Skewed Parton Distributions and $F_2^D$ at $\beta \to 1$

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Abstract

We show that the diffractive structure function is perturbatively calculable in the domain where the diffractive mass is small but still outside the resonance region. In this domain, which can be characterized by $\Lambda^2/Q^2 \ll 1-\beta \ll \sqrt{\Lambda^2/Q^2}$, the structure function represents a new observable, which is highly sensitive to the small-$x$ skewed gluon distribution. Our leading order calculation and the estimate of next-to-leading order corrections are consistent with available data and demonstrate the potential of more precise data to put further constraints on skewing effects.
1 Introduction

During the last few years, the concept of skewed parton distributions (see [1] and refs. therein) has attracted much interest. These quantities, which are also known as off-diagonal or non-forward parton distribution functions, represent an extension of conventional parton distributions and allow for a QCD analysis of hard processes where the hadronic target is scattered elastically [2].

In this letter, we focus on the role of the skewed gluon distribution in small-mass inclusive diffraction, i.e. the $\beta \to 1$ limit of $F_D^2(\beta, \xi, Q^2)$. The diffractive structure function $F_D^2$ is defined by the process $\gamma^* p \to p' X$. Our variables are the photon virtuality $Q^2$, the invariant mass $M$ of the diffractive system $X$, and the two dimensionless quantities $\beta = Q^2/(Q^2 + M^2)$ and $\xi = x_F = x_{Bj}/\beta$.

We propose the measurement of the diffractive cross section in the kinematic domain $Q^2 \gg M^2 \gg \Lambda^2$ (where $\Lambda \sim \Lambda_{QCD}$ is a hadronic scale) as a new method for the extraction of the skewed gluon distribution at small $x$. At the same time we point out that, given the skewed gluon distribution, the diffractive cross section in this region is perturbatively calculable. This corresponds to the calculation of $F_D^2$ for $\beta$ approaching 1 but still outside the resonance region $M \sim \Lambda$. We compare our approach with the limited large-$\beta$ data available at present and discuss expected next-to-leading order corrections on the basis of the unintegrated gluon distribution.

2 $F_D^2$ at very large $Q^2$ and $\beta \to 1$

The following analysis is based on the well-known formulae [3] for the $q\bar{q}$ pair production by a virtual photon scattering off a hadronic target. At lowest order, the energetic photon fluctuates into a $q\bar{q}$ pair with quark, antiquark momentum fractions $z$ and $1-z$ before it reaches the target. For a fixed value of $z$, the typical transverse size of the pair is $\sim 1/\varepsilon$, where $\varepsilon^2 = z(1-z)Q^2 + m^2$ with $m$ the mass of the quark [4]. If $1/\varepsilon$ is sufficiently small, the proton target can be characterized by its gluon distribution at the scale $\varepsilon$, as is known from vector meson electroproduction [5, 6].

The relevant formulae for longitudinal and transverse incoming photons producing a pair of quarks with electric charge $e_q$ read

$$\xi F_{L,T}^D(\beta, \xi, Q^2) = \frac{e_q^2 \alpha_s}{6bQ^2} \beta^3 (2\beta - 1)^2 \int_{z_{\text{min}}}^{1-z_{\text{min}}} \frac{dz}{z(1-z)} \left[\xi G(\xi, \varepsilon^2)\right]^2 f_{L,T}(z), \quad (1)$$

where

$$f_L(z) = \left\{1 + \frac{r}{1 - 1/(2\beta)}\right\}^2, \quad r = \frac{m^2}{z(1-z)Q^2}, \quad (2)$$

$$f_T(z) = \frac{f_L(z)}{z(1-z)} \left\{z^2 + (1-z)^2\right\} \left(1 - (1 + r)\right) \left(2 - \frac{1}{\beta(1+r)}\right)^{-2} + \frac{r}{4}, \quad (3)$$
and the kinematical limits of the \( z \) integration are determined by

\[
    z_{\text{min}} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{m^2}{Q^2(1/\beta - 1)}}. \tag{4}
\]

The prefactor \( 1/b \) is due to the \( t \) integration performed under the assumption of a \( t \) dependence \( \sim \exp[bt] \).

Equation (1) is valid in the leading \( \log(1/x) \) approximation, where the difference between the longitudinal momenta carried by the two exchanged gluons is irrelevant [7] and the conventional gluon distribution appears. To take this difference into account, the gluon distribution has to be replaced by the skewed gluon distribution \( H_g \) according to

\[
    G(x, \mu^2) = H_g(x, x, \mu^2) \rightarrow H(x, x', \mu^2), \tag{5}
\]

where \( x \) and \( x' \) are the momentum fractions of the proton carried by the first and second exchanged gluons. With the main contribution coming from the region \( x' \ll x \), the resulting correction can be accounted for by introducing the multiplicative factor \( R_g = H_g(x, x' \ll x, \mu^2)/H_g(x, x, \mu^2) \) into Eq. (1) (cf. the analyses of [8,9]). This skewing factor has been calculated for \( x \ll 1 \) in [10] to be

\[
    R_g = \frac{2^{2\lambda+3} \Gamma(\lambda + \frac{5}{2})}{\sqrt{\pi} \Gamma(\lambda + 4)}, \quad \text{where} \quad \lambda = \frac{\ln(xG(x, \mu^2))}{\ln(1/x)}. \tag{6}
\]

In deriving Eq. (1) the underlying amplitudes were assumed to be purely imaginary. This can only be justified in the leading \( \log(1/x) \) approximation. Therefore, in addition to skewing, the effect of the real parts of these amplitudes has to be taken into account. Under the assumption of appropriate crossing properties of the amplitude \( T \) and of a purely power-like behaviour of its imaginary part (i.e. a power-like behaviour of the gluon distribution),

\[
    \text{Im} T \sim s^\lambda \quad \text{(where} \quad s = Q^2/x_{\text{Bj}} \text{)}, \tag{7}
\]

the relation

\[
    C \equiv \frac{\text{Re} T}{\text{Im} T} = \tan \left( \frac{\lambda \pi}{2} \right) \tag{8}
\]

can be derived (see, for example, [6,11]).

Equation (1) with the correction factor \( R_g^2(1 + C^2) \) evaluated at \( x = \xi \) and \( \mu^2 = \varepsilon^2 \) forms the basis of our leading order analysis.

Consider first the simple case when \( m = 0 \) and \( \beta \to 1 \). In this limit, \( f_L(z) = 1 \) and \( z_{\text{min}} = 0 \) so that Eq. (1) provides a parameter-free prediction for the longitudinal cross section at asymptotically large \( Q^2 \). This prediction is not affected by the endpoints of the \( z \) integration since the effective anomalous dimension \( \gamma \) of the gluon distribution in this region,

\[
    xG(x, \mu^2) \sim (\mu^2)\gamma \quad (\gamma > 0), \tag{9}
\]

suppresses the integrand (cf. [12]).
The transverse cross section at $m = 0$ and $\beta \to 1$ is suppressed with respect to the longitudinal one by an explicit factor $\sim (1 - \beta)$ contained in $f_T(z)$. However, the $z$ integration in the transverse case is divergent even for $\gamma > 0$. Since no free quarks can be produced, it is natural to use a cutoff $z_{\text{min}} \sim \Lambda^2 / M^2$, which follows from Eq. (4) if $m$ is replaced by a hadronization scale $\sim \Lambda$. (Note that for sufficiently large $\beta$ the scale $\varepsilon^2 \geq z_{\text{min}}(1 - z_{\text{min}})Q^2$ always stays in the perturbative domain.) The $z$ integral in $F_D^T$ then gives a result $\sim 1 / z_{\text{min}}^{1-2\gamma}$ leading to an overall ratio

$$\frac{F_D^T}{F_D^L} \sim \frac{M^2}{Q^2} \left( \frac{M^2}{\Lambda^2} \right)^{1-2\gamma}.$$  (10)

This ratio is small as long as

$$M^4 \ll \left[ \left( \frac{\Lambda^2}{Q^2} \right)^{1-2\gamma} \right]^{1-\gamma},$$  (11)

which, even when combined with the condition $M^2 \gg \Lambda^2$, leaves a window for perturbative calculability of $F_D^T$. A particularly simple, conservative expression for this window is obtained by setting $\gamma = 0$ in Eq. (11), which on switching to the kinematic variable $\beta$ leads to

$$\Lambda^2 / Q^2 \ll 1 - \beta \ll \sqrt{\Lambda^2 / Q^2}.$$  (12)

In this region, the diffractive structure function is dominated by the perturbatively calculable, longitudinal, higher-twist contribution, the phenomenological importance of which has been emphasized before [13].

Contributions associated with the production of $q\bar{q}g$ final states are known to be important for $F_D^2$ [14]. However, they are not relevant in the limit $\beta \to 1$. This can be seen by relating the $q\bar{q}g$ final state to the diffractive gluon distribution [15], which is known to fall like $(1 - \beta)^2$ at large $\beta$ in different QCD-based models [16].

Thus, we have established a region at large $Q^2$ and $\beta \sim 1$ where the diffractive structure function $F_D^2(\beta, \xi, Q^2)$ is perturbatively calculable on the basis of the skewed gluon distribution.

### 3 $F_D^2$ at realistic values of $Q^2$ and $\beta$

In this section, the diffractive cross section at high $\beta$ and at realistic values of $\xi$ and $Q^2$ is calculated on the basis of Eq. (1), corrected for skewing and the non-zero real parts of the amplitudes. Figure 1 shows the longitudinal structure function obtained with three different parametrizations of the leading order gluon distribution.

In our calculation, we use a charm mass $m_c = 1.5$ GeV and a leading order running coupling defined by $\Lambda_{\text{LO},n_f=3} = 144$ MeV ($\alpha_s(M_Z) = 0.118$) and evaluated at a fixed scale $\mu^2 = Q^2 / 8$. This choice is justified by the variation of $\varepsilon^2$, which determines the hardness of the process, between 0 and $Q^2 / 4$ in the massless case. Furthermore, we use the slope $b = 8$ observed in the closely related hard process of elastic $\rho$ meson electroproduction [21].
The longitudinal structure function $F_L^D(\beta, \xi, Q^2)$ at $Q^2 = 75 \text{ GeV}^2$ and $\xi = 0.003$ calculated on the basis of the leading order GRV [17, 20] (grv98lo), CTEQ [18, 20] (cteq5l) and MRST [19, 20] (mrs98lo, central gluon) gluon distribution functions. The charm threshold is clearly visible.

Figure 1: The longitudinal structure function $F_L^D(\beta, \xi, Q^2)$ at $Q^2 = 75 \text{ GeV}^2$ and $\xi = 0.003$ calculated on the basis of the leading order GRV [17, 20] (grv98lo), CTEQ [18, 20] (cteq5l) and MRST [19, 20] (mrs98lo, central gluon) gluon distribution functions. The charm threshold is clearly visible.

In the longitudinal case, no light quark masses are introduced. The endpoints of the $z$ integration are cut off by demanding $\varepsilon^2 > \varepsilon_{\text{min}}^2 = 1.25 \text{ GeV}^2$, which is the lowest scale at which the MRST gluon distribution is defined. If the cutoff is raised to $\varepsilon_{\text{min}}^2 = 2.5 \text{ GeV}^2$, the result changes by $\lesssim 5\%$. This is much less than other theoretical errors, e.g. the considerable uncertainty of the small-$x$ gluon distribution (cf. Fig. 1).

In the following, we use the CTEQ gluon distribution because of its central position in the comparative plot of Fig. 1. For this gluon distribution, the effects of skewing and of the real part at $\beta = 1$ are given by $R_g^2 = 1.88$ and $1 + C^2 = 1.42$.

Figure 2 shows our leading order results together with $F_L^D$ data from H1. Unfortunately, at the values of $\beta$ and $Q^2$ available at present, the transverse contribution cannot be neglected. Therefore we have also included an estimate of the transverse contribution in the plot. This contribution was calculated on the basis of Eq. (1) with skewing and real-part corrections. Unlike the prediction for $F_L^D$, it is necessary to impose a regulator to estimate $F_T^D$. Here we make the $z$ integration finite by giving the light quarks a non-zero mass $m_q \sim \Lambda$, which is justified by the production of massive hadrons in the final state. According to Eq. (4), this mass gives rise to a cutoff $z_{\text{min}}$ and therefore to a minimal virtuality $\varepsilon_{\text{min}}^2 = m_q^2/(1 - \beta)$. At sufficiently large $\beta$, this minimal virtuality is in the perturbative domain and a calculation on the basis of parametrizations of the gluon distribution becomes possible. We have plotted the corresponding curves in the $\beta$ range where $\varepsilon_{\text{min}}^2 > 1.25 \text{ GeV}^2$.

Even though we have argued that $\varepsilon^2$ is always in the perturbative domain, our transverse cross section should only be interpreted as an estimate. The reason for this is the unknown value of the regulator $m_q$ or, more generally, the lack of a quantitative understanding of the way in which confinement effects in the final state limit the possible kinematics of the $q\bar{q}$ pair.
Figure 2: The diffractive structure functions at $Q^2 = 75 \text{ GeV}^2$ and $\xi = 0.003$ (based on the CTEQ leading order gluon) compared with $F_2^D$ data from H1 [22]. The estimates of the transverse contribution are based on mass regulators $m_q = 0.3 \text{ GeV}$ (upper curves) and $m_q = 0.6 \text{ GeV}$ (lower curves).

Figure 3: Same as Fig. 2, but for $Q^2 = 60 \text{ GeV}^2$ and $\xi = 0.0042$ with corresponding data from ZEUS [22].

In Fig. 3, the analysis of Fig. 2 is repeated for $F_2^D$ data from ZEUS. In both figures we have used the data extrapolated by the experimental collaborations to fixed $\xi$ and $Q^2$ values, because we are interested in the $\beta$ shape at given $\xi$ and $Q^2$. With more data points and smaller errors one could, in principle, extrapolate the data to $\beta \simeq 1$ and thus directly compare with the perturbative $F_L^D$ calculation.

At present, we can only say that the data are approaching the region where the calculable longitudinal contribution dominates. Our calculations, which include a skewing enhancement of almost a factor 2, are compatible with the data. Thus, the prospects for a quantitative description of $F_2^D$ data in this domain and for a model-independent test of skewing effects are good.
At present there exists no complete next-to-leading order (NLO) calculation of diffractive scattering. However, it is possible to improve the predictions beyond the leading approximation. Above we have already discussed the important contributions from the real part of the amplitudes and the effect from skewed parton distributions. Both are corrections to the leading log(1/x) approximation and lead to a significant enhancement of $F_D^2$. Therefore, unless explicitly indicated, they will be taken into account (on amplitude level, see [9] for details) in all the numerical results discussed below. In addition, the simple form of Eq. (1) was obtained in the leading log($Q^2/k^2_T$) approximation, where $k_T$ is the transverse momentum of the exchanged gluons that mediate the diffractive scattering. Only in this limit can the (loop-)integral over $k_T$ be performed immediately, leading to the integrated gluon distribution. In the more general case, the amplitudes contain a non-trivial $k_T$-integration over the unintegrated gluon distribution $f(\xi,k^2_T)$.

$$\xi G(\xi,Q^2) = \int Q^2 \frac{dk^2_T}{k^2_T} f(\xi,k^2_T).$$

(13)

This approach is an application of the so-called $k_T$ or high-energy factorization [24] which is an extension of ordinary collinear factorization. It was already used in [23] for the prediction of diffractive open charm production and found to increase the cross section considerably. However, this effect is nearly compensated by the fact that the NLO gluon parametrizations, which are used for consistency in our NLO calculations, are smaller than the leading order ones.

Two additional comments are in order here: when aiming at a full NLO prediction, in principle also $q\bar{q}g$ contributions have to be taken into account. While this is certainly true for arbitrary kinematics, additional real gluon emission is kinematically suppressed for large $\beta$, which is the case we are interested in. Virtual gluon exchange, on the other hand, might be important. Up to now, such corrections to the $gqg\bar{q}$ vertex have not been calculated yet. They might lead to an additional $K$ factor of numerical relevance (see also [23]).

Let us now discuss the numerical results of our NLO predictions for $F_D^2$ obtained in the $k_T$-factorization approach. Figure 4 shows the contributions to $F_D^2$ from $F_D^{L}$ and $F_D^{T}$ as continuous and dashed lines, respectively. Here $Q^2 = 60 \text{ GeV}^2$, $\xi = 0.0042$, and we have used the CTEQ5M gluon as input in our calculations. The three dotted curves correspond to three different gluon parametrizations, as indicated in the plot; they demonstrate the strong dependence of our results on the gluon distribution used as input. For comparison we have shown two ZEUS data points, which are well described by the CTEQ5M prediction. We should, however, keep it in mind that our calculation is plagued by an uncertainty from the IR regime. The size of this uncertainty can be estimated by varying the light quark mass, which serves as an IR cutoff. Figure 5 shows predictions

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1For the full amplitudes including the integration over the gluon transverse momentum we refer the reader to [23,9].
2In a first step the finiteness of the impact factors was demonstrated, see [25].
Figure 4: \( F_2^D \) (at \( Q^2 = 60 \text{ GeV}^2 \) and \( \xi = 0.0042 \)) in NLO approximation as discussed in the text. The continuous (dashed) line shows the longitudinal (transverse) contribution calculated with the CTEQ5M gluon [18], the dotted lines display the sum \( F_L^D + F_T^D \) for the three different NLO gluon parametrizations GRV [17] (grv98nlm), CTEQ [18] (cteq5m) and MRST [19] (mrs99, central gluon) as indicated in the plot. ZEUS data [22] are shown for comparison.

for \( F_2^D \) (using the CTEQ5M gluon) for the three different choices \( m_q = 0/0.3/0.6 \text{ GeV} \). (In Fig. 4 \( m_q = 0.3 \text{ GeV} \) was taken as a default value.) Note that a cutoff given by the requirement \( \varepsilon^2 + \kappa_T^2 > (0.3 \text{ GeV})^2 \), where \( \kappa_T \) is the transverse momentum of the produced quarks, together with \( m_q = 0 \) leads to results only slightly smaller than the upper curve of Fig. 5, obtained with \( m_q = 0 \) and no other ‘confinement cutoff’.

As long as (trivial) phase space suppression is not dominant, the variation of \( m_q \) affects mainly the overall normalization. Together with the unknown \( K \) factor mentioned above this uncertainty at present makes it difficult to ‘extract’ the skewed gluon distribution with high accuracy. Therefore, precise data at higher \( Q^2 \) and the calculation of the \( K \) factor are highly desirable. Nevertheless the strong sensitivity of \( F_2^D \) on skewing effects is clearly demonstrated in Fig. 5 by the dash-dotted curve, which does not include skewing and has to be compared with the continuous line. Also note that the enhancement due to the real part contributions amounts to roughly 40 per cent, in good agreement with the enhancement factor found in the leading order analysis.

\footnote{Here we can calculate \( F_2^D \) even for a vanishing quark mass. This is because no IR divergences appear in the integrals over the unintegrated gluon distribution, which is continued linearly for very small scales \( k_T^2 < 1.5 \text{ GeV}^2 \) as described in [23].}
Figure 5: $F_D^2$ (at $Q^2 = 60$ GeV$^2$ and $\xi = 0.0042$) for three different choices of the light quark mass as indicated in the plot. For comparison two ZEUS data [22] points are shown. The dash-dotted line is the prediction if the effects from skewing are neglected.

5 Conclusions

In the present letter, we have suggested a new way of extracting the small-$x$ skewed gluon distribution from diffractive electroproduction data. We have demonstrated that inclusive diffraction in the kinematic domain $Q^2 \gg M^2 \gg \Lambda^2$ is, in principle, a perturbatively calculable quantity, which is highly sensitive to skewing effects. Our leading order numerical analysis, which includes a skewing factor $\lesssim 2$, is consistent with $F_D^2$ data now available in the region of interest. Next-to-leading order effects have been estimated using the unintegrated gluon distribution. The results reinforce the leading order situation since the enhancement that goes with the explicit integration over transverse gluon momenta is largely compensated by the smaller next-to-leading order gluon distribution. It is clear that future, more precise data have the potential to constrain the small-$x$ skewed gluon distribution significantly.

Our method has the advantage that, unlike the case of diffractive vector meson electroproduction, no effects of non-perturbative final state wave functions affect the theoretical prediction. Furthermore, our calculation of inclusive diffraction at $\beta \to 1$ can be used to supplement the familiar leading twist partonic analyses of $F_D^2$ with a higher twist correction, which is known to be numerically important. Thus, we expect that the specific kinematic domain of $F_D^2$ at large $Q^2$ and $\beta \to 1$ will play an important role in the future development of hard diffraction, both theoretically and experimentally.
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