_{(1)} \wedge F_{(1)} - \frac{1}{2} \text{tr} R_{(1)} \wedge R_{(1)} \right] \delta(y) \right)$$

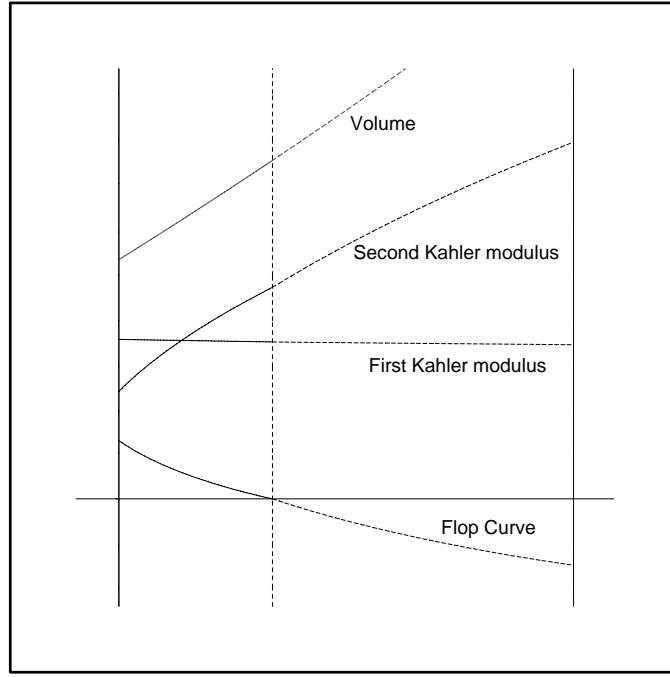


Figure 7: Profile of the areas of the two cycles and the Calabi-Yau volume along y . Solid lines belong to the validity region $y < y_*$ of the Calabi-Yau $\widetilde{\mathcal{M}}$. Dashed lines belong to the Calabi-Yau \mathcal{M} and hold for $y > y_*$.

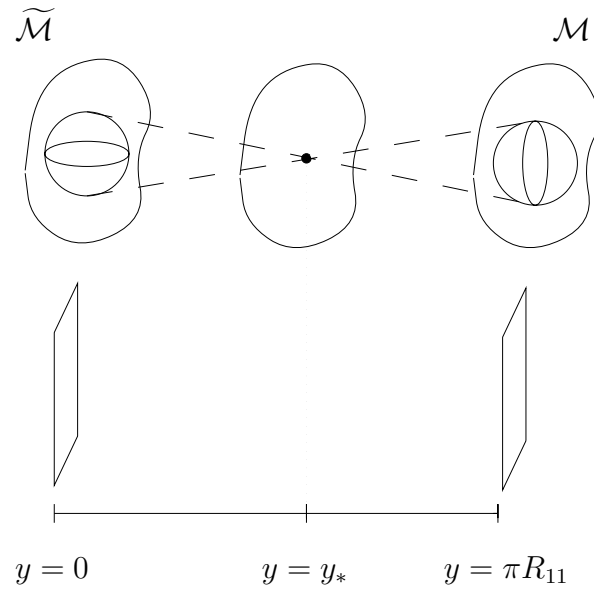


Figure 8: Calabi-Yau configuration in strongly coupled heterotic theory which undergoes a flop.

$$+ \left[\text{tr} F_{(2)} \wedge F_{(2)} - \frac{1}{2} \text{tr} R_{(2)} \wedge R_{(2)} \right] \delta(y - \pi R_{11}) \Big) \wedge dy , \quad (9)$$

where we are now taking care to distinguish the curvatures of the different Calabi-Yau spaces at each end of the interval. In the usual case, where $\text{tr} R_{(1)} \wedge R_{(1)} = \text{tr} R_{(2)} \wedge R_{(2)}$, we have the familiar standard embedding solution

$$\text{tr} F_{(1)} \wedge F_{(1)} = \text{tr} R \wedge R , \quad \text{tr} F_{(2)} \wedge F_{(2)} = 0 , \quad (10)$$

to the global consistency constraint that

$$\int_{S_1/\mathbb{Z}_2 \times D} dG = \int_{S_1/\mathbb{Z}_2 \times D} dy \left(\frac{1}{2} \text{tr} R \wedge R \delta(y) - \frac{1}{2} \text{tr} R \wedge R \delta(y - \pi R_{11}) \right) = 0. \quad (11)$$

But if $\text{tr} R_{(1)} \wedge R_{(1)} \neq \text{tr} R_{(2)} \wedge R_{(2)}$ (cohomologically) then the mismatch implies that solely embedding either spin connection into the gauge group is no longer a solution.

A natural suggestion, then, is to seek out different holomorphic stable bundles to place at $y = 0$ and $y = \pi R_{11}$ with different second Chern classes, so as to find new consistent solutions to the Bianchi identity. In [4], though, we give evidence that such an approach will overlook an essential contribution. Namely, when $\text{tr} R_{(1)} \wedge R_{(1)} \neq \text{tr} R_{(2)} \wedge R_{(2)}$ because the Calabi-Yau has flopped somewhere along y , there is a new contribution to the Bianchi identity associated with the collapsed flop curves.

In particular, a theorem of Tian and Yau [31] states that starting from a Calabi-Yau manifold $\widetilde{\mathcal{M}}$ and a collection of holomorphic curves $\{C^\beta\}$ on $\widetilde{\mathcal{M}}$, the second Chern numbers of \widetilde{M} and its flopped cousin M are related by

$$c_2(\mathcal{M}) = c_2(\widetilde{\mathcal{M}}) + 2 \sum_{\beta} \int_D [C^\beta] , \quad (12)$$

with D an arbitrary divisor and $[C^\beta] \in H^4(\widetilde{\mathcal{M}})$, the Poincare dual of C^β . This implies that

$$-\frac{1}{2} \text{tr}_{\mathcal{M}} R_{(2)} \wedge R_{(2)} + \sum_{\beta} \delta_{C_{(1)}^\beta} = -\frac{1}{2} \text{tr}_{\widetilde{\mathcal{M}}} R_{(1)} \wedge R_{(1)} . \quad (13)$$

As suggested in [4] this jump should be compensated by associating a magnetic charge with the collapsed flop curve, (a charge that might well be interpretable — in a manner described precisely in [4] — as that of a 5-brane wrapped on the curve, but no definitive conclusion on this interpretation was reached).

This work suggests that the study of G_2 manifolds with Calabi-Yau boundaries (a study that has been initiated in [32]) would be a fruitful way of learning more about these kind of string solutions.

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