PROCEEDINGS OF THE WORKSHOP ON
STANDARD MODEL PHYSICS (AND MORE) AT THE LHC

Editors:
G. Altarelli, M.L. Mangano
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ISSN 0007-8328
ISBN 92–9083–164–2
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ABSTRACT

This Report documents the results of the 1999 CERN Workshop on "Standard Model Physics (and more) at the LHC". The physics potential of the 14 TeV pp Collider in the areas of QCD, electroweak interactions and heavy quark physics is reviewed. The current state of the art in the theoretical predictions is presented, and the most up-to-date experimental strategies and measurement goals are illustrated.
FOREWORD

The Workshop on "Standard Model Physics (and more) at the LHC", whose proceedings are presented here, was promoted and financially supported jointly by the Particle Physics Experiments and the Theory Divisions of CERN.

The specific goal of the Workshop, not directly evident from the somewhat mysterious title, was to promote physics studies at the LHC in areas beyond the Higgs and new particles search (especially supersymmetric particles). That is, the purpose was to explore additional possibilities of the experiments beyond the well-studied subjects that are the main focus of the physics programme at the LHC. A strong encouragement to promote this Workshop came from the physicists community, which is very much interested in keeping the discussion on physics alive and focused during the long years of machine and detector construction.

The LHC Committee strongly supported the initiative and closely followed its development. An Organizing Committee was appointed in the fall of 1998, composed of Daniel Denegri, Daniel Froidevaux, Tatsuya Nakada (the physics coordinators of CMS, ATLAS and LHCb) plus Michelangelo Mangano and Guido Altarelli. Five Working Groups were set up, on QCD, Electroweak Interactions, Top Quark Physics, Beauty Production, and Beauty Decays. In each Group, theorists together with experimentalists from ATLAS, CMS and LHCb acted as Conveners. Heavy-Ion Physics was covered in a separate Workshop in 1999.

The Workshop was held at CERN. The first plenary meeting took place on 25 and 26 May 1999, the second and final on 14 and 15 October 1999. The response from the community was quite impressive. A large number of participants from outside CERN were present, including a strong involvement of theorists. Among the participants a conspicuous number of distinguished visitors from the United States were attracted to CERN, some of them taking a leading role as Conveners, many more of them contributing to the Working Groups and presenting talks at the plenary meetings. The participation of experimentalists was essential and qualified, although numerically not so large with respect to the size of the collaborations, signalling that most of the LHC forces are at the moment either busy with making the detectors or still working on other experiments.

Last but not least, a special thankful acknowledgement goes to Jeanne Rostant and Suzy Vascotto for their invaluable help in the organization and the running of the Workshop.

On the whole, the experience with this Workshop was quite positive and similar meetings will certainly be repeated.

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(8) Supported in part by TMR project FMRX-CT98-0169.
(9) Supported by the Austrian Ministry for Science and Transport.
(10) Supported by the Russian Foundation of Basic Research, grants RFBI 99-01-00091 and RFBI 00-02-17432.
(11) On leave from Yerevan Physics Institute, Yerevan 375036, Armenia; supported by the Bundesministerium für Bildung und Forschung (BMBF) under contract No. 05 7WZ91P 0.
(12) On leave from CPT, Marseille. Warm thanks to Jonathan Flynn, David Lin and Guido Martinelli for informative discussions.
(13) N.N. acknowledges useful discussions with D.I. Melikhov.
(14) Supported by grants from the Xunta de Galicia (No. XUGA-20604A98) and CICYT (No. AEN99-0589-C02-02).
(15) Supported in part by the Polish Goverment grant KBN 2P03B14715 and by the Polish--American Maria Sklodowska-Curie Joint Fund II, in cooperation with PAA and DOE under project PAA/DOE-97-316.
(16) Deceased.
(17) Now at MPI, Munich, Germany.
(18) Supported in part by the Polish Goverment grant KBN 2P03B12218.
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Abstract
We discuss issues of QCD at the LHC including parton distributions, Monte Carlo event generators, the available next-to-leading order calculations, resummation, photon production, small $x$ physics, double parton scattering, and backgrounds to Higgs production.

1. INTRODUCTION

It is well known that precision QCD calculations and their experimental tests at a proton–proton collider are inherently difficult. “Unfortunately”, essentially all physics aspects of the LHC, from particle searches beyond the Standard Model (SM) to electroweak precision measurements and studies of heavy quarks are connected to the interactions of quarks and gluons at large transferred momentum. An optimal exploitation of the LHC is thus unimaginable without the solid understanding of many aspects of QCD and their implementation in accurate Monte Carlo programs.

This review on QCD aspects relevant for the LHC gives an overview of today’s knowledge, of ongoing theoretical efforts and of some experimental feasibility studies for the LHC. More aspects related to the experimental feasibility and an overview of possible measurements, classified according to final state properties, can be found in Chapter 15 of Ref. [1]. It was impossible, within the time-scale of this Workshop, to provide accurate and quantitative answers to all the needs for LHC measurements. Moreover, owing to the foreseen theoretical and experimental progress, detailed quantitative studies of QCD will have necessarily to be updated just before the start of the LHC experimental program. The aim of this review is to update Ref. [2] and to provide reference work for the activities required in preparation of the LHC program in the coming years.

Especially relevant for essentially all possible measurements at the LHC and their theoretical interpretation is the knowledge of the parton (quark, anti-quark and gluon) distribution functions (pdf’s), discussed in Sect. 2. Today’s knowledge about quark and anti-quark distribution functions comes from lepton-hadron deep-inelastic scattering (DIS) experiments and from Drell-Yan (DY) lepton-pair production in hadron collisions. Most information about the gluon distribution function is extracted from hadron–hadron interactions with photons in the final state. The theoretical interpretation of a large number of experiments has resulted in various sets of pdf’s which are the basis for cross section predictions at the LHC. Although these pdf’s are widely used for LHC simulations, their uncertainties are difficult to estimate and various quantitative methods are being developed now (see Sects. 2.1 – 2.4).

The accuracy of this traditional approach to describe proton–proton interactions is limited by the possible knowledge of the proton–proton luminosity at the LHC. Alternatively, much more precise information might eventually be obtained from an approach which considers the LHC directly as a parton–parton collider at large transferred momentum. Following this approach, the experimentally cleanest and theoretically best understood reactions would be used to normalize directly the LHC parton–parton
luminosities to estimate various other reactions. Today’s feasibility studies indicate that this approach might eventually lead to cross section accuracies, due to experimental uncertainties, of about ± 1%. Such accuracies require that in order to profit, the corresponding theoretical uncertainties have to be controlled at a similar level using perturbative calculations and the corresponding Monte Carlo simulations. As examples, the one-jet inclusive cross section and the rapidity dependence of $W$ and $Z$ production are known at next-to-leading order, implying a theoretical accuracy of about 10%. To improve further, higher order corrections have to be calculated.

Section 3 addresses the implementation of QCD calculations in Monte Carlo programs, which are an essential tool in the preparation of physics data analyses. Monte Carlo programs are composed of several building blocks, related to various stages in the interaction: the hard scattering, the production of additional parton radiation and the hadronization. Progress is being made in the improvement and extension of matrix element generators and in the prediction for the transverse momentum distribution in boson production. Besides the issues of parton distributions and hadronization, another non-perturbative piece in a Monte Carlo generator is the treatment of the minimum bias and underlying events. One of the important issue discussed in the section on Monte Carlo generators is the consistent matching of the various building blocks. More detailed studies on Monte Carlo generators for the LHC will be performed in a foreseen topical workshop.

The status of higher order calculations and prospects for further improvements are presented in Sect. 4. As mentioned earlier, one of the essential ingredients for improving the accuracy of theoretical predictions is the availability of higher order corrections. For almost all processes of interest, containing a (partially) hadronic final state, the next-to-leading order (NLO) corrections have been computed and allow to make reliable estimates of production cross sections. However, to obtain an accurate estimate of the uncertainty, the calculation of the next-to-next-to-leading order (NNLO) corrections is needed. These calculations are extremely challenging and once performed, they will have to be matched with a corresponding increase in accuracy in the evolution of the pdf’s.

Section 5 discusses the summations of logarithmically enhanced contributions in perturbation theory. Examples of such contributions occur in the inclusive production of a final-state system which carries a large fraction of the available center-of-mass energy (“threshold resummation”) or in case of the production of a system with high mass at small transverse momentum (“$p_T$ resummation”). In case of threshold resummations, the theoretical calculations for most processes of interest have been performed at next-to-leading logarithmic accuracy. Their importance is two-fold: firstly, the cross sections at LHC might be directly affected; secondly, the extraction of pdf’s from other reactions might be influenced and thus the cross sections at LHC are modified indirectly. For transverse momentum resummations, two analytical methods are discussed.

The production of prompt photons (as discussed in Sect. 6) can be used to put constraints on the gluon density in the proton and possibly to obtain measurements of the strong coupling constant at LHC. The definition of a photon usually involves some isolation criteria (against hadrons produced close in phase space). This requirement is theoretically desirable, as it reduces the dependence of observables on the fragmentation contribution to photon production. At the same time, it is useful from the experimental point of view as the background due to jets faking a photon signature can be further reduced. A new scheme for isolation is able to eliminate the fragmentation contribution.

In Sect. 7 the issue of QCD dynamics in the region of small $x$ is discussed. For semi-hard strong interactions, which are characterized by two large, different scales, the cross sections contain large logarithms. The resummation of these at leading logarithmic (LL) accuracy can be performed by the BFKL equation. Available experimental data are however not described by the LL BFKL, indicating the present of large sub-leading contributions and the need to include next-to-leading corrections. Studies of QCD dynamics in this regime can be made not only by using inclusive observables, but also through the study of final state properties. These include the production of di-jets at large rapidity separation (studying the azimuthal decorrelation between the two jets) or the production of mini-jets (studying their multiplicity).
An important topic at the LHC is multiple (especially double) parton scattering (described in Sect. 8), i.e. the simultaneous occurrence of two independent hard scattering in the same interaction. Extrapolations to LHC energies, based on measurements at the Tevatron show the importance of taking this process into account when small transverse momenta are involved. Manifestations of double parton scattering are expected in the production of four jet final states and in the production of a lepton in association with two b-quarks (where the latter is used as a final state for Higgs searches).

The last section (Sect. 9) addresses the issue of the present knowledge of background for Higgs searches, for final states containing two photons or multi-leptons. For the case of di-photon final states (used for Higgs searches with $90 < m_H < 140$ GeV), studies of the irreducible background are performed by calculating the (single and double) fragmentation contributions to NLO accuracy and by studying the effects of soft gluon emission. The production of rare five lepton final states could provide valuable information on the Higgs couplings for $m_H > 200$ GeV, awaiting further studies on improving the understanding of the backgrounds.

During the workshop, no studies of diffractive scattering at the LHC have been performed. This topic is challenging both from the theoretical and the experimental point of view. The study of diffractive processes (with a typical signature of a leading proton and/or a large rapidity gap) should lead to an improved understanding of the transition between soft and hard process and of the non-perturbative aspects of QCD. From the experimental point of view, the detection of leading protons in the LHC environment is challenging and requires adding additional detectors to ATLAS and CMS. If hard diffractive scattering (leading proton(s) together with e.g. jets as signature for a hard scattering) is to be studied with decent statistical accuracy at large $p_T$, most of the luminosity delivered under normal running conditions has to be utilized. A few more details can be found in Chapter 15 of Ref. [1], some ideas for detectors in Ref. [3]. Much more work remains to be done, including a detailed assessment of the capabilities of the additional detectors.

### 1.1 Overview of QCD tools

All of the processes to be investigated at the LHC involve QCD to some extent. It cannot be otherwise, since the colliding quarks and gluons carry the QCD color charge. One can use perturbation theory to describe the cross section for an inclusive hard-scattering process,

$$h_1(p_1) + h_2(p_2) \rightarrow H(Q, \{ \ldots \}) + X .$$

Here the colliding hadrons $h_1$ and $h_2$ have momenta $p_1$ and $p_2$, $H$ denotes the triggered hard probe (vector bosons, jets, heavy quarks, Higgs bosons, SUSY particles and so on) and $X$ stands for any unobserved particles produced by the collision. The typical scale $Q$ of the scattering process is set by the invariant mass or the transverse momentum of the hard probe and the notation $\{ \ldots \}$ stands for any other measured kinematic variable of the process. For example, the hard process may be the production of a Z boson. Then $Q = M_Z$ and we can take $\{ \ldots \} = y$, where $y$ is the rapidity of the Z boson. One can also measure the transverse momentum $Q_T$ of the the Z boson. Then the simple analysis described below applies if $Q_T \sim M_Z$. In the cases $Q_T \ll M_Z$ and $M_Z \ll Q_T$, there are two hard scales in the process and a more complicated analysis is needed. The case $Q_T \ll M_Z$ is of particular importance and is discussed in Sects. 3.3, 3.4 and 5.3.

The cross section for the process (1) is computed by using the factorization formula [4, 5]

$$\sigma(p_1, p_2; Q, \{ \ldots \}) = \sum_{ab} \int dx_1 dx_2 f_{a/h_1}(x_1, Q^2) f_{b/h_2}(x_2, Q^2) \hat{\sigma}_{ab}(x_1 p_1, x_2 p_2; Q, \{ \ldots \}; \alpha_s(Q)) + O((\Lambda_{QCD}/Q)^6) .$$

Here the indices $a, b$ denote parton flavors, $\{g, u, \bar{u}, d, \bar{d}, \ldots \}$. The factorization formula (2) involves the convolution of the partonic cross section $\hat{\sigma}_{ab}$ and the parton distribution functions $f_{a/h}(x, Q^2)$ of
the colliding hadrons. The term $O((A_{QCD}/Q)^p)$ on the right-hand side of Eq. (2) generically denotes non-perturbative contributions (hadronization effects, multiparton interactions, contributions of the soft underlying event and so on).

Evidently, the pdf’s are of great importance to making predictions for the LHC. These functions are determined from experiments. Some of the issues relating to this determination are discussed in Sect. 2. In particular, there are discussions of the question of error analysis in the determination of the pdf’s and there is a discussion of the prospects for determining the pdf’s from LHC experiments.

The partonic cross section $\hat{\sigma}_{ab}$ is computable as a power series expansion in the QCD coupling $\alpha_S(Q)$:

$$\hat{\sigma}_{ab}(p_1, p_2; Q, \ldots; \alpha_S(Q)) = \alpha_S^2(Q) \left\{ \sigma_{ab}^{(LO)}(p_1, p_2; Q, \ldots) \right. $$

$$+ \alpha_S(Q) \sigma_{ab}^{(NLO)}(p_1, p_2; Q, \ldots) $$

$$+ \alpha_S^2(Q) \sigma_{ab}^{(NNLO)}(p_1, p_2; Q, \ldots) + \cdots \right\} . \tag{3}$$

The lowest (or leading) order (LO) term $\sigma^{(LO)}$ gives only a rough estimate of the cross section. Thus one needs the next-to-leading order (NLO) term, which is available for most cases of interest. A list of the available calculations is given in Sect. 4.1. Cross sections at NNLO are not available at present, but the prospects are discussed in Sect. 4.2.

The simple formula (2) applies when the cross section being measured is “infrared safe.” This means that the cross section does not change if one high energy strongly interacting light particle in the final state divides into two particles moving in the same direction or if one such particle emits a light particle carrying very small momentum. Thus in order to have a simple theoretical formula one does not typically measure the cross section to find a single high-$p_T$ pion, say, but rather one measures the cross section to have a collimated jet of particles with a given total transverse momentum $p_T$. If, instead, a single high-$p_T$ pion (or, more generally, a high-$p_T$ hadron $H$) is measured, the factorization formula has to include an additional convolution with the corresponding parton fragmentation function $d_{q/H}(z, Q^2)$. An example of a case where one needs a more complicated treatment is the production of high-$p_T$ photons. This case is discussed in Sect. 6.

As an example of a NLO calculation, we display in Fig. 1 the predicted cross section $d\sigma / dE_T dy$ at the LHC for the inclusive production of a jet with transverse energy $E_T$ and rapidity $y$ averaged over the rapidity interval $-1 < y < 1$. The calculation uses the program in Ref. [6] and the pdf set CTEQ5M [7]. As mentioned above, the “jets” must be defined with an infrared safe algorithm. Here we use the $k_T$ algorithm [8, 9] with a joining parameter $R = 1$. The $k_T$ algorithm has better theoretical properties than the cone algorithm that has often been used in hadron collider experiments.

In Eq. (2) there are integrations over the parton momentum fractions $x_1$ and $x_2$. The values of $x_1$ and $x_2$ that dominate the integral are controlled by the kinematics of the hard-scattering process. In the case of the production of a heavy particle of mass $M$ and rapidity $y$, the dominant values of the momentum fractions are $x_{1,2} \sim (Me^{\pm y})/\sqrt{s}$, where $s = (p_1 + p_2)^2$ is the square of the centre-of-mass energy of the collision. Thus, varying $M$ and $y$ at fixed $\sqrt{s}$, we are sensitive to partons with different momentum fractions. Increasing $\sqrt{s}$ the pdf’s are probed in a kinematic range that extends towards larger values of $Q$ and smaller values of $x_{1,2}$. This is illustrated in Fig. 2. At the LHC, $x_{1,2}$ can be quite small. Thus small $x$ effects that go beyond the simple formula (2) could be important. These are discussed in Sect. 7.

In Fig. 3 we plot NLO cross sections for a selection of hard processes versus $\sqrt{s}$. The curves for the lower values of $\sqrt{s}$ are for $p\bar{p}$ collisions, as at the Tevatron, while the curves for the higher values of $\sqrt{s}$ are for $pp$ collisions, as at the LHC. An approximation (based on an extrapolation of a standard Regge parametrization) to the total cross section is also displayed. We see that the cross sections for production of objects with a fixed mass or jets with a fixed transverse energy $E_T$ rise with $\sqrt{s}$. This is
Fig. 1: Jet cross section at the LHC, averaged over the rapidity interval $-1 < y < 1$. The cross section is calculated at NLO using CTEQ5M partons with the renormalization and factorization scales set to $\mu_R = \mu_F = E_T/2$. Representative values at $E_T = 0.5, 1, 2, 3$ and $4$ TeV are $(6.2 \times 10^{3}, 8.3 \times 10^{3}, 4.0 \times 10^{-1}, 5.1 \times 10^{-3}, 5.9 \times 10^{-5})$ fb/GeV with about 3% statistical errors.

because the important $x_{1,2}$ values decrease, as discussed above, and there are more partons at smaller $x$. On the other hand, cross sections for jets with transverse momentum that is a fixed fraction of $\sqrt{s}$ fall with $\sqrt{s}$. This is (mostly) because the partonic cross sections $\sigma_{part}$ like $\sigma_{part}$.

The perturbative evaluation of the factorization formula (2) is based on performing power series expansions in the QCD coupling $\alpha_S(Q)$. The dependence of $\alpha_S$ on the scale $Q$ is logarithmic and it is given by the renormalization group equation [4]

$$Q^2 \frac{d\alpha_S(Q)}{dQ^2} = \beta(\alpha_S(Q)) = -b_0 \alpha_S^2(Q) - b_1 \alpha_S^3(Q) + \cdots,$$  \hspace{1cm} (4)

where the first two perturbative coefficients are

$$b_0 = \frac{33 - 2N_f}{12\pi}, \quad b_1 = \frac{153 - 19N_f}{24\pi^2},$$  \hspace{1cm} (5)

and $N_f$ is the number of flavours of light quarks (quarks whose mass is much smaller than the scale $Q$). The third and fourth coefficients $b_2$ and $b_3$ of the $\beta$-function are also known [11, 12]. If we include only the LO term, Eq. (4) has the exact analytical solution

$$\alpha_S(Q) = \frac{1}{b_0 \ln(Q^2/\Lambda^2_{QCD})},$$  \hspace{1cm} (6)

where the integration constant $\Lambda_{QCD}$ fixes the absolute size of the QCD coupling. From Eq. (6) we can see that a change of the scale $Q$ by an arbitrary factor of order unity (say, $Q \rightarrow Q/2$) induces a variation in $\alpha_S$ that is of the order of $\alpha_S^3$. This variation is uncontrollable because it is beyond the accuracy at which Eq. (6) is valid. Therefore, in LO of perturbation theory the size of $\alpha_S$ is not unambiguously defined.

The QCD coupling $\alpha_S(Q)$ can be precisely defined only starting from the NLO in perturbation theory. To this order, the renormalization group equation (4) has no exact analytical solution. Different approximate solutions can differ by higher-order corrections and some (arbitrary) choice has to be made. Different choices can eventually be related to the definition of different renormalization schemes. The
The most popular choice [13] is to use the \( \overline{\text{MS}} \)-scheme to define renormalization and then to use the following approximate solution of the two loop evolution equation to define \( \Lambda_{QCD} \):

\[
\alpha_s(Q) = \frac{1}{b_0 \ln(Q^2/\Lambda_{QCD}^2)} \left[ 1 - \frac{b_1 \ln[\ln(Q^2/\Lambda_{QCD}^2)]}{b_0 \ln(Q^2/\Lambda_{QCD}^2)} + O\left(\frac{\ln^2[\ln(Q^2/\Lambda_{QCD}^2)]}{\ln^2(Q^2/\Lambda_{QCD}^2)}\right)\right]. \tag{7}
\]

Here the definition of \( \Lambda_{QCD} \) (\( \Lambda_{QCD} = \Lambda_{\overline{\text{MS}}} \)) is contained in the fact that there is no term proportional to \( 1/\ln^2(Q^2/\Lambda_{QCD}^2) \). In this expression there are \( N_f \) light quarks. Depending on the value of \( Q \), one may want to use different values for the number of quarks that are considered light. Then one must match between different renormalization schemes, and correspondingly change the value of \( \Lambda_{\overline{\text{MS}}} \) as discussed in Ref. [13]. The constant \( \Lambda_{\overline{\text{MS}}} \) is the one fundamental constant of QCD that must be determined from experiments. Equivalently, experiments can be used to determine the value of \( \alpha_S \) at a fixed reference scale \( Q = \mu_0 \). It has become standard to choose \( \mu_0 = M_Z \). The most recent determinations of \( \alpha_S \) lead [13] to the world average \( \alpha_S(M_Z) = 0.119 \pm 0.002 \). In present applications to hadron collisions, the value of \( \alpha_S \) is often varied in the wider range \( \alpha_S(M_Z) = 0.113 - 0.123 \) to conservatively estimate theoretical uncertainties.

The parton distribution functions \( f_{a/h}(x, Q^2) \) at any fixed scale \( Q \) are not computable in perturbation theory. However, their scale dependence is perturbatively controlled by the DGLAP evolution.
Fig. 3: Cross sections for hard scattering versus $\sqrt{s}$. The cross section values at $\sqrt{s} = 14$ TeV are: $\sigma_{\text{tot}} = 99.4$ mb, $\sigma_b = 0.633$ mb, $\sigma_t = 0.888$ nb, $\sigma_W = 187$ nb, $\sigma_Z = 55.5$ nb, $\sigma_H(M_H = 150$ GeV) = 23.8 pb, $\sigma_H(M_H = 500$ GeV) = 3.82 pb, $\sigma_{\text{jet}}(E_T^{\text{jet}} > 100$ GeV) = 1.57 µb, $\sigma_{\text{jet}}(E_T^{\text{jet}} > \sqrt{s}/20) = 0.133$ nb, $\sigma_{\text{jet}}(E_T^{\text{jet}} > \sqrt{s}/4) = 0.10$ fb. All except the first of these are calculated using the latest MRST pdf’s [10].

Having determined $f_{a/h}(x, Q_0^2)$ at a given input scale $Q = Q_0$, the evolution equation can be used to compute the pdf’s at different perturbative scales $Q$ and larger values of $x$.

The kernels $P_{ab}(\alpha_S, z)$ in Eq. (8) are the Altarelli–Parisi (AP) splitting functions. They depend on the parton flavours $a, b$ but do not depend on the colliding hadron $h$ and thus they are process-independent. The AP splitting functions can be computed as a power series expansion in $\alpha_S$:

$$P_{ab}(\alpha_S, z) = \alpha_S P_{ab}^{(LO)}(z) + \alpha_S^2 P_{ab}^{(NLO)}(z) + \alpha_S^3 P_{ab}^{(NNLO)}(z) + O(\alpha_S^4) .$$

The LO and NLO terms $P_{ab}^{(LO)}(z)$ and $P_{ab}^{(NLO)}(z)$ in the expansion are known [18–24]. These first two terms (their explicit expressions are collected in Ref. [4]) are used in most of the QCD studies. Partial calculations [25, 26] of the next-to-next-to-leading order (NNLO) term $P_{ab}^{(NNLO)}(z)$ are also available (see Sects. 2.5, 2.6 and 4.2).
As in the case of $\alpha_S$, the definition and the evolution of the pdf’s depends on how many of the quark flavors are considered to be light in the calculation in which the parton distributions are used. Again, there are matching conditions that apply. In the currently popular sets of parton distributions there is a change of definition at $Q = M$, where $M$ is the mass of a heavy quark.

The factorization on the right-hand side of Eq. (2) in terms of (perturbative) process-dependent partonic cross sections and (non-perturbative) process-independent pdf’s involves some degree of arbitrariness, which is known as factorization-scheme dependence. We can always ‘re-define’ the pdf’s by multiplying (convoluting) them by some process-independent perturbative function. Thus, we should always specify the factorization-scheme used to define the pdf’s. The most common scheme is the MS factorization-scheme [4]. An alternative scheme, known as DIS factorization-scheme [27], is sometimes used. Of course, physical quantities cannot depend on the factorization scheme. Perturbative corrections beyond the LO to partonic cross sections and AP splitting functions are thus factorization-scheme dependent to compensate the corresponding dependence of the pdf’s. In the evaluation of hadronic cross sections at a given perturbative order, the compensation may not be exact because of the presence of yet uncalculated higher-order terms. Quantitative studies of the factorization-scheme dependence can be used to set a lower limit on the size of missing higher-order corrections.

The factorization-scheme dependence is not the only signal of the uncertainty related to the computation of the factorization formula (2) by truncating its perturbative expansion at a given order. Truncation leads to additional uncertainties and, in particular, to a dependence on the renormalization and factorization scales. The renormalization scale $\mu_R$ is the scale at which the QCD coupling $\alpha_S$ is evaluated. The factorization scale $\mu_F$ is introduced to separate the bound-state effects (which are embodied in the pdf’s) from the perturbative interactions (which are embodied in the partonic cross section) of the partons. In Eqs. (2) and (3) we took $\mu_R = \mu_F = Q$. On physical grounds these scales have to be of the same order as $Q$, but their value cannot be unambiguously fixed. In the general case, the right-hand side of Eq. (2) is modified by introducing explicit dependence on $\mu_R, \mu_F$ according to the replacement

$$f_{a/h_1} (x_1, Q^2) f_{a/h_2} (x_2, Q^2) \to f_{a/h_1} (x_1, \mu_R^2) f_{a/h_2} (x_2, \mu_F^2) \to \hat{\sigma}_{ab} (x_1 p_1, x_2 p_2; Q, \{\ldots \}; \alpha_S (Q))$$

$$\downarrow$$

$$f_{a/h_1} (x_1, \mu_R^2) f_{a/h_2} (x_2, \mu_F^2) \to \hat{\sigma}_{ab} (x_1 p_1, x_2 p_2; Q, \{\ldots \}; \mu_R, \mu_F; \alpha_S (\mu_R)) .$$

(10)

The physical cross section $\sigma (p_1, p_2; Q, \{\ldots \})$ does not depend on the arbitrary scales $\mu_R, \mu_F$, but parton densities and partonic cross sections separately depend on these scales. The $\mu_R, \mu_F$-dependence of the partonic cross sections appears in their perturbative expansion and compensates the $\mu_R$ dependence of $\alpha_S (\mu_R)$ and the $\mu_F$-dependence of the pdf’s. The compensation would be exact if everything could be computed to all orders in perturbation theory. However, when the quantities entering Eq. (10) are evaluated at, say, the $n$-th perturbative order, the result exhibits a residual $\mu_R, \mu_F$-dependence, which is formally of the $(n + 1)$-th order. That is, the explicit $\mu_R, \mu_F$-dependence that still remains reflects the absence of yet uncalculated higher-order terms. For this reason, the size of the $\mu_R, \mu_F$ dependence is often used as a measure of the size of at least some of the uncalculated higher-order terms and thus as an estimator of the theoretical error caused by truncating the perturbative expansion.

As an example, we estimate the theoretical error on the predicted jet cross section in Fig. 1. We vary the renormalization scale $\mu_R$ and the factorization scale $\mu_F$. In Fig. 4, we plot

$$\Delta (\mu_R/E_T, \mu_F/E_T) = \frac{\langle d\sigma (\mu_R/E_T, \mu_F/E_T) / dE_T dy \rangle}{\langle d\sigma (0.5, 0.5) / dE_T dy \rangle},$$

(11)

versus $E_T$ for four values of the pair $\{\mu_R/E_T, \mu_F/E_T\}$, namely $\{0.25, 0.25\}$, $\{1.0, 0.25\}$, $\{0.25, 1.0\}$, and $\{1.0, 1.0\}$. We see about a 10% variation in the cross section. This suggests that the theoretical uncertainty is at least 10%.

The issue of the scale dependence of the perturbative QCD calculations has received attention in the literature and various recipes have been proposed to choose ‘optimal’ values of $\mu$ (see the references...
in [13]). There is no compelling argument that shows that these ‘optimal’ values reduce the size of the yet unknown higher-order corrections. These recipes may thus be used to get more confidence on the central value of the theoretical calculation, but they cannot be used to reduce its theoretical uncertainty as estimated, for instance, by scale variations around $\mu \sim Q$. The theoretical uncertainty ensuing from the truncation of the perturbative series can only be reduced by actually computing more terms in perturbation theory.

We have so far discussed the factorization formula (2). We should emphasize that there is another mode of analysis of the theory available, that embodied in Monte Carlo event generator programs. In this type of analysis, one is limited (at present) to leading order partonic hard scattering cross sections. However, one simulates the complete physical process, beginning with the hard scattering and proceeding through parton showering via repeated one parton to two parton splittings and finally ending with a model for how partons turn into hadrons. This class of programs, which simulate complete events according to an approximation to QCD, are very important to the design and analysis of experiments. Current issues in Monte Carlo event generator and other related computer programs are discussed in Sect. 3.

2. PARTON DISTRIBUTION FUNCTIONS

Parton distributions (pdf’s) play a central role in hard scattering cross sections at the LHC. A precise knowledge of the pdf’s is absolutely vital for reliable predictions for signal and background cross sections. In many cases, it is the uncertainty in the input pdf’s that dominates the theoretical error on the prediction. Such uncertainties can arise both from the starting distributions, obtained from a global fit to DIS, DY and other data, and from DGLAP evolution to the higher $Q^2$ scales typical of LHC hard scattering processes.

To predict LHC cross sections we will need accurate pdf’s over a wide range of $x$ and $Q^2$ (see Fig. 2). Several groups have made significant contributions to the determination of pdf’s both during and after the workshop. The MRST and CTEQ global analyses have been updated and refined, and small numerical problems have been corrected. The ‘central’ pdf sets obtained from these global fits are, not surprisingly, very similar, and remain the best way to estimate central values for LHC cross sections. Specially constructed variants of the central fits (exploring, for example, different values of $\alpha_s$ or different theoretical treatments of heavy quark distributions) allow the sensitivity of the cross sections to some of the input assumptions.

A rigorous and global treatment of pdf uncertainties remains elusive, but there has been significant progress in the last few years, with several groups introducing sophisticated statistical analyses into quasi-global fits. While some of the more novel methods are still at a rather preliminary stage, it is hoped that over the next few years they may be developed into useful tools.

One can reasonably expect that by LHC start-up time, the precision pdf determinations will have improved from NLO to NNLO. Although the complete NNLO splitting functions have not yet been

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calculated, several studies have made use of partial information (moments, \( x \to 0, 1 \) limiting behaviour) to assess the impact of the NNLO corrections.

At the same time, accurate measurements of Standard Model (SM) cross sections at the LHC will further constrain the pdf’s. The kinematic acceptance of the LHC detectors allows a large range of \( x \) and \( Q^2 \) to be probed. Furthermore, the wide variety of final states and high parton-parton luminosities available will allow an accurate determination of the gluon density and flavour decomposition of quark densities.

All of the above issues are discussed in the individual contributions that follow. Lack of space has necessarily restricted the amount of information that can be included, but more details can always be found in the literature.

2.1 MRS: pdf uncertainties and W and Z production at the LHC

There are several reasons why it is very difficult to derive overall ‘one sigma’ errors on parton distributions of the form \( f_i \pm \delta f_i \). In the global fit there are complicated correlations between a particular pdf at different \( x \) values, and between the different pdf flavours. For example, the charm distribution is correlated with the gluon distribution, the gluon distribution at low \( x \) is correlated with the gluon at high \( x \) via the momentum sum rule, and so on. Secondly, many of the uncertainties in the input data or fitting procedure are not ‘true’ errors in the probabilistic sense. For example, the uncertainty in the high–\( x \) gluon in the MRST fits [28] derives from a subjective assessment of the impact of ‘intrinsic \( k_T \)’ on the prompt photon cross sections included in the global fit. Despite these difficulties, several groups have attempted to extract meaningful \( \pm \delta f_i \) pdf errors (see [29, 30] and Sects. 2.3,2.4). Typically, these analyses focus on subsets of the available DIS and other data, which are statistically ‘clean’, i.e. free from undetermined systematic errors. As a result, various aspects of the pdf’s that are phenomenologically important, the flavour structure of the sea and the sea and gluon distributions at large \( x \) for example, are either only weakly constrained or not determined at all.

Faced with the difficulties in trying to formulate global pdf errors, one can adopt a more pragmatic approach to the problem by making a detailed assessment of the pdf uncertainty for a particular cross section of interest. This involves determining which partons contribute and at which \( x \) and \( Q^2 \) values, and then systematically tracing back to the data sets that constrained the distributions in the global fit. Individual pdf sets can then be constructed to reflect the uncertainty in the particular partons determined by a particular data set.

Fig. 6: Predictions for the $W$ and $Z$ total cross sections times leptonic branching ratio in $pp$ collisions at 14 TeV using the various MRST parton sets from Ref. [10]. The error bars on the default MRST prediction correspond to a scale variation of $\mu = M_v/2 \rightarrow 2M_v$, $V = W, Z$.

We have recently performed such an analysis for $W$ and $Z$ total cross sections at the Tevatron and LHC [10]. The theoretical technology for calculating these is very robust. The total cross sections are known to NNLO in QCD perturbation theory [31–33], and the input electroweak parameters ($M_{W,Z}$, weak couplings, etc.) are known to high accuracy. The main theoretical uncertainty therefore derives from the input pdf’s and, to a lesser extent, from $\alpha_S$.\(^{3}\)

For the hadro-production of a heavy object like a $W$ boson, with mass $M$ and rapidity $y$, leading-order kinematics give $x = M \exp(\pm y) / \sqrt{s}$ and $Q = M$. For example, a $W$ boson ($M = 80$ GeV) produced at rapidity $y = 3$ at the LHC corresponds to the annihilation of quarks with $x = 0.00028$ and 0.11, probed at $Q^2 = 6400$ GeV$^2$. Notice that $u, d$ quarks with these $x$ values are already more or less directly ‘measured’ in deep inelastic scattering (at HERA and in fixed-target experiments respectively), but at much lower $Q^2$, see Fig. 2. Therefore the first two important sources of uncertainty in the pdf’s relevant to $W$ production are

(i) the uncertainty in the DGLAP evolution, which except at high $x$ comes mainly from the gluon and $\alpha_S$;

(ii) the uncertainty in the quark distributions from measurement errors on the structure function data used in the fit.

This is illustrated in Fig. 5.\(^4\) Only 75% of the total $W$ cross section at the LHC arises from the scattering of $u$ and $d$ (anti)quarks. Therefore also potentially important is

(iii) the uncertainty in the input strange ($s$) and charm ($c$) quark distributions, which are relatively poorly determined at low $Q^2$ scales.

\(^{3}\)The two are of course correlated, see for example [28].

\(^{4}\)The ‘feed-down’ error represents a possible anomalously large contribution at $x \approx 1$ affecting the evolution at lower $x$. It is not relevant, however, for $W$ production at the Tevatron or LHC.
In order to investigate these various effects we have constructed ten variants of the standard MRST99 distributions [10] that probe approximate $\pm 1\sigma$ variations in the gluon, $\alpha_s$, the overall quark normalisation, and the $s$ and $c$ pdf’s. The corresponding predictions for the $W$ total cross section at the LHC are shown in Fig. 6. Evidently the largest variation comes from the effect of varying $\alpha_s(M_Z^2)$, in this case by $\pm 0.005$ about the central value of 0.1175. The higher the value of $\alpha_s$, the faster the (upwards) evolution, and the larger the predicted $W$ cross section. The effect of a $\pm 2.5\%$ normalisation error, as parameterised by the $q^\dagger$ pdf’s, is also significant. The uncertainties in the input $s$ and $c$ distributions get washed out by evolution to high $Q^2$, and turn out to be numerically unimportant.

In conclusion, we see from Fig. 6 that $\pm 5\%$ represents a conservative error on the prediction of $\sigma(W)$ at LHC. We arrive at this result without recourse to complicated statistical analyses in the global fit. It is also reassuring that the latest (corrected) CTEQ5 prediction [7] is very close to the central MRST99 prediction, see Fig. 8 below. Finally, it is important to stress that the results of our analysis represent a ‘snap-shot’ of the current situation. As further data are added to the global fit in coming years, the situation may change. However it is already clear that LHC $W$ and $Z$ cross sections can already be predicted with high precision, and their measurement will therefore provide a fundamental test of the SM.

2.2 CTEQ: studies of pdf uncertainties

Status of Standard Parton Distribution Functions

The widely used pdf sets all have been updated recently, driven mainly by new experimental inputs. Largely due to differences in the choices of these inputs (direct photon vs. jets) and their theoretical treatment, the latest MRST [10] and CTEQ [7] distributions have noticeable differences in the gluon distribution for $x > 0.2$. Details are described in the original papers.

The accuracy of modern DIS measurements and the expanding $(x, Q)$ range in which pdf’s are applied require accurate QCD evolution calculations. Previously known differences in the QCD evolution codes have now been corrected; all groups now agree with established results [34] with good precision. The differences between updated pdf’s obtained with the improved evolution code and the original ones are generally small; and the differences between the physical cross sections based on the two versions of pdf’s are insignificant, by definition, since both have been fitted to the same experimental data sets. However, accurate predictions for physical processes not included in the global analysis, especially at values of $(x, Q)$ beyond the current range, can differ and require the improved pdf’s. Figs. 7a,b compare the pdf sets CTEQ5M (original) and CTEQ5M1 (updated) at scales $Q = 5$ and 80 GeV respectively.

A comparison of the predicted $W$ production cross sections at the Tevatron and at LHC, using the historical CTEQ parton distribution sets, as well as the most recent MRST sets are given in Figs. 8. We see

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that the predicted values of $\sigma_W$ agree very well. However, the spread of $\sigma_W$ from different “best fit” pdf sets does not give a quantitative measure of the uncertainty of $\sigma_W$!

**Studies of pdf Uncertainties**

It is important to quantify the uncertainties of physics predictions due to imprecise knowledge of the pdf’s at future colliders (such as the LHC): these uncertainties may strongly affect the error estimates in precision SM measurements as well as the signal and backgrounds for new physics searches.

Uncertainties of the pdf’s themselves are strictly speaking unphysical, since pdf’s are not directly measurable. They are renormalization and factorization scheme dependent; and there are strong correlations between different flavours and different values of $x$ which can compensate each other in physics predictions. On the other hand, since pdf’s are universal, if one can obtain meaningful estimates of their uncertainties based on global analysis of existing data, they can then be applied to all processes that are of interest for the future.

An alternative approach is to assess the uncertainties on specific physical predictions for the full range (i.e. the ensemble) of pdf’s allowed by available experimental constraints which are used in current global analyses, without explicit reference to the uncertainties of the parton distributions themselves. This clearly gives more reliable estimates of the range of possible predictions on the physical variable under study. The disadvantage is that the results are process-specific; hence the analysis has to be carried out for each process of interest.

In this short report, we present first results from a systematic study of both approaches. In the next section we focus on the $W^\pm$ production cross section, as a proto-typical case of current interest. A technique of Lagrange multiplier is incorporated in the CTEQ global analysis to probe its range of uncertainty at the Tevatron and the LHC. This method is directly applicable to other cross sections of interest, e.g. Higgs production. We also plan to extend it for studying the uncertainties of $W$-mass measurements in the future. In the following section we describe a Hessian study of the uncertainties of the non-perturbative pdf parameters in general, followed by application of these to the $W^\pm$ production cross section study and a comparison of this result with that of the Lagrange-multiplier approach.

First, it is important to note the various sources of uncertainty in pdf analysis.

- **Statistical errors** of experimental data. These vary over a wide range, but are straightforward to treat.
- **Systematic experimental errors** within each data set typically arise from many sources, some of which are highly correlated. These errors can be treated by standard methods provided they are precisely known, which unfortunately is often not the case – either because they are not randomly distributed or their estimation may involve subjective judgements. Since strict quantitative statistical methods are based on idealized assumptions, such as random errors, one faces an important trade-off in pdf uncertainty analysis. If emphasis is put on the “rigor” of the statistical method, then most experimental data sets can not be included the analysis (see Sect. 2.3). If priority is placed on using the maximal experimental
constraints from available data, then standard statistical methods need to be supplemented by physical considerations, taking into account existing experimental and theoretical limitations. We take the latter tack.

- **Theoretical uncertainties** arise from higher-order PQCD corrections, resummation corrections near the boundaries of phase space, power-law (higher twist) and nuclear target corrections, etc.

- Uncertainties of pdf’s due to the **parametrization of the non-perturbative pdf’s**, \( f_a(x, Q^2) \), at some low energy scale \( Q_0 \). The specific functional form used introduces implicit correlations between the various \( x \)-ranges, which could be as important, if not more so, than the experimental correlations in the determination of \( f_a(x, Q^2) \) for all \( Q \).

In view of these considerations, the preliminary results reported here can only be regarded as the beginning of a continuing effort which will be complex, but certainly very important for the next generation of collider programs.

**The Lagrange multiplier method**

Our work uses the standard CTEQ5 analysis tools and results [7] as the starting point. The “best fit” is the CTEQ5M1 set. There are 15 experimental data sets, with a total of \( \sim 1300 \) data points; and 18 parameters \( a_i, i = 1, \ldots, 18 \) for the non-perturbative initial parton distributions. A natural way to find the limits of a physical quantity \( F \), such as \( \sigma_W \) at \( \sqrt{s} = 1.8 \) TeV, is to take \( X \) as one of the search parameters in the global fit and study the dependence of \( \chi^2 \) for the 15 base experimental data sets on \( X \).

Conceptually, we can think of the function \( \chi^2 \) that is minimized in the fit as a function of \( \{a_1, a_{17}, X\} \) instead of \( \{a_1, a_{18}\} \). This idea could be implemented directly in principle, but Lagrange’s method of undetermined multipliers does the same thing in a more efficient way. One minimizes

\[
F(\lambda) = \chi^2 + \lambda X(a_1, \ldots, a_{18})
\]

for fixed \( \lambda \). By minimizing \( F(\lambda) \) for many values of \( \lambda \), we map out \( \chi^2 \) as a function of \( X \).

Figs. 9a,b show the \( \chi^2 \) for the 15 base experimental data sets as a function of \( \sigma_W \) at the Tevatron and the LHC energies respectively. Two curves with points corresponding to specific global fits are included in each plot\(^6\): one obtained with all experimental normalizations fixed; the other with these included as fitting parameters (with the appropriate experimental errors). We see that the \( \chi^2 \)'s for the best fits corresponding to various values of the \( W \) cross section are close to being parabolic, as expected. Indicated on the plots are 3% and 5% ranges for \( \sigma_W \). The two curves for the Tevatron case are farther apart than for LHC, reflecting the fact that the \( W \)-production cross section is more sensitive to the quark/anti-quark distributions and these are tightly constrained by existing DIS data.

The important question is: how large an increase in \( \chi^2 \) should be taken to define the likely range of uncertainty in \( X \). The elementary statistical theorem that \( \Delta \chi^2 = 1 \) corresponds to 1 standard deviation

\(^6\)The third line in Figs. 9a refers to results of the next section.
of the measured quantity $X$ relies on assuming that the errors are Gaussian, uncorrelated, and with their magnitudes correctly estimated. Because these conditions do not hold for the full data set (of 1300 points from 15 different experiments), this theorem cannot be naively applied quantitatively.\footnote{As shown by Giele et al.\cite{35}, taken literally, only one or two selected experiments satisfy the standard statistical tests.} We plan to examine in some detail how well the fits along the parabolas shown in Fig.9a,b compare with the individual precision experiments included in the global analysis, in order to arrive at reasonable quantitative estimates on the uncertainty range for the $W$ cross section. In the meantime, based on past (admittedly subjective) experience with global fits, we believe a $\chi^2$ difference of 40-50 represents a reasonable estimate of current uncertainty of parton distributions. This implies that the uncertainty of $\sigma_W$ is about 3% at the Tevatron, and 5% at the LHC. These estimates certainly need to be put on a firmer basis by the on-going detailed investigation mentioned above.

The Hessian matrix method

The Hessian matrix is a standard procedure for error analysis. At the minimum of $\chi^2$, the first derivatives with respect to the parameters $a_i$ are zero, so near the minimum $\chi^2$ can be approximated by

$$\chi^2 = \chi_0^2 + \frac{1}{2} \sum_{i,j} F_{ij} y_i y_j$$

where $y_i = a_i - a_{0i}$ is the displacement from the minimum, and $F_{ij}$ is the Hessian, the matrix of second derivatives. It is natural to define a new set of coordinates using the complete orthonormal set of eigenvectors of the symmetric matrix $F_{ij}$. These vectors can be ordered by their eigenvalues $e_i$. The eigenvalues indicate the uncertainties for displacements along the eigenvectors. For uncorrelated Gaussian statistics, the quantity $\ell_i = 1/\sqrt{e_i}$ is the distance in the 18 dimensional parameter space that gives a unit increase in $\chi^2$ in the direction of eigenvector $i$.

From calculations of the Hessian we find the eigenvalues vary over a wide range. There are “steep” directions of $\chi^2$ – combinations of parameters that are well determined – e.g. parameters for $u$ and $d$, which are well-constrained by DIS data. There are also “flat” directions where $\chi^2$ changes little over large distance in $a_i$ space, some of them associated with the gluon distribution. These flat directions are inevitable in global fitting, because as the data improve it makes sense to maintain enough flexibility for $f_a(x, Q^2)$ to be determined by the available experimental constraints. The Hessian method gives an analytic picture of the region in parameter space around the minimum, hence allows us to identify the particular degrees of freedom which need further experimental input in future global analyses.

We have calculated how the $W$ cross section $\sigma_W$ varies along the eigenvectors of the Hessian. Details will be described elsewhere. This provides another way to calculate the relation between the minimum $\chi^2$ for the base experimental data sets and the value of $\sigma_W$. The results are shown as the third line in Fig. 9a. We see that there is approximate agreement between this method and the Lagrange multiplier method. Armed with the Hessian, one can in principle make similar calculations on other physical cross sections without having to do repeated global fits as in the Lagrange multiplier method. The latter, however, gives more reliable bounds for each individual process.

Conclusion

We have just begun the task of determining quantitative uncertainties for the parton distribution functions and their physics predictions. The methods developed so far look promising. Related work reported in this Workshop (see \cite{10, 35–37} and Sects. 2.1,2.3,2.4) share the same objectives, but have rather different emphases, some of which are briefly mentioned in the text. These complementary approaches should lead to eventual progress which is critical for the high-energy physics program at LHC, as well as at other colliders.
2.3 Pdf uncertainties

Introduction

The goal of our work is to extract pdf’s from data with a quantitative estimation of the uncertainties. There are some qualitative tools that exist to estimate the uncertainties, see e.g. [28]. These tools are clearly not adequate when the pdf uncertainties become important. One crucial example of a measurement that will need a quantitative assessment of the pdf uncertainty is the planned high precision measurement of the mass of the $W$-vector boson at the Tevatron.

The method we have developed in [35] is flexible and can accommodate non-Gaussian distributions for the uncertainties associated with the data and the fitted parameters as well as all their correlations. New data can be added in the fit without having to redo the whole fit. Experimenters can therefore include their own data into the fit during the analysis phase, as long as correlation with older data can be neglected. Within this method it is trivial to propagate the pdf uncertainties to new observables, there is for example no need to calculate the derivative of the observable with respect to the different pdf parameters. The method also provides tools to assess the goodness of the fit and the compatibility of new data with current fit. The computer code has to be fast as there is a large number of choices in the inputs that need to be tested.

It is clear that some of the uncertainties are difficult to quantify and it might not be possible to quantify all of them. All the plots presented here are for illustration of the method only, our results are preliminary. At the moment we are not including all the sources of uncertainties and our results should therefore be considered as lower limits on the pdf uncertainties. Note that all the techniques we use are standard, in the sense that they can be found in books and papers on statistics [38,39] and/or in Numerical Recipes.

Outline of the Method

We only give a brief overview of the method in this section. More details are available in [35]. Once a set of core experiments is selected, a large number of uniformly distributed sets of parameters $\lambda \equiv \lambda_1, \lambda_2, \ldots, \lambda_{N_{\text{par}}}$ (each set corresponds to one pdf) can be generated. The probability of each set, $P(\lambda)$, can be calculated from the likelihood (the probability) that the predictions based on $\lambda$ describe the data, assuming that the initial probability distribution of the parameters is uniform, see [38,39].

Knowing $P(\lambda)$, the probability of the possible values of any observable (quantity that depends on $\lambda$) can be calculated using a Monte Carlo integration. For example, the average value and the pdf uncertainty of an observable $x$ are given by:

$$\mu_x = \int \left( \prod_{i=1}^{N_{\text{par}}} d\lambda_i \right) x(\lambda) P(\lambda), \quad \sigma^2_x = \int \left( \prod_{i=1}^{N_{\text{par}}} d\lambda_i \right) (x(\lambda) - \mu_x)^2 P(\lambda)$$

Note that the average value and the standard deviation represents the distribution only if the latter is a Gaussian. The above is correct but computationally inefficient, instead we use a Metropolis algorithm to generate $N_{\text{pdf}}$ unweighted pdf’s distributed according to $P(\lambda)$. Then:

$$\mu_x \approx \frac{1}{N_{\text{pdf}}} \sum_{j=1}^{N_{\text{pdf}}} x(\lambda_j), \quad \sigma^2_x \approx \frac{1}{N_{\text{pdf}}} \sum_{j=1}^{N_{\text{pdf}}} (x(\lambda_j) - \mu_x)^2.$$

This is equivalent to importance sampling in Monte Carlo integration techniques and is very efficient. Given the unweighted set of pdf’s, a new experiment can be added to the fit by assigning a weight (a new probability) to each of the pdf’s, using Bayes’ theorem. The above summations become weighted. There is no need to redo the whole fit if there is no correlation between the old and new data. If we know how to calculate $P(\lambda)$ properly, the only uncertainty in the method comes from the Monte-Carlo integrations.

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Fig. 10: Plot of the distribution (histogram) of four of the parameters. The first one is $\alpha_S$, the strong coupling constant at the mass of the $Z$-boson. The line is a Gaussian distribution with same average and standard deviation as the histogram.

Calculation of $P(\lambda)$

Given a set of experimental points $\{x^e\} = x_1^e, x_2^e, \ldots, x_{N_{\text{obs}}}^e$, the probability of a set of pdf is proportional to the likelihood, the probability of the data given that the theory is derived from that set of pdf: $P(\lambda) \approx P(\{x^e\} | \lambda)$. If all the uncertainties are Gaussian distributed, then it is well known that: $P(x^e | \lambda) \approx e^{-\frac{\chi^2}{2}}$, where $\chi^2$ is the usual chi-square. It is only in this case that it is sufficient to report the size of the uncertainties and their correlation. When the uncertainties are not Gaussian distributed, it is necessary for experiments to report the distribution of their uncertainties and the relation between these uncertainties the theory and the value of the measurements. Unfortunately most of the time that information is not reported, or difficult to extract from papers. This is a very important issue that has been one of the focus of the pdf working group at a Fermilab workshop in preparation for run II [40]. In other words, experiments should always provide a way to calculate the likelihood of their data given a theory prediction for each of their measured data point ($P(\{x^e\} | \lambda)$). This was also the unanimous conclusion of a recent workshop on confidence limits held at CERN [41]. This is particularly crucial when combining different experiments together: the pull of each experiment will depend on it and, as a result, so will the central values of the deduced pdf’s. Another problem that is sometimes underestimated is the fact that some if not all systematic uncertainties are in fact proportional to the theory. Ignoring this fact while fitting for the parameters can lead to serious bias.

Sources of uncertainties

There are many sources of uncertainties beside the experimental uncertainties. They either have to be shown to be small enough to be neglected or they need to be included in the pdf uncertainties. For examples: variation of the renormalization and factorization scales; non-perturbative and nuclear binding effects; the choice of functional form of the input pdf at the initial scale; accuracy of the evolution; Monte-Carlo uncertainties; and the theory cut-off dependences.

Current fit

Draconian measures were needed to restart from scratch and re-evaluate each issue. We fixed the renormalization and factorisation scales, avoided data affected by nuclear binding and non-perturbative effects, and use a MRS-style parametrization for the input pdf’s. The evolution of the pdf is done by Mellin transform method, see [42, 43]. All the quarks are considered massless. We imposed a positivity constraint on $F_2$. A positivity constraint on other “observables” could also be imposed.

At the moment we are using H1 and BCDMS(proton) measurement of $F_2^p$ for our core set. The full correlation matrix is taken into account. Assuming that all the uncertainties are Gaussian distributed $^9$ we calculate the $\chi^2(\lambda)$ and $P(\lambda) \approx \exp(-\chi^2/2)$. We generated 50000 unweighted pdf’s according to the probability function. For 532 data points, we obtained a minimum $\chi^2 = 530$ for 24 parameters. We have plotted in Fig. 10, the probability distribution of some of the parameters. Note that the first

$^9$No information being given about the distribution of the uncertainties.
Fig. 11: The relative uncertainties for selected set of parton luminosities (full lines: experimental errors (stat+syst); short-dashed lines: RS; dotted-dashed lines: TS; sparse-dotted lines: DC; dense-dotted lines: MC; long-dashed lines: SS). Here $L_{GG}$ is gluon-gluon luminosity; $L_{qq} = L_{uu} + L_{dd} + L_{u\bar{d}}$; $L_{q\bar{q}} = L_{u\bar{u}} + L_{d\bar{d}}$; $L_{(u+\bar{u})G} = L_{uG} + L_{\bar{u}G} + L_{dG} + L_{\bar{d}G}$.

The parameter is $\alpha_S$. The value is smaller than the current world average. However, it is known that the experiments we are using prefer a lower value of this parameter, see [44], and as already pointed out, our current uncertainties are lower limits. Note that the distribution of the parameter is not Gaussian, indicating that the asymptotic region is not reached yet. In this case, the blind use of a so-called chi-squared fitting technique is not appropriate. From this large set of pdf’s, it is straightforward to plot, for example, the correlation between different parameters and to propagate the uncertainties to other observables.

2.4 Uncertainties on pdf’s and parton-parton luminosities

An important quantity for LHC physics is the uncertainty of pdf’s used for the cross section calculations. The modern widely used pdf’s parametrizations do not contain complete estimate of their uncertainties. This estimate is difficult partially due to the lack of experimental information on the data points correlations, partially due to the fact that the theoretical uncertainties are conventional, and partially due to the fundamental problem of restoring the distribution from the finite number of measurements. These problems are not completely solved at the moment and a comprehensive estimate of the pdf’s uncertainties is not available so far. The study given below is based on the NLO QCD analysis of the world charged leptons DIS data of Refs. [45–51] for proton and deuterium targets. The analysed data span the region $bcedfhgjilkmfnpoq$, $rts$ and allows for precise determination of pdf’s at low $b$, which is important for LHC since the most of accessible processes are related to small $x$. The data are accompanied by the information on point-to-point correlations due to systematic errors. This allows the complete inference of systematic errors, that was performed using the covariance matrix approach, as in Ref. [36]. The pdf’s uncertainties due to the variation of the strong coupling constant $\alpha_S$ and the high twists (HT) contribution are automatically accounted for in the total experimental uncertainties since $\alpha_S$ and HT are fitted. Other theoretical errors on pdf’s were estimated as the pdf’s variation after the change of different fit ansatzes:

RS – the change of renormalization scale in the evolution equations from $Q^2$ to $4Q^2$. This uncertainty is evidently connected with the influence of NNLO corrections.

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11More details of the analysis can be found in Ref. [29].

12The value of $\alpha_S(M_Z) = 0.1165 \pm 0.0017$(stat + syst) is obtained, that is compatible with the world average.
Fig. 12: The ratios of the experimental pdf’s errors calculated with some fitted parameters fixed to the pdf’s errors calculated with all parameters released (αS fixed – a; HT fixed – b). The similar ratio for the systematic errors omitted/included is also given – c). Full lines correspond to gluons, dashed ones – to total sea, dotted ones – to d-quarks, dashed-dotted ones – to u-quarks.

**TS** – the change of threshold value of $Q^2$ for the QCD evolution loops with heavy quarks from $m_Q^2$ to $6.5m_Q^2$. The variation is conventional and was chosen following the arguments of Ref. [52].

**DC** – the change of correction on nuclear effects in deuterium from the ansatz based on the Fermi motion model of Ref. [53] to the phenomenological formula from Ref. [54]. Note that this uncertainty may be overestimated in view of discussions [55, 56] on the applicability of the model of Ref. [54] to light nuclei.

**MC** – the change of c-quark mass by 0.25 GeV (the central value is 1.5 GeV).

**SS** – the change of strange sea suppression factor by 0.1, in accordance with recent results by the NuTeV collaboration [57] (the central value is 0.42).

One can see that the scale of the theoretical errors is conventional and can change with improvements in the determination of the fit input parameters and progress in theory. Moreover, the uncertainties can be correlated with the uncertainties of the partonic cross sections, e.g. the effect of RS uncertainty on pdf’s can be compensated by the NNLO correction to parton cross section. Thus the theoretical uncertainties should not be applied automatically to any cross section calculations, contrary to experimental ones.

The pdf’s uncertainties have different importance for various processes. The limited space does not allow us to review all of them. We give the figures for the most generic ones only. The uncertainties of a specific cross section due to pdf’s are entirely located in the uncertainties of the parton-parton luminosity $L_{ab}$, that is defined as

$$L_{ab}(M) = \frac{1}{s} \int_{\tau}^{1} \frac{dx}{x} f_a(x, M^2) f_b(\tau/x, M^2),$$

where $M$ is the produced mass and $\tau = M^2/s$. In Fig. 11 the uncertainties for selected set of parton luminosities calculated using the pdf’s from Ref. [29] are given. The upper bound of $M$ was chosen so that the corresponding luminosity is $\sim 0.01$ pb. One can see that in general at $M \gtrsim 1$ TeV experimental uncertainties dominate, while at $M \lesssim 1$ TeV theoretical ones dominate. Of the latter the most important are the RS uncertainty for the gluon luminosity and MC uncertainty for the quark luminosities. At the largest $M$ the DC uncertainty for quark-quark luminosity is comparable with the experimental one. In the whole the uncertainties do not exceed 10% at $M \lesssim 1$ TeV. As for the quark-quark luminosity, its uncertainty is less than 10% in the whole $M$ range. The uncertainties are not so large in view of the fact that only a small subset of data relevant for the pdf’s extraction was used in the analysis. Adding data on prompt photon production, DY process, and jet production can improve the pdf’s determination at large $x$. Meanwhile it is worth to note that high order QCD corrections are more important for these processes than for DIS and the decrease of experimental errors due to adding data points can be accompanied by the increase of theoretical errors.
As it was noted above, the experimental pdf’s errors by definition include the statistical and systematic errors, as well as errors due to $\alpha_S$ and HT. To trace the effect of $\alpha_S$ variation on the pdf’s uncertainties the latter were re-calculated with $\alpha_S$ fixed at the value obtained in the fit. The ratios of obtained experimental pdf’s errors to the errors calculated with $\alpha_S$ released are given in Fig. 12. It is seen that the $\alpha_S$ variation takes some effect on the gluon distribution errors only. Similar ratios for the HT fixed are also given in Fig. 12. One can conclude, that the account of HT contribution have significant impact on the pdf’s errors. Meanwhile it is evident that these ratios hardly depend on the scale of pdf’s error and are specific for the analysed data set. For instance, in the analysis of CCFR data on the structure function $F_3$ no significant influence of HT on the pdf’s was observed [58, 59]. The contribution of systematic errors to the total experimental pdf’s uncertainties is also given in Fig. 12: the systematic errors are most essential for the u- and d-quark distributions.

Except uncertainties itself the pdf correlation are also important (see Fig. 13). The account of correlations can lead to cancellation of the pdf’s uncertainties in the calculated cross section. The luminosities uncertainties can also cancel in the ratios of cross sections. An example of such cancellation is given in Table 2.4, where the uncertainties of luminosities for the $W/Z$ production cross sections and their ratios are given.

The pdf set discussed in this subsection can be obtained by the code [60]. The pdf’s are DGLAP evolved in the range $x = 10^{-7} \div 1$, $Q^2 = 2.5 \div 5.6 \cdot 10^7$ GeV$^2$. The code returns the values of u-, d-, s-quark, and gluon distributions Gaussian-randomized with accordance of their dispersions and correlations including both experimental and theoretical ones.
2.5 Approximate NNLO evolution of parton densities\textsuperscript{13}

In order to arrive at precise predictions of perturbative QCD for the LHC, for example for the total W\,-\,production cross section discussed in Sects. 2.1 and 2.2, the calculations need to be extended beyond the NLO. Indeed, the NNLO coefficient functions for the above cross section have been calculated some time ago [32, 33]. The same holds for the structure functions in DIS [61–64] which form the backbone of the present information on the parton densities. On the other hand, the corresponding NNLO splitting functions have not been computed so far. Partial results are however available, notably the lowest four and five even-integer moments, respectively, for the singlet and non-singlet combinations [25, 26]. When supplemented by results on the leading $x \to 0$ terms [65–69] derived from small-$x$ resummations, these constraints facilitate effective parametrisations [70, 71] which are sufficiently accurate for a wide range in $x$ (and thus a wide range of final-state masses at the LHC). In this section, we compile these expressions and take a brief look at their implications. For detailed discussions the reader is referred to refs. [70, 71].

In terms of the flavour non-singlet (NS) and singlet (S) combinations of the parton densities (here $f_{q_f} \equiv q$ and $f_g \equiv g$),

\begin{align}
q^{±}_{NS,i} &= q_i \pm q_i - (q_k \pm q_k) , \\
q^V_{NS} &= \sum_{r=1}^{N_f} (q_r - q_r) , \\
q^S &= \left( \sum_q \right) ,
\end{align}

with $\Sigma = \sum_{r=1}^{N_f} (q_r + q_r)$, the evolution equations (8) consist of $2N_fN_f$ scalar non-singlet equations and the $2 \times 2$ singlet system. The LO and NLO splitting functions $p^{(LO)}(x)$ and $p^{(NLO)}(x)$ in Eq. (9) are known for a long time. For each of the NNLO functions $p^{(2)}(x) = (4\pi)^3 p^{(NNLO)}(x)$ two approximate expressions (denoted by ‘$A$’ and ‘$B$’) are given below in the $\overline{\text{MS}}$ scheme, which span the estimated residual uncertainty. The central results are represented by the average $1/2 (p_{A}^{(2)} + p_{B}^{(2)})$.

The NS$^+$ parametrisations [70] read, using $\delta \equiv \delta(1-x)$, $L_1 \equiv \ln(1-x)$ and $L_0 \equiv \ln x$,

\begin{align}
P_{NS,A}^{(2)+}(x) &= \frac{1137.897}{1-x} + 1099.754 \delta - 2975.371 x^2 - 125.243 - 64.105 L_0^2 + 1.580 L_0^4 \\
&\quad - N_f \left( \frac{184.4098}{1-x} + 180.6971 \delta + 98.5885 L_1 - 205.7690 x^2 - 6.1618 - 5.0439 L_0^2 \right) + p_{NS,N_f}^{(2)} , \\
P_{NS,B}^{(2)+}(x) &= \frac{1347.207}{1-x} + 2283.011 \delta - 722.137 L_1 - 1236.264 - 332.254 L_0 + 1.580 (L_0^4 - 4L_0^2) \\
&\quad - N_f \left( \frac{184.4098}{1-x} + 180.6971 \delta + 98.5885 L_1 - 205.7690 x^2 - 6.1618 - 5.0439 L_0^2 \right) + p_{NS,N_f}^{(2)} \\
\end{align}

with

\begin{align}
p_{NS,N_f}^{(2)}(x) &= \frac{1}{81} \left[ \frac{64}{1-x} - \frac{204 + 192 \zeta(3) - 320 \zeta(2) \delta(1-x) + 64}{1-x} \\
&\quad + x \ln x \left( 96 \ln x + 320 \right) + (1-x)(48 \ln^2 x + 352 \ln x + 384) \right] .
\end{align}

Here $\zeta(l)$ denotes Riemann’s $\zeta$-function. Equation (16) is an exact result, derived from large-$N_f$ methods [72]. The corresponding expressions for $p_{NS}^{(2)\sim}$ are

\begin{align}
P_{NS,A}^{(2)\sim}(x) &= P_{NS,A}^{(2)+}(x) + 20.687 x^2 - 18.466 + 66.866 L_0^2 - 0.148 L_0^4 \\
&\quad + N_f \left( 0.0163 L_1 - 0.402 x^2 + 0.4122 - 1.4965 L_0^2 \right) , \\
P_{NS,B}^{(2)\sim}(x) &= P_{NS,B}^{(2)+}(x) - 0.101 L_2^2 + 1.508 + 4.775 L_0 - 0.148 (L_0^4 - 4L_0^3) \\
&\quad + N_f \left( 0.0163 L_1 - 0.402 x^2 + 0.4122 - 1.4965 L_0^2 \right) .
\end{align}

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The difference between $P^{(2)}_{NS}$ and $P^{(2)}_{NS}^{\perp}$ is unknown, but expected to have a negligible effect ($\ll 1\%$).

The effective parametrisations for the singlet sector are given in Ref. [71]. Besides the $1/x \ln x$ terms of $P^{(2)}_{g}$, $P^{(2)}_{qg}$ and $P^{(2)}_{gg}$ [66, 67], only the $N_f^2$ contribution $\propto 1/|1 - x|_+$ to $P^{(2)}_{gg}$ is exactly known here [73].

The evolution equations (8) are written for a factorization scale $\mu_f = Q$. Their form can be straightforwardly generalized to include also the dependence on the renormalization scale $\mu_r$.

The expansion of Eq. (8) is illustrated in the left part of Fig. 14 for $\mu_r = \mu_f$, $\alpha_S = 0.2$ and parton densities typical for $\mu_f^2 \approx 30$ GeV$^2$. Under these conditions, the NNLO effects are small ($< 2\%$) at medium and large $x$. This also holds for the non-singlet evolution not shown in the figure. The approximate character of the our results for $P^{(2)}$ does not introduce relevant uncertainties at $x \gtrsim 2 \cdot 10^{-3}$. The third-order corrections increase with decreasing $x$, reaching $(12 \pm 4)\%$ and $(-6 \pm 3)\%$, respectively, of the NLO predictions for $\bar{\Sigma}$ and $\bar{g}$ at $x = 10^{-4}$.

The renormalization-scale uncertainty of these results is shown in the right part of Fig. 14 in terms of $\Delta_{\mu_r} \hat{q} \equiv (\hat{q}_{\text{max}} - \hat{q}_{\text{min}})/[2 \hat{q}_{\text{average}}]$, as determined over the range $0.5 \mu_f \leq \mu_r \leq 2 \mu_f$. Note that the spikes slightly below $x = 0.1$ arise from $\hat{q}_{\text{average}} \approx 0$ and do not represent enhanced uncertainties. Thus the inclusion of the third-order terms in Eq. (8), already in its approximate form, leads to significant improvements of the scale stability, except for the gluon evolution below $x = 10^{-3}$.

2.6 The NNLO analysis of the experimental data for $xF_3$ and the effects of high-twist power corrections

During the last few years there has been considerable progress in calculations of the perturbative QCD corrections to characteristics of DIS. Indeed, the analytic expressions for the NNLO perturbative QCD corrections to the coefficient functions of structure functions $F_2$ [61, 62, 64] and $xF_3$ [63, 74] are now

\footnote{Contributing authors: A.L. Kataev, G. Parente and A.V. Sidrovov.}
known. However, to perform the NNLO QCD fits of the concrete experimental data it is also necessary to know the NNLO expressions for the anomalous dimensions of the moments of $F_2$ and $xF_3$. At present, this information is available in the case of $n = 2, 4, 6, 8, 10$ moments of $F_2$ [25, 26]. The results of Refs. [25, 26, 61–64, 74] are forming the theoretical background for the study of the effects, contributing to scaling violation at the level of new theoretical precision, namely with taking into account the effects of the NNLO perturbative QCD contributions.

In the process of these studies it is rather instructive to include the available theoretical information on the effects of high-twist corrections, which could give rise to scaling violation of the form $1/Q^2$. The development of the infrared renormalon (IRR) approach (for a review see Ref. [75]) and the dispersive method [76] (see also [77, 78]) made it possible to construct models for the power-suppressed corrections to DIS structure functions (SFs). Therefore, it became possible to include the predictions of these models to the concrete analysis of the experimental data.

In this part of the Report the results of the series of works [58, 59, 79, 80] will be summarized. These works are devoted to the analysis of the experimental data of $xF_3$ SF of $\nu N$ DIS, obtained by the CCFR collaboration [81]. They have the aim to determine the NNLO values of $\Lambda^{(4)}_{\overline{MS}}$ and $\alpha_S(M_Z)$ with fixation of theoretical ambiguities due to uncalculated higher-order perturbative QCD terms and transitions from the case of $f = 4$ number of active flavours to the case of $f = 5$ number of active flavours. The second task was to extract the effects of the twist-4 contributions to $xF_3$ [58, 80] and compare them with the IRR-model predictions of Ref. [82]. Some estimates of the influence of the twist-4 corrections to the constants of the initial parametrization of $xF_3$ [59] are presented. These constants are related to the parton distribution parameters.

The analysis of Refs. [58, 59, 79, 80] is based on reconstruction of the non-singlet (NS) SF $xF_3$ from the finite number of its moments $M_n(Q^2) = \int_0^1 x^{n-1}F_3(x, Q^2)dx$ using the Jacobi polynomial method, proposed in Ref. [83] and further developed in Refs. [84–87]. Within this method one has

$$xf_3(x, Q^2) = x^\alpha (1 - x)\beta \sum_{n=0}^{N_{max}} \Theta_n^{(n)}(x) \sum_{j=0}^{n} c_j^{(n)}(\alpha, \beta) M_{\beta + 2}^{JMC}(Q^2)$$

(18)

where $\Theta_n^{(n)}$ are the Jacobi polynomials, $c_j^{(n)}(\alpha, \beta)$ are combinatorial coefficients given in terms of Euler $\Gamma$-functions and the $\alpha, \beta$-weight parameters. In view of the reasons, discussed in Ref. [58] they were fixed to 0.7 and 3 respectively, while $N_{max} = 6$ was taken. Note, that the expressions for Mellin moments were corrected by target mass contributions (TMC), taken into account as $M_{\beta + 2}^{JMC}(Q^2) = M_n(Q^2) + (n(n+1)/(n+2))(M_{n+2}^{JMC}/Q^2)M_{n+2}(Q^2)$. The QCD evolution of the moments is defined by the solution of the corresponding renormalization group equation

$$\frac{M_n(Q^2)}{M_n(Q_0^2)} = \exp\left[-\int_{Q_0^2}^{Q^2} \frac{\alpha_S(Q^2)}{\beta(x)} dx\right] \left[\frac{C_{NS}(A_s(Q^2))}{C_{NS}(A_s(Q_0^2))}\right]$$

(19)

The QCD running coupling constant enters this equation through $A_s(Q^2) = \alpha_S(Q^2)/(4\pi)$ and is defined as the expansion in terms of inverse powers of $ln(Q^2/\Lambda^{(4)}_{\overline{MS}})$-terms in the LO, NLO and NNLO. The NNLO approximation of the coefficient functions of the moments $C_{NS}(A_s(Q^2)) = 1 + C^{(1)}(n)A_s(Q^2) + C^{(2)}(n)A_s^2(Q^2)$ were determined from the results of Ref. [63, 74]. The related anomalous dimension functions are defined as

$$\mu \frac{\partial n Z_n^{NS}}{\partial \mu} = \gamma_n^{(n)}(A_s) = \sum_{i=0}^n \gamma_i^{(n)}(A_s) A_s^{i+1}$$

(20)

where $Z_n^{NS}$ are the renormalization constants of the corresponding NS operators. The expression for the QCD $\beta$-function in the $\overline{MS}$-scheme is known analytically at the NNLO [11, 88]. However, as was already mentioned, the NNLO corrections to $\gamma_n^{(n)}$ are known at present only in the case of $n = 2, 4, 6, 8, 10$ NS
moments of $F_2$ SF of $eN$ DIS [25, 26]. Keeping in mind that in these cases the difference between
the NLO expressions for $\gamma_{NS,F_2}^{(1)}$ and $\gamma_{NS,xF_3}^{(1)}$ is rather small [79], it was assumed that the similar feature is true at the NNLO also. The $xF_3$ fits of Refs. [58, 59, 79, 80] were done within this approximation. The one more approximation, entering onto these analysis, was the estimation of the anomalous dimensions of odd moments with $n = 3, 5, 7, 9$ by means of smooth interpolation of the results of Refs. [25, 26], originally proposed in Ref. [89]. In view of the basic role of the NNLO corrections to the coefficient functions of $x F_3$ moments, revealed in the process of the concrete fits [58, 59, 79, 80], it is expected that neither the calculations of the NNLO corrections to $x F_3$ odd anomalous dimensions (which are now in progress [90]) and further interpolation to even values of $n$, nor the fine-tuning of the reconstruction method of Eq. (18), which depends on the values of $\alpha, \beta$ and $N_{max}$, will not affect significantly the accuracy of the main results of Refs. [58, 59, 80].

The power corrections were included in the analysis using two different approaches. First, following the ideas of Ref. [91], the term $h(x)/Q^2$ was added onto the r.h.s. of Eq. (18). The function $h(x)$ was parameterized by a set of free constants $h_i$ for each $x$-bin of the analysed data. These constants were extracted from the concrete LO, NLO and NNLO fits. The resulting behaviour of $h(x)$ is presented in Fig. 15, taken from Ref. [58]. Secondly, the IRR model contribution $M^{IRR}_n(Q^2) = C(n)M_n(Q^2)A^{2}_Q/Q^2$ was added into the reconstruction formula of Eq. (18), where $A^{2}_Q$ is the free parameter and was estimated in Ref. [82]. The factor $M_n(Q^2_0)$ in the l.h.s. of Eq. (19) was defined at the initial scale $Q^2_0$ using the parametrization $x F_3(x, Q^2_0) = A(Q^2_0) x^{b(Q^2_0)}(1 - x)^{c(Q^2_0)}(1 + \gamma(Q^2_0)x)$. In Table 2 the combined results of the fits of Refs. [58, 59] of CCFR’97 data are presented. The twist-4 terms were switched off and retained following the discussions presented above.

The comments on the extracted behaviour of $h(x)$ (see Fig. 15) are now in order. Its $x$-shape, obtained from LO and NLO analysis of Ref. [58] is in agreement with the IRR-model formula of Ref. [82]. Note also, that the combination of quark counting rules [92, 93] with the results of Ref. [94, 95] predict the following $x$-shape of $h(x)$: $h(x) \sim A^{2}_Q(1 - x)^2$. Taking into account the negative values of $A^{2}_Q$, obtained in the process of LO and NLO fits (see Table 2), one can conclude, that the related behaviour of $h(x)$ is in qualitative agreement with these predictions. Though a certain indication of the twist-4 terms survives even at the NNLO, the NNLO part of Fig. 15 demonstrates that the $x$-shape of $h(x)$ starts to deviate from the IRR model of Ref. [82]. Notice also, that within the statistical error bars the NNLO value of $A^{2}_Q$ is indistinguishable from zero (see Table 2). This feature might be related to the interplay between NNLO perturbative and $1/Q^2$ corrections. Moreover, at the used reference scale $Q^2_0 = 20$ GeV$^2$ the high-twist parameters cannot be defined independently from the effects of perturbation theory, which at the NNLO can mimic the contributions of higher-twists provided the experimental data is not precise enough and the value of $Q^2_0$ is not too small (for the recent discussion of this subject see Refs. [29, 30]).

<table>
<thead>
<tr>
<th>Order</th>
<th>$A^{(1)}_{M_S}$</th>
<th>$A$</th>
<th>$b$</th>
<th>$c$</th>
<th>$\gamma$</th>
<th>$A^{2}_Q$/(GeV$^2$)</th>
<th>$\chi^2$/points</th>
</tr>
</thead>
<tbody>
<tr>
<td>LO</td>
<td>264±36</td>
<td>4.98±0.23</td>
<td>0.68±0.02</td>
<td>4.05±0.05</td>
<td>0.96±0.18</td>
<td>$-$</td>
<td>113.1/86</td>
</tr>
<tr>
<td></td>
<td>433±51</td>
<td>4.69±0.13</td>
<td>0.64±0.01</td>
<td>4.03±0.04</td>
<td>1.16±0.12</td>
<td>$-$</td>
<td>83.1/86</td>
</tr>
<tr>
<td></td>
<td>331±162</td>
<td>5.33±1.33</td>
<td>0.69±0.08</td>
<td>4.21±0.17</td>
<td>1.15±0.94</td>
<td>$-$</td>
<td>66.3/86</td>
</tr>
<tr>
<td>NLO</td>
<td>339±35</td>
<td>4.67±0.11</td>
<td>0.65±0.01</td>
<td>3.96±0.04</td>
<td>0.95±0.09</td>
<td>$-$</td>
<td>87.6/86</td>
</tr>
<tr>
<td></td>
<td>369±37</td>
<td>4.62±0.16</td>
<td>0.64±0.01</td>
<td>3.95±0.05</td>
<td>0.98±0.17</td>
<td>$-$</td>
<td>82.3/86</td>
</tr>
<tr>
<td></td>
<td>440±183</td>
<td>4.71±1.14</td>
<td>0.66±0.08</td>
<td>4.09±0.14</td>
<td>1.34±0.86</td>
<td>h(x) in Fig. 15</td>
<td>65.7/86</td>
</tr>
<tr>
<td>NNLO</td>
<td>326±35</td>
<td>4.70±0.34</td>
<td>0.65±0.03</td>
<td>3.88±0.08</td>
<td>0.80±0.28</td>
<td>$-$</td>
<td>77.0/86</td>
</tr>
<tr>
<td></td>
<td>327±35</td>
<td>4.70±0.34</td>
<td>0.65±0.03</td>
<td>3.88±0.08</td>
<td>0.80±0.29</td>
<td>$-$</td>
<td>76.9/86</td>
</tr>
<tr>
<td></td>
<td>373±133</td>
<td>4.79±0.75</td>
<td>0.66±0.05</td>
<td>3.95±0.19</td>
<td>0.96±0.57</td>
<td>h(x) in Fig. 15</td>
<td>65.0/86</td>
</tr>
</tbody>
</table>

Table 2: The results of the fits of the CCFR’97 data with the cut $Q^2 > 5$ GeV$^2$. The parameters $A, b, c, \gamma$ are normalized at $Q^2_0 = 20$ GeV$^2$, which is initial scale of the QCD evolution. Statistical errors are indicated.
The results of Table 2 demonstrate, that despite the correlation of the NLO values $\Lambda_{MS}^{(4)}$ with the values of the twist-4 coefficient $A'_n$, the parameters of the adopted model for $xF_3(x, Q_0^2)$ remain almost unaffected by the inclusion of the $1/Q^2$-term via the IRR-model of Ref. [82]. Thus, the corresponding parton distributions are less sensitive to twist-4 effects, than the NLO value of $\Lambda_{MS}^{(4)}$. At the NNLO level the similar feature is related to already discussed tendency of the effective minimization of the $1/Q^2$-contributions to $xF_3$ (see also NNLO part of Fig. 15).

For the completeness the NLO and NNLO values of $\alpha_S(M_Z)$, obtained in Ref. [58] from the results of Table 2 with twist-4 terms modelled through the IRR approach are also presented:

\[
\begin{align*}
\text{NLO} & \quad \alpha_S(M_Z) = 0.120 \pm 0.003 \text{(stat)} \pm 0.005 \text{(syst)} \pm 0.009 \\
\text{NNLO} & \quad \alpha_S(M_Z) = 0.118 \pm 0.003 \text{(stat)} \pm 0.005 \text{(syst)} \pm 0.003
\end{align*}
\] (21)

The systematical uncertainties in these results are determined by the pure systematical uncertainties of the CCFR’97 data for $xF_3$ [81]. The theoretical errors are fixed by variation of the factorization and renormalization scales [58]. The incorporation into the $\overline{MS}$-matching formula for $\alpha_S$ [96–98] of the proposal of Ref. [52] to vary the scale of smooth transition to the world with $f = 5$ number of active flavours from $m_t^2$ to $(6.5m_t)^2$ was also taken into account. The theoretical uncertainties, presented in Eq. (22) are in agreement with the ones, estimated in Ref. [70] using the DGLAP equation. The NNLO value of $\alpha_S(M_Z)$ is in agreement with another NNLO result $\alpha_S(M_Z) = 0.1172 \pm 0.0024$, which was obtained in Ref. [99] from the analysis of SLAC, BCDMS, E665 and HERA data for $F_2$ with the help of the Bernstein polynomial technique [100].

### 2.7 Measuring Parton Luminosities and Parton Distribution Functions at the LHC\(^{15}\)

The traditional approach for cross section calculations and measurements at hadron colliders uses the proton–proton luminosity, $L_{\text{proton–proton}}$, and the “best” known quark, anti-quark and gluon parton–distribution functions, $PDF(x_1, x_2, Q^2)$ to predict event rates $N_{\text{events}}$ for a particular parton parton process with a calculable cross section $\sigma_{\text{theory}}(q, \bar{q}, g \rightarrow X)$, using:

\[
N_{\text{events}}(pp \rightarrow X) = L_{\text{proton–proton}} \times PDF(x_1, x_2, Q^2) \times \sigma_{\text{theory}}(q, \bar{q}, g \rightarrow X).
\] (22)

The possible quantitative accuracy of such comparisons depends not only on the statistical errors, but also on the knowledge of $L_{\text{proton–proton}}$, the $PDF(x_1, x_2, Q^2)$ and the theoretical and experimental uncertainties for the observed and predicted event rates for the studied process.

\(^{15}\)Contributing authors: M. Dittmar, K. Mazumdar and N. Skachkov.
For many interesting reactions at the LHC one finds that statistical uncertainties become quickly negligible when compared to today's uncertainties. Besides the technical difficulties to perform higher order calculations, limitations arise from the knowledge of the proton–proton luminosity and the parton distribution functions. Estimates for proton–proton luminosity measurements at the LHC assign typically uncertainties of $\pm 5\%$. Similar uncertainties are expected from the limited knowledge of parton distribution functions. Consequently, the traditional approach to cross section predictions and the corresponding measurements will be limited to uncertainties of at best $\pm 5\%$.

A more promising method [101], using only relative cross section measurements, might lead eventually to accuracies of $\pm 1\%$. The new approach starts from the idea that for high $Q^2$ processes one should consider the LHC as a parton–parton collider instead of a proton–proton collider. Consequently, one needs to determine the different parton–parton luminosities from experimentally clean and theoretical well understood reactions.

The production of the vector bosons $W^\pm$ and $Z^0$ with their subsequent leptonic decays fulfill these requirements. Taking today’s experimental results, the vector boson masses are precisely known and their couplings to fermions have been measured with accuracies of better than 1%. Furthermore, $W^\pm$ and $Z^0$ bosons with leptonic decays have 1) huge cross sections (several nb’s) and 2) can be identified over a large rapidity range with small backgrounds.

From the known mass and the number of “counted” events as a function of the rapidity $Y$ one can use the relations $M^2 = s x_1 x_2$ and $Y = \frac{1}{2} \ln \frac{x_1}{x_2}$ to measure directly the corresponding quark and antiquark luminosities over a wide $x$ range (see fig 2). Simulation studies indicate that the leptonic $W$ and $Z$ decays can be measured with good accuracies up to lepton pseudorapidities $|\eta| < 2.5$, corresponding roughly to quark and anti-quark $x$ ranges between 0.0003 to 0.1. The sensitivity of $W$ and $Z$ production data at the LHC even to small variations of the pdf’s is indicated in Figure 16.

Once the quark and anti-quark luminosities are determined from the $W$ and $Z$ data over a wide $x$ range, their combinations with the parton–parton luminosities lead to a unique combination of parton distribution functions that are in agreement with experimentally clean and theoretically well understood processes.
range, SM event rates of high mass Drell–Yan lepton pairs and other processes dominated by quark–anti-quark scattering can be predicted. The accuracy for such predictions is only limited by the theoretical uncertainties of the studied process relative to the one for $W$ and $Z$ production.

The approach can also be used to measure the gluon luminosity with unprecedented accuracies. Starting from gluon dominated “well” understood reactions within the SM, one finds that the cleanest experimental conditions are found for the production of high mass $\gamma$–Jet, $Z^0$–Jet and perhaps, $W^\pm$–Jet events. However, the identification of these final states requires more selection criteria and includes an irreducible background of about 10–20% from quark–anti-quark scattering. Some experimental observables to constrain the gluon luminosity from these reactions have been investigated previously [102]. The study, using rather restrictive selection criteria to select the above reactions with well defined kinematics, indicated the possibility to extract the gluon luminosity function with negligible statistical errors and systematics which might approach errors of about ±1% over a wide $x$ range.

Furthermore, the use of the different rapidity distributions for the Vector bosons and the associated jets has been suggested in [103]. The proposed measurement of the rapidity asymmetry improves the separation between signals and backgrounds and should thus improve the accuracies to extract the gluon luminosity.

For this workshop, previous experimental simulations of photon–jet final states have been repeated with much larger Monte Carlo statistics and more realistic detector simulations [104]. These studies select events with exactly one jet recoiling against an isolated photon with a minimum $p_t$ of 40 GeV. With the requirement that, in the plane transverse to the beam direction the jet is back–to–back with the photon, only the photon momentum vector and the jet angle needs to be measured. Using the selected kinematics, the mass of the photon–jet system can be reconstructed with good accuracy. These studies show that several million of photon–jet events with the above kinematics will be detected for a typical LHC year of 10 fb$^{-1}$ and thus negligible statistical errors for the luminosity and $x$ between 0.0005 to ≈ 0.2. This $x$ range seems to be sufficient for essentially all high $Q^2$ reactions involving gluons. In addition, it might however be possible using dedicated trigger conditions, to select events with photon $p_t$ as low as 10–20 GeV, which should enlarge the $x$ range to values as low as 0.0001. The above reactions are thus excellent candidates to determine accurately the parton luminosity for light quarks, anti-quarks and gluons.

To complete the determination of the different parton luminosities one needs also to constrain the luminosities for the heavier $s$,$c$ and $b$ quarks. The charm and beauty quarks can be measured from a quark flavour tagged subsample of the photon–jet final states. One finds that the photon–jet subsamples with charm or beauty flavoured jets are produced dominantly from the heavy quark–gluon scattering $(c(b)g \rightarrow c(b)\gamma)$. For this additional study of photon–jet final states, the jet flavour has been identified as being a charm or beauty jet, using inclusive high $p_t$ muons and in addition $b$-jet identification using standard lifetime tagging techniques [105]. The simulation indicates that clean photon–charm jet and photon–beauty jet event samples with high $p_t$ photons (>40 GeV) and jets with inclusive high $p_t$ muons. The muon $p_t$ spectrum from the different initial quark flavours is shown in Figure 17.

Assuming that inclusive muons with a minimum $p_t$ of 5–10 GeV can be clearly identified, a PYTHIA Monte Carlo simulation shows that a few $10^5$ $c$-photon events and about $10^5$ $b$-photon events per 10 fb$^{-1}$ LHC year should be accepted. These numbers correspond to statistical errors of about ±1% for a $x_c$ and $x_b$ range between 0.001 and 0.1. However, without a much better understanding of charm and beauty fragmentation functions such measurements will be limited to systematic uncertainties of ± 5–10%.

Finally, the strange quark luminosity can be determined from the scattering of $sg \rightarrow Wc$. The events would thus consist of $W^\pm$ charm–jet final states. Using inclusive muons to tag charm jets and the leptonic decays of $W$’s to electrons and muons we expect about an accepted event sample with a cross section of 2.1 pb leading to about 20k tagged events per 10 fb$^{-1}$ LHC year. Again, it seems that the corresponding statistical errors are much smaller than the expected systematic uncertainties from the
charm tagging of $\pm 5$–10%.

In summary, we have identified and studied several final states which should allow to constrain the light quarks and anti-quarks and the gluon luminosities with statistical errors well below 1% for an $x$ range between 0.0005 to at $\approx 0.2$. However, experimental systematics for isolated charged leptons and photons, due to the limited knowledge of the detector acceptance and selection efficiencies will be the limiting factor which optimistically limit the accuracies to perhaps $\pm 1\%$ for light quarks and gluons. The studied final states with photon–jet events with tagged charm and beauty jets should allow to constrain experimentally the luminosities of $s$, $c$ and $b$ quarks and anti-quarks over a similar $x$ range and systematic uncertainties of perhaps 5–10%.

These promising experimental feasibility studies need now to be combined with the corresponding theoretical calculations and Monte Carlo modelling. In detail one has to study how well uncertainties from scale dependence, $\alpha_S$ and higher order corrections change expected cross section ratios. Figure 6 gives an example of today’s uncertainties for $W$ and $Z$ cross sections at the LHC [10]. Similar estimates for all studied processes need to be done during the coming years in order to know the real potential of this approach to precision cross section measurements and their interpretation at the LHC.

2.8 Lepton Pair Production at the LHC and the Gluon Density in the Proton

The production of lepton pairs in hadron collisions $h_1 h_2 \rightarrow \gamma^* X; \gamma^* \rightarrow ll$ proceeds through an intermediate virtual photon via $q\bar{q} \rightarrow \gamma^*$, and the subsequent leptonic decay of the virtual photon. Interest in this DY process is usually focused on lepton pairs with large mass $Q$ which justifies the application of perturbative QCD and allows for the extraction of the anti-quark density in hadrons [106]. Prompt photon production $h_1 h_2 \rightarrow \gamma X$ can be calculated in perturbative QCD if the transverse momentum $Q_T$ of the photon is sufficiently large. Because the quark-gluon Compton subprocess is dominant, $gg \rightarrow \gamma X$, this reaction provides essential information on the gluon density in the proton at large $x$ [28]. Alternatively, the gluon density can be constrained from the production of jets with large transverse momentum at hadron colliders [7].

In this report we exploit the fact that, along prompt photon production, lepton pair production is dominated by quark-gluon scattering in the region $Q_T > Q/2$. This realization means that new independent constraints on the gluon density may be derived from DY data in kinematical regimes that are accessible at the LHC but without the theoretical and experimental uncertainties present in the prompt photon case.

At LO, two partonic subprocesses contribute to the production of virtual and real photons with non-zero transverse momentum: $q\bar{q} \rightarrow \gamma(*)g$ and $gg \rightarrow \gamma(*)q$. The cross section for lepton pair production is related to the cross section for virtual photon production through the leptonic branching ratio of the virtual photon $\alpha/(\beta^2 3\pi Q^2)$. The virtual photon cross section reduces to the real photon cross section in the limit $Q^2 \rightarrow 0$.

The NLO corrections arise from virtual one-loop diagrams interfering with the LO diagrams and from real emission diagrams. At this order 2 $\rightarrow$ 3 partonic processes with incident gluon pairs $(gg)$, quark pairs $(qq)$, and non-factorizable quark-anti-quark $(q\bar{q}_2)$ processes contribute also. An important difference between virtual and real photon production arises when a quark emits a collinear photon. Whereas the collinear emission of a real photon leads to a $1/\epsilon$ singularity that has to be factored into a fragmentation function, the collinear emission of a virtual photon yields a finite logarithmic contribution since it is regulated naturally by the photon virtuality $Q$. In the limit $Q^2 \rightarrow 0$ the NLO virtual photon cross section reduces to the real photon cross section if this logarithm is replaced by a $1/\epsilon$ pole. A more detailed discussion can be found in Ref. [107, 108].

The situation is completely analogous to hard photo-production where the photon participates in the scattering in the initial state instead of the final state. For real photons, one encounters an initial-

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\footnote{Contributing authors: E. L. Berger and M. Klasen.}
state singularity that is factored into a photon structure function. For virtual photons, this singularity is replaced by a logarithmic dependence on the photon virtuality $Q$. 

A remark is in order concerning the interval in $Q$ in which our analysis is appropriate. In general, in two-scale situations, a series of logarithmic contributions will arise with terms of the type $\alpha_s^a \ln^b(Q/Q)$. Thus, if either $Q_T >> Q$ or $Q_T << Q$, resummations of this series must be considered. For practical reasons, such as event rate, we do not venture into the domain $Q_T >> Q$, and our fixed-order calculation should be adequate. On the other hand, the cross section is large in the region $Q_T << Q$. In previous papers [107, 108], we compared our cross sections with available fixed-target and collider data on massive lepton-pair production, and we were able to establish that fixed-order perturbative calculations, without resummation, should be reliable for $Q_T > Q/2$. At smaller values of $Q_T$, non-perturbative and matching complications introduce some level of phenomenological ambiguity. For the goal we have in mind, viz., constraints on the gluon density, it would appear best to restrict attention to the region $Q_T \approx Q/2$, but below $Q_T >> Q$.

We analyze the invariant cross section $Ed^3\sigma/dp^3$ averaged over the rapidity interval $-1.0 < y < 1.0$. We integrate the cross section over various intervals of pair-mass $Q$ and plot it as a function of the transverse momentum $Q_T$. Our predictions are based on a NLO calculation [110] and are evaluated in the $\overline{MS}$ renormalization scheme. The renormalization and factorization scales are set to $\mu = \mu_R = \mu_F = \sqrt{Q^2 + Q_T^2}$. If not stated otherwise, we use the CTEQ4M parton distributions [111] and the corresponding value of $\Lambda$ in the two-loop expression of $\alpha_s$ with four flavours (five if $\mu > m_b$). The DY factor $\alpha/(3\pi Q^2)$ for the decay of the virtual photon into a lepton pair is included in all numerical results.

In Fig. 18 we display the NLO cross section for lepton pair production at the LHC as a function of $Q_T$ for $pp \rightarrow \gamma^* X$ at $\sqrt{s} = 14$ TeV. The $qg$ channel dominates in the region $Q_T > Q/2$. For $\gamma g$ and $qg$ to the invariant cross section $Ed^3\sigma/dp^3$ as a function of $Q_T$ for $pp \rightarrow \gamma^* X$ at $\sqrt{s} = 14$ TeV.
$Q_T$ for four regions of $Q$ chosen to avoid resonances, \textit{i.e.} from threshold to 2.5 GeV, between the $J/\psi$ and the $\Upsilon$ resonances, above the $\Upsilon$'s, and a high mass region. The cross section falls both with the mass of the lepton pair $Q$ and, more steeply, with its transverse momentum $Q_T$. The initial LHC luminosity is expected to be $10^{33} \text{ cm}^{-2} \text{ s}^{-1}$, or 10 fb$^{-1}$/year, and to reach the design luminosity of $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ after three or four years. Therefore it should be possible to analyze data for lepton pair production to at least $Q_T \simeq 100$ GeV where one can probe the parton densities in the proton up to $x_T = 2Q_T/\sqrt{s} \simeq 0.014$. The UA1 collaboration measured the transverse momentum distribution of lepton pairs at $\sqrt{s} = 630$ GeV to $x_T = 0.13$ [112], and their data agree well with our expectations [107, 108].

The fractional contributions from the $gg$ and $q\bar{q}$ subprocesses through NLO are shown in Fig. 19. It is evident that the $gg$ subprocess is the most important subprocess as long as $Q_T > Q/2$. The dominance of the $gg$ subprocess increases somewhat with $Q$, rising from over 80 % for the lowest values of $Q$ to about 90 % at its maximum for $Q \simeq 30$ GeV. Subprocesses other than those initiated by the $gg$ initial channels are of negligible import.

The full uncertainty in the gluon density is not known. We estimate the sensitivity of LHC experiments to the gluon density in the proton from the variation of different recent parametrizations. We choose the latest global fit by the CTEQ collaboration (5M) as our point of reference [7] and compare results to those based on their preceding analysis (4M) [111] and on a fit with a higher gluon density (5HJ) intended to describe the CDF and D0 jet data at large transverse momentum. We also compare to results based on global fits by MRST [28], who provide three different sets with a central, higher, and lower gluon density, and to GRV98 [113]$^1$.

$^1$In this set a purely perturbative generation of heavy flavours (charm and bottom) is assumed. Since we are working in a massless approach, we resort to the GRV92 parametrization for the charm contribution [114] and assume the bottom contribution to be negligible.
In Fig. 20 we plot the cross section for lepton pairs with mass between the $J/\psi$ and $\Upsilon$ resonances at the LHC in the region between $Q_T = 50$ and 100 GeV ($x_T = 0.007\ldots0.014$). For the CTEQ parametrizations we find that the cross section increases from 4M to 5M by 5% and does not change from 5M to 5HJ in the whole $Q_T$-range. The largest differences from CTEQ5M are obtained with GRV98 (minus 18%).

The theoretical uncertainty in the cross section can be estimated by varying the renormalization and factorization scale $\mu_R = \mu_F$ about the central value $\sqrt{Q^2 + Q_T^2}$. In the region between the $J/\psi$ and $\Upsilon$ resonances, the cross section drops from $\pm39\%$ (LO) to $\pm16\%$ (NLO) when $\mu$ is varied over the interval interval $0.5 < \mu/\sqrt{Q^2 + Q_T^2} < 2$. The $K$-factor ratio (NLO/LO) is approximately 1.3 at $\mu/\sqrt{Q^2 + Q_T^2} = 1$.

We conclude that the hadroproduction of low mass lepton pairs is an advantageous source of information on the parametrization and size of the gluon density. With the design luminosity of the LHC, regions of $x_T \approx 0.014$ should be accessible. The theoretical uncertainty has been estimated from the scale dependence of the cross sections and found to be small at NLO.

3. MONTE CARLO EVENT GENERATORS

The event generation package is the first link of the event simulation/reconstruction software suite which is central to any experimental data analysis. Physics results are obtained by a direct comparison of simulated and observed data. Therefore, precision analyses rely on an accurate and detailed implementation of the underlying physics model in the generation of signal as well as background processes.

An event generator is built from various pieces whose object and nature are quite different. Some are perturbative: the hard-scattering matrix element (ME) which can be calculated exactly, the parton shower (PS) which approximates, through the evolution equations, the initial parton conditions and final-state jet structure, and some are non-perturbative and probabilistic like the parton distribution in the composite initial particles and the fragmentation of the final partons. The main difficulty in writing event generator programs lies on the consistent matching of those different components.

Several multi-process parton shower event generators (PSEG) have been developed to cover the physics programme at $e^+e^-$, $pp$ or $p\bar{p}$ colliders: PYTHIA [115], HERWIG [116–118], ISAJET [119, 120]. These Monte Carlo programs provide an accurate description of jet physics at existing high-energy colliders, which allow the simulation of a large variety of final-state processes within and beyond the SM. These programs have been essential to demonstrate the impressive LHC potential on many different and detailed physics questions, to develop new analysis strategies and also to optimize the performance of the LHC experiments.

Nevertheless, the increasing potential of very accurate measurements at the LHC and the sensitivity to exotic physics processes using specific and rare kinematics demand for the implementation of higher-order processes and thus a rethinking of the organisation and probably an extensive rewriting of many specific Monte Carlo generators.

In the first section, we list the major points of concern or pending issues in the development of event generators for the LHC physics. The next section discusses the present treatments of minimum bias and underlying events. The following two contributions address the implementation of transverse momentum effects in boson production. The last three sections present a short description of some of the currently available ME generators.

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2Section coordinator: D. Perret-Gallix.
3.1 QCD event generators: major issues

3.11 Multi-particle final states

Matrix element

PSEGS are essentially limited to the simulation of $2 \rightarrow 2$ processes based on analytic matrix element expressions. However, the LHC center of mass energy is large enough to open many high multiplicity channels. In addition, new particle searches in the Higgs and Susy sectors require the simulation of $2 \rightarrow 4, 2 \rightarrow 6$ or even $2 \rightarrow 10$ jet processes for which a precise knowledge of the SM background processes is mandatory.

QCD multi-jets events $pp \rightarrow n_1$ jets and $pp \rightarrow Z/W + n_2$ jets have been computed at LO, for $n_1 \leq 6$ by using the SPHEL approximation [121] (i.e. assuming all helicity amplitudes give similar contributions), and for $n_1 \leq 6$ (NJETS) [122] and $n_2 \leq 4$ (VECBO) [123] by using exact recurrence relations [124].

In the PSEG, partonic final states are mimicked through the PS mechanism based on the leading logarithmic (LL) approximation. It properly describes parton radiations only in the soft and collinear region leading to a crude estimate of the multi-parton dynamics of the event. The remedy for a better multi-parton event generator is two-fold: (i) to improve the simulation of the PS by introducing ME corrections (see Sects. 3.3 and 3.4), (ii) to implement the complete multi-parton hard scattering ME process.

The evaluation of ME for multi-particle QCD processes has been reviewed in [125]. A powerful technique is the use of helicity amplitudes in the massless limit [126–128]. Recent developments in this direction were done in [129] where the Weyl-van-der-Waarden spinor calculus was generalized to the massive fermions. At this level of complexity where so many sub-processes must be calculated, the analytic hand-made approach becomes literally intractable unless stringent approximations are imposed, as the narrow width approximation, massless fermions, averaging/summing over initial/final helicity state or selecting only a subset of gauge invariant diagrams.

A more systematic approach is needed: (i) to provide all required channels, (ii) to allow for a detail study of finite width effects and helicity and color correlations, (iii) to generate complete ME expressions in order to match the experimental precision. For example, the LHC statistics will allow to measure the top quark mass with negligible uncertainty. This implies that both top quark and $W$ finite widths must be taken into account in the evaluation of the interference between signal and background diagrams.

The automatic Feynman diagram generator packages, largely used for the $e^+e^-$ physics analysis, generate complete and approximation-free tree-level ME codes, in principle, for any final state multiplicity and with a higher reliability level than hand written procedures. They are gradually upgraded to $pp$ physics. GRACE [130–132], MADGRAPH [133], ALPHA [134] and PHACT [135, 136] are based on tree level helicity amplitude algorithms in arbitrary massive gauge theories. The evaluation is purely numerical and the code size scales linearly with the number of external particles. In ALPHA (see Sect. 3.7), an iterative algorithm, based on Green functional methods, evaluates the amplitudes for any given Lagrangian and leads to more compact expressions allowing, for example, the generation of $gg/q\bar{q} \rightarrow n$ with $n \leq 9$ [137]. The COMHEP [132, 138] package is based on the squared amplitude technique. Here, the size of the ME code grows exponentially with the number of external particles, but it produces more powerful symbolic expressions. This method has shown good efficiency for the evaluation $2 \rightarrow 3, 4$ processes, comparable to the helicity amplitude algorithms.

However, the completeness of the automatically produced matrix elements and the poor optimization of the code (when compared to hand coding) often translate into computationally intensive and

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$^3$Contributing authors: V.A. Ilyin, D. Perret-Gallix and A.E. Pukhov.

$^4n \rightarrow m$ represents processes where $n$ initial particles decays or scatter to produce $m$ particles in the final state.

$^5$In R-parity non-conserving models.

$^6$The packages automatically generate checks for gauge invariance and gauge independence.
memory hungry expressions, sometimes reaching the limit of computability on conventional workstations.

The development effort is focused on two directions: (i) to improve the code efficiency by the introduction of new computational algorithm, by a better optimization and by the “automated” introduction of approximations, (ii) to develop code taking advantage of massively parallel systems [139, 140].

**Multi-dimensional integration**

The cross section computation and the event generation stage are based on the multi-dimensional integration procedure. It needs to be focused to the phase space region where the amplitude is large. The amplitude behavior on those regions can be sharp and multi-variate due to complex singularity patterns. Integration packages including VEGAS [141, 142], BASES/SPRING [143, 144], MILXy [145], FOAM [146] use self-adapting techniques based on importance and/or stratified sampling. However, a faster integration convergence is obtained by providing the integration algorithm with information on the location and behavior of the singularities. This is usually done by the so-called “kinematics” routine performing the mapping of the integration variables to the physics parameters. Not yet fully automated [147], it is aiming by appropriate variable transforms at smoothing the singularities and reducing their dimensionality.

For many important processes, it is impossible to match all singularities within a single set of variable transforms (e.g. $pp \rightarrow u\bar{d}d\bar{u}$ with $W,Z$ decays and $t$-channel singularities). In those cases, one relies on a multichannel algorithm [148, 149] where each peaking structure has its own appropriate mapping.

**Interface to the PSEG package**

The implementation of automatically-produced hard-process ME in PSEG is a delicate but essential task to benefit from the implementation of the complex QCD machinery reproducing the initial and final states.

The ultimate goal is to embed the full ME with its appropriate kinematics mapping into the kernel of the PSEG through some automated procedure. Although some progress has been achieved toward this end, a simpler approach is to generate parton level event sample using a program dedicated to a given ME, then let them fragment through the PS and hadronization scheme of the selected PSEG. For example, in PYTHIA the routines PYUPIN and PYUFEV are available for the implementation of externally produced event processes. Similar facilities exist or can be implemented in other PSEG. This technique already used by the LHC experiments (see section 3.5) may raise consistent parameter and parton distribution bookkeeping issues.

### 3.12 Heavy-quark production and parton shower

Keeping the fermion masses at their on-shell value, although making the expressions more complex, is always a good practice to get rid of the propagator pole divergence. At LHC, from a phenomenological point of view, light $u, d$ and $s$ quark masses can be neglected, but heavy $c, b$ and $t$ quark should be implemented not only to reproduce threshold effects, but also for a correct treatment of spin correlation and NLO corrections. Beside the basic $t$-quark physics studies, the heavy-quark event generation plays an important role as the dominant background to the Higgs search ($W/Zb\bar{b}, t\bar{t} + 2jets$ and $t\bar{t}tt, b\bar{b}b\bar{b}, b\bar{b}t\bar{t}$). Those computations require the use of multi-particle massive ME as developed in the automatic approach.

The simulation of the PS developed by a massive quark is similar to the massless case above an angular cut-off of $\theta = m_q/E_q$, while below no radiation is emitted. This is true only in the soft and collinear region, if the physics observable is sensitive to high-$p_T$ effects (e.g. top mass reconstruction) full massive radiative heavy-quark decay ME (i.e. $t \rightarrow bWg$) must be embedded in the PS code [150,
3.13 Color and helicity implementation

Color and spin effects are important at LHC. Color correlations beside driving the fragmentation of partons lead to color reconnection effects acting on the local event multiplicity. Spin effects in the top physics, for example, provide a useful handle on the nature of the couplings [152].

The procedure to assign helicity and color to the initial/final partons requires similar implementations in an event generator. For $2 \rightarrow 2$ processes, the number of possible color flows is small and can be handled easily through an overall factor for the single diagram case and through a slightly more elaborated treatment when dealing with the interference of 2 diagrams with different color flows [153]. For higher multiplicity [154], in the super-symmetric QCD [155] and in the $R$-parity violated processes [156], the selection of the color final state is more involved. In the helicity amplitude approach, each diagram must be decomposed over a color flow reference base. The cross sections for all possible color/helicity combinations $(8^{n_q} \times 3^{n_s} \times 2^{n_g+n_j})$ are then evaluated. Adding more final-state particles drastically increases the number of cross section computations.

3.14 NLO and NNLO corrections

In QCD, talking about corrections concerning the NLO and NNLO contributions is an understatement. Higher-order computations are very important not only due to the rather large coupling constant $\alpha_s$ inducing substantial corrections, but mainly because they reduce the renormalization and factorization scale dependence. Furthermore, analysis or experimental-cut dependencies (like the cone-size dependence in jet analysis) are better reproduced when higher-order corrections are included. Roughly speaking if one can say that NLO is the first order giving a sensible perturbative result, NNLO can be seen as the error estimate on this result.

In principle, computing NLO matrix elements is straightforward using loop integral reduction techniques, but the number of involved diagrams and their complexity have lead to the development of automatic coding programs like FeynArt/FeynCalc Formcalc/Looptools [157–159] or GRACE (see Sect. 3.6). The latter is geared to provide 1-loop $n$-body final-state ME while, in practice, a maximum of $n = 4$ and further approximations are imposed by computational limitations.

But the main problem lies in the cancellation of soft and collinear infinities present at NLO precision. Fully inclusive computations generate the so-called $K$-factor as a global scaling factor, but detailed analyses need phase-space dependent corrections. Two techniques (see the general discussion in Sect. 4.) have been developed to handle the cancellations: the phase-space slicing method [160] and the subtraction method [161,162]. In the former, the cancellation is performed by approximate integration within regions delimited by some unphysical cut-off (the approximation becomes better as the cut-off becomes smaller), in the latter the divergent terms are replaced by a suitable analytically-integrable expression plus its finite difference with the original expression. For these two approaches, Monte-Carlo integration techniques are used, allowing for a precise implementation of the experimental cuts. These NLO programs (see Sect. 4.) can be seen as “pseudo-event generators”. Phase space points (pseudo-events) after being tested against the cuts have their corresponding weights accumulated to form the observable. Single or multi differential distributions can be built in one go. But two issues prevent the use of these packages as true event generators: (i) the handling of negative weighted events and (ii) the interface to the PS and fragmentation stage. No definite scheme currently exists to properly implement LO+NLO processes in a stochastic event generator.

The negative weighted events arise from the virtual corrections cancelling the soft and collinear divergences. Several attempts are on trial. One approach is to treat those events as the usual positive weighted events and to observe the cancellation only after the reconstruction stage where the experimental resolution will have introduced a natural cut-off. This implies the generation, the simulation and the
reconstruction of many events which finally cancel, not contributing to the statistical significance and therefore leads to unstable results. More advanced attempts have been based on a re-weighting of event generated by showering from the LO matrix elements [150, 151, 163–166]. Recently, a modified subtraction method is exercised to built NLO event generators [167, 168] by point-by-point cancellation of the singularities. This approach looks quite encouraging although final implementations have not been realized yet.

The second problem is the matching of a NLO ME to the PS. A consistent approach would be to interface a NLO ME to next-to-leading logarithmic (NLL) order parton shower, but no such algorithm exist yet (see Sect. 3.15) and therefore one has to find the least damaging approach to connect NLO ME and LL PS and final hadronization.Basically the ordered evolution PS variable should be matched to the ME regularization parameter. Remaining double counting effects will be removed by the rejection algorithm for each event topology [167].

3.15 Parton shower

In hadronic collision, the parton showering occurs both in the initial and in the final state. In the latter, the high-virtuality partons are evolved using the DGLAP equations down to quasi-real objects ready to undergo final hadronization. The initial partons selected from the parton distribution functions with a relative momentum fraction $x$ and virtuality $Q^2$ follow a backward evolution [169–171] to bring back the virtuality down to values compatible with the confinement of partons in a fast hadron (cloud of quasi-real particles). In this process, gluons and quarks are emitted (absorbed in the backward-evolution time frame) by quark radiation or gluon splitting. This radiation contributes to the final-state multiplicity (beam remnants). In addition, the parton acquires a transverse momentum and the full kinematic of the initial centre-of-mass of the hard scattering will be uniquely defined (see Sects. 3.3 and 3.4).

The parton shower model implemented in the PSEG is essentially a LL approximation, even if some NLL corrections have been added through exact energy-momentum conservation, angular ordering and ‘optimal scheme’ definition for $\alpha_S$ [172]. The dominant logarithmic singularities are resummed in the Sudakov form factors.

As seen in the previous section, the need for a NLL parton shower is high. The problem is that resumming higher-order correction breaks one major “raison d’être” of the PS: the universality. At LL level, the hard scattering and the parton showering are 2 independent processes (factorization between the short an the long range) and the success of the PSEG is based on this feature. Incorporating higher-order corrections may break universality and each type of hard scattering process may require a specific NLL PS evaluation (see also the last paragraph in Sect. 3.6).

3.16 Multi-parton scattering

PSEG for rare events usually include single-scattering processes only. At the LHC, one expect, due to the unitary bound, multi-parton interactions to give important contributions to several processes [173, 174]. As an example the cross section for the production of four jets with double-parton collisions dominates the single-scattering process when the minimum of the produced jets transverse momenta is $p_{T_{\text{min}}} < 20\text{ GeV}$ (see Sect. 8.). These processes, observed by CDF [175, 176], are largely discussed in Sect. 8., in the Bottom Production Chapter of this Report and in the ATLAS TDR [1]. Information related to the PSEG implementation of multi-parton scattering can be found in the PYTHIA [115] and HERWIG v6.1 [118] manuals.

Under the simplifying assumptions of no correlation between the longitudinal-momentum fractions of the initial partons, and of the process-independence of parton correlations in transverse-momentum space, double-parton interactions are easily implementable into PSEG codes, in terms of a single universal parameter $\sigma_{\text{eff}}$ (see Sect. 8.). However, none of those hypotheses can be taken for granted. It is therefore important to implement those effects in PSEG programs by using different dynamical models.
In addition to their contributions to the background to new particle searches, the multi-parton interactions at the LHC can provide insights on the dynamical structure of the hadrons [177–179].

3.17 Standardization and language issues
The availability of several independent event generation packages although aiming at similar scopes is a big advantage for the experimental community. It makes possible comparative checks and leads to a deeper understanding of the various approximations used and implementation dependent issues.

However, one must strongly stress that the definition of a common interface scheme between the event generators and the simulation/analysis experimental packages would be extremely valuable. Such a standardization would cover the following issues: (i) parameter naming convention, (ii) parameter database management, (iii) event output format, (iv) event sample database.

Although the standardization scheme can already be exercised on the existing Fortran PSEG, it takes its full meaning with the current transition to the object oriented (OO) methodology. The maintenance issue of those large and complex packages over the long expected lifetime of the LHC experiments is the main reason for using the OO technology, but the built-in object modularity opens the door to a finer grained standardization at least to the level of the interfaces of the main procedures (random number generator, diagram generation, diagram display, matrix element code, integrator, parton shower, fragmentation, structure functions). This would allow the building of event generators using procedures from various origins. Most of the PSEG package developers have endorsed C++ as the language for the future developments. Design and implementation studies are already in progress [180–182].

On these last issues, the setting up of a dedicated working group with all concerned authors and users would be quite timely.

3.2 Minimum bias and underlying events
A crucial area of physics for the LHC is the structure of final states in soft minimum-bias collisions and the soft underlying event in hard processes. At present very little is understood about these matters on the basis of QCD starting from first principles. The three principal event generators in use for LHC physics, ISAJET, HERWIG and PYTHIA, use quite different models for this type of physics, although each uses basically the same model to generate both minimum-bias and underlying events.

Simulation of minimum-bias events starts with a parametrization of the total cross section. HERWIG and PYTHIA both use the Donnachie-Landshoff fit [183]

\[
\sigma_{tot} = 21.70 s^{0.0808} + 56.08 s^{-0.4525}
\]

(where \( \sigma \) is in mb and \( \sqrt{s} \) in GeV), whereas ISAJET uses a \( \log^2 s \) form:

\[
\sigma_{tot} = 25.65 \left[ 1 + 0.0102 \log^2(s/1.76) \right]
\]

Notice (see Fig. 21) that, although smaller asymptotically, the ISAJET value is larger at LHC energies.

To model soft final states, HERWIG uses the UA5 minimum-bias Monte Carlo [184], adapted to its own cluster fragmentation model. See the HERWIG manuals [118] for further details. The model is based on a negative binomial parametrization of the overall charged multiplicity. This has the property of generating large multiplicity fluctuations with long range in rapidity, in addition to short-range correlations due to cluster decay. For true minimum-bias simulation, the soft events generated by HERWIG should be mixed with an appropriate fraction of QCD hard-scattering events. For the underlying event in

\footnote{Maintenance here means much more than a mere bug correcting process; it refers to the ability to implement new physics models, processes or features on request.}

\footnote{Contributing author: B.R. Webber.}
hard collisions, the same model is used to simulate a soft collision between beam clusters containing the spectator partons.

The minimum-bias/underlying event model used in ISAJET is based on a mechanism of multiple Pomeron exchange [185], with a fluctuating number of ‘cut Pomerons’ acting as sources of final-state hadrons. Each cut Pomeron fragments directly into hadrons according to the ISAJET independent fragmentation model, with the fragmentation axis along the beam direction. The model again produces large long-range multiplicity fluctuations, but short-range correlations are weak due to the absence of clustering.

In PYTHIA a multiple interaction model is used to generate hard, soft and underlying events in a unified manner. Multiple interactions are discussed in more detail below. The number \( n \) and distribution \( P(n) \) of interactions per event is controlled by the minimum transverse momentum allowed in each interaction and, optionally, by a model for the impact parameter profile. Long-range fluctuations may be somewhat weaker in this model, with short-range correlations somewhere between the two other generators. In minimum-bias events the choice \( n = 0 \) can occur, in which case a two-string fragmentation model linking a quark in each beam proton to a diquark in the other is used.

A study of energy-flow correlations between well-separated phase-space regions would be helpful in understanding the underlying event and in separating its contribution from that of the hard subprocess [186]. Such a study is currently being undertaken by the CDF Collaboration.

3.3 Matrix-element corrections to vector boson production and transverse-momentum distributions

Vector boson production will be a fundamental process to test QCD and the SM of the electroweak interactions. Monte Carlo event generators [115–117] simulate the initial-state radiation in vector boson production processes in the soft/collinear approximation, but can have ‘dead zones’ in phase space, where no parton emission is allowed. The radiation in the dead zone is physically suppressed, since it is not soft or collinear logarithmically enhanced, but not complete absent as nevertheless happens in standard PS algorithms. Matrix-element corrections to the HERWIG simulation of Drell–Yan processes have been implemented in [164] following the method described in [163]: the dead zone is populated by the using of the exact first-order amplitude and the cascade in the already-populated phase-space region is corrected using the exact matrix element every time an emission is capable of being the hardest so far. A somewhat different procedure is followed to implement matrix-element corrections to the PYTHIA

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event generator [165, 166]: the PS probability distribution is applied over the whole phase space, the previous algorithm having a cut $q_T < m_V$ on the vector boson $V$ transverse momentum to avoid double counting, and the exact $\mathcal{O}(\alpha_S)$ matrix element is used only to generate the closest branching to the hard vertex. Referring hereinafter to the HERWIG event generator, in Fig. 22 the distribution of the $W$ transverse momentum $q_T$ is plotted at the LHC by running HERWIG 5.9, the latest public version, and HERWIG 6.1 [118], the new version including matrix-element corrections to vector boson production, for an intrinsic transverse momentum $q_{T\text{Int}} = 0$, its default value. A big difference can be seen at large $q_T$, where the 6.1 version has many more events which are generated via the exact $\mathcal{O}(\alpha_S)$ amplitude. In the PS soft/collinear approximation, on the contrary, $q_T$ is constrained to be $q_T < m_W$. A suppression can be seen at small $q_T$, due to the fact that, even though we are providing the Monte Carlo shower with the tree-level $\mathcal{O}(\alpha_S)$ matrix-element corrections, virtual contributions are missing and, by default, we still get the total leading-order cross section. No next-to-leading order parton shower algorithm is presently available.

In Fig. 23 some recent DØ data [187] on the $W$ $q_T$ spectrum at the Tevatron is compared with the HERWIG 6.1 results, which are corrected for detector smearing effects. A good agreement is found after hard and soft matrix-element corrections; the options $q_T = 0$ and 1 GeV are investigated, but no relevant effect is visible after detector corrections, which have been shown in [164] to be pretty strong.

In Fig. 24, we compare HERWIG with some CDF data [188] on $Z$ production, already corrected for detector effects, which are however much smaller than the $W$ case. We consider the options $q_{T\text{Int}} = 0,$
1 and 2 GeV. The overall agreement is good, with a crucial role of matrix-element corrections to fit in with the data at large $q_T$. At low $q_T$, the best fit is the one corresponding to $q_{T\text{int}} = 2$ GeV. Even though, as can be seen from Fig. 24, the $Z$ distribution is strongly dependent on the intrinsic transverse momentum at low $q_T$, in [189] and in Fig. 25 it is shown that the ratio of the $W$ and $Z$ differential cross sections, both normalized to one, is roughly independent of $q_{T\text{int}}$, which means that the effect of a non-zero $q_{T\text{int}}$ is approximately the same for both $W$ and $Z$ spectra. This ratio is one of the main inputs for the experimental analyses and the fact that it is not strongly dependent on unknown non-perturbative effects is good news for studies on the $W$ mass measurement.

It is also worthwhile comparing the HERWIG 6.1 $q_T$ distributions with some available calculations which resum the logarithms $l = \log(m_V/q_T)$, $m_V$ being the vector boson mass, in a Sudakov-like exponential form factor (see Sect. 5. for a review of theoretical aspects of Sudakov resummation). Such logarithms are large in the low $q_T$ range. In [164] the Monte Carlo results are compared with the resummation approaches of [190], where all terms down to the next-to-leading logarithmic order $\approx \alpha_s^2 l^n$ are kept in the Sudakov exponent, both in $q_T$- and impact parameter $b$-space, and of [191], where the authors expand the Sudakov exponent and keep in the differential cross section all terms down to the order $\approx \alpha_s^2 l^{2n-3}$, which are next-to-next-to-leading logarithms after the expansion of the form factor. Such resummations are also matched to the exact first-order result in [164]. In Figs. 26 and 27 the $W$ $q_T$ distributions are plotted according to HERWIG 6.1 and the resummed calculations at small $q_T$ and over the whole $q_T$ range respectively. The overall agreement at low $q_T$ is reasonable and the HERWIG plots lie well within the range of the resummed approaches. At large $q_T$ the matching of the resummed calculations to the exact $\mathcal{O}(\alpha_S)$ result works well only for the approach of [190] in the $q_T$-space, as we have a continuous distribution at the point $q_T = m_V$, the other distributions showing a step due to uncompensated contributions of order $\alpha_s^2$ or higher.

In [164], it is also shown that matrix-element corrections to vector boson production have a negligible effect on rapidity distributions, the latest version HERWIG 5.9 agreeing well with the CDF data on the $Z$ rapidity. The implemented hard and large-angle gluon radiation has nevertheless a marked impact on jet distributions both at the Tevatron and LHC, as many more events with high transverse energy jets are now generated. While these analyses are performed assuming that the produced vector boson decays into a lepton pair, the implementation of matrix-element corrections to the HERWIG simulation of the hadronic $W$ decay $W \rightarrow q\bar{q}'$ is in progress, however it is expected to be a reasonably straightforward extension of the corrections already applied to the process $Z \rightarrow q\bar{q}$. Furthermore, the method applied to improve the initial-state shower for $W/Z$ production could be extended to many other processes which
are relevant for the LHC. Among these, we expect that the implementation of matrix-element corrections to top and Higgs production may have a remarkable phenomenological effect at the LHC. This is in progress as well.

3.4 A comparison of the predictions from Monte Carlo programs and transverse momentum resummation

For many physical quantities, the predictions from PS Monte Carlo programs should be nearly as precise as those from analytic theoretical calculations. This is expected, among others, for calculations which resum logs with the transverse momentum of partons initiating the hard scattering (resummed calculations are described in Sect. 5.). In the recent literature, most calculations of this kind are either based on or originate from the formalism developed by J. Collins, D. Soper, and G. Sterman [192], which we choose as the analytic ‘benchmark’ of this Section. In this case, both the Monte Carlo and analytic calculations should accurately describe the effects of the emission of multiple soft gluons from the incoming partons, an all orders problem in QCD. The initial state soft gluon emission can affect the kinematics of the final state partons. This may have an impact on the signatures of physics processes at both the trigger and analysis levels and thus it is important to understand the reliability of such predictions. The best method for testing the reliability is the direct comparison of the predictions to experimental data. If no experimental data is available for certain predictions, then some understanding of the reliability may be gained from the comparison of the predictions from the two different methods.

Parton showering resums primarily the leading logarithms, which are universal, i.e. process independent, and depend only on the given initial state. In this lies one of the strengths of Monte Carlos, since parton showering can be incorporated into a wide variety of physical processes. As discussed in Sect. 5., an analytic calculation, in comparison, can resum all large logarithms, since all (in principle) are included in the Sudakov exponent given in Eq. (46).

If we try to interpret parton showering in the same language as resummation, which is admittedly risky, then we can say that the Monte Carlo Sudakov exponent always contains terms analogous to \( A^{(1)} \) and \( B^{(1)} \) in Eq. (47). It was shown in Ref. [172] that a suitable modification of the Altarelli–Parisi splitting function, or equivalently the strong coupling constant \( \alpha_s \), also effectively approximates the \( A^{(2)} \) coefficient. 11

Both Monte Carlo and analytic calculations describe the effects of the emission of multiple soft gluons from the incoming partons, an all orders problem in QCD. The initial state soft gluon emission affects the kinematics of the final state partons, which, in turn, may have an impact on the signatures of physics processes at both the trigger and analysis levels. Thus it is important to understand the reliability of such predictions. The best method for testing the reliability is the direct comparison of the predictions to experimental data. If no experimental data is available for certain predictions, then some understanding of the reliability may be gained from the comparison of the predictions from the two different methods.

In particular, one quantity which should be well-described by both calculations is the transverse momentum \( (p_T) \) of the final state electroweak boson in a subprocess such as \( q\bar{q} \rightarrow WX, ZX \) or \( gg \rightarrow HX \), where most of the \( p_T \) is provided by initial state parton showering. The parton showering supplies the same sort of transverse kick as the soft gluon radiation in a resummation calculation. This correspondence between the Sudakov form factors in resummation and Monte Carlo approaches may seem trivial, but there are many subtleties in the relationship between the two approaches relating to both the arguments of the Sudakov factors as well as the impact of sub-leading logs [164, 166, 188].

At a point in its evolution corresponding to (typically) the virtuality of a few GeV\(^2\), the parton shower is cut off and the effects of gluon emission at softer scales must be parameterized and inserted

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11Reference [172] deals only with the high-\( x \) (or \( \sqrt{s} \)) region, but the same results apply to the small-\( p_T \) region in transverse momentum distributions.
by hand. This is similar to the (somewhat arbitrary) division between perturbative and non–perturbative regions in a resummation calculation. The parametrization is typically done with a Gaussian smearing similar to that used for the non–perturbative $k_T$ in a resummation program. In general, the value for the non–perturbative $\langle k_T \rangle$ needed in a Monte Carlo program will depend on the particular kinematics being investigated.\footnote{Note that this is unlike the case of the resummation calculations in Refs. \cite{192,194,195}, where the non–perturbative physics is determined from fits to fixed target data and then automatically evolved to the kinematic regime of interest.}

A value for the average non–perturbative $k_T$ greater than 1 GeV does not imply that there is an anomalous intrinsic $k_T$ associated with the parton size; rather, this amount of $\langle k_T \rangle$ needs to be supplied to provide what is missing in the truncated parton shower. If the shower is cut off at a higher virtuality, more of the ‘non–perturbative’ $k_T$ will be needed.

\subsection{Vector boson production and comparison with PYTHIA and RESBOS}

The (resolution corrected) $p_T$ distribution for $Z^0$ bosons (in the low $p_T$ region) for the CDF experiment \cite{188} is shown in Figure 28 \cite{193}, compared to both the resummed prediction from ResBos \cite{194}, and to two predictions from PYTHIA (version 6.125). One PYTHIA prediction uses the default (rms) value of intrinsic $k_T$ of 0.44 GeV and the second a value of 2.15 GeV per incoming parton. The latter value was found to give the best agreement for PYTHIA with the data.\footnote{For a Gaussian distribution, $k_T^{\text{rms}} = 1.13 \langle k_T \rangle$.} All of the predictions use the CTEQ4M parton distributions \cite{111}. Good agreement is observed between ResBos, PYTHIA and the CDF data.

\subsection{Higgs boson production and comparison with PYTHIA}

A comparison of the Higgs $p_T$ distribution at the LHC \cite{193}\footnote{See Sect. 3.3 and Fig. 24 for comparisons of the CDF $Z^0$ $p_T$ data with HERWIG.}, for a Higgs mass of 150 GeV, is shown in Figure 29, for ResBos \cite{195} and the two recent versions of PYTHIA. PYTHIA has been rescaled to agree with the normalization of ResBos to allow for a better shape comparison. Note that the peak of the

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig28.png}
\caption{The $Z^0$ $p_T$ distribution (at low $p_T$) from CDF for Run 1 compared to predictions from ResBos and from PYTHIA. The two PYTHIA predictions use the default (rms) value for the non–perturbative $k_T$ (0.44 GeV) and the value that gives the best agreement with the shape of the data (2.15 GeV).}
\end{figure}
resummed distribution is at $p_T \approx 11$ GeV (compared to about 3 GeV for $Z^0$ production at the Tevatron). This is partially due to the larger mass (150 GeV compared to 90 GeV), but is primarily because of the larger color factors associated with initial state gluons ($C_A = 3$) rather than quarks ($C_F = 4/3$), and also because of the larger phase space for initial state gluon emission at the LHC. The newer version of PYTHIA agrees well with ResBos at low to moderate $p_T$, but falls below the resummed prediction at high $p_T$. This is easily understood: ResBos switches to the NLO Higgs + jet matrix element [197] at high $p_T$ while the default PYTHIA can generate the Higgs $p_T$ distribution only by initial state gluon radiation, using as maximum virtuality the Higgs mass squared. High $p_T$ Higgs production is another example where a $2 \to 1$ Monte Carlo calculation with parton showering can not completely reproduce the exact matrix element calculation, without the use of matrix element corrections as already discussed in section 3.3. The high $p_T$ region is better reproduced if the maximum virtuality $Q^2_{\text{max}}$ is set equal to the squared partonic center of mass energy, $s$, rather than $m_H^2$. This is equivalent to applying the PS to all of phase space. However, this has the consequence of depleting the low $p_T$ region as ‘too much’ showering causes events to migrate out of the peak. The appropriate scale to use in PYTHIA (or any Monte Carlo) depends on the $p_T$ range to be probed. If matrix element information is used to constrain the behavior, the correct high $p_T$ cross section can be obtained while still using the lower scale for showering. The incorporation of matrix element corrections to Higgs production (involving the processes $qg \to qH, qg \to gH, gg \to gH$) is the next logical project for the Monte Carlo experts, in order to accurately describe the high $p_T$ region.

The older version of PYTHIA produces too many Higgs events at moderate $p_T$ (in comparison to ResBos). Two changes have been implemented in the newer version. The first change is that a cut is placed on the combination of $z$ and $Q^2$ values in a branching: $\hat{u} = Q^2 - s(1 - z) < 0$, where $\hat{s}$ refers to the subsystem of the hard scattering plus the shower partons considered to that point. The association with $\hat{u}$ is relevant if the branching is interpreted in terms of a $2 \to 2$ hard scattering. The corner of emissions that do not respect this requirement occurs when the $Q^2$ value of the space-like emitting parton is little changed and the $z$ value of the branching is close to unity. This effect is mainly for the
This difference in the $p_T$ distribution between the two versions, 5.7 and 6.1, of PYTHIA could have an impact on the analysis strategies for Higgs searches at the LHC [199]. For example, for the CMS simulation of the Higgs search and the decay into two photons it is envisaged to optimize the efficiency and the mass resolution for the high-luminosity running phase using charged particles with relatively large $p_T$, which balance the Higgs $p_T$ spectrum. These associated charged particles will allow to distinguish the Higgs event vertex from other vertices of unrelated proton–proton interactions with good accuracy. The efficiency of such an analysis strategy depends obviously on the knowledge of the Higgs $p_T$ spectrum and is thus somewhat sensitive to the used Monte Carlo parametrisation.

3.43 Comparison with HERWIG

The variation between versions 5.7 and 6.1 of PYTHIA gives an indication of the uncertainties due to the types of choices that can be made in Monte Carlos. The requirement that $\hat{u}$ be negative for all branchings is a choice rather than an absolute requirement. Perhaps the better agreement of version 6.1 with ResBos is an indication that the adoption of the $\hat{u}$ restrictions was correct. Of course, there may be other changes to PYTHIA which would also lead to better agreement with ResBos for this variable.

Since there are a variety of choices that can be made in Monte Carlo implementations, it is instructive to compare the predictions for the $p_T$ distribution for Higgs production from ResBos and PYTHIA

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16Such branchings are kinematically allowed, but since matrix element corrections would assume initial state partons to have $Q^2 = 0$, a non-physical $\hat{u}$ results (and thus no possibility to impose matrix element corrections). The correct behavior is beyond the predictive power of LL Monte-Carlos.
with that from HERWIG (version 5.6, also using the CTEQ4M parton distribution functions). The HERWIG prediction is shown in Figure 30 along with the PYTHIA and ResBos predictions, all normalized to the ResBos prediction\(^\text{17}\). In all cases, the CTEQ4M parton distribution was used. The predictions from HERWIG and PYTHIA 6.1 are very similar, with the HERWIG prediction matching the ResBos shape somewhat better at low \(p_T\). An understanding of the signature for Higgs boson production at either the Tevatron or LHC depends upon the understanding of the details of soft gluon emission from the initial state partons. This soft gluon emission can be modelled either in a Monte Carlo or in a resummation calculation, with various choices possible in both implementations. A comparison of the two approaches is useful to understand the strengths and weaknesses of each. The data from the Tevatron that either exists now, or will exist in Run 2, will be extremely useful to test both approaches.

In contrast to the case for \(Z\) production at the Tevatron, the Higgs cross section at the LHC is not particularly sensitive to the non–perturbative \(k_T\) added at the scale \(Q_0\). In the evolution to the hard scatter scale \(Q\), the \(k_T\) is ‘radiated away’, given the enhanced gluon radiation probability present for a \(gg\) initial state. For a more thorough discussion of the comparison between analytic methods and parton showers, see Ref. [193].

### 3.5 COMPHEP for LHC\(^\text{18}\)

The COMPHEP package is available from: http://theory.npi.msu.su/~comphep/. A version adapted to the LHC physics COMPHEP V.33 [138], including executable Linux modules is available at CERN from: /afs/cern.ch/cms/physics/COMPHEP-Linux.

The current COMPHEP version performs all calculation at tree level (LO). Three issues must be discussed as they open several setting options: a) the parton distributions, b) the QCD scale, and c) the running strong coupling.

In COMPHEP v.33, the following parton distribution sets are implemented: MRS(A’) and MRS(G) [200], CTEQ4l and CTEQ4m [111]. Note that CTEQ4l is a LO parametrization, while in all others the evolution of parton distributions is treated at NLO. Dedicated routines are available to allow the addition of any other defined parton distribution (e.g. CTEQ5).

As discussed in Sect. 1., the factorization theorem states that the parton distribution depends not only on Bjorken variable \(x\) but also on its virtuality \(Q^2\) or, equivalently, on the factorization scale. This parameter is related to the energy (or momentum) scale which characterizes the hard subprocess, but it cannot be unambiguously fixed (see Sect. 1.). Therefore it can be experimentally tuned. It can be set by the user for each specific QCD process as either fixed or running. In the latter case, \(Q^2\) can be set to any linear combination squared of the external particles momenta (e.g. \((p_1 - p_3)^2\), \((p_1 - p_3 - p_4)^2\), \((p_3 + p_4)^2\) . . . where initial and outgoing momenta enter with opposite signs).

In COMPHEP V.33, the QCD coupling \(\alpha_S\) can be computed at LO, NLO or NNLO precision. All the corresponding formulas are written in terms of \(\Lambda^{(6)}_{\text{MS}}\), the fundamental QCD scale for \(N_f = 6\) flavours of massless quarks (see Sect. 1. and [13]). In COMPHEP, to evaluate a QCD process, one first fixes the \(\alpha_S\) normalization point (e.g. a popular normalization point is the mass of \(Z\) boson, \(Q = M_Z\)) to which correspond an experimental fit (e.g. \(\alpha_S^{NLO}(M_Z) = 0.118\)). Then, the corresponding \(\Lambda^{(N_f)}_{\text{MS}}(N_f = 5\) at \(Q = M_Z\)) can be deduced from the \(\alpha_S\) expression at the selected precision order. The COMPHEP input parameter \(\Lambda^{(6)}_{\text{MS}}\) is then obtained from \(\Lambda^{(N_f)}_{\text{MS}}\). Finally, the choice of the QCD scale \(Q\) determines \(\alpha_S\) and the factorization scale for the pdf’s. Therefore, complete LO calculations of LHC processes are made available for a consistent phenomenological analysis of the influence of higher order contributions.

\(^{17}\)The normalization factors (ResBos/Monte Carlo) are PYTHIA (both versions)(1.61) and HERWIG (1.76). Figures of the absolutely normalized predictions from ResBos, PYTHIA and HERWIG for the \(p_T\) distribution of the Higgs at the LHC can be found in Ref. [193].

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3.51 COMPHEP-PYTHIA interface

An interface between COMPHEP and PYTHIA can be found in:
/afs/cern.ch/cms/physics/COMPPYTH.

A library of COMPHEP based partonic event generators for LHC processes has been initiated and various samples of event are available at: /afs/cern.ch/cms/physics/PEVLIB for Zb\bar{b}, Wb\bar{b}, t\bar{t}b\bar{b} and some others. Unweighted event sample files, located in the corresponding directories (see the files README for details) when handled by the COMPHEP-PYTHIA interface code, generate complete LHC events, ready to be fed to the detector simulation software. For example, the Zb\bar{b} process can be found in: /afs/cern.ch/cms/physics/PEVLIB/Z_b\_b. The file _pevZb\bar{b} contains about 200K unweighted events. Each event is represented by the Lorentz momenta of all external particles. In the current version of the package, there is no color information associated to the events. Thus, only the Independent Fragmentation Model can be invoked. One can always require the Lund model option for the fragmentation, as long as the corresponding color strings have been set by an external algorithm in the routine PYUPEV. The same remark applies also to the final state radiations (FSR), which are, by default, switched off in COMPHEP-PYTHIA interface although initial state radiations (ISR) are switched on. In the upcoming version of the COMPHEP package [201] color strings will be generated from the matrix element factors allowing for the use of the Lund fragmentation model.

3.6 GRACE for LHC\(^{19}\)

The URL of web page for the GRACE system is http://www-sc.kek.jp/minami/ where the latest information, the reports and manuals [130, 131], the GRACE version.2 and the other products are available.

The automated system allows us to create event generators for complicated processes which are hard to calculate by hand. For instance the process \(gg \rightarrow b\bar{b}b\bar{b}\) has been calculated without any approximation (e.g. accounting for massive fermions) by use of the GRACE system [130, 131].

The intrinsic function of the GRACE system is to generate the amplitude for a specified parton interaction. The system has been tested for many reactions and it was confirmed to be able to manage 2-body to 6-body final state processes. The interface with the pdf’s, PS and the fragmentation tools will be implemented in the coming versions (see for example GRAPE for \(e\bar{p} \rightarrow \ell\ell X\) [202]). For the parton showering and the fragmentation, two kinds of approach can be followed. The first is, like in GRAPE a 2 step procedure: the BASES/SPRING package including pdf’s is used for the integration over the phase space and for the generation of unweighted events. If the “kinematics” code is appropriate, SPRING generates events with high efficiency and writes the four-vectors of the final-state particles on a temporary file. Then the generated momenta are passed to PYTHIA for PS and fragmentation. The other approach is more convenient but more complex. Here the code including the kinematics and the generated matrix element is prepared so that PYTHIA can drive them directly. This type of interface is tested till now only for the processes whose final state consist of 2-, 3- and 4-bodies.

The GRACE system can automatically deal with one-loop processes (NLO) for the electroweak and QED-like QCD interactions. For the final two-body processes the performance has been shown to be good. The application to the multi-body final states, however, would be limited because of the huge CPU time required when the code is used as event generators. For such cases a practical use of the generated code will be to evaluate the cross sections and to give the distribution of several physical quantities rather than providing event generators.

As mentioned the contributions beyond LO are crucial for a detailed QCD study. Since the PS method is based on the renormalization group equation, it works as a bridge between the “hard” parton collision and the fragmentation. This bridge is built on the solid and reliable ground of perturbative QCD. In other words the parton shower provides an unambiguous theoretical understanding of \(pp(\bar{p})\) interac-

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tions except for the “soft” component which cannot be controlled by the perturbative QCD. However, the PS in LL order is not enough. One of the shortcomings is as follows. The pdf’s for the initial state, products of elaborated works, are parameterized according to the NLO QCD formulas. On the other hand the corresponding PS, implemented in the existing programs like PYTHIA, is evolved using only the LL algorithm at least in their current status. Then the systematic summation of large logarithms up to NLL order must solve this annoying situations. Though the basic technology has been already established and known for many years [203–205], its implementation is not a trivial task as simply imagined. First it is process-dependent. Once the idea evolves and is realized as one of the environments of GRACE, it should allow more precise prediction for LHC. Thus this must be the biggest issue to us.

3.7 ALPHEA for LHC

As discussed in the introduction to this Section, the ability to evaluate production rates for multi-jet final states will be fundamental at the LHC to study a large class of processes, within and beyond the SM. As was also discussed in the Sect. 3.1, a necessary feature of any multi-jet calculation is the possibility to properly evolve the purely partonic final state, for which exact fixed-order perturbative calculations can be performed, into the observable hadronic final state. This evolution is best performed using shower Monte Carlo calculations. The accurate description of color-coherence effects, furthermore, requires as noticed in the introduction a careful bookkeeping of the contribution to the matrix elements of all possible color configurations. The goal of the algorithm [137] described in this Section is to allow the effective calculation of multi-parton matrix elements, allowing the separation, to the leading order in \(1/N_c^2\) (\(N_c = 3\) being the number of colors), of the independent color configurations. This technique allows an unweighting of the color configurations, and allows the merging of the parton level calculation with the HERWIG Monte Carlo.

The key element of the strategy is the use of the algorithm ALPHEA, introduced in Ref. [134] for the evaluation of arbitrary multi-parton matrix elements. This algorithm determines the matrix elements from a (numerical) Legendre transform of the effective action, using a recursive procedure which does not make explicit use of Feynman diagrams. The algorithm has a complexity growing like a power in the number of particles, compared to the factorial-like growth that one expects from naive diagram counting. This is a necessary feature of any attempt to evaluate matrix elements for processes with large numbers of external particles, since the number of Feynman diagrams grows very quickly beyond any reasonable value. For example, this calculation allowed [137] the evaluation of the matrix elements for the production of 8-gluon final states. The number of Feynman diagrams which describe this process exceeds 7 billion.

The interface of the parton level scattering matrix element with the PS requires the capability to reconstruct the appropriate color flow for a given event. The strategy to deal with this issue is described in detail in [137]. The following points have to be noticed:

1. Dual amplitudes [206–208] can be easily evaluated using the ALPHEA algorithm. Since the dual amplitudes are independent of the number \(N_c\) of colors, they can be calculated exactly by taking \(N_c\) sufficiently large.

2. With an appropriate choice for the color of the external partons, the full amplitude is proportional to a single dual amplitude.

We explicitly calculated \(n\)-gluon dual amplitudes using the large-\(N_c\) Lagrangian. The correctness of the calculation was checked for \(n\) up to 11 by comparing the results for maximally helicity violating (MHV) amplitudes [209] (e.g., \(g^+g^+ \rightarrow g^+\cdots g^+\)) with the analytic expressions known exactly for arbitrary \(n\) [124, 206–208]. The input of the numerical evaluation of the matrix element is a string containing the total number of gluons, their helicity state, and their momenta. From these data, the amplitude is evaluated automatically.

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20 Contributing authors: M.L. Mangano and M. Moretti.
The prescription to correctly generate the parton-shower associated to a given event in the large-$N_c$ limit is therefore the following:

1. Calculate the $(n-1)!$ dual amplitudes corresponding to all possible planar color configurations.
2. Extract the most likely color configuration for this event on a statistical basis, according to the relative contribution of the single configurations to the total event weight \(^{21}\). Since each dual amplitude is gauge invariant, the choice of color-configurations is also a gauge-invariant operation.
3. Develop the PS out of each initial and final-state parton, starting from the selected color configuration. This step can be carried out by feeding the generated event to a Monte-Carlo program such as HERWIG, which is precisely designed to turn partons into jets starting from an assigned color-ordered configuration.

Notice that, if the dual amplitudes are evaluated for a specific helicity configuration, HERWIG will also include spin-correlation effects in the evolution of the parton shower \([116, 117, 171, 210, 211]\).

As a result, use of the dual-amplitude representation of a multi-gluon amplitude allows to accurately describe not only the large-angle correlations in multi-jet final states, but also the full shower evolution of the initial- and final-state partons with the same accuracy available in HERWIG for the description of 2-jet final states.

In alternative to the above prescription, one can use ALPHA to calculate the matrix elements for external states with assigned colors. Since these states are all orthogonal, such an approach is particularly efficient if one wants to use a Monte Carlo approach to the summation over all possible color states. The program will then extract through a standard unweighting (at the leading order in $1/N_c^2$) a specific color flow from all possible color flows contributing to a given orthogonal color state. This color flow is then suitable as an initial condition for the shower evolution. Further details can be found in \([137]\). At this time, the program is only available in its parton-level form, and allows the calculation of matrix elements for $gg \rightarrow g \ldots g$ and $q\bar{q} \rightarrow g \ldots g$ processes, with up to 8 final-state gluons. A full version including the interface with HERWIG is being prepared.

4. AVAILABLE NLO CALCULATIONS AND PROSPECTS AT NNLO\(^{22}\)

4.1 Available NLO calculations of multijet processes\(^{23}\)

QCD calculations of multijet\(^{24}\) processes beyond LO in the strong coupling constant $\alpha_S$ are quite involved. Nowadays we know (see below) how to perform in general calculations of the NLO corrections to multijet processes, and almost every process of interest has been computed to that accuracy. Instead, the calculation of the NNLO corrections is still at an organisational stage and represents a main challenge. Why should we perform calculations which are technically so complicated?

The general motivation is that the calculation of the NLO corrections allows us to estimate reliably a given production rate, while the NNLO corrections allow us to estimate the theoretical uncertainty on the production rate. That comes about because higher-order corrections reduce the dependence of the cross section on the renormalization scale, $\mu_R$, and for processes with strongly-interacting incoming particles the dependence on the factorization scale, $\mu_F$, as well.

An example is the determination of $\alpha_S$ from event shape variables in $e^+e^- \rightarrow 3$ jets \([212–215]\). The calculation of the NNLO contributions to this process would be needed to further reduce the theoretical uncertainty in the determination of $\alpha_S$. An additional motivation for performing calculations at NNLO is to obtain a more accurate theoretical determination of signal and QCD background to Higgs production (for further details, see Sect. 9.).

\(^{21}\)Defining $w_i = |A_i|^2$ for each color flow $i$, and $W_i = \sum_{k=1}^{i-1} w_k/\sum_{k=1}^{i} w_k$, the $j$-th color structure will be selected if $W_{j-1} \leq \xi < W_j$, for a random number $\xi$ uniformly distributed over the interval $[0, 1]$.

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\(^{23}\)Contributing authors: V. Del Duca and S. Frixione.

\(^{24}\)For the sake of brevity, in this section we will term as multijet any kind of (partly) hadronic final state.
In recent years it has become clear how to construct general-purpose algorithms for the calculation of multijet processes at NLO accuracy. The crucial point is to organise the cancellation of the infrared (i.e. collinear and soft) singularities of the QCD amplitudes in a universal, i.e. process- and observable-independent, way. The universal terms in a NLO calculation are given by the tree-level collinear \[14, 16, 17, 216\] and soft \[217–219\] functions, and by the universal structure of the poles of the one-loop amplitudes \[160,220,221\]. The universal NLO terms and the process-dependent amplitudes are combined into effective matrix elements, which are devoid of singularities. The various NLO algorithms (phase-space slicing \[160,222–224\] and subtraction method \[161,162,225–227\]) provide different methods to construct the effective matrix elements. These can be integrated in four dimensions, in practice almost always numerically, due to the complexity of the integrand. The integration can be performed with arbitrary experimental acceptance cuts.

We now outline how to perform a NLO calculation of a generic physical observable. As is well known from Bloch-Nordsieck and Kinoshita-Lee-Nauenberg theorems, QCD (like QED) does not have an infinite-resolution power; any attempt to compute the kinematical properties of a fixed number of final-state quarks and gluons results in an infrared-divergent cross section. In order to obtain finite quantities, all the partonic subprocesses which contribute to the same order in \(\alpha_S\) to the squared amplitude have to be included in the computation, regardless of the number of final-state particles. In addition, one is forced to consider variables which are inclusive enough to be infrared safe. Roughly speaking, an observable is said to be infrared safe when its value, computed with the kinematical variables of the final-state partons, does not change abruptly when a soft gluon is emitted, or a parton splits almost collinearly into a pair of partons. More technically, an infrared-safe observable must have a smooth limit (that is, must behave continuously) in the following three configurations: a) when a gluon in the final state gets soft; b) when two partons in the final state tend to get collinear to each other; c) when an initial-state parton emits collinearly another parton.

At NLO (assuming that the LO cross section gets contributions from the \(n\)-parton amplitudes), this implies that one has simply to consider two contributions, denoted as virtual and real. The former is the product of the \(n\)-parton one-loop amplitudes with the \(n\)-parton tree amplitudes, while the latter is the square of the \((n+1)\)-parton tree amplitudes. In order to deal with finite quantities in the intermediate steps of the calculation, we adopt dimensional regularization – i.e. we change the dimensionality of space-time to \(d = 4 - 2\epsilon\). Thus, we can schematically write the virtual and real contributions to the cross section as follows:

\[
\left(\frac{d\sigma}{dx}\right)_v = \frac{1}{2\epsilon}\delta(1-x), \quad \left(\frac{d\sigma}{dx}\right)_R = \frac{1}{1-x};
\]

(23)

here, \(1-x\) represents the radiated energy. So, \(x = 1\) means no radiation, and \(x = 0\) is the maximum of radiation. The relevant physical quantity will be the average value \(<F>\) of a certain function \(F(x)\); for example, we can think of \(F\) as being the product of theta functions representing a histogram bin. Then, the NLO contribution to \(<F>_\text{NLO}\) is

\[
<F>_{\text{NLO}} = \int_0^1 dx \left(\frac{d\sigma}{dx}\right)_v F(x) + \int_0^1 dx (1-x)^{-2\epsilon} \left(\frac{d\sigma}{dx}\right)_R F(x)
\]

(24)

\[
= \frac{1}{2\epsilon} \int_0^1 dx \delta(1-x) F(x) + \int_0^1 dx (1-x)^{-1-2\epsilon} F(x)
\]

(25)

\[
= \frac{1}{2\epsilon} F(1) + <F>_R.
\]

(26)

The factor \((1-x)^{-2\epsilon}\) in the real contribution comes from the necessity of performing the computation in \(d\) dimensions, in order to regulate the divergences arising when performing the integration over the phase space. As it is apparent from eq. (25), the most difficult task is the computation of the real contribution. In practice, the form of \(F(x)\) is too complicated to perform an analytical integration. On the other hand, we cannot proceed straightforwardly, and compute the integral numerically; in fact, the integral is
divergent in the limit $\epsilon \rightarrow 0$, and the pole in $1/\epsilon$ will exactly cancel that explicitly displayed in the virtual contribution (provided that $F$ describes an infrared-safe quantity).

Two strategies have been developed to tackle this problem. In the framework of the *slicing* method, the real contribution is rewritten as follows:

$$< F >_R = \int_{-\delta}^{1-\delta} dx \frac{F(x)}{(1-x)^{1+2\epsilon}} + \int_{1-\delta}^{1} dx \frac{F(x)}{(1-x)^{1+2\epsilon}}, \quad \text{for} \quad 0 < \delta \leq 1.$$  

(27)

where $\delta$ is an arbitrary parameter, $0 < \delta \leq 1$. The first term on the right hand side of this equation is free of divergences ($F(x)$ is regular in the limit $x \rightarrow 1$); in this term, one can therefore set $\epsilon = 0$, and compute the integral with standard numerical methods. On the other hand, the second term is still divergent for $\epsilon \rightarrow 0$; however, if $\delta$ is small enough, one can approximate $F(x)$ with $F(1)$ (that is, with the first term of its Taylor expansion around $x = 1$). Therefore

$$< F >_R = \int_{-\delta}^{1-\delta} dx \frac{F(x)}{1-x} + F(1) \int_{1-\delta}^{1} dx \frac{1}{(1-x)^{1+2\epsilon}} + \mathcal{O}(\delta) \quad \text{for} \quad \epsilon = 0.$$  

(28)

$$= \int_{0}^{1-\delta} dx \frac{F(x)}{1-x} - \frac{\delta-2\epsilon}{2\epsilon} F(1) + \mathcal{O}(\delta). \quad \text{for} \quad \epsilon = 0.$$  

(29)

Eq. (29) can now be substituted into eq. (26). Expanding eq. (29) in powers of $\epsilon$, keeping only the terms which do not vanish in the limit $\epsilon \rightarrow 0$, and neglecting the contributions of the terms of $\mathcal{O}(\delta)$, we see that the pole terms in $1/\epsilon$ cancel, and one is left with a finite result:

$$< F >_{\text{slicing}}_{\text{NLO}} = \int_{0}^{1-\delta} dx \frac{F(x)}{1-x} + F(1) \log \delta.$$  

(30)

At a first glance, this expression is seemingly puzzling: the parameter $\delta$ is arbitrary, and the physical results should not depend on it. However, it is easy to see that the upper bound of the integral gives a contribution behaving (approximately) like $-F(1) \log \delta$. It has to be stressed that the slicing method is based on the approximation performed in eq. (28); for this approximation to hold, it is crucial that $\delta$ is as small as possible; otherwise, the terms collectively denoted with $\mathcal{O}(\delta)$ in eq. (29) are not negligible. On the other hand, in practical computations, the integral in eq. (30) is performed numerically; due to the divergence of the integrand for $x \rightarrow 1$, $\delta$ cannot be taken too small, because of the loss of accuracy of the numerical integration. Thus, the value of $\delta$ is a compromise between these two opposite requirements, being neither too small nor too large. Of course, “small” and “large” are meaningful only when referred to a specific computation. Therefore, when using the slicing method, it is mandatory to check that the physical results are stable against the variation of the value of $\delta$, chosen in a suitable range. In principle, this check would have to be performed for each observable $F$ computed; in practice, only one observable is checked, generally chosen to be rather inclusive (such as a total rate).

Another possibility to compute $< F >_R$ is given by the *subtraction* method. One writes

$$< F >_R = \int_{0}^{1} dx \frac{F(x) - F(1)\theta(x - 1 + x_c)}{(1-x)^{1+2\epsilon}} + F(1) \int_{0}^{1} dx \frac{\theta(x - 1 + x_c)}{(1-x)^{1+2\epsilon}}, \quad \text{for} \quad 0 < x_c \leq 1.$$  

(31)

where $x_c$ is an arbitrary parameter $0 < x_c \leq 1$. The first term on the right hand side of this equation is convergent, and we can set $\epsilon = 0$. The second term is formally identical to the one appearing in eq. (28). Notice, however, that no approximation has been made in eq. (31); the price to pay is a more complicated expression for the first integral. Proceeding as before, we get:

$$< F >_{\text{sub}}_{\text{NLO}} = \int_{0}^{1} dx \frac{F(x) - F(1)\theta(x - 1 + x_c)}{1-x} + F(1) \log x_c.$$  

(32)

This equation has to be compared to eq. (30); although the two are quite similar, there are two important differences that have to be stressed. Firstly, the parameter $x_c$ introduced in the subtraction method does...
not need to be small (actually, in the original formulation of the method $x_c$ was not even introduced, which corresponds to set $x_c = 1$ here). This is due to the fact that in the subtraction method no approximation has been performed in the intermediate steps of the computation. This in turn implies the second point: there is no need to check that the physical results are independent of the value of $x_c$, since this is true by construction.

The universal algorithms previously mentioned allow the computation of any infrared-safe observable in a straightforward manner; the matrix elements do not need any algebraic manipulation, and can be computed in four dimensions. It is therefore relatively easy to construct computer codes, accurate to NLO in QCD, that are flexible enough to become a useful tool in the analysis of the experimental data. In the following, we will list the codes which are of direct interest for the physics of high-energy hadronic collisions. We do not intend to give a complete list of references to the papers relevant for the calculation of a given production process, but rather only to quote the computer codes which will have a chance to be used by the experimental collaborations at the LHC. Most of the codes listed here are available as free software.

- **Jets**
  - S.D. Ellis, Z. Kunszt and D.E. Soper [6, 220], *subtraction*, computes one- and two-jet observables.
  - W.T. Giele, E.W.N. Glover and D.A. Kosower (JETRAD) [222], *slicing*, computes one- and two-jet observables.
  - S. Frixione [227], *subtraction*, computes one- and two-jet observables.
  - W. Kilgore and W.T. Giele [228], *slicing*, computes three-jet observables.

- **Single Isolated Photon (plus one jet)**
  - H. Baer, J. Ohnemus and J.F. Owens [229], *slicing*, fragmentation contribution computed to LO accuracy.
  - L.E. Gordon and W. Vogelsang [230], analytical integration over the variables of the recoiling partons: no information on the accompanying jet; dependence on the isolation variables treated to logarithmic approximation.
  - S. Frixione [231], *subtraction*, only effective with the isolation prescription of ref. [232].
  - M. Werlen (PHONLL) [http://home.cern.ch/~monicaw/phonll.html], *slicing*, based on ref. [233, 234].

- **Isolated-Photon Pairs**
  - B. Bailey, J.F. Owens and J. Ohnemus [235], *slicing*, fragmentation contributions computed to LO accuracy.
  - C. Balazs, E.L. Berger, S. Mrenna and C.P. Yuan [236], *slicing*, resummation effects included, fragmentation contributions computed with parton shower methods.
  - T.Binoth, J.Ph. Guillet, E. Pilon and M.Werlen (DIPHOX) [237], *slicing*, all contributions computed to NLO accuracy.

- **Single Heavy Vector Boson (plus one jet)**
  - W.T. Giele, E.W.N. Glover and D.A. Kosower (DYRAD) [222], *slicing*.

- **Single Heavy Vector Boson plus one photon**
  - U. Baur, T. Han, J. Ohnemus [238, 239], *slicing*.
  - D. de Florian and A. Signer [240], *subtraction*, includes spin correlations in the decay of the bosons; fragmentation contributions computed to LO accuracy.

- **Heavy Vector Boson Pairs**
  - U. Baur, T. Han, J. Ohnemus and J.F. Owens [241–245], *slicing*.

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25Further details on codes involving the production of a single vector boson and of a Higgs boson can be found in Sect. 6. and 9., respectively.
- S. Frixione, B. Mele, P. Nason and G. Ridolfi [246–248], *subtraction*.
- J.M. Campbell and R.K. Ellis (MCFM) [249], *subtraction*, includes spin correlations in the decay of the bosons.
- L. Dixon, Z. Kunszt and A. Signer [250], *subtraction*, includes spin correlations in the decay of the bosons.

- **Higgs Boson at large transverse momentum (plus one jet)**
  - D. de Florian and M. Grazzini and Z. Kunszt [197], *subtraction*, computes Higgs-boson production in the infinite top-quark-mass limit.

- **Heavy Quarks**
  - M. Mangano, P. Nason and G. Ridolfi [251], *subtraction*, computes single-inclusive distribution and correlations between $Q$ and $Q'$.

Since the universal algorithms accomplish the task of cancelling the infrared divergences of the virtual and real contributions in a process-independent way, the remaining work that has to be performed to calculate a production rate at NLO is the computation of the appropriate tree and one-loop amplitudes. As we said previously, to compute $n$-jet production at NLO, two sets of amplitudes are required: 

1. $n$-particle production amplitudes at tree level and one loop;
2. $(n+1)$-particle production amplitudes at tree level. If the one-loop amplitudes are regularised through dimensional regularisation, it suffices at NLO to compute them to $O(\epsilon^0)$.

Efficient methods based on the color decomposition [125, 252–254] of an amplitude in color-ordered subamplitudes, which are then projected onto the helicity states of the external partons, have largely enhanced the ability of computing tree [125] and one-loop [255] amplitudes. Accordingly, tree amplitudes with up to seven massless partons [125, 256, 257] and with a vector boson and up to five massless partons [258] have been computed analytically. In addition, efficient techniques to evaluate numerically tree multi-parton amplitudes have been introduced [137, 259] (see Sect. 3, for a description of available numerical codes), and have been used to compute tree amplitudes with up to eleven massless partons [137]. The calculation of one-loop amplitudes can be reduced to the calculation of one-loop $n$-point scalar integrals [260–262]. The reduction method [260] allowed the computation of one-loop amplitudes with four massless partons [263] and with a vector boson and three massless partons [264]. However, one-loop scalar integrals present infrared divergences, induced by the massless external legs. For one-loop multi-parton amplitudes, the infrared divergences hinder the reduction methods of ref. [260–262]. This problem has been overcome in ref. [265, 266]. Accordingly, one-loop amplitudes with five massless partons [267–269] and with a vector boson and four massless partons [270–274] have been computed analytically. The reduction procedure of ref. [265, 266] has been generalised in ref. [275], where it has been shown that any one-loop $n$-point scalar integral, with $n > 4$, can be reduced to box scalar integrals. The calculation of one-loop multi-parton amplitudes thus can be pushed a step further in the near future.

### 4.2 Prospects for NNLO calculations

Eventually, a procedure similar to the one followed at NLO will permit the construction of general-purpose algorithms at NNLO accuracy. It is mandatory then to fully investigate the infrared structure of the matrix elements at NNLO. The universal pieces needed to organise the cancellation of the infrared singularities are given by the tree-level triple-collinear [253, 276, 277], double-soft [219, 278] and soft-collinear [276, 278] functions, by the one-loop splitting [271, 279–281] and eikonal [271] functions, and by the universal structure of the poles of the two-loop amplitudes [282]. These universal pieces have yet to be assembled together, to show the cancellation of the infrared divergences at NNLO.

Then to compute $n$-jet production at NNLO, three sets of amplitudes are required: 

1. $n$-particle production amplitudes at tree level, one loop and two loops;
2. $(n+1)$-particle production amplitudes

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at tree level and one loop; c) \((n+2)\)-particle production amplitudes at tree level. In dimensional regularisation at NNLO, the two-loop amplitudes need be computed to \(\mathcal{O}(\epsilon^0)\), while the one-loop amplitudes must be evaluated to \(\mathcal{O}(\epsilon^2)\) \cite{271,283}. The main challenge is the calculation of the two-loop amplitudes. At present, the only amplitude known at two loops is the one for \(V \leftrightarrow q\bar{q}\) \cite{32,284,285}, with \(V\) a massive vector boson, which depends only on one kinematic variable. It has been used to evaluate the NNLO corrections to Drell-Yan production \cite{32,33} and to deeply inelastic scattering (DIS) \cite{63,64}. Two-loop computations for configurations involving two kinematic variables, which are needed in the case of parton-parton scattering, exist only in the special cases of maximal supersymmetry \cite{286}, and of maximal helicity violation \cite{287}. The latter contributes only beyond NNLO. One of the main obstacles for configurations involving two kinematic variables is the analytic computation of the two-loop four-point functions with massless external legs, where significant progress has just been achieved. These consist of planar double-box integrals \cite{288,289}, non-planar double-box integrals \cite{290}, single-box integrals with a bubble insertion on one of the propagators \cite{291} and single-box integrals with a vertex correction \cite{292}. Finally, processes such as \(e^+e^- \rightarrow 3\) jets and \(pp \rightarrow H\ jet\) sport configurations involving three kinematic variables and require the analytic computation of two-loop four-point functions with a massive external leg. Some of the required two-loop four-point functions of this kind have been derived recently \cite{293}. Another obstacle is the color decomposition of two-loop amplitudes, which is not generally known yet. Substantial progress is expected in the near future on all the issues outlined above, which should make the present note soon outdated.

Finally, we mention that in the factorization of collinear singularities for strongly-interacting incoming particles, the evolution of the pdf’s in the jet cross section should be determined to an accuracy matching the one of the parton cross section. For hadroproduction of jets computed at NLO, one needs the NLO AP splitting functions for the evolution of the pdf’s (see Eqs. (8) and (9)). Accordingly, for hadroproduction at NNLO the evolution of the pdf’s should be computed using the NNLO AP splitting functions. Except for the lowest five (four) even-integer moments of the NNLO non-singlet (singlet) AP splitting functions \cite{25,26}, no calculation of the NNLO evolution of the pdf’s exists yet. Some NNLO analyses based on the finite set of known moments have been performed for the DIS structure functions \(x F_2\) and \(F_2\) (see Sects. 2.5 and 2.6 and Ref. \cite{99}). Furthermore, in ref. \cite{70} a quantitative assessment of the importance of the yet unknown higher-order terms has been performed, with the conclusion that they should be numerically significant only for Bjorken \(x\) smaller than \(10^{-2}\).

The computation of the evolution kernels of the pdf’s at NNLO accuracy is a major challenge in QCD. The NLO computation was performed with two different methods, one using the operator product expansion (OPE) in a covariant gauge \cite{18–21,24}, the other using the light-cone axial (LCA) gauge with principal value prescription \cite{22,23}. However, the prescription used in ref. \cite{22,23} has certain shortcomings. Accordingly, the calculation has been repeated in the LCA gauge using a prescription \cite{294,295} which makes it amenable to extensions beyond NLO, whereas the principal value prescription does not seem to be applicable beyond NLO \cite{296}. On the other hand, using the OPE method, there had been a problem with operator mixing in the singlet sector, which has been fixed \cite{297–299} only recently, and the result finally coincides with the one obtained in the LCA gauge in ref. \cite{23}. Thus the result for the AP splitting functions at NLO accuracy is fully under control. Recent proposals for their calculation beyond NLO include extensions of the OPE technique, which have been used to recompute the NNLO corrections to DIS \cite{300}, and a computation based on combining universal gauge-invariant collinear pieces \cite{301}.
5. SUMMATIONS OF PERTURBATION THEORY

5.1 Summations of logarithmically-enhanced contributions

The calculation of hard–scattering cross sections in hadron collisions requires the knowledge of partonic cross sections \( \hat{\sigma} \), as well as that of parton densities (see the factorization formula in Eq. (2)). The partonic cross sections \( \hat{\sigma}(p_1, p_2; Q, \{ Q_1, \ldots \}; \mu^2) \) are usually computed by truncating their perturbative expansion at a fixed order in \( \alpha_S \), as in Eq. (3). However, fixed–order calculations are quantitatively reliable only when all the kinematical scales \( Q, \{ Q_1, \ldots \} \) are of the same order of magnitude. When the hard–scattering process involves two (or several) very different scales, say \( Q \gg Q_1 \), the \( n \)-th term in Eq. (3) can contain double– and single–logarithmic contributions of the type \((\alpha_S L^2)^n\) and \((\alpha_S L)^n\) with \( L = \ln(Q/Q_1) \gg 1 \). These terms spoil the reliability of the fixed–order expansion and have to be summed to all orders, systematically improving on the logarithmic accuracy of the expansion.

Typical examples of such large logarithms are the terms \( L = \ln Q/Q_0 \) related to the evolution of parton densities (and parton fragmentation functions) from a low input scale \( Q_0 \) to the hard–scattering scale \( Q \). These logarithms are produced by collinear radiation from the colliding partons and give single–logarithmic contributions. They never explicitly appear in the calculation of the partonic cross section, because they are systematically (LO, NLO and so forth) resummed in the evolved parton densities \( f_{a/h}(x, Q^2) \) and parton fragmentation functions \( d_{a/H}(x, Q^2) \) by using DGLAP equations (8).

A different sort of large logarithm, \( L = \ln \sqrt{s}/Q \), arises when the centre–of–mass energy \( \sqrt{s} \) of the collision is much larger than the hard scale \( Q \). These small–\( x \) \( (x = Q/\sqrt{s}) \) logarithms are produced by multiple gluon radiation over the wide rapidity range that is available at large energy. For sufficiently inclusive processes in singlet channels these give single–logarithmic (LLx) contributions that can be calculated by using the BFKL equation [302–306]. The subleading (NLLx) contributions have also been calculated recently [67, 307] and turn out to be very large. This is understood to be due to contamination by collinear logarithms of \( Q^2/Q_0^2 \), which must be simultaneously resummed to obtain reliable predictions at small \( x \) [308, 309]. Various resummation procedures have been suggested, and will be briefly discussed in Sects. 5.4 and 7. Unfortunately there are as yet no substantial phenomenological analyses which use these resummations. The resummation of small–\( x \) logarithms will be important for the accurate determination of the behaviour of singlet parton densities \( f_{a/h}(x, Q^2) \) at small values of the parton momentum fraction \( x \), and thus for making reliable predictions of any process that is sensitive to the hard–scattering of low–momentum partons (for example \( b \)–quark production\(^{29} \) and inclusive production of low–\( E_T \) jets and prompt photons at the LHC). The BFKL equation is however also relevant for understanding the structure of final states, for example when there are jets with large rapidity intervals, or diffractive processes with large rapidity gaps. These more general aspects of small–\( x \) physics are discussed in Sect. 7.

Yet another class of large logarithms is associated to the bremsstrahlung spectrum of soft gluons. Since soft gluons can be radiated collinearly, they give rise to double–logarithmic contributions to the partonic cross section, which takes the form

\[
\hat{\sigma} \sim \alpha_S^k \hat{\sigma}^{(LO)} \left\{ 1 + \sum_{n=1}^{\infty} \alpha_S^n \left( C_{2n}^{(n)} L^{2n} + C_{2n-1}^{(n)} L^{2n-1} + C_{2n-2}^{(n)} L^{2n-2} + \ldots \right) \right\} . \tag{33}
\]

Double–logarithmic terms due to soft gluons arise in all the kinematic configurations where the contributions of real and virtual partons are highly unbalanced (see Ref. [218] and references therein).

When partons (particles or jets) with low momentum fraction \( z \) are directly triggered in the final state, the rôle of (real) soft radiation is evidently enhanced. The low–momentum region of the fragmentation spectra of particles and subjets in jet final–states is thus particularly sensitive to the resummation...

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\(^{29}\)See the Bottom Production Chapter of this Report.
of small–z logarithms. The calculations based on the resummation of these logarithms are probably the perturbative predictions that are most sensitive to the coherence properties [218,310] of QCD. Detailed studies of fragmentation processes have been performed in $e^+e^-$ annihilation, DIS and at the Tevatron (see the recent review in Ref. [311]). Although this topic is not included in these proceedings, similar studies at the LHC would certainly be valuable.

In different kinematic configurations, (real) radiation in the final state can instead be strongly inhibited. For instance, this happens in the case of transverse momentum distributions at low transverse momentum, in the case of hard–scattering production near threshold or when the structure of the final state is investigated with high resolution (internal jet structure, shape variables).

Soft–gluon resummation for jet shapes has extensively been studied and applied to hadronic final states produced by $e^+e^-$ annihilation [214,312,313]. Applications to hadron–hadron collisions have just begun to appear [314–316] and have a large, yet uncovered, potential (from $\alpha_S$ determinations to studies of non–perturbative dynamics). Future studies of this topic are certainly warranted.

Threshold logarithms, $L = \ln(1 - x)$, occur when the tagged final state produced by the hard scattering is forced to carry a very large fraction $x$ ($x \to 1$) of the available centre–of–mass energy $\sqrt{s}$. Outstanding examples of hard processes near threshold are DIS at large $x$ (here $x$ is the Bjorken variable), production of DY lepton pairs or di-jets with large total invariant mass $Q = M_W$ or $M_{jj}$ ($x = Q/\sqrt{s}$), production of $W$, $Z$ and Higgs bosons ($x = M_{W,Z,H}/\sqrt{s}$), production of heavy quark–anti-quark pairs ($x = 2m_Q/\sqrt{s}$), inclusive production of single jets and single photons at large transverse energy $E_T$ ($x = 2E_T/\sqrt{s}$).

Transverse–momentum logarithms, $L = \ln Q^2/p_T^2$, occur in the distribution of transverse momentum $p_T$ of systems with high mass $Q$ ($Q \gg p_T$) that are produced with a vanishing $p_T$ in the LO subprocess. Examples of such systems are DY lepton pairs, lepton pairs produced by $W$ and $Z$ decay, heavy quark–anti-quark pairs, photon pairs and Higgs bosons.

Studies of soft–gluon resummation for transverse–momentum distributions at low transverse momentum and hard–scattering production near threshold were pioneered two decades ago [317–327]. The physical bases for a systematic all–order summation of the soft–gluon contributions are dynamics and kinematics factorizations [328,329]. The first factorization follows from gauge invariance and unitarity: in the soft limit multigluon amplitudes fulfil factorization formulae given in terms of universal (process independent) soft contributions. The second factorization regards kinematics and strongly depends on the actual cross section to be evaluated. When phase–space kinematics is factorizable, resummation is analytically feasible in the form of a generalized exponentiation of the universal soft contributions that appear in the factorization formulae of QCD amplitudes.

Typically, phase–space factorization does not occur in the space of the kinematic variables where the cross section is defined. It is thus necessary to introduce a conjugate space to overcome phase space constraints. This is the case for hard–scattering production near threshold, where the relevant kinematical constraint is (one–dimensional) energy conservation, which can be factorized performing a Laplace (or Mellin) transformation (see Sect. 5.2). Analogously, the relevant kinematical constraint for $p_T$–distributions is (two–dimensional) transverse–momentum conservation and it can be factorized by performing a Fourier transformation (see Sect. 5.3). In the conjugate space, the logarithms $L$ of the relevant ratio of momentum scales are replaced by logarithms $\hat{L}$ of the conjugate variable.

The resummed cross section is thus typically of the form
\[
\hat{\sigma}_{\text{res.}} = \alpha_S^\kappa \int_{\text{inv.}} \hat{\sigma}^{(LO)} \cdot C \cdot S,
\]  
where the integral $\int_{\text{inv.}}$ denotes the inverse transformation from the conjugate space where resummation is actually carried out. The factor $C$ contains all constant contributions in the limit $\hat{L} \to \infty$. The singular
dependence on $\bar{L}$ is entirely exponentiated in the effective form factor $S$:

$$S = \exp \left\{ \bar{L} \ g_1(\alpha_s(\mu)\bar{L}) + g_2(\alpha_s(\mu)\bar{L}; \mu^2) + \alpha_s(\mu) \ g_3(\alpha_s(\mu)\bar{L}; \mu^2) + \ldots \right\}. \quad (35)$$

The structure of the exponent is formally analogous to that of the fixed–order expansion of the partonic cross sections (see Eq. (3)). The function $\bar{L} g_1$ resums all the leading logarithmic (LL) contributions $\alpha_s^n L^{n+1}$, while $g_2$ contains the next–to–leading logarithmic (NLL) terms $\alpha_s^n L^n$ and so forth. Note that the NLL terms are formally suppressed by a power of $\alpha_s$ with respect to the LL ones, and the same is true for the successive classes of logarithmic terms. Thus, this logarithmic expansion is as systematic as the fixed–order expansion in Eq. (3).

In general, a resummed expression such as Eq. (34) must be properly combined with the best available fixed–order result. Using a shorthand notation, this is achieved by writing the partonic cross section $\hat{\sigma}$ as

$$\hat{\sigma} = \hat{\sigma}_{\text{res.}} + \hat{\sigma}_{\text{rem.}}. \quad (36)$$

The term $\hat{\sigma}_{\text{res.}}$ embodies the all–order resummation, while the remainder $\hat{\sigma}_{\text{rem.}}$ contains no large logarithmic contributions. The latter has the form

$$\hat{\sigma}_{\text{rem.}} = \hat{\sigma}^{(\text{f.o.})} - [\hat{\sigma}_{\text{res.}}]^{(\text{f.o.})}, \quad (37)$$

and it is obtained from $\hat{\sigma}^{(\text{f.o.})}$, the truncation of the perturbative expansion for $\hat{\sigma}$ at a given fixed order (LO, NLO, . . .), by subtracting the corresponding truncation $[\hat{\sigma}_{\text{res.}}]^{(\text{f.o.})}$ of the resummed part. Thus, the expression on the right–hand side of Eq. (36) includes soft–gluon logarithms to all orders and it is matched to the exact (with no logarithmic approximation) fixed–order calculation. It represents an improved perturbative calculation that is everywhere as good as the fixed–order result, and much better in the kinematics regions where the soft–gluon logarithms become large ($\alpha_s L \sim 1$). Eventually, when $\alpha_s L \gtrsim 1$, the resummed perturbative contributions are of the same size as the non–perturbative contributions and the effect of the latter has to be implemented in the resummed calculation.

Using a matched NLL+NLO calculation as described above, we can consistently introduce a precise definition (say $\overline{\text{MS}}$) of $\alpha_s(\mu)$ and investigate the theoretical accuracy of the calculation by studying its dependence on the renormalization/factorization scale $\mu$.

Resummed calculations for hadron collisions near threshold and for $p_T$–distributions are discussed in Sects. 5.2 and 5.3, respectively. Some overviews can also be found in Ref. [196]. We refer the reader to Sects. 3.3 and 3.4 for comparisons of resummed calculations with parton shower event generators.

5.2 Threshold resummations

Large logarithms arise in any inclusive cross section for the production of an object with a large mass $Q$, whenever the partonic energy $\sqrt{s}$ available for the process is close to $Q$, the production threshold. The physical mechanism responsible for these logarithms is simple. Close to threshold the phase space for the emission of gluon radiation in the final state is kinematically restricted; soft real radiation is, however, responsible for the cancellation of infrared divergences associated with virtual gluon exchange; whenever radiation is inhibited, the cancellation is partially spoiled: finite but large contributions are left over, in the form of logarithms of the ratio of the two relevant energy scales, $\ln[(\hat{s} - Q^2)/\hat{s}]$. Close to partonic threshold these logarithms become large and must be resummed. Processes for which this resummation

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30This has to be contrasted with the tower expansion sketched on the right–hand side of Eq. (33). Within the framework of the tower expansion that sums the double-logarithmic terms $\alpha_s L^n$, then the terms $\alpha_s^n L^{2n-1} \sim \alpha_s L(\alpha_s L^2)^{n-1}$ and so forth, the ratio of two successive towers is, roughly speaking, of the order of $\alpha_s L$. More precisely, the tower expansion allows us to formally extend the applicability of perturbative QCD to the region $\alpha_s L^2 \lesssim 1$, and the exponentiation in Eq. (35) extends it to the wider region $\alpha_s L \lesssim 1$.

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is relevant are ubiquitous, as noted in the previous subsection. Techniques to perform threshold resummations have been developed and progressively extended for well over a decade; references in which these techniques are explained in some detail include [330–337]; here we will briefly review the basic theoretical issues, and sketch the status of phenomenological applications of relevance to the LHC.

As described in the introduction to the present Section, the resummation of threshold logarithms is performed in Mellin space. To illustrate the structure of a typical resummation of threshold logarithms, let us concentrate on the simplest and best known example: the DY cross section. In this case the resummed formula for the Mellin transform of the partonic cross section, in the DIS factorization scheme, takes the form [330, 331]

\[ \hat{\sigma}_{\text{res.}}(N, Q^2) = C(\alpha_S(Q^2)) \exp \left[ E(N, Q^2) \right], \tag{38} \]

where the function \( C \) collects terms independent of the Mellin variable \( N \), while the exponent can be written as

\[ E(N, Q^2) = -2 \int_0^1 dz \frac{z^N - 1}{1 - z} \left[ B(\alpha_S((1 - z)Q^2)) + \int_{(1-z)^2Q^2}^{(1-z)Q^2} \frac{dq^2}{q^2} A(\alpha_S(q^2)) \right]. \tag{39} \]

Equations (38) and (39) resum, in principle, all logarithms of \( N \) to all orders in perturbation theory, in the sense that all such logarithms exponentiate and are calculable from the functions \( A \) and \( B \), for which Feynman rules can be derived. In practice, the functions \( A \) and \( B \) are known only to two loops, so that the resummation can explicitly be performed only for leading and next-to-leading logarithms in the exponent. Performing the integrals in \( E(N, Q^2) \), after expansion of the running couplings in terms of \( \alpha_S(Q^2) \) to the desired accuracy, yields in general an expression of the form

\[ E(N, Q^2) = \ln N \, g_1(\alpha_S \ln N) + g_2(\alpha_S \ln N) + \sum_{k=1}^{\infty} \alpha_S^k g_{k+2}(\alpha_S \ln N) , \tag{40} \]

where the functions \( g_1 \) and \( g_2 \) are known in terms of the coefficients \( A^{(1)} \), \( A^{(2)} \) and \( B^{(1)} \) of the perturbative expansion of the functions \( A \) and \( B \), together with the one- and two-loop coefficients of the QCD \( \beta \) function. The (unknown) function \( g_3 \), giving the NNLL logarithms, would require the determination of \( A^{(3)} \), as well as \( B^{(2)} \) and the three-loop \( \beta \) function.

Several comments are necessary in order to introduce the practical applications of resummed formulas such as Eq. (39).

- At the present level of accuracy (NLL) the dependence on the renormalization scale and on the factorization scheme is under control. A change in renormalization scale shifts the function \( g_2 \) by an amount proportional to the derivative of the function \( g_1 \). A change in factorization scheme changes both \( g_1 \) and \( g_2 \), because it affects the way in which the DIS process is subtracted from DY to construct a finite cross section, however the change is well understood and both functions can be translated from one scheme to another [172, 335].

- To understand the effects of resummation, one should keep in mind that it is performed at the level of the partonic cross section. One consequence of this fact is that resummation generically enhances the cross section, although one might expect a Sudakov suppression, since the probability of having a nearly radiation-less hard scattering is exponentially suppressed. This is easily understood in the DIS scheme: there one computes the (factorized) partonic DY cross section by taking the ratio of the DY process to the square of the DIS process, since there are two partons in the DY initial state. In this ratio, the denominator is Sudakov suppressed twice as much as the numerator, resulting in an overall Sudakov enhancement.

- The fact that the resummed partonic cross section must be folded with parton distributions to extract a physical prediction also means that the effects of resummation are felt quite far away from
the hadronic threshold. In fact, given a hadronic centre–of–mass energy $S$, the typical partonic energy available for the production process will be $\langle \delta \rangle = \langle x_1 x_2 \rangle > S$, where $x_1$ and $x_2$ are the momentum fractions of the scattered partons. Clearly $\delta$ becomes close to threshold long before $S$ does.

- The resummed partonic cross section by construction contains a subset of the finite order perturbative calculations available for the process at hand. One should then work with a “matched” cross section, as described in the previous subsection (see Eqs. (36) and (37)).
- The alert reader will have noticed that Eq. (39), although well–defined order by order if the running couplings depending on variable arguments are re-expanded in terms of a fixed large scale, is actually ill–defined in the leading–logarithm (of $Q^2$) approximation, because the integration contour runs over the Landau pole. This is a general feature of most known resummations of perturbation theory: in fact, perturbation theory is pointing us to its own limitations, and to the need to include information concerning the non–perturbative structure of QCD [75]. This fact has two consequences. On the one hand, it is possible to exploit partial resummations such as Eq. (39) to estimate the size of the first relevant non–perturbative corrections: in the case of the DY process, two independent approaches [76, 338] lead to the conclusion that the first power correction to Eq. (38) is $O((N/Q)^2)$. On the other hand, experience has shown that the necessary inversion of the Mellin transform back to momentum space can generate unjustified (and stronger) power corrections that are not present in the original resummed expression. Methods to circumvent this problem have been developed [334], so that Eq. (38) can be used confidently, with a definite understanding of the size of expected corrections.
- In the general case of colored final states, a comparatively simple expression for the resummed cross section, such as Eq. (39), is not available to all logarithmic orders, because the corresponding evolution equations are in matrix form, and their solution involves a scale–dependent mixing of color tensors. To NLL accuracy, however, a simple exponentiation can still be achieved, by diagonalizing a matrix of anomalous dimensions in the space of available color configurations [336,337]. This results in a matrix of exponentials, each similar to Eq. (39), with two new color–dependent functions of the running coupling. These new functions also carry the necessary dependence on the angles between incoming and outgoing colored partons.
- It should be emphasized that further improvements are possible, and in some cases have already been achieved. In the case of the DY process, the terms independent of $N$ contained in the factor $C$ in Eq. (38) can also be resummed: in the DIS scheme, they contain the absolute value of the ratio of the time-like to the space-like Sudakov form factor, which is known to exponentiate [339]. Methods to resum classes of terms of the form $\ln N/N$ have recently been suggested [340]. Finally, a technique to resum simultaneously threshold logarithms and recoil enhancements in single particle inclusive cross sections has been introduced [341].

Turning to practical applications, we observe that resummations of threshold logarithms have been performed to NLL accuracy for most of the processes of interest at the LHC, ranging from DIS and DY [172, 330, 331, 335, 342, 343] to Higgs boson [340] production, to include more recently studies of processes with hard colored particles in the final states, such as heavy quark [336,337,344], prompt photon [345–348], $W$ boson [349] and di-jet [350] production; applications of the formalism to quarkonium production have been proposed [351]. Detailed phenomenological calculations, however, are presently available only for a subset of these processes.

It is important to note that at the LHC threshold resummation can be important for two reasons. On one side, it can directly be applied to LHC processes through the corresponding partonic cross sections. On the other side, it can be applied to the lower–energy processes that are typically used to determine the parton densities, and thus it can indirectly affect LHC predictions through the use of (evolved) parton distributions reevaluated in this manner.

We shall illustrate the phenomenological effects of the application of these techniques with few ex-
Fig. 31: Scale dependence of $\frac{d\sigma}{dE_T}$ for single prompt–photon production in $pN$ collisions. The solid lines represent the NLO result for different choices of $\mu = \mu_R = \mu_F$ ($\mu = E_T/2$ and $2E_T$), normalized to the result for $\mu = E_T$. The dashed lines represent the NLO+NLL results for different choices of $\mu$, normalized to NLO result for $\mu = E_T$. See Ref. [347] for details.

amplifies, which will serve to point out another relevant feature of NLO+NLL calculations: their increased stability with respect to scale variations.

As discussed in Sect. 2., present data and NLO calculations do not constrain very well the determination of the parton distributions at large values of the parton momentum fraction $x$. This is particularly true for the gluon density $f_g(x, Q^2)$ at $x \gtrsim 10^{-1}$ and $Q \sim 5 - 10$ GeV. The uncertainty on $f_g$ in this kinematic region propagates (although with a reduced overall size) to smaller values of $x$ and larger values of $Q^2$ in LHC processes. Threshold resummation can help to extract parton distributions at large $x$ with more confidence than is at present in NLO analyses. Consider, for instance, the production of prompt photons with high transverse energy $E_T$ at fixed–target experiments. This process is very sensitive to the behaviour of the gluon density at large $x$ ($x \sim x_T = 2E_T/\sqrt{s}$). The corresponding theoretical calculations at fixed perturbative order, however, are not very accurate, as can be argued by studying their dependence on the factorization/renormalization scale $\mu$. When NLL resummation is applied [347], the scale dependence of the calculation is highly reduced and the resummed NLL contributions lead to large corrections at high $x_T$ (and smaller corrections at lower $x_T$). The scale dependence of the theoretical cross section in $pN$ collisions is shown in Fig. 31 as a function of $E_{beam}$, the energy of the proton beam. Fixing $\mu_R = \mu_F = \mu$ and varying $\mu$ in the range $E_T/2 < \mu < 2E_T$ with $E_T = 10$ GeV and $E_{beam} = 530$ GeV (this corresponds to the largest value of $x_T$ that is reachable by the E706 kinematics [352]), the cross section varies by a factor of $\sim 6$ at LO (the result of the LO calculation is not shown in the plot), by a factor of $\sim 4$ at NLO and by a factor of $\sim 1.3$ after NLL resummation. The central value (i.e. with $\mu = E_T$) of the NLO cross section increases by a factor of $\sim 2.5$ after NLL resummation. As expected, the size of these effects is reduced by decreasing $x_T$ (e.g. by increasing $\sqrt{s}$ at fixed $E_T$). This (extreme) example clearly illustrates how NLO+NLL resummed calculations can improve the present NLO determinations of parton distributions. The method of Ref. [341] can also be applied to investigate the relevance of recoil effects in prompt-photon production.

NLL resummations of threshold logarithms are now available for all the most important processes (DIS, DY, and prompt–photon production) used to determine the parton densities via global fits. It is thus possible to consistently [346, 353] take into account all threshold effects affecting the different hadronic cross sections. Preliminary studies [353–355] suggest that NLO+NLL fits are not likely to make drastic differences in the parton densities that are strongly constrained by DIS data, at least so long as the region
of small $Q$ ($Q \sim \text{few GeV}$) is avoided at very large $x$. At the same time, they suggest that resummed fits can make some difference where the pdf’s are not so well known (gluon density at large $x$ and quark densities at larger values of $x$). In particular, NLO+NLL fits, if implemented, are likely to reduce scale dependence, and thus further improve our confidence in the theoretical predictions for LHC cross sections.

As for direct effects of NLL threshold resummation at the LHC, we briefly discuss top pair production, which is currently the best studied process in LHC kinematics [337]. One could argue that threshold resummation effects in this case should not be expected, since at the LHC we have $x = 2m_t / \sqrt{s} \sim 0.03$. This would however be incorrect since, as explained above, partonic threshold can be, on average, quite far from hadronic threshold. In the case of top production at the LHC, the dominant partonic subprocess is gluon fusion. The gluon density is steeply falling at large $x$ and quite large at small $x$, so that the average momentum fraction of gluons entering the partonic hard subprocess is relatively small and $\hat{s} \ll s$. As a consequence, the effect of NLL resummation is still visible at the LHC: the NLO+NLL resummed cross section is larger than the NLO estimate by about $5\%$. Moreover, NLL resummation reduces the scale dependence of the cross section by approximately a factor of two (from about $10\%$ to about $5\%$). This can be relevant, because the uncertainty due to the present knowledge of the parton densities is estimated to be twice as large. We refer the reader to the Top Physics Chapter of this Report for full details.

Other topical LHC processes are Higgs production, DY production of $W$, $Z$ and lepton pairs, as well as production of high-$E_T$ jets. Since the Higgs mass $M_H$ is expected to be of the same order as the top-quark mass, Higgs production will be dominated by gluon fusion. Thus, the effects of threshold resummation on this process should be at least as important as for top-pair production. The results of Ref. [340], based on the expansion at NNLO of threshold resummation, support this conclusion. Complete quantitative studies to NLL accuracy are not yet available and would be valuable. The production of $W$ and $Z$ at the LHC is less close to threshold than top production. Moreover, its dominant partonic subprocess is $q\bar{q}$ annihilation. The large--$x$ behaviour of the quark densities is less steep than that of the gluon density, and soft--gluon radiation from initial--state quarks is depleted by the colour charge factor $C_F / C_A \sim 1/2$ with respect to radiation from gluons. Thus, the effects of threshold resummation on $W$, $Z$ production should be small. Their size could however increase in the case of production of high-mass (say, $Q \gtrsim 1$ TeV) DY lepton pairs. The inclusive production of high-$E_T$ jets and di-jets with large invariant mass at the Tevatron and at the LHC can be sensitive to threshold logarithmic contributions. Nonetheless, phenomenological analyses to NLL accuracy are not available for these processes. An important conceptual reason for that is the fact that the cone algorithms used so far to experimentally define jets are not infrared and collinear safe [315,356]. Although their unsafety may show up only at some high order in perturbation theory, it prevents all--order summations. The future use [357] of safe algorithms, such as the $k_L$-algorithm [8, 9] and the improved cone algorithm studied at the Workshop on Physics at the Tevatron in Run II, will overcome this problem. For the definition of different jet algorithms, we refer the reader to Ref. [357].

### 5.3 Resummation of transverse momentum distributions

The description of vector and scalar boson production properties, in particular their transverse momentum ($p_T$) distribution, is likely to be one of the most investigated topics at the LHC, especially in the context of Higgs searches. To obtain a reliable theoretical prediction for the $p_T$ distribution, the corrections due to soft gluon radiation have to be taken into account. At small transverse momentum the $p_T$ distribution is dominated by large logarithms $\ln(Q^2/p_T^2)$, which are directly related to the emission of gluons by the incoming partons. Therefore, at sufficiently small $p_T$, fixed--order perturbation theory breaks down and the logarithms must be resummed. The origin of the large logarithms is visible already at leading--order: in fact, the contribution from real emission diagrams for $q\bar{q} \rightarrow Vg$ contains a term of

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the form $\alpha_S C_F \ln \left( Q^2/p_T^2 \right) / (\pi p_T^2)$. When more gluons are emitted, the logarithmic divergence becomes stronger. It can be shown that in the approximation of soft and collinear gluons with strongly ordered transverse momenta $k_T$, i.e.

$$k_{T,1}^2 \ll k_{T,2}^2 \ll \ldots \ll k_{T,n}^2 \ll p_T^2 \ll Q^2$$

the dominant contributions to the $q\bar{q} \rightarrow VX$ cross section can be resummed, giving a so-called Sudakov factor [319], of the form

$$\frac{1}{\sigma_0} \frac{d\sigma}{dp_T^2} = \frac{\alpha_S A}{2\pi p_T^2} \ln \left( \frac{Q^2}{p_T^2} \right) \exp \left( -\frac{\alpha_S A}{4\pi} \ln^2 \left( \frac{Q^2}{p_T^2} \right) \right),$$

where $A = 2C_F$, and $\sigma_0$ is the total LO $q\bar{q} \rightarrow V$ cross section. This approximation is commonly known as the Double Leading Logarithm Approximation (DLLA).

The resummation in Eq. (42) gives a finite but unphysically suppressed result in the small $p_T$ limit. This suppression is caused by the vanishing of strongly–ordered phase space, in which overall transverse momentum conservation is ignored. The result in (42) corresponds to a configuration in which a single soft gluon balances the vector boson transverse momentum, giving the overall $\ln(Q^2/p_T^2)/p_T^2$ term, while all other gluons have transverse momenta $\ll p_T$. This is not the dominant configuration in the small $p_T$ limit. Equally important are non–strongly–ordered contributions corresponding to the emission of soft ($\sim p_T$) gluons whose transverse momenta add vectorially to give the overall $p_T$ of the vector boson. Although such contributions are formally sub-leading order–by–order, they do dominate the cross section in the region where the Sudakov form factor suppresses the (formally) leading DLLA contributions. The non–leading ‘kinematical’ logarithms are correctly taken into account by imposing transverse momentum conservation (rather than strong ordering), and this is most easily achieved by means of a Fourier transform to impact parameter ($b$–)space.

We next discuss analytic methods for resumming large logarithms in $b$–space and $p_T$–space. As already mentioned, comparisons of resummed calculations with the predictions coming from parton shower Monte Carlo approaches are presented in Sects. 3.3 and 3.4.

5.31 Analytic methods: $b$–space

In the $b$–space method [317] one imposes transverse momentum conservation by Fourier transforming the $p_T$ distribution to impact parameter space and using the identity

$$\delta^{(2)} \left( \sum_{i=1}^N k_{T_i} - p_T \right) = \frac{1}{4\pi^2} \int d^2 b e^{-ib\cdot p_T} \prod_{i=1}^N e^{ib\cdot k_{T_i}}.$$  (43)

This allows for the derivation of a general expression resumming all terms of the perturbation series which are at least as singular as $1/p_T^2$ when $p_T \rightarrow 0$ [192, 358, 359]. The resummed expression is of the form

$$\frac{d\sigma(AB \rightarrow V (\rightarrow l\bar{l})\,X)}{dp_T^2} dQ^2 dy d\cos \theta d\phi = \frac{Q^2}{256\pi N_c s} \left( Q^2 - M_V^2 \right)^2 + M_V^2 \Gamma_V^2 \times [Y_r(p_T^2, Q^2, y, \theta) + Y_f(p_T^2, Q^2, y, \theta, \phi)],$$  (44)

where $M_V$ and $\Gamma_V$ are the mass and the width of the vector boson, and $\theta$ and $\phi$ stand for the lepton polar and azimuthal angles in the Collins–Soper frame [192, 358, 359]. $Y_r$ denotes the resummed part of the cross section, while $Y_f$ is the remainder (that is, the fixed–order expression minus terms which are already taken into account in $Y_r$, as in Eq. (36)). The exact expression for $Y_f$ can be found in [360],
\begin{align*}
Y_r(p_T^2, Q^2, y, \theta) &= \Theta(Q^2 - p_T^2) \frac{1}{2\pi} \int_0^\infty db \, J_0(p_T b) \sum_{a,b} F_{ab}^{NP}(Q, b, x_A, x_B) \\
&\times H_{ab}(\theta) f'_a/(x_A, \frac{b_0}{b_a}) f'_b/(x_B, \frac{b_0}{b_b}) \exp[S(b, Q)].
\end{align*}

Here \(f'\) denotes a modified parton distribution, \(H_{ab}(\theta)\) includes coupling factors and the angular dependence of the lowest order cross section \([360]\), and \(b_0\) and \(F_{ab}^{NP}\) are discussed below. The Sudakov factor has the form

\begin{align}
S(b, Q^2) &= -\int_{b_0^2}^{Q^2} \frac{d\mu^2}{\mu^2} \left[ \ln \left( \frac{Q^2}{\mu^2} \right) A(\alpha_S(\mu^2)) + B(\alpha_S(\mu^2)) \right], \\
A(\alpha_S) &= \sum_{i=1}^\infty \left( \frac{\alpha_S}{2\pi} \right)^i A^{(i)}, \quad B(\alpha_S) = \sum_{i=1}^\infty \left( \frac{\alpha_S}{2\pi} \right)^i B^{(i)},
\end{align}

with \(b_0 = 2\exp(-\gamma_E)\). The form in Eq. (46) is equally valid for processes initiated by \(q\bar{q}\)-annihilation (e.g. production of DY lepton pairs, \(W\) and \(Z\)) and by \(gg\)-fusion (e.g. Higgs production). The coefficients \(A^{(1)}, A^{(2)}\) and \(B^{(1)}\) in each series (47) were computed in Ref. [361] for \(q\bar{q}\)-annihilation and in Ref. [362] for \(gg\)-fusion. These coefficients\(^{33}\) can also be obtained [363] from the exact fixed–order perturbative calculation in the high \(p_T\) region by comparing the logarithmic terms therein with the corresponding logarithms generated by the first three terms of the expansion of \(\exp(S(b, Q^2))\) in Eq. (45).

Although the \(b\)-space method succeeds in recovering a finite, positive result in the \(p_T \to 0\) limit, there are drawbacks associated with the need to work in impact parameter space. The first is the difficulty of matching the resummed and fixed–order predictions. Since the resummation is performed in \(b\)-space one loses control over which logarithmic terms (in \(p_T\)-space) are taken into account. Therefore there is no unambiguous prescription for matching; existing prescriptions require switching from resummed to fixed–order calculation at some value of \(p_T\). Secondly, since the integration in (45) extends from 0 to \(\infty\), it is impossible to make predictions for any \(p_T\) without having a prescription for how to deal with the non–perturbative regime of large \(b\). One prescription is to artificially prevent \(b\) from reaching large values by replacing it with a new variable \(b_\ast\) and by parametrising the non–perturbative large–\(b\) region in terms of the form factor \(F_{ab}^{NP}\). The ‘freezing’ of \(b\) at \(b_\ast\) is achieved by

\[
b_\ast = \frac{b}{1 + (b/b_{\ast\text{im}})^2}, \quad b_\ast < b_{\ast\text{im}},
\]

with the parameter \(b_{\ast\text{im}} \sim 1/\Lambda_{\text{QCD}}\) separating perturbative and non–perturbative physics. The detailed form of the non–perturbative function \(F_{ab}^{NP}\) remains a matter of theoretical dispute (for a review see [360]), although it is assumed to have the general form [192, 358, 359]

\[F_{ab}^{NP}(Q, b, x_A, x_B) = \exp \left\{ - \left[ h_Q(b) \ln \left( \frac{Q}{2Q_0} \right) + h_a(b, x_A) + h_b(b, x_B) \right] \right\}.\]

In a very simple model in which the non–perturbative contribution arises from a Gaussian ‘intrinsic’ \(k_T\) distribution, one would have \(F \sim \exp(-\kappa b^2)\). The data are not inconsistent with such a form, but suggest that the parameter \(\kappa\) may have some dependence on \(Q\) and \(x\).

Phenomenological studies and numerical calculations based on the \(b\)-space formalism are presented in Refs. [110, 194, 360, 364, 365] (for DY lepton pair, \(W\) and \(Z\) production) and in Refs. [195, 366–368] (for Higgs production).

\(^{33}\)In Ref. [363] the coefficient \(B^{(2)}\) for \(q\bar{q}\)-annihilation was also computed. The coefficient \(B^{(2)}\) for \(gg\)-fusion is not yet known.
5.32 Analytic methods: $p_T$–space

The difficulties mentioned above could in principle be overcome if one had a method of performing the calculations directly in transverse momentum space. Given an insight into which logarithmic terms are resummed, it should be fairly straightforward to perform matching with the fixed–order result. Moreover, the non–perturbative input would be required in (and would affect) only the small $p_T$ region.

Three techniques have been proposed for carrying out resummation in $p_T$–space [190, 191, 369]. The main difference lies in the selection of subsets of logarithmic terms which each method resums; for a detailed discussion the reader is referred to [370]. The starting point for all techniques is the general expression in impact parameter space for the vector boson transverse momentum distribution in the DY process [192, 358, 359], at the quark level. To illustrate the results, we consider the approach of [369], section. The result is too lengthy to reproduce here, but can be found in [369, 371].

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Strong ordering is equivalent to replacing the δ function by $\delta^{(2)}(k_{T1} - p_T) \times \theta(k_{T1}^2 - k_{T2}^2) + (1 \leftrightarrow 2)$. This gives only the leading $L^3$ term on the right–hand side. The $\zeta(3)$ term represents the first appearance of the (kinematic) $\tau_3$ coefficient of Eq. (48).

In principle the formalism presented above allows for an inclusion of any number of such sub-leading kinematic logarithms. In practice, we use Eq. (48) with a finite number of terms by introducing $N_{\text{max}}$ as the upper limit of the first summation. $N_{\text{max}}$ corresponds to the number of towers of logarithms which are fully resummed. Figure 32 shows that for small values of $p_T$ the approximation of the $b$–space result improves with increasing $N_{\text{max}}$. Therefore by retaining sufficiently many terms one can obtain a good approximation (i.e. adequate for phenomenological purposes) to the $b$–space result by summing logarithms directly in $p_T$ space.\footnote{Notice however that, due to the lack of knowledge of $A^{(3)}$, $B^{(3)}$, etc., it is only possible to obtain the complete result for the first four ‘towers’ of logarithms; subsequent towers can be included only in the approximations leading to Eq. (48), see [369].}

The technique developed so far can be extended to include sub-leading $A$ and $B$ coefficients, the running coupling and parton distributions, thus yielding a ‘realistic’ expression for the hadron–level cross section. The result is too lengthy to reproduce here, but can be found in [369, 371].
Fig. 32: The $b$–space result (parton level, fixed coupling, only $A^{(1)}$) compared to the expression (48), calculated for various values of $N_{\text{max}}$. Here $\eta = p_T^2/Q^2$ and $N_{\text{max}}$ is the upper limit of the first summation in Eq. 48.

Although the $p_T$–space method provides a simple matching prescription, the form of the non–perturbative function in this approach (as well as in $b$–space approach) remains an open theoretical issue. In particular, the current lack of understanding of the $x$ and $Q^2$ dependence of the non–perturbative contribution is a limiting factor in predicting the $p_T \rightarrow 0$ behaviour of the distribution at the LHC. However, it seems that the dependence on the amount of non–perturbative smearing weakens with increasing $Q$ (see Ref. [193] and the discussion in Sect. 3.4). It has also been shown [370] that the quality of the approximation to the $b$–space result achieved by various resummation approaches in $p_T$–space changes significantly only for small values of $p_T^2/Q^2$. This in turn would suggest that the differences between these approaches may become relevant for obtaining an accurate theoretical description of very heavy boson (e.g. Higgs) production in the small $p_T$ regime.

5.4 Small–$x$ resummations

If we are to make accurate predictions for LHC ‘background’ processes with partonic centre–of–mass energy below 1 TeV, we need to extrapolate cross sections measured at HERA and the Tevatron forward by between one and three orders of magnitude in $Q^2$, and back by between one and three orders of magnitude in $x$. Since away from thresholds these cross sections are generally rather smooth functions of $x$ and $Q^2$ one might try to do this by simply extrapolating parametric fits [372, 373]. However the uncertainties in such extrapolations are very difficult to quantify. Adding an assumption that the dominant singularities are Regge poles is not very helpful, since even with current data more than one ‘Pomeron’ singularity is needed for a satisfactory fit [374, 375]. Moreover in this kind of approach it is not possible to relate all the various cross sections of interest, or for example calculate heavy quark production, or jet cross sections: each must be fitted individually. Clearly we need more dynamics. Strong interaction dynamics at high energies inevitably means perturbative QCD, and it is the current understanding of perturbative QCD at small $x$ that we summarise here.

Provided there is a hard scale in the process, strong interaction processes may generally be factorized into a hard partonic cross section, computable in perturbative QCD, and parton densities which must be determined empirically. At large scales $Q^2$ and not too small but fixed $x$ the QCD evolution equations [14, 16, 216, 376, 377] provide a reliable framework for the extrapolation of these parton densities from some initial scale $Q_0^2$ to higher values of $Q^2$. The complete AP splitting functions have been computed in perturbation theory at order $\alpha_S$ (LO) and $\alpha_S^2$ (NLO). For the first few moments the AP

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splitting functions at order $\alpha_S^3$ (NNLO) are also known [25, 26]. Once we have the parton distributions, it is straightforward to compute hadronic cross sections at LO or NLO: potentially large contributions of the form $(\alpha_S \ln Q^2/Q_0^2)^n$ (LLQ), $\alpha_S(\alpha_S \ln Q^2/Q_0^2)^n$ (NLLQ), . . . , have been resummed by solving the evolution equations, so all that is necessary is the convolution of the evolved parton densities with the hard partonic cross section.

If we start with initial parton distributions that rise less steeply than a power in $1/x$ as $x$ decreases, then fixed order evolution to higher $Q^2$ inevitably leads to distributions that become progressively steeper in $1/x$ as $Q^2$ increases [378], in agreement with the rise in the $F_2$ data from HERA. More significantly the specific form and steepness of the rise is precisely [379–381] as predicted. This is a major triumph for perturbative QCD, since it can be interpreted as direct evidence for asymptotic freedom [382]: the coefficient $\beta_0$ which determines the slope of the rise is the first coefficient of the QCD $\beta$–function. This has now been confirmed many times by successful NLO fits (see [383, 384] and Sect. 2.) to increasingly precise HERA $F_2$ datasets. From these fits a gluon distribution may be extracted, and predictions made for $F_2^g$, di-jet production, and $F_L$, all of which have now been confirmed by direct measurements [385, 386]. Clearly fixed order perturbative QCD works well at HERA: none of these predictions is trivial, and all are successful. Extrapolation to the LHC region, and the calculation of relevant NLO cross sections, can then be performed in the same way as at large $x$, with the added bonus that besides extrapolating up in $Q^2$ one can simultaneously extrapolate backwards in $x$. The errors in such predictions are the usual mix of experimental and parametrization uncertainties (see the discussion in Sects. 2.3, 2.4 and in [387]), and theoretical errors predominantly due to missing sub-leading corrections, which may be estimated by partial calculations of NNLO terms [70, 388] (see also Sect. 2.5).

However to obtain truly reliable predictions for processes at the LHC it is not sufficient to confirm NLO QCD within errors at HERA: we must also be convinced that new sources of theoretical uncertainty do not arise as the kinematic region is extended. In particular, as one goes to smaller values of $x$ it is not clear that retaining only the first few terms in the expansion (9) of the splitting functions in powers of $\alpha_S$ will be and remain a good approximation: as soon as $\xi = \ln 1/x$ is sufficiently large that $\alpha_S \xi \sim 1$, terms of order $\alpha_S(\alpha_S \xi)^n$ (LLX), $\alpha_S^2 (\alpha_S \xi)^n$ (NLLX), . . . must also be considered in order to achieve a result which is reliable up to terms of order $\alpha_S^3$. In fact $\alpha_S \xi \gtrsim 1$ throughout much of the kinematic region available at both HERA and the LHC, so one might naively expect these effects to be significant when extrapolating from one to the other. The fact that at HERA they seem to be small empirically is a mystery which must be solved if reliable predictions are to be made for the LHC.

Using the BFKL kernel $K(Q^2,k^2)$ [302–306] (see also Sect. 7) calculated to $O(\alpha_S)$ (LO) it is possible [389–391] to deduce the coefficients of the LLX singularities of the AP splitting function to all orders in perturbation theory. Summing these up, the splitting function (and thus the structure function) is predicted to grow as $x^{-\lambda}$ as $x \to 0$, where (at LLX) $\lambda = \lambda_0 \equiv (12 \alpha_S \ln 2)/\pi$. This procedure may be extended to NLLX singularities, using calculations of the coefficient function and gluon normalization [66, 392] and of the NLLX kernel [67, 271, 307, 393–403], to give all the NLLX terms in the splitting function [404–410]. It was known some time ago that reconciling these summed logarithms with the HERA data was going to be difficult [379–381, 411–413], simply because there is no evidence in the data for a rise with a fixed power $\lambda_0$. Once all the NLLX corrections were known it became clearer why: the expansion in summed anomalous dimensions at LLX,NLLX,... is unstable [69, 414, 415], the ratio of NLLX to LLX contributions growing without bound as $x \to 0$. It follows that the previous theoretical estimates [404–413] of the size of the effects of the small $x$ logarithms based on the fixed order BFKL equation, either at LO or NLO, were all hopelessly unreliable. Indeed any calculation which resums LO and NLO logs of $Q^2$, but sums up only LO and NLO logarithms of $x$ is seen to be insufficient: some sort of all order resummation of the small $x$ logarithms is necessary. Clearly there are many ways in which such a resummation might be attempted: what are needed are guiding principles to keep it under control.

There are two distinct strands to this problem. The first is the stability of the BFKL equation itself (see the discussion in Sect. 7.3). Various proposals have been put forward: for example a partic-
ular choice of the renormalization scale [416], or a different identification of the large logs which are resummed [417,418]. However the root of the problem [308] is that the perturbative contributions to the kernel $K(Q^2, k^2)$ contain unresummed logarithms of the form $\alpha_S(\alpha_S t)^n$ (LLQ), $\alpha_S^2(\alpha_S^2 t)^n$ (NLLQ),..., where $t \equiv \ln Q^2/k^2$, which destabilise the fixed order expansion both in the ultraviolet region $Q^2 \gg k^2$ and in the infrared $Q^2 \ll k^2$. These logarithmic contributions turn out to be so large that the fixed order expansion is useless, even in the small $x$ region, unless $\alpha_S$ is unrealistically small. In order to obtain a realistic approximation to the kernel, the large logarithms of $Q^2$ must be resummed to all orders in perturbation theory. Fortunately the ultraviolet logarithms not associated with the running of the coupling may be determined at LLQ and NLLQ from the LO and NLO Altarelli–Parisi splitting functions [419]. Summing them up, longitudinal momentum is automatically conserved: the relevant part of the kernel then satisfies the all order sum rule [419] $\int_{-\infty}^{\infty} dt K(t) = 1$. Furthermore, it turns out that when the LLQ and NLLQ contributions to the LO and NLO BFKL kernels are resummed, the expansion stabilises in the perturbative ($Q^2 >> k^2$) region, and the residual part of the kernel which resums the remaining small $x$ logarithms is relatively small.

However before we can use this resummed BFKL kernel to compute small $x$ resummation corrections we need to resolve a second issue: the inherent perturbative instability of the LLx and NLLx contributions to the splitting functions first noted in [69,414]. This is quite distinct from the previous problem: it can be shown (see [415] and Sect. 7.3) to follow inevitably from the shift in the value of $\lambda$ from its LLx value $\lambda_0$ to $\lambda_0 + \Delta \lambda$ at NLLx. This shift must be accounted for exactly if a sensible resummed perturbative expansion is to be obtained. Since in practice the correction $\Delta \lambda$ is of the same order as $\lambda_0$, it seems probable that $\lambda = \lambda_0 + \Delta \lambda$ is not calculable in perturbation theory: rather the value of $\lambda$ may be used to parameterise the uncertainty in the value of the kernel $K(Q^2, k^2)$ when $Q^2 \sim k^2$.

Putting together the two principles of momentum conservation and perturbative stability, we can compute fully resummed NLO splitting functions [419]. The result depends on the unknown parameter $\lambda$. Provided $\lambda \leq 0$, the corrections to conventional NLO evolution in the HERA region are tiny: this in itself is sufficient to explain the success of NLO evolution in describing the HERA data, and furthermore means that effect of resummed small $x$ logarithms on the extrapolation upwards in $Q^2$ from HERA to the LHC should also be rather small. More significant effects might be expected in the extrapolation down to smaller $x$, particularly if $Q^2$ is also small and $\lambda$ is positive. It should now be possible to quantify such uncertainties by a phenomenological analysis, using available HERA data to constrain $\lambda$.

One might have hoped that eventually it would be possible to compute $\lambda$ perturbatively. The main uncertainty in current calculations is due to the unresummed infrared logarithms in the kernel $K(Q^2, k^2)$, which destabilise the fixed order perturbative expansion in the region $Q^2 \ll k^2$. In Refs. [309,420,421] an attempt is made to resum these logarithms through a symmetrization of $K(Q^2, k^2)$ in $Q^2$ and $k^2$: the idea is to deduce the infrared logarithms from the ultraviolet ones. The main shortcoming of this approach is that it makes implicit assumptions about the validity of perturbation theory when $Q^2$ is very small: symmetrization only works when running coupling effects are included, but making the coupling run with $Q^2$ or $k^2$ is not only very model dependent but seems inevitably to destabilise the small $x$ evolution [422–427], suggesting that effects beyond the reach of the usual perturbative expansion become important in this region.

It seems that to make further progress we require either genuine nonperturbative input, or a substantial extension of the perturbative domain. A possible way in which this might be done through a new factorization procedure was explored in Ref. [428], from which the main conclusion was that at small $x$ the coupling should run not with $Q^2$, but with $W^2 \sim Q^2/x$. Preliminary calculations [429] suggest that this is not phenomenologically unacceptable. An alternative approach to factorization in high energy QCD based on Wilson lines may be found in Refs. [430,431]. Clearly much work remains to be done.
6. PROMPT PHOTON PRODUCTION

6.1 General features of photon production

When mentioning the photon in the framework of high-energy collider physics, one is immediately led to think – with good reasons – to Higgs searches through the gold-plated channel $H \rightarrow \gamma \gamma$. However, the production of photons also deserves attention on its own. Firstly, a detailed understanding of the continuum two-photon production is crucial in order to clearly disentangle any Higgs signals from the background. Secondly, in hadronic collisions, where a very large number of strong-interacting particles is produced, photon signals are relatively clean, since the photon directly couples only to quarks. Therefore, prompt-photon data can be used to study the underlying parton dynamics, in a complementary way with respect to analogous studies performed with hadrons or jets. For the same reason, these data represent a very important tool in the determination of the gluon density in the proton, $f_g(x)$. Indeed, in recent years almost all the direct information (that is, not obtained through scaling violations as predicted by the DGLAP equations) on the intermediate- and high-$x$ behaviour of $f_g(x)$ came from prompt-photon production, $pp \rightarrow \gamma X$ and $pN \rightarrow \gamma X$, in fixed-target experiments. The main reason for this is that, at LO, a photon in the final state is produced in the reactions $qg \rightarrow \gamma q$ and $q\bar{q} \rightarrow \gamma g$, with the contribution of the former subprocess being obviously sensitive to the gluon and usually dominant over that of the latter. It is the ‘point-like’ coupling of the photon to the quark in these subprocesses that is responsible for a much cleaner signal than, say, for the inclusive production of a $\pi^0$, which proceeds necessarily through a fragmentation process.

There is, however, a big flaw in the arguments given above. In fact, photons can also be produced through a fragmentation process, in which a parton, scattered or produced in a QCD reaction, fragments into a photon plus a number of hadrons. The problem with the fragmentation component in the prompt-photon reaction is twofold: first, it introduces in the cross section a dependence upon non-perturbative fragmentation functions, similar to those relevant in the case of single-hadron production, which are not calculable in perturbative QCD: they depend on non-perturbative initial conditions \[432, 433\], and only their asymptotic behavior at very large scales is perturbatively calculable \[434\]. These functions are, at present, very poorly determined by the sparse LEP data available. Secondly, all QCD partonic reactions contribute to the fragmentation component; thus, when addressing the problem of the determination of the gluon density, the advantage of having a priori only one partonic reaction ($q\bar{q} \rightarrow \gamma g$) competing with the signal ($qg \rightarrow \gamma q$) is lost, even though some of the subprocesses relevant to the fragmentation part at the same time result from a gluon in the initial state.

The relative contribution of the fragmentation component with respect to the direct component (where the photon participates in the short-distance, hard-scattering process) is larger the larger the centre-of-mass energy and the smaller the final-state transverse momentum\[38\]: at the LHC, for transverse momenta of the order of few tens of GeV, it can become dominant. However, here the situation is saved by the so-called ‘isolation’ cut, which is imposed on the photon signal in experiments. Isolation is an experimental necessity: in a hadronic environment the study of photons in the final state is complicated by the abundance of $\pi^0$’s, eventually decaying into pairs of $\gamma$’s. The isolation cut simply serves to improve the signal-to-noise ratio: if a given neighbourhood of the photon is free of energetic hadron tracks, the event is kept; it is rejected otherwise. Fortunately, by requiring the photon to be isolated, one also severely reduces the contribution of the fragmentation part to the cross section. This is because fragmentation is an essentially collinear process: therefore, photons resulting from parton fragmentation are usually accompanied by hadrons, and are therefore bound to be rejected after the imposition of an isolation cut.

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\[36\]Session coordinators: M. Fontannaz, S. Frixione and S. Tapprogge.
\[37\]Contributing authors: P. Aurenche, M. Fontannaz and S. Frixione.
\[38\]Actually, in the fixed-target $pp \rightarrow \gamma X$ reaction, one can see the fragmentation component increasing relatively to the direct one also at very large $p_T$, because of the direct cross section dying out very quickly at such momenta. This effect is of no phenomenological relevance at the LHC.
It has to be stressed that, at fixed-target energies, the size of the average transverse momentum allows to resolve the two photons coming from $\pi^0$ decay and therefore to identify the $\pi^0$. It seems therefore appropriate to recall some fixed target results before turning to prompt photon production at the LHC. A recent review on the comparisons between data and theory may be found in [435]. Theory means NLO predictions including the direct and the bremsstrahlung contributions [229, 233, 234, 436, 437]. A Fortran code which puts together both contributions and allows simple changes of parameters is now available [438]. The conclusion reached in ref. [435] is that some data sets are incompatible with each other, or that theory must be modified. A modification proposed in ref. [352] consists in introducing a transverse momentum of initial partons with a large average value $< \kappa_\perp > \sim 1.4$ GeV. If this average value varies with $\sqrt{s}$, then it is possible to adjust theory to data. The resummation of threshold effects [347] (see also Sect. 5.) increases the cross section at large $x_\perp = 2p_\perp / \sqrt{s}$, but it cannot remove the discrepancy between theory and data. Clearly an unsettled problem remains in this fixed target energy range, which questions the possibility to determine the gluon contents of the proton from prompt photon data (see Sect. 2.).

We now turn to the case of photon production at high-energy colliders; after some general introductory remarks, we will present phenomenological predictions relevant to the LHC; we remind the reader that the production of prompt photons at LHC was first studied at the Aachen workshop [2]. No NLO corrections to the bremsstrahlung terms were available at that time, and the isolation prescriptions were implemented only at LO accuracy. Since then, theoretical computations progressed toward a fully consistent NLO framework, which we will discuss in the following.

### 6.2 Isolation prescriptions

As mentioned before, the fragmentation contribution, that threatened to spoil the cleanliness of the photon signals at colliders, is relatively well under control in the case of isolated-photon cross sections. There is of course a price to pay for this gain: the isolation condition poses additional problems in the theoretical computations, which are not present in the case of fully-inclusive photon cross sections. To be specific, we write the cross section for the production of a single isolated photon in hadronic collisions as follows:

\[
\frac{d\sigma}{d^2p_\gamma} = \int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) d\hat{\sigma}_{ab, c}(x_1p_1, x_2p_2; p_\gamma, \mu_R, \mu_F, \mu_c) + \int dx_1 dx_2 f_{a/h_1}(x_1, \mu_F) f_{b/h_2}(x_2, \mu_F) d\hat{\sigma}_{ab, c}(x_1p_1, x_2p_2; p_\gamma, \mu_R, \mu_F, \mu_c) d_{\gamma/c}(z, \mu_c), \tag{51}
\]

where $h_1$ and $h_2$ are the incoming hadrons, with momenta $p_1$ and $p_2$ respectively, and a sum over the parton indices $a$, $b$ and $c$ is understood. In the first term on the right hand side of eq. (51) (the direct component) the subtracted partonic cross sections $d\hat{\sigma}_{ab, c}^{isol}$ get contributions from all the diagrams with a photon leg. On the other hand, the subtracted partonic cross sections $d\hat{\sigma}_{ab, c}^{frag}$ appearing in the second term on the right hand side of eq. (51) (the fragmentation component), get contribution from the pure QCD diagrams, with one of the partons eventually fragmenting in a photon, in a way described by the partonto-photon fragmentation function $d_{\gamma/c}$. As the notation in eq. (51) indicates, the isolation condition is embedded into the partonic cross sections.

It is a well-known fact that, in perturbative QCD beyond LO, and for all the isolation prescriptions known at present, with the exception of that of ref. [232], neither the direct nor the fragmentation components are separately well defined at any fixed order in perturbation theory: only their sum is physically

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40The production of pairs of isolated photons can be described in the very same manner; we will consider this case later. Here we stick to a simpler case in order to have as simple as notation as possible.
meaningful. In fact, the direct component is affected by quark-to-photon collinear divergences, which are subtracted by the bare fragmentation function that appears in the unsubtracted fragmentation component. Of course, this subtraction is arbitrary as far as finite terms are concerned. This is formally expressed in eq. (51) by the presence of the same scale $\mu_r$ in both the direct and fragmentation components: a finite piece may be either included in the former or in the latter, without affecting the physical predictions. The need for introducing a fragmentation contribution is physically better motivated from the fact that a QCD hard scattering process may produce, again through a fragmentation process, a $\rho$ meson that has the same quantum numbers as the photon and can thus convert into a photon, leading to the same signal.

As far as the isolation prescriptions are concerned, here we will restrict to those belonging to the class that can be denoted as ‘cone isolations’ [229, 230, 439–442]. In the framework of hadronic collisions, where the need for invariance under longitudinal boosts (which is necessary for collinear factorizability) suggests not to define physical quantities in terms of angles, the cone is drawn in the pseudorapidity–azimuthal angle plane, and corresponds to the set of points

$$C_R = \left\{ (\eta, \phi) \mid \sqrt{(\eta - \eta_\gamma)^2 + (\phi - \phi_\gamma)^2} \leq R \right\},$$  \hspace{1cm} (52)

where $\eta_\gamma$ and $\phi_\gamma$ are the pseudorapidity and azimuthal angle of the photon, respectively, and $R$ is the aperture (or half-angle) of the cone. After having drawn the cone, one has to actually impose the isolation condition. We consider here two sub-classes of cone isolation, whose difference lies mainly in the behaviour of the fragmentation component. Prior to that, we need to define the total amount of hadronic transverse energy deposited in a cone of half-angle $R$ as

$$E_{T,\text{had}}(R) = \sum_{i=1}^{n} E_{T,i} \theta(R - R_{\eta\gamma}),$$  \hspace{1cm} (53)

where

$$R_{\eta\gamma} = \sqrt{(\eta_i - \eta_\gamma)^2 + (\phi_i - \phi_\gamma)^2},$$  \hspace{1cm} (54)

and the sum runs over all the hadrons in the event (or, alternatively, $i$ can be interpreted as an index running over the towers of a hadronic calorimeter). For both the isolation prescriptions we are going to define below, the first step is to draw a cone of fixed half-angle $R_0$ around the photon axis, as given in eq. (52). We will denote this cone as the isolation cone.

**Definition A.** The photon is isolated if the total amount of hadronic transverse energy in the isolation cone fulfils the following condition:

$$E_{T,\text{had}}(R_0) \leq \epsilon_c p_{\gamma T},$$  \hspace{1cm} (55)

where $\epsilon_c$ is a fixed (generally small) parameter, and $p_{\gamma T}$ is the transverse momentum of the photon.

**Definition B.** The photon is isolated if the following inequality is satisfied:

$$E_{T,\text{had}}(R) \leq \epsilon_s p_{\gamma T} \mathcal{Y}(R),$$  \hspace{1cm} (56)

for all the cones lying inside the isolation cone, that is for any $R \leq R_0$. The function $\mathcal{Y}$ is arbitrary to a large extent, but must at least have the following property:

$$\lim_{R \to 0} \mathcal{Y}(R) = 0,$$  \hspace{1cm} (57)

and being different from zero everywhere except for $R = 0$.

Definition A was proven to lead to an infrared-safe cross section at all orders of perturbation theory in ref. [443]. The smaller $\epsilon_c$, the tighter the isolation. Loosely speaking, for vanishing $\epsilon_c$ the direct
component behaves like $\log \epsilon_c$, while the fragmentation component behaves like $\epsilon_c \log \epsilon_c$. Thus, for $\epsilon_c \to 0$ eq. (51) diverges. This is obvious since the limit $\epsilon_c \to 0$ corresponds to a fully-isolated-photon cross section, which cannot be a meaningful quantity, whether experimentally (because of limited energy resolution) or theoretically (because soft-particle emission inside the cone cannot be forbidden without spoiling the infrared safety of the cross section).

Definition B was proposed and proven to lead to an infrared-safe cross section at all orders of perturbation theory in ref. [232]. Eq. (57) implies that the energy of a parton falling into the isolation cone $C_{R_0}$ is correlated to its distance (in the $\eta-\phi$ plane) from the photon. In particular, a parton becoming collinear to the photon is also becoming soft. When a quark is collinear to the photon, there is a collinear divergence; however, if the quark is also soft, this divergence is damped by the quark vanishing energy. When a gluon is collinear to the photon, then either it is emitted from a quark, which is itself collinear to the photon – in which case, what was said previously applies – or the matrix element is finite. Finally, it is clear that the isolation condition given above does not destroy the cancellation of soft singularities, since a gluon with small enough energy can be emitted anywhere inside the isolation cone. The fact that this prescription is free of final-state QED collinear singularities implies that the direct part of the cross section is finite. As far as the fragmentation contribution is concerned, in QCD the fragmentation mechanism is purely collinear. Therefore, by imposing eq. (56), one forces the hadronic remnants collinear to the photon to have zero energy. This is equivalent to saying that the fragmentation variable $z$ is restricted to the range $z = 1$. Since the parton-to-photon fragmentation functions do not contain any $\delta(1 - z)$, this means that the fragmentation contribution to the cross section is zero, because an integration over a zero-measure set is carried out. Therefore, only the first term on the right hand side of eq. (51) is different from zero, and it does not contain any $\mu_\gamma$ dependence.

We stress again that the function $\mathcal{Y}$ can be rather freely defined. Any sufficiently well-behaved function, fulfilling eq. (57), could do the job, the key point being the correlation between the distance of a parton from the photon and the parton energy, which must be strong enough to cancel the quark-to-photon collinear singularity. Throughout this paper, we will use

$$\mathcal{Y}(R) = \left( \frac{1 - \cos R}{1 - \cos R_0} \right)^n, \quad n = 1. \quad (58)$$

We also remark that the traditional cone-isolation prescription, eq. (55), can be formally recovered from eq. (56) by setting $\mathcal{Y} = 1$ and $\epsilon_\gamma = \epsilon_c$.

### 6.3 Single isolated photons at the LHC

In this section, we will present results for isolated-photon cross sections in $pp$ collisions at 14 TeV. These results have been obtained with the fully-exclusive NLO code of ref. [231], and are relevant to the isolation obtained with definition B; the actual parameters used in the computation are given in eq. (58), together with $\epsilon_\gamma = 1$. We set $R_0 = 0.4$. We will comment in the following on the outcome of definition A. Benchmark rates for isolated photons over different ranges of rapidity are given in Fig. 33.

Any sensible perturbative computation should address the issue of the perturbative stability of its results. A rigorous estimate of the error affecting a cross section at a given order can be given if the next order result is also available. If this is not the case, it is customary to study the dependence of the physical observables upon the renormalization ($\mu_R$) and factorization ($\mu_F$) scales. It is important to stress that the resulting spread should not be taken as the ‘theoretical error’ affecting the cross section; to understand this, it is enough to say that the range in which $\mu_R$ and $\mu_F$ are varied is arbitrary. Rather, one should compare the spread obtained at the various perturbative orders; only if the scale dependence decreases when including higher orders the cross section can be regarded as perturbatively stable and sensibly compared to data.

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Usually, $\mu_R$ and $\mu_F$ are imposed to have the same value, $\mu$, which is eventually varied. However, this procedure might hide some problems, because of a possible cancellation between the effects induced by the two scales. It is therefore desirable to vary $\mu_R$ and $\mu_F$ independently. Here, an additional problem arises at the NLO. The expression of any cross section in terms of $\mu$ (that is, when $\mu = \mu_R = \mu_F$) is not ambiguous, while it is ambiguous if $\mu_R \neq \mu_F$. In fact, when $\mu_R \neq \mu_F$, the cross section can be written as the sum of a term corresponding to the contribution relevant to the case $\mu_R = \mu_F$, plus a term of the kind:

$$\alpha_S(\mu_A) B(\alpha_S(\mu_R)) \log \frac{\mu_R}{\mu_F},$$

(59)

where $B$ has the same power of $\alpha_S$ as the LO contribution, say $\alpha_S^{k+1}$. The argument of the $\alpha_S$ in front of eq. (59), $\mu_A$, can be chosen either equal to $\mu_R$ or equal to $\mu_F$, since the difference between these two choices is of NNLO. Thus, it follows that the dependence upon $\mu_R$ or $\mu_F$ of a NLO cross section reflects the arbitrariness of the choice made in eq. (59), which is negligible only if the NNLO ($\alpha_S^{k+2}$) corrections are much smaller than the NLO ones ($\alpha_S^{k+1}$). This leads to the conclusion that a study of the dependence upon $\mu_R$ or $\mu_F$ only can be misleading. In other words: $B$ in eq. (59) is determined through DGLAP equations in order to cancel the scale dependence of the parton densities up to terms of order $\alpha_S^{k+2}$. This happens regardless of the choice made for $\mu_A$ in eq. (59). However, here we are not discussing the cancellation to a given perturbative order of the effects due to scale variations; we are concerned about the coefficient in front of the $\mathcal{O}(\alpha_S^{k+2})$ term induced by such variations, whose size is dependent upon the choice made for $\mu_A$ and therefore, to some extent, arbitrary. We have to live with this arbitrariness, if we decide to vary $\mu_R$ or $\mu_F$ only. However, we can still vary $\mu_R$ and $\mu_F$ independently, but eventually putting together the results in some sensible way, that reduces the impact of the choice made for $\mu_A$. In this section, we will consider the quantities defined as follows:

$$\left( \frac{\delta \sigma}{\sigma} \right)_\pm = \pm \left\{ \frac{\sigma(\mu_R = \mu_0, \mu_F = \mu_0) - \sigma(\mu_R = \alpha \pm \mu_0, \mu_F = \mu_0)}{\sigma(\mu_R = \mu_0, \mu_F = \mu_0) + \sigma(\mu_R = \alpha \pm \mu_0, \mu_F = \mu_0)} \right\} \left( \frac{\sigma(\mu_R = \mu_0, \mu_F = \mu_0) - \sigma(\mu_R = \mu_0, \mu_F = a \pm \mu_0)}{\sigma(\mu_R = \mu_0, \mu_F = \mu_0) + \sigma(\mu_R = \mu_0, \mu_F = a \pm \mu_0)} \right)^2 + \frac{1}{2},
$$

(60)

where $a_+$ and $a_- = 1/a_+$ are two numbers of order one, which we will take equal to 1/2 and 2 respectively; the $\pm$ sign in front of the right hand side of eq. (60) is purely conventional. We can evaluate $(\delta \sigma / \sigma)_\pm$ by using $\mu_A = \mu_R$ or $\mu_A = \mu_F$ in eq. (59). The reader can convince himself, with the help of the renormalization group equation (4), that the difference between these two choices is of order $\alpha_S^4$ in the expansion of the contribution to $(\delta \sigma / \sigma)_\pm^2$ due to eq. (59); on the other hand, this difference is only of order $\alpha_S^3$ in each of the two terms under the square root in the right hand side of eq. (60). This is exactly
what we wanted to achieve: a suitable combination of the cross sections resulting from independent $\mu_R$ and $\mu_F$ variations is less sensitive to the choice for $\mu_A$ made in eq. (59) than the results obtained by varying $\mu_R$ or $\mu_F$ only.

In table 3 we present the results for the total isolated-photon rates, both at NLO and at LO. The latter cross sections have been obtained by retaining only the LO terms ($\mathcal{O}(\alpha_s)$) in the short-distance cross section, and convoluting them with NLO-evolved parton densities. Also, a two-loop expression for $\alpha_s$ has been used. There is of course a lot of freedom in the definition of a Born-level result. However, we believe that with this definition one has a better understanding of some issues related to the stability of the perturbative series. To obtain the rates entering table 3, we required the photon transverse momentum to be in the range $40 < p_T \gamma < 400$ GeV, and we considered the rapidity cuts $|\eta_\gamma| < 1.5$ and $|\eta_\gamma| < 2.5$, in order to simulate a realistic geometrical acceptance of the LHC detectors. We first consider the scale dependence of our results (last column), evaluated according to eq. (60). We see that the NLO results are clearly more stable than the LO ones; this is reassuring, and implies the possibility of a sensible comparison between NLO predictions and the data. Notice that the size of the radiative corrections ($K$ factor, defined as the ratio of the NLO result over the LO result) is quite large. From the table, we see that the cross sections obtained with different parton densities differ by 6% at the most (relative to the result obtained with MRST99-1 [10], which we take as the default set). MRST99 sets 2 and 3 are meant to give an estimate of the effects due to the current uncertainties affecting the gluon density (see sect. 2.), whereas sets 4 and 5 allow to study the sensitivity of our predictions to the value of $\alpha_s(M_Z)$ (sets 1, 4 and 5 have $\Lambda_{\alpha_s}^{(5)} = 220, 164$ and 288 MeV respectively). On the other hand, the difference between MRST99-1 and CTEQ5M [7] results is due to the inherent difference between these two density sets (CTEQ5M has $\Lambda_{\alpha_s}^{(5)} = 226$ MeV, and therefore the difference in the values of $\alpha_s(M_Z)$ plays only a very minor role).

From inspection of table 3, we can conclude that isolated-photon cross section at the LHC is under control, both in the sense of perturbation theory and of the dependence upon non-calculable inputs, like $\alpha_s(M_Z)$ and parton densities. The relatively weak dependence upon the parton densities, however, is not a good piece of news if one aims at using photon data to directly access the gluon density. On the other hand, the expected statistics is large enough to justify attempts of a direct measurement of such a quantity. In the remainder of this section, we will concentrate on this issue. We will consider

$$R_x = \frac{d\sigma_0/dx - d\sigma/dx}{d\sigma_0/dx + d\sigma/dx},$$

(61)

where $x$ is any observable constructed with the kinematical variables of the photon and, possibly, of the accompanying jets. $\sigma$ and $\sigma_0$ are the cross sections obtained with two different sets of parton densities, the latter of which is always the default one (MRST99-1). We can imagine a gedanken experiment, where it is possible to change at will the parton densities; in this way, we can assume the relative statistical errors affecting $\sigma$ and $\sigma_0$ to decrease as $1/\sqrt{N}$ and $1/\sqrt{N_0}$, $N$ and $N_0$ being the corresponding number of events. It is then straightforward to calculate the statistical error affecting $R_x$; by imposing $R_x$ to be
larger than its statistical error, one gets

$$\mathcal{R}_x > (\mathcal{R}_x)_{\text{min}} \equiv \frac{1}{\sqrt{2 L \epsilon \sigma(x, \Delta x)}}$$

(62)

where $\mathcal{L}$ is the integrated luminosity, $\epsilon \leq 1$ collects all the experimental efficiencies, and

$$\sigma(x, \Delta x) = \int_{x-\Delta x/2}^{x+\Delta x/2} \frac{d\sigma}{dx}$$

(63)

is the total cross section in a range of width $\Delta x$ around $x$.

In fig. 34 we present our predictions for $\mathcal{R}_x$. In the left panel of the figure we have chosen $x = p_{T\gamma}$, while in the right panel we have $x = x_{\gamma j}$, where

$$x_{\gamma j} = \frac{p_{T\gamma} \exp(\eta_j) + p_{Tj} \exp(\eta_j)}{\sqrt{s}}.$$  

(64)

In this equation $\sqrt{s}$ is the centre-of-mass energy of the colliding hadrons, and $p_{T\gamma}$ and $\eta_j$ are the transverse momentum and rapidity of the hardest jet recoiling against the photon. In order to reconstruct the jets, we adopted here a $k_t$-algorithm [8], in the version of ref. [9] with $D = 1$. Notice that $x_{\gamma j}$ exactly coincides at the LO with the longitudinal momentum fraction $x$ of the partons in one of the incoming hadrons; NLO corrections introduce only minor deviations. For all the density sets considered, the dependence of $\mathcal{R}$ upon $p_{T\gamma}$ is rather mild. The values in the low-$p_{T\gamma}$ region could also be inferred from table 3, since the cross section is dominated by small $p_{T\gamma}$'s. Analogously to what happens in the case of total rates, the sets MRST99-4 and MRST99-5 give rise to extreme results for $\mathcal{R}_{p_{T\gamma}}$, since the value of $\sigma_{\text{min}}(M_Z)$ is quite different from that of the default set. From the figure, it is apparent that, by studying the transverse momentum spectrum, it will not be easy to distinguish among the possible shapes of the gluon density. On the other hand, it seems that, as far as the statistics is concerned, a distinction between any two sets can be performed. Indeed, the symbols in the figure display the quantity defined in eq. (62), for $L = 100$ fb$^{-1}$, $\Delta p_{T\gamma} = 10$ GeV and $\epsilon = 1$. Of course, the latter value is not realistic. However, a smaller value (leading to a larger $(\mathcal{R})_{\text{min}}$), can easily be compensated by enlarging $\Delta p_{T\gamma}$ and by the fact that the total integrated luminosity is expected to be much larger than that adopted in fig. 34.

Turning to the right panel of fig. 34, we can see a much more interesting situation. Actually, it can be shown that the pattern displayed in the figure is rather faithfully reproduced by plotting the analogous quantity, where one uses the gluon densities instead of the cross sections. This does not come as a surprise. First, $x_{\gamma j}$ is in an almost one-to-one correspondence with the $x$ entering the densities.
Secondly, photon production is dominated by the gluon-quark channel, and therefore the cross section has a linear dependence upon \( f_g(x) \), which can be easily spotted. It does seem, therefore, to be rather advantageous to look at more exclusive variables, like photon-jet correlations (this is especially true if one considers the procedure of unfolding the gluon density from the data: in the case of single-inclusive variables, the unfolding requires a de-convolution, which is not needed in the case of correlations). Of course, there is a price to pay: the efficiency \( \epsilon \) will be smaller in the case of photon-jet correlations, with respect to the case of single-inclusive photon observables, mainly because of the jet-tagging. However, from the figure it appears that there should be no problem with statistics, except in the very large \( x_{\gamma j} \) region.

Finally, we would like to comment on the fact that, for the case of single-inclusive photon observables, we also computed the cross section by isolating the photon according to definition A, using \( \epsilon_c = 2 \text{ GeV}/p_{T\gamma} \). The two definitions return a \( p_{T\gamma} \) spectrum almost identical in shape, with definition B higher by a factor of about 9\%. It is only at the smallest \( p_{T\gamma} \) values that we considered, that definition B returns a slightly steeper spectrum. The fact that such different definitions produce very similar cross sections may be surprising. This happens because, prior to applying the isolation condition, partons tend to be radiated close to the photon; therefore, most of them are rejected when applying the isolation, no matter of which type. This situation has already been encountered in the production of photons at much smaller energies. The reader can find a detailed discussion on this point in ref. [444].

In the previous paragraphs, we concentrated on the possibility that isolated-photon data can be used to constrain or measure the gluon density in the proton. However, it is well known that \( f_g(x) \) is rather strongly correlated to \( \alpha_S \). This is not a problem if one is interested in observables that only depend upon the quantity \( \alpha_S f_g(x) \). On the other hand, the determination of the gluon density alone is important in many respects. Thus, one has to assume an accurate knowledge of \( \alpha_S \) to extract \( f_g(x) \) from the data. It is of course possible to turn this argument the other way round: that is, to assume a good knowledge of \( f_g(x) \) to measure \( \alpha_S \). The sensitivity of the isolated-photon cross section at the LHC upon the value of \( \alpha_S \) can be inferred from table 3 and fig. 34, looking at the results obtained with the sets MRST99-4 and MRST99-5. Unfortunately, since the gluon-initiated processes dominate the cross section, and the gluon is the least known among the parton densities, this procedure will probably result in sizeable systematic errors; on the other hand, thanks to the size of the production rate, we should expect a precise result on a statistical basis. These considerations should encourage us to find alternative ways of measuring \( \alpha_S \) by using photon data. Since the main problem is in the dependence of the cross section upon \( f_g(x) \), the guide line is that of considering observables that are less sensitive to the parton densities than the isolated-photon cross section.

In what follows, we will argue that an observable of this kind is given by the ratio

\[
\mathcal{X}(p_T) = \frac{d\sigma_{j}}{dp_{Tj}}(p_T) / \frac{d\sigma_{\gamma}}{dp_{T\gamma}}(p_T).
\]  

(65)

Here, \( d\sigma_{j}/dp_{Tj} \) is the single-inclusive jet transverse momentum spectrum, while \( d\sigma_{\gamma}/dp_{T\gamma} \) is the transverse momentum spectrum of the isolated photon.

It is immediate to see that, at the LO, \( \mathcal{X} \) is proportional to \( \alpha_S \). In the ratio that defines \( \mathcal{X} \), one expects that the dependence upon the parton densities cancel to a good extent, thus giving an observable suited to measure \( \alpha_S \), regardless of the precision to which \( f_g(x) \) is known. In hadronic physics, the trick of considering ratios of cross sections (instead of the cross sections themselves) in order to reduce the dependence on the parton densities is frequently used. In particular, for the measurement of \( \alpha_S \) at hadron colliders, one can think to the \( W + 1 \)-jet over \( W + 0 \)-jet ratio (\( \mathcal{A} \)), and to the 3-jet over 2-jet ratio (\( \mathcal{B} \)). We have to stress an important difference between these two quantities and \( \mathcal{X} \): in the ratio that defines \( \mathcal{A} \) and \( \mathcal{B} \), the numerator requires the definition (through final-state cuts) of an hard object in addition to those already present in the denominator. This implies that the kinematical configurations in the numerator and denominator can be sizably different. Therefore, one faces the following problem: even if \( \mathcal{A} \) and
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$p_T^{min}$ (GeV) & 40 & 100 & 200 \\
\hline
MRST99-2 & 1.006 ± 0.009 & 1.003 ± 0.025 & 0.991 ± 0.051 \\
MRST99-3 & 1.002 ± 0.009 & 1.009 ± 0.023 & 1.007 ± 0.048 \\
\hline
|$\eta$| < 1.5 & & & \\
\hline
MRST99-2 & 1.003 ± 0.008 & 1.002 ± 0.023 & 0.998 ± 0.042 \\
MRST99-3 & 1.009 ± 0.008 & 1.009 ± 0.023 & 0.999 ± 0.046 \\
\hline
|$\eta$| < 2.5 & & & \\
\hline
\end{tabular}
\caption{NLO predictions for the double ratio $D$ defined in eq. (66), for various $p_T^{min}$ and two ranges in rapidity.}
\end{table}

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|}
\hline
$p_T^{min}$ (GeV) & 40 & 100 & 200 \\
\hline
MRST99-2 & 0.974 ± 0.003 & 0.966 ± 0.010 & 0.984 ± 0.027 \\
MRST99-3 & 1.019 ± 0.003 & 1.016 ± 0.010 & 1.012 ± 0.025 \\
\hline
|$\eta$| < 1.5 & & & \\
\hline
MRST99-2 & 0.976 ± 0.002 & 0.973 ± 0.008 & 0.987 ± 0.019 \\
MRST99-3 & 1.017 ± 0.002 & 1.010 ± 0.008 & 1.010 ± 0.018 \\
\hline
|$\eta$| < 2.5 & & & \\
\hline
\end{tabular}
\caption{NLO predictions for the ratio defined in eq. (68). This table has to be compared to table 4.}
\end{table}

$B$ are formally proportional (at the LO) to $\alpha_S$, it is not straightforward to determine the scale at which $\alpha_S$ is calculated. Furthermore, since the numerator and the denominator have different hard scales, the parton densities appearing in these two quantities will be probed at different momenta: this of course will partially destroy the cancellation that one is willing to achieve when considering such ratios. One the other hand, this problem does not affect $\mathcal{A}$: both the isolated-photon and the single-inclusive cross sections are dominated by two-body, back-to-back configurations: it is therefore pretty intuitive that $\alpha_S$ will be evaluated at a scale equal to the transverse momentum of the observed photon and jet. On the other hand, the partonic subprocesses contributing to the numerator and the denominator of $\mathcal{A}$ and $\mathcal{B}$ are basically the same. This is not true for $\mathcal{X}$, because of the different hard production processes involved. Therefore, one might argue that in the latter case the cancellation of the dependence on parton densities will not take place. We can however observe the following: at the LHC, and if one does not consider too large values in $p_T$, the average momentum fraction $x$ probed is small: thus, the quark densities are dominated by the sea, which is in turn related to $f_q(x)$. In this way, we can expect to recover the cancellation.

Of course, there is no way to tell beforehand which observable displays the smallest dependence upon the parton density choice. In order to study this issue in the case of $\mathcal{X}$, we will consider in the following the double ratio

$$D(p_T^{min}) = \mathcal{X}(p_T^{min}) / \mathcal{X}_0(p_T^{min}),$$

where

$$\mathcal{X}(p_T^{min}) = \int_{p_T^{min}}^{p_T^{max}} dp_T \frac{d\sigma}{d\eta} \int_{p_T^{min}}^{p_T^{max}} dp_T \frac{d\sigma}{d\eta}.$$  \hfill (67)

In eq. (66), $\mathcal{X}_0$ is computed with our default parton density set (MRST99-1), while $\mathcal{X}$ is computed with the other sets. Notice that we considered $\mathcal{X}$ instead of $\mathcal{X}$ just because we collected a limited amount of statistics in the MC runs performed so far, and $\mathcal{X}$ stands a better chance than $\mathcal{X}$ to be insensitive to fluctuations. Notice, however, that the relevant transverse momentum spectra are quite steep, and therefore $\mathcal{X}(p_T^{min})$ is dominated by $\mathcal{X}(p_T^{min})$. In eq. (67), the upper limit $p_T^{max}$ can be chosen at will. A possible choice is to set it equal to the kinematical limit; in the results presented in this section, we have set $p_T^{max} = 400$ GeV.
Our NLO predictions for the double ratio $D$ are presented in table 4. By inspection of the table, we can see that $D$ is remarkably stable with respect to the choice of the density set; it has to be stressed, however, that an increase of the statistics is mandatory at the highest $p_T^{min}$ considered. In the table, we limited ourselves to considering only the sets MRST99-2 and MRST99-3. The reason is the following: by construction, these sets gauge the current uncertainty affecting the determination of $f_g(x)$, with MRST99-1 being assumed to return the “true” densities. Thus, since $D$ is compatible with one, we are indeed checking that the dependence upon the parton densities in $\chi$ (actually, $\overline{\chi}$) almost perfectly cancels. If we were considering other sets, like MRST99-4, we would expect $D \simeq \alpha_S(\Lambda_{MRST99-4})/\alpha_S(\Lambda_{MRST99-1})$. However, the strong correlation between $\alpha_S$ and $f_g(x)$ might spoil this naive expectation. The same can be said when considering the sets of the CTEQ group: in this case, a further bias can be introduced by the fact that MRST and CTEQ use different parametrizations and evolution codes. We postpone a more careful analysis of this problem to a forthcoming work.

It can be argued that the results displayed in table 4 are due to the fact that the densities used are actually not that different in the $x$ range of interest. This, however, is not true. In fact, at the level of cross sections, the differences between the predictions obtained with the default set or with the other sets are much larger. This can be seen from table 3. More precisely, we can consider the ratio

$$\int_{p_T^{min}}^{p_T^{max}} \frac{d\sigma_0}{dp_T \gamma} \frac{d\sigma_0}{dp_T \gamma} \int_{p_T^{min}}^{p_T^{max}} \frac{d\sigma}{dp_T \gamma} \frac{d\sigma}{dp_T \gamma},$$

(68)

where $d\sigma_0$ is calculated using MRST99-1, and $d\sigma$ with all the other density sets. The results for this quantity are presented in table 5. Each entry of this table has to be compared with the corresponding entry in table 4. From this exercise, it is indeed evident that $\chi$ is much less sensitive than the isolated-photon cross section to the choice of the density set, at least at small $p_T^{min}$. When $p_T^{min}$ approaches larger values, no firm conclusion can be reached, given the statistics collected; as mentioned before, one can suspect that, the higher $p_T^{min}$, the larger the dependence of $\overline{\chi}$ upon the densities. One the other hand, it can be observed that smaller momenta allow an easier observation of the running of $\alpha_S$.

### 6.4 Pairs of isolated photons: infrared sensitivity with standard cone isolation

In the discussion given before, we restricted to the case of the production of a single isolated photons. Of course, the considerations we made can be extended with obvious modification in eq. (51) to the case of the production of photon pairs. In such a case, the cross section splits naturally in three unphysical components: direct, single-fragmentation and double-fragmentation, corresponding to the processes where both photons, one photon and none of the photons are directly entering the hard subprocess. As far as the isolation prescription is concerned, things are unchanged: this cut has to be imposed on both photons, and possibly supplemented by the requirement that the photons be isolated from each other.

In Sect. 9., the production of photon pairs is described with a special emphasis on its role as a background to Higgs searches. Here we would like to concentrate on a different, more technical aspect, which is more relevant to pure-QCD studies. We investigate appearance of infrared divergences inside the physical spectrum. An example of such divergences appears in the transverse momentum ($q_T$) spectrum of a pair of isolated photons - or of a jet+isolated photon system. This can be seen in Fig. 35, which shows $d\sigma/dq_T$ vs. $q_T$ for isolated photon pairs, computed at NLO accuracy [237]. The rather large value of isolation cut used here, $E_{T_{max}} = 15$ GeV, is not motivated by any phenomenological consideration: it instead allows to split the well known infrared issue in the vicinity of $q_T \rightarrow 0$ from the new one at $q_T \rightarrow E_{T_{max}}$.

The trouble comes from the “single fragmentation” contribution (the contribution where only one photon comes from the fragmentation of a hard parton, the other being emitted by the partonic subprocess). In the QCD improved parton model framework, the fragmentation is a strictly collinear pro-

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42 Contributing authors: T. Binoth, J.P. Guillet and E. Pilon.
Di-Photon differential cross section $d\sigma/dq_T$ at LHC, $\sqrt{s} = 14$ TeV, with the kinematic cuts $p_T(\gamma_1) \geq 40$ GeV, $p_T(\gamma_2) \geq 25$ GeV, $|y(\gamma_1,2)| \leq 2.5$, and with isolation criterion $E_{T_{\text{max}}} = 15$ GeV in $R = 0.4$. The scale choice for initial state factorization scale ($M$), fragmentation scale ($M_f$) and renormalization scale ($\mu$) is $M = M_f = \mu = m_{\gamma}/2$.

Fig. 35: Di-Photon differential cross section $d\sigma/dq_T$ at LHC, $\sqrt{s} = 14$ TeV, with the kinematic cuts $p_T(\gamma_1) \geq 40$ GeV, $p_T(\gamma_2) \geq 25$ GeV, $|y(\gamma_1,2)| \leq 2.5$, and with isolation criterion $E_{T_{\text{max}}} = 15$ GeV in $R = 0.4$. The scale choice for initial state factorization scale ($M$), fragmentation scale ($M_f$) and renormalization scale ($\mu$) is $M = M_f = \mu = m_{\gamma}/2$.

cess, hence all the hadronic debris of the parton-to-photon fragmentation fall inside the cone of the photon from fragmentation. At LO, both photons are back-to-back in the transverse plane, so, due to transverse momentum conservation, $q_T = E_{T_{\text{had}}}$. Since the transverse hadronic energy deposited in the isolation cone has to be less than $E_{T_{\text{max}}}$, the LO “single fragmentation” contribution of the $q_T$ distribution has a stepwise behavior. Then, as shown in [445], at NLO such an observable gets an infrared double logarithmic divergence at the critical point $q_T = E_{T_{\text{max}}}$. The details of this infrared structure are very sensitive to the kinematic constraints and the observable considered. In the case at hand, the NLO contribution to $d\sigma/dq_T$ gets a double logarithm below the critical point, which is produced by the convolution of the lowest order stepwise term with the probability distribution for emitting a soft and collinear gluon, yielding:

$$
\left( \frac{d\sigma}{dq_T} \right)_{NLO} \sim \left( \frac{d\sigma}{dq_T} \right)_{LO} \Theta(E_{T_{\text{max}}} - q_T) \times \alpha_S \ln^2 \left( 1 - \frac{q_T^2}{E_{T_{\text{max}}}^2} \right) + \cdots
$$

(69)

More generally, at each order in $\alpha_S$, up to two powers of such logarithms will appear, making any fixed order calculation diverge at $q_T = E_{T_{\text{max}}}$, so that the spectrum computed by any fixed order calculation is unreliable in the vicinity of this critical value. In principle, an all order resummation has to be carried out if possible, in order to restore any predictability. In practice, the phenomenologically relevant values of $E_{T_{\text{max}}}$ are fairly lower than 15 GeV, so that this problem may affect only the very first bins of the $q_T$ distribution.

6.41 Mismatch theory/experiment with very severe isolation cuts

Another issue deserves some care, when isolated photons are selected by mean of the above standard cone criterion. In an actual prompt photon event the transverse energy deposited inside the isolation cone has several physical origins. One is when hadrons coming from the hadronization of hard partons involved in the subprocess fall into the cone. A second one is given by the debris of the fragmentation producing the photon, when the latter comes from such a mechanism. A third source of accompanying transverse energy is provided by “minimum bias”. Moreover at high luminosity, piled-up events may also contaminate the hadronic environment of a previous photon event. From an experimental point of
view, the value of $E_{T_{\text{max}}}$ has to be as low as possible in order to suppress background events and events with photons from fragmentation, while retaining most of the “true” direct photons. The goal is thus to use an experimental value of $E_{T_{\text{max}}}$ basically saturated by “minimum bias” - and pile-up. For example this is nearly achieved by CDF at the Tevatron requiring $E_{T_{\text{max}}} = 1$ GeV in $R = 0.4$. In partonic calculations, the first two sources of accompanying transverse energy are taken into account, whereas the last two are ignored. However if the accompanying $E_{T_{\text{ad}}}$ is to be saturated by “minimum bias” and pile-up, then in a partonic calculation, this leaves almost no room for accompanying partonic $E_T$ coming from the hard subprocess itself. Therefore, a partonic calculation meant to incorporate the effect of such an experimental cut should use an effective value for $E_{T_{\text{max}}}$ in the calculation, which is much smaller than the one experimentally used, e.g. at most a few hundred MeV for CDF. The correspondence between the values used in experiments, or full Monte Carlo simulations (which model the “minimum bias”), and their counterparts in higher order partonic calculations has to be further studied. Such a comparison is worthwhile especially because the actual isolation cuts used by colliders experiments are more exclusive and sophisticated than the schematic criterion defined above.

However when the experimental value of $E_{T_{\text{max}}}$ is nearly saturated by “minimum bias”, such a study is complicated by an infrared problem. Indeed, an infrared divergence appears in partonic calculations, when photons are required to be absolutely isolated, i.e. accompanied by a vanishing amount of partonic transverse energy inside a cone of finite size, because this amputation of gluon phase space prevents the cancellation of the infrared singularities associated with soft gluon emission. With a finite value $E_{T_{\text{max}}}$, this would translate into the appearance of $\ln \left( E_{T_{\text{max}}}/Q \right)$ (where $Q$ is some large scale, of the order of the photon’s $p_T$) which would become large with a tiny $E_{T_{\text{max}}}$. Whereas the “fragmentation” contribution to, e.g. the $p_T$ distribution of direct photons [230,446], or the invariant mass distribution of photon pairs, is roughly

$$\sigma_{\text{frag}} \sim \varepsilon \left( \ln^2 \varepsilon + \ln \varepsilon \ln R + \cdots \right)$$

(with $\varepsilon = E_{T_{\text{max}}}/Q$), the “direct” contribution behaves as

$$\sigma_{\text{dir}} \sim R^2 \ln \varepsilon + \mathcal{O}(1)$$

The theoretical partonic calculation would then become unstable and unreliable, when $\varepsilon \ll 1$ with finite $R$. Moreover, this problem is not localized in the sole vicinity of some isolated point, at the border of or inside the spectrum, but in principle it plagues the calculation over the whole spectrum - at least some extended range of it - for observables such as, e.g., the $p_T$ distribution of direct photons, or the invariant mass distribution of photon pairs. The dependence of theoretical partonic calculations on the isolation parameters, especially on $E_{T_{\text{max}}}$, has still to be studied in detail [447] in order to fix this puzzle.

7. SMALL X PHYSICS

7.1 Jet physics at large rapidity intervals and the BFKL equation

The LHC offers a unique opportunity to explore semi-hard strong-interaction processes, which are characterized by two large and disparate kinematic scales. In inclusive jet production, jets of transverse energy $E_\perp = 50$ GeV can span a kinematic range of up to 11 units of rapidity. Processes with two large and disparate kinematic scales typically lead to cross sections containing large logarithms. Examples of this type of process are di-jet production in hadron collisions at large rapidity intervals [448], forward jet production in DIS [449–451], and $\gamma^*\gamma^*$ collisions in double-tag events, $e^+e^- \rightarrow e^+e^- + $ hadrons [452]. In large-rapidity di-jet production the large logarithm is the rapidity interval between the jets, $\Delta y \approx \ln(\hat{s}/|\hat{t}|)$, with $\hat{s}$ the squared parton center-of-mass energy and $|\hat{t}|$ of the order of the squared jet transverse energy. In forward jet production in DIS the large logarithm is $\ln(x/x_{bj})$, where $x_{bj}$ is the Bjorken scaling variable and $x$ the momentum fraction of the parton entering the hard scattering. These

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logarithms will arise in a perturbative calculation at each order in the coupling constant $\alpha_S$. Alternatively, if the logarithms are large enough, it is possible to include them through an all-order resummation in the leading logarithmic (LL) approximation performed by means of the Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation [304–306].

In the high-energy limit, $\hat{s} \gg |\hat{t}|$, the BFKL theory assumes that any scattering process is dominated by gluon exchange in the crossed channel \[45\] which for a given scattering occurs at $O(\alpha_S^2)$. This constitutes the leading-order (LO) term of the BFKL resummation. The corresponding QCD amplitude factorizes into a gauge-invariant effective amplitude formed by two scattering centers, the LO impact factors, connected by the gluon exchanged in the crossed channel. The impact factors are characteristic of the scattering process at hand. The BFKL equation then resums the universal LL corrections, of $O(\alpha_S^3 \ln^n(\hat{s}/|\hat{t}|))$, to the gluon exchange in the crossed channel. These are obtained in the limit of a strong rapidity ordering of the emitted gluon radiation, i.e. for $n$ gluons produced in the scattering,

$$ y_1 \gg y_2 \gg \cdots \gg y_{n-1} \gg y_n. $$

(72)

Di-Jet production in hadron collisions at large rapidity intervals is the simplest process to which to apply the BFKL resummation, and one of the topical BFKL processes at the LHC, thus we shall use it as the paradigm process. Since di-jet production at large rapidity intervals is dominated by gluon exchange in the crossed channel, the functional form of the QCD amplitudes for gluon-gluon, gluon-quark or quark-quark scattering at LO is the same; they differ only by the colour strength in the parton-production vertices. We can then write the cross section in the following factorized form [458–460]

$$ \frac{d\sigma}{d^2p_{a\perp}d^2p_{b\perp}dy_au_dy_{b\nu}} = x_a^0 f_{a\text{eff}}(x_a^0, \mu_F^2) x_b^0 f_{b\text{eff}}(x_b^0, \mu_F^2) \frac{d\hat{\sigma}_{gg}}{d^2p_{a\perp}d^2p_{b\perp}}, $$

(73)

where $\mu_F$ is the factorisation scale, $a'$ and $b'$ label the forward and backward outgoing jet, respectively, and $p_{\perp}$ are two-dimensional vectors in the plane transverse to the collision axis, the azimuthal plane. $x_{a}^{0}$, $x_{b}^{0}$ are the parton momentum fractions in the high-energy limit,

$$ x_{a}^{0} = \frac{[p_{a\perp}]}{\sqrt{s}} e^{\eta_{a'}}, \quad x_{b}^{0} = \frac{[p_{b\perp}]}{\sqrt{s}} e^{-\eta_{b'}}, $$

(74)

and the effective parton distribution functions are [461]

$$ f_{a\text{eff}}(x, \mu_F^2) = f_{g}(x, \mu_F^2) + \frac{4}{9} \sum_{f} \left[ f_{q_{f}}(x, \mu_F^2) + f_{\bar{q}_{f}}(x, \mu_F^2) \right], $$

(75)

where the sum is over the quark flavours. In the high-energy limit, the gluon-gluon scattering cross section becomes [458]

$$ \frac{d\hat{\sigma}_{gg}}{d^2p_{a\perp}d^2p_{b\perp}} = \left[ \frac{C_A \alpha_S}{p_{a\perp}^2} \right] \hat{f}(q_{a\perp}, q_{b\perp}, \Delta y) \left[ \frac{C_A \alpha_S}{p_{b\perp}^2} \right], $$

(76)

with $C_A = N_c = 3$, $\Delta y = \eta_{a'} - \eta_{b'}$ and $q_{a\perp}$ the momenta transferred in the $t$-channel, with $q_{a\perp} = -p_{a\perp}$ and $q_{b\perp} = p_{b\perp}$, and where we use the shorthand for the magnitude squared, $[p_{\perp}]^2 \equiv p_{\perp}^2$. The quantities in square brackets are the LO impact factors for jet production. The function $\hat{f}(q_{a\perp}, q_{b\perp}, \Delta y)$ is the Green’s function associated with the gluon exchanged in the crossed channel. It is process independent and given in the LL approximation by the solution of the BFKL equation. This equation is a diagnostic tool for discriminating between different dynamical models for parton scattering. In the measurement of di-jet angular distributions, models which feature gluon exchange in the crossed channel, like QCD, predict a characteristic $\sin^{-1}(\theta^*/2)$ di-jet angular distribution [453–455], while models featuring contact-term interactions, which do not have gluon exchange in the crossed channel, predict a flattening of the di-jet angular distribution at large $\hat{s}/|\hat{t}|$ [456,457].
two-dimensional integral equation which describes the evolution in transverse momentum of the gluon propagator exchanged in the crossed channel. If we transform to moment space via

$$ f(q_{a\perp}, q_{b\perp}, \Delta y) = \int \frac{d\omega}{2\pi i} e^{\omega \Delta y} f_{\omega}(q_{a\perp}, q_{b\perp}) $$

(77)

we can write the BFKL equation as

$$ \omega f_{\omega}(q_{a\perp}, q_{b\perp}) = \frac{1}{2} \delta^2(q_{a\perp} - q_{b\perp}) + \frac{\alpha_S}{\pi} K[f_{\omega}(q_{a\perp}, q_{b\perp})], $$

(78)

with $\alpha_S = \alpha S N_c / \pi$, and where the kernel $K$ is given by

$$ K[f_{\omega}(q_{a\perp}, q_{b\perp})] = \int \frac{d^2 k_{\perp}}{k_{\perp}^2} \left[ f_{\omega}(q_{a\perp} + k_{\perp}, q_{b\perp}) - \frac{q_{a\perp}^2}{k_{\perp}^2 + (q_{a\perp} + k_{\perp})^2} f_{\omega}(q_{a\perp}, q_{b\perp}) \right]. $$

(79)

The first term in the kernel accounts for the emission of a real gluon of transverse momentum $k_{\perp}$ and the second term accounts for the virtual radiative corrections, which reggeise the gluon exchanged in the crossed channel. The solution to the BFKL equation is,

$$ f(q_{a\perp}, q_{b\perp}, \Delta y) = \frac{1}{(2\pi)^2} \sqrt{q_{a\perp} q_{b\perp}} \sum_{n=-\infty}^{\infty} e^{in\phi_{ab}} \int_{-\infty}^{\infty} d\nu e^{\omega(\nu,n)\Delta y} \left( \frac{q_{a\perp}^2}{q_{b\perp}^2} \right)^{i\nu}, $$

(80)

with $\phi_{ab}$ the azimuthal angle between $q_{a\perp}$ and $q_{b\perp}$, and $\omega(\nu,n)$ the eigenvalue of the BFKL equation

$$ \omega(\nu,n) = -\frac{1}{\alpha_S} \left[ \psi \left( \frac{|n| + 1}{2} + i\nu \right) + \psi \left( \frac{|n| + 1}{2} - i\nu \right) + 2\gamma_e \right], $$

(81)

with $\psi$ the digamma function, $\gamma_e = -\psi(1)$ the Euler constant, and with maximum at $\omega(0,0) \equiv \lambda = 4\alpha_S \ln 2$. Thus the solution of the BFKL equation resums powers of $\Delta y$. The resulting gluon-gluon cross section grows with $\Delta y$ as $f(q_{a\perp}, q_{b\perp}, \Delta y) \sim \exp(\lambda \Delta y)$ [305, 306], in contrast to the leading-order ($O(\alpha_S^2)$) cross section which is constant at large $\Delta y$.

In order to detect evidence of a BFKL-type behaviour in a scattering process, we need to have $\Delta y$ as large as possible. In di-jet production it can be done by minimizing the jet transverse energy, and maximizing $\hat{s}$. Since $\hat{s} = x_a x_b \hat{s}$, in a fixed-energy collider this is achieved by increasing the parton momentum fractions $x_{a,b}$, and then measuring e.g. the di-jet production rate $d\sigma / d\Delta y$. However, as the $x$’s grow the parton luminosity falls off, making it difficult to disentangle the eventual BFKL-driven rise of the parton cross section from the pdf’s fall off [459, 460]. One way to circumvent this problem is to use a variable-energy collider: the increase in $\hat{s}$ can then be achieved by fixing the $x$’s (and hence the pdf’s) and by letting the hadron center-of-mass energy $s$ grow. The advantage of this set-up is that variations in the pdf’s are minimised, while variations in the parton dynamics, and thus in the eventual underlying BFKL behaviour, are stressed [458, 462]. The D0 collaboration have recently attempted to uncover BFKL behavior in this way by comparing di-jet cross sections measured at $\sqrt{s} = 630$ GeV and 1.8 TeV [463]. In a contribution to this Workshop [464], the possibility of testing for BFKL-type behaviour by comparing di-jet cross sections at the Tevatron (2 TeV) and the LHC (14 TeV) has been investigated. The difficulty here is that one is comparing jets measured in two very different detectors, with resulting systematic uncertainties in the relative cross sections. One could also, of course, contemplate running the LHC at a lower collision energy. Note that a variable-energy configuration can be more easily realised: in forward-jet production in DIS, since a fixed-energy $ep$ collider is nonetheless a variable-energy collider in the photon-proton frame [465–470]; in $e^+ e^-$ collisions in double-tag events, $e^+ e^- \rightarrow e^+ e^- +$ hadrons, by varying the energy in the photon-photon frame [471, 472].

As a more practical alternative to varying the collider energy, one can study less inclusive observables. In particular, the correlation between the tagging jets, which at LO are supposed to be back to back,
is smeared by gluon radiation induced by parton showers and by hadronization. However, if we look at the correlation also as a function of $\Delta y$, we expect the (BFKL) gluon radiation in the rapidity interval between the jets to further blur the information on the mutual position in transverse momentum space, and thus the decorrelation to grow with $\Delta y$. Accordingly, the transverse momentum imbalance [459,473], and the azimuthal angle decorrelation [459,460,474–476] have been proposed as BFKL observables. In particular, it is straightforward to derive from (80) the prediction for the dependence of $\langle \cos \phi_{ab} \rangle$ on $\Delta y$:\footnote{In practice one integrates the di-jet transverse momenta above some threshold, $|p_{a\perp}|, |p_{b\perp}| > p_{\perp}^{\text{min}}$.} $\langle \cos \phi_{ab} \rangle \approx 0$. One finds [459,460,474–476] that $\langle \cos \phi_{ab} \rangle$ decreases rapidly from 1 at small $\Delta y$ (back-to-back jets), and approaches zero as $\Delta y \to \infty$. Such an azimuthal angle decorrelation has indeed been observed at the Tevatron Collider [448]. However, the LL BFKL formalism predicts a much stronger decorrelation than that observed in the data. On the other hand a NLO partonic Monte Carlo generator (JETRAD [222,477]), in which the exact $2 \to 2$ and $2 \to 3$ matrix elements are taken into account, predicts too little decorrelation. In fact the data are well described by the HERWIG Monte Carlo generator [116,171,211], which ‘dresses’ the basic $2 \to 2$ parton scattering with parton showers and also includes hadronization. Thus the present conclusion is that at least for di-jets with transverse momenta $> 20$ GeV and with rapidity intervals $< 6$ units, as analysed by the D0 Collaboration at the Tevatron, there is no evidence for LL BFKL-induced gluon radiation in the azimuthal angle decorrelation.

A possible explanation of the failure of the LL BFKL prediction to describe the Tevatron data is that the sub-leading corrections are large. There are various sources of such corrections: next-to-leading order corrections to the BFKL kernel in (79), which have recently been calculated (see Sect. 7.3), related running coupling effects\footnote{Note that the solution given in (80) assumes a fixed value for $\alpha_s$.}, and finally kinematic corrections that take into account the limited phase space available for BFKL-type gluon emission. In the derivation leading to the result (80), the transverse momentum of each emitted gluon is unbounded, and it is this unrestricted emission of gluons with transverse momenta $\sim |p_{a\perp}|, |p_{b\perp}|$ that leads to the strong decorrelation in azimuthal angle.

In an attempt to go beyond the analytic LL BFKL results, a Monte Carlo approach has been adopted [462,476,478]. By solving the BFKL equation (78) by iteration, which amounts to ‘unfolding’ the summation over the intermediate radiated gluons and making their contributions explicit, it is possible to include the effects of both the running coupling and the overall kinematic constraints. It is also straightforward to implement the resulting iterated solution in an event generator.

The first step in this procedure is to separate the $k_\perp$ integral in (78) into ‘resolved’ and ‘unresolved’ contributions, according to whether they lie above or below a small transverse energy scale $\mu$. The scale $\mu$ is assumed to be small compared to the other relevant scales in the problem (the minimum transverse momentum $p_{\perp}^{\text{min}}$ for example). The virtual and unresolved contributions are then combined into a single, finite integral. The BFKL equation becomes

$$
\omega f_\omega(q_{a\perp}, q_{b\perp}) = \frac{1}{2} \delta^2(q_{a\perp} - q_{b\perp}) + \frac{\alpha_s}{\pi} \int_{k_\perp^2 > \mu^2} \frac{d^2k_\perp}{k_\perp^2} f_\omega(q_{a\perp} + k_\perp, q_{b\perp}) 
+ \frac{\alpha_s}{\pi} \int \frac{d^2k_\perp}{k_\perp^2} \left[ f_\omega(q_{a\perp} + k_\perp, q_{b\perp}) \theta(\mu^2 - k_\perp^2) - \frac{q_{a\perp}^2 f_\omega(q_{a\perp}, q_{b\perp})}{k_\perp^2 + (q_{a\perp} + k_\perp)^2} \right].
$$

(82)

The combined unresolved/virtual integral can be simplified by noting that since $k_\perp^2 \ll q_{a\perp}^2, q_{b\perp}^2$ by construction, the $k_\perp$ term in the argument of $f_\omega$ can be neglected, giving

$$
(\omega - \omega_0) f_\omega(q_{a\perp}, q_{b\perp}) = \frac{1}{2} \delta^2(q_{a\perp} - q_{b\perp}) + \frac{\alpha_s}{\pi} \int_{k_\perp^2 > \mu^2} \frac{d^2k_\perp}{k_\perp^2} f_\omega(q_{a\perp} + k_\perp, q_{b\perp}),
$$

(83)

where

$$
\omega_0 = \frac{\alpha_s}{\pi} \int \frac{d^2k_\perp}{k_\perp^2} \left[ \theta(\mu^2 - k_\perp^2) - \frac{q_{a\perp}^2}{k_\perp^2 + (q_{a\perp} + k_\perp)^2} \right] = \alpha_s \ln \left( \frac{\mu^2}{q_{a\perp}^2} \right).
$$

(84)
The virtual and unresolved contributions are now contained in $\omega_0$ and we are left with an integral over resolved real gluons. We can now solve (83) iteratively, and performing the inverse transform we have

$$f(q_a, q_b, \Delta y) = \sum_{n=0}^{\infty} f^{(n)}(q_a, q_b, \Delta y).$$

where

$$f^{(0)}(q_a, q_b, \Delta y) = \left[ \frac{\mu^2}{q_{a\perp}} \right]^{\Delta y} \frac{1}{2} \delta^2(q_a - q_b)$$

$$f^{(n \geq 1)}(q_a, q_b, \Delta y) = \left[ \frac{\mu^2}{q_{a\perp}} \right]^{\Delta y} \left\{ \prod_{i=1}^{n} \int d^2k_{i\perp} dy_i \mathcal{F}_i \right\} \frac{1}{2} \delta^2(q_a - q_b - \sum_{i=1}^{n} k_{i\perp})$$

$$\mathcal{F}_i = \frac{\alpha_s}{\pi k_{i\perp}^2} \theta(k_{i\perp}^2 - \mu^2) \theta(y_i - y_i') \left[ \frac{(q_a + \sum_{j=1}^{n-1} k_{j\perp})^2}{(q_a + \sum_{j=1}^{n} k_{j\perp})^2} \right]^{\Delta y}$$

Thus the solution to the BFKL equation is recast in terms of phase space integrals for resolved gluon emissions, with form factors representing the net effect of unresolved and virtual emissions. Unlike in the case of DGLAP evolution, there is no strong ordering of the transverse momenta $k_{i\perp}$. Strictly speaking, the derivation given above only applies for fixed coupling because we have left $\alpha_s$ outside the integrals. The modifications necessary to account for a running coupling $\alpha_s(k_{i\perp}^2)$ are straightforward [476].

The expression for $f$ in (85,86) above is amenable to numerical integration, and one can for example reproduce the analytic result given in (80). More importantly, having made explicit the BFKL gluon emission phase space, we can impose overall energy and momentum conservation. In particular the parton momentum fractions in the presence of BFKL gluon emission become

$$x_a = \frac{e^{y_a'}}{\sqrt{s}} \left( |p_{a'} \perp| + |p_{b'} \perp| e^{-\Delta y} + \sum_i |k_i \perp| e^{-y_i - y_a'} \right),$$

$$x_b = \frac{e^{-y_b'}}{\sqrt{s}} \left( |p_{b'} \perp| + |p_{a'} \perp| e^{-\Delta y} + \sum_i |k_i \perp| e^{-y_i + y_b'} \right).$$

The momentum fractions in the high-energy limit given in (74) are recovered by imposing strong rapidity ordering, eq. (72). Note that the requirement $x_a, x_b \leq 1$ effectively imposes an upper limit on the transverse momentum ($k_{i\perp}$) integrals. This in turn means that the analytic result (80) is not reproduced in the presence of such a constraint, since they require the internal transverse momenta integrals to extend to infinity. Formally, the kinematic constraints $x_a, x_b \leq 1$ induce an infinite sequence of sub-leading logarithms $\alpha_s^n \Delta y^{-n-1}, \alpha_s^n \Delta y^{-n-2}, \ldots$ that suppress the growth of the parton scattering cross section with $\Delta y$.

Applying kinematic constraints and including the running coupling suppresses the emission of energetic BFKL gluons, and therefore weakens the azimuthal decorrelation predicted at LL level [476, 478]. As a result, reasonable agreement with the D0 decorrelation data is recovered. It is clear, therefore, that one needs a higher-energy collider such as the LHC in order to discriminate between the BFKL and parton shower (DGLAP) dynamics.

Figure 36 shows the mean value of $\cos \Delta \phi$ in di-jet production in an improved BFKL MC approach [479] that includes kinematic constraints and running couplings (upper curves). The jets are completely correlated (i.e. back-to-back in the azimuthal plane) at $\Delta y = 0$, and as $\Delta y$ increases we see the characteristic BFKL decorrelation, followed by a flattening out and then an increase in $\langle \cos \Delta \phi \rangle$ as the kinematic limit is approached$^{48}$.

$^{48}$For any given transverse momentum threshold, there is some $\Delta y$ at which the jet pair $(a', b')$ alone saturates the kinematic limit, and emission of additional (real) gluons is completely suppressed and the azimuthal correlation returns. As we approach that limiting value of $\Delta y$ we therefore expect to see a transition back towards correlated jets.
effect when the $p_{T}^{\min}$ threshold is set at 50 GeV (dashed curve) than at 20 GeV (solid curve); in the latter case more phase space is available to radiate gluons. We also show for comparison the decorrelation for di-jet production at the Tevatron for $p_T > 20$ GeV. There we see that the lower collision energy (1.8 TeV) limits the allowed rapidity difference and substantially suppresses the decorrelation at large $\Delta y$. Note that the larger center-of-mass energy compared to transverse momentum threshold at the LHC would seem to give it a significant advantage as far as observing BFKL effects is concerned.

The lower set of curves in Fig. 36 refer to Higgs production via the $W W, ZZ$ fusion process $qq \rightarrow qqH$, and are included for comparison [479]. This process automatically provides a 'BFKL-like' di-jet sample with large rapidity separation, although evidently the jets are significantly less correlated in azimuthal angle.

In summary, the LHC offers an important test of BFKL dynamics in the production of relatively low transverse momentum jet pairs with a large rapidity separation. In this section we have given an overview of the relevant theory. An important next step is to include the effects of the next-to-leading order contributions to the BFKL kernel, and to consider other related processes with gluon exchange in the crossed channel\footnote{Examples include $qq \rightarrow Wqg, gg \rightarrow b\bar{b}g$ etc.}. On the experimental side, it remains a challenge to trigger on such low $p_T$ jets in the far forward regions of the detector.

### 7.2 Small-$x$ Effects in Final States\footnote{Contributing authors: C. Ewerz and B.R. Webber.}

To understand the special features of QCD dynamics at small $x$, it will be essential not only to study the fully inclusive cross sections for small-$x$ processes at the LHC, such as the Drell-Yan process at dilepton mass-squared $Q^2$ much smaller than the c.m. energy-squared, but also to investigate the structure of the associated final states. One important aspect of the final state is the number of mini-jets produced. By mini-jets we mean jets with transverse momenta above some resolution scale $\mu_R$, where $\mu_R^2 \ll Q^2$. Thus the mini-jet multiplicity at small $x$ involves not only $\ln x \gg 1$ but also another large logarithm, $T = \ln(Q^2/\mu_R^2)$, which needs to be resummed. The results presented below include all terms of the form $(\alpha_S \ln x)^m T^m$ where $1 \leq m \leq n$. Terms with $m = n$ are called double-logarithmic (DL) while those with $1 \leq m < n$ give single-logarithmic (SL) corrections. The DL contributions to the mini-jet multiplicity have been obtained in [480], and the SL terms have been included in [481, 482]. In these
calculations the BFKL formalism [302, 306] has been used, but the results are expected to hold [483]
also in the CCFM formalism [390, 391, 484, 485] based on angular ordering of gluon emissions.

We start by considering the gluon structure function at the momentum scale \( Q^2, F(x, Q^2) \). It is the
sum of contributions \( F^{(\text{jet})}(x, Q^2, \mu^2) \) in which different numbers \( r \) of final-state mini-jets are resolved
with transverse momentum greater than \( \mu_r \),

\[
F^{(\text{jet})}(x, Q^2, \mu^2) = F(x, \mu^2) \otimes G^{(r)}(x, T) = \int_x^1 \frac{dz}{z} F(z, \mu^2) G^{(r)}(x/z, T) .
\]

To determine the coefficient function \( G^{(r)} \) to leading logarithmic order in \( x \), it is convenient to apply a
Mellin transformation,

\[
f_\omega(\ldots) = \int_0^1 dx \, x^\omega f(x, \ldots) .
\]

In \( \omega \)-space the evolution of the structure function is \( F_\omega(Q^2) = \exp[\gamma_L(\alpha_s/\omega)T] F_\omega(\mu^2) \), where \( \gamma_L \) is the
Lipatov anomalous dimension, i.e. the solution obtained from eq. (81) by setting \( n = 0 \) and \( \gamma = 1/2 + i \nu \),

\[
\omega = -\alpha_s \left[ \psi(\gamma) + \psi(1 - \gamma) + 2\gamma \right] \equiv \alpha_s \chi(\gamma) .
\]

The Lipatov anomalous dimension can be written as an expansion in powers of \( \alpha_s/\omega \),

\[
\gamma_L(\alpha_s/\omega) = \left( \frac{\alpha_s}{\omega} \right)^4 + 2\left( \frac{\alpha_s}{\omega} \right)^6 + \ldots .
\]

In [482] it has been shown that the generating function \( G_\omega(u, T) = \sum_{r=0}^{\infty} u^r G^{(r)}_\omega(T) \) can be written as

\[
G_\omega(u, T) = \frac{I_\omega(u, 0)}{I_\omega(u, T)} ,
\]

where

\[
I_\omega(u, T) = \int_\Gamma \frac{d\gamma}{\gamma} e^{-\gamma T + \phi_\omega(u, \gamma)} ,
\]

\( \Gamma \) being a contour parallel to the imaginary axis on the left of all singularities of the integrand, and

\[
\phi_\omega(u, \gamma) = \frac{u}{u - 1} \int_{\frac{1}{2}}^\gamma \frac{d\gamma'}{u} \left[ \frac{\omega}{\alpha_s u} - \chi(\gamma') \right] .
\]

One can obtain the moments of the jet multiplicity distribution from the generating function as follows:

\[
\overline{r} \ldots (r-s+1) = \exp[-\gamma_L(\alpha_s/\omega)T] \frac{\partial^n G_\omega}{\partial u^n} \bigg|_{u=1} .
\]

Using the expressions (92)-(94) we thus find for the mean number of jets

\[
\overline{r}_\omega = -\frac{1}{\chi'} \left( \frac{1}{\gamma_L} + \frac{\chi''}{2\chi'} + \chi \right) T - \frac{1}{2\chi'} T^2
\]

where \( \chi' \) means the derivative of \( \chi(\gamma) \) evaluated at \( \gamma = \gamma_L \). The corresponding expression for the variance in the number of jets, \( \sigma^2_\omega \equiv r^2 \omega - \overline{r}^2_\omega \), is more complicated [482]. Interestingly, the variance is a polynomial of third degree in \( T \). This implies that the distribution in the number of jets remains narrow for large \( T \) in the sense that its width grows slower than its mean.

Considered as functions of \( \omega \) the coefficients of the powers of \( T \) in eq. (96) and in the corre-
sponding expression for \( \sigma^2_\omega \) [482] exhibit bad behaviour at large values of \( \alpha_s/\omega \). This is associated
with the singularity of the leading-order Lipatov anomalous dimension \( \gamma_L \) at \( \alpha_s/\omega = (4\ln 2)^{-1} \). We
would expect this behaviour to be modified strongly by higher order corrections. Although the next-to-leading corrections to $\gamma_1$ are known [67, 307, 400] a full calculation of the corresponding corrections to the associated jet multiplicity has not been performed and would appear very difficult.

For practical purposes it is necessary to determine the multiplicity moments as functions of $x$. This can be done using (90) and the perturbative expansion (91) of the anomalous dimension. The inverse Mellin transformation can then be applied to this series term by term using

$$\frac{1}{2\pi i} \int_C d\omega \, x^{-\omega-1} \left( \frac{\bar{\sigma}_S}{\omega} \right)^n = \bar{\sigma}_S \frac{\bar{\sigma}_S \ln(1/x)]^{n-1}}{x (n-1)!}. \tag{97}$$

In this way one easily finds a series for the inverse Mellin transform $\bar{\tau}(x)$ of $\bar{\tau}_\omega$, for example. We note that the factorial in the denominator makes the resulting series in $x$-space converge very rapidly. It is then straightforward to compute the mini-jet multiplicity associated with point-like scattering on the gluonic component of the proton at small $x$ using

$$n(x) = \frac{F(x, Q^2) \otimes \bar{\tau}(x)}{F(x, Q^2)}. \tag{98}$$

To illustrate the effects of BFKL resummation we compute the number of associated jets in central Higgs production at the LHC. The dominant production process for a SM Higgs boson at the LHC is expected to be gluon-gluon fusion. The production cross section for a Higgs boson of mass $M_H$ and rapidity $y$ by gluon-gluon fusion in proton-proton collisions at centre-of mass energy $\sqrt{s}$ takes the form

$$\frac{d\sigma}{dy} = F(x_1, M_H^2) \ F(x_2, M_H^2) \ C(M_H^2), \tag{99}$$

where for central production of the Higgs ($y = 0$) we have $x_1 = x_2 = M_H/\sqrt{s}$, and for LHC $\sqrt{s} = 14$ TeV. $C$ represents the $gg \to H$ vertex, which is perturbatively calculable as an intermediate top-quark loop. A more careful treatment would involve replacing the Higgs production vertex $C(M_H^2)$ by an impact factor $C(M_H^2, k_t^2, k_T^2)$ and convoluting it with unintegrated gluon densities taken at the off-shell gluon virtualities $k_t^2$ and $k_T^2$, respectively. The dependence of the impact factor $C(M_H^2, k_t^2, k_T^2)$ on these virtualities is expected to be weak, and we have neglected it to arrive at eq. (99). Then $C$ cancels in the mean number of mini-jets and its dispersion, and we do not need to know its detailed form.

Since the gluon emissions in the regions of positive and negative rapidity are independent, we can simply add the numbers $n_1 = n(x_1)$ and $n_2 = n(x_2)$ of mini-jets produced in these regions. The mean multiplicity $N$ of associated mini-jets becomes

$$N(x) = n_1 + n_2 = 2n(x), \tag{100}$$

where $n(x)$ can be calculated as in (98) after replacing $Q^2$ by $M_H^2$. Similarly, the variance is

$$\sigma^2_N(x) = \sigma^2_n(x_1) + \sigma^2_n(x_2) = 2\sigma^2_n(x). \tag{101}$$

The variance $\sigma^2_n$ can be obtained in a similar way as the mean (for details, see ref. [482]).

We have calculated the dependence of $N$ and $\sigma_N$ on the Higgs mass $M_H$ using the leading-order MRST gluon distribution [28]. Our numerical results are shown in fig. 37. The DL results, obtained by keeping only the first term in eq. (91), give an excellent approximation and the SL terms are less significant. We see that the mini-jet multiplicity and its dispersion are rather insensitive to the Higgs mass at the energy of the LHC. The mean number of associated mini-jets is rather low, such that the identification of the Higgs boson should not be seriously affected by them. In view of the rapid convergence of the perturbative series in $x$-space we do not expect the result for the mini-jet multiplicity to be strongly modified by higher order corrections.

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$^{51}$We do not count any jets emerging from the proton remnants.
7.3 The next-to-leading corrections

As has already been discussed, in practically all experimental contexts, the LL BFKL equations fail to reproduce the data. It is likely that the problem is due to the presence of significant sub-leading corrections.

The next-to-leading logarithmic (NLL) correction terms $\alpha_S(\alpha_S \ln s)^n$ are therefore of particular interest. Such terms can arise for example from configurations containing a pair of particles which are close in rapidity, or due to the running of the coupling. We write the kernel of the BFKL equation (78) as

$$\mathcal{K} [f_\omega(q_{a\perp}, q_{b\perp})] = \mathcal{K}_0 [f_\omega(q_{a\perp}, q_{b\perp})] + \alpha_S \mathcal{K}_1 [f_\omega(q_{a\perp}, q_{b\perp})] + \mathcal{O}(\alpha_S^2),$$

(102)

where $\mathcal{K}_0$ is the LL kernel (79), and $\mathcal{K}_1$ contains the NLL corrections. A number of different pieces contribute to $\mathcal{K}_1$: the emission of two close-in-rapidity partons (two gluons [401, 486] or a $q\bar{q}$ pair [398, 399, 402, 487, 488]) from the gluon ladder; the one-loop corrections [395–397, 489, 490] to the emission of a gluon from the ladder; the NLL corrections to a reggeised gluon [393, 394, 491, 492]. The various pieces were put together in [67, 307, 400].

The resulting corrections have a number of interesting features, such as the fact that they imply the emitted transverse momentum as being the appropriate scale for $\alpha_S$, and certain parts of the resulting kernel can be associated with physical contributions such as the finite-$z$ part of the DGLAP splitting functions. However from the point of view of their direct use in phenomenology, the NLL corrections present problems: applying the NLL kernel to the LL eigenfunctions, $(k^2_\perp)^\gamma$, with $\gamma$ as in eq. (90), the BFKL exponent becomes [67, 307]

$$\lambda \simeq 4 \ln 2 \alpha_S (1 - 6.2 \alpha_S),$$

(103)

and inserting a value of $\alpha_S = 0.2$ relevant for many BFKL studies leads to a negative power. A detailed study of the resummation of the kernel reveals the even worse property that for $\alpha_S > 0.05$ the NLL corrections lead to negative cross sections [493].

7.3.1 Beyond NLL

At first sight one might therefore conclude that the NLL corrections remove all predictive power from BFKL physics. Various groups have however proposed rather different approaches for the inclusion and

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\(52\)Contributing author: G.P. Salam.
resummation of higher-order terms with a view to stabilising the perturbative series. Three basic strategies have been suggested: BLM resummation together with an appropriate scheme change, a rapidity veto, and resummation of collinearly enhanced terms.

A standard approach in situations where the perturbative series converges slowly is to apply a scale change. One such procedure is BLM scale setting \cite{494}, where it is argued that for any given observable, some of the NLL corrections come from the natural scale being different from $Q^2$, and that the appropriate scale can be deduced from the coefficient of the $N_f$-dependent part of the NLL correction. In \cite{416} the procedure is applied to the BFKL NLL corrections. The authors find that in the $\overline{\text{MS}}$ scheme, BLM scale setting makes little difference to the poor convergence of the series. They then show that in certain other schemes, notably the MOM (based on the symmetric triple-gluon vertex) and $\Upsilon$ (based on $\Upsilon \to ggg$ decay) schemes, the coefficient of the $N_f$ dependence is significantly modified — the BLM resummation then has a much larger effect leading to an estimate for the exponent, $\lambda \approx 0.15$ fairly independently of $Q^2$. The problem of negative cross sections still persists however, albeit to a lesser extent. There are also questions regarding the naturalness of the particular scheme choices that are required in order to obtain a stable answer, there being arguments both for and against.

The rapidity veto approach has been studied in detail in \cite{417}. The background of this approach is that the BFKL kernel is formally valid only for branchings separated by a large rapidity — but to obtain the high-energy power-growth one then normally integrates over all possible rapidity intervals between successive branchings, including small rapidities. One can equally well place a rapidity veto, i.e. integrate only over rapidities beyond some cut, $\Delta y$, of order 1 or 2. This corresponds to introducing a set of corrections at NLL and beyond, and one argues that part of the actual NLL corrections may come from something akin to such a rapidity veto. One then studies the effect of the rapidity veto at all orders (while fixing the NLL corrections). This was done in \cite{417} where it was found that for large rapidity vetoes ($\Delta y > 2.2$) the exponent $\lambda$ is quite stable against variations in $\Delta y$ and that the problems of negative cross sections disappear. But for smaller rapidity vetoes, the usual problems persist.

The two above approaches conjecture some new physical effect (natural non-Abelian scheme, rapidity veto). The third approach is a little different in that it takes the small-$x$ kernel and supplements it in such a way as to render it consistent with DGLAP evolution in the collinear and anti-collinear limits, i.e. where one of the interacting objects has a much larger transverse scale than the other. The motivation for doing this comes from the observation that while the convergence of the small-$x$ expansion is poor for normal high-energy scattering (both objects of the same transverse scale), for (anti)collinear high-energy scattering the expansion becomes far worse and so must be resummed: technically speaking, the LL characteristic function\footnote{In the notation of Sect. 7.1 and generalising eq. (90), $\omega(\nu, 0) = \pi \chi_0(1/2 + \hat{\nu}) + \pi \chi_1(1/2 + \hat{\nu}) + \ldots$. Higher azimuthal components $\omega(\nu, n \geq 1)$ are not included. However, they contribute only to azimuthal angle correlations such as those discussed in Sect. 7.1.} $\chi_0(\gamma)$ diverges as $1/\gamma$ in the collinear limit $\gamma \to 0$, while the NLL function, $\chi_1(\gamma)$, diverges as $1/\gamma^3$. Since the structure of these divergences is governed by collinear physics, it can be calculated at all orders. It turns out that there are double and single collinear logs and alone they are responsible for most of the NLL correction even outside the collinear region. They have been resummed respectively in \cite{308, 495} and \cite{309, 420}, leading to a stable result for the exponent $\lambda$, free of the problem of negative cross sections. The dependence of $\lambda$ on $\alpha_S$ is shown in figure 38, together with the leading and next-to-leading results, for comparison. There is relatively little dependence on changes of scheme and scale \cite{420} and on the additional introduction of a rapidity veto \cite{418}. This approach therefore seems to be the most likely candidate for practical phenomenology.

### 7.32 Spin-offs from the NLL results: understanding running coupling

One of the spin-offs of the NLL corrections was that they identified the correct scale to be used in the kernel: $\alpha_S(q^2)$, where $q$ is the emitted transverse momentum. However to understand the effects of running coupling in high-energy cross sections it is necessary to understand the iteration of the kernel
with running coupling. The two contexts of interest are for quantities such as Mueller-Navelet jets, and for anomalous dimensions.

In the former, one has a situation where diffusion takes place both above and below the scale set by the jets. The running of the coupling causes diffusion below the typical scale $E_T^2$ of the jets to be enhanced compared to that above — as a result, as the rapidity separation increases and diffusion increases, evolution below $E_T^2$ is increasingly favoured, and the cross section grows faster than $e^{a(E_T^2)Y}$; an extra term appears in the exponent, proportional to $a_S^5(E_T^2)Y^3$ [426, 496]. This causes the effective power growth to increase gradually. A second, recently hypothesised effect called tunneling [421], should at a certain point cause a sudden increase in the observed power growth, as the contribution from very-low-scale evolution becomes larger than that from evolution at scales of order $E_T^2$. This happens at a rapidity of $Y \approx \ln Q^2/\lambda_P$, where $\lambda_P$ is the exponent characteristic of low scales. It remains to be seen whether such an effect will be phenomenologically observable.

Another quantity for which running coupling effects turn out to be very important is anomalous dimensions, or equivalently small-$x$ splitting functions. Very schematically, anomalous dimensions at a scale $Q^2$ seem to involve small-$x$ branching only above $Q^2$: branching below that scale has already been factorized out. Consequently they sample a region where the running coupling is smaller than $a_S(Q^2)$. Thus the observed small-$x$ exponent of the anomalous dimension, $\lambda_\gamma(Q^2)$, is smaller than the exponent $\lambda(Q^2)$ relevant in say Mueller-Navelet jets with scale $E_T^2 = Q^2$ [420, 421, 497]. An alternative point of view [415, 419] is discussed in Sect. 5.4.

8. DOUBLE PARTON SCATTERING

8.1 Introduction

The large flux of partons, which becomes available for hard collisions at high energies, justifies the expectation, at the LHC, of sizeable effects due to the unitarization of the hard component of the interaction. In fact it is not difficult to foresee hard collision processes with a cross section larger than the total cross section itself [498, 499]. Such a result is not inconsistent, if one keeps into account that the inclusive cross section, described by the single scattering expression of the QCD-parton model, includes a multiplicity factor which keeps into account the possibility of having several partonic interactions in the same hadronic inelastic event [500, 501]. The possibility of hard processes with multiple parton in-

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Fig. 38: The high-energy exponent in various approaches; $\lambda$ is the exponent relevant to processes such as Mueller-Navelet jets, including the NLL corrections and collinear improvements; the equivalent exponent relevant to anomalous dimensions is $\lambda_\gamma$.
teractions, namely different pair of partons interacting independently with a large momentum transfer in the same hadronic collision, was on the other hand foreseen long ago by several authors [502–514]. In a multi-parton interaction the different pairs of interacting partons are separated in transverse space by a distance of the order of the hadron radius. As a consequence the transverse momenta have to be balanced independently in the different partonic collisions, giving in this way a well defined characterization to the process. The simplest event of that kind, the double parton scattering, has been a topic of experimental search of all high energy hadron collider experiments since several years [515–517]. While initially the results have been sparse and not very consistent, recently CDF has reported the observation of a large number of events with double parton scatterings [175, 176].

8.2 Cross section for double parton scattering

The inclusive cross section of a double parton scattering has a simple probabilistic expression. Interference effects between the two partonic collisions are in fact negligible, since the partonic interactions are localized in a much smaller region, with a size of the order of the inverse of the momentum transfer, as compared to the distance in transverse space between the different partonic interactions. The non-perturbative component of the process gets factorized, as a consequence, into a function which depends on the fractional momenta of the partons taking part the interaction and on their distance in transverse space, which has to be the same for both the target and the projectile partons, in order to have the alignment which is needed for the interaction to occur. One obtains therefore for the double parton scattering cross section the expression (see fig. 40)

\[
\sigma_D = \frac{1}{2} \int_{p_T} \Gamma_A(x_1, x_2; b) \hat{\sigma}(x_1, x'_1) \hat{\sigma}(x_2, x'_2) \Gamma_B(x'_1, x'_2; b) dx_1 dx_1' dx_2 dx_2' d^2b, \tag{104}
\]

where the non-perturbative input is the two-body parton distribution \( \Gamma(x_1, x_2; b) \), whose arguments are the two fractional momenta, \( x_1 \) and \( x_2 \), and the distance of the two partons in transverse space \( b \). The partonic cross sections, \( \hat{\sigma}(x, x') \), are integrated on the momentum transfer, at a fixed value of the partonic center of mass energy, and the cutoff \( p_T^{cut} \) is introduced to regularize the singularity at small \( p_T \) and at small \( x \) values. The two-body parton distributions \( \Gamma(x_1, x_2; b) \) represent the new property of the hadron structure which becomes accessible through the observation of the double parton collision processes. It is a non-perturbative quantity which is independent on the one-body parton distributions, namely on the non-perturbative input to the large \( p_T \) processes usually considered. The two-body parton distributions are in fact related directly to the two-body parton correlations in the hadron structure.

If the two pairs of partons undergoing the hard interactions are not correlated in \( x \) and if the dependence on \( b \) can be factorized, the two-body parton distributions are nevertheless expressed as \( \Gamma(x_1, x_2; b) = f(x_1) f(x_2) F(b) \), where \( f(x) \) is the usual one-body parton distribution, appearing in
large $p_T$ inclusive processes, and $F(b)$ is a function which describes the distribution of the partons in transverse space. With these assumptions the cross section for a double parton collision leads, in the case of two indistinguishable parton interactions, to the simplest factorized expression

$$
\sigma_D(p_T^{\text{int}}) = \frac{[\sigma_S(p_T^{\text{int}})]^2}{2\sigma_{\text{eff}}},
$$

(105)

where $\sigma_S$ is the usual inclusive cross section of the perturbative QCD, i.e. the convolution of parton distributions with the partonic cross section, $p_T^{\text{int}}$ is the lower integration threshold and $\sigma_{\text{eff}}$ is a scale factor, with dimensions of a cross section. It is the result of the integration on the transverse distance $b$, actually $1/\sigma_{\text{eff}} = \int d^2 b F^2(b)$. All the information on the parton correlation in transverse space is summarized in $\sigma_{\text{eff}}$ [518]. The geometrical origin of $\sigma_{\text{eff}}$ justifies the expectation that its value is both a energy and cutoff independent quantity.

The double parton scattering process has been measured at Fermilab by CDF by looking at final states with three mini-jets and one photon [175, 176]. The measured value of the scale factor is:

$$
\sigma_{\text{eff}} = 14.5 \pm 1.7^{+1.7}_{-2.3} \text{ mb}.
$$

(106)

In the limited range of $x$ experimentally accessible, $\sigma_{\text{eff}}$ does not show evidence of dependence on the fractional momenta, which indicates that the simplest hypotheses above are not in contradiction with the experiment.

The qualitative features of the double parton scattering process are easily read in the factorized expression in Eq. (105). As a consequence of the proportionality of $\sigma_D$ with $\sigma_S^2$, the double parton scattering cross section is characterized by a rapid decrease for $p_T \rightarrow \infty$ and by a rapid growth for $p_T \rightarrow 0$. As for the energy behavior, $\sigma_D$ increases faster with $s$ as compared to the single scattering cross section (it goes as $\sigma_S^2$). Multiple parton collisions are therefore enhanced at the LHC.

### 8.3 Four jet production

The most obvious case where multiple parton collisions play a role at high energy is in the production of jets, since the integrated cross section can easily exceed the unitarity limit at large energies and with a fixed value of $p_T^{\text{int}}$. One has in fact that, for any value of $p_T^{\text{int}}$, when $s$ is sufficiently large $\sigma_S > \sigma_{\text{inel}}$. The simplest case to consider is the production of four large $p_T$ jets, where one can compare the leading $(2 \rightarrow 4)$ process with the power suppressed $(2 \rightarrow 2)^2$ double parton collision.

In fig. (41) we show the expected rates of production of four large $p_T$ jets in the central rapidity region ($|y| < 3$) with the two different production mechanisms, as a function of the lowest value of the transverse momenta of the produced jets $p_T^\text{min}$. The continuous curve is the expected cross section as from the leading QCD production mechanism $(2 \rightarrow 4)$ [122, 519]. The dashed curve is the double parton collisions $(2 \rightarrow 2)^2$ cross section. The curve representing the double parton collision in fig. (41) has to be regarded as a lower limit, rather than as the expected rate of the double parton collision process, since no factor $K$, accounting for higher order correction terms in $\alpha_S$, has been included in the evaluation. Notice that higher order corrections in $\alpha_S$ will contribute with a factor $K^2$ in the double parton collision cross section. The overall qualitative feature is that, at the LHC, the double parton collision dominates, with respect to the leading QCD single scattering interaction, when one of the jets has a transverse momentum which becomes as low as 20 GeV.

### 8.4 $l + b\bar{b}$ production

Although multi-parton collisions have been mostly considered to describe the multiplicity distributions in high energy hadronic interactions (for a discussion of multi-parton interactions at LHCb, we refer the reader to the Bottom Production Chapter of this Report), the role of multi-parton collisions is not limited to the case of production of large or relatively large $p_T$ jets. One may find in fact various other
processes of interest at the LHC where multiple parton collisions are relevant \cite{173, 174}. While $\sigma_{eff}$ may depend in principle on the different species of partons involved in the interaction, $\sigma_{eff}$ should not vary much in the different processes and one would expect that it is, to a large extent, a process independent quantity \cite{178}. We will therefore consider it, in the following, as a universal quantity and we will use for $\sigma_{eff}$ the value which has been measured in the CDF experiment. The cross section of a double parton interaction, resulting from the two distinguishable parton collisions 1 and 2, is therefore expressed as

$$\sigma_D = \frac{\sigma_A \sigma_B}{\sigma_{eff}}.$$  \hspace{1cm} (107)

As a meaningful example we have considered the production of an isolated lepton and of a $b\bar{b}$ pair \cite{520}, which represents an interesting channel to detect the Higgs boson production at the LHC in the intermediate Higgs mass range, $80\text{GeV} < M_H < 150\text{GeV}$. A background to the process $p + p \rightarrow WH + X$, with $W \rightarrow l\nu_l$ and $H \rightarrow b\bar{b}$, is represented by the double parton scattering interaction where the intermediate vector boson $W$ and the $b\bar{b}$ pair are created in two independent parton interactions. If one uses $\sigma(W) \times BR(W \rightarrow l\nu_l) \approx 40\text{nb}$ \cite{10} and $\sigma(b\bar{b}) \approx 5 \times 10^2\mu\text{b}$, one obtains for the double collision cross section the value of 1.4 nb. The Higgs production cross sections, $p + p \rightarrow WH + X$, with $W \rightarrow l\nu_l$ and $H \rightarrow b\bar{b}$, has been estimated to be rather of order of 1 pb \cite{521, 522}. Obviously the three orders of magnitude of difference in the integrated cross section are mainly due to the configurations where the $b\bar{b}$ pair is produced with an invariant mass close to the threshold of $b\bar{b}$ production. The expected background to the Higgs production signal, caused by the double parton collision process, is shown in fig. (42) as a function of the invariant mass of the $b\bar{b}$ pair.

In fig. (42) we have plotted the expected signal in the $b\bar{b}$ invariant mass due to the Higgs boson production for three possible values of the Higgs mass, 80, 100 and 120 GeV. The dashed line is the double parton scattering background at the LO in perturbation theory. The continuous line is the result for the double parton scattering background when computing the $b\bar{b}$ cross section at order $\alpha_3^S$ \cite{251}.

In fig. (43) we compare the signal and the background after applying all the typical cuts considered to select the Higgs signal in this channel \cite{521}:

- for the lepton we require: $p_{T,l} > 20\text{GeV}, |\eta_l| < 2.5$ and isolation from the $b$’s, $\Delta R_{l,b} > .7$

![Fig. 41: Integrated cross section for production of four jets with $|y| < 3$ as a function of the lowest transverse momentum of the jets $p_T^{min}$. The continuous curve is the expected cross section as from the leading QCD production mechanism $2 \rightarrow 4$, the dashed curve is the expected cross section due to the contribution of double parton collisions $(2 \rightarrow 2)^2$.](image-url)
Fig. 42: Double parton scattering background to Higgs boson production in association with a $W$ as a function of the $b\bar{b}$ invariant mass. The expected Higgs signal is for three possible values of the Higgs mass, 80, 100 and 120 GeV. The dashed line is the background at the LO in perturbation theory. The continuous line is the result for the double parton scattering background when computing the cross section at order $\alpha_S^3$ [251].

- - for the two $b$ partons: $p_T^b > 15$ GeV, $|\eta^b| < 2$ and $\Delta R_{b,\bar{b}} > .7$

As in the previous figure the Higgs signal in the $b\bar{b}$ invariant mass corresponds to three possible values for the mass of the Higgs boson, 80, 100 and 120 GeV. The dotted line is the single parton scattering background, where the $Wb\bar{b}$ state is created directly in a single partonic interaction. The dashed line is the expected background originated by the double parton scattering process, evaluated by estimating the $b\bar{b}$ production cross section at $O(\alpha_S^3)$. The continuous line is the total expected background. In the calculations of the background and signal we used, for the LO matrix elements, the packages MadGraph [133] and HELAS [523]. The integration was performed by VEGAS [141] with the parton distributions MRS99 [10].

Also after using the more realistic cuts just described, the double parton scatterings process remains a rather substantial component of the background, as one may see by comparing in fig. (43) the total background estimate (continuous curve) with the more conventional single scattering background estimate (dotted curve).

8.5 Summarizing remarks

At the LHC one has to expect large effects from multiple parton collisions in various processes of interest. To the purpose of illustration, we have presently studied the production of a $b\bar{b}$ pair in association with a $W$ boson, followed by the decay $W \rightarrow l\nu$, in the mass range $M_{b\bar{b}} \approx 100$ GeV. The channel is of interest for the observation of the Higgs boson production when the Higgs mass is below the threshold of $W^+W^-$ production. We find that, if one applies the standard cuts to the final state usually considered to isolate the Higgs signal in this channel, the background due to double parton scatterings ($b\bar{b}$ pair and $W$ boson produced in two different partonic interactions) is comparable to the more traditional background, where the $b\bar{b}$ pair and the $W$ boson are produced in a single parton collision. A similar situation can be expected with several other final states:
• $Zb\bar{b}$,
• $W + \text{jets}$, $Wb + \text{jets}$ and $Wb\bar{b} + \text{jets}$,
• $t\bar{t} \rightarrow llb\bar{b}$,
• $t\bar{b} \rightarrow b\bar{b}\nu\nu$,
• $b\bar{b} + \text{jets}$,
• final states with many jets when $p_T^{\text{min}} \simeq 20, 30 \text{ GeV}$.

The well definite characterization of the states produced by the multiple parton scattering processes allows nevertheless one to figure out more efficient selection criteria to get rid of this further background source, or to measure it in a precise way. The present analysis however points out that, as a consequence of the enhanced role of multiple parton collisions at high energy, a detailed and systematic study of the expected rates and backgrounds, due to multiple parton collision processes, is of great importance at the LHC and it represents one of the topics which have to be addressed seriously in the next future.

9. BACKGROUNDS TO NEUTRAL HIGGS BOSONS SEARCHES

9.1 Introduction

The most important goal of the physics programme of the LHC experiments ATLAS [1] and CMS [524] is to perform measurements which lead to the understanding of the mechanism of electroweak symmetry breaking. In the framework of the SM, as well as its extensions, e.g. super-symmetric (SUSY), it translates into the major topic of Higgs boson searches. The SM assumes one doublet of scalar fields, implying the existence of one neutral scalar particle. In SUSY models, the Higgs sector is extended to contain at least two doublets of scalar fields leading to the prediction of five Higgs particles, three electrically neutral and two charged. The following discussion focuses on neutral bosons.

The Higgs boson mass remains largely unconstrained in the SM. From perturbative unitarity arguments an upper limit of $\sim 1 \text{ TeV}$ can be derived. The requirements of stability of electroweak vacuum, and of perturbative validity of the SM seen as an effective theory, allow to set upper and lower bounds depending on the cut-off value chosen for the energy scale up to which the SM is assumed to be valid [525–535]. If the cut-off is assumed to be about the Planck mass, which means that no new physics appears up to that scale, the Higgs boson is predicted to be in the range $130 - 190 \text{ GeV}$. This bound becomes weaker if new physics appears at a lower mass scale. A global fit to all electroweak data in the SM framework seems to favour a rather light Higgs boson: $m_H = 76^{+85}_{-47} \text{ GeV}$ [536]. Moreover, SUSY extensions of the SM generically predict the existence of one rather light neutral Higgs boson (e.g. roughly $m_H \leq 130 \text{ GeV}$ in the minimal SUSY extension). The LEP2 experiments are searching Higgs bosons with masses up to about $110 \text{ GeV}$ [537]. Assuming that no Higgs boson will be found at LEP, the above indications raise even more interest in the Higgs boson searches at LHC in the intermediate mass range from $95 \text{ GeV}$ to $2m_Z$.

The Higgs boson searches scenarios prepared by the ATLAS [1] and CMS [524] Collaborations cover a large spectrum of final state signatures in this mass range. The rare $H \rightarrow \gamma \gamma$ decay mode is expected to be accessible in inclusive Higgs production in the mass range $90 - 140 \text{ GeV}$ already for an integrated luminosity of $100 \, fb^{-1}$. This observability can be also complemented by looking at an additional jet (production in association with jets) or lepton in the final state ($t\bar{t}H, WH, ZH$ associated production). The additional isolated lepton in the final state will also allow to access the dominant $H \rightarrow b\bar{b}$ decay mode, and such observability has been established in the ATLAS searches scenarios for the $t\bar{t}H$ production channel. Higgs decay into $WW$ in inclusive or associated production lead to the clean signature of 2 or 3 leptons in the final state. A signature with even higher lepton multiplicity is provided by the $H \rightarrow ZZ^\pm$ channel in the inclusive and associated production. The possible observability of

\footnote{Section coordinators: J.-P. Guillet, E. Pilon and E. Richter-Was.}

the latest one is still under investigation, as presented below. A rich spectrum of final state signatures was proposed recently, which explored $WW$ and $ZZ$ fusion mechanisms producing a Higgs boson in association with two forward/backward jets. The observability of the $H \rightarrow \gamma\gamma$, $H \rightarrow \tau^+\tau^-$ and $H \rightarrow WW^*$ as established so far in [538–541] at the particle level seems very promising.

Given the very large spectrum of final state signatures which have become of interest in the intermediate mass range, this section will be focused on recent progress in the evaluation of backgrounds to two-photon and multi-lepton signatures, and in the observability of the latter in associated production. Recent results concerning the two-photon background in the mass range 90 - 140 GeV, together with the NLO contribution to the signal of associated production $H +$ jet, are given in Sect. 9.2. A recent investigation on $WH$ associated production for $m_H \geq 140$ GeV is presented in Sect. 9.3.

9.2 The two-photon channel in the mass range 90 - 140 GeV

In this range, the most promising channel is $H \rightarrow \gamma\gamma$. The branching ratio is however small\(^{58}\), typically $B(H \rightarrow \gamma\gamma) \sim \mathcal{O}(10^{-3})$, and initially the background is eight orders of magnitude larger than the signal. This background is split into two components, called irreducible and reducible.

9.2.1 Irreducible background: prompt photon pairs.

This class of background comes from prompt photon pair production, where “prompt” means that the photons do not come from the decay of high-$p_T$ $\pi^0$ or $\eta$, but from hard partonic interactions. A large amount of this background, which we therefore call irreducible, passes the photon isolation cuts. Further kinematic cuts have to be used to suppress it. Regarding the efficiency of background rejection, one may distinguish between the signal processes of inclusive production, and of associated production (and corresponding backgrounds). The first class yields higher rates than the second one. On the other hand, kinematical cuts are more efficient in the case of associated production, and the background may be theoretically better controlled than in the inclusive case. These issues are discussed in the following.

Mechanisms of prompt photon pair production.

Schematically, three mechanisms produce prompt photon pairs with a large invariant mass: the “direct” mechanism produces both photons directly from the hard subprocess; the “single-fragmentation” mechanism, instead, involves precisely one photon resulting from the fragmentation of a hard parton; the “double-fragmentation” mechanism yields both photons by fragmentation. Topologically, a photon from fragmentation is most probably accompanied by a jet of hadrons, therefore will be more strongly rejected by the isolation criterion. From a calculational point of view, this schematic classification emerges from the QCD factorization procedure described in Sect. 1. (see [237] for more details). Although this classification is convenient, one has to keep in mind that the splitting between these different contributions is arbitrary: none of these contributions is separately measurable, only their sum is. Due to the high gluon density at LHC, “single-fragmentation” dominates the inclusive production of prompt photon pairs. Beyond NLO, a new process of the “direct” type appears, the so-called box $gg \rightarrow \gamma\gamma$ contribution. Strictly speaking, it is a NNLO contribution. However, the large gluon luminosity at LHC magnifies it to a size comparable with the Born term $q\bar{q} \rightarrow \gamma\gamma$ in the invariant mass range 90 - 140 GeV. Therefore it is usually included in LHC phenomenological studies [235–237,544–548].

Recent improvements

Early calculations [544, 545] of photon-pair production were not suitable to estimate the background to Higgs boson production. A first improvement [235, 546, 547] implemented these results in a more flexible way by combining analytical and Monte-Carlo techniques. Following a similar approach, recent work goes further along two directions.

\(^{58}\)The cross section for the production of a SM Higgs boson at the Tevatron in this range is $\sim 1 pb$, not enough to allow a search in this mode given its small branching ratio. A search for a non SM Higgs Boson in this mode has been carried out by both CDF and D$\phi$ with negative conclusions [542,543].
In [236, 548], multiple soft gluons effects in the “direct” contribution are summed to next-to-leading logarithmic accuracy in the Collins-Soper framework. This provides a prediction for semi-inclusive observables such as the transverse momentum ($q_T$) distribution of photon pairs that extends over the whole spectrum, thanks to a matching between the resummed part (suited for the low $q_T$ peak) and a fixed order calculation for the high $q_T$ tail. These features are encoded in the computer program RESBOS [236, 548]. In this calculation, the “single-fragmentation” contribution is evaluated at LO and “double-fragmentation” is neglected.

Another recent improvement is the computation of the NLO corrections to both fragmentation contributions (using the set of NLO fragmentation functions of [433]), which provides a consistent NLO approximation suitable for inclusive observables. This calculation, also implemented in a computer code DIPHOX of Monte Carlo type, is described in [237]. No soft gluon summation has so far been implemented in [237].

Effects of isolation
Actually, the isolation requirements, imposed experimentally to suppress the reducible background, severely reduce the fragmentation components, too (which, properly speaking, are thus not really irreducible\textsuperscript{59}). The isolation criterion commonly used is schematically the following\textsuperscript{60}. A photon is called isolated if, inside a cone about the photon, defined in rapidity and azimuthal angle by $(\eta - \eta_\gamma)^2 + (\phi - \phi_\gamma)^2 \leq R^2$, the deposited transverse hadronic energy $E_T^{\text{had}}$ is less than some specified value $E_T^{\text{max}}$. Severe isolation requirements, as $E_T^{\text{max}} = 5$ GeV inside a cone of radius $R = 0.4$, suppress the "single-fragmentation" component by a factor 20 to 50, and kill the "double-fragmentation" contribution, so that the production of isolated photon pairs is dominated by the "direct" mechanism\textsuperscript{61}. Isolation implies however that one is not really dealing with inclusive quantities anymore. Although the factorization property of collinear singularities still holds in this case [443,446], infrared divergences can appear inside the physical spectrum for some distributions calculated at fixed order, e.g. NLO, accuracy, due to isolation. The appearance and the pattern of these singularities depend strongly on the kinematics and on the type of isolation criterion used. Moreover, potential infrared instabilities may affect the reliability of the predictions, when a very low value of $E_T^{\text{max}}$ compared to the $p_T$ of the isolated photon, is used. A better understanding of these problems is required (see [237] and Sect. 6. for a more detailed discussion).

Phenomenology
Our understanding of photon pair production is already tested at the Tevatron [553–555]. A comparison of the CDF di-photon cross section to NLO and resummed predictions is shown in Fig. 44 (for a recent comparison with DØ data see, e.g., [237]). Measured inclusive observables, such as the invariant mass distribution, each photon's $p_T$ distribution, the azimuthal angle ($\phi_{\gamma\gamma}$) distribution of pairs, agree reasonably well with NLO calculations [235,237,544–547]. However, the measured di-photon $q_T$ distribution is noticeably broader than the NLO prediction, but it is in agreement with the resummed prediction of [236, 548]. This is expected since the $q_T$ distribution is particularly sensitive to soft gluon effects\textsuperscript{62} [196].

\textsuperscript{59}This misleading terminology sometimes [549,550] leads to call irreducible only the “direct” component, and reducible the $\pi^0$, $\eta$, etc plus the “fragmentation” components. Although it seems intuitive at LO, this alternative classification is ill defined beyond LO, as the splitting between “fragmentation” components and higher order corrections to the “direct” one is theoretically ambiguous.

\textsuperscript{60}This isolation criterion for single prompt photon production is discussed in the theoretical literature in Refs. [442,443,551,552] ($e^+e^-$ collisions) and in Refs. [229,230,439,440,446] (hadronic collisions). An alternative criterion has been recently proposed in [232]. More discussion on the issue of isolation can be found in Sects. 6.

\textsuperscript{61}The situation is essentially the same for a less severe cut as $E_T^{\text{max}} = 10$ GeV. Note however that such a partonic calculation ignores the hadronic transverse energy splashed in by underlying events. The value of $E_T^{\text{max}}$ used in this type of calculation may then be considered as an effective parameter, smaller than the actual value used experimentally. This issue has still to be clarified, especially when the experimental value is nearly saturated by underlying events and pile-up effects.

\textsuperscript{62}Infrared sensitive distributions, such as the $q_T$ distribution near $q_T \to 0$, and the $\phi_{\gamma\gamma}$ distribution near $\phi_{\gamma\gamma} \to \pi$, can be reliably estimated only with resummed calculations. Note that, for the $\phi_{\gamma\gamma}$ distribution near $\phi_{\gamma\gamma} \to \pi$, not only the “direct”
Fig. 44: A comparison of the NLO and ResBos predictions for di-photon production at the Tevatron for the di-photon mass, the di-photon azimuthal angle (denoted here by $\Delta \phi$) and the di-photon transverse momentum (denoted here by $K_T$).

Fig. 45: Top: di-photon differential cross section $d\sigma/dm_{\gamma\gamma}$ vs. $m_{\gamma\gamma}$ at LHC, with isolation criterion $E_{T,\text{max}} = 5$ GeV in $R = 0.4$, for the scale choice $\tilde{M} = \mu = m_{\gamma\gamma}/2$. Bottom: factorization ($\tilde{M}$) and renormalization ($\mu$) scale dependences of the NLO cross section $d\sigma/dm_{\gamma\gamma}$ vs. $m_{\gamma\gamma}$, normalized by $d\sigma/dm_{\gamma\gamma}|_{\tilde{M} = \mu = m_{\gamma\gamma}/2}$.

Component diverges order by order and requires a soft gluon summation, but also both fragmentation contributions. This much more complicated case has not been treated yet.
The results from Run 1 at the Tevatron were obtained with less than 100 pb$^{-1}$ of data. During Run 2, a data sample approximately 20 times as large will be available, allowing both the di-photon signal and its background to be studied in detail. In particular, the di-photon $q_T$ distribution will be measured to much greater precision, allowing a study of the $q_T$ resummation techniques for a $gg$ initial state, necessary for both Higgs and di-photon production at the LHC [196].

On the theoretical side, scale ambiguities as well as the uncertainties from unknown beyond NLO corrections plague the predictions. A study of scale uncertainties has been performed [237] for inclusive observables such as the invariant mass distribution of photon pairs at LHC in the range 90 - 140 GeV, (Fig. 45). In the isolated case with $E_T^{\text{max}} = 5$ GeV inside a cone with $R = 0.4$, the scale uncertainties are dominated by the dependences on the factorization and renormalization scales $M$ and $\mu$; while the fragmentation scale ($M_f$) dependence is negligible due to the strong suppression of the fragmentation contribution. The scale uncertainties are rather small (less than 5%) when the factorization and renormalization scales are set to be equal and are varied between $m_{\gamma\gamma}/2$ and $2m_{\gamma\gamma}$. On the other hand, anti-correlated variations of $M$ and $\mu$ in the same range lead to still rather large (up to 20%) uncertainties. In summary, the higher order corrections in prompt photon pair production are not fully under control yet. The consistent calculation at full NNLO accuracy would involve, in particular, two-loop $q\bar{q} \rightarrow \gamma\gamma$ amplitudes and the NNLO evolution of the parton distributions. Despite recent progress [70, 288–290] in this direction$^{63}$, such a NNLO description is not yet available. Furthermore, the box contribution $gg \rightarrow \gamma\gamma$ is the lowest order term of a new subprocess. Reducing its scale dependence would involve the calculation of N$^3$LO corrections$^{64}$. Meanwhile, preliminary numerical comparisons have been initiated between these new NLO (and resummed) partonic calculations, and Monte Carlo event generators [196]. They have to be pushed further.

9.22 Reducible background

Before any cut is applied, most of the $H \rightarrow \gamma\gamma$ background comes from large-$p_T$ $\pi^0$, $\eta$ or $\omega$, decaying into photons. It can be severely reduced by imposing combined geometric and calorimetric isolation criteria. A small fraction of this huge background, consisting in large-$p_T$ isolated $\pi^0$ or $\eta$ may still pass such cuts. Earlier estimations of this background rely on Monte Carlo event generators, in which the tails of fragmentation distributions near the end point are rather poorly known. An improvement can be provided by using isolated $\pi^0$ pairs and $\gamma\pi^0$ Tevatron data, compared with Monte-Carlo type NLO calculations, such as [556], to improve NLO fragmentation functions at large $z$.

Like continuum di-photon production, its background from $\gamma\pi^0$ and $\pi^0\pi^0$ production has been extensively studied at the Tevatron [553–555]. This study can serve as a useful benchmark for the reducible background prediction, as well as for very useful tests of QCD. The inclusive $\pi^0\pi^0$ and $\gamma\pi^0$ cross sections are orders of magnitude larger than the $\gamma\gamma$ cross sections, making an extraction of the latter difficult, unless additional selection criteria are applied. As in essentially all collider photon measurements, an isolation cut needs to be applied to each of the di-photon candidates$^{65}$. In the case of CDF (in Run 1B), the isolation cut requires that any additional energy in a cone of radius $R = 0.4$ ($R = \sqrt{\Delta \eta^2 + \Delta \phi^2}$) around the photon direction be less than 1 GeV. This requirement is basically saturated by the energy deposited by the di-photon underlying event and any additional minimum bias interactions that may have occurred during the same crossing. Such a strict isolation requirement rejects the majority of the $\gamma\pi^0$ and $\pi^0\pi^0$ backgrounds while retaining the true di-photon events with 80% efficiency$^{66}$.

The isolation cut suppresses the di-photon backgrounds to the point where they are comparable

$^{63}$For more details, see also Sect. 4.

$^{64}$Although incomplete, the N$^3$LO corrections to sole $gg$ initiated subprocesses, especially the first correction to the box, might already reduce the scale uncertainties. A complete N$^3$LO calculation goes beyond the scope of available techniques.

$^{65}$Other cuts are applied as well but the main impact on the background is from the isolation cut.

$^{66}$For the sake of compactness, only $\pi^0$ backgrounds are listed, but other backgrounds, for example, from $\eta$ and $\omega$ production, are also considered.
to the di-photon signal. One still needs a technique that allows for the separation of the di-photon signal from the background, in a Monte Carlo independent manner. CDF uses two such techniques: a measurement of the electro-magnetic shower width using a wire chamber placed at the EM shower maximum position, and a measurement of the fraction of the photon candidates that have converted in the magnet coil. The two photons from the $\pi^0$ cannot be separately reconstructed given the tower granularity, but they do have a different shower width distribution and a different conversion probability than single photons. These differences allow the extraction of the di-photon signal, not on an event-by-event basis, but on a statistical basis, at each kinematic point being considered. The latter consideration is important since the background fraction does vary with the kinematics of the events being considered.

With the 1 GeV isolation cut for each photon, the di-photon signal fraction varies from about 30% at low $E_T$ to essentially 100% at high $E_T$ (50 GeV). The dominant source of background was determined to be from $\pi^0\pi^0$ production. Note that if the leakage of the electro-magnetic shower energy into the isolation cone is correctly accounted for, there is no reason to have a fractional isolation scale (some fixed fraction of the photon energy) rather than a fixed amount of energy allowed in the isolation cone. A fixed energy isolation cut provides a discrimination against jet backgrounds that increases in effectiveness as the energy of the photon candidate increases. At higher transverse energies, the isolation cut requires the jet to fragment into a $\pi^0$ at larger values of the fragmentation variable $z$, a process greatly suppressed by the steeply falling fragmentation function. The large $z$ ($z > 0.95$) region is poorly known since inclusive measurements of jet fragmentation have few statistics in this region. This statement is even more true for the case of gluon jets, which form the bulk of the background source at the LHC. The di-photon trigger at the Tevatron selects those rare jets that fragment into isolated $\pi^0$'s. Thus, it would be useful to try to normalize the predictions of the event generators such as PYTHIA, which are used for background studies at the LHC to the background data at the Tevatron. Such a comparison is now in progress.

9.23 Production in association with jets

In order to improve the signal/background ratio, it has been suggested to study the associated production of $H(\rightarrow \gamma\gamma)+jet$. For this process, both signal $S$ and background $B$ are reduced but still remain at the level of $\sim 100$ signal events at low LHC luminosity. The LO estimate has shown that the $S/B$ ratio is improved critically with the same level of significance $S/\sqrt{B}$.

Furthermore, higher order corrections to the background have been shown recently to be under better control than in the inclusive case.

Background: associated vs. inclusive

Indeed, the situations in the inclusive and associated channels are quite different. In the inclusive case, the main reason why the magnitude of the NNLO box contribution is comparable to the LO cross section is that the latter is initiated by $q\bar{q}$, whereas the former involves $gg$. The $gg$ luminosity, much larger than the $q\bar{q}$ one, compensates numerically the extra $\alpha_s^2$ factor of the box. This is not the case in the channel $\gamma\gamma+jet$, since the LO cross section is dominated instead by a $qg$ initiated subprocess. The $gg$ luminosity is sizably larger than the $q\bar{q}$ one, which guarantees that the corresponding NNLO contribution remains small (less than 20% for $p_T > 30$ GeV) compared to the LO result. Thus, expecting that the subprocess $gg \rightarrow \gamma\gamma g$ gives the main NNLO correction, a quantitative description of the background with an accuracy better than 20% could be achieved already at NLO in the $\gamma\gamma+jet$ channel for a high-$p_T$ jet. All the helicity amplitudes needed for the implementation of the (“direct” contribution to the) background to NLO accuracy are now available.

Signal vs. background

The 3-body kinematics of the process allows more refined cuts to improve the $S/B$ ratio up to $1/2 - 1/3$ (to be compared with $S/B \geq 1/7$ for the inclusive channel). Due to helicity and total

$^{67}$ A study of the di-photon backgrounds at ATLAS found the $\gamma\pi^0$ and $\pi^0\pi^0$ backgrounds to be of roughly equal size in the low mass Higgs signal region, with each of the backgrounds being of the order of 20% of the di-photon continuum [1].
angular momentum conservation the $s$-wave state does not contribute to the dominant signal subprocess $gg \rightarrow Hg$. On the contrary, all angular momentum states contribute to the subprocesses $gq \rightarrow \gamma \gamma q$ and $q\bar{q} \rightarrow \gamma \gamma g$. Therefore, the signal has a more suppressed threshold behaviour compared to the background. The $S/B$ ratio can thus be improved by increasing the partonic c.m.s. energy $\sqrt{s}$ far beyond threshold. Indeed, a cut $\sqrt{s} > 300$ GeV has been found to give the best $S/B$ ratio for the LHC. The effect can not be fully explained by the threshold behavior only, since that would result in a uniform suppression factor. It was shown in [549, 550] (see Figs. 5 and 6 there) that the dependences of the mass and 5000 for a well separated and 5000 for a well separated dependence of the factorization and renormalization scales. Fig. 46(a) displays the present discussion is based on a LO analysis, and concerns only what was defined above as the “direct” component of the irreducible background. One now has to understand how this works at NLO.

Other, reducible, sources of background are potentially dangerous. The above-defined “single-fragmentation” component to the so-called irreducible background, and the reducible background coming from misidentification of jet events were treated on a similar footing in the LO analysis of [549, 550] as a de facto reducible background (see footnote 9.21). In [549, 550], a rough analysis found that this reducible background is less than 20% of the irreducible one after cuts are imposed. The misidentification rate is given mainly by the subprocesses $gg \rightarrow \gamma gg$, $gg \rightarrow \gamma q\bar{q}$ and $q\bar{q} \rightarrow \gamma q'(g)\bar{q}'(g)$, when the final state parton produces an energetic isolated photon but other products of the hadronization escape the detection as a jet. There, a $\gamma(\pi^0)/$jet rejection factor equal to 2500 for a jet misidentified as a photon and 5000 for a well separated $\gamma(\pi^0)$ production by a jet were used. No additional $\pi^0$ rejection algorithms were applied. Furthermore, this reducible background is expected to be suppressed even more strongly than the irreducible background of “direct” type when a cut on $\sqrt{s}$ is applied.

In summary, the associated channel $H(\rightarrow \gamma\gamma)+$ jet with jet transverse energy $E_T > 30$ GeV and rapidity $|\eta| < 4.5$ (thus involving forward hadronic calorimeters) opens a promising possibility for discovering the Higgs boson with a mass of 100-140 GeV at LHC even at low luminosity. However, to perform a quantitative analysis, the NLO calculations of the background have to be completed and included in a more realistic final state analysis.

**Signal at NLO**

The exact calculation of the NLO corrections to the signal is very complex, since the gluons interact with the Higgs boson via virtual quark loops. Fortunately, the effective field theory approach [561, 562] applicable in the large top mass limit with effective gluon-gluon-Higgs boson coupling gives an accurate approximation with an error less than 5%, provided $m_H \leq 2m_t$. Recently, in this approximation and using the helicity method, the transition amplitudes relevant to the NLO corrections have been analytically calculated for all contributing subprocesses (loop corrections [563] and bremsstrahlung [564, 565]). The subtraction method of [161, 227] has been used to cancel analytically the soft and collinear singularities and to implement the amplitudes into a numerical program of Monte-Carlo type which allows to calculate any infrared-safe observable for the production of a Higgs boson with one jet at NLO accuracy [197].

One of the main results of the calculation is that the NLO corrections are large and increase considerably the cross section, with a $K$ factor $\sim 1.5-1.6$ ($K = \sigma^{NLO}/\sigma^{LO}$) and almost constant for a large kinematical range of $p_T$ and rapidity of the Higgs boson. Furthermore, the NLO result is less dependent on variations of the factorization and renormalization scales. Fig. 46(a) displays the $p_T$ distribution at both LO and NLO for a Higgs boson with $m_H = 120$ GeV. The curves correspond to three different renormalization/factorization scale choices $Q = \mu (m_H^2 + p_T^2)^{1/2}$, with $\mu = 0.5, 1, 2,$ and
show that the scale dependence is reduced at NLO. The same features can be observed in more detail in Fig. 46(b), where the LO and NLO cross sections integrated for $p_T$ larger than 30 and 70 GeV are shown as a function of the renormalization/factorization scale. Both the LO and NLO cross sections increase monotonically with decreasing $\mu$, down to the limiting value where perturbative QCD can still be applied, indicating that the stability of the NLO result is not completely satisfactory. However, in the usual range of variation of $\mu$ from 0.5 to 2, the LO scale uncertainty amounts to $\pm 35\%$, whereas at NLO it is reduced to $\pm 20\%$.

![Fig. 46: Scale dependence of LO and NLO distributions for Higgs boson production. (a) $p_T$ distributions at different scales and (b) the scale dependence of the integrated cross sections for $p_T > 30$ and 70 GeV. The MRST parton distributions are used.](image)

### 9.3 Multi-lepton channels in the mass range $m_H \geq 140$ GeV.

Above 140 GeV, the most promising channel is $H \rightarrow ZZ^{(*)} \rightarrow 4 \ell$ leptons. The corresponding irreducible background comes mainly from the non resonant $ZZ^{(*)}$ production. Severe isolation cuts are needed to suppress reducible $t\bar{t}$ and $Zb\bar{b}$ backgrounds for Higgs boson masses below the $ZZ$ threshold. The topic of weak boson pair production is presented in a dedicated Section of the Electroweak Physics Chapter of this Report. In particular, the latter gathers the effects of NLO contributions to distributions of invariant mass, or transverse momentum of weak boson pairs, and comparisons between Monte Carlo event generators and recent NLO partonic calculations.

The $H \rightarrow WW \rightarrow 2\ell+\not{E}_T$ channels was recently found [566, 567] to be very promising in this mass range around 170 GeV, where the significance of the $H \rightarrow ZZ^{(*)} \rightarrow 4\ell$ channel is relatively small due to the suppression of $ZZ^{(*)}$ branching ratio as the $WW$ mode opens up. In this mass range, the leptonic branching ratio of the $WW$ mode is approximately 100 times larger than the $ZZ^{(*)} \rightarrow 4\ell$ mode. Although the Higgs boson mass peak cannot be directly reconstructed in this case, the transverse mass distribution can be used to sign the Higgs boson and extract information on its mass.

The multileptonic channels $H \rightarrow WW^{(*)}$ and $H \rightarrow ZZ^{(*)}$ are also of great interest for the associated $WH$ production. Although the cross section for the associated production is a factor 50 to 100 lower than for the inclusive production, the $S/B$ ratio is substantially improved. They are also interesting to determine the Higgs boson couplings, since only the couplings to gauge bosons appear in the production and decay chain. The observability of $WH$ with $H \rightarrow WW^{(*)} \rightarrow 2\ell 2\nu$ has been recently proposed in [568] and experimentally studied in [1]. The observability of the associated production $WH$, $H \rightarrow ZZ^{(*)} \rightarrow 4\ell$ has been recently considered in [569] and is sketched below. Due to the small number of events expected for $ZH$ and $t\bar{t}H$ production, only the $WH$ process has been investigated.
Table 6: Number of events in the 5 leptons channel for \( L = 10^{5} \text{pb}^{-1} \), \( p_T \) cut = 10 GeV. No mass window on 4 leptons is applied.

<table>
<thead>
<tr>
<th></th>
<th>no cut</th>
<th>isolation cut</th>
<th>Z mass cut</th>
<th>all cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td>( WH, M_H = 150 \text{GeV} )</td>
<td>3.56</td>
<td>3.42</td>
<td>2.89</td>
<td>2.69</td>
</tr>
<tr>
<td>( t\bar{t} ) background</td>
<td>141.</td>
<td>3.10</td>
<td>26.1</td>
<td>0.098</td>
</tr>
<tr>
<td>( Zbb ) background</td>
<td>17.3</td>
<td>3.46</td>
<td>13.8</td>
<td>3.46</td>
</tr>
<tr>
<td>( WH, M_H = 200 \text{GeV} )</td>
<td>5.92</td>
<td>5.55</td>
<td>3.95</td>
<td>3.76</td>
</tr>
<tr>
<td>( WH, M_H = 300 \text{GeV} )</td>
<td>1.45</td>
<td>1.30</td>
<td>0.91</td>
<td>0.86</td>
</tr>
<tr>
<td>( t\bar{t} ) background</td>
<td>141.</td>
<td>3.10</td>
<td>0.098</td>
<td>0.098</td>
</tr>
<tr>
<td>( Zbb ) background</td>
<td>17.3</td>
<td>3.46</td>
<td>1.73</td>
<td>0</td>
</tr>
</tbody>
</table>

9.31 Associated \( WH \) production, five lepton channel

Selection criteria

All simulations of Higgs boson and background events have been made with the PYTHIA 5.702 and JETSET 7.408 Monte Carlo programs implemented in the CMSIM/CMANA package [570]. The processes implemented in PYTHIA were simulated with parton showers, with the exception of internal bremsstrahlung, generated by PHOTOS [571]. No K factors were used, so the final numbers of signal events may be underestimated by about a factor 1.3 [572]. The experimental resolution of CMS for lepton reconstruction was simulated by a Gaussian smearing:

\[
\frac{\Delta p_T}{p_T} = 4.5\% \sqrt{p_T/1000} \quad \text{for muons,}
\]

\[
\frac{(\Delta E/E)^2}{(4\%/\sqrt{E})^2 + (0.230/E)^2 + (0.55\%)^2} \quad \text{for electrons,}
\]

where \( p_T \) and \( E \) are expressed in GeV. Dedicated programs calculate the dependence on \( \eta \) and \( p_T \) of the geometrical and kinematical acceptances, the invariant mass cuts to select the \( Z \) or \( Z^* \), and the rejection of non isolated leptons in jets with cuts selecting leptons without charged tracks above \( p_T > 2 \text{ GeV} \) in a cone \( R < 0.1 \) \( (R^2 = \Delta \eta^2 + \Delta \phi^2) \). A few events were also fully generated and visualized in CMS by CMSIM. The reactions \( W^\pm H \rightarrow \mu^\pm \nu_\mu ZZ^*(s) \rightarrow 5\mu^\pm \nu_\mu \) and \( W^\pm H \rightarrow e^\pm \nu_e ZZ^*(s) \rightarrow 5e^\pm \nu_e \) have been studied in details. Although the branching ratios are identical, some differences between these channels are expected due to differences in acceptances and trigger efficiencies. The generated leptons are sorted in decreasing \( p_T \) order, from 1 to 5, then the following cuts are applied.

For muon events:

- \(| \eta | < 2.1 \) for \( \mu_1 \) and \( \mu_2 \)
- \(| \eta | < 2.5 \) for \( \mu_3 \) to \( \mu_5 \)
- \( p_T > 20 \text{ GeV} \) for \( \mu_1 \)
- \( p_T > 10 \text{ GeV} \) for \( \mu_2 \)
- \( p_T > 5 \) or \( 10 \text{ GeV} \) for \( \mu_3, \mu_4 \) and \( \mu_5 \)

For electron events:

- \(| \eta | < 2.5 \) for \( e_1 \) to \( e_5 \)
- \( p_T > 20 \text{ GeV} \) for \( e_1 \)
- \( p_T > 15 \text{ GeV} \) for \( e_2 \)
- \( p_T > 7, 10 \) or \( 15 \text{ GeV} \) for \( e_3, e_4 \) and \( e_5 \)

Leptons 1 and 2 are the ones used to trigger events, leptons 3 to 5 \( p_T \) thresholds can be set at lower values. Almost no difference is observed when the trigger threshold is set at a higher value (30 and 20 GeV), as expected since leptons 1 and 2 produced by \( W \) and \( Z \) decays are very energetic. The other possible final states: \( 2e + 3\mu, 2\mu + 3e, 4e + 1\mu \) and \( 4\mu + 1e \) are also good candidates. Since only small numerical differences were found in the results between the pure electronic and muonic final states, the 4 mixed ones were not simulated and the total number of expected events was multiplied by a factor 8. As the expected cross section is very low, the present search would be meaningful at high luminosity only. The pile-up at high luminosity has a minor impact for the detection of leptons. Nevertheless it has to be taken into account when using the isolation cuts.
$H \rightarrow ZZ^*$

This channel concerns the mass range $m_H < 2m_Z$. The irreducible background, due to the non resonant $WZZ^*$ production, is not included in PYTHIA. In order to get a rough order of magnitude, the $S/B$ ratio was then assumed to be of the same order as the one of direct production of $H \rightarrow ZZ^*$, compared to non resonant $ZZ^*$. This ratio has been estimated in [573] to be lower than 10% for $m_H = 150$ GeV. The reducible background comes from the $t\bar{t}$ and $Zb\bar{b}$ channels with three leptons coming from semi-leptonic decays of $B$ and $D$ mesons. The initial cross sections of these processes are very high and, without cut, this background is much higher than the irreducible one.

The selection requests one pair of opposite sign muons or electrons with a mass equal to $m_Z \pm 5$ GeV, and one pair of opposite sign muons or electrons with a mass below $m_Z$. This removes only 19% of the signal events which fall in the tails of the mass distributions. An additional effect of widening the $Z$ mass would come from the $e^{\pm}$ bremsstrahlung in the tracker material [574] and contribute to decrease the acceptance. The lepton pair mass spectra of the $t\bar{t}$ and $Zb\bar{b}$ backgrounds exhibit a peak at low mass. A cut at $m_{Z^*} > 10$ GeV would further reduce these backgrounds by 20% without affecting the signal. No detector reconstruction inefficiency was considered at this level. The isolation cut is used to reject leptons from $b$ or $c$ quark decay, in the reducible background channels. The events exhibiting tracks with $p_T > 2$ GeV contained in a cone $R < 0.1$ around any of the five leptons are rejected (Fig. 47). Actually a better rejection is expected in the CMS detector when using the information from the $b$ vertex position [575].

Another reducible background was considered: the non resonant production of $ZZ^*$ where one of the $Z^{(*)}$ decays into two leptons and the other decays into $b\bar{b}$, the $b$ quarks decaying semi-leptonically. The number of events before acceptance, mass and isolation cut is about 70% of the signal, but as we expect the leptons from the $b$’s to be very soft and non isolated, that this background can be considered as negligible.

$H \rightarrow ZZ$

This channel is similar to the previous one except that we request two pairs of opposite sign muons or electrons with masses equal to $m_Z \pm 5$ GeV. This cut removes 32 to 34% of the signal events. It is now much more efficient against the $t\bar{t}$ background than against the $Zb\bar{b}$, because the $Zb\bar{b}$ channel involves a real $Z$. The calculations were made for Higgs bosons with $m_H = 200$ and 300 GeV (Fig. 47). The acceptances of the signal vary only slightly as a function of $p_T$ cut and other selection cuts. The four leptons mass spectrum for the background is a wide distribution centered around 150 GeV. A cut on this spectrum can be used to obtain an additional rejection factor of the order of 10 to 50, after the Higgs boson mass has been previously measured in a more sensitive channel, like the inclusive $H \rightarrow 4l$ [575].

Results

The number of expected 5 muons or 5 electrons events for one year of running at high luminosity $100 fb^{-1}$ is low: 0.34 for a Higgs boson mass of 150 GeV, 0.47 for 200 GeV and 0.11 for 300 GeV/c. Considering all the possible 5 leptons channels, these numbers must be multiplied by a factor 8. They are summarized in table 1, together with the corresponding backgrounds (not including the cut on the four leptons mass spectrum described above). The $S/B$ ratio is better for $m_H = 200$ GeV and is unacceptable for $m_H = 150$ GeV. Thus this channel can be considered almost hopeless for the discovery of the Higgs boson below the $ZZ$ threshold. On the other hand, if the Higgs boson is in the mass range 200 to 300 GeV, the detection of these rare 5 lepton events above a low background would be a valuable information for the study of the Higgs boson couplings.

However, before drawing any definitive conclusion, several issues should be improved concerning the backgrounds. Firstly the irreducible $WZZ^*$ background has to be calculated, e.g. using an automatized calculation like [138] and included in the analysis. Moreover the reducible $Zb\bar{b}$ process should be revisited with another Monte Carlo generator, as the implementation in PYTHIA 5.7 for the $Zb\bar{b}$ process is known to suffer from an instability in the phase space generation (this implementation has been removed from the version PYTHIA 6.1 for this reason). Finally, another source of 5 leptons events, not
Fig. 47: Number of events at $L = 10^9$ pb$^{-1}$ for $W H \rightarrow \mu \mu$ channel and background for $M_H = 150$ GeV (top), 200 and 300 GeV (bottom) after kinematical cuts (left), isolation and $ZZ$ mass cuts combined (right). $P_T$ cut refers to the softest of the five muons. Dotted line is an upper limit (no Monte Carlo event survive the cuts).

evaluated with enough statistics so far is the semi-leptonic decay of $b\bar{b}$ or $c\bar{c}$ generated by initial or final gluon radiation.

An extension of this study would also be the investigation of the associated production of a higher mass Higgs boson using other decay modes with larger branching ratios like $Z \rightarrow$ jet jet.

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Abstract
In this review, we consider four main topics:

1. The prospects for a significant improvement in the precise measurement of the electroweak parameters.
2. NLO QCD description of the production $W^+W^-$, $W^\pm Z$, $ZZ$, $W^\pm \gamma$ or $Z\gamma\gamma$ pairs with leptonic decays and with anomalous triple gauge-boson couplings.
3. The prospects for significant improvement in the direct measurement of the non-Abelian gauge-coupling, with direct limits on triple and quartic anomalous couplings.
4. Gauge-boson scattering at large centre of mass energy.

1. INTRODUCTION

1.1 Electroweak parameters
At the LHC, substantial improvement in the precise determination of electroweak parameters, such as the $W$ boson mass, the top-quark mass and the electroweak mixing angle, will become feasible, as well as an accurate measurement of the vector-boson self couplings and of the mass of the Higgs boson. This opens promising perspectives towards very comprehensive and challenging tests of the electroweak theory.

Electroweak precision observables provide the basis for important consistency tests of the Standard Model (SM) or its extensions, in particular the Minimal Supersymmetric Standard Model (MSSM). By comparing precision data with the predictions of specific models, it is possible to derive indirect constraints on the parameters of the model. In the case of the top-quark mass, $m_t$, the indirect determination from the precision observables in the framework of the SM turned out to be in remarkable agreement with the direct experimental measurement of $m_t$. Since the Higgs boson mass, $M_H$, enters the predictions for the precision observables only logarithmically in leading order, the indirect determination of $M_H$ requires very accurate experimental data as well as high precision of the theoretical predictions. The uncertainties of the predictions arise from the following sources: a) the unknown higher-order corrections - since the perturbative evaluation is truncated at a certain order, and b) the parametric uncertainties induced by the experimental errors of the input parameters.

The most important universal top-quark contribution to the electroweak precision observables enters via the $\rho$ parameter, which deviates from unity by a loop contribution $\Delta \rho$. At the one-loop level, the $(t, b)$ doublet yields a term proportional to $m_t^2$ [1], namely $\Delta \rho = 3G_\mu m_t^2/(8\pi^2\sqrt{2})$ in the limit $m_b \to 0$. Therefore, it is to be expected that the precision measurement of the top-quark mass at the LHC (see Section 3.1) will significantly improve the theoretical prediction of the $W$ mass, $M_W$ – at

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present, the experimental error on $m_t$ is a limiting factor for the accuracy in the theoretical predictions of the precision observables. $M_W$ itself will be measured at the LHC with a sizably improved accuracy.

The theoretical prediction for $M_W$ is obtained from the relation between the vector-boson masses $M_{W,Z}$ and the Fermi constant $G_\mu$, which is conventionally written in the form

$$M_W^2 \left( 1 - \frac{M_Z^2}{M_W^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_\mu} \frac{1}{1 - \Delta r}.$$  \quad (1)

The quantity $\Delta r = \Delta r(\alpha, M_Z, M_W, m_t, M_H)$, first derived in [2, 3] in one-loop order, summarises the quantum corrections to the vector-boson mass correlation; it is obtained from the calculation of the muon lifetime in the SM beyond the tree-level approximation. At one-loop order, $\Delta r$ can be written as

$$\Delta r = \Delta \alpha - \frac{\alpha_W^2}{s_W^2} \Delta \rho + (\Delta r)_{\text{rem}}.$$  \quad (2)

$\Delta \alpha$ contains the large logarithmic contributions from the light fermions, and $\Delta \rho$ the $m_t^2$ dependence; the non-leading terms are collected in $(\Delta r)_{\text{rem}}$ where also the dependence on $M_H$ enters. In Equation 1, $\Delta r$ is a quantity that accounts also for terms of higher order than just one-loop. Moreover, a partial resummation of large contributions from light fermions and from the $\rho$ parameter is contained in the expression. For a discussion see for example the section on the Electroweak Working Group Report in [4]. Results for $M_W$ that were not yet available at the time of the report [4] are the next-to-leading two-loop terms of $O(G_\mu^2 m_t^2 M_Z^2)$ [5, 6] in an expansion for asymptotically large $m_t$ and the result for the Higgs mass dependence of the fermionic two-loop contributions [7]. Recently, the complete result for the fermionic two-loop contributions has been obtained [8]. Furthermore, the QCD corrections to $\Delta r$ of $O(\alpha \alpha_s^2)$ have been derived [9].

The most recent theoretical prediction [8] for $M_W$ within the SM is displayed in Figure 1 as a function of $M_H$. To illustrate the comparison between theory and experiment, the experimental result is included in the figure for the current uncertainty $\delta M_W = \pm 0.042$ GeV [10] and the estimated LHC uncertainty $\delta M_W = \pm 0.015$ GeV (see Section 3.1) (assuming the same central value). The uncertainty for the current status and for the case where the LHC will have measured the top-quark mass with much higher accuracy is also displayed, in combination with the theoretical uncertainty from unknown higher-order corrections. It is clear that both improvements, in $M_W$ and in $m_t$, will lead to a substantial increase in the significance of Standard Model tests, with stringent bounds on the Higgs boson mass to be confronted with the directly measured value of $M_H$.

Besides the $W$ boson mass, the improvement in $m_t$ will also have an effect on the predictions of the $Z$ pole observables. They are conveniently described in terms of effective couplings

$$g_V^f = \sqrt{\rho_f} (I_3^f - 2 Q_f \sin^2 \theta_{\text{eff}}^f), \quad g_A^f = \sqrt{\rho_f} I_3^f$$  \quad (3)

in the neutral-current vertex at the $Z$ resonance for a given fermion species $f$, normalised according to $J^{\text{NC}}_\mu = (\sqrt{2} G_\mu M_Z^2)^{1/2} (g_V^f \gamma_\mu - g_A^f \gamma_\mu \gamma_5)$. Besides the overall normalisation factor $\rho_f = 1 + \Delta \rho + \cdots$, we mention in particular the effective mixing angle, which is usually chosen as the on-resonance mixing angle for the leptons $f = e, \mu, \tau$ in Equation 3 and denoted as $\sin^2 \theta_{\text{eff}}^\text{lept}$. This quantity also depends sensitively on the top-quark mass, mainly through $\Delta \rho$. The theoretical prediction of $\sin^2 \theta_{\text{eff}}^\text{lept}$ will definitely be sharpened by the precise measurement of the top-quark mass; a sizable improvement concerning the internal consistency test can be anticipated. The on-resonance mixing angle for the light quarks $\neq b$ is numerically very close to the leptonic one. $\sin^2 \theta_{\text{eff}}^\text{lept}$ can therefore be measured at the LHC in the Drell-Yan production of charged-lepton pairs around the $Z$ resonance, via $q \bar{q} \rightarrow l^+ l^-$, where an accuracy of $1.4 \times 10^{-4}$ on $\sin^2 \theta_{\text{eff}}^\text{lept}$ may be feasible (see Section 3.2).

Besides these internal consistency checks of the SM, the electroweak precision observables may be useful to distinguish between different models as candidates for the electroweak theory. In Figure 2,
Fig. 1: The dependence of $M_W$, predicted by means of Equation 1, on $M_H$ is shown for the SM. The uncertainty of the predictions corresponds to the present and expected parametric uncertainty owing to the top mass, in combination with the theoretical uncertainty. The central lines (solid) correspond to the present central values of $M_W = 80.394$ GeV and $m_t = 174.3$ GeV.

the SM prediction of $M_W$ as a function of $m_t$ is compared with the prediction within the MSSM, where the MSSM prediction is based on results up to $\mathcal{O}(\alpha_5)$ [11, 12]. The SM uncertainty arises from the only unknown parameter, the Higgs boson mass. On the other hand, within the MSSM, the Higgs boson mass is not a free parameter [13], and the uncertainty originates from the unknown SUSY mass scales. In the small overlap region, the MSSM behaves like the SM, i.e. all SUSY particles are heavy and decouple from the precision observables, and the $M_H$ value of the SM stays below 130 GeV, the upper bound on the lightest MSSM Higgs boson mass for $m_t = 175$ GeV (see [14] and references therein). Figure 2 shows the clear improvement from the current status to the LHC era, where eventually, besides direct experimental evidence, a distinction between SM and MSSM might become feasible.

1.2 Vector-boson pair production and scattering

At the LHC, the precise measurement of the production of $W^+W^-$, $W^\pm Z$, $ZZ$, $W^\pm\gamma$ or $Z\gamma$ pairs is also an important physics goal. In the simplest studies, the gauge-bosons will be detected via their leptonic decays. Already a couple events have been obtained by CDF and D0 for $W^+W^-$ and $WZ$ production and D0 has seen about 100 $W\gamma$ and 30 $Z\gamma$ events. The data set at Run II will be about 20 times larger and about 1000 times larger at the LHC. For a summary of the experimental situation see [15, 16].

The production of gauge-boson pairs provide us with the best test of the non-Abelian gauge-symmetry of the Standard Model (SM). Deviation from the SM predictions may come either from the presence of anomalous couplings or the production of new heavy particles and their decays into vector-boson pairs. If the particle spectrum of the SM has to be enlarged with new particles (as in the Minimal Supersymmetric Standard Model (MSSM)) with mass values of $\geq 0.5 - 1$ TeV, small anomalous couplings are generated at low energy. If the Higgs boson is very heavy, it will decay mainly into $W^+W^-$ and $ZZ$ pairs. If the symmetry breaking mechanism is dynamical (technicolor models, BESS models), large anomalous couplings might be generated or new heavy particles may be produced. In both of these cases, vector-boson pair production will show deviations from the Standard Model predictions. At the same time, vector-boson pair production gives the most important background for a number of new physics signals. For example, one of the most important physics signal for supersymmetry at hadron
colliders is the production of three charged leptons and missing transverse momentum. The dominant background for this process is the production of $W^+$ plus a $Z$ (real or virtual) or $\gamma$.

The leading order production mechanism of gauge-boson pair production is $q\bar{q}$ annihilation. The precise calculation of the cross sections in the QCD improved parton model have received recently a lot of attention. The cross sections of the gauge-boson pair production and its decay into lepton pairs have been calculated in next-to-leading order (NLO) accuracy retaining the full spin correlations of the leptonic decay products. A significant achievement was that the theoretical results in NLO QCD for the production of $W^+W^-$, $W^\pm Z$, $ZZ$, $W^{\pm}\gamma$ or $Z\gamma$ pairs could be documented in short analytic formulae [17] allowing for independent numerical implementations. Subsequently, several so called NLO numerical Monte Carlo programs have been developed and the complete one loop corrections became available for the first time for $W^+W^-$, $W^\pm Z$, $ZZ$ in [18, 19], and for $W^\pm\gamma$, or $Z\gamma$ pairs in [20]. These new results have superseded and confirmed previous NLO results on spin averaged production gauge-boson pair production [21, 22, 23, 24, 25, 26, 27, 28], as well the approximate results where spin correlation have been neglected in the virtual corrections [29, 30, 31, 32, 33]. The agreement between the well documented results in [19] and in [22, 24, 26] is within the precise integration error and the agreement between the results of [19] and the recent programs of [29, 30, 31, 32, 33] is about 3%. Therefore, previous experimental simulation studies based on these programs (see Section 6.5) should not be repeated.

Simple analytic NLO results exist also for the anomalous coupling contributions at NLO accuracy in [19, 20]. Again, the agreement with previous approximate NLO results [29, 30, 31, 32, 33] is also good (see Section 5.5). Future anomalous coupling studies may like to use the more accurate packages. At the LHC, contrary to LEP, the phenomenological studies of anomalous triple gauge-boson coupling constants cannot be treated as constant couplings since they lead to violation of $SU(2)$ gauge-symmetry and unitarity. The difficulty comes from truncation of the contribution of an infinite series of higher dimensional non-renormalisable gauge-invariant operators. In the case of $q\bar{q}$ annihilation to gauge-boson pairs, a suitable phenomenological approach is the introduction of form factors for the anomalous couplings (which in principle are calculable in the true underlying theory). As long as we do not obtain deviations from the Standard Model, for practical purposes, simple dipole form factors with various cut-

Fig. 2: The dependence of $M_W$ on $m_t$ is shown for the SM and the MSSM. It is compared to the current errors and to the errors expected from the LHC.
off parameters can be used. With better data, one can put limits on the form factor values in small $\sqrt{s}$ intervals, assuming constant couplings for each interval. In the case of positive signals, such a form factor measurement will provide us with important information on the underlying theory (see Sections 3., 4. and 5.).

At higher energies, the higher order production processes of $WW$ and $ZZ$ scattering (the weak boson are emitted from the incoming quarks) will become more and more important. These interactions are the most sensitive to the mechanism of the electroweak symmetry breaking. In particular, if the breaking of the electroweak symmetry is due to new particles with strong interactions at the TeV scale, enhanced production of longitudinal gauge-boson pairs will be the most typical signal [34, 35]. The minimal model to describe this alternative is obtained by assuming that the new particles are too heavy to be produced at LHC and the linear $\sigma$-model Higgs-sector of the Standard Model is replaced by the non-renormalisable non-linear $\sigma$-model which can also be considered as an effective chiral vector-boson Lagrangian with non-linear realisation of the gauge-symmetry [36, 37]. The question is whether this more phenomenological approach is consistent with the precision data. In a recent analysis, a positive answer was obtained [38]. It has been found that due to the screening of the symmetry breaking sector [39], this alternative still has enough flexibility to be in perfect agreement with the precision data up to a cut-off scale of $3\text{ TeV}$ (see Sections 5. and 6.). In the chiral approach, the gauge-boson observables are obtained as truncated series in powers of the external momenta $p^n/(4\pi)^n$ with $M^2_{\text{LV}} \approx g^2v^2/8$. The approximation is valid up to energy scales of $E = 4\pi v \approx 3\text{ TeV}$. At the LHC, the partonic centre of mass energy can be higher and the phenomenological implementation is confronted with the problem of unitarisation [40, 41, 42]. Although unitarisation is not unique, the use of the K-matrix formalism [40] or the $\mathcal{O}(p^4)$ Inverse Amplitude Method [42] appear to give reasonable model independent framework to explore the various possibilities. When extrapolating to higher energies in particular, the masses of resonances are rather sensitive to the actual value of additional chiral parameters. An alternative approach for the phenomenological formulation of the dynamical symmetry breaking consistent with the precision data is offered by the BESS model [43] with an extended strongly interacting gauge-sector with enhanced global symmetries and with important decoupling properties at low energies. The phenomenologically acceptable technicolor models [44] also require an enhanced global symmetry in the spectrum of the theory. In the most pessimistic parameter ranges, it is rather difficult to detect the signals of the strong $WW$ and $WZ$ scattering; therefore, one has to push the LHC analysis to its limits. In the future, further clever strategies have to be pursued for this case (see Section 6.).

2. ELECTROWEAK CORRECTIONS TO DRELL-YAN PROCESSES

The basic parton processes for single vector-boson production are $q\bar{q} \rightarrow W \rightarrow l\nu_l$ and $q\bar{q} \rightarrow Z \rightarrow l^+l^-$, with charged leptons $l$ in the final state. Investigations around the $W$ and $Z$ resonance allow a precise measurement of the $W$ mass and of the electroweak mixing angle from the forward-backward asymmetry. At high invariant masses of the $l^+l^-$ pair, deviations from the standard cross section and $A_{FB}$ could indicate scales of new physics, e.g. associated with an extra heavy $Z'$ or extra space dimensions. For the envisaged precision, a discussion of the electroweak higher-order contributions is necessary, on top of the QCD corrections. The electroweak corrections consist of the set of electroweak loop contributions, including virtual photons, and of the emission of real photons.

With respect to QCD, the cross sections in this section are all of lowest order, evaluated with parton distribution functions at factorisation scales $M_{W,V}$ (for $W$ production) and $M_Z$ (for $Z$ production). Hence, the numerical values are not yet directly the physical ones. They are given here to point out the structure and the size of the higher-order electroweak contributions. The QCD corrections are considered in the QCD chapter of this report, where a QCD-related uncertainty of $\sim 5\%$ is estimated. For illustration, we give the values (in nb) for $[\sigma(pp \rightarrow W^+) + \sigma(pp \rightarrow W^-)] \cdot BR(W \rightarrow e\nu)$ and

\textsuperscript{2}Section coordinator: W. Hollik.
\( \sigma(pp \to Z) \cdot BR(Z \to e^+ e^-) \) in the purely electroweak calculation (EW) and with NNLO QCD [50]:

\[
\begin{align*}
W : & \quad 17.9 \text{ (EW)} \quad \text{and} \quad 20.3 \pm 1.0 \text{ (NNLO)}, \\
Z : & \quad 1.71 \text{ (EW)} \quad \text{and} \quad 1.87 \pm 0.09 \text{ (NNLO)}.
\end{align*}
\]

2.1 Universal initial-state QED corrections

![Graph showing QED corrections](image)

Fig. 3: QED corrections to the parton distribution functions for up-type quarks, \( U(x, \mu^2) = \sum_{\text{gen}} (u + \bar{u}) \), down-type quarks, \( D(x, \mu^2) = \sum_{\text{gen}} (d + \bar{d}) \) and the gluon \( g(x, \mu^2) \) in per cent for the scale \( \mu = M_W \) (a) and \( \mu = m_t \) (b).

QED corrections related to the emission of (real or virtual) photons from quarks contain mass singularities which factorise and therefore can be absorbed by a redefinition (renormalisation) of parton distribution functions [45]. This redefinition is well-known in the calculation of QCD radiative corrections where in complete analogy to photon radiation, the emission of gluons leads to mass singularities as well. By the redefinition, the mass singularities disappear from the observable cross section and the renormalised distribution functions become dependent on the factorisation scale \( \mu \) which is controlled by the well-known Gribov-Lipatov-Altarelli-Parisi (GLAP) equations [46, 47]. The factorisation scale should be identified with a typical scale of the process, i.e. a large transverse momentum, or the mass of a produced particle.

Since mass singularities are universal, i.e. independent of the process under consideration, the definition of renormalised parton distributions is also universal. Therefore it is possible to discuss the bulk of initial-state QED radiative corrections in terms of parton distribution functions. This will be true if there is only one large scale in the process.

The treatment of mass singularities due to gluonic or photonic radiation is identical. Photonic corrections can therefore be taken into account by a straightforward modification [48, 49] of the standard GLAP equations which describe gluonic corrections only. The modification corresponds to the addition of a term of the order of the electromagnetic fine-structure constant, \( \alpha \). Apart from small non-singular contributions, the resulting modified scale dependence of parton distribution functions is the only observable effect of initial-state QED corrections in high-energy scattering of hadrons.
The modified evolution equation for the charged parton distribution functions, $q_f(x, \mu^2)$ for quarks with flavour $f$, can be written as:

$$
\frac{d}{dt}q_f(x,t) = \frac{\alpha_s(t)}{2\pi} \int_x^1 \frac{dz}{z} \left[ P_{q/g}(z,t)q_f(x/z,t) + P_{q/g}(z,t)g(x/z,t) \right] + \frac{\alpha(t)}{2\pi} \int_x^1 \frac{dz}{z} \frac{d^\gamma}{q/g}(z,t)q_f(x/z,t)
$$

In the leading logarithmic approximation, the splitting functions $P_{i/j}$ are independent of the scale $t = \ln \mu^2/\Lambda^2$, and the QED splitting function is given by

$$
P_{q/g}^\gamma(z) = Q_f^2 \left[ \frac{1 + z^2}{(1 - z)^2} + \frac{3}{2} \delta(1 - z) \right] = \frac{Q_f^2}{C_F} P_{q/g}.
$$

Since quarks are coupled through the splitting function $P_{q/g}^\gamma(z) = \frac{1}{2} [z^2 + (1 - z)^2]$ to gluons, the gluon distribution $g(x, \mu^2)$ is affected by QED corrections as well, although only indirectly, by terms of the order of $O(\alpha_s)$. $\alpha(t)$ is the running electromagnetic fine-structure constant and $Q_f$ are the fermion charges in units of the positron charge.

The proper treatment of the mass-singular initial-state QED corrections would require not only the solution of the evolution equations with the QED term, but also to correct all data that are used to fit the parton distributions for those QED effects. Apart from a few exceptions, experimental data have not been corrected for photon emission from quarks. However, one can illustrate the QED radiative corrections by comparing the modification of the parton distributions relative to the distribution functions obtained from the evolution equations without the QED terms, which are used as an input.

The solution of the evolution equations corresponds to the resummation of terms containing factors $\alpha(\alpha_s \ln \mu^2)^n$ with arbitrary power $n$. In Figures 3a and 3b, we show numerical results for the corrections $\Delta Q_{\text{ED}}$ to the distribution functions $U(x, \mu^2)$ ($D(x, \mu^2)$) for the sum of all up-(down)-type quarks, and the gluon distribution $g(x, \mu^2)$. The figures show the QED corrections in per cent relative to the distribution functions obtained from the GLAP equations without the QED term. The input distributions were taken from [50]. One finds small, negative corrections at the per-mille level for all values of $x$ and $\mu^2$ relevant in the LHC experiments. Only at large $x \approx 0.5$ and large $\mu^2 \approx 10^3$ GeV$^2$ do the corrections reach the magnitude of one per cent. The increase of corrections for $x \to 1$ is due to the $\ln(1 - x)$ terms appearing in the evaluation of the “+” distributions.

The largest corrections are obtained for up-type quarks due to the larger charge factor $4/9$ as compared to $1/9$ for down-type quarks. The gluon distribution, being of order $O(\alpha_s)$, is corrected by less than 0.1% up to values of $x$ of about 0.2.

The corrections vanish for $\mu^2 \to \mu_0^2$ since it was assumed that the input distributions $q_f(x, \mu_0^2)$ and $g(x, \mu_0^2)$ have been extracted from experiment at the reference scale $\mu_0^2$ without subtracting quarkonic QED corrections.

The asymptotic behaviour for $x \to 0$ can be checked analytically. The singular behaviour of distributions $\propto x^{-\eta}$ for $x \to 0$ remains unchanged by the GLAP equations if $\eta > 1$. Thus the $O(\alpha)$ corrected distributions have the same power behaviour as the uncorrected ones, the ratio consequently reaching a constant value for $x \to 0$. The valence parts of $U(x)$ and $D(x)$, however, which vanish at $x = 0$, receive positive corrections at small $x$, thus producing the well-known physical picture: radiation of gluons as well as of photons leads to a depletion at large $x$ and an enhancement at small $x$, i.e. partons are shifted to smaller $x$.

Other input distribution functions lead to differences of QED corrections at the per-mille level which are again irrelevant when compared with the expected experimental precision of structure-function measurements.
2.2 Electroweak corrections to $W$ production

2.21 Physical goals of single $W$ production

The Drell-Yan-like production of $W$ bosons represents one of the cleanest processes with a large cross section at the LHC. This reaction is not only well suited for a precise determination of the $W$ boson mass $M_W$, it also yields valuable information on the parton structure of the proton. Specifically, the target accuracy of the order of $15$ MeV [53] in the $M_W$ measurement exceeds the precision of roughly $30$ MeV achieved at LEP2 [51] and Tevatron Run II [52], and thus competes with the one of a future $e^+e^-$ collider [54]. Concerning quark distributions, precise measurements of rapidity distributions provide information over a wide range in $x$ [50]; a measurement of the $d/u$ ratio would, in particular, be complementary to HERA results. The more direct determination of parton-parton luminosities instead of single parton distributions is even more precise [55]; extracting the corresponding luminosities from Drell-Yan-like processes allows us to predict related $q\bar{q}$ processes at the per-cent level.

Owing to the high experimental precision outlined above, the predictions for the processes $pp \to W \to l\nu_l$ should attain per-cent accuracy. To this end, radiative corrections have to be included. In the following some basic features of this processes and recent progress [56, 57, 58, 59, 60] on electroweak corrections are summarised; a discussion of QCD corrections can be found in the QCD chapter of this report.

2.22 Lowest-order cross section and preliminaries

We consider the parton process $u\bar{d} \to \nu_l l^+ (+\gamma)$, where $u$ and $d$ generically denote the light up- and down-type quarks, $u = u, c$ and $d = d, s$. The lepton $l$ represents $l = e, \mu, \tau$. In lowest order, only the Feynman diagram shown in Figure 4 contributes to the scattering amplitude, and the Born amplitude reads

$$
\mathcal{M}_0 = \frac{e^2 V^*_{ud}}{2s_w^2} \left[ \bar{\nu}_l \gamma^\mu \omega_- u_u \right] \frac{1}{s - M_W^2 + iM_W \Gamma_W(s)} \left[ \bar{\nu}_q \gamma^\mu \omega_- v_l \right],
$$

with $s$ being the squared centre-of-mass (CM) energy of the parton system. The notation for the Dirac spinors $\bar{\nu}_d$, etc., is obvious, and $\omega_- = \frac{1}{2}(1 - \gamma_0)$ is the left-handed chirality projector. The electric unit charge is denoted by $e$, the weak mixing angle is fixed by the ratio $c_w^2 = 1 - s_w^2 = M_W^2 / M_Z^2$ of the $W$ and $Z$ boson masses $M_W$ and $M_Z$, and $V_{ud}$ is the CKM matrix element for the $u \bar{d}$ transition.

Strictly speaking, Equation (6) already goes beyond lowest order, since the $W$ boson width $\Gamma_W(s)$ results from the Dyson summation of all insertions of the (imaginary parts of the) $W$ self-energy. Defining the mass $M_W$ and the width $\Gamma_W$ of the $W$ boson in the on-shell scheme (see e.g. [61, 62]), the Dyson summation directly leads to a running width, i.e. $\Gamma_W(s)_{\text{run}} = \Gamma_W \times (s / M_W^2)$. On the other hand, a description of the resonance by an expansion about the complex pole in the complex $s$ plane corresponds to a constant width, i.e. $\Gamma_W(s)_{\text{const}} = \Gamma_W$. In lowest order these two parametrisations of the resonance region are fully equivalent, but the corresponding values of the line-shape parameters $M_W$ and $\Gamma_W$ differ in higher orders [56, 63, 64]. The numerical difference is given by $M_W|_{\text{run}} - M_W|_{\text{const}} \approx 26$ MeV so

![Fig. 4: Lowest-order diagram for $u\bar{d} \to W^+ \to \nu_l l^+ (+\gamma)$.](image-url)
that it is necessary to state explicitly which parametrisation is used in a precision determination of the W boson mass from the W line shape.

The differential lowest-order cross section is easily obtained by squaring the lowest-order matrix element $\mathcal{M}_0$ of (6),

$$\left( \frac{d\sigma}{d\Omega} \right) = \frac{1}{12} \frac{1}{64\pi^2s} |\mathcal{M}_0|^2 = \frac{\alpha^2 |V_{ud}|^2}{192\pi^2 s} \frac{\hat{u}^2}{|\hat{s} - M_W^2 + iM_W\Gamma_W(\hat{s})|^2},$$

(7)

where $\hat{u} = (p_u - p_l)^2$ is the squared momentum difference between the up-type quark and the lepton. The explicit factor $1/12$ results from the average over the quark spins and colours, and $\hat{\Omega}$ is the solid angle of the outgoing $l^+$ in the parton CM frame. The electromagnetic coupling $\alpha = e^2/(4\pi)$ can be set to different values according to different renormalisation schemes. It can be directly identified with the fine-structure constant $\alpha(0)$ or the running electromagnetic coupling $\alpha(Q^2)$ at a high energy scale $Q$. For instance, it is possible to make use of the value of $\alpha(M_Z^2)$ that is obtained by analysing the experimental ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/(e^+e^- \rightarrow \mu^+\mu^-)$. These choices are called $\alpha(0)$-scheme and $\alpha(M_Z^2)$-scheme, respectively, in the following. Another value for $\alpha$ can be deduced from the Fermi constant $G_\mu$, yielding $\alpha G_\mu = \sqrt{2}G_\mu M_W^2 s_W^2/\pi$; this choice is referred to as $G_\mu$-scheme.

2.23 Electroweak corrections

The electroweak $\mathcal{O}(\alpha)$ corrections consist of virtual one-loop corrections and real-photonic bremsstrahlung. The corrections to resonant $W$ production have already been studied in [56, 57]; detailed discussions of the full calculation, including non-resonant corrections, can be found in [59, 60]. Since in $\mathcal{O}(\alpha^2)$ only two-photon bremsstrahlung [58] has been studied so far, the following discussion is restricted to $\mathcal{O}(\alpha)$ corrections.

The algebraic structure of the virtual corrections allows for a factorisation of the one-loop amplitude $\mathcal{M}_1$ into the Born amplitude $\mathcal{M}_0$ and a relative correction factor $\delta^{\text{virt}}$. Thus, in $\mathcal{O}(\alpha)$ the correction to the squared amplitude reads

$$|\mathcal{M}_0 + \mathcal{M}_1|^2 = (1 + 2\text{Re}\{\delta^{\text{virt}}\})|\mathcal{M}_0|^2 + \ldots.$$  

(8)

Since only the real part of $\delta^{\text{virt}}$ appears, there is no double-counting of the $\mathcal{O}(\alpha)$ correction that is already included in $\mathcal{M}_0$ by the $iM_W\Gamma_W$ term. Moreover, the factorisation trivially avoids potential problems with gauge-invariance after the introduction of the $W$ decay width in the resonant terms. Besides the Breit-Wigner factor in $|\mathcal{M}_0|^2$, there are logarithmic terms $\ln(\hat{s} - M_W^2)$ in $\delta^{\text{virt}}$ which are singular on resonance. The consistent replacement $\ln(\hat{s} - M_W^2) \rightarrow \ln(\hat{s} - M_W^2 + i\Gamma_W M_W)$ accounts for a Dyson summation of resonant $W$ propagators in loop diagrams, without introducing problems with gauge-invariance.

The real corrections are included by adding the lowest-order cross section for the process $u\bar{d} \rightarrow \nu l^+ + \gamma$. The only non-trivial condition induced by gauge-invariance is the Ward identity for the external photon, i.e. electromagnetic current conservation. If the $W$ width is zero, this identity is trivially fulfilled. This remains true even for a constant width, since the $W$ boson mass appears only in the $W$ propagator denominators, i.e. the substitution $M_W^2 \rightarrow M_W^2 - i\Gamma_W M_W$ is a consistent reparametrisation of the amplitude in this case. However, if a running $W$ width is introduced naively, i.e. in the $W$ propagators only, the Ward identity is violated. The identity can be restored by taking into account those part of the fermion-loop correction to the $\gamma W W$ vertex that corresponds to the fermion loops in the $W$ self-energy leading to the width in the propagator [64, 65, 66]. For an external photon, this modification simply amounts to the multiplication of the $\gamma W W$ vertex by the factor $f_{\gamma WW}\big|_{\text{run}} = 1 + i\Gamma_W/M_W$.

Adding virtual and real corrections, all IR divergences cancel. Mass singularities of the form $\alpha \ln m_t$ related to a final-state lepton drop out for all observables in which photons within a collinear cone around the lepton are treated inclusively, in accordance with the KLN theorem. As already discussed in...
Section 2.1 (see also [57]), mass singularities to the initial-state quarks are absorbed into renormalised quark distribution functions.

As long as one is interested in observables that are dominated by resonant $W$ boson production, the radiative corrections can be approximated by the corrections to the production and decay subprocesses to resonant $W$ bosons. Formally such an approximation can be carried out by a systematic expansion of all amplitudes about the resonance pole and is, therefore, called pole approximation (PA). In PA, the virtual correction consists of two parts. The first contribution is provided by the (constant) correction factors to the $W f ar{f}$ vertex for stable (on-shell) $W$ bosons and is called factorisable. The second contribution, which is called non-factorisable, comprises all remaining resonant corrections. It is entirely due to photonic effects and includes, in particular, the $\ln(s - M_{W}^{2} + i\Gamma_{W} M_{W})$ terms. The difference between PA and the exact result can be estimated by $\delta_{\text{PA}}^{\text{virt}} - \delta^{\text{virt}} \sim (\alpha/\pi) \ln(s/M_{W}^{2}) \ln(\ldots)$, where $\ln(\ldots)$ indicates any logarithmic enhancements. In principle, also the real corrections can be treated in PA. However, since a reliable error estimate is not obvious, they are usually calculated exactly. More details about PA can be found in [56, 60].

2.24 Numerical results

The following numerical results have been obtained with the input parameters of [60] and a constant $W$ width; in particular, we have $M_{W} = 80.35$ GeV and $\Gamma_{W} = 2.08$ GeV. The QED factorisation is performed in the $\overline{\text{MS}}$ scheme with $M_{W}$ being the factorisation scale, and the CTEQ4L [67] quark distributions are used in the evaluation of the $pp$ cross section. For the partonic cross section, the CKM matrix element $V_{uq}$ is set to 1; for the $pp$ cross section a non-trivial CKM matrix is included in the parton luminosities (see [60]).

![Figure 5: Total parton cross section $\hat{\sigma}$ in $G_{\mu}$ parametrisation and relative corrections $\delta$ for different parametrisations (results based on [60]).](image)

Figure 5 shows the total partonic cross section $\hat{\sigma}$ and the corresponding relative correction $\delta$ for intermediate energies. Note that the total cross section and its correction is the same for all final-state leptons $l = e, \mu, \tau$ in the limit of vanishing lepton masses. As expected, the $G_{\mu}$ parametrisation of the Born cross section minimises the correction at low energies, since the universal corrections induced by the running of $\alpha$ and by the $\rho$ parameter are absorbed in the lowest-order cross section. Moreover, the naive error estimate for the PA taken from above turns out to be realistic. The PA describes the correction in the resonance region within a few per mille. Table 1 contains some results on the partonic
Table 1: Total lowest-order parton cross section $\hat{\sigma}_0$ in $G_\mu$ parametrisation and corresponding relative correction $\delta$, exact and in PA (results based on [60]).

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (GeV)</th>
<th>40</th>
<th>80</th>
<th>120</th>
<th>200</th>
<th>500</th>
<th>1000</th>
<th>2000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\sigma}_0$ (pb)</td>
<td>2.646</td>
<td>7991.4</td>
<td>8.906</td>
<td>1.388</td>
<td>0.165</td>
<td>0.0396</td>
<td>0.00979</td>
</tr>
<tr>
<td>$\delta$ (%)</td>
<td>0.7</td>
<td>2.42</td>
<td>-12.9</td>
<td>-3.3</td>
<td>12</td>
<td>19</td>
<td>23</td>
</tr>
<tr>
<td>$\delta_{PA}$ (%)</td>
<td>0.0</td>
<td>2.40</td>
<td>-12.3</td>
<td>-0.7</td>
<td>18</td>
<td>31</td>
<td>43</td>
</tr>
</tbody>
</table>

Cross section and its correction up to energies in the TeV range. Far above resonance, the PA cannot follow the exact correction anymore, since non-resonant corrections become more and more important. The leading corrections are due to Sudakov logarithms of the form $\alpha \ln^2(\hat{s}/M_W^2)$.

![Graph](image)

Figure 6: Transverse-momentum distribution $(d\sigma/dp_{T,i})$ and relative corrections $\delta$ (results based on [60]).

Figure 6 shows the transverse-momentum distribution for the lepton $l^+$ produced in $pp \rightarrow W^+ \rightarrow \nu_l l^+ (+\gamma)$ for the $pp$ CM energy $\sqrt{s} = 14$ TeV of the LHC. The transverse momenta $p_T$ and the lepton pseudorapidity $\eta_l$ are restricted by $p_{T,i}, \hat{p}_T > 25$ GeV and $|\eta_l| < 1.2$. Since we do not recombine collinear photons and leptons, the corrections for different leptons do not coincide, but differ by corrections of the form $\ln(m_l/M_W)$. In the total cross section without any cuts these logarithms cancel, and the correction is again universal for all leptons in the massless limit. Since the $\ln m_l$ corrections are strongest for electrons, and since collinear photon emission reduces the momentum of the produced lepton, the correction $\delta$ for electrons is more negative (positive) for large (small) momenta than in the case of the muon. In particular, Figure 6 demonstrates the reliability of the PA for transverse lepton momenta $p_{T,i} \lesssim M_W/2$, where resonant $W$ bosons dominate. However, high $p_{T,i}$ values may also be interesting in searches for new physics. Table 2 shows the contributions to the total cross section divided by different ranges in $p_{T,i}$. From the above discussion of the parton cross section it is clear that the PA is not applicable for very large $p_{T,i}$, where the $W$ boson is far off shell.

The above results underline the importance of electroweak radiative corrections in a precise description for the $W$ boson cross section at the LHC. Although the corrections of $O(\alpha)$ are well under control now, there are still some topics to be studied, such as the impact of realistic detector cuts and photon recombination procedures or the inclusion of higher-order effects.
The impact of final state photon radiation on $W$ observables strongly depends on the lepton identification requirements imposed by the experiment. In addition to the lepton $p_T$, $p_T^*$ and pseudorapidity cuts, one usually imposes requirements on the separation of the charged lepton and the photon. For muons, the energy of the photon is required to be less than a critical value, $E^\gamma_c$, in a cone of radius $R^\gamma_c$ around the muon. For electrons, the finite resolution of the electromagnetic calorimeter makes it difficult to separate electrons and photons for small opening angles between the particles. Their four momentum vectors are therefore recombed if their separation is smaller than a critical value $R^\gamma_e$. Finally, uncertainties in the energy and momentum measurements of the charged lepton and the missing transverse energy need to be taken into account. They can be simulated by Gaussian smearing of the particle four-momentum vectors with standard deviation $\sigma$ which depends on the particle type and the detector.

To illustrate how finite detector resolution affects the size of the electroweak corrections, we show in Figure 7 the ratio of the NLO and lowest-order cross sections as a function of the $p_T$ of the electron in $pp \rightarrow \nu_\ell e^+ (\gamma)$ obtained with the Monte Carlo generator WGRAD [57]. The solid histogram shows the cross section ratio taking only transverse-momentum and pseudorapidity cuts into account. The dashed histogram displays the result obtained when, in addition, the four-momentum vectors are smeared according to the ATLAS specifications [53], and electron and photon momenta are combined if $\Delta R (e, \gamma) < 0.07$ [53]. Recombining the electron and photon four-momentum vectors eliminates the mass-singular logarithmic terms of the form $\alpha_\text{em} m_e$, and strongly reduces the size of the electroweak corrections.

### Table 2: Integrated lowest-order $pp$ cross sections $\sigma_0$ for different ranges in $p_{T,\ell}$ and corresponding relative corrections $\delta$, exact and in PA (results based on [60]).

<table>
<thead>
<tr>
<th>$p_{T,\ell}$ (GeV)</th>
<th>25–∞</th>
<th>25–45</th>
<th>45–∞</th>
<th>50–∞</th>
<th>80–∞</th>
<th>200–∞</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_0$ (pb)</td>
<td>1933.3(2)</td>
<td>1909.9(2)</td>
<td>23.52(2)</td>
<td>11.47(2)</td>
<td>1.682(3)</td>
<td>0.1014(1)</td>
</tr>
<tr>
<td>$\delta e^- e^+$ (%)</td>
<td>-5.51(5)</td>
<td>-5.45(7)</td>
<td>-11.8(5)</td>
<td>-9.7(4)</td>
<td>-11.7(3)</td>
<td>-17.7(2)</td>
</tr>
<tr>
<td>$\delta e^- \nu_e$ $\text{PA}$ (%)</td>
<td>-5.51(5)</td>
<td>-5.45(7)</td>
<td>-10.9(5)</td>
<td>-8.2(3)</td>
<td>-8.3(3)</td>
<td>-9.0(2)</td>
</tr>
<tr>
<td>$\delta \mu^- \nu_\mu$ (%)</td>
<td>-2.98(5)</td>
<td>-2.94(7)</td>
<td>-6.3(6)</td>
<td>-5.7(4)</td>
<td>-8.1(3)</td>
<td>-14.2(3)</td>
</tr>
<tr>
<td>$\delta \mu^- \nu_\mu$ $\text{PA}$ (%)</td>
<td>-2.97(5)</td>
<td>-2.94(7)</td>
<td>-5.7(6)</td>
<td>-4.6(4)</td>
<td>-4.9(3)</td>
<td>-5.6(2)</td>
</tr>
</tbody>
</table>

2.3 Electroweak corrections to $Z$ production and continuum neutral-current processes

#### 2.3.1 QED corrections

The mass-singular universal QED corrections from initial-state radiation from quarks have already been discussed in Section 2.1. They are part of the quark distribution functions. The residual QED initial-state corrections, together with final-state corrections and interference of initial-final radiation are treated separately by an explicit diagrammatic computation.

A complete calculation of the QED $\mathcal{O}(\alpha)$ radiative corrections to $pp \rightarrow Z, \gamma \rightarrow l^+ l^-$ has been carried out in [68]. The calculation is based on an explicit diagrammatic approach. The collinear singularities associated with initial-state photon radiation are factorised into the parton distribution functions (see Section 2.1). Absorbing the initial-state mass singularities into the pdf’s introduces a QED factorisation-scale dependence. The results presented here are obtained within the QED DIS scheme which is defined analogously to the QCD DIS factorisation scheme. The MRS(A) parton distributions are used, with a factorisation scale $M_Z$. Due to mass-singular logarithmic terms associated with photons emitted collinear with one of the final-state leptons, QED radiative corrections strongly affect the...
shape of the di-lepton invariant mass distribution, the lepton transverse momentum spectrum, and the forward-backward asymmetry, $A_{FB}$.

The effect of the QED corrections on the di-muon invariant mass distribution in the region $45$ GeV $< m(\mu^+\mu^-) < 105$ GeV is shown in Figure 8a where we plot the ratio of the $\mathcal{O}(\alpha^3)$ and lowest-order differential cross sections as a function of $m(\mu^+\mu^-)$. The lowest-order cross section has been evaluated in the effective Born approximation (EBA) which already takes into account those higher-order corrections which can be absorbed into a redefinition of the coupling constants and the effective weak mixing angle. More details on the EBA can be found in Section 2.32. In the region shown in the figure, the cross-section ratio is seen to vary rapidly. Below the $Z$ peak, QED corrections significantly enhance the cross section. At the $Z$ pole, the differential cross section is reduced by about 20%. Photon radiation from one of the leptons lowers the di-lepton invariant mass. Therefore, events from the $Z$ peak region are shifted towards smaller values of $m(\mu^+\mu^-)$, thus reducing the cross section in and above the peak region, and increasing the rate below the $Z$ pole. Final-state radiative corrections completely dominate over the entire mass range considered. They are responsible for the strong modification of the di-lepton invariant mass distribution. In contrast, initial-state corrections are uniform and small ($\approx +0.4\%$ in the QED DIS scheme).

As pointed out earlier, at the LHC a precise measurement of the effective mixing angle $\sin^2\theta_{\text{eff}}^{\text{kept}}$ using the forward-backward asymmetry may be possible. In Figure 8b, the forward-backward asymmetry is shown in the EBA (dashed line), and including QED corrections (solid line) for $pp \rightarrow \mu^+\mu^-(\gamma)$ in the di-muon invariant mass range from 45 GeV to 105 GeV. Here, $A_{FB}$ is defined by [68]

$$A_{FB} = \frac{F - B}{F + B}$$

where

$$F = \int_0^1 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*, \quad B = \int_{-1}^0 \frac{d\sigma}{d\cos\theta^*} d\cos\theta^*.$$  \hspace{1cm} (10)

$\cos\theta^*$ is given by

$$\cos\theta^* = \frac{|p_z(\mu^+\mu^-)|}{p_z(\mu^+\mu^-)} \frac{2}{m(\mu^+\mu^-) \sqrt{m^2(\mu^+\mu^-) + p_T^2(\mu^+\mu^-)}} \left[ p^+(\mu^-) p^- (\mu^+) - p^-(\mu^-) p^+ (\mu^+) \right]$$

\hspace{1cm} (11)
Fig. 8: Ratio of the $O(\alpha^3)$ and lowest-order differential cross sections, and the forward-backward asymmetry, $A_{FB}$, as a function of the $\mu^+\mu^-$ invariant mass. The cuts imposed are described in the text.

in the Collins-Soper frame [69], with

$$p^\pm = \frac{1}{\sqrt{2}} (E \pm p_z) ,$$

where $E$ is the energy and $p_z$ is the longitudinal component of the momentum vector. As expected, the $O(\alpha)$ QED corrections to $A_{FB}$ are large in the region below the $Z$ peak. Since events from the $Z$ peak, where $A_{FB}$ is positive and small, are shifted towards smaller values of $m(\mu^+\mu^-)$ by photon radiation, the forward-backward asymmetry is significantly reduced in magnitude by radiative corrections for $55 \text{ GeV} < m(\mu^+\mu^-) < 90 \text{ GeV}$. It should be noted that the forward-backward asymmetry is rather sensitive to the rapidity cuts imposed on the leptons. More details on $A_{FB}$ and the measurement of the effective weak mixing angle can be found in Section 3.24.

The mass singular terms arising from final-state photon radiation are proportional to $\alpha \log(s/m_l^2)$, where $m_l$ is the lepton mass. Thus, the corrections to the $Z$ line shape and $A_{FB}$ for electrons in the final state are considerably larger than those in the muon case [68].

To simulate detector acceptances, we have imposed a $p_T(\mu) > 20 \text{ GeV}$ and a $|p(\mu)| < 3.2$ cut in Figure 8. Except for the threshold region, the effects of the lepton acceptance cuts approximately cancel in the cross section ratio. In a more realistic simulation of how QED corrections affect observables in Drell-Yan production, lepton and photon identification requirements need to be taken into account in addition to the lepton acceptance cuts. Muons are identified in a hadron collider detector by hits in the muon chambers. In addition to a hit in the muon chambers, one requires that the associated track is consistent with a minimum ionising particle. This limits the energy of a photon which traverses the same calorimeter cell as the muon to be smaller than a critical value $E_\gamma^c$. For electrons, the finite resolution of the electromagnetic calorimeter makes it difficult to separate electrons and photons for small opening angles between their momentum vectors. Therefore, electron and photon four-momentum vectors are recombined if their separation in the azimuthal angle–pseudorapidity plane is smaller than a critical value, $R_c$. This eliminates the mass-singular terms associated with final-state photon radiation (KLN theorem) and thus may reduce significantly the effect QED corrections have on physical observables in $pp \rightarrow e^+e^-(\gamma)$ [68]. Specific results sensitively depend on the value of $R_c$, which is detector dependent.

2.32 Non-QED corrections and effective Born description

The amplitude for the parton process $q(p) + \bar{q}(\bar{p}) \rightarrow l^+(k_+) + l^- (k_-)$ of quark-antiquark annihilation into charged-lepton pairs is in lowest order described by photon and $Z$ boson exchange. In the kinematical
variables for the parton system

\[ \hat{s} = (k_+ + k_-)^2, \quad t = (p - k_-)^2, \quad u = (p - k_+)^2 \]  

the differential parton cross section can be written as follows (\( \theta \) denotes the scattering angle in the parton CMS):

\[
64\pi^2 \hat{s} \frac{d\hat{\sigma}}{d\Omega} = 2A_0 \frac{u^2 + t^2}{\hat{s}^2} + A_1 \frac{u^2 - t^2}{\hat{s}^2} = A_0 (1 + \cos^2 \theta) + A_1 \cos \theta
\]  

with

\[
A_0 = Q_q^2 Q_t^2 e(\hat{s})^4 + 2v_q v_t Q_q Q_t e(\hat{s})^2 \Re \chi(\hat{s}) + (v_q^2 + a_q^2) (v_t^2 + a_t^2) |\chi(\hat{s})|^2;
\]

\[
A_1 = 4Q_q Q_t a_q a_t e(\hat{s})^2 \Re \chi(\hat{s}) + 8v_q a_q v_t a_t |\chi(\hat{s})|^2.
\]  

This expression is an effective Born approximation, which incorporates several entries from higher-order calculations: the effective (running) electromagnetic charge containing the photon vacuum polarisation (real part)

\[
e(\hat{s})^2 = \frac{4\pi\alpha}{\Gamma(\hat{s})};
\]  

the Z propagator, together with the overall normalisation factor of the neutral-current couplings in terms of the Fermi constant \( G_\mu \),

\[
\chi(\hat{s}) = (G_\mu M_Z^2 \sqrt{2})^2 \frac{\hat{s} - M_Z^2}{\hat{s} - M_Z^2 + i\hat{\sigma}_Z/M_Z},
\]  

containing the Z width as measured from the Z resonance at LEP; the vector and axial-vector coupling constants for \( f = l, q \)

\[
v_f = I_f^3 - 2Q_f \sin^2 \theta_{\text{eff}}, \quad a_f = I_f^3,
\]  

which contain the effective ( leptonic) mixing angle at the Z peak, which is measured at LEP and SLC. Taking \( \Gamma_Z \) and \( \sin^2 \theta_{\text{eff}} \) from higher-order calculations, the formulae above yield a good description in the region around the Z resonance.

From the cross section (14) a forward-backward asymmetry for the produced \( l^+l^- \) system can be derived, which at the parton level is given by

\[
\hat{A}_{FB} = \frac{\hat{\sigma}_F - \hat{\sigma}_B}{\hat{\sigma}_F + \hat{\sigma}_B} = \frac{3A_1}{8A_0}.
\]  

Around the Z peak, this quantity depends sensitively on \( \sin^2 \theta_{\text{eff}} \). Using a parametrisation of the Born-like expressions in Equation 15, a measurement of \( \hat{A}_{FB} \) allows a determination of the mixing angle (see Section 3.). Below we give a quantitative evaluation of the higher-order electroweak effects in the integrated cross section and in \( \hat{A}_{FB} \) to demonstrate the quality of the approximation around the Z pole and to point out deviations at higher invariant masses of the lepton pairs.

Besides the universal and non-universal QED corrections, the following IR-finite next-order electroweak terms contribute, which are schematically depicted in Figure 9: self-energy contributions to the photon and Z propagators, vertex corrections to the \( \gamma/Z\ll \) and \( \gamma/Z\q\bar{q} \) 3-point couplings, and box diagrams with two massive boson exchanges. Details of the treatment of the resonance region at higher order is equivalent to that in \( e^+e^- \) annihilation in fermion pairs and can be found e.g. in [4]. Around the Z pole, the box graphs are negligible, but they increase strongly with the energy and hence contribute sizeably at high invariant masses of the lepton pair. A description in terms of an effective-Born cross section far away from the Z pole becomes insufficient for two reasons: the effective couplings (based on self-energies and vertex corrections only) are not static but grow as functions of \( \hat{s} \), and the presence of the box contributions, which cannot be absorbed in effective vector and axial-vector couplings in a Born-like structure.
Fig. 9: Born and higher-order electroweak contributions to $q\bar{q} \rightarrow e^+e^-$ in symbolic notation.

In Figures 10 and 11 we compare the integrated cross section $\hat{\sigma}$ and the asymmetry $A_{FB}$ at the parton level in the approximation corresponding to Equations 14 and 15 with results obtained by a complete one-loop calculation with proper treatment of higher-order terms around the $Z$ resonance. For demonstrational purpose, the effect of the box diagrams is displayed separately. As one can see, the region where the effective Born description starts to become unsatisfactory is at rather high values of the parton energy.

In order to give an idea of the effects remaining in the hadronic cross section after convolution with the quark distribution functions, Table 3 contains the relative deviations of the cross section based on the higher-order parton results from those based on the Born approximation Equation 15. Also listed are the estimated experimental accuracies with which the cross section in the various bins can be measured. The comparison shows that at high invariant masses the radiative corrections remain sizeable and should be taken into account for studies at high $s$, for example in the search for new physics effects originating from a heavy extra gauge-boson $Z'$.

2.33 The full electroweak $\mathcal{O}(\alpha)$ corrections: Monte Carlo simulations with ZGRAD2

The QED corrections described in Section 2.31 have been combined with the weak corrections summarised in the previous section in a new Monte Carlo program called ZGRAD2 [71]. In Figure 12a we show the ratio of the full $\mathcal{O}(\alpha^3)$ electroweak and the $\mathcal{O}(\alpha^3)$ QED differential cross sections for $pp \rightarrow \mu^+\mu^-(\gamma)$ obtained with ZGRAD2 as a function of the $\mu^+\mu^-$ invariant mass. As in Section 2.31, we have imposed a $p_T(\mu) > 20$ GeV and a $|\eta(\mu)| < 3.2$ cut, and used the EBA to evaluate the lowest-order contribution to the $\mathcal{O}(\alpha^3)$ QED cross section. Thus, the ratio directly displays the effect of the weak box-diagrams and the energy dependence of the weak coupling form factors. While the additional weak contributions only change the differential cross section by 0.6% at most, they do modify the shape of the $Z$ resonance curve.

Figure 12b compares the effect of the $\mathcal{O}(\alpha^3)$ QED corrections and the full $\mathcal{O}(\alpha^3)$ electroweak corrections on the di-muon invariant mass distribution for $m(\mu^+\mu^-)$ values between 200 GeV and 2 TeV. Due to the presence of logarithms of the form $\log(s/M_Z^2)$, the weak corrections become significantly larger than the QED corrections at large values of $m(\mu^+\mu^-)$, and, eventually, may have to be resummed [70]. For $m(\mu^+\mu^-) = 2$ TeV, the full $\mathcal{O}(\alpha^3)$ electroweak corrections are found to reduce the
Fig. 10: $u\bar{u} \rightarrow e^+e^-$. Energy dependence of $\hat{\sigma}$ in various steps of the approximation. $M_H = 100$ GeV and $m_t = 174$ GeV.
Finally, in Figure 13 we show how the $\mathcal{O}(\alpha^3)$ corrections affect the forward-backward asymmetry (see Equations 9 to 11). Both QED and weak corrections reduce $A_{FB}$, and their size increases with growing di-muon masses. For $m(\mu^+\mu^-) = 2 \text{ TeV}$, the weak corrections are about twice as large as the QED corrections. Note that the electroweak corrections affect $A_{FB}$ much less than the lepton pair invariant mass distribution. In the $Z$ pole region, $75 \text{ GeV} < m(\mu^+\mu^-) < 105 \text{ GeV}$, the weak corrections change the forward-backward asymmetry by at most $5 \times 10^{-4}$. Results qualitatively similar to those shown in Figures 12 and 13 are obtained for $pp \to e^+e^-(\gamma)$.

ZGRAD2 includes the complete weak one-loop corrections and the full non-universal QED $\mathcal{O}(\alpha)$ corrections. The collinear singularities associated with initial-state photon radiation are factorised into the parton distribution functions. However, QED corrections to the evolution of the parton distribution functions (see Section 2.1) are not included in ZGRAD2. These corrections should be part of a complete global fit of the pdf’s including all QED effect - this is beyond the scope of the calculation presented here. None of the current fits to the pdf’s include QED corrections.
Table 3: Hadronic cross section for $e^+e^-$ pairs with invariant mass in certain energy ranges. Columns two and three show the predicted cross sections in the effective Born approximation and the full one-loop calculation. Columns four and five show the relative corrections to the effective Born approximation arising from the full one-loop calculation as well as the estimated experimental errors for the cross section measurements in the various bins.

<table>
<thead>
<tr>
<th>Energy range (for $e^+e^-$ pairs)</th>
<th>Born cross section (fb)</th>
<th>Full (non-QED) cross section (fb)</th>
<th>Relative correction to Born cross section (%)</th>
<th>Relative experimental error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.9 - 1.1</td>
<td>6.2299</td>
<td>5.6524</td>
<td>-9.3</td>
<td>3</td>
</tr>
<tr>
<td>1.1 - 1.5</td>
<td>3.5205</td>
<td>3.1491</td>
<td>-10.5</td>
<td>4</td>
</tr>
<tr>
<td>1.5 - 1.75</td>
<td>0.6076</td>
<td>0.5317</td>
<td>-12.5</td>
<td>9.5</td>
</tr>
<tr>
<td>1.75 - 2.0</td>
<td>0.2681</td>
<td>0.2314</td>
<td>-13.7</td>
<td>14</td>
</tr>
<tr>
<td>2.0 - 2.5</td>
<td>0.1886</td>
<td>0.1590</td>
<td>-15.7</td>
<td>17</td>
</tr>
<tr>
<td>2.5 - 3.0</td>
<td>0.04895</td>
<td>0.04031</td>
<td>-17.7</td>
<td>30</td>
</tr>
<tr>
<td>3.0 - 4.0</td>
<td>0.01837</td>
<td>0.01464</td>
<td>-20.3</td>
<td>50</td>
</tr>
</tbody>
</table>

Fig. 12: a) Ratio of the full $\mathcal{O}(\alpha^3)$ electroweak and the $\mathcal{O}(\alpha^3)$ QED differential cross sections in the vicinity of the Z pole. b) Differential cross section ratios, displaying the size of the full $\mathcal{O}(\alpha^3)$ electroweak and the $\mathcal{O}(\alpha^3)$ QED corrections for large values of $m(\mu^+\mu^-)$. The cuts imposed are described in the text.
2.4 Z' indication from new APV data in cesium and searches at LHC

The weak charge $Q_W$ for a heavy atom is defined in terms of the number of $u, d$ quarks $N_u = 2Z + N, N_d = 2N + Z$ in the nucleus $(Z, N)$ and the coefficients $C_{1u,d}$ in the parity-violating part of the electron-quark Hamiltonian,

$$\mathcal{H}_{PV} = -\frac{G_F}{\sqrt{2}} \bar{e}_\mu \gamma^\mu e\left(C_{1u} \bar{u} \gamma^\mu u + C_{1d} \bar{d} \gamma^\mu d\right),$$

(20)

via the relation

$$Q_W = 2(N_u C_{1u} + N_d C_{1d}).$$

(21)

In the SM: $C_{1q} = I_3^q - 2Q_q \sin^2 \theta_W$.

In a recent paper [72] a new determination of the weak charge of atomic cesium has been reported. The most precise atomic parity violating (APV) experiment compares the mixing among $S$ and $P$ states due to neutral weak interactions to an induced Stark mixing [73]. The 1.2% uncertainty on the previous measurement of the weak charge $Q_W$ was dominated by the theoretical calculations on the amount of Stark mixing and on the electronic parity violating matrix elements. In [72] the Stark mixing was measured and, incorporating new experimental data, the uncertainty in the electronic parity violating matrix elements was reduced. The new result $Q_W^{133^\text{Cs}} = -72.06 \pm (0.28)_{\text{expt}} \pm (0.34)_{\text{theor}}$ represents a considerable improvement with respect to the previous determination [73, 74, 75, 76]. The discrepancy between the standard model (SM) and the experimental data is now given by $Q_W^{\text{exp}} - Q_W^{SM} = 1.18(1.28) \pm 0.46$ (for $m_t = 175$ GeV and $M_H = 100(300)$ GeV). This corresponds to 2.6(2.8) standard deviations [77], excluding the SM at 99% CL and, a fortiori, all the models leading to negative additional contributions to $Q_W$, as for example models with a sequential $Z'$ [77]. This deviation could be explained by assuming the existence of an extra $Z'$ from $E_6$ or $O(10)$ or from $Z'_{LR}$ of left-right (LR) models [72, 77, 78]. The high-energy data at the $Z$ resonance strongly bound the $Z - Z'$ mixing [79]; for this reason we will assume zero mixing. In this case, the new physics contribution to $Q_W$ is due to the direct exchange of the $Z'$ and is completely fixed by the $Z'$ parameters, $\delta_N Q_W = 16a'_{le}(2Z + N)v'_u + (Z + 2N)v'_d/M^2_{Z'}$, where $a'_{le}, v'_f$ are the couplings $Z'$ to fermions and, for $^{133^\text{Cs}}, Z = 55$ and $N = 78$. The relevant couplings of the $Z'$ to the electron and to the up and down quarks are given in the Table 1 of [77].

In the case of the LR model considered in [77], the extra contribution to the weak charge is $\delta_N Q_W = -M^2_{Z'}/M^2_{Z'} Q_W^{SM}$. For this model one has a 95% CL lower bound on $M_{Z'_{LR}}$ from the Tevatron [80] given by $M_{Z'_{LR}} \geq 630$ GeV. An LR model could then explain the APV data allowing for a mass of the $Z'_{LR}$ varying between the intersection from the 95% CL bounds $540 \leq M_{Z'_{LR}}$ (GeV) $\leq 1470$ deriving
Fig. 14: The 95% CL lower and upper bounds for $M_{Z'}$ for the extra-U(1) models versus $\theta_6$. The solid (dash) line corresponds to $H = 100(300)$ GeV.

from $Q_W$ and the lower bound of 630 GeV. In the case of the extra-U(1) models, the CDF experimental lower bounds for the masses vary according to the values of the angle $\theta_6$ which parameterises different extra-U(1) models, but in general they are about 600 GeV at 95% CL [80]. For the particular models $\eta$, $\psi$, $\chi$, corresponding to $\theta_6 = \arctan(-\sqrt{5}/3)$, $\pi/2$, $0$, the 95% CL lower bounds are $M_{Z'\eta} \approx 620$ GeV, $M_{Z'\psi} \approx 590$ GeV, $M_{Z'\chi} \approx 595$ GeV. In Figure 14, the 95% CL bounds on $M_{Z'}$ from APV are plotted versus $\theta_6$ (the direct lower bounds from the Tevatron are about 600 GeV). We see that an extra $Z'$ can explain the discrepancy with the SM prediction for the $Q_W$ for a wide range of $\theta_6$ angle. In particular, the models $\eta$ and $\psi$ are excluded, whereas the $\chi$ model is allowed for $M_{Z'\chi}$ less than about 1.2 TeV.

In the near future, the Tevatron upgrade and LHC can confirm or disprove this indication coming from $Q_W$. The existing bounds for $E_6$ models from direct searches at the Tevatron will be upgraded by the future run with $\sqrt{s} = 2$ TeV and 1 fb$^{-1}$ to $M_{Z'} \sim 800 - 900$ GeV and pushed to $\sim$ 1 TeV for 10 fb$^{-1}$. The bounds are based on 10 events in the $e^+e^- + \mu^+\mu^-$ channels and decays to SM final-states only are assumed [81]. At the LHC with an integrated luminosity of 100 fb$^{-1}$, one can explore a mass range up to $4 - 4.5$ TeV depending on the $\theta_6$ value. Concerning LR models, the 95% CL lower limits from the Tevatron run with $\sqrt{s} = 2$ TeV and 1(10) fb$^{-1}$ are $\sim$ 900(1000) GeV and extend to $\sim$ 4.5 TeV at LHC [81]. Ratios of $Z'$ couplings to fermions can be probed at LHC, by considering the forward-backward asymmetries, ratios of cross sections in different rapidity bins and other observables. For example, for $M_{Z'} = 1$ TeV, the LHC can determine the magnitude of normalised $Z'$ quark and lepton couplings to around 10 – 20% [81]. Therefore, if the deviation for the weak charge $Q_W$ with respect to the SM prediction is not due to a statistical fluctuation, the new physics described by an extra gauge-boson model like $Z'_\chi$ can explain the discrepancy and the LHC will be able to verify this possible evidence.
3. PRECISION MEASUREMENTS

3.1 Measurement of the $W$ mass

At the time of the LHC start-up, the $W$ mass will be known with a precision of about 30 MeV from measurements at LEP2 [82] and Tevatron [83]. The motivation to improve on such a precision is discussed briefly below. The $W$ mass, which is one of the fundamental parameters of the Standard Model, is related to other parameters of the theory, i.e. the QED fine structure constant $\alpha$, the Fermi constant $G_F$ and the Weinberg angle $\sin \theta_W$, through the relation

$$M_W = \sqrt{\frac{\pi \alpha}{G_F \sqrt{2}}} \cdot \frac{1}{\sin \theta_W \sqrt{1 - \Delta r}} \quad (22)$$

where $\Delta r$ accounts for the radiative corrections which amount to about 4%. The radiative corrections depend on the top mass as $\sim m_t^2$ and on the Higgs mass as $\sim \log M_H$. Therefore, precise measurements of both the $W$ mass and the top mass constrain the mass of the Standard Model Higgs boson or of the $h$ boson of the MSSM. This constraint is relatively weak because of the logarithmic dependence of the radiative corrections on the Higgs mass. When it comes to making a comparison of the measurements of $(M_W, m_t)$ with the SM predictions, it is not very useful if one measurement is much more restrictive than the other. To ensure that the two mass determinations have equal weight in a $\chi^2$ test, the precision on the top mass and on the $W$ mass should be related by the expression

$$\Delta M_W \approx 0.7 \times 10^{-2} \Delta m_t \quad (23)$$

Since the top mass will be measured with an accuracy of about 2 GeV at the LHC [53], the $W$ mass should be known with a precision of about 15 MeV, so that it does not become the dominant error in the test of the radiative corrections and in the estimation of the Higgs mass. Such a precision is beyond the sensitivity of Tevatron and LEP2.

A study was performed to assess whether the LHC will be able to measure the $W$ mass to about 15 MeV [84, 85]. The ATLAS experiment was taken as an example, but similar conclusions hold also for CMS. Such a precise measurement, which will be performed already in the initial phase at low luminosity as will the top mass measurement, would constrain the mass of the Higgs boson to better than 30%. When and if the Higgs boson will be found, such constraints would provide an important consistency check of the theory, and in particular of its scalar sector. Distinguishing between the Standard Model and the MSSM might be possible, since the radiative corrections to the $W$ mass are expected to be a few percent larger in the latter case.

The measurement of the $W$ mass at hadron colliders is sensitive to many subtle effects which are difficult to predict before the experiments start. However, based on the present knowledge of the LHC detector performance and on the experience from the Tevatron, it is possible to make a reasonable estimate of the total uncertainty and of the main contributions to be expected. In turn, this will lead to requirements for the detector performance and the theoretical inputs which are needed to achieve the desired precision. This is the aim of the study which is described in the next sections.

3.11 The method

The measurement of the $W$ mass at hadron colliders is performed in the leptonic channels. Since the longitudinal momentum of the neutrino cannot be measured, the transverse mass $m_T^W$ is used. This is calculated using the transverse momenta of the neutrino and of the charged lepton, ignoring the longitudinal momenta:

$$m_T^W = \sqrt{2p_T^{\nu} p_T^e (1 - \cos \Delta \phi)} \quad (24)$$

Section coordinator: S. Haywood
where \( l = e, \mu \). The lepton transverse momentum \( p_T^l \) is measured, whereas the transverse momentum of the neutrino \( p_T^\nu \) is obtained from the transverse momentum of the lepton and the momentum \( \vec{u} \) of the system recoiling against the \( W \) in the transverse plane (hereafter called “the recoil”):

\[
p_{T}^{l} = -|p_{T}^{l} + \vec{u}|
\]  

(25)

The angle between the lepton and the neutrino in the transverse plane is denoted by \( \Delta \phi \). The distribution of \( m_T^W \), and in particular the trailing edge of the spectrum, is sensitive to the \( W \) mass. Therefore, by fitting the experimental distribution of the transverse mass with Monte Carlo samples generated with different values of \( M_W \), it is possible to obtain the mass which best fits the data. The trailing edge is smeared by several effects, such as the \( W \) intrinsic width and the detector resolution. This is illustrated in Figure 15, which shows the distribution of the \( W \) transverse mass as obtained at particle level (no detector resolution) and by including the energy and momentum resolution as implemented in a fast particle-level simulation and reconstruction of the ATLAS detector (ATLFAST, [85]). The smearing due to the finite resolution reduces the sharpness of the end-point and therefore the sensitivity to \( M_W \).

![Distribution of the W transverse mass](image)

Fig. 15: Distribution of the \( W \) transverse mass as obtained at particle level and by including the expected ATLAS detector resolution.

When running at high luminosity, the pile-up will smear significantly the transverse mass distribution, therefore the use of the transverse-mass method will probably be limited to the initial phase at low luminosity. Alternative methods are mentioned in Section 3.14.

### 3.12 W production and selection

At the LHC, the cross-section for the process \( pp \rightarrow W + X \) with \( W \rightarrow l\nu \) and \( l = e, \mu \) is 30 nb. Therefore, about 300 million events are expected to be produced in each experiment in one year of operation at low luminosity (integrated luminosity 10 fb\(^{-1}\)). Such a cross-section is a factor of ten larger than at the Tevatron (\( \sqrt{s} = 1.8 \) TeV).

To extract a clean \( W \) signal, one should require:

- An isolated charged lepton (\( e \) or \( \mu \)) with \( p_T > 25 \text{ GeV} \) inside the pseudorapidity region devoted to precision physics \( |\eta| < 2.4 \).
- Missing transverse energy \( E_T^{\text{miss}} > 25 \text{ GeV} \).
- No jets in the event with \( p_T > 30 \text{ GeV} \).
- The recoil should satisfy \( |\vec{u}| < 20 \text{ GeV} \).
The last two cuts are applied to reject $W$’s produced with high $p_T$, since for large $p_T^W$ the transverse mass resolution deteriorates and the QCD background increases. The acceptance of the above cuts is about 25%. By assuming a lepton reconstruction efficiency of 90% and an identification efficiency of 80% [86], a total selection efficiency of about 20% should be achieved. Therefore, after all cuts about 60 million $W$’s are expected in one year of data taking at low luminosity in each experiment, which is a factor of about 50 larger than the statistics expected from the Tevatron Run 2.

3.13 Expected uncertainties

Due to the large event sample, the statistical uncertainty on the $W$ mass should be smaller than 2 MeV for an integrated luminosity of $10 \text{ fb}^{-1}$.

Since the $W$ mass is obtained by fitting the experimental distribution of the transverse mass with Monte Carlo samples, the systematic uncertainty will come mainly from the Monte Carlo modelling of the data, i.e. the physics and the detector performance. Uncertainties related to the physics include the knowledge of: the $W$ $p_T$ spectrum and angular distribution, the parton distribution functions, the $W$ width, the radiative decays and the background. Uncertainties related to the detector include the knowledge of: the lepton energy and momentum scale, the energy and momentum resolution, the detector response to the recoil and the effect of the lepton identification cuts. At the LHC, as now at the Tevatron, most of these uncertainties will be constrained in situ by using data samples such as $Z \rightarrow ll$ decays. The latter will be used to determine the lepton energy scale, to measure the detector resolution, to model the detector response to the $W$ recoil and the $p_T$ spectrum of the $W$, etc.

The advantages of the LHC with respect to the Tevatron experiments are:

- The large number of $W$ events mentioned above.
- The large size of the ‘control samples’. About six million $Z \rightarrow ll$ decays, where $l = e, \mu$, are expected in each experiment in one year of data taking at low luminosity after all selection cuts. This is a factor of about 50 larger than the event sample from the Tevatron Run 2.
- ATLAS and CMS are in general more powerful than CDF and D0 are, in terms of energy resolution, particle identification capability, geometrical acceptance and granularity. What may be more important for this measurement is the fact that ATLAS and CMS will benefit from extensive and detailed simulations and test-beam studies of the detector performance, undertaken even before the start of data-taking.

Nevertheless, the LHC experiments have complex detectors, which will require a great deal of study before their behaviour is well understood.

To evaluate the systematic uncertainty on the $W$ mass to be expected in ATLAS, $W \rightarrow l\nu$ decays were generated with PYTHIA 5.7 and processed with ATLASFD. After applying the selection cuts discussed above, a transverse mass spectrum was produced for a reference mass value (80.300 GeV). All sources of systematic uncertainty affecting the measurement of the $W$ mass from CDF Run 1 [87, 88] were then considered as an example. Their magnitude was evaluated in most cases by extrapolating from the Tevatron results, on the basis of the expected ATLAS detector performance. The resulting error on the $W$ mass was determined by generating new $W$ samples, each one including one source of uncertainty, and by comparing the resulting transverse mass distributions with the one obtained for the reference mass. A Kolmogorov test [90] was used to evaluate the compatibility between distributions.

Since the goal is a total error of $\sim 20$ MeV per experiment, the individual contributions should be much smaller than 10 MeV. A large number of events was needed to achieve such a sensitivity. With three million events after all cuts, corresponding to twelve million events at the generation level, a sensitivity at the level of 8 MeV was obtained.

\[^4\text{Similar results have been obtained by the D0 experiment [88, 89].}\]
The main sources of uncertainty and their impact on the $W$ mass measurement are discussed one by one in the remainder of this section. The total error and some concluding remarks are presented in Section 3.14.

**Lepton energy and momentum scale** This is the dominant source of uncertainty on the measurement of the $W$ mass from Tevatron Run 1, where the absolute lepton scale is known with a precision of about 0.1% [87, 88, 89]. Most likely, this will be the dominant error also at the LHC. In order to measure the $W$ mass with a precision of better than 20 MeV, the lepton scale has to be known to 0.02%. The latter is the most stringent requirement on the energy and momentum scale from LHC physics. It should be noted that a very high precision (0.04%) must be achieved also by the Tevatron experiments in Run 2, in order to measure the $W$ mass to 40 MeV [83]. If such a precision will indeed be demonstrated at the Tevatron, it would represent a good benchmark for the LHC experiments.

The lepton energy and momentum scale will be calibrated in situ at the LHC by using physics samples, which will complement the information coming from the hardware calibration, from the magnetic field mapping of solenoids and toroids, and from test-beam measurements. The muon scale will be calibrated by using mainly $Z \to \mu\mu$ events, and the electromagnetic calorimeter scale will be calibrated by using mainly $Z \to ee$ events or $E/p$ measurements for isolated electrons, where $E$ and $p$ are the electron energy and momentum as measured in the electromagnetic calorimeter and in the inner detector respectively. Leptonic decays of other resonances ($\Upsilon, J/\psi$) should provide additional constraints which minimise the extrapolation error to lower masses than the $Z$ boson mass.

Similar methods are used today at the Tevatron, where the uncertainty on the absolute lepton scale is dominated by the statistical error due to the limited $Z$ data sample. The main advantage of the LHC compared to the Tevatron is the large sample of $Z \to ll$ decays. The $Z$ boson is close in mass to the $W$ boson, therefore the extrapolation error from the point where the scale is determined to the point where the measurement is performed is small.

A preliminary study of the error on the absolute electron scale to be expected in ATLAS was performed by using a sample of 500000 $Z \to ee$ decays processed through a full GEANT-based simulation of the ATLAS detector [86]. Several possible sources of uncertainties were considered: the knowledge of the amount of material in the inner detector, which affects the electromagnetic calorimeter scale because of photon bremsstrahlung; radiative $Z$ decays, which distort the reconstructed mass spectrum; the modelling of the underlying event and of the pile-up at low and high luminosity. Table 4 shows that the impact of these uncertainties on the electron scale in the calorimeter can most likely be kept below 0.02%. The most stringent requirement to achieve this goal is the knowledge of the material in the inner detector to 1%, which will require scrutiny during construction plus in situ measurements with photon conversions and $E/p$ for isolated electrons. More details can be found in [86].

<table>
<thead>
<tr>
<th>Source</th>
<th>Requirement</th>
<th>Uncertainty on scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material in inner detector</td>
<td>Known to 1%</td>
<td>&lt; 0.01%</td>
</tr>
<tr>
<td>Radiative decays</td>
<td>Known to 10%</td>
<td>&lt; 0.01%</td>
</tr>
<tr>
<td>Underlying event</td>
<td>Calibrate and subtract</td>
<td>≲ 0.03%</td>
</tr>
<tr>
<td>Pile-up at low luminosity</td>
<td>Calibrate and subtract</td>
<td>≲ 0.01%</td>
</tr>
<tr>
<td>Pile-up at high luminosity</td>
<td>Calibrate and subtract</td>
<td>≲ 0.01%</td>
</tr>
</tbody>
</table>

Table 4: Expected contributions to the uncertainty on the electron energy scale of the ATLAS electromagnetic calorimeter, as determined using a fully-simulated sample of $Z \to ee$ decays (from [86]).
Several experimental constraints will be needed to achieve a 0.02% uncertainty on the inner detector muon scale: the solenoidal magnetic field in the inner cavity must be known locally to better than 0.1%, the alignment must be understood locally to $\sim 1\,\mu\text{m}$ in the bending plane, etc. A detailed discussion on how to meet these goals can be found in [86, 91].

The scale calibration of the external muon spectrometer depends on the knowledge of the magnetic field, on the chamber alignment and on the knowledge of the muon energy losses in the calorimeters. The latter must be understood to a precision of 0.25% in order to achieve the goal uncertainty of 0.02% on the absolute scale. A preliminary study based on a full GEANT simulation of the ATLAS detector demonstrated that with a sample of only 10000 $Z \rightarrow \mu\mu$ decays a scale uncertainty of 0.1% should be attained in the muon spectrometer. More details can be found in [86, 92].

In conclusion, to achieve the needed precision on the lepton scale, several experimental constraints will have to be satisfied. In addition, cross-checks and combined fits between different sub-detectors (inner detector and electromagnetic calorimeter for the electron scale, inner detector and muon system for the muon scale) will be needed. Indeed, only in an over-constrained situation will it be possible to disentangle the various contributions to the detector response, and therefore to derive a reliable systematic error.

**Lepton energy and momentum resolution** To keep the uncertainty on the $W$ mass from the lepton resolution to less than 10 MeV, the energy resolution of the electromagnetic calorimeter and the momentum resolution of the inner detector and muon system have to be known with a precision of better than 1.5%.

The lepton energy and momentum resolutions will be determined at the LHC by using information from test-beam data and from Monte Carlo simulations of the detector, as well as in situ measurements of the $Z$ width in $Z \rightarrow ll$ final states. The $E/p$ distribution for electrons from $W$ decays provides an additional tool. These methods are used presently at the Tevatron. As an example, the statistical error on the momentum resolution obtained by CDF in Run 1A is 10%, whereas the systematic error is only 1% and is dominated by the uncertainty on the radiative decays of the $Z$ [87]. Since the ATLAS performance in terms of momentum resolution is expected to be similar to that of CDF in the momentum range relevant to $W$ production and decays, and since the statistical error at the LHC will be negligible, a total error of much less than 1.5% should be achieved. This uncertainty might further be decreased if improved theoretical calculations of radiative $Z$ decays will become available.

**Recoil modelling** The transverse momentum of the system recoiling against the $W$, together with the lepton transverse momentum, is used to determine the $p_T$ of the neutrino (see Equation 25). The recoil is mainly composed of soft hadrons from the underlying event, for which neither the physics nor the detector response are known with enough accuracy. Therefore, in order to get a reliable recoil distribution in the Monte Carlo, information from data is used at the Tevatron. By exploiting the similar production mechanisms of $W$ and $Z$ bosons, in each Monte Carlo event with a given $p_T^W$ (determined from the truth information) the recoil is replaced by the recoil measured in the data for $Z$ events characterised by a $p_T^Z$ (measured by the leptons) similar to $p_T^W$. The resulting error on the $W$ mass from CDF Run 1A is 60 MeV per channel, and is dominated by the limited statistics of $Z$ data. The result obtained from Run 1B (about 30 MeV) shows that this uncertainty scales with $\sqrt{N}$, where $N$ is the number of events. Extrapolating to the LHC data sample, an error of smaller than 10 MeV per channel should be achieved. It should be noted that the recoil includes the contribution of the pile-up expected at low luminosity (two minimum-bias events per bunch crossing on average).

**$W$ $p_T$ spectrum** The modelling of $p_T^W$ in the Monte Carlo is affected by both theoretical and experimental uncertainties. Theoretical uncertainties arise from the difficulty in predicting the non-perturbative
regime of soft-gluon emission, as well as from missing higher-order QCD corrections. Experimental uncertainties are mainly related to the difficulty of simulating the detector response to low-energy particles.

Therefore, the method used at the Tevatron to obtain a reliable estimate of $p_T^W$ consists of measuring the $p_T$ distribution of the $Z$ boson from $Z \rightarrow \ell \ell$ events in the data, exploiting the fact that both gauge-bosons have similar $p_T$ distributions, and using the theoretical prediction for the ratio $p_T^W/p_T^Z$ (in this ratio several uncertainties cancel) to convert the measured $p_T^Z$ into $p_T^W$. The resulting error on the $W$ mass obtained by CDF is 20 MeV, dominated by the limited $Z$ statistics.

At the LHC, the average transverse momentum of the $W$ ($Z$) is 12 GeV (14 GeV), as given by PYTHIA 5.7. Over the range $p_T$ ($W,Z$) < 20 GeV, both gauge-bosons have $p_T$ spectra which agree to within ±10%. By assuming a negligible statistical error on the knowledge of $p_T^Z$, which will be measured with high-statistics data samples, and by using the $p_T^Z$ spectrum instead of the $p_T^W$ distribution, an error on the $W$ mass of about 10 MeV per channel was obtained without any further tuning. Although the leading-order parton shower approach of PYTHIA is only an approximation to reality, this result is encouraging. Furthermore, improved theoretical calculations for the ratio of the $W$ and $Z$ $p_T$ distributions should become available at the time of the LHC, so that the final uncertainty will most likely be smaller than 10 MeV.

**Parton distribution functions** Parton momentum distributions inside the protons determine the $W$ longitudinal momentum, and therefore affect the transverse mass distribution through lepton acceptance effects. At the Tevatron, parton distribution functions (pdf), in particular the $u/d$ ratio, are constrained by measuring the forward-backward charge asymmetry of the $W$ rapidity distribution. Such an asymmetry, which is typical of $p\bar{p}$ collisions, is not present in $pp$ collisions and therefore cannot be used at the LHC. However, it has been shown [55] that pdf can be constrained to a few percent at the LHC by using mainly the pseudorapidity distributions of leptons produced in $W$ and $Z$ decays. The resulting uncertainty on the $W$ mass should be smaller than 10 MeV.

**$W$ width** At hadron colliders, the $W$ width can be obtained from the measurement of $R$, the ratio between the rate of leptonically decaying $W$’s and leptonically decaying $Z$’s:

$$R = \frac{\sigma_W}{\sigma_Z} \times \frac{BR(W \rightarrow \ell \nu)}{BR(Z \rightarrow \ell \ell)}$$  \hspace{1cm} (26)

where the $Z$ branching ratio ($BR$) is obtained from LEP measurements, and the ratio between the $W$ and the $Z$ cross-sections is obtained from theory. By measuring $R$, the leptonic branching ratio of the $W$ can be extracted from the above formula, and therefore $\Gamma_W$ can be deduced assuming Standard Model couplings for $W \rightarrow \ell \nu$. The precision achievable with this method is limited by the theoretical knowledge of the ratio of the $W$ to the $Z$ cross-sections. Another method consists of fitting the high-mass tails of the transverse mass distribution, which are sensitive to the $W$ width.

By using these methods, the $W$ width was measured with a precision of about 60 MeV by CDF in Run 1, which translates into an error of 10 MeV per channel on the $W$ mass measurement.

In Run 2, the $W$ width should be measured with a precision of 30 MeV [83], which contributes an error of 7 MeV per channel on the $W$ mass. This is however a conservative estimate for the LHC, where the $W$ width should be measured with higher precision than at Tevatron by using the high-mass tails of the transverse mass distribution. The measurement of $R$, on the other hand, in addition to being model-dependent would require very precise theoretical inputs. It should be noted that one could also use the value of the $W$ width predicted by the Standard Model.

**Radiative decays** Radiative $W \rightarrow \ell \nu \gamma$ decays produce a shift in the reconstructed transverse mass, which must be precisely modelled in the Monte Carlo. Uncertainties arise from missing higher-order
corrections, which translate into an error of 20 MeV on the W mass as measured by CDF in Run 1. Improved theoretical calculations have become recently available [93]. Furthermore, the excellent granularity of the ATLAS electromagnetic calorimeter, and the large statistics of radiative Z decays, should provide useful additional information. Therefore, a W mass error of 10 MeV per channel was assumed in this study. This is a conservative estimate, since the D0 error from Run 1 is smaller than 10 MeV [88].

Backgrounds distort the W transverse mass distribution, contributing mainly to the low-mass region. Therefore, uncertainties on the background normalisation and shape translate into an error on the W mass. This error is at the level of 5 MeV (25 MeV) in the electron (muon) channel for the measurement performed by CDF in Run 1, where the background is known with a precision of about 10%.

A study was made of the main backgrounds to \( W \rightarrow l \nu \) final states to be expected in ATLAS. The contribution from \( W \rightarrow \tau \nu \) decays should be of order 1.3% in both the electron and the muon channel. The background from \( Z \rightarrow ee \) decays to the \( W \rightarrow \mu \nu \) channel is expected to be negligible, whereas the contribution of \( Z \rightarrow \mu \mu \) decays to the \( W \rightarrow l \nu \) channel should amount to 4%. The difference between these two channels is due to the fact that the calorimetry coverage extends up to \( |\eta| \sim 5 \), whereas the coverage of the muon spectrometer is limited to \( |\eta| < 2.7 \). Therefore, muons from Z decays which are produced with \( |\eta| > 2.7 \) escape detection and thus give rise to a relatively large missing transverse momentum. On the other hand, electrons from Z decays produced with \( |\eta| > 2.4 \) are not efficiently identified, because of the absence of tracking devices and of fine-grained calorimetry, however their energy can be measured up to \( |\eta| \sim 5 \). Therefore these events do not pass the \( E_{T}^{\text{miss}} \) cut described in Section 3.12. Finally, \( t\bar{t} \) production and QCD processes are expected to give negligible contributions.

In order to limit the error on the W mass to less than 10 MeV, the background to the electron channel should be known with a precision of 30%, which is easily achievable, and the background to the muon channel should be known with a precision of 7%. The latter could be monitored by using \( Z \rightarrow ee \) decays.

3.14 Results
The expected contributions to the uncertainty on the W mass measurement, of which some are discussed in the previous sections, are summarised in Table 5. For comparison, the errors obtained by CDF in Run 1A (integrated luminosity \( \sim 20 \text{ pb}^{-1} \)) and Run 1B (integrated luminosity \( \sim 90 \text{ pb}^{-1} \)) are also shown separately. The evolution of the uncertainty between Run 1A and Run 1B shows the effect of the increased statistics and of the improved knowledge of the detector performance and of the physics, and provides a solid basis for the LHC results presented here.

With an integrated luminosity of 10 fb\(^{-1} \), which should be collected in one year of LHC operation, and by considering only one lepton species (e or \( \mu \)), a total uncertainty of smaller than 25 MeV should be achieved by each LHC experiment. By combining both lepton channels, which should also provide useful cross-checks since some of the systematic uncertainties are different for the electron and the muon sample, and taking into account common uncertainties, the total error should decrease to less than 20 MeV per experiment. Finally, the total LHC uncertainty could be reduced to about 15 MeV by combining ATLAS and CMS together. Such a precision would allow the LHC to compete with the expected precision at a Next Linear Collider [94].

The most serious experimental challenge in this measurement is the determination of the lepton absolute energy and momentum scale to 0.02%. All other uncertainties are expected to be of the order of (or smaller than) 10 MeV. However, to achieve such a goal, improved theoretical calculations of radiative decays, of the \( W \) and \( Z \) \( p_{T} \) spectra, and of higher-order QCD corrections will be needed.

The results presented here have to be considered as preliminary and far from being complete. It may be possible that, by applying stronger selection cuts, for instance on the maximum transverse
Table 5: Expected contributions to the uncertainty on the $W$ mass measurement in ATLAS for each lepton family and for an integrated luminosity of 10 $fb^{-1}$ (fourth column). The corresponding uncertainties of the CDF measurement in the electron channel, as obtained in Run 1A [87] and Run 1B [88], are also shown for comparison (second and third column).

<table>
<thead>
<tr>
<th>Source</th>
<th>$\Delta M_W$ (CDF Run 1A) (MeV)</th>
<th>$\Delta M_W$ (CDF Run 1B) (MeV)</th>
<th>$\Delta M_W$ (ATLAS) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Statistics</td>
<td>145</td>
<td>65</td>
<td>&lt; 2</td>
</tr>
<tr>
<td>$E - p$ scale</td>
<td>120</td>
<td>75</td>
<td>15</td>
</tr>
<tr>
<td>Energy resolution</td>
<td>80</td>
<td>25</td>
<td>5</td>
</tr>
<tr>
<td>Recoil model</td>
<td>60</td>
<td>33</td>
<td>5</td>
</tr>
<tr>
<td>Lepton identification</td>
<td>25</td>
<td>–</td>
<td>5</td>
</tr>
<tr>
<td>$p_T^W$</td>
<td>45</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>Parton distribution functions</td>
<td>50</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>$W$ width</td>
<td>20</td>
<td>10</td>
<td>7</td>
</tr>
<tr>
<td>Radiative decays</td>
<td>20</td>
<td>20</td>
<td>&lt; 10</td>
</tr>
<tr>
<td>Background</td>
<td>10</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>TOTAL</td>
<td>230</td>
<td>113</td>
<td>&lt; 25</td>
</tr>
</tbody>
</table>

momentum of the $W$, the systematic uncertainties may be reduced further. Moreover, two alternative methods to measure the $W$ mass can be envisaged. The first one uses the $p_T$ distribution of the charged lepton in the final state. Such a distribution features a Jacobian peak at $p_T^1 \sim M_W/2$ and has the advantage of being affected very little by the pile-up, therefore it could be used at high luminosity. However, the lepton momentum is very sensitive to the $p_T$ of the $W$ boson, whereas the transverse mass is not, and hence a very precise theoretical knowledge of the $W$ $p_T$ spectrum would be needed to use this method. Another possibility is to use the ratio of the transverse masses of the $W$ and $Z$ bosons [95]. The $Z$ transverse mass can be reconstructed by using the $p_T$ of one of the charged leptons, while the second lepton is treated like a neutrino whose $p_T$ is measured by the first lepton and the recoil. By shifting the $m_T^Z$ distribution until it fits the $m_T^W$ distribution, it is possible to obtain a scaling factor between the $W$ and the $Z$ masses and therefore the $W$ mass. The advantage of this method is that common systematic uncertainties cancel in the ratio. The main disadvantage is the loss of a factor of ten in statistics, since the $Z \rightarrow ll$ sample is a factor of ten smaller than the $W \rightarrow l\nu$ sample (and only events near to the Jacobian peak contribute significantly to the mass determination). Furthermore, differences in the production mechanism between the $W$ and the $Z$ ($p_T$, angular distribution, etc.), and possible biases coming from the $Z$ selection cuts, will give rise to a non-negligible systematic error.

The final measurement will require using all the methods discussed above, in order to cross-check the systematic uncertainties and to achieve the highest precision.

### 3.15 Conclusions

Preliminary studies indicate that measuring the $W$ mass at the LHC with a precision of about 15 MeV should be possible, although very challenging. The biggest single advantage of the LHC is the large statistics, which will result in small statistical errors and good control of the systematics. To achieve such unprecedented precision, improved theoretical calculations in many areas will be needed (e.g. radiative decays, pdf’s, $p_T^W$), and many stringent experimental requirements will have to be satisfied.
3.2 Drell-Yan production of lepton pairs

3.21 Introduction

Parton level: In the Standard Model (SM), the production of lepton pairs in hadron-hadron collisions (the Drell-Yan process) is described by s-channel exchange of photons or Z bosons. The parton cross section in the centre-of-mass system has the form:

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} [A_0(1 + \cos^2 \theta) + A_1 \cos \theta]
\]

(27)

where \( \hat{\sigma} = \frac{4\pi \alpha^2}{3s} A_0 \) and \( A_{FB} = \frac{3}{8} \frac{A_1}{A_0} \) give the total cross section and the forward-backward asymmetry, respectively. The terms \( A_0 \) and \( A_1 \) are fully determined by the electroweak couplings of the initial- and final-state fermions. At the \( Z \) peak, the \( Z \) exchange dominates while the interference term is vanishing. At higher energies, both photon and \( Z \) exchange contribute and the large value of the forward-backward asymmetry arises from the interference between the neutral currents.

Fermion-pair production above the \( Z \) pole is a rich search field for new phenomena at present and future high-energy colliders [96]. The differential cross section is given by

\[
\frac{d\hat{\sigma}}{d\Omega} \sim |\gamma_s + Z_s + \text{New Physics}?!|^2
\]

(28)

where many proposed types of new physics can lead to observable effects by adding new amplitudes or through their interference with the neutral currents of the SM.

At hadron colliders: The parton cross sections are folded with the parton distribution functions (pdf’s):

\[
\frac{d^2\sigma}{dM_{ll}dy}(pp \rightarrow l_1l_2) \sim \sum_{ij} \left( f_{i/p}(x_1)f_{j/p}(x_2) + (i \leftrightarrow j) \right) \hat{\sigma}
\]

(29)

where \( \hat{\sigma} \) is the cross section for the partonic subprocess \( ij \rightarrow l_1l_2 \), \( M_{ll} = \sqrt{s} = \sqrt{t} \) and \( y \) are the invariant mass and rapidity of the lepton pair, \( x_1 = \sqrt{t}e^y \) and \( x_2 = \sqrt{t}e^{-y} \) are the parton momentum fractions, and \( f_{i/p}(x_i) \) is the probability to find a parton \( i \) with momentum fraction \( x_i \) in the (anti)proton.

\[
\sigma_{F\pm B}(y, M) = \int_{1}^{0} \int_{-1}^{0} \sigma_{lld}(\cos \theta^*)
\]

(30)

\[
A_{FB}(y, M) = \frac{\sigma_{F- B}(y, M)}{\sigma_{F+ B}(y, M)}
\]

(31)

The total cross section and the forward-backward asymmetry are functions of observables which are well measured experimentally: the invariant mass and the rapidity of the final state lepton-pair. For a pair of partons \( (x_1 \geq x_2) \), there are four combinations of quarks initiating Drell-Yan production: \( u\bar{u}, \bar{u}u, d\bar{d}, \bar{d}d \). In \( pp \) collisions, the antiquarks come always from the sea while the quarks can have valence or sea origin. The \( x \)-range probed depends on the mass and rapidity of the lepton-pair as shown in Table 6. Going to higher rapidities increases the difference between \( x_1 \) and \( x_2 \) and hence the probability that the first quark is a valence one.

3.22 Event rates

The expected numbers of events for the Tevatron Run 2 (TEV2) and the LHC are shown in Table 7 and Figure 16. The estimation is based on simulations with PYTHIA 5.7 [97] by applying the following cuts:

1. For LHC: both leptons \( |\eta| < 2.5 \); for TEV2: one lepton \( |\eta| < 1 \), the other \( |\eta| < 2.5 \).
Table 6: $x_1$ and $x_2$ for different masses and rapidities.

<table>
<thead>
<tr>
<th>$M$ (GeV)</th>
<th>$y$</th>
<th>$91.2$</th>
<th>$200$</th>
<th>$1000$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>$x_1$</td>
<td>0.0065</td>
<td>0.0481</td>
<td>0.3557</td>
<td>0.0143</td>
</tr>
<tr>
<td>$x_2$</td>
<td>0.0065</td>
<td>0.0009</td>
<td>0.0001</td>
<td>0.0143</td>
</tr>
</tbody>
</table>

2. For both leptons, $p_T > 20$ GeV.

The data sample can be divided into three classes:

Events near the $Z$ pole:
- There will be a huge sample of $Z$ events at the LHC. These will allow study of the interplay between $\sin^2\theta_{\text{eff}}(M_Z^2)$ and the pdf’s.

High mass pairs (110-400 GeV):
- LEP2 will study this region up to 200 GeV.
- TEV2 will collect a sizeable sample of events in this region.
- LHC will be able to do precision studies.

Very high mass pairs (400-4000 GeV):
- TEV2 will have a first glance.
- LHC will collect a sizeable sample for tests of the SM at the highest momentum transfers ($Q^2$) and for searches of new phenomena at the TeV scale.

Table 7: PYTHIA estimate: expected number of events for one experiment in the $e^+e^-$ and $\mu^+\mu^-$ channels. For LEP2 and CDF the observed number of events is shown.

<table>
<thead>
<tr>
<th>Pair Mass</th>
<th>LEP2</th>
<th>CDF</th>
<th>TEV2</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 pb⁻¹</td>
<td>110 pb⁻¹</td>
<td>10 fb⁻¹</td>
<td>100 fb⁻¹</td>
<td></td>
</tr>
<tr>
<td>SM / Data</td>
<td>Data</td>
<td>PYTHIA</td>
<td>PYTHIA</td>
<td></td>
</tr>
<tr>
<td>Z pole</td>
<td>-</td>
<td>-</td>
<td>$\sim 1.5 \times 10^6$</td>
<td>$\sim 134 \times 10^6$</td>
</tr>
<tr>
<td>&gt; 110 GeV</td>
<td>12500</td>
<td>148 (&gt; 150 GeV)</td>
<td>46000</td>
<td>2.6 $\times 10^6$</td>
</tr>
<tr>
<td>&gt; 400 GeV</td>
<td>-</td>
<td>1</td>
<td>250</td>
<td>33000</td>
</tr>
</tbody>
</table>

3.23 Measurements of $\sigma$ and $A_{FB}$

The experimental signature for Drell-Yan events is distinctive: a pair of well isolated leptons with opposite charge. This should be straight forward for the ATLAS and CMS detectors to identify. The backgrounds are low: $W^+W^-, \tau^+\tau^-, c\bar{c}, b\bar{b}, t\bar{t}$, fakes, cosmics etc.. If the need arises, they can be further suppressed by acoplanarity and isolation cuts. The selection cuts used in this study have already been described in the section on simulations.

An important ingredient in the cross section measurement is the precise determination of the luminosity. A promising possibility is to go directly to the parton luminosity [55] by using the $W^\pm$ (Z) production of single (pair) leptons:

<table>
<thead>
<tr>
<th>Pair Mass</th>
<th>LEP2</th>
<th>CDF</th>
<th>TEV2</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>600 pb⁻¹</td>
<td>110 pb⁻¹</td>
<td>10 fb⁻¹</td>
<td>100 fb⁻¹</td>
<td></td>
</tr>
<tr>
<td>SM / Data</td>
<td>Data</td>
<td>PYTHIA</td>
<td>PYTHIA</td>
<td></td>
</tr>
<tr>
<td>Z pole</td>
<td>-</td>
<td>-</td>
<td>$\sim 1.5 \times 10^6$</td>
<td>$\sim 134 \times 10^6$</td>
</tr>
<tr>
<td>&gt; 110 GeV</td>
<td>12500</td>
<td>148 (&gt; 150 GeV)</td>
<td>46000</td>
<td>2.6 $\times 10^6$</td>
</tr>
<tr>
<td>&gt; 400 GeV</td>
<td>-</td>
<td>1</td>
<td>250</td>
<td>33000</td>
</tr>
</tbody>
</table>
- Constrain the pdf’s.
- Measure directly the parton-parton luminosity.

In this way, the systematic error on $\sigma^{\text{high}}_{\text{DY}} Q^2$ relative to $\sigma_Z$ can be reduced to $\sim 1\%$.

In order to measure the forward-backward asymmetry, it is necessary to tag the directions of the incoming quark and antiquark. At the Tevatron, the $p\bar{p}$ collisions provide a natural label for the valence (anti)quark. In contrast at the LHC, the $\sqrt{s}$ initial state is symmetric. But in the reaction $q\bar{q} \rightarrow l^+ l^-$ only $q$ can be a valence quark, carrying on average a higher momentum compared to the sea antiquarks. Therefore at the LHC, $A_{FB}$ will be signed according to the sign of the rapidity of the lepton pair $y(\ell\ell)$. Consequently, $A_{FB}$ increases as a function of $y(\ell\ell)$ [98, 99] (see Figure 18).

A precise measurement of $\sigma$ and $A_{FB}$ at large $\hat{s}$ requires good knowledge of the different types of electroweak radiative corrections to the DY process: vertex, propagator, EW boxes. A complete one-loop parton cross section calculation has been performed [71]. The size of these corrections after folding with the pdf’s and the expected experimental precision on the cross section measurement are compared in Figure 17. The LHC experiments can probe these corrections up to $\sim 2$ TeV.

3.24 Determination of $\sin^2 \theta_{\text{eff}}^{\text{lept}} (M_Z^2)$

A very precise determination of $\sin^2 \theta_{\text{eff}}^{\text{lept}} (M_Z^2)$ will constrain the Higgs mass or, if the Higgs boson is discovered, will check the consistency of the SM [100]. The latest result of the LEP Electroweak Working Group from the summer of 1999 is:

$$\sin^2 \theta_{\text{eff}}^{\text{lept}} (M_Z^2) = 0.23151 \pm 0.00017$$  \hspace{1cm} (32)

**Event selection**  A careful study [101] of the precision which can be obtained from the $Z \rightarrow ee$ decay by ATLAS and CMS has been made using PYTHIA 5.7 and JETSET 7.2. Background processes
from $pp \rightarrow 2 \text{jets}$ and $pp \rightarrow t\bar{t} \rightarrow e^+e^-$ have been included. In the regions of precision measurements ($|\eta| \leq 2.5$), the precision which can be obtained from $Z \rightarrow \mu\mu$ decays should be comparable to that from the electron channel. In addition, the detectors have calorimetry extending to $|\eta| \sim 5$ and hence, if it is possible to tag very forward electrons, albeit with significantly lower quality, it may be possible to improve dramatically the measurement of $\sin^2 \theta_{\text{eff}}^{\text{lep}} (M_Z^2)$.

The following cuts were made:

1. $p_T^{\text{electron}} > 20 \text{ GeV/c}$
2. $85.2 \text{ GeV/c}^2 < M(e^+e^-) < 97.2 \text{ GeV/c}^2$

In all cases, one electron was required in the precision calorimetry $|\eta| \leq 2.5$. Efficiencies after typical electron identification cuts were taken from detailed studies reported in [86]. These are typically around 70%, with corresponding jet rejections of $> 10^3$ (there was no advantage for this measurement of larger rejection factors). For the second electron, the possibility for it to be identified in the forward calorimetry, albeit with significantly lower quality, may be possible to improve dramatically the measurement of $\sin^2 \theta_{\text{eff}}^{\text{lep}} (M_Z^2)$.

Statistical sensitivity

The sensitivity of $A_{FB}$ to $\sin^2 \theta_{\text{eff}}^{\text{lep}} (M_Z^2)$ can be parametrised as follows:

$$A_{FB} = b(a - \sin^2 \theta_{\text{eff}}^{\text{lep}} (M_Z^2))$$

$$a^{O(\alpha^3)} = a^{\text{Born}} + \Delta a^{QED} + \Delta a^{QCD}$$

$$b^{O(\alpha^3)} = b^{\text{Born}} + \Delta b^{QED} + \Delta b^{QCD}$$

Values of $a$ and $b$ were calculated in [68] and have been re-evaluated by Baur corresponding to the above cuts - see Table 8.
Fig. 18: Forward-backward asymmetry vs rapidity for $e^+e^-$ pairs from Z decays satisfying the selection cuts described in Section 3.24. The asymmetry is shown where both electrons have $|\eta| < 2.5$ (triangles) and where one electron is allowed to have $|\eta| < 4.9$ (squares). The results are the same for both sets of cuts in the first bin.

A summary of the statistical errors which can be obtained with 100 fb$^{-1}$ are indicated in Table 9. With the best rejection factors shown in the table, the effect of the background is negligible. If no jet rejection is possible in the forward calorimetry, the statistical precisions which can be obtained on $\sin^2 \theta_{\text{eff}} (M_Z^2)$ are $3.4 \times 10^{-4}$ and $4.1 \times 10^{-4}$ for no $y$ cut and $|y(e^+e^-)| > 1.0$ respectively. While the sensitivity to $\sin^2 \theta_{\text{eff}} (M_Z^2)$ is increased by cutting on $|y(e^+e^-)|$ (see Table 8), the gain is reduced by the loss of acceptance and increased significance of the background when the forward calorimetry is used. It is probable that greater sensitivity could be obtained by fitting $A_{FB}$ as a function of $|y(e^+e^-)|$.

From Table 9, it can be seen that for a single lepton species from one LHC experiment, using leptons measured in $|\eta| < 2.5$, a statistical precision of $4.0 \times 10^{-4}$ on $\sin^2 \theta_{\text{eff}} (M_Z^2)$ could be obtained. With the combination of electrons and muons in two experiments, $2.0 \times 10^{-4}$ could be obtained.

The table shows that for moderate jet rejection ($\gtrsim 10^2$) in the forward calorimetry, a statistical precision of $1.4 \times 10^{-4}$ could be reached by a single experiment using just the electron channel (cannot include the muons). Even a poor rejection $\sim 10$, would provide a useful measurement. While no studies with full detector simulation have been done, it seems likely that both the ATLAS and CMS forward calorimetry will be able to provide useful electron identification because of moderate longitudinal and transverse segmentation. Combining both experiments will permit a further $\sqrt{2}$ reduction in the statistical uncertainty.

**Systematic uncertainties** In order to be able to exploit the possibility of measuring $\sin^2 \theta_{\text{eff}} (M_Z^2)$ with such high precision, the systematic errors have to be comparably small. Quick estimates indicate that the following factors are the most important ones:

1. *pdf’s:* affect both the lepton acceptance as well as the results of radiative correction calculations.
2. *Lepton acceptance and reconstruction efficiency as a function of lepton rapidity:* while there is some cancellation in the determination of the asymmetry, the product will need to be known to better than 0.1%. CDF [102] has shown that it is possible to achieve a precision of about 1%, with
Table 8: Parameters $a$ and $b$ in Equation 33.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>$a^{\text{Born}}$</th>
<th>$\Delta a^{QED}$</th>
<th>$\Delta a^{QCD}$</th>
<th>$a^{O(\alpha^2)}$</th>
<th>$b^{\text{Born}}$</th>
<th>$\Delta b^{QED}$</th>
<th>$\Delta b^{QCD}$</th>
<th>$b^{O(\alpha^2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$ both $e^\pm$</td>
<td>.2481</td>
<td>.0025</td>
<td>-.0026</td>
<td>.2480</td>
<td>0.48</td>
<td>-0.01</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$ both $e^\pm$</td>
<td>.2503</td>
<td>-.0009</td>
<td>-.0069</td>
<td>.2425</td>
<td>0.74</td>
<td>0.05</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 4.9$ the other $e^\pm$</td>
<td>.2483</td>
<td>-.0005</td>
<td>-.0015</td>
<td>.2463</td>
<td>1.18</td>
<td>0.15</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$ one $e^\pm$</td>
<td>.2486</td>
<td>.0011</td>
<td>-.0028</td>
<td>.2469</td>
<td>1.66</td>
<td>0.01</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 4.9$ the other $e^\pm$</td>
<td>.2483</td>
<td>-.0005</td>
<td>-.0015</td>
<td>.2463</td>
<td>1.18</td>
<td>0.15</td>
</tr>
</tbody>
</table>

the largest contribution being due to the uncertainty in the pdf’s.

3. Effects of higher order QCD (and electroweak) corrections: can be estimated by varying the errors on the parameters $a$ and $b$.

4. Mass Scale: $A_{FB}$ varies as a function of the invariant mass of the lepton pair. Since the measured asymmetry corresponds to an integration over the $Z$ resonance, it is important to understand the mass scale. It is expected that this will be known to $\sim 0.02\%$ (see 3.13) by direct comparison of the $Z$ peak with the measured LEP parameters.

The most important systematic contribution is that coming from the uncertainties in the pdf’s. A study using several “modern” pdf’s (MRST, CTEQ3 and CTEQ4) gave agreement between the resulting values of $A_{FB}$ within the $1\%$ statistical errors of the study ($5 \times 10^5$ events were generated for each pdf set). This uncertainty must be reduced by a factor of 10 if it is to be smaller than the expected statistical precision on $A_{FB}$ shown in Table 9. It remains to be seen whether (a) the differences arising from the various pdf’s will shrink with increased statistical sensitivity of the study and (b) whether the current pdf’s actually describe the measured data sufficiently well (since the pdf’s are fitted to common data, variations are not necessarily indicative of the actual uncertainties). New measurements from the Tevatron (and ultimately the LHC itself) will improve the understanding of the pdf’s, but it is unclear at this stage whether this will be sufficient. It may be possible to fit simultaneously $\sin^2 \theta_{\text{eff}}^{\text{pt}} (M_Z^2)$ and the pdf’s, or alternatively, it may be necessary to reverse the strategy and use the measurement of $A_{FB}$ combined with existing measurements of $\sin^2 \theta_{\text{eff}}^{\text{pt}} (M_Z^2)$ to constrain the pdf’s.

3.25 Search for new phenomena

Contact interactions Contact interactions offer a general framework for a new interaction with coupling $g$ and typical energy scale $\Lambda \gg \sqrt{s}$. At LEP2, the current limits [96, 103] for quark-lepton compositeness at 95% CL vary between 3 and 8 TeV, depending on the model. At the LHC scales up to 25-30 TeV are reachable, as illustrated in Figure 19.

Search for resonances The other extreme is the search for resonances like $Z’$ or $\tilde{\nu}$, which produce peaks in the mass distributions. A neutral heavy gauge-boson $Z’$ is characterised by its mass $m_{Z’}$, by its couplings and by its mixing angle $\theta_M$ with the standard $Z$ boson. If $\theta_M = 0$ and the $Z’$ has SM couplings, the current limit is $m_{Z’} > 1050$ GeV [104]. For other coupling scenarios the lower limits
Table 9: Statistical precision which can be obtained on $\sin^2 \theta_{\text{eff}}(M_Z^2)$ from measurements of $A_{FB}$ in $Z \to ee$ from one LHC experiment with 100 fb$^{-1}$. Results are given for different jet rejection factors $\rho$ for the forward calorimetry $2.5 < |\eta| < 4.9$.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>$\rho$</th>
<th>$A_{FB}$ (%)</th>
<th>$\Delta A_{FB}$ (%)</th>
<th>$\Delta \sin^2 \theta_{\text{eff}}(M_Z^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$ both $e^\pm$</td>
<td>-</td>
<td>0.774</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$ both $e^\pm$</td>
<td>-</td>
<td>1.66</td>
</tr>
<tr>
<td>$</td>
<td>y(e^+e^-)</td>
<td>&gt; 1.0$</td>
<td>$10^4$</td>
<td>2.02</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$ one $e^\pm$</td>
<td>$10^2$</td>
<td>1.98</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 4.9$ the other $e^\pm$</td>
<td>$10^1$</td>
<td>1.68</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 2.5$ one $e^\pm$</td>
<td>$10^4$</td>
<td>3.04</td>
</tr>
<tr>
<td>$</td>
<td>\eta</td>
<td>&lt; 4.9$ the other $e^\pm$</td>
<td>$10^2$</td>
<td>2.94</td>
</tr>
<tr>
<td>$</td>
<td>y(e^+e^-)</td>
<td>&gt; 1.0$</td>
<td>$10^1$</td>
<td>2.31</td>
</tr>
</tbody>
</table>

are model dependent and typically of the order of several hundred GeV. Resonances with masses up to $\sim 4$-5 TeV can be probed at LHC, as shown in Figure 19.

**R-parity violation**  In SUSY theories with R-parity violation, it is possible to couple sleptons to pairs of SM leptons or quarks through new independent Yukawa couplings (9 $\lambda$ couplings for the slepton-lepton sector and 27 $\lambda'$ couplings for the slepton-quark sector). This makes the resonance formation of single scalar neutrino $\tilde{\nu}$ in $d\bar{d}$ scattering possible. It can be observed through the decay of the $\tilde{\nu}$ to lepton pairs, if a suitable combination of two couplings (e.g. $\lambda'_{311}\lambda'_{131}$) is present [105]. The K-factor for slepton production is not calculated yet, leading to an uncertainty $\sim 10\%$ in the estimate of the $\lambda\lambda'$ sensitivity.

**Low-scale gravity**  An exciting possibility is the search for low-scale gravity effects in theories with extra spatial dimensions, leading to virtual graviton exchange. The best limits at LEP2 come from combined analysis of Bhabha scattering [106]:

$$\Lambda_T = 1.412(1.077) \, \text{TeV for } \lambda = +1(-1) \text{ at 95\% CL}$$

In the Drell-Yan process there is an unique contribution from $s$-channel graviton exchange [107], which modifies the form of the differential cross section and gives a distinct signature:

$$gg \rightarrow l^+l^-$$

$$\frac{d\sigma}{d\cos\theta} = \frac{\lambda'^2 \sin^3}{64\pi M_s^8} (1 - \cos^4 \theta)$$

(35)

The large parton luminosity for gluons at LHC may also compensate the $M_s^{-8}$ suppression. Scales up to $\sim 5$ TeV can be probed with luminosity 100 fb$^{-1}$. 


Fig. 19: Left: Contact interaction sensitivity (CMS study). Error bars show SM spectrum; histograms show effect of contact term with $\Lambda = 25$ TeV, the sign corresponds to the sign of the amplitude. Right: $Z'$ mass reach for 100 fb$^{-1}$ (ATLAS study) [53].

3.26 Summary

The main results of this study are:

- A competitive measurement of $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z^2)$ is hard due to the central acceptance of the experiments and the difficulty of controlling the pdf’s (parton distribution functions) with the required precision. However, a detector with extended forward acceptance for one of the leptons offers the possibility to measure $\sin^2 \theta_{\text{eff}}^{\text{lept}}(M_Z^2)$ with a statistical precision of $1.4 \times 10^{-4}$.
- The total cross-section can be measured with systematic error $\frac{\Delta \sigma}{\sigma} < 1\%$.
- A non-zero forward-backward asymmetry $A_{FB}$ can be measured up to 2 TeV with statistical precision $> 3 \sigma$.
- The Drell-Yan process can probe electroweak radiative corrections up to 1.5 TeV with statistical precision at the 2 $\sigma$ level as a function of $Q^2$.
- The high energy and luminosity of LHC offers a rich search field at the TeV scale in the Drell-Yan channel: contact interactions, resonance formation ($Z'$, scalar neutrinos), low-scale gravity, etc.

Further studies will refine the following points:

- The effect of higher order QED corrections (initial- and final-state radiation and their interference).
- The effect of experimental cuts on the electroweak corrections.
- The careful separation of the $\sigma_{u\bar{u}}$ and $\sigma_{d\bar{d}}$ contributions.

3.3 Tau physics

The $\tau$ lepton is a member of the third generation which decays into particles belonging to the first and second ones. Thus, $\tau$ physics could provide some clues to the puzzle of the recurring families of leptons and quarks. One naively expects the heavier fermions to be more sensitive to whatever dynamics is responsible for the fermion-mass generation. The pure leptonic or semileptonic character of $\tau$ decays...
provides a clean laboratory to test the structure of the weak currents and the universality of their couplings to the gauge-bosons.

The last few years have witnessed a substantial change in our knowledge of the $\tau$ properties [108, 109]. The large (and clean) data samples collected by the most recent experiments have improved considerably the statistical accuracy and, moreover, have brought a new level of systematic understanding.

A high-energy hadron collider does not provide a very good environment to perform precision $\tau$ physics. Nevertheless, there are a few topics where LHC could contribute in a relevant and unique way. Moreover, since the $\tau$ is the heaviest known lepton, it can play a very important role in searches for new particles (for example, as in Section 6.1).

### 3.31 Charged-current universality

Table 10: Present constraints on charged-current lepton universality [109].

|                | $|g_\mu/g_e|$   | $|g_\tau/g_\mu|$ | $|g_\tau/g_e|$   |
|----------------|----------------|-----------------|----------------|
| $B_\tau\rightarrow e\mu / B_\tau\rightarrow e\bar{e}$ | 1.0009 ± 0.0022 | —               | —               |
| $B_\tau\rightarrow e\mu / B_\tau\rightarrow e\bar{e}$ | —               | 0.9993 ± 0.0023 | —               |
| $B_\tau\rightarrow \mu / B_\tau\rightarrow \tau$   | —               | —               | 1.0002 ± 0.0023 |
| $B_\pi\rightarrow e / B_\pi\rightarrow \mu$       | 1.0017 ± 0.0015 | —               | —               |
| $\Gamma_\tau\rightarrow \pi / \Gamma_\tau\rightarrow \mu$ | —               | 1.005 ± 0.005  | —               |
| $\Gamma_\tau\rightarrow K / \Gamma_\tau\rightarrow \mu$ | —               | 0.981 ± 0.018  | —               |
| $B_{W\rightarrow l\bar{l}} / B_{W\rightarrow l\bar{l}}$ (p$\bar{p}$) | 0.98 ± 0.03    | —               | 0.987 ± 0.025  |
| $B_{W\rightarrow l\bar{l}} / B_{W\rightarrow l\bar{l}}$ (LEP2) | 1.002 ± 0.016  | 1.008 ± 0.019  | 1.010 ± 0.019  |

Table 10 shows the present experimental tests on the universality of the leptonic charged-current couplings. The leptonic $\tau$ branching ratios are already known with a quite impressive precision of 0.3%; this translates into a test of $g_\mu/g_e$ universality at the 0.22% level. However, in order to test the ratios $g_\tau/g_\mu$ and $g_\tau/g_e$, one needs precise measurements of the $\tau$ mass and lifetime, in addition. At present, these quantities are known with a precision of 0.016% ($m_\tau = 1777.05 ^{+0.20}_{-0.26}$ MeV) and 0.34% ($\tau_\tau = 290.77 \pm 0.99$ fs), respectively [109], which leads to a sensitivity of 0.23% for the three $g_l/g_{l'}$ ratios.

Future high-luminosity $e^+e^-$ colliders running near the $\tau^+\tau^-$ production threshold could perform more precise measurements of the leptonic $\tau$ branching fractions and the $\tau$ mass. However, one needs a high-energy machine to measure the $\tau$ lifetime. Clearly, the future tests of lepton universality will be limited by the $\tau$ accuracy. It is not clear whether the $B$-factories would be able to improve the present $\tau_\tau$ measurement in a significant way. Thus, it is important to know how well $\tau_\tau$ can be determined at LHC.

A less precise but more direct test on the universality of the leptonic $W^{\pm}$ couplings is provided by the comparison of the different $W^+ \rightarrow l^+\nu_l$ branching fractions. LEP2 has already achieved a better sensitivity than the Tevatron collider, and a further improvement is expected when the full LEP2 statistics will be available. It is an open question whether LHC could be competitive at this level ($\sim 1\%$) of precision.
3.32 Tau lifetime

The current world average for the $\tau$ lifetime is $290.8 \pm 1.0$ fs ($\tau = 87 \mu m$) [109]. Improvements in this measurement would be welcome in order to give better tests of the Standard Model, in particular lepton universality and electroweak calculations. In this section, the results of a preliminary study to examine the LHC potential are given.

In LEP experiments, $\tau$ pairs are produced back-to-back with well defined momenta - this will not be the case at the LHC. The first feature allows valuable correlations to be made between the two $\tau$ decays, while the second provides the boost required to obtain proper lifetime estimates. At the LHC, $Z \rightarrow \tau\tau$ events will be triggered by requiring one $\tau$ to decay to an electron or muon, while the lifetime is estimated from the other $\tau$ which is required to decay to three charged particles.

**Tau reconstruction** A study was made using fully simulated events in the ATLAS detector (see [86] for more details). When the $Z$ has some transverse momentum, the momenta of the $\tau$'s can be deduced by projecting the recoil momentum vector measured by the calorimetry along the lines of flight of the two $\tau$'s (determined from the direction of the lepton and the hadronic jet, respectively). Due to resolution effects, this procedure works best when the $\tau$'s are not back-to-back. The following cuts were made:

- The lepton should have $p_T > 24$ GeV, $|\eta| < 2.5$.
- The identified hadronic jet should contain three charged tracks and satisfy $E_T > 30$ GeV, $|\eta| < 2.5$.
- Transverse mass of lepton and missing energy should be $< 50$ GeV.
- The angle $\Delta\phi$ between the $\tau$'s in the transverse plane should satisfy: $1.8 < \Delta\phi < 2.7$ or $3.6 < \Delta\phi < 4.5$.
- The invariant mass of the $\tau$ pair should satisfy: $60 < m_{\tau\tau} < 120$ GeV.

These cuts result in an efficiency of 1.5%. For these events, the $\tau$ momenta could be estimated with a resolution of 15%.

A vertex was formed from the charged tracks in the hadronic jet. It was required that the vertex should be within 2 cm of the interaction point and the invariant mass of the particles should be between 0.4 and 1.78 GeV. The efficiency for this was 70% and resulted in a resolution on the vertex position in the transverse plane of $490 \mu m$, corresponding to a resolution on the proper decay length of $17 \mu m$.

**Lifetime estimate** The statistical resolution on the proper decay length from the combination of the vertexing and the estimate of the tau momentum is of the order of $21 \mu m$ (corresponding to 55 fs). A simple Monte Carlo study was made to estimate the statistical uncertainty on the $\tau$ lifetime ($\tau_\tau$) which could be achieved with $N$ hadronic $\tau$ decays. Since the resolution of the lifetime for a single event (55 fs) is a fair bit smaller than the $\tau$ lifetime (291 fs), the statistical error which can be obtained is dominated by the number of events: $\sigma(\tau_\tau) \approx \tau_\tau/\sqrt{N}$.

At the LHC, the cross section for $Z \rightarrow \tau\tau$ will be 1.5 nb, with a branching ratio of 11% for a lepton and a three-prong hadronic decay. The reconstruction and selection described above results in an efficiency of 0.54%. If 30 fb$^{-1}$ were collected in a low luminosity run, then 26,000 reconstructed $\tau$'s could be used, leading to a statistical error on the lifetime of 1.8 fs. To make this competitive would require increased efficiency for selecting the $\tau$ decays - this is probably a low luminosity measurement and so cannot benefit from the statistics of a high luminosity run.

Increasing the efficiency may not be simple, since the cuts were designed to control the background. $W+\text{jet}$ events will be removed by the mass cuts, and apart from a small amount of gluon splitting to heavy flavour, the jets should not contain significant lifetime information, hence this background should not be a problem. The $B$ lifetime is a factor of five larger than that of the $\tau$, and hence more care will be required with $b\bar{b}$ events.
Concerning systematic errors coming from the determination of the decay length in the silicon tracking, the average radial position of the detectors in the vertexing layer will need to be understood to better than 10 $\mu$m. This will be challenging but studies suggest this may be feasible [91]. It should be possible to control the systematics on the measurement of recoil momentum of the $Z$ by comparison with $Z \rightarrow ee$ or $Z \rightarrow \mu\mu$ events, where the recoil can be measured accurately by the leptons.

The use of $W \rightarrow \tau\nu$ It may be possible to use the decays $W \rightarrow \tau\nu$ which have a higher cross section than $Z \rightarrow \tau\tau$. In ATLAS, such events could be triggered by a special $\tau$-jet and missing $E_T$ trigger [86]. Information about the $\tau$ momentum can be deduced by comparing the energy and direction of the hadronic jet with the direction of the $\tau$ and using the $\tau$ mass constraint, where the $\tau$ direction can be determined from the reconstructed decay vertex. In principle, it is possible to solve completely for the $\tau$ momentum, although resolution effects on the vertex position and complications arising from $\pi^0$'s in the hadronic jet mean that sometimes solutions are not physical. Alternatively, an approximate estimator can be formed which does not employ the mass constraint [110]. This uses the $\tau$-jet energy, mass and $p_T$ relative to the $\tau$ direction - all three quantities being determined from the charged tracks alone. This is more robust but its behaviour is sensitive to the selection cuts. It is yet to be proved that a $W \rightarrow \tau\nu$ signal can be identified with sufficient efficiency above the huge QCD (and in particular, $bb\bar{b}$) background.

3.33 Rare decays
Owing to the huge backgrounds, it will not be possible to make a general search for rare decay modes of the $\tau$. However, the lepton-number violating decay $\tau^- \rightarrow \mu^+\mu^-\mu^-$ has a clean signature, which is well suited for the LHC detectors. The present experimental bound [111] is

$$BR(\tau^- \rightarrow \mu^+\mu^-\mu^-) < 1.9 \times 10^{-6} \; \; (90\% \; CL)$$

This limit reflects the size of the existing $\tau$ data samples. LHC will produce a huge statistics, several orders of magnitude larger than the present one. The achievable limit will then be set by systematics and backgrounds, which need to be properly estimated. A sensitivity at the level of $10^{-8}$ does not seem out of reach. This could open a very interesting window into new physics phenomena, since many extensions of the Standard Model framework can lead to signals in the $10^{-8}$ to $10^{-5}$ range.

Although more difficult to detect, other lepton-number violating decays such as $\tau \rightarrow \mu\mu e,\mu e e, e e e$, $\mu\gamma$ are worth studying.

4. VECTOR-BOSON PAIR PRODUCTION

4.1 $W^+W^-, W^\pm Z, ZZ$ production

4.11 Recent numerical implementations
As already is noted in the introduction, for the description of $W^+W^-, W^\pm Z, ZZ$ production with their subsequent decays into lepton pairs two new numerical parton-level Monte Carlo programs have recently become available [18](MCFM), [19](DKS). These packages consider the production of four leptons in the double resonance approximation with complete $O(\alpha_s)$ corrections. They can be used to compute any infra-red safe quantity with arbitrary experimental cuts on the leptonic decay products. These packages have already been used for updating and cross-checking previous results. The DKS program is available in fortran90 and fortran77 versions and includes anomalous triple gauge-boson couplings. The MCFM program is more complete in the sense that single resonance background diagrams are also added and finite width effects are included in some approximation which respects gauge-invariance. However, it does not include anomalous triple gauge-boson couplings. The results of the MCFM and DKS programs agree with each other within the integration error of $\leq 0.5\%$. Similar agreement is found with the spin

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Section coordinator: Z. Kunszt
averaged cross section indicated in [22, 24, 26]. In the past years the majority of the experimental studies used the programs described in [29, 31, 30] (BHO). A recent comparison between the DKS and BHO programs finds agreement at the level of 1% for $WZ$ production and 3-4% for $WW$ production (further details see Section 5.5). This confirms the assumptions of [29] that the spin correlations effects coming from virtual corrections are small. Note that recently a new $O(\alpha_s)$ package has been written also for $W\gamma$ and $Z\gamma$ production with anomalous couplings [20] and for the first time the complete one loop QCD corrections are available also for these processes (see Section 4.2).

### 4.12 Input parameters and benchmark cross sections

In using these packages, one should be careful with input parameters. The QCD input is standard: the latest next-to-leading order parton number densities have to be used with the corresponding running coupling constant in the spirit of the “improved Born approximation” [112, 113]. Universality means that their contributions can modify only the leading order relation between $M_Z, M_W$ and $\sin^2\theta_W$ which can be taken into account with the use of the effective coupling

$$\sin^2\theta_W = \frac{\pi\alpha(M_Z)}{\sqrt{2}G_FM_W^2},$$

where $G_F = 1.16639 \times 10^{-5}$ GeV$^{-2}$ is the Fermi constant and $\alpha(\mu)$ is the running QED coupling. With the values of the gauge-boson masses of $M_Z = 91.187$ GeV and $M_W = 80.41$ GeV, one obtains $\alpha = \alpha(M_Z) = 1/128$ and $\sin^2\theta_W = 0.230$. Ignoring this correlation may lead to about 5-6% discrepancy in the cross section values. The remaining electroweak corrections are estimated to be less than 2% as long the parton sub-energy is below $0.5-1$ TeV. However, above the 1 TeV scale double logarithmic corrections of $O(\alpha_W \log^2 s/M_W^2)$ become non-negligible. The origin of these large contributions is the incomplete cancellation of the soft singularities of massless gauge-boson emission (the Bloch-Nordsieck theorem is not valid for non-Abelian theories [115]). Since the physical cross section decreases strongly with the increase of the invariant mass of the gauge-boson pairs, these corrections are not important at the LHC. The validity of the improved Born approximation and the presence of the double logarithmic corrections has been tested for $W$ pair production at LEP2 where the full next-to-leading order corrections are available [112, 113].

Additional electroweak input parameters are the matrix elements of the CKM mixing matrix. In the light quark sector, one should use the best experimental values [116]. In the case of the heavy quark contributions, the calculation is approximate since the $O(\alpha_s)$ helicity amplitudes have been calculated assuming massless quarks [17]. This assumption is clearly not valid for the top contributions. $WW$ pair production receives contributions from diagrams with the $t$-channel exchange of the top quark (with $|V_{td}| = |V_{ts}| = 0$ and $|V_{tb}| = 1$). However, it is suppressed due to the large top mass and small $b$-quark parton densities; therefore, it is reasonable to use $|V_{tb}| = 0$. The contribution of the subprocess $b\bar{b} \rightarrow W^+W^-$ (treating the top as massless) is of the order of 2% for the LHC [17] giving an upper limit on the theoretical ambiguity coming from this source. In the case of $W^\pm Z$ production, one can neglect the subprocess $bg \rightarrow W^-Zt$. It is present at next-to-leading order but again it is strongly suppressed by the large top quark mass, as well as the small $b$-quark distribution function. For the numerical results presented here, values $|V_{ud}| = |V_{cd}| = 0.975; |V_{us}| = |V_{cs}| = 0.222$ and $|V_{ub}| = |V_{cb}| = |V_{td}| = |V_{ts}| = |V_{tb}| = 0$ are used. We present cross-section values without including the branching ratios. To get event signals, they have to be multiplied with the leptonic branching ratios of the vector-bosons. We
\[ BR(Z \rightarrow e^+e^- \text{ or } \mu^+\mu^-) = 3.37\% \quad BR(Z \rightarrow \sum_{i=e,\mu,\tau} \nu_i \bar{\nu}_i) = 20.1\% \]

\[ BR(W^+ \rightarrow e^+\nu_e \text{ or } \mu^+\nu_\mu) = 10.8\% \]

These ratios implicitly incorporate QCD corrections to the hadronic decay widths of the \( W \) and \( Z \).

Most of the results are obtained with some “standard cuts” defined as follows: a transverse momentum cut of \( p_T > 20 \text{ GeV} \) and a pseudorapidity cut of \( |\eta| \leq 2.5 \) is applied for all charged leptons and \( p_T^{\text{miss}} \geq 20 \text{ GeV} \) is required for \( WZ \) production while \( p_T^{\text{miss}} \geq 25 \text{ GeV} \) for \( W \) pair production. We use two different parton distributions, MRST [114] with \( M_W = 80.41 \text{ GeV} \) and CTEQ(4M) [67] with \( M_W = 80.33 \text{ GeV} \) which we refer to simply as MRST and CTEQ. \( \alpha_s(M_Z) = 0.1175 \) is used for MRST and \( \alpha_s(M_Z) = 0.116 \) is used for CTEQ. In all computations, we set the renormalisation and factorisation scales equal to each other.

In Table 11, we present the total cross section values for the various processes at the LHC, for the MRST and CTEQ parton distributions. We tabulated the results for \( \sigma^{\text{lo}} \) (the cross sections without any cuts applied) as well as \( \sigma^{\text{cut}} \) (the cross sections with the standard cuts defined above). The cross section values are given for the scale

\[ \mu = (M_{V_1} + M_{V_2})/2, \]

(37)

where \( M_{V_i} \) are the masses of the two produced vector bosons.

Table 11: Cross sections in pb for pp collisions at \( \sqrt{s} = 14 \text{ TeV} \). The statistical errors are \( \pm 1 \) on the last digit.

<table>
<thead>
<tr>
<th></th>
<th>ZZ</th>
<th>W^+W^-</th>
<th>W^+Z</th>
<th>W^-Z</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>LO</td>
<td>NLO</td>
<td>LO</td>
<td>NLO</td>
</tr>
<tr>
<td>( \sigma^{\text{lo}} ) (MRST)</td>
<td>11.6</td>
<td>15.5</td>
<td>78.7</td>
<td>117</td>
</tr>
<tr>
<td>( \sigma^{\text{lo}} ) (CTEQ)</td>
<td>11.8</td>
<td>15.8</td>
<td>81.3</td>
<td>120</td>
</tr>
<tr>
<td>( \sigma^{\text{cut}} ) (MRST)</td>
<td>4.07</td>
<td>5.47</td>
<td>25.0</td>
<td>40.18</td>
</tr>
<tr>
<td>( \sigma^{\text{cut}} ) (CTEQ)</td>
<td>4.09</td>
<td>5.51</td>
<td>25.6</td>
<td>42.0</td>
</tr>
</tbody>
</table>

In previous publications [22, 24, 26, 29, 31, 30, 18, 19] a number of phenomenologically interesting questions have been considered. Here we restrict ourselves to recall two interesting and typical features: the scale dependence of the radiative corrections for \( WW \) production and radiation zeros for \( WZ \) production.

### 4.13 Scale dependence

The one-loop corrections to the total cross sections are of the order 50% of the leading order term and they can be much larger for the kinematical range of larger transverse momenta or invariant mass of the vector-boson pair. For differential distributions where \( p_T \) is not integrated out completely, the scale choice

\[ \mu^2 = \mu^2_{\text{cut}} \equiv \frac{1}{2}(p_T^2(V_1) + p_T^2(V_2) + M_{V_1}^2 + M_{V_2}^2) \]

appears to be appropriate. For the total cross section, the difference between the two scale choices expressed in Equations 37 and 38 is very small since it is dominated by low-\( p_T \) vector-bosons. However, for more exclusive quantities, the differences can be substantial. At the LHC, the huge one-loop corrections in the tails of the distributions are dominated by the bremsstrahlung contributions; therefore it
is natural to consider the cross sections with and without the jet veto (that is, with or without the cut $E_T^{\text{jet}} < 40 \text{ GeV}$).

In Figure 20, the scale dependence of $\sigma^{\text{cut}}$ is shown for standard cuts, with a jet veto and with stronger cuts on the transverse momenta of the charged leptons. We can see that the corrections are large and increase with the additional cuts applied. The scale dependence at LO is reduced at NLO and it is reduced further when a jet veto is applied. In particular, the size of the correction is strongly reduced when applying the jet veto - an important feature for background studies.

### 4.14 Approximate radiation zeros in $WZ$ production

In leading order, the angular distribution of $WZ$ production exhibits an approximate radiation zero for $\cos\theta = (g_1 + g_2)/(g_1 - g_2)$ [31] where $g_1, g_2$ denote the $Z$ boson couplings to the left handed up and down quarks, respectively. Since the precise flight direction of the $W$ boson is not known (due to the uncertainty in the longitudinal momentum carried by the neutrino) it is convenient to plot a distribution in the (true) rapidity difference between the $Z$ boson and the charged lepton coming from the decay of the $W$: $\Delta y_{Z\ell} \equiv y_Z - y_\ell$. This quantity is similar to the rapidity difference $\Delta y_{WZ} \equiv |y_W - y_Z|$ studied in [24], but uses only the observable charged-lepton variables. It is the direct analogue of the variable $y_{\gamma} - y_{\gamma^\prime}$ considered in [117] for the case of $W\gamma$ production. It is possible to determine $\cos\theta$ in the $W\gamma$ or $WZ$ rest frame, by solving for the neutrino longitudinal momentum using the $W$ mass as a constraint, up to a two-fold discrete ambiguity for each event [118, 119, 120]. However, it has been found [117] that the ambiguity degrades the radiation zero - at least if each solution is given a weight of 50% - so that the rapidity difference $y_{\gamma} - y_{\gamma^\prime}$ is more discriminating than $\cos\theta$. As one can see from Figure 21, there is a residual dip in the $\Delta y_{Z\ell}$ distribution, even at order $\alpha_s$. This dip can be enhanced easily by requiring a minimal energy for the decay lepton from the $W$ and by cutting on the rapidity of the $Z$ boson. In Figure 21, we have chosen $E(l) > 100 \text{ GeV}$ with and without $y_{Z\ell} < 0$. Note that the latter two curves are scaled up by a factor of 5. At the LHC, for the first time, we shall have enough statistics to test experimentally for the presence of approximate radiation zeros.

New physics contributions can modify the self-interactions of vector-bosons, in particular the triple gauge-boson vertices. If new physics occurs at an energy scale well above that being probed experimentally, it can be integrated out, and the result expressed as a set of anomalous (non-Standard Model) interaction vertices. (The physics of anomalous coupling will be considered in detail in Section 5.
Fig. 21: $WZ$ production followed by leptonic decays of both the $W$ and $Z$ bosons. We plot the distribution, in picobarns, in the rapidity difference between the $Z$ and the charged lepton $l$ from the decay of the $W$, $\Delta y_{WZ} = y_Z - y_l$. Leptonic branching ratios are not included and the scale has been set to $\mu = (M_W + M_Z)/2$. The basic cuts used are $p_T(l) > 20 \text{ GeV}$ and $|\eta(l)| < 2$ for all three charged leptons, and a missing transverse momentum cut of $p_T^{\text{miss}} > 20 \text{ GeV}$. We plot the $\Delta y_{WZ}$ distribution with these cuts (blue, upper pair), with an additional cut on the $W$ decay lepton, $E(l) > 100 \text{ GeV}$ (green, middle pair) and with a further cut on the rapidity of the $Z$ boson $y_Z < 0$ (red, lower pair); the latter curves have been scaled up by a factor of 5. The dashed curves are Born-level results; the solid curves include the $\mathcal{O}(\alpha_s)$ corrections.

and our standard notation for the anomalous triple gauge-boson couplings is given there.) It is interesting to know what is the effect of the anomalous $W^+W^-Z$ couplings on the approximate radiation zero of $WZ$ production [121]. In Figure 22, the $\Delta y_{WZ}$ distribution is plotted for two different sets of anomalous couplings at vanishing $q^2$ ($\Delta g_1 = -0.013$, $\lambda^Z = 0.02$, $\Delta \kappa^Z = -0.028$) and ($\Delta g_1 = 0.065$, $\lambda^Z = 0.04$, $\Delta \kappa^Z = 0.071$). For the $q^2$ dependence we assumed dipole form factors of the generic form

$$\hat{a}(q^2) = \frac{a}{\left(1 + \frac{q^2}{\Lambda^2}\right)^2}$$

with $\Lambda = 2 \text{ TeV}$. As one can see in Figure 22, the contributions of anomalous couplings have the tendency to make the dip less pronounced.

4.15 Future improvements

The present state of art of the description of gauge-boson pair production is not completely satisfactory yet. Of the various issues, there are three which require further theoretical studies. First, the double resonant approximation is expected to be correct only up to a few percent accuracy - it is important to go beyond this approximation. A first attempt has been made by Campbell and Ellis [18] where, as already mentioned above, the singly-resonant diagrams have also been included. These additions are obviously relevant in the off-resonant regions. The inclusion of finite width effect is not completely straightforward because of possible conflict with gauge-invariance. This issue requires further theoretical study. Secondly, we need NLO results also for the semi-leptonic channels when one of the gauge-bosons decays hadronically. This requires the inclusion of the contributions of diagrams describing the gluonic interactions.
Fig. 22: $WZ$ production followed by leptonic decays of both the $W$ and $Z$ bosons. We plot the NLO distribution, in picobarns, in the rapidity difference between the $Z$ and the charged lepton $l$ from the decay of the $W$: $\Delta y_{Zl} \equiv y_Z - y_l$. Leptonic branching ratios are not included and the scale has been set to $\mu = (M_W + M_Z)/2$. The standard cuts $p_T(l) > 20$ GeV, $|\eta(l)| < 2.5$ for all three charged leptons and a missing transverse momentum cut of $p_T^{\text{miss}} > 20$ GeV are applied. We plot the $\Delta y_{Zl}$ distribution without anomalous couplings (red, lower pair) and with two sets of anomalous couplings ($\Delta g_1 = -0.013$, $\Delta \lambda^2 = 0.02$, $\Delta \kappa^2 = -0.028$) (green, middle pair) and ($\Delta g_1 = 0.065$, $\Delta \lambda^2 = 0.04$, $\Delta \kappa^2 = 0.071$) (blue, upper pair). The $q^2$ dependence of the couplings is given by the dipole form of Equation 39 with $\Lambda = 2$ TeV. Also we plot the same quantities supplementing the standard cuts with the additional cut on the the $W$ decay lepton, $E(l) > 100$ GeV and with the rapidity cut $y_Z < 0$; the latter curves have been scaled up by a factor of 5. The dashed curves are Born-level results; the solid curves include the $\mathcal{O}(\alpha_s)$ corrections.
corrections to the final-state quarks. Thirdly, fixed order perturbative QCD description is not applicable for the description of the low-$p_T$ behaviour of the gauge-boson pair. The technique for the resummation of the low-$p_T$ contributions is well known and it can be applied also to the case of gauge-boson pair production. For example, one calculation for the $ZZ$ has been carried out [122].

4.16 Comparison with Pythia

In most of the studies carried out so far for the LHC, where the production of vector boson pairs played an important role, the usual Monte Carlo simulation tool has been Pythia [123] based on LO matrix elements [124] with parton shower. In particular, it is expected that for some optimisation cuts, where the large corrections provided by NLO diagrams (for example by choosing high-$p_T$ ($V$) or high-$M_{VV}$ regions) its predictions are not acceptable. By making comparison between the predictions of Pythia and the the DKS parton level NLO Monte Carlo [19], we investigate here how accurate does Pythia simulate the di-boson cross sections at the LHC, especially in some kinematic regions. We relate our analysis to the special case of the CMS detector [125].

In all results presented in this analysis, we assume that the vector-bosons always decay leptonically. We use the CTEQ(4M) parton distribution [67] in both Monte Carlo and the cross section values are for the scale $\mu = (M_{V_1} + M_{V_2})/2$, where $M_{V_i}$ are the masses of the two produced vector-bosons. If the DKS Monte Carlo is run at Born-level, we obtain very good agreement with the total cross sections given by Pythia.

Figure 23 shows the transverse momentum of the $W$ bosons. The comparison between Pythia and DKS indicates the large difference in cross section observables at high-$p_T$ values. This is related to the fact that at NLO, the sub-processes $qg \rightarrow V_1 V_2 q$ have to be taken into account [26, 30]. This is also reported in Table 12. The leptons are selected following the CMS criteria, where a $p_T$ larger than 20 GeV and a pseudorapidity $|\eta| < 2.5$ are required. Jets are selected by: $p_T > 20$ GeV and $|\eta| < 3$. The K-factor increases then from 1.5 for the total cross sections up to values of about 60 if the jets are required to have a $p_T$ larger than 150 GeV. The same effect is shown in figure 24 for the $WZ$ production, where the $p_T$ of the jets is shown (the jet balances the $p_T^{V}$). For this process the K-factors at large $p_T$-values are even larger than in the $WW$ case (as shown in the table). The transverse momentum
Fig. 25: Missing transverse energy in the $WW$ production. The events are obtained by running the DKS generator with and without including the NLO corrections.

Fig. 26: Smallest angle between one of the $W$'s and the jet in $WW$ pair production. The two leptons are required to be within the detector acceptance and the jet to have a $p_T$ larger than 150 GeV.

Fig. 27: Transverse momentum of $ZZ$ pairs originating from a Higgs ($M_H = 250$ GeV), where the two leptons fall into the detector acceptance and the $M_{ZZ}$ is consistent with the Higgs mass. The non-resonant $ZZ$ background is simulated with (DKS) and without (PYTHIA) NLO corrections.
of the di-boson system (or of the jet(s)) are not the only variables affected by large NLO corrections. Other variables can show significant differences within their distributions: for example the lepton $p_T$, the invariant mass of the lepton pair $M_{ll}$, the missing transverse energy $E_T$ (as shown in Figure 25), the maximal transverse momentum of the two charged leptons $p_T^{\text{max}}$, the lepton pseudorapidities $\eta^l$, their difference $\Delta\eta_l = \eta^{l-} - \eta^{l+}$, the angle between leptons $\cos\theta_{ll}$, the transverse angle between leptons $\cos\phi_{ll}$ and so on.

Therefore, it is extremely important to take into account the possible influence of NLO corrections for the vector-boson production at the LHC energy. Every time one is performing an optimisation of signal selection, one should be aware of the possible deviations due to the use of a LO generator like PYTHIA. This is especially true for complicated cuts, where it is difficult to judge whether the effects are large or not. An example is shown for $WW$ events in Figure 26, where the smallest angle between one of the $W$’s and the jet is shown for events with a high-$p_T$ jet. Not only is the cross section clearly smaller in PYTHIA but also the shape of the distribution is quite different, changing the result of a possible cut. Another good example is the Higgs search through the decay channel $H \rightarrow ZZ \rightarrow 4l$ (see Figure 27). The idea of using $p_T$-cuts to improve the signal-to-background ratio may not be as effective as one would expect from using only PYTHIA. The figure shows indeed that, if the NLO corrections are included, the $p_T$ distribution of the non-resonant background follows much more closely those of the signal, reducing the gain considerably.

Table 12: Cross sections in pb for $pp$ collision at $\sqrt{s}=14$ TeV. The leptons are selected by requiring a $p_T$ larger than 20 GeV and a pseudorapidity $|\eta| < 2.5$. The jets should have a $p_T > 20$ GeV and $|\eta| < 3$.

<table>
<thead>
<tr>
<th>(pb)</th>
<th>Selected</th>
<th>Jet</th>
<th>$p_T^j$ selection (in GeV):</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma^{tot} \times BR$</td>
<td>leptons</td>
<td>veto</td>
</tr>
<tr>
<td>$\sigma_{PYTHIA}^{W^+W^-\rightarrow l^+l^-}$</td>
<td>3.704</td>
<td>1.704</td>
<td>1.125</td>
</tr>
<tr>
<td>$\sigma_{DKS,LO}^{W^+W^-\rightarrow l^+l^-}$</td>
<td>3.79</td>
<td>1.71</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{DKS,NLO}^{W^+W^-\rightarrow l^+l^-}$</td>
<td>5.56</td>
<td>2.58</td>
<td>1.49</td>
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<tr>
<td>K-factor</td>
<td>1.5</td>
<td>1.54</td>
<td>1.32</td>
</tr>
<tr>
<td>$\sigma_{DKS}^{W^\pm Z^\mp\rightarrow l^+l^-}$</td>
<td>$4.35 \times 10^{-1}$</td>
<td>$1.45 \times 10^{-1}$</td>
<td>$9.47 \times 10^{-2}$</td>
</tr>
<tr>
<td>$\sigma_{DKS,LO}^{W^\pm Z^\mp\rightarrow l^+l^-}$</td>
<td>$4.34 \times 10^{-1}$</td>
<td>$1.48 \times 10^{-1}$</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{DKS,NLO}^{W^\pm Z^\mp\rightarrow l^+l^-}$</td>
<td>$7.42 \times 10^{-1}$</td>
<td>$2.77 \times 10^{-1}$</td>
<td>$1.31 \times 10^{-1}$</td>
</tr>
<tr>
<td>K-factor</td>
<td>1.71</td>
<td>1.91</td>
<td>1.39</td>
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<tr>
<td>$\sigma_{DKS}^{ZZ\rightarrow l^+l^-}$</td>
<td>$5.13 \times 10^{-2}$</td>
<td>$1.79 \times 10^{-2}$</td>
<td>$1.15 \times 10^{-2}$</td>
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<tr>
<td>$\sigma_{DKS,LO}^{ZZ\rightarrow l^+l^-}$</td>
<td>$5.31 \times 10^{-2}$</td>
<td>$1.84 \times 10^{-2}$</td>
<td>-</td>
</tr>
<tr>
<td>$\sigma_{DKS,NLO}^{ZZ\rightarrow l^+l^-}$</td>
<td>$7.07 \times 10^{-2}$</td>
<td>$2.55 \times 10^{-2}$</td>
<td>$1.58 \times 10^{-2}$</td>
</tr>
<tr>
<td>K-factor</td>
<td>1.38</td>
<td>1.42</td>
<td>1.38</td>
</tr>
</tbody>
</table>

4.2 $W\gamma$ and $Z\gamma$ production at NLO

In this section, we present order $\alpha_s$ results for $W\gamma$ and $Z\gamma$ production at the LHC, including the full leptonic correlations and anomalous couplings in the narrow-width approximation [126]. Previous analyses [32, 127, 33] included decay correlations only in the bremsstrahlung amplitudes implementing, as an approximation, the finite part of the spin-summed one-loop amplitudes.
To perform the calculation, we use the helicity amplitudes presented in [17]. The amplitudes relevant for the inclusion of anomalous couplings are given in [126]. In order to cancel analytically the soft and collinear singularities coming from the bremsstrahlung and one loop parts, we have used the version of the subtraction method presented in [128]. Therefore, the amplitudes are implemented into a numerical Monte Carlo style program which allows calculation of any infrared-safe physical quantity with arbitrary cuts.

The results presented in this section correspond to $pp$ scattering at $\sqrt{s} = 14$ TeV using the following cuts: a transverse momentum cut of $p_T^\gamma > 25$ GeV for the charged leptons is imposed and the pseudorapidity is limited to $|\eta| < 2.4$ for all detected particles. The photon transverse momentum cut is $p_T^\gamma > 50\,(100)$ GeV for $W\gamma\,(Z\gamma)$ production. For the $W\gamma$ case, we require a minimum missing transverse momentum carried by the neutrinos $p_T^{\text{miss}} > 50$ GeV. Additionally, charged leptons and the photons must be separated in the pseudorapidity-azimuthal angle by $\Delta R_{\gamma\ell} = \sqrt{(\eta_{\ell} - \eta_{\gamma})^2 + (\phi_{\ell} - \phi_{\gamma})^2} > 0.7$. In order to suppress the contribution from the off-resonant diagrams, we require the transverse mass $M_T > 90$ GeV for $W\gamma$ production and the invariant mass of the $ll\gamma$ system $M_{ll\gamma} > 100$ GeV for the $Z\gamma$ case.

Finally, in order to suppress the contribution from the fragmentation of partons into photons, computed only to LO accuracy, the photons are required to be isolated from hadrons: the transverse momentum cut is

$$\sum_{\Delta R < R_0} p_T^{\text{had}} \leq 0.15 p_T^\gamma.$$  \hfill (40)

This completes the definition of the “standard” cuts.

In the results presented here, the branching ratios of the vector-bosons into leptons are not included. For both the LO and NLO results, we use the latest set of parton distributions of MRST(cor01) [114] and the two loop expression for the strong coupling constant. For the fragmentation component, we use the fragmentation functions from [129].

The “standard” scale for both the factorisation and renormalisation scales is

$$\mu^2 = \mu^2_{\text{st}} \equiv M^2_V + \frac{1}{2} \left[ (p_T^V)^2 + (p_T^\gamma)^2 \right].$$  \hfill (41)

The masses of the vector-bosons have been set to $M_Z = 91.187$ GeV and $M_W = 80.41$ GeV and the following values have been used for the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements: $|V_{ud}| = |V_{cd}| = 0.975$ and $|V_{us}| = |V_{cs}| = 0.222$. We do not include any QED or electroweak corrections but choose the coupling constants $\alpha$ and $\sin^2\theta_W$ in the spirit of the “improved Born approximation” [112, 113], with $\sin^2\theta_W = 0.230$. Notice that the observable is order $\alpha^2$; within the same spirit, we use the running $\alpha = \alpha(M_Z) = 1/128$ for the coupling between the vector-boson and the quarks (to take into account effectively the EW corrections) whereas we keep $\alpha = 1/137$ for the photon coupling. It is worth noticing that this modification results already in more than a 6% change in the normalisation of the cross section with respect to the standard approach of using both running coupling constants.

### 4.21 Results at NLO

For future checks, and for an estimate of the number of events to be observed at the LHC, some benchmark total cross section numbers are presented in Table 13. The first ones were obtained by imposing only the cut on the transverse momentum of the photon $p_T^\gamma > 50\,(100)$ GeV for $W\gamma\,(Z\gamma)$ production. The importance of the NLO corrections, as well as the size of the fragmentation contribution before applying the isolation cut prescription, can be seen from the table. Furthermore, we also include the result for the total cross section obtained after the implementation of the standard cuts.
Table 13: Cross sections for \( pp \) collisions at \( \sqrt{s} = 14 \) TeV. The statistical errors are ±1 within the last digit. LO* corresponds to the direct component only.

<table>
<thead>
<tr>
<th>( \sigma ) (pb)</th>
<th>LO*</th>
<th>Frag.</th>
<th>NLO</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W^+\gamma ) ( (p_T^\gamma &gt; 50 ) GeV)</td>
<td>4.79</td>
<td>3.02</td>
<td>13.89</td>
</tr>
<tr>
<td>( W^-\gamma ) ( (p_T^\gamma &gt; 50 ) GeV)</td>
<td>3.08</td>
<td>3.55</td>
<td>10.15</td>
</tr>
<tr>
<td>( Z\gamma ) ( (p_T^\gamma &gt; 100 ) GeV)</td>
<td>1.29</td>
<td>0.412</td>
<td>2.37</td>
</tr>
<tr>
<td>( W^+\gamma ) (std. cuts)</td>
<td>0.436</td>
<td>0.094</td>
<td>1.71</td>
</tr>
<tr>
<td>( W^-\gamma ) (std. cuts)</td>
<td>0.310</td>
<td>0.095</td>
<td>1.20</td>
</tr>
<tr>
<td>( Z\gamma ) (std. cuts)</td>
<td>0.524</td>
<td>0.041</td>
<td>0.877</td>
</tr>
</tbody>
</table>

In what follows, we will estimate the theoretical uncertainty of the results by analysing the changes on different distributions when varying the scale by a factor of two in both directions \( \frac{\mu}{2} < \mu < 2\mu_{\text{true}} \).

In Figure 28, we show the scale dependence of the \( p_T \) distribution of the photon in \( W^+\gamma \) production with the standard cuts (upper curves) and also with the additional requirement of a jet-veto. As can be observed, the scale dependence is still large (±10%) but is considerably reduced when the jet-veto is applied. The situation is similar to what has been observed in the case of \( WW \) production [19] and is caused by the suppression of the contribution from the quark initial state appearing for the first time at NLO. Since this initial state dominates the cross section, the NLO result behaves effectively like a LO one, as far as the scale dependence is concerned.

In the inset plot, we present the ratio between the NLO and LO results (with the standard scale), which remains larger than 3 and increases with the photon transverse momentum. This clearly shows that the LO calculation is not even sufficient for an understanding of the shape of the distribution, since the NLO effect goes beyond a simple normalisation. As is well known [28], the relevance of the NLO corrections for this process is mainly due to the breaking of the radiation amplitude zero appearing at LO and to the large quark initial state parton luminosity at the LHC. It is worth mentioning that the scale dependence of the LO result turns out to be very small. This is an artificial effect and illustrates that a small scale dependence is by no means a guarantee for small NLO corrections. Furthermore, we present the ratio of the NLO jet-veto and the LO result. As expected, this ratio is closer to 1, again due to the fact that most of the contributions coming from the new subprocesses appearing at NLO are suppressed by the jet-veto.

In Figure 29, we study the lepton correlation in the azimuthal angle for \( Z\gamma \) production \( \Delta\phi_{\mu} = |\phi_{\mu^-} - \phi_{\mu^+}| \). Notice that this observable can be studied at NLO since the spin correlations between the leptons are fully taken into account in the implementation of the one-loop corrections. In this case, we observe that the NLO corrections are rather sizeable and increase the cross section by 50% for small \( \Delta\phi_{\mu} \). The region \( \Delta\phi_{\mu} > 2 \) (with the standard cuts) is kinematically forbidden unless a jet with a high transverse momentum is produced; therefore, the cross section vanishes at LO and it is strongly suppressed for the NLO calculation with jet-veto. In this region, the full NLO calculation is effectively LO and its scale dependence becomes larger, as expected.

Because there is no radiation amplitude zero appearing at LO for \( Z\gamma \) production, the NLO corrections are under better control in the kinematical region where the LO cross section does not vanish. Nevertheless, for large transverse momentum, the quark initial state again dominates the NLO contribution and the corrections increase considerably.
Fig. 28: Scale dependence of $\sigma^{NLO}$ without (upper curves) and with (lower curves) jet-veto. The scale has been varied according to $\frac{\mu}{2}$ (dashed) $< \mu < 2\mu_{\text{sat}}$ (dots). The inset plot shows the ratio $\sigma^{NLO}/\sigma^{LO}$, again without (solid) and with (dots) jet-veto.

Fig. 29: Scale dependence of $\sigma^{NLO}$ without jet-veto (upper solid curves), $\sigma^{NLO}$ with jet-veto (lower solid curves) and $\sigma^{LO}$ (dotted curves). The scale has been varied according to $\frac{\mu}{2} < \mu < 2\mu_{\text{sat}}$. 
The study of triple vector-boson couplings is motivated by the hope that some physics beyond the Standard Model leads to a modification of these couplings which eventually could be detected. In order to quantify the effects of the new physics, an effective Lagrangian is introduced which contains all Lorentz invariant terms, in principle. The new terms spoil the gauge-cancellation in the high energy limit and, therefore, will lead to violation of unitarity for increasing partonic centre of mass energy $\hat{s}$. Usually, in an analysis of anomalous couplings from experimental data in hadronic collisions, this problem is circumvented by supplementing the anomalous couplings $\alpha_{AC}$ with form factors. A common choice for the form factor is

$$\alpha_{AC} \rightarrow \frac{\alpha_{AC}}{(1 + \frac{\hat{s}}{\Lambda^2})^n}$$

(42)

where $n$ has to be large enough to ensure unitarity and $\Lambda$ is interpreted as the scale for new physics. Obviously, this procedure is rather ad hoc and introduces some arbitrariness. Therefore, it would be very convenient to avoid it in an analysis of anomalous couplings at hadron colliders. This would bring these analyses more into line with those at $e^+e^-$ colliders. In order to do so, one should analyse the data at fixed values of $\hat{s}$, as it is done at LEP. This results in limits for the anomalous parameters which are a function of $\hat{s}$.

Clearly, it is possible to do such analysis for the production of $Z\gamma$ when both leptons are detected [130], since the partonic centre of mass energy can be reconstructed from the kinematics of the final state particles and therefore the cross section can be measured for different bins of fixed $\hat{s}$.

The situation is more complicated for $W\gamma$ production since the neutrino is not observed. Nevertheless, by identifying the transverse momentum of the neutrino with the missing transverse momentum, and assuming the $W$ boson to be on shell, it is possible to reconstruct the neutrino kinematics (particularly the longitudinal momentum) with a two-fold ambiguity. In the case of the Tevatron, since it is a $p\bar{p}$ collider, it is possible to choose the “correct” neutrino kinematics 73% of the times by selecting the maximum (minimum) of the two reconstructed values for the longitudinal momentum of the neutrino for $W^+\gamma(W^\gamma)$.

This is not true at the LHC where, due to the symmetry of the colliding beams, both reconstructed kinematics have equal chances to be correct. Fortunately, in the case of anomalous couplings, we are interested in an efficient way to reconstruct the $\hat{s}$ rather than the full kinematics. Again there are two possible values of $\hat{s}$. It turns out that there is a simple method to choose the “correct” one 66% of the times at the LHC (73% of the times at Tevatron) by selecting the minimum $\hat{s}$, $\hat{s}_{\text{min}}$, of the two reconstructed values (for both $W^+\gamma$ and $W^\gamma$). Furthermore, we checked that the selected value $\hat{s}_{\text{min}}$ differs in almost 90% of the events by less than 10% from the exact value $\hat{s}$. This is likely to be enough precision, since the data will be collected in sizeable bins of $\hat{s}$ and the anomalous parameters are not expected to change very rapidly with the energy in any case.

To quantify the advantage of the method, we show in Figure 30 the correlations of $\sqrt{s_{\text{min}}}$ with $\sqrt{\hat{s}}$. The left plot corresponds to the case of pure Standard Model, whereas the right plot presents results for (already experimentally ruled out) huge values of anomalous couplings $\Delta\kappa = 0.8$ and $\lambda = 0.2$ with an ordinary form factor ($n = 2, \Lambda = 1$ TeV).

The cross section drops very rapidly for increasing $\sqrt{\hat{s}} - \sqrt{s_{\text{min}}}$. This correlation clearly holds in the particularly interesting large $\sqrt{\hat{s}}$ region and for both Standard Model and anomalous contribution.

As a result of this investigation, we conclude that even in the case of $W\gamma$ production, reliable bounds for anomalous couplings as a function of $\hat{s}$ (using $s_{\text{min}}$) can be obtained. Such a procedure would certainly allow a comparison of various bounds from different experiments.
Fig. 30: The cross section for $W^+\gamma$ production (in pb/bin) as a function of $\sqrt{s}$ and $\sqrt{s_{\text{min}}}$ (in GeV) in order to illustrate the steep fall of $\sigma$ for increasing $|\sqrt{s} - \sqrt{s_{\text{min}}}|$. The left plot corresponds to the Standard Model, whereas the right plot includes anomalous couplings (see text).

5. ANOMALOUS VECTOR-BOSON COUPLINGS

The principle of gauge-invariance is used as the basis for the Standard Model. The non-Abelian gauge-group structure of the theory of electroweak interactions predicts very specific couplings between the electroweak gauge-bosons. Measurements of these triple gauge-boson couplings (TGCs) of the $W$, $Z$ and $\gamma$ gauge-bosons therefore provide powerful tests of the Standard Model.

In the most general Lorentz invariant parametrisation, the three gauge-boson vertices, $WW\gamma$ and $WWZ$, can be described by fourteen independent couplings [131], seven for each vertex. The possible four quadruple gauge-boson vertices: $\gamma\gamma WW$, $Z\gamma WW$, $ZZWW$ and $WWWW$ require 36, 54, 81 and 81 couplings, respectively for a general description. Assuming electromagnetic gauge-invariance, C- and P-conservation, the set of 14 couplings for the three gauge-boson vertices is reduced to 5: $g_1^Z$, $\kappa_\gamma$, $\kappa_Z$, $\lambda_\gamma$ and $\lambda_Z$ [132], where their Standard Model values are equal to $g_1^Z = \kappa_\gamma = \kappa_Z = 1$ and $\lambda_\gamma = \lambda_Z = 0$ at tree level.

The TGCs related to the $WW\gamma$ vertex determine properties of the $W$, such as its magnetic dipole moment $\mu_W$ and electric quadrupole moment $q_W$:

\[
\mu_W = \frac{e}{2M_W}(g_1^Z + \kappa_\gamma + \lambda_\gamma) \tag{43}
\]

\[
q_W = \frac{e}{M_W^2}(\kappa_\gamma - \lambda_\gamma) \tag{44}
\]

In the following, the anomalous TGCs are denoted by $\Delta g_1^Z$, $\Delta \kappa_\gamma$, $\Delta \kappa_Z$, $\lambda_\gamma$ and $\lambda_Z$, where the $\Delta$ denotes the deviations of the respective quantity from its Standard Model value.

5.1 Introduction

The Standard Model is well established by the experiments at LEP and the Tevatron. Any deviations of the Standard Model can therefore be introduced only with care. Changes to the Standard Model come

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6Section coordinators: P.R. Hobson, W. Hollik
with different forms of severity. In order to see at what level anomalous vector-boson couplings can be reasonably discussed, one has to consider these cases separately. Changes to the gauge-structure of the theory, that do not violate the renormalisability of the theory, i.e. the introduction of extra fermions or possible extensions of the gauge-group are the least severe. They will typically generate small corrections to vector-boson couplings via loop effects. In this case also, radiative effects will be generated at lower energies. For the LHC, the important thing in this case is not to measure the anomalous couplings precisely, but to look for the extra particles. However, this is beyond the scope of this chapter. In the other case, a more fundamental role is expected for the anomalous couplings, implying strong interactions. In this case, one has to ask oneself whether one should study a model with or without a fundamental Higgs boson.

Simply removing the Higgs boson from the Standard Model is a relatively mild change. The model becomes non-renormalisable, but the radiative effects grow only logarithmically with the cut-off at the one-loop level. The question is whether this scenario is ruled out by the LEP1 precision data. The LEP1 data appear to be in agreement with the Standard Model, preferring a low Higgs mass. One is sensitive to the Higgs mass in three parameters, labelled $S$, $T$, $U$ or $\epsilon_1, \epsilon_2, \epsilon_3$. These receive corrections of the form $g^2 (\log(M_H/M_W) + \text{constant})$, where the constants are of order one. The logarithmic enhancement is universal and would also appear in models without a Higgs as $\log(\Lambda)$, where $\Lambda$ is the cut-off at which new interactions should appear. Only when one can determine the three different constants independently, can one say that one has established the Standard Model. At present, the data do not provide sufficient precision to do this.

A much more severe change to the Standard Model is the introduction of vector-boson couplings not of the gauge-interaction type. These new couplings violate renormalisability much more severely than simply removing the Higgs boson. Typically, quadratically and quartically divergent corrections would appear to physical observables. Therefore, it is questionable as to whether one should study models with a fundamental Higgs boson, but with extra anomalous vector-boson couplings. It is hard to imagine a form of dynamics that could do this. If the vector-bosons become strongly interacting, the Higgs probably would exist at most in an “effective” way. Therefore, the most natural way is to study anomalous vector-boson couplings in models without a fundamental Higgs. Actually when one removes the Higgs boson, the Standard Model becomes a gauged non-linear sigma-model. It is well known that the nonlinear sigma-model describes low-energy pion physics. The “pions” correspond to the longitudinal degrees of freedom of the vector-bosons and $f_{\pi}$ corresponds to the vacuum expectation value of the Higgs field. Within this description, the Standard Model corresponds to the lowest-order term quadratic in the momenta, anomalous couplings to higher derivative terms. The systematic expansion in terms of momenta is known as chiral perturbation theory and is extensively used in meson physics.

Writing down the most general non-linear chiral Lagrangian containing up to four derivatives gives rise to a large number of terms, which are too general to be studied effectively. One therefore has to look for dynamical principles that can limit the number of terms. Of particular importance are approximate symmetry principles. In the first place one, expects CP-violation to be small. We limit ourselves therefore to CP-preserving terms. In order to see what this means in practice, it is advantageous to describe the couplings in a manifestly gauge-invariant way, using the Stückelberg formalism [133, 37]. One needs the following definitions:

\begin{equation}
F_{\mu\nu} = \frac{i\tau_1}{2} (\partial_\mu W^i_\nu - \partial_\nu W^i_\mu + ge^{ijk} W^j_\mu W^k_\nu) \tag{45}
\end{equation}

is the $SU(2)$ field strength with the $SU(2)$ gauge-coupling $g$;

\begin{equation}
D_\mu U = \partial_\mu U + \frac{ig}{2} \tau_1 W^i_\mu U + ig \tan \theta W U \tau_3 B_\mu \tag{46}
\end{equation}

is the gauge-covariant derivative of the $SU(2)$-valued field $U$, which describes the longitudinal degrees
of freedom of the vector fields in a gauge-invariant way:

\[ B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \]  

(47)

is the hypercharge field strength. In addition,

\[ V_\mu = (D_\mu U)U^\dagger / g, \]

(48)

\[ T = U T_\mu U^\dagger / g \]

(49)

are auxiliary quantities having simple transformation properties. Excluding CP violation, the non-standard three and four vector-boson couplings are described in this formalism by the following set of operators:

\[ \mathcal{L}_1 = \text{Tr}(F_{\mu\nu}[V_\mu, V_\nu]) \]
\[ \mathcal{L}_2 = i \frac{B_{\mu\nu}}{2} \text{Tr}(T[V_\mu, V_\nu]) \]
\[ \mathcal{L}_3 = \text{Tr}(TF_{\mu\nu}) \text{Tr}(T[V_\mu, V_\nu]) \]
\[ \mathcal{L}_4 = (\text{Tr}[V_\mu V_\nu])^2 \]
\[ \mathcal{L}_5 = (\text{Tr}[V_\mu V_\nu])^2 \]
\[ \mathcal{L}_6 = \text{Tr}(V_\mu V_\nu) \text{Tr}(TV_\mu) \text{Tr}(TV_\nu) \]
\[ \mathcal{L}_7 = \text{Tr}(V_\mu V_\nu)(\text{Tr}[TV_\nu])^2 \]
\[ \mathcal{L}_8 = \frac{1}{2}[(\text{Tr}[TV_\mu]) (\text{Tr}[TV_\nu])^2 \]

(50) \hspace{1cm} (51) \hspace{1cm} (52) \hspace{1cm} (53) \hspace{1cm} (54) \hspace{1cm} (55) \hspace{1cm} (56) \hspace{1cm} (57)

In the unitary gauge \( U = 1 \), one has (with \( c_W = \cos \theta_W, s_W = \sin \theta_W \))

\[ \mathcal{L}_1 = i (c_W Z_{\mu\nu} + s_W F_{\mu\nu}) W_\mu^+ W_\nu^- + Z_\nu / c_W (W_\mu^+ W_\nu^- - W_{\mu\nu} W_\mu^+) \]

\[ + \text{gauge-induced four boson vertices,} \]

\[ \mathcal{L}_2 = i (c_W F_{\mu\nu} - s_W Z_{\mu\nu}) W_\mu^+ W_\nu^- \]

\[ \mathcal{L}_3 = i (c_W Z_{\mu\nu} + s_W F_{\mu\nu}) W_\mu^+ W_\nu^- . \]

(58) \hspace{1cm} (59) \hspace{1cm} (60)

where \( Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \text{ and } W_{\mu\nu}^+ = \partial_\mu W_{\nu}^{+\text{--}} - \partial_\nu W_{\mu}^{+\text{--}}. \) The Standard Model without a Higgs corresponds to

\[ \mathcal{L}_{EW} = \frac{1}{2} \text{Tr}(F_{\mu\nu} F^{\mu\nu}) - \frac{i}{4} B_{\mu\nu} B^{\mu\nu} + \frac{g^2 s^2}{4} \text{Tr}(V_\mu V_\nu). \]

(61)

### 5.2 Dynamical constraints

The list given in the previous section contains terms that give rise to vertices with minimally three or four vector-bosons. Already with the present data a number of constraints and/or consistency conditions can be put on the vertices. The most important of these come from the limits on the breaking of the so-called custodial symmetry. If the hypercharge is put to zero, the effective Lagrangian has a larger symmetry than \( SU_L(2) \times U_Y(1) \), i.e. it has the symmetry \( SU_L(2) \times SU_R(2). \) The \( SU_R(2) \) invariance is a global invariance. Within the Standard Model this invariance is an invariance of the Higgs potential, but not of the full Lagrangian. It is ultimately this invariance that is responsible for the fact that the \( \rho \) parameter, which is the ratio of charged to neutral current strength, is equal to one at the tree level. Some terms in the Lagrangian, i.e. the ones containing the hypercharge field explicitly or the terms with \( T \), that project out the third isospin component violate this symmetry explicitly. These terms, when inserted in a loop graph, give rise to quartically divergent contributions to the \( \rho \) parameter. Given the measurements, this means that the coefficients of these terms must be extremely small. It is therefore reasonable to limit oneself to a Lagrangian, where hypercharge appears only indirectly via a minimal coupling, so without explicit \( T \). This assumption means physically that the ultimate dynamics that is
Using a naive analysis one finds \[134\] should be taken as such. The current data show that Lagrangian. As it stands, one can use the LEP1 data to put a limit on the terms in the two point vertices.

This particular combination does not seem to have any natural interpretation from underlying dynamics. Eliminating the custodial symmetry violating interactions, we are left with the simplified Lagrangian, containing \(\mathcal{L}_1, \mathcal{L}_4, \mathcal{L}_5\). Besides the vertices, there are also propagator corrections, in principle. We take the two-point functions without explicit \(T\). Specifically, we add to the theory \[134\]

\[
\mathcal{L}_{h,c,\text{tr}} = -\frac{1}{2\Lambda_X^4} \text{Tr}[(D_\alpha F^\mu_{\nu})(D^\alpha F_{\mu\nu})] + \frac{1}{2\Lambda_X^4} \text{Tr}[(\partial_\alpha B^\mu_{\nu})(\partial^\alpha B_{\mu\nu})]
\]  

(62)

for the transverse degrees of freedom of the gauge-fields, and

\[
\mathcal{L}_{h,c,\text{lg}} = -\frac{q_0^2 v^2}{4\Lambda_X^4} \text{Tr}[(D^\alpha V^\mu)(D_\alpha V_\mu)]
\]  

(63)

for the longitudinal ones, where the \(\Lambda_X\) parametrise the quadratic divergences and are expected to represent the scales where new physics comes in. In phenomenological applications, these contributions give rise to form factors in the propagators \[134, 139\]. Introducing such cut-off dependent propagators in the analysis of the vector-boson pair production is similar to having \(s\)-dependent triple vector-boson couplings, which is the way the data are usually analysed.

This effective Lagrangian is very similar to the one in pion-physics. Indeed, if one takes the limit vacuum expectation value (vev) fixed and gauge-couplings to zero, one finds the standard pion Lagrangian. As it stands, one can use the LEPI data to put a limit on the terms in the two point vertices. Using a naive analysis one finds \[134\] \(1/\Lambda_X^2 = 0\). For the other two cut-offs one has:

A. The case \(\Lambda_V^2 > 0, \Lambda_W^2 < 0\): \(\Lambda_V > 0.49\) TeV, \(|\Lambda_W| > 1.3\) TeV.

B. The case \(\Lambda_V^2 < 0, \Lambda_W^2 > 0\): \(|\Lambda_V| > 0.74\) TeV, \(\Lambda_W > 1.5\) TeV.

This information is important for further limits at high-energy colliders, as it tells us, how one has to cut off off-shell propagators. We notice that the limits on the form factors are different for the transverse, longitudinal and hypercharge form factors. The precise limits are somewhat qualitative and should be taken as such. The current data show that \(\Lambda = 0.5\) TeV, which thus has to be considered as a minimal possible value as long as a dipole form factor is used. Further information comes from the direct measurements of the three-point couplings at LEP2, which tell us that they are small. Similar limits at the Tevatron have to be taken with some care, as there is a cut-off dependence. As there is no known model that can give large three-point interactions, we assume for the further analysis of the four-point vertices, that the three-point anomalous couplings are absent. Two more constraints can be put on the remaining two four-point vertices. The first comes from consistency of chiral perturbation theory \[135\]. Not every effective chiral Lagrangian can be generated from a physical underlying theory.

A second condition comes from the \(\rho\) parameter. Even the existing violation of the custodial symmetry, though indirect via the minimal coupling to hypercharge, gives a contribution to the \(\rho\) parameter. It constrains the combination \(5g_4 + 2g_5\). The remaining combination \(2\mathcal{L}_4 - 5\mathcal{L}_5\) is fully unconstrained by experiment and in principle gives a possibility for very strong interactions to be present. However, this particular combination does not seem to have any natural interpretation from underlying dynamics. Therefore, one can conclude presumably that both couplings \(g_4, g_5\) are small. There is a loophole to this conclusion, namely when the anomalous couplings are so large that the one-loop approximation, used to arrive at the limits, is not consistent and resummation has to be performed everywhere. This is a somewhat exotic possibility that could lead to very low-lying resonances and which ought to be easy to discover at the LHC \[41\].
5.3 LHC processes

Given the situation described above, one has to ask oneself, what the LHC can do and in which way the data should be analysed. There are essentially three processes that can be used to study vector-boson vertices: vector-boson pair production, vector-boson scattering, triple vector-boson production. About the first two we have only a few remarks to make. They are discussed more fully in other contributions to the workshop.

5.31 Vector-boson pair production

Vector-boson pair production can be studied in a relatively straightforward way. The reason is that here the Higgs boson does not play a role in the Standard Model, as we take the incoming quarks to be massless. Therefore naive violations of unitarity can be compensated by the introduction of smooth form-factors.

One produces two vector-bosons via normal Standard Model processes with an anomalous vertex added. The extra anomalous coupling leads to unitarity-violating cross sections at high energy. As a total energy of 14 TeV is available this is a serious problem, in principle. It is cured by introducing a form factor for the incoming off-shell line connected to the anomalous vertex. Naively this leads to a form-factor dependent limit on the anomalous coupling in question. The LEP1 data gives a lower limit on the cut-off to be used inside the propagator. When one wants an overall limit on the anomalous coupling, one should use this value. This is particularly relevant for the Tevatron. Here one typically takes a cut-off of 2 TeV. This might give too strict a limit, as the LEP1 data indicate that the cut-off can be as low as 500 GeV. For practical purposes the analysis at the Tevatron should give limits on anomalous couplings for different values of the cut-off form factors, including low values of the cut-off. For the analysis at the LHC, one has much larger statistics. This means that one can do better and measure limits on the anomalous couplings as a function of the invariant mass of the produced system. This way one measures the anomalous form factor completely.

5.32 Vector-boson scattering

Here the situation is more complicated than in vector-boson pair production. The reason is that within the Standard Model the process cannot be considered without intermediate Higgs contribution. This would violate unitarity. However the incoming vector-bosons are basically on-shell and this allows the use of unitarisation methods, as are commonly used in chiral perturbation theory in pion physics. These methods tend to give rise to resonances in longitudinal vector-boson scattering. The precise details depend on the coupling constants. The unitarisation methods are not unique, but generically give rise to large $I = J = 0$ and/or $I = J = 1$ cross section enhancements. The literature is quite extensive: a good introduction is [136]; a recent review is [137].

5.33 Triple vector-boson production

In this case it is not clear how one should consistently approach an analysis of anomalous vector-boson couplings. Within the Standard Model the presence of the Higgs boson is essential in this channel. Leaving it out, one has to study the unitarisation. This unitarisation has to take place not only on the two-to-two scattering subgraphs, as in vector-boson scattering, but also on the incoming off-shell vector-boson, decaying into three real ones. The analysis here becomes too arbitrary to derive very meaningful results. One cannot calculate confidently anything here without a fully known underlying model of new strong interactions. Also measurable cross sections tend to be small, so that the triple vector-boson production is best used as corroboration of results in vector-boson scattering. Deviations of Standard Model cross sections could be seen, but the vector-boson scattering would be needed for interpretation.

One therefore needs the Standard Model results. The total number of events with three vector-bosons in the final state is given in Table 14. We used an integrated luminosity of 100 fb$^{-1}$ and an
energy of 14 TeV throughout.

Table 14: Number of events: before cuts and all decays ($\sqrt{s} = 14$ TeV, 100 fb$^{-1}$).

<table>
<thead>
<tr>
<th>$M_{\text{Higgs}}$ (GeV)</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+W^-W^-$</td>
<td>11675</td>
<td>5084</td>
<td>4780</td>
<td>4800</td>
</tr>
<tr>
<td>$W^+W^+W^-$</td>
<td>20250</td>
<td>9243</td>
<td>8684</td>
<td>8768</td>
</tr>
<tr>
<td>$W^+W^-Z$</td>
<td>20915</td>
<td>11167</td>
<td>10638</td>
<td>10685</td>
</tr>
<tr>
<td>$W^-ZZ$</td>
<td>2294</td>
<td>1181</td>
<td>1113</td>
<td>1113</td>
</tr>
<tr>
<td>$W^+ZZ$</td>
<td>4084</td>
<td>2243</td>
<td>2108</td>
<td>2165</td>
</tr>
<tr>
<td>$ZZZ$</td>
<td>4883</td>
<td>1332</td>
<td>1087</td>
<td>1085</td>
</tr>
</tbody>
</table>

One sees from this table that a large part of the events comes from associated Higgs production, when the Higgs is light. However for the study of anomalous vector-boson couplings, the heavier Higgs results are arguably more relevant. Not all the events can be used for the analysis. If we limit ourselves to events, containing only electrons, muons and neutrinos, assuming just acceptance cuts we find the results shown in Table 15.

Table 15: Number of events containing only leptonic decays. Cuts on leptons: $|\eta| < 3$, $p_T > 20$ GeV; no cuts on missing energy ($\sqrt{s} = 14$ TeV, 100 fb$^{-1}$).

<table>
<thead>
<tr>
<th>$M_{\text{Higgs}}$ (GeV)</th>
<th>200</th>
<th>400</th>
<th>600</th>
<th>800</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W^+W^-W^-$</td>
<td>68</td>
<td>28</td>
<td>25</td>
<td>25</td>
</tr>
<tr>
<td>$W^+W^+W^-$</td>
<td>112</td>
<td>49</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>$W^+W^-Z$</td>
<td>32</td>
<td>17</td>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>$W^-ZZ$</td>
<td>1.0</td>
<td>0.51</td>
<td>0.46</td>
<td>0.45</td>
</tr>
<tr>
<td>$W^+ZZ$</td>
<td>1.7</td>
<td>0.88</td>
<td>0.79</td>
<td>0.79</td>
</tr>
<tr>
<td>$ZZZ$</td>
<td>0.62</td>
<td>0.18</td>
<td>0.13</td>
<td>0.12</td>
</tr>
</tbody>
</table>

We see that very little is left, in particular in the processes with at least two $Z$ bosons, where the events can be fully reconstructed. In order to see how sensitive we are to anomalous couplings, we assumed a 4$Z$ coupling with a form factor cut-off at 2 TeV. We make here no correction for efficiencies etc.. Using the triple $Z$ boson production, assuming no events are seen in 100 fb$^{-1}$, we find a limit $|g_4 + g_5| < 0.09$ at the 95% CL, where $g_4$ and $g_5$ are the coefficients multiplying the operators $\mathcal{L}_4$ and $\mathcal{L}_5$. This is to be compared with $-0.15 < 5g_4 + 2g_5 < 0.14$ [138] or $-0.066 < (5g_4 + 2g_5)\Lambda^2$ [134, 139]. So the sensitivity is not better than present indirect limits. Better limits exist in vector-boson scattering [140] or at a linear collider [141, 142, 143].

In the following tables we present numbers for observable cross sections in different decay modes of the vector-bosons. We used the following cuts.

$$|\eta|_{\text{lepton}} < 3, \quad |\eta|_{\text{jet}} < 2.5,$$

$$|p_T|_{\text{lepton}} > 20 \text{ GeV}, \quad |p_T|_{\text{jet}} > 40 \text{ GeV}, \quad |p_T|_{2\nu} > 50 \text{ GeV},$$

$$\Delta R_{\text{jet,lepton}} > 0.3, \quad \Delta R_{\text{jet,jet}} > 0.5.$$
States with more than two neutrinos are not very useful because of the background from two vector-boson production. We did not consider final states containing \( \tau \)-leptons.

With the given cuts, the total number of events to be expected is rather small. In particular, this is the case because we did not consider the reduction in events due to experimental inefficiencies, which may be relatively large because of the large number of particles in the final state. For the processes containing jets in the final state, there will be large backgrounds due to QCD processes. A final conclusion on the significance of the triple vector-boson production for constraining the four vector-boson couplings will need more work, involving detector Monte Carlo calculations.

However it is probably fair to say from the above results, that no very strong constraints will be found from this process at the LHC, but it is useful as a cross-check with other processes. It may provide complementary information if non-zero anomalous couplings are found.

<table>
<thead>
<tr>
<th>( M_{\text{Higgs}} ) (GeV)</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>6( \ell )</td>
<td>0.62</td>
<td>0.29</td>
<td>0.18</td>
<td>0.14</td>
<td>0.13</td>
</tr>
<tr>
<td>4( \ell ), 2( \nu )</td>
<td>5.1</td>
<td>2.5</td>
<td>1.5</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>4( \ell ), 2( j )</td>
<td>6.6</td>
<td>3.8</td>
<td>2.2</td>
<td>1.7</td>
<td>1.4</td>
</tr>
<tr>
<td>2( \ell ), 2( j ), 2( \nu )</td>
<td>34</td>
<td>20</td>
<td>12</td>
<td>9.0</td>
<td>7.7</td>
</tr>
<tr>
<td>2( \ell ), 4( j )</td>
<td>24</td>
<td>19</td>
<td>11</td>
<td>7.6</td>
<td>6.0</td>
</tr>
<tr>
<td>2( \nu ), 4( j )</td>
<td>37</td>
<td>34</td>
<td>21</td>
<td>15</td>
<td>11</td>
</tr>
<tr>
<td>6( j )</td>
<td>25</td>
<td>31</td>
<td>19</td>
<td>12</td>
<td>8.7</td>
</tr>
</tbody>
</table>

Table 17: Number of events from \( WWZ \) production in different decay modes (\( \sqrt{s} = 14 \text{ TeV}, 100 \text{ fb}^{-1} \)).

<table>
<thead>
<tr>
<th>( M_{\text{Higgs}} ) (GeV)</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>4( \ell ), 2( \nu )</td>
<td>31</td>
<td>20</td>
<td>17</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>3( \ell ), 2( j ), 1( \nu )</td>
<td>51</td>
<td>40</td>
<td>31</td>
<td>28</td>
<td>26</td>
</tr>
<tr>
<td>2( \ell ), 4( j )</td>
<td>19</td>
<td>22</td>
<td>17</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>2( \nu ), 4( j )</td>
<td>63</td>
<td>74</td>
<td>60</td>
<td>51</td>
<td>48</td>
</tr>
<tr>
<td>2( \ell ), 2( j ), 2( \nu )</td>
<td>102</td>
<td>68</td>
<td>54</td>
<td>49</td>
<td>48</td>
</tr>
<tr>
<td>1( \ell ), 4( j ), 1( \nu )</td>
<td>262</td>
<td>196</td>
<td>140</td>
<td>127</td>
<td>127</td>
</tr>
<tr>
<td>6( j )</td>
<td>86</td>
<td>104</td>
<td>78</td>
<td>62</td>
<td>56</td>
</tr>
</tbody>
</table>

5.4 Unitarity limits and form factors

Unitarity in the Standard Model depends directly on its gauge-structure. Departure from this structure can violate unitarity at relatively low energies and so protection is provided in the effective Lagrangian for triple gauge-boson vertices by expressing the anomalous couplings as energy dependent form factors. For experimental results at a given subprocess energy \( \bar{s} \) (i.e. \( e^+ e^- \) colliders), the choice of form
Table 18: Number of events from $ZZW^-(\text{upper})$ and $ZZW^+(\text{lower})$ production in different decay modes ($\sqrt{s} = 14$ TeV, 100 fb$^{-1}$).

<table>
<thead>
<tr>
<th>$M_{Higgs}$ (GeV)</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5l, 1\nu$</td>
<td>0.45</td>
<td>1.04</td>
<td>0.63</td>
<td>0.52</td>
<td>0.47</td>
</tr>
<tr>
<td></td>
<td>0.80</td>
<td>1.69</td>
<td>1.08</td>
<td>0.91</td>
<td>0.81</td>
</tr>
<tr>
<td>$3l, 2j, 1\nu$</td>
<td>3.37</td>
<td>6.89</td>
<td>5.36</td>
<td>4.18</td>
<td>3.73</td>
</tr>
<tr>
<td></td>
<td>5.9</td>
<td>11.5</td>
<td>9.3</td>
<td>7.4</td>
<td>6.5</td>
</tr>
<tr>
<td>$1l, 4j, 1\nu$</td>
<td>7.6</td>
<td>11.5</td>
<td>12.4</td>
<td>10.0</td>
<td>8.4</td>
</tr>
<tr>
<td></td>
<td>13.3</td>
<td>20.0</td>
<td>21.6</td>
<td>18</td>
<td>15</td>
</tr>
<tr>
<td>$4l, 2j$</td>
<td>0.29</td>
<td>1.0</td>
<td>0.54</td>
<td>0.38</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td>0.49</td>
<td>1.6</td>
<td>0.91</td>
<td>0.65</td>
<td>0.54</td>
</tr>
<tr>
<td>$2l, 2j, 2\nu$</td>
<td>2.0</td>
<td>6.5</td>
<td>3.5</td>
<td>2.5</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>3.4</td>
<td>10.7</td>
<td>6.1</td>
<td>4.4</td>
<td>3.7</td>
</tr>
<tr>
<td>$2l, 4j$</td>
<td>2.5</td>
<td>7.4</td>
<td>5.4</td>
<td>3.6</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td>4.7</td>
<td>9.5</td>
<td>9.5</td>
<td>6.9</td>
<td>5.6</td>
</tr>
<tr>
<td>$4j, 2\nu$</td>
<td>8.9</td>
<td>27</td>
<td>18</td>
<td>12.6</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>195</td>
<td>54</td>
<td>38</td>
<td>28</td>
<td>23</td>
</tr>
<tr>
<td>$6j$</td>
<td>5.3</td>
<td>12.3</td>
<td>13.3</td>
<td>8.8</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>9.1</td>
<td>20.7</td>
<td>23</td>
<td>16</td>
<td>12.5</td>
</tr>
</tbody>
</table>

Factor parametrisation is not important since one can unambiguously translate between parametrisations. However, when results are integrated over a range of $\hat{s}$ as they will be at the LHC, no simple translation is possible and results depend crucially on the choice of the form factors. The form factor behaviour of anomalous couplings should not be neglected, particularly in regions of $\hat{s}$ near to unitarity limits. Any measurement of anomalous couplings over integrated energies carries with it assumptions on the parametrisation of the form factor.

This section outlines the considerations which influence the choice of form factor and suggests a method for measuring energy dependent anomalous couplings.

5.41 Form factor parametrisation

Triple gauge-boson vertices in di-boson production arise in the $J = 1$ partial wave amplitude only ($s$-channel exchange of a gauge-boson coupled to massless fermions). $S$-matrix unitarity implies a constant bound to any partial wave amplitude. This means unitarity is violated at asymptotically high energies if constant anomalous couplings are assumed. Unambiguous and model-independent constant unitarity constraints for $WW$ production have been derived$^7$ [144].

To conserve unitarity at arbitrary energies, anomalous couplings must be introduced as form factors. Thus, an arbitrary anomalous coupling $\tilde{A} = A_0 \times F(q_1^2, q_2^2, P^2)$ vanishes when $q_1^2$, $q_2^2$, or $P^2$ becomes large, where $q_1^2$ and $q_2^2$ are the invariant masses squared of the production bosons and $P^2 = \hat{s}$ is

$^7$Cancellations may occur if more than one anomalous coupling is allowed non-zero at a time, which weakens the unitarity limits somewhat.
Table 19: Number of events from $W^-W^+W^+$ production in different decay modes ($\sqrt{s} = 14$ TeV, 100 fb$^{-1}$).

<table>
<thead>
<tr>
<th>$M_{Higgs}$ (GeV)</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\ell, 3\nu$</td>
<td>66</td>
<td>44</td>
<td>37</td>
<td>35</td>
<td>33</td>
</tr>
<tr>
<td>$l^-l^+, 2j, 2\nu$</td>
<td>57</td>
<td>43</td>
<td>31</td>
<td>26</td>
<td>24</td>
</tr>
<tr>
<td>$l^-l^+, 2j, 2\nu$</td>
<td>13</td>
<td>7.9</td>
<td>5.3</td>
<td>4.4</td>
<td>4.0</td>
</tr>
<tr>
<td>$l^+, 4j, 1\nu$</td>
<td>148</td>
<td>129</td>
<td>86</td>
<td>66</td>
<td>58</td>
</tr>
<tr>
<td>$l^-, 4j, 1\nu$</td>
<td>99</td>
<td>61</td>
<td>36</td>
<td>26</td>
<td>23</td>
</tr>
<tr>
<td>$6j$</td>
<td>50</td>
<td>74</td>
<td>46</td>
<td>32</td>
<td>25</td>
</tr>
</tbody>
</table>

Table 20: Number of events from $W^+W^-W^-$ production in different decay modes ($\sqrt{s} = 14$ TeV, 100 fb$^{-1}$).

<table>
<thead>
<tr>
<th>$M_{Higgs}$ (GeV)</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3\ell, 3\nu$</td>
<td>40</td>
<td>26</td>
<td>22</td>
<td>21</td>
<td>20</td>
</tr>
<tr>
<td>$l^-l^-, 2j, 2\nu$</td>
<td>34</td>
<td>25</td>
<td>17</td>
<td>14</td>
<td>13</td>
</tr>
<tr>
<td>$l^+l^-, 2j, 2\nu$</td>
<td>78</td>
<td>45</td>
<td>30</td>
<td>25</td>
<td>23</td>
</tr>
<tr>
<td>$l^-, 4j, 1\nu$</td>
<td>90</td>
<td>76</td>
<td>49</td>
<td>37</td>
<td>33</td>
</tr>
<tr>
<td>$l^+, 4j, 1\nu$</td>
<td>59</td>
<td>35</td>
<td>20</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>$6j$</td>
<td>29</td>
<td>43</td>
<td>26</td>
<td>18</td>
<td>14</td>
</tr>
</tbody>
</table>

the virtual exchange boson invariant mass squared. We refer to $\tilde{A}_0$ as the “bare coupling” and $\tilde{A}$ as the form factor ($\tilde{A} \in \lambda V, \Delta \kappa V, h_Y V, \ldots$). For di-boson production, the final state bosons are nearly on-shell $q_1^2, q_2^2 \approx M_V^2$, even when finite width effects are taken into account, though large virtual exchange boson masses $\sqrt{s}$ will be probed at the LHC.

The choice of parametrisation for the form factors is arbitrary provided unitarity is conserved at all energies for a sufficiently small value of anomalous coupling. A step function operating at a cutoff scale $\Lambda_{\text{FF}}$ is sufficient\(^8\) though discontinuous and thus unphysical. More common in the literature is a generalised dipole form factor which is motivated by the well known nucleon form factors and has further appeal because it enters the Lagrangian in a form similar to that of a propagator of mass $\Lambda_{\text{FF}}$. The parametrisation is

$$\tilde{A} = \frac{\tilde{A}_0}{(1 + \frac{\sqrt{s}}{\Lambda_{\text{FF}}})^n}$$

where $n > 1/2$ ($n > 1$) is sufficient for the $WWV$ vertex anomalous couplings $\Delta \kappa V (\lambda V, \Delta g_Y V)$ which grow like $\tilde{s}^{1/2}$, ($\tilde{s}$). For the $ZV\gamma$ vertex $n > 3/2$ ($n > 5/2$) is sufficient for anomalous couplings $h_1 (h_{2,4})$ which grow like $\tilde{s}^{3/2}$, ($\tilde{s}^{5/2}$). The usual assumptions are $n = 2$ for $g_Y V$, $\lambda V$, $\kappa V$ [31, 32, 30] and $n = 3$ ($n = 4$) for $h_{1,3} (h_{2,4})$ [145]. Unitarity limits for generalised dipole form factors have been enumerated [146, Equations 22-26].

The form factor scale $\Lambda_{\text{FF}}$ can be regarded as a regularisation scale. It is related to (but not neces-

\(^8\)i.e. assuming a step function form factor operating at 2 TeV, the $\lambda^\gamma$ coupling conserves unitarity for $\lambda^\gamma < 0.99$ [144, Equation 23].
sarilly identical to) the energy scale at which new physics becomes important in the weak boson sector.

5.42 Impact of form factor on $\hat{s}$ dependent distributions

The impact of the form factor parametrisation on $\hat{s}$ dependent distributions is illustrated in Figure 31 where the reconstructed $M_{\text{inv}}(WZ)$ and $p_T(Z)$ spectra are plotted for LHC $W^+Z$ production with leptonic decays at $O(\alpha_s)$. The Standard Model expectation is compared to scenarios with a modest $\lambda_0^Z = 0.05$ coupling for various generalised dipole form factor parametrisations.

For the region of low invariant mass where $\sqrt{\hat{s}} \ll \Lambda_{\text{pp}}$, the form factors remain essentially constant and distributions with the same bare coupling agree well. As the form factor scale $\Lambda_{\text{pp}}$ is approached, the distributions begin to be pushed back to the SM expectation (visible at about $M_{\text{inv}}(WZ) = 500$ GeV for the $\Lambda_{\text{pp}}=2$ TeV case). For $\sqrt{\hat{s}} > \Lambda_{\text{pp}}$ the distribution returns to the SM expectation. The exponent of the form factor $\eta$ dictates how fast the “pushing” occurs as $\Lambda_{\text{pp}}$ is approached. Thus distributions sensitive to the $ZV\gamma$ vertex (for which $n = 3$ or 4 is the usual choice) exhibit a more pronounced form factor behaviour than distributions sensitive to the $WWV$ vertex (for which $n = 2$ is usual).

Since distributions are constrained to the SM expectation at invariant masses above the form factor scale, great care should be taken when fitting to a form factor parametrised model in a region with data where $\sqrt{\hat{s}} \geq \Lambda_{\text{pp}}$. Effectively, since the anomalous couplings are constrained near zero above $\Lambda_{\text{pp}}$ by the parametrisation model, there are no free parameters for the fit in this $\hat{s}$ region. For the case of observable non-zero anomalous couplings, an analysis assuming a parametrisation of the form factor with fixed $\Lambda_{\text{pp}}$ smaller than that provided by nature but within the $\hat{s}$ accessible by the machine would overestimate the

---

9Reconstructing $M_{\text{inv}}(WZ)$ requires knowledge of the neutrino longitudinal momentum which is obtained up to a two-fold ambiguity using the $W^+$ mass constraint. Each solution is given half weight in the $M_{\text{inv}}(WZ)$ spectrum.
The dependence of anomalous coupling limits on the form factor scale $\Lambda_{FF}$ is illustrated in Figure 32 where the 95% confidence limits for $W W \gamma$ vertex anomalous $\lambda^0_{ij}$, $\Delta\kappa^0_{ij}$ couplings in $W \gamma$ production with $W \rightarrow e\nu, \mu\nu$ are presented as a function of $\Lambda_{FF}$ for a dipole form factor with $n = 2$. The limits are for illustrative purposes only and have been derived at NLO generator level using a binned maximum likelihood fit to the $p_T(\gamma)$ distribution. No detector simulation has been applied and the specific choice of cuts are unimportant.

The unitarity limit curve is superimposed. The region above this is non-physical (violates unitarity). The curve is independent of experiment and analysis but depends on the form factor parametrisation. It goes asymptotically to zero for large $\Lambda_{FF}$ indicating TGC couplings are restricted to SM values at extreme energies.

Simulated experimental limits for the Tevatron (2 TeV $p\bar{p}$ collisions, $\mathcal{L} = 100$ pb$^{-1}$) and the LHC (14 TeV $pp$ collisions, $\mathcal{L} = 300$ fb$^{-1}$) are presented. The limits depend on the analysis and machine parameters. The restricted $\hat{s}$ accessible by the machines result in an asymptotic behaviour wherein an optimal limit for anomalous couplings is reached. We refer to the scale at which this occurs as $\Lambda_{\text{machine}}$. 

---

Fig. 32: Limits for $W W \gamma$ vertex anomalous couplings at the 95% confidence level as a function of $\Lambda_{FF}$ for a $n = 2$ dipole form factor parametrisation are presented. The limits are derived at NLO generator level for the $W \gamma \rightarrow e\nu\gamma, \mu\nu\gamma$ channel using a binned maximum likelihood fit to the $p_T(\gamma)$ distribution. The limits are for illustrative purposes only. Further details are provided in the text.
A measurement with this scale reflects the maximal discovery potential for anomalous couplings for a
given machine (since the full spectra in $s$ contributes to the limit). It occurs at about 2 TeV for the
Tevatron and about 5-10 TeV for the LHC for $\gamma^\gamma$, $\Delta \epsilon^\gamma$ and lies below the unitarity limit in both cases.
The experimental limits are not sensitive to changes in $\Lambda_{\text{FF}}$ for $\Lambda_{\text{FF}} \geq \Lambda_{\text{machine}}$. Indeed, in this region the
distributions behave exactly as if the form factors were constants $\Lambda \equiv \Lambda_0$. There is no contradiction with
unitarity in approximating them as such, provided we consider sufficiently small anomalous couplings so as to
remain far from the unitary limit at the energy regimes accessible by the machines. This is consistent
with the basic assumption ($\Lambda \gg \sqrt{s}$) which allows for the effective Lagrangian parametrisation of the
TGC vertex keeping only the lowest dimensions: it is sufficient to assume the form factor behaviour
commences above the observable scale so as to regulate the distributions before the unitarity limit.

There is also a region on the extreme left side of the plots in Figure 32 (although not indicated)
which is excluded by direct experimental searches. This is the region where physics is believed to be
well described by the SM.

Experimentally it is desirable to report confidence limits as a function of $\Lambda_{\text{FF}}$. A result using
$\Lambda_{\text{FF}} = \Lambda_{\text{machine}}$ should be included (so long as $\Lambda_{\text{machine}}$ lies below the unitarity limit) as it is motivated
by machine parameters and provides a reasonable point of reference for comparisons between different
experiments. Other scales (particularly those of theoretical interest) should not be neglected\(^{10}\).

### 5.44 Measuring form factors

For a machine of sufficient luminosity such as the LHC, it is possible to measure the energy dependence
of anomalous couplings\(^{11}\) by grouping the data into bins of invariant mass and extracting constant anomalous couplings within these restricted domains. Such a measurement does not carry any assumptions about the form factor (until a fit to a given parametrisation is performed). It is a viable method for measuring form factors, but due to the restricted number of events in each bin, will not produce competitive limits. The method is best employed in the case where non-zero anomalous couplings have been observed.

The method is illustrated in Figure 33 for the case of the $W\gamma$ channel with $W \rightarrow e\nu_e, \mu\nu_\mu$ assuming nature provides an anomalous $\lambda_0^\gamma = 0.025$ coupling described by an $n = 2$ dipole form factor with $\Lambda_{\text{FF}} = 2$ TeV. Three years of high luminosity (300 fb\(^{-1}\)) LHC events generated at NLO are binned according to the reconstructed $M_{\text{inv}}(W\gamma)$. The corresponding points derived using the generated (unobservable) $M_{\text{inv}}(W\gamma)$ are superimposed for comparison. Bin widths (denoted by arrows along the x-axis) are chosen so as to ensure sufficient data in each $M_{\text{inv}}(W\gamma)$ domain. A measurement of the anomalous coupling (assumed constant) is performed within each domain using a binned maximum likelihood fit to the $p_T(\gamma)$ distribution. No detector simulation has been applied and the specific choice of cuts is unimportant for this illustration. The results of the likelihood fits are plotted as a function of $M_{\text{inv}}(W\gamma)$ and a fit to an $n = 2$ dipole form factor is performed. With this simple illustration, the bare coupling and form factor scale are reconstructed as $\lambda_0^\gamma = 0.029$ and $\Lambda_{\text{FF}} = 1.67$ TeV. Sensitivity to the anomalous coupling increases in the larger invariant mass domains, reflecting the $s$ growth of the $\lambda_0^\gamma$ coupling (indeed the measurement in the first bin is consistent with zero). Systematic effects related to the fit method (such as the non-uniform distribution of events within the bins) have not been accounted for in this illustration.

### 5.5 Partonic simulation tools for di-boson production

Several Monte Carlo programs for hadronic di-boson event simulation are in common use. General
purpose programs such as \textsc{Pythia} \cite{123} evaluate the matrix element at leading order (LO) with no spin
correlations for boson decay products. Limited or no anomalous couplings are included. In the past

\(^{10}\)It should be noted that particularly for small choices of $\Lambda_{\text{FF}}$, a change in the analysis strategy may be necessary to increase sensitivity to the relevant regions of $s$.

\(^{11}\)The suggestion of making such a measurement is not new \cite{130} but has received little attention in the literature.
The $\lambda^7$ form factor is extracted in restricted invariant mass domains for $300 \, \text{fb}^{-1}$ of LHC data in the $W\gamma$ channel with $W \rightarrow \nu\ell, \mu\nu$, assuming nature provides an anomalous $\lambda^7_{\text{phys}} = 0.025$ coupling described by an $n = 2$ dipole form factor with $\Lambda_{\text{FF}} = 2 \, \text{TeV}$. A fit to an $n = 2$ dipole form factor is performed to reconstruct the bare coupling and form factor scale. Arrows along the $x$-axis denote bin widths. Further details are provided in the text.

decade, programs have been implemented to calculate di-boson production with leptonic decays to next-to-leading order (NLO) in QCD. The diagrams contributing to $O(\alpha_s)$ are: the squared Born (LO) graphs, the interference of the Born with the virtual one-loop graphs, and the squared real emission graphs.

The NLO generators by Baur, Han, and Ohnemus [32, 31, 30, 33] (BHO) have been available for several years. They employ the phase space slicing method [147] and the calculation is performed in the narrow width approximation for the leptonically decaying gauge-bosons. Non-standard TGC couplings are included. Spin correlations in the leptonic decays are included everywhere except in the virtual contribution. The authors expect a negligible overall effect from neglecting the spin correlations in the virtual corrections as compared to the uncertainty from parton distribution functions and the choice of factorisation scale. More recently Dixon, Kunszt, and Signer [19] (DKS) have implemented a program with full lepton decay spin correlations (helicity amplitudes are presented in [17]). The subtraction method [128, 149] is employed in the narrow width approximation including non-standard TGC couplings. A third Monte Carlo program, MCFM, by Campbell and Ellis [18] exists. It does not assume the narrow width approximation and includes singly resonant diagrams but does not allow for non-standard TGC couplings. The effects of these improvements in MCFM are largest in off-resonant regions - such as near di-boson production thresholds. The regions are of importance to studies of SM backgrounds to new physics but contribute negligibly to the cross section in TGC studies for typical choices of kinematic cuts [30].

A common feature of the NLO generators is the inability to produce unweighted events. Both the phase space slicing and subtraction methods produce events for which the weight may be either positive or negative - thus it is only the integrated cross section over a region of phase space (i.e. histogram bin) which is physical. This makes traditional Monte Carlo techniques for unweighting events (such as hit-and-miss) difficult to apply, and we are aware of no universally satisfactory technique for producing unweighted events using the NLO generators. Computationally this can render analyses very slow, since a large fraction of CPU time can be spent processing events with near-vanishing cross sections.

One method involves reweighting events from a LO generator using a “look-up table” constructed at NLO.
5.51 Comparison of NLO particle level generators

In this section, we present a comparison of the predictions from the BHO and DKS generators, for which no published consistency check exists, restricting ourselves to $W^+Z$ and $WW$ production for simplicity. The DKS and MCFM packages have been found to be in good agreement [19].

The comparison is performed at LHC energy (14 TeV $pp$ collisions) using CTEQ4M [67] structure functions. Input parameters are taken as $\alpha_{EM} = \frac{1}{127}$, $\sin^2 \theta_W = 0.23$, $\alpha_s(M_Z) = 0.116$, $M_W = 80.396$ GeV, $M_Z = 91.187$ GeV, factorisation scale $Q^2 = M_W^2$, and Cabibbo angle $\cos \theta_C = 0.975$ with no 3rd generation mixing. Branching ratios are taken as $BR(Z\rightarrow l^+l^-) = 3.36\%$, $BR(W^\pm\rightarrow l^+\nu) = 10.8\%$. The $b$ quark contribution to parton distributions has been taken as zero ($b\bar{b} \rightarrow W^+W^-$ contributes $O(2\%)$ at LHC [19]). Kinematic cuts motivated by TGC analyses are chosen. The transverse momentum of all leptons must exceed 25 GeV and the rapidity of all leptons must be less than 3. Missing transverse momentum must be greater than 25 GeV. A jet is defined when the transverse momentum of a parton exceeds 30 GeV in the pseudorapidity interval $|\eta| < 3$.

For $W^+Z$ production, the transverse momentum distribution of the $Z$ boson $p_T(Z)$, the distribution of rapidity separation between the $W^+$ decay lepton and the $Z$ boson $y(l) - y(Z)$, and total cross section are compared at LO, inclusive NLO, and NLO with a jet veto. Branching ratios to $e, \mu$-type leptons are applied. For $WW$ production, the transverse momentum distribution of the lepton pair from the $W^\pm$ decays $|p_T(e^-) + p_T(e^+)|$, the distribution of rapidity separation between the $W$ decay leptons $y(e^-) - y(e^+)$, the angle between the $W$ decay leptons in the transverse plane $\cos \Phi(e^-, e^+)$, and the total cross section are compared at LO, inclusive NLO, and NLO with a jet veto. Branching ratios to one lepton flavour are applied.

The cross section results are presented in Table 21 and the distributions in Figure 34. Consistency between generators is at the 1% level for $WZ$ production and 3-4% level for $WW$ production. Qualitative agreement is observed in the distribution shapes.

![Fig. 34: Distributions for $W^+Z$ production (left) and $WW$ production (right) from the Baur/Han/Ohnemus and Dixon/Kunszt/Signer generators are superimposed at Born level, inclusive NLO, and NLO with a jet veto (defined as $p_T$(jet) > 30 GeV, $|\eta$(jet) | < 3).](image)

The choice of parton distribution function has an $O(5\%)$ effect on the cross section.
Table 21: $W^+ Z$ and $WW$ cross section predictions are tabulated for the BHO and DKS generators at LO, inclusive NLO, and NLO with a jet veto. A jet is defined for $p_T$ (jet) $>$ 30 GeV, $\eta$(jet) $<$ 3. Statistical precision is $O(1 \text{ fb})$.

### W$^+ Z$ Production

<table>
<thead>
<tr>
<th></th>
<th>Baur/Han/Ohnemus</th>
<th>Dixon/Kunszt/Signer</th>
<th>% diff.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Model</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{NLO inclusive}}$</td>
<td>127.9 fb</td>
<td>129.8 fb</td>
<td>1.4%</td>
</tr>
<tr>
<td>$\sigma_{\text{NLO 0jet}}$</td>
<td>74.7 fb</td>
<td>75.1 fb</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\sigma_{\text{Born}}$</td>
<td>70.5 fb</td>
<td>70.9 fb</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\Delta g_Z^1 = 0, \Delta \kappa_Z = 0.5, \lambda_Z = 0.1$ ((\Lambda = 2 \text{ TeV}))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{NLO inclusive}}$</td>
<td>198.5 fb</td>
<td>199.9 fb</td>
<td>0.7%</td>
</tr>
<tr>
<td>$\sigma_{\text{NLO 0jet}}$</td>
<td>107.5 fb</td>
<td>106.8 fb</td>
<td>0.7%</td>
</tr>
<tr>
<td>$\sigma_{\text{Born}}$</td>
<td>119.7 fb</td>
<td>119.9 fb</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

### WW Production

<table>
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<th></th>
<th>Baur/Han/Ohnemus</th>
<th>Dixon/Kunszt/Signer</th>
<th>% diff.</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{NLO inclusive}}$</td>
<td>500.5 fb</td>
<td>483.2 fb</td>
<td>3.5%</td>
</tr>
<tr>
<td>$\sigma_{\text{NLO 0jet}}$</td>
<td>321.0 fb</td>
<td>309.6 fb</td>
<td>3.6%</td>
</tr>
<tr>
<td>$\sigma_{\text{Born}}$</td>
<td>294.0 fb</td>
<td>295.5 fb</td>
<td>0.5%</td>
</tr>
<tr>
<td>$\Delta g_Z^1 = 0.25, \Delta \kappa_Z = \Delta \kappa_\gamma = 0.1, \lambda_Z = \lambda_\gamma = 0.1$ ((\Lambda = 2 \text{ TeV}))</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_{\text{NLO inclusive}}$</td>
<td>594.2 fb</td>
<td>575.0 fb</td>
<td>3.3%</td>
</tr>
<tr>
<td>$\sigma_{\text{NLO 0jet}}$</td>
<td>363.0 fb</td>
<td>349.6 fb</td>
<td>3.8%</td>
</tr>
<tr>
<td>$\sigma_{\text{Born}}$</td>
<td>351.6 fb</td>
<td>353.7 fb</td>
<td>0.6%</td>
</tr>
</tbody>
</table>
5.52 Effects of NLO corrections

NLO corrections in hadronic di-boson production are large at LHC energies, particularly in the region of high transverse momentum and small rapidity separation (see Figure 34) which is the same region of maximum sensitivity to anomalous TGCs. The corrections can amount to more than an order of magnitude. The high quark-gluon luminosity at the LHC and a logarithmic enhancement at high transverse momentum in the $qg$ and $gg$ real emissions subprocesses are primarily responsible [32, 31, 30]. In the channels which exhibit radiation zero behaviour (i.e. $W\gamma$ and $WZ$), the Born contribution is suppressed and NLO corrections are even larger [32, 31]. Since the $O(\alpha_s)$ subprocesses responsible for the enhancement at large transverse momentum do not involve TGCs, the overall effect of NLO corrections is a spoiling of sensitivity to anomalous TGCs.

**Jet veto**  Distributions obtained by vetoing hard jets in the central rapidity region for one possible choice of jet definition ($p_T(jet) > 30$ GeV, $|\eta(jet)| < 3$) are shown in Figure 34. The jet veto is effective in recovering the qualitative shape of the LO distributions including the approximate radiation zero in $WZ$ production (Figure 34, bottom left). The jet veto serves to recover anomalous TGC sensitivity which is otherwise lost when introducing NLO corrections. A 10-30% improvement in anomalous TGC coupling sensitivity limits in $WZ$ production can be achieved [31] when a jet veto is applied as compared to the inclusive NLO case. These limits are often close to those obtained at LO. In general results derived at LO can be considered approximate zero jet results and their conclusions remain interesting. A jet veto also reduces the scale dependence of NLO results [32, 31, 30, 19].

5.6 Determination of TGCs

At the LHC the measurement of TGCs will benefit from both the large statistics and the high centre-of-mass energy. The large available statistics will allow the use of multi-dimensional distributions to increase the sensitivity to the TGCs.

This section discusses the experimental observables sensitive to TGCs and describes the analysis methods employed to measure the TGCs.

5.61 Experimental observables

The experimental sensitivity to the TGCs comes from the increase of the production cross section and the modification of differential distributions with non-standard TGCs. The sensitivity is enhanced at high centre-of-mass energies of the hard scattering process, more significantly for $\lambda$-type TGCs than for $\kappa$-type TGCs in the case of $W\gamma$ and $WZ$ production. As an example, the increase in the number of events with large di-boson invariant masses is a clear signature of non-standard TGCs as illustrated in Figure 35, where the invariant mass of the hard scattering is shown for $W\gamma$ events, simulated with a parametric description of the ATLAS detector, for the Standard Model and non-standard TGCs. A form factor of 10 TeV was used.

For the event generation employing non-standard values of the TGCs, leading order (LO) [150] as well as next to leading order (NLO) [32, 33] calculations have been used (see Section 5.5). Limits on the TGCs can be obtained from event counting in the high invariant mass region. The disadvantage of such an approach alone is that the behaviour of the cross section as function of the TGCs makes it difficult to disentangle the contributions from different TGCs and even their sign (with respect to SM). It is therefore advantageous to combine it with information from angular distributions of the bosons and possibly their decay angles; this improves the sensitivity and improves the separation of contributions from different non-standard TGCs.

In general it is possible experimentally to reconstruct up to four (six) angular variables in the di-boson rest-frame describing an $W\gamma$ or $Z\gamma$ ($WZ$) event:
Fig. 35: The distribution of the invariant mass of the $W\gamma$ system from $pp \rightarrow W\gamma$. Standard Model data (shaded histogram) and a non-standard value of 0.01 for $\chi_\gamma$ (white histogram) are shown. Both charges of $W$ were generated using a parameterised Monte Carlo and summed. The number of events corresponds to an integrated luminosity of 30 fb$^{-1}$.

- Boson production angles, $\Theta$ and $\Phi$, of the di-boson system with respect to the beam-axis in the di-boson rest-frame.
- Decay angles of bosons, $\theta_1^{\gamma}(2)$ and $\phi_1^{\gamma}(2)$, in the rest-frame of the decaying bosons.

The azimuthal boson production angle, $\Phi$, has no sensitivity to the TGCs. In case of $W\gamma/WZ$, $\Theta$ is the most sensitive kinematical variable. The enhanced sensitivity to the TGCs in $WV$ production is due to the vanishing of helicity amplitudes in the Standard Model prediction at $\cos \Theta \sim 1/3$, affecting the small $|\eta|$ region [150]. Non-standard TGCs may partially eliminate the radiation zero, although the zero radiation prediction is less significant when including NLO corrections [32]. In $Z\gamma$ production, no radiation amplitude zero is present.

In contrast, the sensitivity to the TGCs from the decay angles is weak; the decay angles primarily serve as projectors of different helicity components, enhancing the sensitivity of other variables.

Fig. 36: Differential cross section for $Z\gamma$ production versus $p_T^\gamma$ for Standard Model (solid line) and two different non-standard couplings (dashed and dotted lines) at LHC.

In the study presented here, several experimentally derived observables and combinations thereof have been studied to assess the possible sensitivity to the TGCs. For both ($W\gamma$, $WZ$) and ($Z\gamma$, $ZZ$) events the observables are very similar; for $WZ$, the $Z$ takes the role of the $\gamma$. The actual behaviour of
the observables as function of the couplings and the energy is different between the processes, due to the different masses of the involved bosons.

One observable, the transverse momentum, $p_T$, of the $\gamma$ or $Z$ (depending on the di-boson process), which has traditionally been used at hadron colliders, has sensitivity from a combination of high mass event counting and the $\Theta$ angular distribution. Figure 36 shows the enhancement of di-boson production cross section for large values of the photon transverse momentum in presence of non-standard couplings.

The distribution of $p_T^{\gamma, Z}$ assuming an integrated luminosity of 30 fb$^{-1}$ is shown in Figure 37 for $W\gamma$ and $WZ$ events, simulated with a parametric detector simulation program, for the Standard Model and non-standard TGCs. The enhancement for non-standard TGCs at high $p_T^{\gamma, Z}$ is clearly visible and, furthermore, the qualitative behaviour is the same for different TGCs.

For the statistics expected at the LHC, even after 3 years running at low luminosity, one may enhance the experimental sensitivity further by separating the different types of information in multi-dimensional distributions. For $W\gamma$ and $WZ$ di-boson production, two sets of variables have been studied (and the equivalent set for $WZ$): $(m_{W\gamma}, |\eta^*_\gamma|)$, and $(p_T^\gamma, \theta^*)$, where $|\eta^*_\gamma|$ is the rapidity of $\gamma$ with respect to the beam direction in the $W\gamma$ system (equivalent to $\Theta$), and $\theta^*$ is the polar decay angle of the charged lepton in the $W$ rest-frame. Both sets consist of one variable sensitive to the energy behaviour and one sensitive to the angular information. For $|\eta^*_\gamma|$ and $\theta^*$, a complete reconstruction of the $W$ is necessary. The momentum of the $W$ can be reconstructed by using the $W$ mass as a constraint and assuming that the missing transverse energy is carried away by the neutrino. This leads to a two-fold ambiguity in the reconstruction. Alternatively, $|\eta^*_\gamma|$, may be approximated by the rapidity difference between the lepton from the $W$ and the $\gamma$. Distributions of $|\eta^*_\gamma|$ and $\theta^*$ are shown in Figure 38, for both the standard model expectation and different non-standard TGCs. The high sensitivity to the TGCs from $|\eta^*_\gamma|$ is due to the characteristic “zero radiation” gap. In contrast, the sensitivity to the TGCs from the decay polar angle, $\theta^*$, is weak.

5.62 Analysis techniques for TGC determination

Depending on the available statistics and the dimensionality of the experimental distributions, different extraction techniques can be used in the determination of the TGCs.

One approach employed in this study determines the couplings by a binned maximum-likelihood fit to distributions of the observables, combined with the total cross section information. The likelihood
function is constructed by comparing the fitted histogram with a reference histogram using Poisson probabilities. The reference distributions can be obtained for different values of the couplings by reweighting Monte Carlo events at generator level or equivalently using several Monte Carlo event samples generated for different values of the TGCs.

Although the expected number of events at the LHC will allow binning in two dimensions, a general multidimensional binned fit using all the TGC sensitive information will not be possible. In the latter case, an unbinned maximum likelihood fit to the observed information can be used, where the probability distribution functions can be constructed by Monte Carlo techniques. In the case of many dimensions, this approach can be time-consuming, but it may be advantageously combined with the reweighting technique. The information from the absolute prediction of the cross section can be included by the so-called “extended maximum likelihood” method [151].

5.7 Sensitivities at LHC

Sensitivity limits have been derived for the triple gauge-couplings $WW\gamma$ (ATLAS, CMS), $WWZ$ (ATLAS) and $ZZ\gamma$ (CMS). The analysis techniques used by ATLAS and CMS are described in Section 5.6. The ATLAS studies assume an integrated luminosity of $\int L \, dt = 30 \, fb^{-1}$, corresponding to three years of LHC low luminosity operation. CMS assumes $100 \, fb^{-1}$, which is the expectation for one year of LHC high luminosity running.

CMS has performed its studies for a range of different form factor scales $\Lambda_{FF}$, as motivated in Section 5.4. The plots in Figure 39 show the expected 95% CL limits on the anomalous $WW\gamma$ and $ZZ\gamma$ coupling parameters together with the corresponding unitarity limits. Only the displayed coupling is considered to deviate from the Standard Model. The points where the experimental curves turn asymptotic with respect to $\Lambda_{FF}$ - or are crossed by the unitarity limit - give an indication on the range of form factor scales accessible by the experiments. While the current Tevatron measurements probe the triple gauge-couplings up to form factors of $\Lambda_{FF} = 0.75 \, TeV$ and around $2 \, TeV$ for $ZZ\gamma$ and $(WW\gamma, WWZ)$, respectively [16], the LHC experiments will be able to study far smaller structures with scales up to $10 \, TeV$, assuming an integrated luminosity of $100 \, fb^{-1}$.

Multi-dimensional fits where several couplings are allowed to vary have also been performed [152]. Here, the sensitivity limits extracted from the log likelihood curves form an ellipse for a particular confidence level. Figure 40 shows the typical $WW\gamma$ sensitivity contours in the two-dimensional CP-conserving $(\kappa \times \lambda)$ coupling space for a form factor scale of $10 \, TeV$. 

Fig. 38: Distribution of $|\eta_\gamma^z|$ (left) and $\theta^*$ (right) from $W\gamma$ and $WZ$ events, respectively, for an integrated luminosity of $30 \, fb^{-1}$. Distributions are shown for the Standard Model (shaded histograms) and for non-standard values (white histograms) $\Delta \kappa_{\gamma} = 0.2$ (left) and $\Delta \kappa_{Z} = 0.2$ (right).
Fig. 39: Sensitivity limits on the $WW\gamma$ (top) and $ZZ\gamma$ (bottom) coupling parameters from a two-dimensional likelihood fit as a function of the form factor scale $\Lambda_{FF}$. 
Table 22 summarises the sensitivity limits obtained by ATLAS and CMS as reported in [53, 152]. In addition, ATLAS has performed a fit using the complete generator level phase space information [53]. The results for this ideal case show that, as the high energy tails of the \( p_T \) distributions exhibit a very strong sensitivity to the \( \lambda \)-like anomalous couplings, the additional information does not improve the limits on this type of couplings considerably. However, the \( \kappa \)-type couplings may profit from a more sophisticated data analysis.

From the numbers in Table 22, we expect an improvement in sensitivity by up to two (four) orders of magnitude for anomalous \( WW\gamma/WWZ \) \( (ZZ\gamma) \) couplings, with respect to the current Tevatron limits. The strong increase in sensitivity is due to the pronounced high \( \hat{s} \) enhancement at the LHC, most prominently for \( ZZ\gamma \) (see Section 5.42). A smaller choice of the form factor scale would cut off this enhancement and diminish the sensitivity considerably, as shown in the lower plots in Figure 39.
5.8 Backgrounds to $W\gamma$

The $W\gamma$ signal has a very small cross section, compared to $W$+jet production for example, and can contain a significant amount of background. The dominant background to the $W\gamma$ signal is from $W$+jet production where the jet is misidentified as a photon, resulting in a fake signal. Radiative $W$ decay also contributes when the electron from the $W$ decay radiates a photon, and both $t\bar{t}\gamma$ and $b\bar{b}\gamma$ quark-gluon fusion processes can also produce a fake signal contributing to the background. $Z\gamma$ production and $W(\tau\nu)\gamma$ also make a small contribution to the backgrounds.

Previous studies [153, 154, 155, 156] have shown that the $W\gamma$ signal will be observable at the LHC provided that the backgrounds can be suppressed. All the backgrounds were generated with PYTHIA 5.7 [123] in conjunction with the CMSJET [157] fast detector simulation for the CMS experiment.

5.8.1 $W$+jet and $W \rightarrow l\nu\gamma$ backgrounds

The dominant background to the process $pp \rightarrow W(e\nu)\gamma$ arises from $W$+jet events where the jet decays electromagnetically and is reconstructed in the calorimeter as a photon. The probability for the jet to fluctuate into an isolated electromagnetic shower is small, but the large number of jets above 10 GeV in the $W$ sample guarantees that some jets will look identical to photons. Even if the jet is not misidentified as a photon, it is possible for a radiative decay of the $W$ to produce the same signature as the signal. If the lepton from the $W$ decay radiates a photon, an event signature of $\gamma, l, \nu$ may be observed. Cuts must therefore be applied to reduce this background.

$W$+jet  Figure 41 shows the $p_T(\gamma)$ spectrum for misidentified photon from the $W$+jet background and the real photon from the $W\gamma$ signal. A photon isolation cut has been applied to both data sets. A rejection power of nearly 7 can be obtained with an efficiency loss of less than 5%, by using an isolation area of $\Delta R = 0.25$ and a $p_T$ threshold of 2 GeV [158]. A greater rejection power with a much smaller efficiency loss is available at low luminosity. Therefore an event is selected if the photon meets the isolation criteria and if it is within $\eta = \pm 2.5$. The isolation cut clearly makes it possible to observe the signal, especially at high $p_T$, however a cut at $p_T(\gamma) = 100$ GeV further reduces the background. This would not harm the sensitivity to anomalous couplings greatly as the anomalies only manifest themselves at high $p_T$.

Radiative $W$  One method of reducing the background of radiative $W$ decays is to make a cut on the invariant mass of the $\gamma l\nu$ system. For the $W\gamma$ signal, $M(\gamma l\nu)$ is always larger than $M_W$ if finite $W$ width effects are ignored.

However, the $M(\gamma l\nu)$ cannot be determined unambiguously as the four-momentum of the neutrino is unknown: even if the transverse momentum is correctly determined from the missing momentum in the event, there is no measurement of the missing longitudinal momentum. Therefore the cluster transverse mass, or minimum invariant mass, may be used instead [159]. The transverse mass is independent of the longitudinal momenta of the parent particle and its decay products.

For $W \rightarrow \gamma l\nu$ the cluster transverse mass sharply peaks at $M_W$ [160] and drops rapidly above the $W$ mass. Thus $\gamma l\nu$ events originating from $W\gamma$ production and radiative $W$ decays can be distinguished if $M_T(\gamma l\nu)$ is cut slightly above $M_W$ [161]. Hence a cut at $M_T(\gamma l\nu) > 90$ GeV should take into account the finite width of the $W$ whilst not significantly affecting the signal.

The $W\gamma$ signal produces the lepton and photon almost back-to-back. Ensuring that they are well separated will further reduce the radiative $W$ background. This can be done using the quantity $\Delta R = \sqrt{(\Delta \phi^2 + \Delta \eta^2)}$. Leading order analysis of the signal and radiative background enabled a study of the optimum value of $\Delta R$ to use for separation. Typically a cut at $\Delta R > 0.5$ is used to ensure separation, but increasing the separation to $\Delta R > 0.7$ makes little difference to the signal whilst greatly reducing the background.
In order to suppress the radiative \( W \) background events, cuts of \( \Delta R(\gamma,l) > 0.7 \) and \( M_T(\gamma l \nu) > 90 \) GeV are used.

5.82 Quark-Gluon fusion background

Quark-gluon fusion is important at the LHC because the rate is extremely high. There are lots of available gluons in the proton at relatively high \( x \), and because the \( WW\gamma \) reaction is suppressed in some regions of phase space.

\( b\bar{b}\gamma \) At the LHC \( 10^{12} \) \( b\bar{b}\) events [162] are expected for a years running at high luminosity. Although the \( b\bar{b}\gamma \) events are not kinematically similar to the signal, the expected number of events is so large that the background will be a problem unless it is reduced by cuts.

The \( b\bar{b}\gamma \) background was generated using the processes: \( qq \rightarrow g\gamma \), and \( q\bar{q} \rightarrow Z\gamma \). Events were generated from \( p_T = 500 \) GeV with a cross section of 1.055 pb. This parton-level requirement was for computational efficiency as only the very highest \( p_T \) events contribute to the background. A cut on missing \( p_T \) can be made at 50 GeV in order to reduce the \( b\bar{b}\gamma \) background.

\( t\bar{t}\gamma \) Since the \( M_t > M_W + M_b \), \( t\bar{t} \) events represent an irreducible background to \( W\gamma \) pair production. \( t\bar{t}\gamma \) production is a copious source of high \( p_T \) photons in association with hard leptons and without cuts has a cross section, \( \sigma \sim 300 \) pb, of at least 3 orders of magnitude more than the \( W\gamma \) signal [163]. The subsequent decay of top quarks into a \( W \) boson and a \( b \) quark and also the \( W \) decay into a \( f\bar{f} \) pair provide the same event signature as the \( W\gamma \) signal. Therefore, due to the very large top quark production cross section at LHC energies, the process \( pp \rightarrow t\bar{t}\gamma \rightarrow W\gamma + X \) represents a potentially significant background.

Events were generated by the process \( qq \rightarrow g\gamma \) and looking for \( t\bar{t} \) production. This method is very inefficient, 4 million events were generated and 489 \( t\bar{t}\gamma \) events were produced, with 10 events passing
all of the cuts. The $t\bar{t}\gamma$ events were generated from $p_T = 500$ GeV (for the same reasons as $b\bar{b}$), with a cross section of 1.049 pb. The large cross section means that although only a few events pass the cuts, this background is a potential problem.

Studies for the SSC [164] showed that the background can be reduced to a manageable level by requiring the photon to be isolated from the hadrons in the event, and by imposing a jet veto (i.e. by considering the exclusive reaction $pp \rightarrow W\gamma + 0$ jets).

Since the top quark decays predominantly into a $Wb$ final state, $t\bar{t}\gamma$ events are characterised by a large hadronic activity which frequently results in one or several high-$p_T$ jets. If the second $W$ boson decays hadronically, up to four jets are possible. This observation suggests that the $t\bar{t}\gamma$ background may be suppressed by vetoing high-$p_T$ jets. Such a “zero jets” requirement has been demonstrated to be very useful in reducing the size of the NLO QCD corrections in $pp \rightarrow W\gamma + X$ at SSC energies [32]. If the second $W$ in the $t\bar{t}\gamma$ events decays hadronically, the number of jets in $pp \rightarrow t\bar{t}\gamma \rightarrow W\gamma + X$ is generally larger than for leptonic $W$ decays, and the jet veto is more efficient.

Unfortunately the jet veto also drastically reduces the number of signal events. Only 10% of the signal survives the jet veto cut alone and only 4% survive all the cuts and the jet veto. This suggests that an alternative method for reducing this background needs to be found for the LHC.

ATLAS [154] studied the possibility of exploiting the number of jets in the $t\bar{t}\gamma$ events by imposing a cut on the second jet in the event. The $W\gamma$ signal will not have a 2nd jet, or if it does, it is a misidentified jet and will be of very low $p_T$. The $t\bar{t}\gamma$ events will have up to four high $p_T$ jets in each event. By cutting all events where the $p_T$ of the second jet is greater that 25 GeV, the majority of the $t\bar{t}\gamma$ events will be eliminated without greatly affecting the signal.

5.83 $Z\gamma$ background

There is a small background to $e\nu\gamma$ that comes from $Z(ee)\gamma$ events in which one of the electrons gives rise to significant missing energy (generally by entering a gap in the detector). As CMS is hermetic and the crystals of the ECAL are off-pointing with respect to the interaction point, this background is very small. ATLAS [154] calculate this background to be $\sim 25$ times smaller than the signal before any cuts are imposed. Thus the $Z\gamma$ background is assumed to be negligible.

5.84 $W(\tau\nu)\gamma$ background

The final background to $pp \rightarrow W(e\nu, \mu\nu)\gamma$ is $pp \rightarrow W(\tau\nu)\gamma$ where the $\tau$ lepton decays into an electron or muon. The background is very small because the decay of the tau lepton results in electrons or muons with significantly reduced $p_T$ and the kinematical threshold for an electron is 25 GeV. Previous studies at Fermilab have shown this background to be negligible [165].

5.85 Summary of backgrounds

Table 23 shows a list of all the cuts proposed to reduce the backgrounds to the $W\gamma$ signal. Having chosen each cut to reduce an individual background, it is important to understand how each cut effects both the signal and the other backgrounds.

Table 24 shows the efficiency of the individual cuts on the signal and the backgrounds. The $W+\text{jet}$ and radiative $W$ backgrounds are treated together. See Figure 42.

5.86 Conclusion

The backgrounds to the $W\gamma$ signal have been studied and cuts have been made in order to reduce the backgrounds to at least an order of magnitude less than the signal for $p_T(\gamma) > 200$ GeV. The $W+\text{jet}$
### Table 23: Proposed cuts to reduce the backgrounds to the $W\gamma$ signal.

| Quantity | $|\eta(\gamma, l, jet)|$ | $p_T(\gamma)$ | $p_T(l)$ | $M_T(\gamma, l, \nu)$ | $\Delta R(\gamma, l)$ | $p_T(\nu)$ | 2nd jet |
|----------|-----------------|-------------|----------|----------------|----------------|----------|--------|
| Cut value | < 2.5 | > 100 | > 25 | > 90 | > 0.7 | > 50 | < 25 |

### Table 24: Efficiency of individual cuts on the signal and backgrounds, errors are statistical.

<table>
<thead>
<tr>
<th>Cut</th>
<th>Signal (%)</th>
<th>Background (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$W+\text{jet/Rad.}$</td>
<td>$t\bar{t}\gamma$</td>
</tr>
<tr>
<td>$p_T(\gamma)$</td>
<td>67±0.49</td>
<td>0.06±0.008</td>
</tr>
<tr>
<td>$p_T(l)$</td>
<td>84±0.52</td>
<td>62±0.25</td>
</tr>
<tr>
<td>$M_T(\gamma, l, \nu)$</td>
<td>85±0.52</td>
<td>19±0.14</td>
</tr>
<tr>
<td>$\Delta R(\gamma, l)$</td>
<td>95±0.55</td>
<td>94±0.3</td>
</tr>
<tr>
<td>$p_T(\nu)$</td>
<td>86±0.53</td>
<td>60±0.25</td>
</tr>
<tr>
<td>2nd jet</td>
<td>89±0.54</td>
<td>42±0.2</td>
</tr>
<tr>
<td>All Cuts</td>
<td>55±0.42</td>
<td>0.033±0.018</td>
</tr>
</tbody>
</table>

and radiative $W$ backgrounds have been well studied and understood and the cuts made reduce these significantly. The quark-gluon fusion backgrounds are not so well understood in this work since a less than optimal generator for $t\bar{t}\gamma$ was used. However, the cuts studied for this channel work well for the low statistic samples presented here. Further study of this background would be interesting.

Backgrounds to $WZ$ production have been studied briefly and are similar, within statistical errors, to those in the $W\gamma$ channel presented here.

### 6. Vector-Boson Fusion and Scattering

#### 6.1 Searching for $VV \to H \to \tau\tau$

##### 6.11 Introduction

The search for the Higgs boson and, hence, for the origin of electroweak symmetry breaking and fermion mass generation, remains one of the premier tasks of present and future high energy physics experiments. Fits to precision electroweak (EW) data have for some time suggested a relatively small Higgs boson mass, of order 100 GeV [166, 167], hence we have studied an intermediate-mass Higgs, with mass in the $\sim 110 - 150$ GeV range, beyond the reach of LEP at CERN and perhaps of the Fermilab Tevatron. Observation of the $H \to \tau\tau$ decay channel in weak boson fusion events at the Large Hadron Collider (LHC) is quite promising, both in the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM). This channel has lower QCD backgrounds compared to the dominant $H \to b\bar{b}$ mode, thus offering the best prospects for a direct measurement of a $Hf\bar{f}$ coupling.

At the LHC, despite the fact that the cross section for Higgs production by weak-boson fusion is significantly lower than that from gluon fusion (by almost one order of magnitude), it has the advantage

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14Section coordinators: Z. Kunszt, R. Mazini, D. Rainwater
of additional information in the event other than the decay products’ transverse momentum and their invariant mass resonance: namely, the observable quark jets. Thus one can exploit techniques like forward jet tagging [168, 169, 170, 171, 172, 173, 174, 175, 176] to reduce the backgrounds. Another advantage is the different colour structure of the signal vs the background. Additional soft jet activity (minijets) in the central region, which occurs much more frequently for the colour-exchange processes of the QCD backgrounds [177, 178], are suppressed via a central jet veto.

We have performed first analyses of intermediate-mass SM $H \rightarrow \tau \tau$ and of the main physics and reducible backgrounds at the LHC, considering separately the decay modes $\tau \tau \rightarrow h^{\pm}l^{\mp}p_T, e^{\pm}\mu^{\mp}p_T$. These modes demonstrate the feasibility of Higgs boson detection in this channel with modest luminosity [179, 180]. We demonstrated that forward jet tagging, $\tau$ identification and reconstruction criteria alone yield a signal-to-background ($S/B$) ratio of approximately 1/1 or better. Additional large background suppression factors can be obtained with the minijet veto, achieving final $S/B$ ratios as good as 6/1, depending on the Higgs mass.

In the MSSM, strategies to identify the structure of the Higgs sector are much less clear. For large $\tan \beta$, the light neutral Higgs bosons may couple much more strongly to the $T_3 = -1/2$ members of the weak isospin doublets than its SM analogue. As a result, the total width can increase significantly compared to a SM Higgs of the same mass. This comes at the expense of the branching ratio $BR(h \rightarrow \gamma \gamma)$, the cleanest Higgs discovery mode, possibly rendering it unobservable over much of MSSM parameter space and forcing consideration of other observational channels. Instead, since $BR(h \rightarrow \tau \tau)$ is enhanced slightly, we have examined the $\tau$ mode as an alternative [180, 181].

### 6.12 Simulations of signal and backgrounds

The analyses used full tree-level matrix elements for the weak boson fusion Higgs signal and the various backgrounds. Extra minijet activity was simulated by adding the emission of one extra parton to the basic signal and background processes, with the soft singularities regulated via a truncated shower approximation (TSA) [182, 183].
We simulated $pp$ collisions at the LHC, $\sqrt{s} = 14$ TeV. For all QCD effects, the running of the strong-coupling constant was evaluated at one-loop order, with $\alpha_s(M_Z) = 0.118$. We employed CTEQ4L parton distribution functions [67] throughout. The factorisation scale was chosen as $\mu_f = \min(p_T)$ of the defined jets, and the renormalisation scale $\mu_r$ was fixed by $(\alpha_s)^n = \prod_{i=1}^n \alpha_s(p_T)$. Detector effects were considered by including Gaussian smearing for partons and leptons according to ATLAS expectations [153, 125].

At lowest order, the signal is described by two single-Feynman-diagram processes, $qq \rightarrow qq(WW; ZZ) \rightarrow qqH$, i.e. $WW$ and $ZZ$ fusion where the weak bosons are emitted from the incoming quarks [184]. From a previous study of $H \rightarrow \gamma\gamma$ decays in weak boson fusion [185], we know several features of the signal which we could exploit directly here: the centrally produced Higgs boson tends to yield central decay products (in this case $\tau^+\tau^-$), and the two quarks enter the detector at large rapidity compared to the $\tau$'s and with transverse momenta in the 20-80 GeV range, thus leading to two observable forward tagging jets.

We considered separately the cases of one $\tau$ decaying leptonically ($e, \mu$) and the other decaying hadronically (with a combined branching fraction of 45%), and both decaying leptonically but with different flavour ($e\mu$ or $\mu e$, with a combined branching fraction of 6.3%). Our analyses critically employed transverse momentum cuts on the charged $\tau$-decay products and, hence, some care was taken to ensure realistic momentum distributions. Because of its small mass, we simulated $\tau$ decays in the collinear and narrow-width approximations and with decay distributions to $\pi, \rho, a_1$ [186], adding the various hadronic decay modes according to their branching ratios. We took into account the anti-correlation of the $\tau^\pm$ polarisations in the decay of the Higgs.

**Lepton-hadron mode** Positive identification of the hadronic $\tau^\pm \rightarrow h^\pm X$ decay requires severe cuts on the charged hadron isolation. We based our simulations on the possible strategies analysed by Cavallo et al. [187]. Considering hadronic jets of $E_T > 40$ GeV in the ATLAS detector, they found non-tau rejection factors of 400 or more while true hadronic $\tau$ decays are retained with an identification efficiency of 26%.

Given the $H$ decay signature, the main physics background to the $\tau^+\tau^-jj$ events of the signal arises from real emission QCD corrections to the Drell-Yan process $q\bar{q} \rightarrow (Z, \gamma) \rightarrow \tau^+\tau^-$, dominated by $t$-channel gluon exchange. All interference effects between virtual photon and $Z$-exchange were included, as was the correlation of $\tau^\pm$ polarisations. The $Z$ component dominates, so we call these processes collectively the “QCD $Zjj$” background.

An additional physics “EW $Zjj$” background arises from $Z$ and $\gamma$ bremsstrahlung in (anti)quark scattering via $t$-channel electroweak boson exchange, with subsequent decay $Z, \gamma \rightarrow \tau^+\tau^-$. Naively, this EW background may be thought of as suppressed compared to the analogous QCD process. However, the EW background includes electroweak boson fusion, $VV \rightarrow \tau^+\tau^-$, which has a momentum and colour structure identical to the signal and thus cannot easily be suppressed via cuts.

Finally, we considered reducible backgrounds, i.e. any event that can mimic the $Hjj$ signature of a hard, isolated lepton and missing $p_T$, a hard, narrow $\tau$-like jet, and two forward tagging jets. Thus we examined $W + jets$, where the $W$ decays leptonically ($e, \mu$) and one jet fakes a hadronic $\tau$, and $b\bar{b} + jets$, where one $b$ decays leptonically and either a light quark or $b$ jet fakes a hadronic $\tau$. We neglected other sources like $t\bar{t}$ events which had previously been shown to give substantially smaller backgrounds [187].

Fluctuations of a parton into a narrow $\tau$-like jet are considered with probability 0.25% for gluons and light-quark jets and 0.15% for $b$ jets (which may be considered an upper bound) [187].

In the case of $b\bar{b} + jj$, we simulated the semileptonic decay $b \rightarrow l\nu e$ by multiplying the $b\bar{b}jj$ cross section by a branching factor of 0.395 and implementing a three-body phase space distribution for the decay momenta to estimate the effects of lepton isolation cuts. We normalised our resulting cross section to reproduce the same factor 100 reduction found in [187].
Dual lepton mode  For the dilepton mode, we consider decay only to \( \ell \ell \) pairs to completely eliminate the backgrounds from real \( Z \) production decaying directly to \( \ell \ell \) or \( \mu \mu \). Tau decays were performed in the same manner as in the lepton-hadron channel. We again considered QCD and EW \( Zjj; Z \rightarrow \tau \tau \) production as the physics backgrounds.

We calculated the primary contributions from reducible backgrounds by considering all significant sources of two \( W \)'s, which decay leptonically to form the signature \( e, \mu \), and two forward jets. This consists of \( t\bar{t} \) production, as well as both QCD and EW \( WWjj \) production. As with the EW \( Zjj \) case, EW \( WWjj \) processes contain an electroweak boson fusion component kinematically similar to the signal, and so cannot be ignored.

We also considered \( b\bar{b}jj \) production, with each \( b \) decaying semileptonically simulated by implementing the \( V-A \) decay distributions of the \( b \)-quarks in the collinear limit, and multiplying the resultant cross section by a branching fraction 0.0218 (for the \( e, \mu \) or \( \mu, e \) final states).

Finally, we considered the overlapping contribution from the signal itself in the decay mode \( R+Ü+Ü \rightarrow e\muT \), which can be significant above \( M_H \geq \sim 130 \text{ GeV} \).

6.13 Standard Model analysis

The basic acceptance requirements must ensure that the two jets and two \( \tau \)'s are observed inside the detector (within the hadronic and electromagnetic calorimeters, respectively), and are well-separated from each other:

\[
P_T > 20 \text{ GeV}, \quad |\eta_j| \leq 5.0, \quad \Delta R_{jj} \geq 0.7, \quad |\eta_j| \leq 2.5, \quad \Delta R_{\tau\tau} \geq 0.7.
\]  

(65)

Tau-tau separation and tau decay product \( p_T \) requirements are slightly different for the two signatures and are discussed separately below.

The \( Hjj \) signal is characterised by two forward jets with large invariant mass, and central \( \tau \) decay products. The QCD backgrounds have a large gluon-initiated component and thus prefer lower invariant tagging jet masses. Also, their \( \tau \) and \( W \) decay products tend to be less central. Thus, to reduce the backgrounds to the level of the signal, we required tagging jets with a combination of large invariant mass, far forward rapidity, and high \( p_T \), as well as \( \tau \) decay products central with respect to the tagging jets [185]:

\[

e_j, \min + 0.7 < \eta_{\tau,1,2} < \eta_{j,\max} - 0.7, \quad \Delta \eta_{\tau\tau} = |\eta_{j_1} - \eta_{j_2}| \geq 4.4, \quad m_{jj} > m_{j\min},
\]  

(66)

where \( m_{j\min} \) is chosen slightly differently for the two scenarios, as discussed below.

Lepton-hadron mode  Here we required two additional cuts to form the tagging jet signature:

\[
P_T > 40, 20 \text{ GeV}, \quad \Delta R_{\tau\tau} \geq 0.7.
\]  

(67)

That is, the \( p_T \) requirement on the tagging jets is staggered, and as one tau decay is hadronic, it must have a large separation from the leptonic tau.

Triggering the event via the isolated \( \tau \)-decay lepton and identifying the hadronic \( \tau \) decay as discussed in [187] requires sizable transverse momenta for the observable \( \tau \) decay products: \( p_{T\tau,lep} > 20 \text{ GeV} \) and \( p_{T\tau,had} > 40 \text{ GeV} \). It is possible to reconstruct the \( \tau \)-pair invariant mass from the observable \( \tau \) decay products and the missing transverse momentum vector of the event [188]. The \( \tau \) mass was
neglected and collinear decays assumed, a condition easily satisfied because of the high \( \tau \) transverse momenta required. The \( \tau \) momenta were reconstructed from the charged decay products’ \( p_T \) and missing \( p_T \) vectors. We imposed a cut on the angle between the \( \tau \) decay products to satisfy the collinear decay assumption, \( \cos \theta_{th} > -0.9 \), and demanded a physicality condition for the reconstructed \( \tau \) momenta (unphysical solutions arise from smearing effects); that is, the fractional momentum \( x_\tau \), a charged decay observables takes from its parent \( \tau \) cannot be negative. Additionally, the \( x_\tau \) distribution of the leptonically decaying \( \tau \)-candidate is softer for real \( \tau \)’s than for the reducible backgrounds, because the charged lepton shares the parent \( \tau \) energy with two neutrinos. Cuts \( x_\tau < 0.75 \) and \( x_{\tau h} < 1 \) proved very effective in suppressing the reducible backgrounds.

Our Monte Carlo predicted a \( \tau \)-pair mass resolution of 10 GeV or better, so we chose \( \pm 10 \) GeV mass bins for analysing the cross sections. To further reduce the QCD backgrounds, which prefer low invariant masses for the tagging jets, we required \( m_{jj} > 1 \) TeV. Additionally, the \( W j + jj \) background exhibits a Jacobian peak in its \( m_T \) distribution [187]; hence a cut \( m_T(l, \not{p}_T) < 30 \) GeV largely eliminates this background.

Finally, to compensate for overall rate loss based on ATLAS and CMS expected detector ID efficiencies, we apply a factor 0.86 to the cross section for each tagging jet, and a factor 0.95 for the charged lepton.

Using all these cuts together, although not in a highly optimised combination, we expect already a signal to background ratio of 2/1 with a signal cross section of 0.4 fb for \( M_H = 120 \) GeV.

A probability for vetoing additional central hadronic radiation was obtained by measuring the fraction of events that have additional radiation in the central region, between the tagging jets, with \( p_T \) above 20 GeV, using the matrix elements for additional parton emission. This minijet veto reduces the signal by about 15\%, but eliminates typically 70\% of the QCD backgrounds; the EW \( Zjj \) background is reduced by about 20\%, indicating the presence of both boson bremsstrahlung and weak boson fusion effects. Because the veto probability for QCD backgrounds is found to be process independent, we applied the same value to the \( bb + jj \) background.

Table 25 summarises the signal and various background cross sections at progressive levels of the cuts, ID efficiencies and minijet veto as described above, for the case \( M_H = 120 \) GeV. Table 26 gives the expected numbers of events for 60 fb\(^{-1} \) integrated luminosity (low luminosity running) at the LHC.

**Table 25: Signal and background cross sections \( \sigma \cdot BR \) (fb) for \( M_H = 120 \) GeV \( H jj \) events in the lepton-hadron channel.**

<table>
<thead>
<tr>
<th>Cuts</th>
<th>( H jj )</th>
<th>QCD ( Zjj )</th>
<th>EW ( Zjj )</th>
<th>( W j + jj )</th>
<th>( b\bar{b} + jj )</th>
<th>( S/B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>forward tagging</td>
<td>68.4</td>
<td>1680</td>
<td>91</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau ) identification</td>
<td>1.99</td>
<td>20.0</td>
<td>1.45</td>
<td>26.4</td>
<td>7.6</td>
<td>1/28</td>
</tr>
<tr>
<td>( 110 &lt; m_{\tau\tau} &lt; 130 ) GeV</td>
<td>1.31</td>
<td>0.95</td>
<td>0.07</td>
<td>1.77</td>
<td>0.59</td>
<td>1/2.6</td>
</tr>
<tr>
<td>( m_{jj} &gt; 1 ) TeV, ( m_T ) ( (l, \not{p}_T) &lt; 30 ) GeV</td>
<td>0.69</td>
<td>0.16</td>
<td>0.04</td>
<td>0.11</td>
<td>0.15</td>
<td>1.5/1</td>
</tr>
<tr>
<td>( x_\tau &lt; 0.75, x_{\tau h} &lt; 1 )</td>
<td>0.54</td>
<td>0.15</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>2.1/1</td>
</tr>
<tr>
<td>ID efficiency ( (\epsilon = 0.70) )</td>
<td>0.38</td>
<td>0.10</td>
<td>0.03</td>
<td>0.03</td>
<td>0.05</td>
<td>2.1/1</td>
</tr>
<tr>
<td>( P_{\text{minijet,20}} )</td>
<td>( \times 0.87 )</td>
<td>( \times 0.28 )</td>
<td>( \times 0.80 )</td>
<td>( \times 0.28 )</td>
<td>( \times 0.28 )</td>
<td>-</td>
</tr>
<tr>
<td>minijet veto</td>
<td>0.33</td>
<td>0.03</td>
<td>0.02</td>
<td>0.004</td>
<td>0.011</td>
<td>5.2/1</td>
</tr>
</tbody>
</table>
Table 26: Number of expected events in the lepton-hadron channel for the signal and backgrounds, for 60 fb\(^{-1}\) at low luminosity running; cuts, ID efficiency (\(\epsilon = 0.70\)) and minijet veto as in the last line of Table 25; for a range of Higgs boson masses. Mass bins of \(\pm 10\) GeV around a given central value are assumed. As a measure of the Poisson probability of the background to fluctuate up to the signal level, the last row gives \(\sigma_{Gauss}\), the number of Gaussian equivalent standard deviations.

<table>
<thead>
<tr>
<th>(M_H) (GeV)</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\epsilon \cdot \sigma_{sig}) (fb)</td>
<td>0.38</td>
<td>0.33</td>
<td>0.25</td>
<td>0.16</td>
<td>0.08</td>
</tr>
<tr>
<td>(N_S)</td>
<td>22.9</td>
<td>19.6</td>
<td>15.2</td>
<td>9.5</td>
<td>4.6</td>
</tr>
<tr>
<td>(N_B)</td>
<td>10.2</td>
<td>3.8</td>
<td>2.4</td>
<td>1.8</td>
<td>1.5</td>
</tr>
<tr>
<td>(S/B)</td>
<td>2.2</td>
<td>5.2</td>
<td>6.4</td>
<td>5.2</td>
<td>3.1</td>
</tr>
<tr>
<td>(\sigma_{Gauss})</td>
<td>5.6</td>
<td>6.6</td>
<td>6.3</td>
<td>4.7</td>
<td>2.6</td>
</tr>
</tbody>
</table>

Fig. 43: Reconstructed \(\tau\) pair invariant mass distribution for the signal (lepton-hadron channel) and backgrounds after all cuts and multiplication by the expected survival probabilities. The solid line represents the sum of the signal and all backgrounds. Individual components are shown as histograms: the \(Hjj\) signal (solid), the irreducible QCD \(Zjj\) background (dashed), the irreducible EW \(Zjj\) background (dotted), and the combined \(Wj + jj\) and \(b\bar{b}jj\) reducible backgrounds (dash-dotted).

It is possible to isolate a virtually background-free \(qq \rightarrow qqH \rightarrow jj\tau\tau\) signal at the LHC, leading to a \(5\sigma\) observation of a SM Higgs boson with a mere 60 fb\(^{-1}\) of data. The expected purity of the signal is demonstrated in Figure 43 showing the reconstructed \(\tau\tau\) invariant mass for a SM Higgs of 120 GeV after all cuts, particle ID efficiency factors and a minijet veto have been applied. While the reducible \(Wj + jj\) and \(b\bar{b}jj\) backgrounds are the most complicated and do require further study, they appear to be easily manageable.

**Dual lepton mode**  For this signature, we simulated tau decays as before, but with both decaying to final-state leptons. As this would form a different final state in experiment, to form the basic tagging jet signature we require the cuts of Equations 65 and 66 as before, but additionally a minimum separation of the charged leptons somewhat less than for the lepton-hadron scenario, \(\Delta R_{\tau\tau} \geq 0.4\). To be able to trigger on the leptons, we require them to have minimum transverse momentum \(p_T \geq 10\) GeV. In the LHC experiments, this may be slightly higher for electrons and slightly lower for muons, but we do not make the distinction here.

Both the \(t\bar{t} + jets\) and \(b\bar{b}jj\) backgrounds are about three orders of magnitude larger than the signal,
but the contribution from $b\bar{b}jj$ may be reduced by a cut on missing transverse energy, $\not{p}_T > 30$ GeV, and that from $t\bar{t} + jets$ may be severely restricted by vetoing additional jets in the central region between the tagging jets, which even before considering additional gluon radiation (minijets) may come from the decays of central final-state $b$-quarks. We veto all events with a central $b$ with $p_T > 20$ GeV. This provides approximately a factor 17 in reduction of the top quark background, which may be substantially improved to even lower $p_T$ threshold via a $b$-tag, which we cannot simulate.

As the dual lepton final state has a lower overall branching ratio than the lepton-hadron case, we retained more overall rate by making a looser cut on the tagging jet invariant mass, $m_{jj} > 800$ GeV. This cut was still necessary to reduce the QCD backgrounds.

Our Monte Carlo again predicted an excellent $\tau$-pair mass resolution, so we retain the mass binning of $\pm 10$ GeV. We also rejected non-tau’s as in the lepton-hadron case, although our exact cut was somewhat differently defined:

$$x_{\tau_1}, x_{\tau_2} > 0, \quad x_{\tau_1}^2 + x_{\tau_2}^2 < 1.$$  

Finally, we found that a cut on the maximal separation of the two charged leptons is very useful in reducing the heavy quark backgrounds: $\Delta R_{e\mu} < 2.6$.

Efficiency factors for detection are the same as in the previous case, although with two final-state leptons an extra factor 0.95 was taken into account. A minijet veto was applied as before, although other analyses we have performed suggest the survival probabilities change slightly due to the lower hardness of the event, which is strongly correlated with $m_{jj}$ (see Table 27).

Table 27 outlines the cross sections of signal and background for progressive levels of cuts as described above, for the case $M_H = 120$ GeV. Table 28 gives the expected numbers of events for $60$ fb$^{-1}$ integrated luminosity (low luminosity running) at the LHC.

Table 27: Signal rates $\sigma \cdot BR(H \to \tau\tau \to e^{\pm}\mu^{\mp}\not{p}_T)$ for a SM Higgs of $M_H = 120$ GeV and progressive levels of cuts as discussed in the text. All rates are given in fb. Note: the fifth line, non-tau rejection, also includes a cut $90$ GeV $< m_{\tau\tau} < 160$ GeV.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>$H \to \tau\tau$</th>
<th>$H \to WW$</th>
<th>QCD</th>
<th>EW</th>
<th>$t\bar{t} + jets$</th>
<th>$b\bar{b}jj$</th>
<th>QCD</th>
<th>EW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>signal</td>
<td>bkgd</td>
<td>$\tau\tau jj$</td>
<td>$\tau\tau jj$</td>
<td>$t\bar{t} + jets$</td>
<td>$b\bar{b}jj$</td>
<td>WW$jj$</td>
<td>WW$jj$</td>
</tr>
<tr>
<td>forward tags</td>
<td>2.2</td>
<td>57</td>
<td>2.3</td>
<td>1230</td>
<td>1050</td>
<td>4.9</td>
<td>3.3</td>
<td>1/1100</td>
</tr>
<tr>
<td>b veto</td>
<td></td>
<td>72</td>
<td>1.73</td>
<td>29</td>
<td>1.57</td>
<td>62</td>
<td>29</td>
<td>4.1</td>
</tr>
<tr>
<td>$p_T &gt; 30$ GeV</td>
<td></td>
<td></td>
<td>1.34</td>
<td>10.3</td>
<td>1.35</td>
<td>16.3</td>
<td>10.4</td>
<td>1.60</td>
</tr>
<tr>
<td>$M_{jj} &gt; 800$ GeV</td>
<td></td>
<td></td>
<td>1.15</td>
<td>5.2</td>
<td>0.63</td>
<td>0.31</td>
<td>0.42</td>
<td>0.032</td>
</tr>
<tr>
<td>non-$\tau$ reject</td>
<td></td>
<td></td>
<td>0.87</td>
<td>0.58</td>
<td>0.10</td>
<td>0.09</td>
<td>0.10</td>
<td>0.009</td>
</tr>
<tr>
<td>$\pm 10$ GeV mass bins</td>
<td></td>
<td></td>
<td>0.84</td>
<td>0.023</td>
<td>0.52</td>
<td>0.086</td>
<td>0.087</td>
<td>0.028</td>
</tr>
<tr>
<td>$\Delta R_{e\mu} &lt; 2.6$</td>
<td></td>
<td></td>
<td>0.56</td>
<td>0.015</td>
<td>0.34</td>
<td>0.058</td>
<td>0.058</td>
<td>0.019</td>
</tr>
<tr>
<td>ID effic. ($\times 0.67$)</td>
<td></td>
<td></td>
<td>$\times 0.89$</td>
<td>$\times 0.89$</td>
<td>$\times 0.89$</td>
<td>$\times 0.29$</td>
<td>$\times 0.29$</td>
<td>$\times 0.75$</td>
</tr>
<tr>
<td>$P_{\text{post}, 20}$</td>
<td>$\times 0.89$</td>
<td>$\times 0.89$</td>
<td>$\times 0.89$</td>
<td>$\times 0.29$</td>
<td>$\times 0.29$</td>
<td>$\times 0.75$</td>
<td>$\times 0.29$</td>
<td>$\times 0.75$</td>
</tr>
<tr>
<td>minijet veto</td>
<td>0.50</td>
<td>0.014</td>
<td>0.100</td>
<td>0.043</td>
<td>0.017</td>
<td>0.006</td>
<td>0.002</td>
<td>0.006</td>
</tr>
</tbody>
</table>

Although the dual lepton channel does not appear to be able to achieve quite as high an $S/B$ ratio as the lepton-hadron channel, it is still better than 1/1 over much of the mass range of interest, which is also clearly evident in the tau pair invariant mass plot of Figure 44. Furthermore, the independent statistical significance of this channel is as good as that found for the lepton-hadron case.
Table 28: Number of expected events for a SM $Hjj$ signal in the $H \rightarrow \tau \tau \rightarrow e^{\pm}\mu^{\mp}\not{p_T}$ channel, for a range of Higgs boson masses. Results are given for 60 fb$^{-1}$ of data at low luminosity running, and application of all efficiency factors and cuts, including a minijet veto. As a measure of the Poisson probability of the background to fluctuate up to the signal level, the last line gives $\sigma_{\text{Gauss}}$, the number of Gaussian equivalent standard deviations.

<table>
<thead>
<tr>
<th>$M_H$ (GeV)</th>
<th>100</th>
<th>105</th>
<th>110</th>
<th>115</th>
<th>120</th>
<th>125</th>
<th>130</th>
<th>135</th>
<th>140</th>
<th>145</th>
<th>150</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon \cdot \sigma_{\text{sig}}$ (fb)</td>
<td>0.62</td>
<td>0.61</td>
<td>0.58</td>
<td>0.55</td>
<td>0.50</td>
<td>0.44</td>
<td>0.37</td>
<td>0.30</td>
<td>0.23</td>
<td>0.16</td>
<td>0.11</td>
</tr>
<tr>
<td>$N_S$</td>
<td>37.4</td>
<td>36.5</td>
<td>35.0</td>
<td>32.8</td>
<td>30.0</td>
<td>26.3</td>
<td>22.3</td>
<td>18.0</td>
<td>13.7</td>
<td>9.9</td>
<td>6.5</td>
</tr>
<tr>
<td>$N_B$</td>
<td>67.7</td>
<td>45.4</td>
<td>27.4</td>
<td>16.8</td>
<td>11.2</td>
<td>8.4</td>
<td>7.1</td>
<td>6.4</td>
<td>6.1</td>
<td>5.9</td>
<td>5.7</td>
</tr>
<tr>
<td>$S/B$</td>
<td>0.6</td>
<td>0.8</td>
<td>1.3</td>
<td>2.0</td>
<td>2.7</td>
<td>3.2</td>
<td>3.1</td>
<td>2.8</td>
<td>2.2</td>
<td>1.7</td>
<td>1.1</td>
</tr>
<tr>
<td>$\sigma_{\text{Gauss}}$</td>
<td>4.1</td>
<td>4.8</td>
<td>5.6</td>
<td>6.4</td>
<td>6.8</td>
<td>6.7</td>
<td>6.1</td>
<td>5.3</td>
<td>4.3</td>
<td>3.2</td>
<td>2.2</td>
</tr>
</tbody>
</table>

Fig. 44: Reconstructed $\tau$ pair invariant mass distribution for a SM $Hjj$ signal and backgrounds after all cuts, particle ID efficiencies and minijet veto. The double-peaked solid line represents the sum of the signal and all backgrounds. Individual components are: the $Hjj$ signal (solid), the irreducible QCD $ZZj$ background (dashed), the irreducible EW $Zjj$ background (dotted), and the combined reducible backgrounds from QCD + EW + Higgs $WWjj$ events and $t\bar{t} + \text{jets}$ and $b\bar{b}jj$ production (dash-dotted).

6.14 MSSM analysis

The production of CP even Higgs bosons in weak boson fusion is governed by the $hWW, HWW$ couplings, which are suppressed by factors $\sin(\beta - \alpha), \cos(\beta - \alpha)$, respectively [189], compared to the SM case. Their branching ratios are modified with slightly more complicated factors. One can simply multiply SM cross section results from our analysis by these factors to determine the observability of $H \rightarrow \tau \tau$ in MSSM parameter space. We used a renormalisation group improved next-to-leading order calculation, which allows a light Higgs mass up to $\sim 125$ GeV, and examined two trilinear term mixing cases, no mixing and maximal mixing [180, 181].

Varying the pseudoscalar Higgs boson mass $M_A$, one finds that $M_h, M_H$ each approach a plateau for the case $M_A \rightarrow \infty, 0$, respectively. Below $M_A \sim 120$ GeV, the light Higgs mass will fall off linearly with $M_A$, while the heavy Higgs will approach $M_H \sim 125$ GeV, whereas above $M_A \sim 120$ GeV, the light Higgs will approach $M_h \sim 125$ GeV and the heavy Higgs mass will rise linearly with $M_A$. The transition region behaviour is very abrupt for large $\tan\beta$, such that the plateau state will go to $\sim 125$ GeV almost immediately, while for small $\tan\beta$ the transition is much softer and the plateau state reaches the limiting value via a more gradual asymptotic approach.
With reasonable integrated luminosity and combination of the lepton-hadron and dual-lepton channels, 40 fb$^{-1}$ in the worst case, it will be possible to observe at the $5\sigma$ level either $h$ or $H$ decays to $\tau$ pairs when they are in their respective plateau region, with the possibility of some overlap in a small region of $M_A$, as shown in Figure 45. Very low values of $\tan\beta$ would be unobservable, but already excluded by LEP2; there should be considerable overlap between this mode at the LHC and the LEP2 excluded region. Furthermore, a parton shower Monte Carlo with full detector simulation should be able to optimise the analysis so that much less data is required to observe or exclude the MSSM Higgs.

### 6.15 Conclusions

The production of a neutral, CP even Higgs via weak boson fusion and decay $H \to \tau\tau$ at the LHC has been studied for the Standard Model and MSSM, utilising parton level Monte Carlo analyses. Each of the decay channels $\tau\tau \to h^\pm l^\mp p_T, e^\pm \mu^\mp p_T$ independently allows a $5\sigma$ observation of a Standard Model Higgs with an integrated luminosity of about 60 fb$^{-1}$ or less, and provides a direct measurement of the $H\tau\tau$ coupling. For the MSSM case, a highly significant signal for at least one of the Higgs bosons with reasonable luminosity is possible over the entire physical parameter space which will be left unexplored by LEP2. Only 40 fb$^{-1}$ of data is required after combining the two channels. We conclude that this mode provides a no-lose strategy for seeing at least one of the CP even neutral MSSM Higgs bosons.

### 6.2 Searching for $VV \to H \to WW$

In the previous section, vector-boson fusion forming a Higgs which then decays to two $\tau$'s was identified as a valuable process by which to find a Higgs boson in the mass range 110 to 150 GeV. Rainwater and Zeppenfeld have shown that a heavier Higgs in the range 130 to 200 GeV could be found by looking for the process $VV \to H \to WW \to e^\pm \mu^\mp p_T$ [190]. As for the lighter Higgs, the forward jet tagging is a powerful tool for removing background ($W$ pairs, $t\bar{t}$ and $Z \to \tau\tau$ accompanied by jets). This approach appears more promising than the a search for an inclusive $H \to WW \to e^\pm \mu^\mp p_T$ signal, yielding a significant result with $\sim 5$ fb$^{-1}$.

Work has started in the context of the Workshop to investigate this with fast detector simulation, but has not yet been completed.
### 6.3 The strongly interacting symmetry breaking sector

One possible scenario for the spontaneous breaking of the electroweak (EW) symmetry is a strongly interacting symmetry breaking sector (SBS), which generically is formed by new particles with strong interactions at the TeV scale. This sector should provide a global $SU(2)_L \times SU(2)_R$ spontaneous symmetry breaking down to the custodial $SU(2)_{L+R}$ subgroup, thus triggering the Standard Model spontaneous breaking from the $SU(2)_L \times U(1)_Y$ gauge-symmetry down to $U(1)_{em}$. This is the minimal symmetry pattern ensuring that $\rho \simeq 1 + O(g^2)$.

By assuming that the new states appear at the TeV scale, we are only left, at low energies, with the three massless Goldstone Bosons (GB) associated to the global symmetry breaking. We will refer to this scenario as the minimal strongly interacting symmetry breaking sector (MSISBS). In this case, the low-energy EW interactions can be well described with the Electroweak Chiral Lagrangian (EChL) [36, 37], which is an $SU(2) \times U(1)$ gauge-invariant effective field theory that couples the GB to the gauge-bosons and fermions, without any further assumptions than those just described. The EChL, inspired in Chiral Perturbation Theory [191], is organised as a derivative (momentum) expansion, with a set of effective operators of increasing dimension. Although the lowest-order Lagrangian is common to all models satisfying the minimal assumptions, at higher orders each effective operator has a coefficient, whose different values will account for different underlying symmetry breaking mechanisms. Within this approach it is possible, not only to calculate at tree level, but to include loops whose divergences will be absorbed in the coefficients of operators of higher dimension, thus yielding finite results order by order in the calculations. The values of these renormalised parameters are expected in the $10^{-3}$ to $10^{-2}$ range.

As far as physics at the LHC is concerned, the most characteristic feature of a strong SBS is the enhanced production of longitudinal gauge-boson pairs. We will review the EChL amplitudes for these processes. However, the EChL perturbative predictions can only describe EW physics at low energies, well below the mass of the heavy states. Indeed, any amplitude calculated with the EChL is obtained as a truncated series in powers of the external momenta. Hence, it will always violate unitarity bounds at high enough energies. In addition, it cannot reproduce any pole associated to new resonant states. Consequently, in order to apply this formalism to study strong SBS phenomenology at the LHC, we have several ways to proceed:

1. Perform studies strictly within the EChL, but restricted to subprocess energies below 1.5 TeV and to very small chiral parameters.
2. Enlarge the EChL introducing explicitly the heavy resonances of each particular model, but this adds new unknown parameters, namely the mass and the width of each resonance.
3. Follow a more model-independent approach, by unitarising the EChL amplitudes and generating heavy resonances from the information contained in the chiral coefficients.

In the last approach, it is possible to describe the different resonant scenarios with just two chiral parameters. Finally we present a study of the LHC sensitivity reach within this parameter space, using the signal of the cleanest leptonic decays of $ZZ$ and $WZ$ pairs.

#### 6.3.1 Effective Chiral Lagrangian description of electroweak interactions

The EChL [36, 37] provides a phenomenological description of EW interactions when the SBS is strongly-interacting. The only degrees of freedom at low energies are the GBs associated to the $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$ global symmetry breaking, which are coupled to the EW gauge and fermion fields in an $SU(2)_L \times U(1)_L$ invariant way. Customarily, the GBs, $\omega^a$ with $a = 1, 2, 3$, are gathered in an $SU(2)$ matrix $U = \exp(i\omega^a \tau^a / \nu)$, where $\tau^a$ are the Pauli matrices and $\nu = 246$ GeV. The C and P invariant effective bosonic operators up to dimension four are (see the appendix for other notations)

$$\mathcal{L}_{\text{EChL}} = \frac{\nu^2}{4} \text{Tr}(D_\mu U(D^\mu U)^\dagger) + a_0 \frac{g^2 \nu^2}{4} [\text{Tr}(T V_\nu)]^2 + a_1 \frac{g g'}{2} B_{\mu \nu} \text{Tr}(T W^{\mu \nu})$$
\[ + a_2 \frac{ig'}{2} \mathcal{B}_{\mu \nu} \text{Tr}(T[V^\mu, V^\nu]) + a_3 g \text{Tr}(\mathcal{W}_{\mu \nu} [V^\mu, V^\nu]) + a_4 \text{Tr}(V_\mu V_\nu)^2 \]
\[ + a_5 [\text{Tr}(V_\mu V^\mu)]^2 + a_6 \text{Tr}(V_\mu V_\nu) \text{Tr}(T V^\mu) \text{Tr}(T V^\nu) + a_7 \text{Tr}(V_\mu V^\mu) [\text{Tr}(T V^\nu)]^2 \]
\[ + a_8 \frac{g^2}{4} [\text{Tr}(T \mathcal{W}_{\mu \nu})]^2 + a_9 \frac{g}{2} \text{Tr}(T \mathcal{W}_{\mu \nu}) \text{Tr}(T [V^\mu, V^\nu]) + a_{10} [\text{Tr}(TV_\mu) \text{Tr}(TV_\nu)]^2 \]
\[ + \text{e.o.m. terms} + \text{standard YM terms} \]  

(68)

where we have defined \( T = U^{-3} U^\dagger \) and \( V_\mu = (D_\mu U) U^\dagger \), as well as

\[
D_\mu U = \partial_\mu U - g W_\mu U + g' U B_\mu, \quad \mathcal{W}_\mu = -\frac{i}{2} \tilde{W}_\mu \cdot \tau, \quad B_\mu = -\frac{i}{2} B_\mu \tau^3;
\]
\[
\mathcal{W}_{\mu \nu} = \partial_\mu \mathcal{W}_\nu - \partial_\nu \mathcal{W}_\mu - g [\mathcal{W}_\mu, \mathcal{W}_\nu], \quad B_{\mu \nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \]  

(69)

The “e.o.m.” terms refer to operators that can be removed using the equations of motion and the “standard YM terms” are the usual Yang Mills Lagrangian together with the gauge-fixing and Faddeev-Popov terms.

The first operator in Equation 68, which provides the \( W \) and \( Z \) masses, has dimension two and has the form of a gauged non-linear sigma model (NL\( \sigma \)M). Note that it is universal, since it only depends on \( v \) - that is why its predictions for longitudinal gauge-boson scattering amplitudes are called “Low Energy Theorems”. In contrast, the \( a_i \) couplings will have different values depending on the underlying theory.

The gauge-boson observables are obtained from \( \mathcal{L}_{\text{EChL}} \) as a double expansion in \( p^n/(4\pi v)^n \), \( p \) being an external momentum, and in the gauge-couplings \( g \) and \( g' \). The lowest-order predictions are given by the tree level NL\( \sigma \)M, whereas the next order corrections are obtained with a one-loop calculation using the NL\( \sigma \)M vertices plus the tree level contributions of the other operators. The \( a_i \) coefficients not only provide a model independent parametrisation of the unknown dynamics, but also some of them are used to absorb all the one-loop NL\( \sigma \)M divergences. This procedure could be carried out to any desired order, adding higher dimensional operators, thus yielding finite results order by order in the expansion.

In principle, the \( a_i \) values for a particular scenario can be obtained by integrating out the heavy degrees of freedom. In fact, they have been determined for the particular cases of the SM with a heavy Higgs [192, 193] and for technicolor theories in the large \( N_{TC} \) limit [194]. In both cases, these couplings lie in the range \( 10^{-2} \) to \( 10^{-3} \), with either sign. They all have a constant contribution, but those needed in the renormalisation also have a logarithmic term.

### 6.32 Present bounds on the chiral parameters

Let us now look at the present experimental constraints on the EChL parameters \( a_i \) from low energy EW data. The best constraints come from the oblique radiative corrections, giving bounds on the \( a_0, a_1 \) and \( a_8 \) parameters that contribute to the gauge-bosons two-point functions up to order \( q^2 \). The EChL calculation of the \( S, T \) and \( U \) [195] self-energy combinations give [196]

\[
S = 16\pi [-a_1(\mu) + \text{EChL loops}(\mu)], \quad T = \frac{8\pi}{c_W} [a_0(\mu) + \text{EChL loops}(\mu)],
\]
\[
U = 16\pi [a_8(\mu) + \text{EChL loops}(\mu)]
\]

Note that the \( a_i \) have been renormalised to absorb the one-loop divergences from the NL\( \sigma \)M chiral loops, so that \( S, T \) and \( U \) are scale independent. Using the \( a_i \) values for a heavy Higgs boson [192, 193], the deviations of EW observables from the SM predictions at a reference value of the Higgs mass \( M_H \) are

\[
\Delta S \equiv S - S_{\text{SM}}(M_H) = 16\pi \left[ -a_1(\mu) + \frac{1}{12} \frac{5/6 - \log M_H^2/\mu^2}{16\pi^2} \right],
\]
\[
\Delta T \equiv T - T_{\text{SM}}(M_H) = \frac{8\pi}{c_W} \left[ a_0(\mu) + \frac{3}{8} \frac{5/6 - \log M_H^2/\mu^2}{16\pi^2} \right], \quad \Delta U \equiv U - U_{\text{SM}}(M_H) = 16\pi a_8.
\]
A global fit with $M_H = 300 \text{ GeV}$ and $m_t = 175 \text{ GeV}$ to the low energy EW data gives $[197]$

$$\Delta S = -0.26 \pm 0.14 \ , \ \Delta T = -0.11 \pm 0.16 \ , \ \Delta U = \ 0.26 \pm 0.24$$

which imply the following bounds for the three chiral couplings

$$a_1(1\text{ TeV}) = (6.8 \pm 2.8) \times 10^{-3}, \ a_0(1\text{ TeV}) = (4.3 \pm 4.9) \times 10^{-3}, \ a_8(1\text{ TeV}) = (4.9 \pm 4.7) \times 10^{-3}.$$ 

Other studies agree with these values $[198]$. These data already disfavour the SM with a heavy Higgs boson and set strong constraints in models with a dominance of vector resonances $[195]$ (like technicolor). With further assumptions on the underlying SBS dynamics, the latter give a negative contribution to $a_1$. However, the precision EW measurements leave room for an strong SBS $[198]$.

Further constraints come from the three-point functions, whose anomalous electroweak effective couplings were traditionally parametrised in terms of $g_1^Z, g_2^Z, \kappa, \kappa_Z, \lambda, \lambda_Z$. A one-loop EChL calculation of these vertices $[199]$ gives

$$g_1^Z - 1 = 0 + \text{EChL loops}, \quad g_2^Z - 1 = -\frac{g^2}{\aleph_W} a_3 + \text{EChL loops}(\mu)$$

$$\kappa - 1 = g_2^Z (a_2 - a_3 - a_1 + a_8 - a_9) + \text{EChL loops}, \quad \lambda = 0$$

$$\kappa_Z - 1 = g_2^Z (a_8 - a_3 - a_9) + g_2^Z (a_1 - a_2) + \text{EChL loops}(\mu), \quad \lambda_Z = 0$$

There are several analyses $[200, 41]$ that constrain these chiral couplings from LEP and Tevatron data. Ignoring the loops from the NL$\sigma$M, we get the following values from present LEP data (the Tevatron precision is comparable) $\lambda = -0.037^{+0.035}_{-0.036}$,

$$\kappa - 1 = 0.038^{+0.070}_{-0.075}, \quad \rightarrow \quad a_2 - a_3 - a_1 + a_8 - a_9 = 0.088^{+0.184}_{-0.174};$$

$$g_2^Z - 1 = -0.010 \pm 0.033 \quad \rightarrow \quad a_3 = 0.018 \pm 0.059.$$ 

Finally, some indirect bounds on quartic couplings have also been found $[201, 202]$. These indirect estimates come from loops containing $a_i$ vertices, but do not include 2-loop diagrams from the NL$\sigma$M. They find bounds on $a_i$ for $i = 4, 5, 6, 7, 10$ ranging from $10^{-1}$ to $10^{-2}$.

In summary, the present data on the oblique EW corrections already sets significant bounds on the $a_0, a_1$ and $a_8$ chiral parameters, but there is not much sensitivity yet to those chiral parameters that contribute to the three or four-point functions. We will see next how, at the LHC, the situation will improve significantly.

6.33 The Effective Chiral description at the LHC

At the next generation of colliders, we will be probing the $W$ and $Z$ interactions at TeV energies. As long as we are only considering the GBs and no other fundamental fields up to the TeV scale, we expect the self-interactions of longitudinal gauge-bosons, $V_L$, to become strong at LHC energies. This can be easily understood since, intuitively, longitudinal gauge-bosons are nothing but the GBs, which interact strongly. This intuitive statement is rigorously given in terms of on-shell amplitudes and is known as the Equivalence Theorem (ET),

$$A(V_L^a, V_L^b, V_L^c \ldots \text{Other fields}) \simeq A(\omega^a, \omega^b, \omega^c \ldots \text{Other fields}) + O \left( M_W^2 / \sqrt{s} \right),$$

which holds for any spontaneously broken non-Abelian theory. Indeed, it was first derived for the SM $[203, 204, 205]$. Its usefulness is twofold: it relates the pure SBS fields with the observables, but also the calculations can now be performed in terms of scalars instead of gauge-bosons, at least in the high energy limit $s \gg M_W^2$. At first sight it may seem that the ET is incompatible with the use of the EChL,
since an effective theory is a low energy limit. Nevertheless, the ET can still be applied with the EChL, only at leading order in g and g', if we only consider energies below 1.5 TeV and small chiral parameters [206, 207, 208].

Hence, in a first approximation, we will simplify the high energy description of the strong SBS by neglecting EW corrections. Thus, due to our assumption that $SU(2)_{L+R}$ is preserved in the SBS, only the operators that respect custodial symmetry once the gauge-symmetries are switched off will be relevant in this regime. These are the universal term and the operators with $a_i$ couplings for $i = 3, 4, 5$.

At the LHC, the two most relevant processes of $V_LV_L$ production are the scattering of two longitudinal vector-bosons in fusion reactions and the $V_L$ pair production from $q\bar{q}$ annihilation. Through the ET, they are identified with GB elastic scattering and $q\bar{q} \rightarrow \omega\omega$, respectively. Customarily, GB elastic scattering is described in terms of partial wave amplitudes of definite angular momentum, $J$, and isospin, $I$, associated to the custodial $SU(2)_{L+R}$ group. With the EChL, these partial waves, $t_{IJ}$, are obtained as

$$t_{IJ}(s) = t^{(2)}_{IJ}(s) + t^{(4)}_{IJ}(s) + \ldots ,$$

where the superscript refers to the corresponding power of momenta. They are given by [191, 209, 210]

$$t^{(2)}_{00} = \frac{s}{16\pi v^2}, \quad t^{(4)}_{00} = \frac{s^2}{64\pi v^4} \left[ \frac{16(11a_5 + 7a_4)}{3} + \frac{101/9 - 50\log(s/\mu^2)/9 + 4i\pi}{16\pi^2} \right],$$

$$t^{(2)}_{11} = \frac{s}{96\pi v^2}, \quad t^{(4)}_{11} = \frac{s^2}{96\pi v^4} \left[ 4(a_4 - 2a_5) + \frac{1}{16\pi^2} \left( \frac{1}{9} + i\frac{\pi}{6} \right) \right],$$

$$t^{(2)}_{20} = \frac{-s}{32\pi v^2}, \quad t^{(4)}_{20} = \frac{s^2}{64\pi v^4} \left[ \frac{32(a_5 + 2a_4)}{3} + \frac{273/54 - 20\log(s/\mu^2)/9 + i\pi}{16\pi^2} \right].$$

Note that, within our approximations, the above amplitudes only depend on $a_4$ and $a_5$. The projection in angular momentum has been defined, from the definite $I$ amplitude $T_I$, as

$$t_{IJ} = \frac{1}{64\pi} \int_{-1}^{1} d(\cos \theta) \, F_I(\cos \theta) T_I(s, t).$$

The $V_LV_L$ production from $q\bar{q}$ annihilation, is very important since vector resonances can also couple to this channel. By means of the ET, we are thus interested in $q\bar{q} \rightarrow \omega\omega$. As far as GBs couple to quarks proportionally to their mass, the only relevant contribution comes from the $s$-channel annihilation through a vector-boson. In practice, for the $WZ$ final state, the $W \rightarrow \omega Z$ interaction is described as $g F_V(s)$, by means of a vector form factor, $F_V(s)$, which is obtained from the EChL as

$$F_V(s) = 1 + F^{(2)}_V(s) + \ldots \quad \text{with} \quad F^{(2)}_V(s) = \frac{s}{(4\pi v)^2} \left[ 64\pi^2a_3(\mu) - \frac{1}{6} \log \frac{s}{\mu^2} + \frac{4}{9} + i\frac{\pi}{6} \right].$$

Let us then review the studies of the LHC sensitivity to the chiral parameters via these two processes.

### 6.34 Non-resonant studies for LHC

The EChL formalism has been applied to study the LHC sensitivity to different non-resonant SBS sectors in [211, 212, 125, 213, 214, 215]. We summarise in Table 29 the results from [125, 213, 214] where the expected number of gold-plated $ZZ$ and $WZ$ from $VV$-fusion and $q\bar{q}$-annihilation was calculated for values of the custodial preserving $a_3, a_4$ and $a_5$ parameters in the $10^{-2}$ to $10^{-3}$ range. Since for values of $a_4$ or $a_5 \geq 5 \times 10^{-3}$ unitarity violations cannot be ignored at energies beyond 1.5 TeV, these studies only include events in the region of low invariant mass $V_LV_L$ pair, i.e. $M_{VV} \leq 1.5$ TeV. The rest of kinematical cuts are similar to those given in Equation 81. To illustrate the agreement between these kinds of studies, we give in Table 29 other estimates [215] of the $a_i$ bounds attainable at the LHC.
Table 29: Expected number of signal and total (signal+background) gold-plated WZ and ZZ events [125, 213, 214]. The statistical significance is defined as $r = (N(a_i) - N(0))/\sqrt{N(0)}$ where $N(a_i)$ is the expected number of events for a given $a_i$. On the bottom right, expected limits on the chiral parameters attainable at the LHC [215] are shown.

<table>
<thead>
<tr>
<th>$\mathcal{L} = 100,\text{fb}^{-1}$</th>
<th>$a_4$</th>
<th></th>
<th>$a_5$</th>
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<td></td>
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<td>$-10^{-2}$</td>
<td>$5 \times 10^{-3}$</td>
<td>$-5 \times 10^{-3}$</td>
</tr>
<tr>
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<td>80</td>
<td>27</td>
<td>47</td>
</tr>
<tr>
<td>total $W^\pm Z$</td>
<td>118</td>
<td>162</td>
<td>109</td>
<td>129</td>
</tr>
<tr>
<td>$r_{WZ}$</td>
<td>0.7</td>
<td>4.8</td>
<td>0.2</td>
<td>1.7</td>
</tr>
<tr>
<td>$r_{WZ,\text{tagging}}$</td>
<td>1.0</td>
<td>7.5</td>
<td>0.3</td>
<td>2.7</td>
</tr>
<tr>
<td>$W^+W^- \rightarrow ZZ$</td>
<td>12</td>
<td>7</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>$ZZ \rightarrow ZZ$</td>
<td>6</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>total $ZZ$</td>
<td>37</td>
<td>32</td>
<td>30</td>
<td>27</td>
</tr>
<tr>
<td>$r_{ZZ}$</td>
<td>1.9</td>
<td>0.9</td>
<td>0.5</td>
<td>$\approx 0$</td>
</tr>
<tr>
<td>$r_{ZZ,\text{tagging}}$</td>
<td>3.5</td>
<td>1.8</td>
<td>0.9</td>
<td>0.1</td>
</tr>
</tbody>
</table>

| $\mathcal{L} = 100\,\text{fb}^{-1}$ | $a_3$ | |
|---|---|
| | $10^{-2}$ | $-10^{-2}$ |
| $qq' \rightarrow W^\pm Z$ | 96 | 139 |
| $r_{WZ\,\text{tagging}}$ | 1.4 | 2.7 |

<table>
<thead>
<tr>
<th>LHC Limits (90% CL)</th>
<th>Process</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-0.0035 \leq a_4 \leq 0.015$</td>
<td>$W^\pm W^\pm, WZ, ZZ$</td>
</tr>
<tr>
<td>$-0.0072 \leq a_5 \leq 0.013$</td>
<td>$W^\pm W^\pm, WZ, ZZ$</td>
</tr>
<tr>
<td>$-0.013 \leq a_6 \leq 0.013$</td>
<td>$WZ, ZZ$</td>
</tr>
<tr>
<td>$-0.013 \leq a_7 \leq 0.011$</td>
<td>$WZ, ZZ$</td>
</tr>
<tr>
<td>$-0.029 \leq a_{10} \leq 0.029$</td>
<td>$ZZ$</td>
</tr>
</tbody>
</table>
It will be very difficult to detect these non-resonant signals over the continuum background, since they just give small enhancements in the high energy region of the $M_{VV}$ and $p_T$ distributions. There is a general agreement that, although the present bounds could be significantly improved, with these non-resonant studies, the LHC would be hardly sensitive to values of the chiral parameters down to the $10^{-3}$ level. Like-sign $W^\pm W^\pm$ production may be better in these channels [171, 216].

Obviously, these studies do not describe one of the most characteristic features of strong interactions: resonances. Moreover, they are limited to moderate energies due to the unitarity violations mentioned already. These caveats can be overcome by means of unitarisation procedures which we explain next.

6.35 Unitarisation and resonances in the SBS

In terms of the partial waves defined in Equation 72, the elastic $V_LV_L$ scattering unitarity condition, (basically, the Optical Theorem) for physical values of $s$, is

$$\text{Im} t_{I,J}(s) = |t_{I,J}(s)|^2 \Rightarrow \text{Im} \frac{1}{t_{I,J}(s)} = -1, \Rightarrow t_{I,J}(s) = \frac{1}{\text{Re} t_{I,J}^{-1}(s) - i}. \quad (75)$$

Hence we only have to use the EChL to approximate

$$\text{Re} t_{I,J}^{-1} = (t_{I,J}^{(2)})^{-1}[1 - \text{Re} t_{I,J}^{(4)}/t_{I,J}^{(2)} + \ldots]. \quad (76)$$

But since the EChL amplitudes satisfy elastic unitarity perturbatively, i.e.

$$\text{Im} t_{I,J}^{(4)}(s) = |t_{I,J}^{(2)}(s)|^2 \Rightarrow \text{Im} \frac{t_{I,J}^{(4)}(s)}{|t_{I,J}^{(2)}(s)|^2} = -1, \quad (77)$$

we find

$$t_{I,J}(s) = \frac{t_{I,J}^{(2)}}{1 - t_{I,J}^{(4)}/t_{I,J}^{(2)}} \quad (78)$$

This is the $O(p^4)$ Inverse Amplitude Method (IAM), which has given remarkable results describing meson interactions, which have a symmetry breaking pattern almost identical to our present case [217, 218, 219, 220]. Note that it respects strict elastic unitarity, while keeping the correct EChL low energy expansion. Furthermore, the extension of Equation 78 to the complex plane can be justified using dispersion theory [217, 218, 219, 220]. In particular, it has the proper analytical structure and, eventually, poles in the second Riemann sheet for certain $a_4$ and $a_5$ values, that can be interpreted as resonances. Thus, EChL+IAM formalism can describe resonances without increasing the number of parameters and respecting chiral symmetry and unitarity.

The EChL+IAM has already been applied to the SBS [221, 222] to study some specific choices of $a_4$ and $a_5$ that mimic models with vector or scalar resonances. The LHC sensitivity to resonances parametrised with $a_4$ and $a_5$ was first studied in [222] and [223], and more recently in [42]. A map of these resonances in the $(a_4, a_5)$ space was first obtained in [224]. We show in Figure 46 the vector and scalar neutral resonances expected in the $(a_4, a_5)$ parameter space. As far as we expect $a_4$ and $a_5$ to lie between $10^{-2}$ and $10^{-3}$, we scan only that range. Furthermore, the poles of the IAM amplitudes will give us the positions and widths of the resonances. Note that, from Equation 72 within our approximations, the $I = J = 1$ and $I = J = 0$ channels only depend on the $a_4 - 2a_5$ and $7a_4 + 11a_5$ combinations, respectively. Thus the straight lines that keep these combinations constant have the same physics in the corresponding channel. We give several examples in the tables within the figure. The fact that each IAM amplitude depends only on one combination of $a_i$ implies that their mass and width are related by the KSFR relation [225, 226]. In addition, we locate five points that we will use later as illustrative examples.
Fig. 46: Resonances in the \((a_4, a_5)\) space [224]. In the tables we give the resonance parameters for several lines. a) Left: Vector resonances. The points with the same \(a_4 - 2a_5\) have the same physics in the \(I = J = 1\) channel. b) Middle: Scalar neutral neutral resonances. Those points with constant \(7a_4 + 11a_5\) have the same physics in this channel. c) Right: General Resonance Spectrum of the strong SBS. \(V\) stands for vector resonances, \(S\) for neutral scalar resonances and \(W_2\), for wide structures that saturate the doubly charged (\(I = 2\)) channel. For illustration, we have also located several simple and familiar models explained in the text.

The white area means that no resonances or saturation of unitarity is reached below \(4\pi v \simeq 3\) TeV, which we expect to be the region of applicability for our approach.

We do not give results for the \(I = 2, J = 0\) channel since we do not expect any heavy resonance with our minimal assumptions. Intuitively this occurs because the \(I = 2, J = 0\) channel is repulsive.

The general resonance spectrum of the MSISBS is gathered in the last plot of Figure 46 [224]. Depending on \(a_4\) and \(a_5\), we find one scalar resonance (\(S\)), one vector resonance (\(V\)), two resonances (\(S, V\)), a resonance and a doubly charged wide saturation effect (\(W_2\)) or even no resonances below 3 TeV (white area). For illustration, we have included points for some simple and familiar scenarios: minimal technicolor models with 3 and 5 technicolors (\(TC3\) and \(TC5\)), and the heavy Higgs SM case, with a tree level mass of 1000 and 1200 GeV (\(H1000\) and \(H1200\)). The black region is excluded by the constraints on the \(I = 2, J = 0\) wave [224]. In the dark “Light Resonances” areas (lighter than 700 GeV), our results should be interpreted cautiously. Outside these areas, we estimate that the predictions of Figure 46 are reliable within \(\sim 20\%\) [42].

Once we have the general spectrum, our aim is to study to what extent the LHC is sensitive to different resonant scenarios via \(V_L V_L\) production. For that purpose, we cannot forget the unitarisation of \(q \bar{q} \rightarrow V_L V_L\), since we expect the final state to re-scatter strongly, in particular when there is a resonance in the \(I = J = 1\) elastic channel. This effect can be parametrised in terms of a vector form factor, \(F_V\). Again, the \(F_V\) obtained from the EChL does not satisfy exactly its unitarity condition

\[
\text{Im} F_V(s) = F_V(s) t_{11}^*(s),
\]

which implies that the phases of \(F_V\) and \(t_{11}\) should be the same (Watson’s Final State Theorem). Moreover, the poles of \(\tilde{F}_V\) should be those of \(t\). Hence, we can relate the combination of \(a_i\) that appears in the perturbative expansion of \(F_V\) (Equation 74) with \(a_4 - 2a_5\). All in all, it is possible to unitarise \(F_V\) using only the \(t_{11}\) EChL result, as follows [42]:

\[
F_V \simeq \frac{1}{1 - \frac{t_{11}^{(4)}}{t_{11}^{(2)}}}.
\]

In summary, \(F_V\) is determined just by \(a_4 - 2a_5\), and we can still use the map of resonances in Figure 46.
We will restrict the study to \( ZZ \) and \( WZ \) production, assuming that their gold-plated decays, \( ZZ \to 4l \) and \( WZ \to lv ll \) (with \( l = e, \mu \)) can be identified and reconstructed with a 100% efficiency. We do not consider like-sign \( W^\pm W^\pm \) production, since, as we have seen, we do not expect \( I = 2 \) resonances.

To evaluate \( VV \) fusion processes, we use the leading-order Effective-W Approximation (EWA) [227]. Non-fusion diagrams are not included since they are expected to be small in our kinematic region. We also use the CTEQ4 [229] parton distribution functions at \( Q^2 = M_W^2 \) for \( VV \) fusion and at \( Q^2 = s \) for \( q\bar{q} \) annihilation and \( gg \) fusion, with \( \sqrt{s} \) being the centre of mass energy of the parton pair. More detail can be found in [42].

Since we do not consider final \( W \) and \( Z \) decays, the cuts are set directly on the gauge-boson variables. A first criterion to enhance the strong \( V_LV_L \) signal over the background is to require high invariant mass \( M_{VV} \) and small rapidities. We have applied the following set of minimal cuts:

\[
500 \text{ GeV} \leq M_{V_1V_2} \leq 10 \text{ TeV}, \quad |y_{lab}(V_1)|, |y_{lab}(V_2)| \leq 2.5, \quad p_T(V_1), p_T(V_2) \geq 200 \text{ GeV},
\]

(81)

which are also required by our approximations, mainly by the ET. An additional invariant mass cut around each resonance will be imposed later.

The \( ZZ \) production signal occurs through the \( W_L^+W_L^- \to Z_LZ_L \) and \( Z_LZ_L \to Z_LZ_L \) fusion processes. In addition, we have included the following backgrounds:

\[
\begin{align*}
q\bar{q} &\to ZZ, \ (61\%), \\
W^+W^- &\to ZZ, \ (18\%), \\
gg &\to ZZ, \ (21\%)
\end{align*}
\]

where we also give their relative contribution to the total background with the minimal cuts. The continuum from \( q\bar{q} \) annihilation has only tree level SM formulae, which is probably too optimistic since the NLO QCD corrections [25, 23, 24, 26] can enhance significantly the tree level cross sections. The second background is calculated in the SM at tree level, with at least one transverse weak boson. Finally, the one-loop \( gg \to ZZ \) amplitude has been taken from [228].

For \( W^\pm Z \) final states, two processes contribute to the signal: \( W_L^\pm Z_L \to W_L^\pm Z_L \) and \( q\bar{q} \to W_L^\pm Z_L \), whereas the backgrounds, calculated at tree level within the SM, are

\[
W^\pm Z \to W^\pm Z, \ (18\%), \quad \gamma Z \to W^\pm Z, \ (15\%), \quad q\bar{q} \to W^\pm Z, \ (67\%).
\]

The \( W^\pm Z \to W^\pm Z \) amplitudes have at least one transverse boson and exclude the Higgs contribution. In the \( q\bar{q} \to W^\pm Z \) background, we have excluded the amplitude with a \( V_LV_L \) pair, which is part of the signal. The QCD corrections to \( q\bar{q} \) annihilation would give an enhancement in both the signal and the background, so we expect that they will not modify considerably our estimates of the statistical significance of vector resonance searches. We have not studied the \( t\bar{t} \) background since it can be efficiently suppressed after imposing kinematic constraints and isolation cuts to high-\( p_T \) leptons [153, 125, 53].

For illustrative purposes, let us first concentrate on the five representative points given in Figure 46. Points 1, 3 and 4 represent models containing a \( J = I = 1 \) resonance with masses in the range 900-2000 GeV. Point 5 represents a model with a scalar resonance with mass 730 GeV and a width of 140 GeV. Finally, point 2 represents both a scalar and a vector resonance. The \( M_{VV} \) distributions for these five models are shown in Figure 47, where we have plotted the signal on top of the background for gold-plated \( ZZ \) and \( WZ \) events, assuming an integrated luminosity of 100 fb\(^{-1}\). The vector resonances in points 1 to 4 can be seen as peaks in the distribution of final \( WZ \) pairs. The scalar resonances in points 2 and 5 give small enhancements of \( ZZ \) pairs. Note that as both \( a_4 \) and \( a_5 \) tend to 0, the resonances become heavier and broader, yielding a less significant signal. It seems evident that it will be much harder to detect scalar than vector resonances. The reasons are that scalars are wider, they are not produced with a significant rate from \( q\bar{q} \) annihilation, and there is a smaller rate of \( ZZ \) production from \( VV \) fusion. Furthermore, the \( ZZ \) branching ratio to leptons is smaller that that of \( WZ \).
Fig. 47: a) Left: Distribution of gold-plated events from $WZ$ and $ZZ$ production [42]. The shaded histogram corresponds to the background as described in the text. On top of it we have plotted the signal as a white histogram. The points labelled $P1$ to $P5$ correspond to those in Figure 46 and are representative of cases which, from top to bottom, present: one narrow vector resonance, a vector and a scalar resonance, an intermediate vector resonance, a very wide vector resonance and, finally, a “narrow” scalar resonance. b) Right: Sensitivity of the LHC to the resonance spectrum of the strong SBS with $WZ$ and $ZZ$ gold plated events [42]. In the $(a_4, a_5)$ parameter space, we show the $3\sigma$ and $5\sigma$ reach with an integrated luminosity of 100 fb$^{-1}$ (solid lines limiting the shaded areas) and 400 fb$^{-1}$ (dashed lines), both for scalar and vector resonances.
The contributions to signal and background for $WZ$ and $ZZ$ production at these representative points are given in Table 30. In order to enhance the signal to background ratio, we have optimised the $M_{VV}$ cut, keeping events within approximately one resonance width around the resonance mass (see the second column of these tables). From the $WZ$ results, it is clear that the LHC will have a very good sensitivity to light vector resonances, due to the $q\bar{q}$-annihilation, which dominates by far the $VV$-fusion process. As the vector resonance mass increases, the $q\bar{q}$ contribution is damped faster than that of $VV$ fusion, and both signals become comparable for vector masses around 2 TeV. Let us remark that, in $ZZ$ production, there is only strong interaction signal in $\ell\ell$ fusion, and therefore to tag forward jets is always convenient in this final state in order to reject non-fusion processes. This is not the case, however, for vector resonance searches since it is mostly due to $q\bar{q}$ annihilation. In these tables, we have also estimated the statistical significance, $\text{Signal}/\sqrt{\text{Bkgd}}$, assuming integrated luminosities of 100 and 400 fb$^{-1}$. In $ZZ$ final states, we also give the significance assuming perfect forward jet-tagging.

Table 30: Expected number of signal and background gold-plated $VV$ events at the LHC with $\mathcal{L} = 100,400$ fb$^{-1}$. a) Top: For $W^\pm Z$ final state and four different $(a_4, a_5)$ values representing vector resonances. b) Bottom: For $ZZ$ and two representative $(a_4, a_5)$ values with scalar resonances. The statistical significance is also given for ideal forward jet-tagging.

<table>
<thead>
<tr>
<th>$M_V, \Gamma_V$ (GeV)</th>
<th>Cuts: $(M_{VV}^{m_{4,5}}, M_{VV}^{a_{4,5}})$</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>$S/\sqrt{B}$</th>
<th>$S/\sqrt{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P1$: 894, 39 (6.25,6.25)</td>
<td>(700,1000)</td>
<td>123</td>
<td>1630</td>
<td>1743</td>
<td>74</td>
<td>150</td>
<td>224</td>
<td>116</td>
<td>232</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P2$: 1150, 85 (-1.25,8.75)</td>
<td>(900, 1300)</td>
<td>65</td>
<td>369</td>
<td>434</td>
<td>50</td>
<td>84</td>
<td>134</td>
<td>37</td>
<td>75</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P3$: 1535, 200 (-1.25,3.75)</td>
<td>(1250, 1700)</td>
<td>24</td>
<td>56</td>
<td>80</td>
<td>21</td>
<td>27</td>
<td>48</td>
<td>11</td>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P4$: 1963, 416 (-1.25,1.25)</td>
<td>(1500, 2350)</td>
<td>10</td>
<td>12</td>
<td>22</td>
<td>14</td>
<td>16</td>
<td>30</td>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$M_S, \Gamma_S$ (GeV)</th>
<th>Cuts: $(M_{VV}^{m_{4,5}}, M_{VV}^{a_{4,5}})$</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Signal</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>Bkgd</th>
<th>$S/\sqrt{B}$</th>
<th>$S/\sqrt{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P2$: 850, 225 (1.25,8.75)</td>
<td>(600, 1050)</td>
<td>15</td>
<td>10</td>
<td>11</td>
<td>34</td>
<td>55</td>
<td>2</td>
<td>5</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P5$: 750, 140 (3.25,3.75)</td>
<td>(550, 900)</td>
<td>21</td>
<td>10</td>
<td>14</td>
<td>39</td>
<td>63</td>
<td>3</td>
<td>6</td>
<td>5</td>
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</tbody>
</table>

Finally, we also show in Figure 47 the regions of the $(a_4, a_5)$ space accessible at the LHC, giving 3 and 5$\sigma$ contours and assuming integrated luminosities of 100 and 400 fb$^{-1}$. In terms of resonance mass reach limits, we find that with 100 fb$^{-1}$, scalar resonances could be discovered (5$\sigma$) in gold-plated $ZZ$ events up to a mass of 800 GeV with forward jet-tagging. Vector resonances could be discovered using gold-plated $WZ$ events up to a mass of 1800 GeV. These numbers are in good agreement with more realistic studies [153, 125, 53] of particular cases. We can also see that there is a central region in the $(a_4, a_5)$ space that does not give significant signals in gold-plated $ZZ$ and $WZ$ events. This region corresponds to models in which either the resonances are too heavy or there are no resonances in the SBS and the scattering amplitudes are unitarised smoothly. It is a key issue as to whether this type of non-resonant $V_\ell V_\ell$ signal could be probed at the LHC. It has been argued that doubly-charged $WW$ production could be relevant to test this non-resonant region. But non-resonant $VV$ distributions would only have slight enhancements at high energies, and a very accurate knowledge of the backgrounds and the detector performance would be necessary in order to establish their existence.
6.37 Appendix

Table 31: Relation between different notations in the literature.

<table>
<thead>
<tr>
<th>Notations</th>
<th>$a_0$</th>
<th>$a_1$</th>
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<th>$a_7$</th>
<th>$a_8$</th>
<th>$a_9$</th>
<th>$a_{10}$</th>
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<tbody>
<tr>
<td>Ours [192, 193]</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>App.&amp; Longh. [36, 37]</td>
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<td>$\frac{g^2}{g} \alpha_1$</td>
<td>$\frac{g^2}{g} \alpha_2$</td>
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<td>$\alpha_4$</td>
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<td>$\alpha_6$</td>
<td>$\alpha_7$</td>
<td>$-\alpha_8$</td>
<td>$-\alpha_9$</td>
<td>$\frac{1}{2} \alpha_{10}$</td>
</tr>
<tr>
<td>S.Alam [200, 41]</td>
<td>$\frac{1}{g^2} \beta_1$</td>
<td>$\alpha_1$</td>
<td>$\alpha_2$</td>
<td>$-\alpha_3$</td>
<td>$\alpha_4$</td>
<td>$\alpha_5$</td>
<td>$\alpha_6$</td>
<td>$\alpha_7$</td>
<td>$-\alpha_8$</td>
<td>$-\alpha_9$</td>
<td>$\frac{1}{2} \alpha_{10}$</td>
</tr>
<tr>
<td>He et al. [211, 212]</td>
<td>$\frac{1}{16\pi^2}$</td>
<td>$\frac{1}{16\pi^2}$</td>
<td>$\frac{1}{16\pi^2}$</td>
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<td>2.3</td>
<td>3.4</td>
<td>4</td>
</tr>
<tr>
<td>$SU(2)_L+R$</td>
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<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

6.4 Vector-boson scattering

The search for a fundamental scalar particle which would be responsible for electroweak symmetry breaking has so far proven unsuccessful. While the existence of a light Standard Model (SM) Higgs alone would be consistent with all precision electroweak measurements, the well known hierarchy problems [230] make the theory unsatisfactory. The model makes \textit{ad hoc} assumptions about the shape of the potential, responsible for electroweak symmetry breaking, and provides no explanation for the values of the parameters. Although supersymmetry is an appealing alternative, no indication exists, yet, of its validity. Therefore, in the absence of a low mass Higgs particle, a strongly coupled theory must be considered. The study of electroweak symmetry breaking will require measurements of the production rate of pairs of longitudinal gauge-bosons, since they are the Goldstone bosons of the symmetry breaking process. It will also be essential to search for the presence of resonances which regularise the vector-boson scattering cross-section. Scalar resonances occur in models with a heavy SM Higgs boson, and vector resonances, in charged or neutral channels, are also predicted in dynamical theories, such as technicolor.

In this section, different channels for scattering of high energy gauge-bosons at the LHC are considered. These include heavy Higgs production and resonant \textit{WW} as well as non-resonant \textit{WW} and \textit{W+W+} production in the Chiral Lagrangian model. High mass gauge-boson pair production in a multi-scale technicolor model is also examined. The possibility of making such measurements at the LHC is evaluated.

6.41 Heavy Higgs signal

It is now generally believed that a SM Higgs should be light, its mass being bound by requirements of vacuum stability and by the validity of the SM to high scales in perturbative calculations [231]. The parameters of the Higgs used in this study were calculated at tree level. One should note that in NNLO, the resonance saturates [232]. Nevertheless, the search for such a resonance at the LHC can serve as a testing ground for the measurement of the production of high mass longitudinal gauge-boson pairs or for the search of a generic resonance. The $H \rightarrow WW \rightarrow l\nu jj$ channel is presented in this section as an example of a typical analysis of a heavy Higgs signal. In fact, $V_L V_L$ fusion is also detectable in the case of a heavy Higgs resonance, through the processes $H \rightarrow ZZ$, up to $M_H \sim 800$ GeV. Simultaneous detection of a heavy Higgs in other signals would not only confirm the discovery but also provide additional information on the Higgs couplings, which are essential for determining the nature of the resonance.

$H \rightarrow WW \rightarrow l\nu jj$ In the vector-boson fusion process of Higgs production, $qq \rightarrow qqH$, the rate for this channel is sufficient to be observed at low luminosity with a very distinctive signature [235, 237, 238]:

- A high-$p_T$ central lepton ($|\eta| < 2$).
- A large $E_T^{miss}$. 
• Two high-$p_T$ jets from the $W \rightarrow jj$ decay in the central region and close-by in space ($\Delta R \sim 0.4$) arising from the large boost of the $W$ boson.
• Two tag jets in the forward regions ($|\eta_j| > 2$).
• No extra jet in the central region (central jet veto).

The main backgrounds are:
• $W$+jet which gives the largest contribution but also suffers from significant theoretical uncertainties due to higher-order corrections [236].
• $t\bar{t} \rightarrow l\nu b jj\bar{b}$, with the presence of a real $W \rightarrow jj$ decay, but also additional hadronic activity from the $b$-jets in the central region.
• $WW \rightarrow l\nu jj$ continuum production, which has a much lower rate but is irreducible in the central region.

In addition to central jet veto and forward tag jets cuts, other cuts (high-$p_T$ cuts) have been used to optimise the statistical significance of the signal. They are:
• Lepton cuts: $p_T^{l}, E_T^{miss} > 100$ GeV, $p_T^{W-l\nu} > 350$ GeV.
• Jet cuts: two jets reconstructed within $\Delta R = 0.2$ with $p_T > 50$ GeV and $p_T^{W-jj} > 350$ GeV.
• $W$ mass window: $m_{jj} = m_W \pm 2\sigma$, where $\sigma$ is the resolution on $m_{jj}$.

Table 32 shows the number of events resulting from this selection, for an integrated luminosity of 30 fb$^{-1}$, for $M_H = 1$ TeV and $M_H = 800$ GeV as evaluated with the ATLAS fast simulation program (ATLFAST, [85]). A significant signal remains above background. Variation of the $E_{tag}$ cut provides the possibility to compare the shape and cross section of the resonance production to the expected parameters of the Higgs signal (see Figure 48).

<table>
<thead>
<tr>
<th>Higgs</th>
<th>$t\bar{t}$</th>
<th>$W$+jets</th>
<th>$WW$</th>
<th>$S/\sqrt{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_H = 1$ TeV</td>
<td>($p_T &gt; 300$ GeV)</td>
<td>($p_T &gt; 250$ GeV)</td>
<td>($p_T &gt; 50$ GeV)</td>
<td></td>
</tr>
<tr>
<td>$M_H = 1$ TeV</td>
<td>37.9</td>
<td>3.3</td>
<td>9.2</td>
<td>1.0</td>
</tr>
<tr>
<td>$M_H = 800$ GeV</td>
<td>43.5</td>
<td>3.3</td>
<td>9.2</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The $H \rightarrow ZZ \rightarrow l\ell\nu\nu$ and $H \rightarrow ZZ \rightarrow l\ell jj$ channels in ATLAS have also been studied [233, 234, 237] over most of the mass range from 300 Gev to 1 TeV. It has been shown that forward jet tagging ($2 < |\eta_j| < 5$), is a powerful method for rejecting background and selecting $qq \rightarrow qqH$ production, i.e. the vector-boson fusion process.

6.42 Strong vector-boson scattering

**Chiral Lagrangian model** In the Chiral Lagrangian model [249], the form of the Lagrangian is only constrained by symmetry considerations which are common to any strong electroweak symmetry breaking sector. Differences among underlying theories appear through the values of the parameters of the Chiral Lagrangian. Within the chiral approach, the low-energy Lagrangian is built as an expansion in derivatives of the Goldstone boson fields. There is only one possible term with two derivatives which respects $SU(2)_{L+R}$ symmetry:

$$\mathcal{L}^{(2)} = \frac{v^2}{4} \text{Tr}(D_\mu U D^\mu U^\dagger)$$

where $D_\mu U = \partial_\mu U - W_\mu U + UB_\mu$, $W_\mu = -ig\sigma^a W_\mu^a/2$, $B_\mu = ig\sigma^3 B_\mu/2$. 

Fig. 48: \( m_{\ell\ell} \) distribution for the summed signal+background obtained with \( M_H = 800 \text{ GeV} \) and \( \mathcal{L} = 30 \text{ fb}^{-1} \) after requiring two tag jets with \( E_{\text{tag}} > 200 \text{ GeV} \) (top) and \( E_{\text{tag}} > 400 \text{ GeV} \) (bottom) [238].

The dependence on the different models appears at next order through two phenomenological parameters \( L_1 \) and \( L_2 \):

\[
\mathcal{L}^{(4)} = L_1 (\text{Tr}(D_\mu U D_\nu U^\dagger))^2 + L_2 (\text{Tr}(D_\mu U D_\nu U^\dagger))^2
\]

The \( SU(2)_{L+R} \) symmetry allows us to define a weak isospin \( I \). The \( W_L W_L \) scattering can then be written in terms of isospin amplitudes, exactly as in low energy hadron physics. We assign isospin indices as follows:

\[
W_L^a W_L^b \to W_L^c W_L^d
\]

where \( W_L \) denotes either \( W_L^\pm \) or \( Z_L \), where \( W_L^\pm = (1/\sqrt{2}) (W_L^1 \mp i W_L^2) \) and \( Z_L = W_L^3 \). The scattering amplitude is given by:

\[
\mathcal{M}(W_L^a W_L^b \to W_L^c W_L^d) = A(s, t, u) \delta^{ab} \delta^{cd} + A(t, s, u) \delta^{ac} \delta^{bd} + A(u, t, s) \delta^{ad} \delta^{bc}
\]

where \( a, b, c, d = 1, 2, 3 \) and \( s, t, u \) are the usual Mandelstam kinematical variables.

In this approach it is possible to compute the function \( A(s, t, u) \) in \( \mathcal{O}(p^4) \) [250, 251]:

\[
A(s, t, u) = \frac{s}{v^2} + \frac{1}{4 \pi v^4} \left( 2 L_1 s^2 + L_2 (t^2 + u^2) \right) + \frac{1}{16 \pi^2 v^4} \left( -\frac{t}{6} (s + 2t) \log(-\frac{t}{\mu^2}) - \frac{u}{6} (s + 2u) \log(-\frac{u}{\mu^2}) - \frac{s^2}{2} \log(-\frac{s}{\mu^2}) \right)
\]

The values of \( L_1 \) and \( L_2 \) depend on the model, but are expected to be in the range \( 10^{-2} \) to \( 10^{-3} \).

The usual Chiral Lagrangian approach does not respect unitarity at high energies. The Inverse Amplitude Method (IAM) [217, 218, 249], which is based on the assumption that the inverse of the amplitude has the same analytic properties as the amplitude itself, has been very successful at describing low energy hadron scattering. The most interesting feature of this approach is that it allows us to describe different reactions by using only the two parameters \( L_1 \) and \( L_2 \).

In analogy to \( \pi\pi \) scattering, there are three possible isospin channels \( I = 0,1,2 \). At low energies, the states of lowest momentum \( J \) are the most important, and thus only the \( a_{00}, a_{11} \) and \( a_{20} \) partial waves are considered. It is possible to reproduce, with the IAM model, the broad Higgs-like resonance in \( (I, J) = (0,0) \) channel as well as resonant and non-resonant scattering in the channel \( (1,1) \) by selecting appropriate values for \( L_1 \) and \( L_2 \). It has been shown [224] that in the \( (I = 1, J = 1) \) channel there may exist narrow resonances up to \( 2500 \text{ GeV} \) and this scattering only depends on the combination of \( (L_2 - 2L_1) \).
Resonant $W_L Z_L \rightarrow W_L Z_L$ channel As a reference for the IAM model, the process $W_L Z_L \rightarrow W_L Z_L$, with $Z \rightarrow ll$ ($l = e, \mu$) and $W \rightarrow jj$ is used [241]. A modified version of PYTHIA 5.7 was used to generate $V_L V_L$ scattering processes for each value of $L_1$ and $L_2$. The simulation was done for two values of $(L_2 - 2L_1) = 0.006$ and 0.01, which yield $\sigma \times BR$ of 1.5 fb and 2.8 fb, with mass peaks at 1.5 TeV and 1.2 TeV respectively.

Irreducible background arises from continuum $WZ$ production and the main QCD background is from $Z + \text{jets}$ production with two final state jets faking the $W$ decay if their invariant mass is close to $m_W$. $t\bar{t}$ production is potentially dangerous but is efficiently suppressed by a cut on the invariant mass of leptons from the $W$ decay [241]. The following cuts were used for background rejection:

- Two isolated leptons with the same flavour and opposite charges in the region $|\eta| < 2.5$ and $p_T > 100$ GeV. Their invariant mass was required to lie in the region $|m_{ll} - m_Z| < 6$ GeV.
- Jets were reconstructed in a cone of width $\Delta R = 0.2$. Only two jets with $p_T > 50$ GeV were allowed in the central region ($|\eta| < 2$) and $|m_{jj} - m_W| < 15$ GeV was required. Only $W$ and $Z$ with $p_T > 200$ GeV were kept.
- In the forward region ($2 < |\eta| < 5$), jets were reconstructed in a cone of width $\Delta R = 0.5$ and events were accepted only if jets with $p_T > 30$ GeV and $E_{jet} > 500$ GeV were present in each hemisphere.

The expected number of signal and background events after all cuts and for $\mathcal{L} = 100$ fb$^{-1}$ are presented in Table 33. The mass spectra obtained after all cuts (Figure 49) shows a clear peak with a width of 75 GeV (100 GeV) for the 1.2 TeV (1.5 TeV) resonance and 14 (8) signal events in the window $|m_{WZ} - m_V| < 2\sigma$. The contribution from irreducible backgrounds is negligible and is below 0.05 events inside the mass window. It is clear that such a narrow resonance could be detected easily after a few years of high luminosity.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>$M_V=1.2$ TeV</th>
<th>$M_V=1.5$ TeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_L Z_L$</td>
<td>284</td>
<td>145</td>
</tr>
<tr>
<td>$Z + \text{jets}$</td>
<td>2187</td>
<td>1781</td>
</tr>
<tr>
<td>$m_{jj} = m_W \pm 15$ GeV</td>
<td>101</td>
<td>46</td>
</tr>
<tr>
<td>Leptonic cuts</td>
<td>70</td>
<td>36</td>
</tr>
<tr>
<td>Forward jet tagging</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>$Z + \text{jets}$</td>
<td>3</td>
<td>1.3</td>
</tr>
</tbody>
</table>

Non-resonant channels If nature does not provide resonances in $V_L V_L$ scattering, the measurement of cross sections at high mass for non-resonant channels becomes the only probe for the mechanism of regularisation of the cross section. It would then be essential to understand very well the magnitude and energy dependence of backgrounds. Those channels can be particularly important since it has been shown that a complementary relationship exists between resonant and non-resonant processes [216, 171, 242]. Both $W_L Z_L$ and $W_L W_L$ scattering have been studied within the ATLAS framework.

$W_L Z_L \rightarrow W_L Z_L$ The non-resonant $W_L Z_L \rightarrow W_L Z_L$ process, with $Z \rightarrow ll$ and $W \rightarrow lv$ ($l = e, \mu$), was incorporated in PYTHIA and used with two values of $L_1$: 0.003 and 0.01, leading to $\sigma \times BR = 0.19$ fb and 0.11 fb respectively. The main features of the signal are:

- The presence of two high-$p_T$ leptons of same flavour and opposite charge in the barrel region, having an invariant mass consistent with the mass of the $Z$ boson.
• One additional high-$p_T$ lepton in the barrel region.
• Significant missing momentum in the event due to the presence of a neutrino.
• The presence of energetic jets in the forward region.

The main irreducible background, coming from continuum $WZ$ production, was generated by PYTHIA with $\sigma \times BR = 13.5$ fb. The main reducible background is the QCD process $Zt\bar{t}$ where one of the $W$ bosons from a $t$-quark decays into a lepton and an anti-neutrino. The value of $\sigma \times BR$ of this process is 26.3 fb. A less important contribution comes from $ZZ$ production with $\sigma \times BR = 1.52$ fb. These different backgrounds were rejected with a high efficiency by using the following cuts:

• Two isolated leptons of same flavour and opposite charge were required in the central region with $p_T > 30$ GeV and invariant mass satisfying $|m_{ll} - m_Z| < 6$ GeV. One additional lepton was required.
• A missing momentum of at least 75 GeV.
• At least one jet with $p_T > 40$ GeV and $E_{jet} > 500$ GeV should be present in the forward region.

In order to analyse $WZ$ scattering in the high-mass region, the transverse mass $M_T$

$$M_T^2 = \left[ \sqrt{M^2(lll) + p_T^2(lll) + |\not{p}_T|} \right]^2 - |\not{p}_T(lll) + \not{p}_T|^2$$

was used. $M(lll)$ and $p_T(lll)$ are the invariant mass and transverse momentum of the three charged leptons and $\not{p}_T$ is the missing momentum in the event. The transverse mass $M_T$ distribution for the $WLZL$ scattering and for $Zt\bar{t}$ background, after the application of cuts, is shown in Figure 50. The number of signal and background events with the invariant mass of $WZ$ system larger than 600 GeV for an integrated luminosity of $\mathcal{L} = 500$ fb$^{-1}$ and applying different cuts, are shown in Table 34. The $ZZ$ background is not shown since it is effectively removed by the requirement of missing transverse momentum.

**Like-sign $W$ pair production** $W^+_L W^-_L$ production has been extensively studied [243]. As possible scenarios for this process by $WLZL$ scattering, the following are considered:

• A $t$-channel exchange of a Higgs with $M_H = 1$ TeV, ($W_L W_L$ only), simulated with PYTHIA with $\sigma \times BR = 1.33$ fb (the same parameters of the resonance as in Section 6.41 were used).
Fig. 50: The transverse mass $M_T$ distribution for $ZW$ system (GeV) for $W^+_L Z_L$ scattering and for $Zt\bar{t}$.

Table 34: Number of expected events for the $WZ$ signal and backgrounds with an integrated luminosity of 500 fb$^{-1}$.

<table>
<thead>
<tr>
<th>Cuts</th>
<th>$L_1=0.003$</th>
<th>$L_1=0.01$</th>
<th>$Zt\bar{t}$</th>
<th>$WZ$</th>
<th>$S/\sqrt{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$L_1=0.003$</td>
<td>$L_1=0.01$</td>
<td></td>
</tr>
<tr>
<td>Leptonic cuts</td>
<td>33.3</td>
<td>18.3</td>
<td>223.</td>
<td>762</td>
<td></td>
</tr>
<tr>
<td>Missing momentum</td>
<td>25.9</td>
<td>14.3</td>
<td>85.1</td>
<td>405</td>
<td></td>
</tr>
<tr>
<td>$p_T(Z) &gt; M_T/4$</td>
<td>22.2</td>
<td>12.2</td>
<td>67.</td>
<td>300</td>
<td></td>
</tr>
<tr>
<td>Forward jet tagging</td>
<td>14</td>
<td>7.3</td>
<td>15</td>
<td>10.8</td>
<td>2.7</td>
</tr>
</tbody>
</table>


The K-matrix unitarised amplitude \[ a_{IJ}^K = \frac{\text{Re}(\sigma_{IJ})}{1 - \text{Re}(\alpha_{IJ})} \], where \( a_{IJ} \) is the low-energy theorem amplitude, proportional to \( s \). This model is constructed to satisfy explicitly elastic unitarity and would yield the maximum expected signal. The \( \sigma \times BR = 1.12 \) fb.

A Chiral Lagrangian model, as in the \( WZ \) resonant channel, with the same parameters: \( L_1 = 0 \), and \( L_2 = 0.006 \) or 0.01, leading to \( \sigma \times BR = 0.484 \) and 0.379 fb, respectively.

Backgrounds from continuum \(WW\) bremsstrahlung produce mostly transverse \( W\)'s. Other backgrounds include processes involving non-Higgs exchange, as well as QCD processes of order \( \alpha \alpha_s \) in amplitude, with gluon exchange and \( W\) bremsstrahlung from interacting quarks. The effects of \( Wt\bar{t} \) and \( WZ \) backgrounds are also considered. The signal was generated with \textsc{Pythia 6.2} and backgrounds were incorporated into \textsc{Pythia} from a Monte Carlo generator based on Barger’s work [244], which takes into account all diagrams. The contribution from electroweak processes not involving the Higgs were estimated by assuming a low-mass Higgs \( (M_H = 100 \text{ GeV}) \).

An analysis was performed using the fast \textsc{Atlas} detector simulation (\textsc{AtlFast}), with parameters set for high luminosity. The following leptonic cuts were first applied:

**L1.** Two positively charged isolated leptons in the central region \( (p_T > 40 \text{ GeV} \text{ and } |\eta| < 1.75) \) must be identified. They will satisfy the trigger requirement.

**L2.** The opening angle between the two leptons, in the transverse plane, must satisfy: \( \cos \Delta \phi < -0.5 \). This cut selects preferentially events with longitudinal \( W\)'s which have high \( p_T \). The invariant mass of the two leptons was further required to satisfy \( m_{ll} > 100 \text{ GeV} \). This latter cut eliminates few events in the low \( m_{ll_{\nu\nu}} \) region.

At the jet level, backgrounds can be reduced by requiring that:

**J1.** No jet having \( p_T > 50 \text{ GeV} \) be present in the central region \( (|\eta| < 2) \). This reduces significantly the background from the \( Wt\bar{t} \) process.

**J2.** Two jets must be present in the forward and backward regions: \( \eta > 2 \) and \( \eta < -2 \), with energies \( > 300 \text{ GeV} \).

**J3.** A lower \( p_T \) was required for the forward jets: \( p_T < 150 \text{ GeV} \) for the first and \( p_T < 90 \text{ GeV} \) for the second.

![Figure 51](https://example.com/figure51.png)

**Figure 51:** Distribution of invariant transverse mass of the two leptons with \( E_T^{\text{miss}} \) in the \( W_L^+W_L^- \rightarrow t^+t^-\nu\nu \) process, after three years of high luminosity running. Full line: K-matrix unitarisation; dashed line: Higgs with \( M_H = 1 \text{ TeV} \), at tree level; hatched area: background from transverse \( W\)'s.

Figure 51 shows expected mass distribution of the \( ll\nu\nu \) system, for an integrated cross section of \( 300 \text{ fb}^{-1} \), after all cuts were applied, accounting only for transverse momentum. No correction was made for pile-up effects in jet tagging or central jet veto. If one counts only events with \( m_{ll_{\nu\nu}} > 400 \text{ GeV} \),
a significant signal to background ratio is obtained (see Table 35). As expected, the K-matrix scenario gives the highest signal [216] - this could be observable after a few years of high luminosity running. By contrast, it was shown in Section 6.42 that if the \( \rho \) resonance is itself clearly observable in the resonant channel, then the signal will be very low. The major remaining background, especially at low values of \( m_{H}\nu\nu \), is from continuum transverse \( W \) pairs. Note that only a \( W_{L}^{+}W_{L}^{+} \) signal was searched for in this analysis. Combining the results with \( W_{T}^{-}W_{T}^{+} \) would add approximately one-half to one-third of the signal and backgrounds. The Chiral Lagrangian model, with its parameters leading to a resonance in the \( WZ \) system, would yield a very weak signal in the \( W^{+}W^{+} \) channel, confirming the complementarity relationship between those two channels [216, 171, 242].

Table 35: Number of events expected for an integrated luminosity of 300 fb\(^{-1}\), after successive applications of cuts. The results are for \( m_{H}\nu\nu > 400 \text{ GeV} \).

<table>
<thead>
<tr>
<th></th>
<th>Lepton cuts</th>
<th>Jet cuts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>L1</td>
<td>L2</td>
</tr>
<tr>
<td>( M_{H}=1 \text{ TeV} )</td>
<td>59</td>
<td>56</td>
</tr>
<tr>
<td>K-matrix</td>
<td>90</td>
<td>86</td>
</tr>
<tr>
<td>Chiral Lagrangian ( L_{2}=0.006 )</td>
<td>22</td>
<td>21</td>
</tr>
<tr>
<td>Chiral Lagrangian ( L_{2}=0.01 )</td>
<td>15.1</td>
<td>14.1</td>
</tr>
<tr>
<td>( W_{T}W_{T} )</td>
<td>350</td>
<td>243</td>
</tr>
<tr>
<td>gluon exchange</td>
<td>76</td>
<td>51</td>
</tr>
<tr>
<td>( Wt\bar{t} )</td>
<td>93</td>
<td>71</td>
</tr>
<tr>
<td>( WZ )</td>
<td>36</td>
<td>35</td>
</tr>
</tbody>
</table>

6.43 Technicolor

Technicolor (TC) provides a framework for dynamical electroweak symmetry breaking [34, 35]. It assumes the existence of techni-fermions possessing a technicolor charge and interacting strongly at high scale. Chiral symmetry is broken by techni-quark condensates giving rise to Goldstone bosons, the techni-pions, which are the longitudinal degrees of freedom of the \( W \) and \( Z \) gauge-bosons. TC has been extended (extended TC, or ETC) to allow the generation of fermion masses [245, 246]. In order to account for the absence of FCNCs, the coupling constant is required to “walk”, rather than “run”. To achieve a walking \( \alpha_{TC} \), multi-scale TC models contain several representations of the fundamental family, and lead to the existence of techni-hadron resonances accessible at LHC energies. Such models [247, 248] are constrained by precision electroweak data [250, 251], but not necessarily excluded [252, 253]. However, the constraints from those data make it unnatural to have a large top quark mass. In top-colour-assisted TC (TC2) models [254, 255], the top quark arises in large part from a new strong top-colour interaction, which is a separate broken gauge-sector.

The possible observation of TC resonances using the ATLAS detector is described in [256]. In particular, the search for a \((I=1, J=1)\) techni-rho resonance, a techni-pion and a techni-omega has been performed. Although certain models, with a given set of parameters, are used as reference, the signals studied can be considered generic in any model which predicts resonances. The model adopted here is that of multi-scale TC [257, 258], with the TC group \( SU(N_{TC}) \) where \( N_{TC} = 4 \) and two isotriplets of techni-pions. The longitudinal gauge-boson and the techni-pions mix

\[
|\Pi_{T}| = \sin \chi |W_{L}| + \cos \chi |\pi_{T}|
\]
with a mixing angle which has a value $\sin \chi = 1/3$. The decay constant of the mixed state is $F_T = F_\pi \sin \chi = 82$ GeV and the charge of the up-type (down-type) techni-fermion is $Q_U = 1$ ($Q_D = 0$). This model is incorporated in PYTHIA 6.1. The decay channels of $\rho_T$ depend on the assumed masses of the techni-particles. Some mass scenarios have been considered to be representative of what one may expect to probe at the LHC and it is also assumed that the $\pi_T$ coupling to the top quark is very small, as may be expected in TC2 models. The following sections present an example showing a typical analysis for extracting TC signals. More channels and an extensive description can be found in [256].

$\rho_T^{\pm} \to W^{\pm} Z \to l^\pm \nu l^\mp l^-$ This decay could be the cleanest channel for the techni-rho detection and complements the study shown in Section 6.42. The good efficiency of the ATLAS and CMS detectors for lepton detection and missing transverse energy measurement will provide good identification of the $W$ and $Z$ bosons. Table 36 shows the parameters for the various sets of events which were generated. For each set, $10^4$ events were generated and the signal was normalised to three years of low luminosity running at the LHC (30 fb$^{-1}$). The branching ratios quoted include a preselection on the transverse mass ($m > 150, 300, 600$ GeV for $m_{\rho_T^\pm} = 220, 500$ and 800 GeV respectively).

Table 36: Signal parameters for the $\rho_T^{\pm} \to W^{\pm} Z \to l^\pm \nu l^\mp l^-$. The last column gives the significance ($S/\sqrt{B}$) for three years of low luminosity running.

<table>
<thead>
<tr>
<th>$m_{\rho_T}$ (GeV)</th>
<th>$m_{\pi_T}$ (GeV)</th>
<th>$\Gamma_{\rho_T}$ (GeV)</th>
<th>$BR$ (GeV)</th>
<th>$\sigma \times BR$ (pb)</th>
<th>$S/\sqrt{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>220</td>
<td>110</td>
<td>0.93</td>
<td>0.13</td>
<td>0.16</td>
<td>31.6</td>
</tr>
<tr>
<td>500</td>
<td>300</td>
<td>4.47</td>
<td>0.21</td>
<td>1.3 x 10^{-2}</td>
<td>14.7</td>
</tr>
<tr>
<td>500</td>
<td>500</td>
<td>1.07</td>
<td>0.87</td>
<td>5.4 x 10^{-2}</td>
<td>64.2</td>
</tr>
<tr>
<td>800</td>
<td>300</td>
<td>52.4</td>
<td>0.032</td>
<td>3.6 x 10^{-4}</td>
<td>1.2</td>
</tr>
<tr>
<td>500</td>
<td>800</td>
<td>7.6</td>
<td>0.22</td>
<td>2.5 x 10^{-3}</td>
<td>10.9</td>
</tr>
</tbody>
</table>

The only background which needs to be considered is the continuum production of $WZ$ gauge-bosons, with $\sigma = 21$ pb. The cuts which were applied are:

- At least three charged leptons were required (with $E_T > 20$ GeV for electrons and $E_T > 6$ GeV for muons), two of which must have the same flavour and opposite charge.
- The invariant mass of the lepton pair with the same flavour and opposite sign should be close to that of the $Z$: $|m_{l^+l^-} - m_Z| < 5$ GeV.
- The longitudinal momentum of the neutrino is calculated (with a 2-fold ambiguity) from the missing transverse energy and the momentum of the unpaired lepton assuming an invariant mass $m_{l_\nu} = m_W$. Once the $W$ and $Z$ were reconstructed, their transverse momentum was required to be larger than 40 GeV.
- Only events for which the decay angle with respect to the direction of the $WZ$ system ($\rho_T$) in its rest frame was $|\cos \theta| < 0.8$ were accepted.

The significance ($S/\sqrt{B}$) of the signal ($S$) above the background ($B$) is shown in Table 36. The number of signal and background events was counted in mass regions around the $\rho_T$ peak: 210 to 240, 460 to 560 and 740 to 870 for $m_{\rho_T} = 220$, 500 and 800 GeV respectively. No evident signal can be observed for cases (b), (e) and (f) (see Figure 52), principally because the $\rho_T$ resonance is too wide.
6.5 The degenerate BESS Model at the LHC

It is well known that naïve Dynamical Symmetry Breaking (DSB) models like standard QCD-scaled technicolor generally tend to provide large corrections to electroweak precision observables. New physics effects are naturally small if decoupling holds. In fact in this case the corrections to electroweak observables are power suppressed in the limit in which the masses of the new particles are made large. It is thus a natural question as to whether examples of DSB models with decoupling do exist.

Here we will focus on a scheme of DSB, called degenerate BESS (D-BESS) [43] in which decoupling is naturally satisfied in the low energy limit. The model predicts the existence of two triplets of new resonances corresponding to the gauge-bosons of an additional gauge-symmetry $SU(2)_L \otimes SU(2)_R$. The global symmetry group of the theory is $(SU(2)_L \otimes SU(2)_R)^3$ breaking down spontaneously to $SU(2)_D \otimes (SU(2)_L \otimes SU(2)_R)$ and giving rise to nine Goldstone bosons. Six of these give mass to the new gauge-bosons, which turn out to be degenerate. As soon as we perform the gauging of the subgroup $SU(2)_L \otimes U(1)_Y$, the three remaining Goldstone bosons disappear giving masses to the SM gauge-bosons.

What makes the model [43] so attractive is the fact that, due to the degeneracy of the masses and couplings of the extra gauge-bosons $(L^\pm, L^3, R^\pm, R^3)$, it decouples, so all the deviations in the low-
energy parameters from their SM values are strongly suppressed. Also, the degeneracy is protected by the additional “custodial” symmetry (SU(2)\textsubscript{L} \otimes SU(2)\textsubscript{R}). The deviations from the SM predictions come from the mixing of (L_\mu, R_\mu) with the standard gauge-bosons. In order to compare with the experimental data, radiative corrections have to be taken into account. Since the model is an effective parametrisation of a strongly interacting symmetry breaking sector, one has to introduce a UV cut-off \Lambda. We neglect the new physics loop corrections and assume for D-BESS the same radiative corrections as for the SM with \( M_H = \Lambda = 1 \text{ TeV} \) [43]. The 95\% CL bounds on the parameter space of the model coming from the precision electroweak data can be expressed by the following approximated relation: \( M(\text{TeV}) \geq 2.4 \frac{g}{g'} \), where \( M \) is the common mass of the new resonances, \( g \) and \( g' \) are the standard SU(2)\textsubscript{L} and the new strong gauge-couplings respectively. Therefore one has a large allowed region available for the model even for the choice \( M_H = \Lambda = 1 \text{ TeV} \) - a value highly disfavoured by the fit within the SM [259]. Also, the bounds on the D-BESS model from the direct search for new gauge bosons performed at Tevatron are very loose [43]. This allows the existence of a strong electroweak sector at relatively low energies such that it may be accessible with accelerators designed for the near future. A peculiar feature of this strong electroweak symmetry breaking model is the absence of \( WW \) enhancement due to the absence of direct couplings of the new resonances to the longitudinal weak gauge-bosons. For this reason, the gold plated channels to consider for discovering (L_\mu, R_\mu) are the fermionic ones.

![Transverse mass distributions](image)

**Fig. 53**: Transverse mass differential distributions for \( pp \rightarrow L^\pm, W^\pm \rightarrow e\nu_e \) events at the LHC within the D-BESS model (dash line) for \( g/g' = 0.1 \) and \( M = 1 \text{ TeV} \) (left), \( M = 2 \text{ TeV} \) (right). The solid line is the SM prediction.

Here we have considered the production of these new resonances at the LHC for the following configuration \( \sqrt{s} = 14 \text{ TeV} \) and \( \mathcal{L} = 10^{34} \text{ cm}^{-2}\text{sec}^{-1} \) and for the electron channel decay (the muon channel was studied in [260]). The events were generated using PYTHIA Monte Carlo (version 6.136) [123]. Only the Drell-Yan mechanism for production was considered since it turns out to be the dominant one. We have analysed the production of the charged resonances in \( pp \rightarrow L^\pm, W^\pm \rightarrow e\nu_e \) (\( R^\pm \) are completely decoupled) and neutral ones in \( pp \rightarrow L_3, R_3, Z, \gamma \rightarrow e^+e^- \). The signal events were compared with the background from SM production. We have performed a rough simulation of the detector, in particular, assuming a 2\% smearing in the momenta of charged leptons and a resolution \( \Delta E_T^{\text{miss}} = 0.6\sqrt{E_T^{\text{miss}}} \) in the missing transverse energy. In the neutral channel, we have assumed an error of 2\% in the reconstruction of the \( e^+e^- \) invariant mass, which includes bremsstrahlung effects [261]. We have considered several choices of the model parameters, in the region allowed by the present bounds, and for each case we have selected cuts to maximise the statistical significance of the signal. In Figure 53 we show the transverse mass distributions for the signal and for the SM background for the case \( M = 1 \text{ TeV} \) (left)
and $M = 2$ TeV (right) and $g/g'' = 0.1$. The following cuts have been applied for $M = 1$ TeV: $p_T^e$ and $|p_T^{miss}| > 0.3$ TeV and $M_T > 0.8$ TeV. The number of signal events per year is 3200, the corresponding background is of 1900 events. The corresponding statistical significance $S/\sqrt{S+B}$ for one year of running is 44. For $M = 2$ TeV, the applied cuts are: $p_T^e$ and $|p_T^{miss}| > 0.7$ TeV and $M_T > 1.8$ TeV, resulting in $S = 108, B = 46$ and $S/\sqrt{S+B} = 8.7$.

Fig. 54: Invariant mass differential distributions for $pp \rightarrow L_3R_3Z, \gamma \rightarrow e^+e^-$ events at the LHC within the D-BESS model (dash line) for $g/g'' = 0.1$ and $M = 1$ TeV (left), $M = 2$ TeV (right). The solid line is the SM prediction.

In Figure 54, we show the results of our simulation for the same choice of the parameters as in Figure 53 for the neutral channel. The following cuts have been applied for $M = 1$ TeV: $p_T^e$ and $|p_T^-|$ > 0.3 TeV and $M_{e^+e^-} > 0.8$ TeV. The number of signal events per year is 620, the background is of 1200 events with a corresponding statistical significance of 15. For $M = 2$ TeV, the cuts are: $p_T^e$ and $|p_T^-|$ > 0.7 TeV and $M_{e^+e^-} > 1.8$ TeV, resulting in $S = 24, B = 30$ and $S/\sqrt{S+B} = 3.3$. It turns out that the cleanest signature is in the neutral channel, but the production rate is lower than for the charged one. Also we observe that, due to the fact that the D-BESS resonances are almost degenerate ($\Delta M/M \sim (g/g'')^2$), it will be impossible to disentangle $L_3$ and $R_3$ which both contribute to the peak of the signal in Figure 54.

Our conclusion is that the LHC will be able to discover a strong electroweak resonant sector as described by the degenerate BESS model for masses up to 2 TeV - in some cases with very significant numbers of events. Furthermore, if no deviations from the SM predictions are seen within the statistical and systematic errors, the LHC with $L = 100$ fb$^{-1}$ will put a 95% CL bound $g/g'' < 0.04 - 0.06$ for $0.5 < M(\text{TeV}) < 2$ [260].

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We review the prospects for bottom production physics at the LHC.

1. INTRODUCTION

In the context of the LHC experiments, the physics of bottom flavoured hadrons enters in different contexts. It can be used for QCD tests, it affects the possibilities of $B$ decays studies, and it is an important source of background for several processes of interest.

The physics of $b$ production at hadron colliders has a rather long story, dating back to its first observation in the UA1 experiment. Subsequently, $b$ production has been studied at the Tevatron. Besides the transverse momentum spectrum of a single $b$, it has also become possible, in recent time, to study correlations in the production characteristics of the $b$ and the $\bar{b}$.

At the LHC new opportunities will be offered by the high statistics and the high energy reach. One expects to be able to study the transverse momentum spectrum at higher transverse momenta, and also to exploit the large statistics to perform more accurate studies of correlations.

This chapter is organized as follows.

Section 2 is mostly theoretical. Its goal is to provide benchmark cross sections and distributions for the LHC, including rates relevant for the trigger requirements of the experiments. Furthermore, a discussion of the present status of $b$ production phenomenology at hadron colliders is given. In this context, one cannot forget that the theoretical status is a mixed success. On one side, the shape of distributions and correlations are reasonably well explained by perturbative QCD. On the other side, however, the observed cross sections at the Tevatron are larger than QCD predictions. It is hoped that further studies may help to understand the nature of the discrepancy. As of now, we see two possible explanations: either the absolute normalization of the cross section is not correctly predicted due to the presence of large higher order terms, or the shape of the distributions is distorted by some perturbative or non-perturbative effects (like, for example, fragmentation effects). With the wide $p_T$ range covered by the LHC experiments, and perhaps also with the possibility of performing more accurate studies of correlations, these two possibilities may be distinguished. The problem of fragmentation effects has been studied in this workshop also from the point of view of hadronization models in Monte Carlo programs, in Section 3. This study deals with the hadronization model in the HERWIG Monte Carlo program. Its aim was to understand whether, in simple realistic models of hadronization, the usual assumption of QCD factorization is really at work. In general, the problem of studying how realistic is the heavy flavour production mechanism implemented in shower Monte Carlo’s is quite important, and probably will require a considerable effort. Along this line, in Section 4, a problem in the heavy flavour production mechanisms implemented in PYTHIA is examined.

Further self-contained theoretical topics are dealt with in Sections 5 and 6. Section 5 deals with the charge asymmetry in $b$ production in $pp$ collisions. In this context, QCD is not of great help, since in
perturbative QCD charge asymmetries turn out to be extremely small. Instead, studies are made within specific hadronization models, that are parametrized in such a way that they fit charm asymmetry data. This topic, besides being interesting in its own, since it deals with a phenomenon which is dominated by non-perturbative physics, has also an impact on $CP$ violation studies in $B$ decays.

In Section 6 quarkonium production is discussed. This subject has been intensively studied in recent years, following an initial CDF observation of a $J/\Psi$ production rate much higher than theoretical predictions. This has triggered, from the theoretical side, the understanding that the fragmentation process is the dominant mechanism in quarkonium production. Besides this, a novel branch of applications of perturbative QCD, the NRQCD approach, has emerged, that may be useful to explain the production process.

In Section 7, the prospects for $b$ detection are discussed. It is shown that there is a complementarity between ATLAS/CMS and the LHCb experiment, with a certain region of overlap. In particular, the LHCb experiment can detect very low momentum heavy quarks, while the other experiments can reach the very high transverse momentum region. Some results on correlations measurements are also given, exploring the possibility of looking at one $b$ decaying into a $J/\Psi$, and the other decaying semileptonically. Double heavy flavour production, charge asymmetry, polarization effects, and doubly-heavy meson production are also discussed.

In Section 8 the tuning of the multiple interaction parameters in PYTHIA is illustrated. The correct treatment of multiple interactions is important to model the multiplicity observables in both minimum-bias and heavy flavour events.

2. **BENCHMARK CROSS SECTIONS**

2.1 **Total cross sections**

It is assumed that heavy flavour production in hadronic collisions can be described in the usual improved parton model approach, where light partons in the incoming hadrons collide and produce a heavy quark-antiquark pair via elementary strong interaction vertices, like, for example, in the diagram of fig. 1.

![Fig. 1: Typical diagram for heavy flavour production](image)

The description is appropriate for all hard processes in hadronic collisions, and thus, in the case of heavy flavours, is applicable as long as the mass of the heavy flavour can be considered sufficiently large. The perturbative QCD cross section for heavy flavour production has been computed to next-to-leading order accuracy (i.e. $O(\alpha_s^3)$) a long time ago [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11], and a large amount of experimental and theoretical work has been done in this field. A relatively recent account of the status of this field can be found in ref. [12]. It can be said that qualitatively the QCD description of heavy flavour production seems to be adequate also for charm production, while quantitatively large uncertainties are present in the calculation of the charm and bottom cross section. Only for a quark as heavy as the top quark the perturbative calculation seems (up to now) to predict the cross section with a good accuracy.
Table 1: Dependence of the $b$ cross section on scale choices.

<table>
<thead>
<tr>
<th>$\mu_F/m_b$</th>
<th>$\mu_R/m_b$</th>
<th>Total ($\mu b$)</th>
<th>Born ($\mu b$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.50</td>
<td>1.00</td>
<td>$0.2779 \times 10^4$</td>
<td>$0.6465 \times 10^2$</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>$0.4960 \times 10^3$</td>
<td>$0.1796 \times 10^3$</td>
</tr>
<tr>
<td>2.00</td>
<td>1.00</td>
<td>$0.6453 \times 10^3$</td>
<td>$0.3253 \times 10^3$</td>
</tr>
<tr>
<td>0.50</td>
<td>0.50</td>
<td>$0.5126 \times 10^3$</td>
<td>$0.1078 \times 10^3$</td>
</tr>
<tr>
<td>1.00</td>
<td>0.50</td>
<td>$0.8289 \times 10^3$</td>
<td>$0.2995 \times 10^3$</td>
</tr>
<tr>
<td>2.00</td>
<td>0.50</td>
<td>$0.9538 \times 10^3$</td>
<td>$0.5426 \times 10^3$</td>
</tr>
<tr>
<td>0.50</td>
<td>2.00</td>
<td>$0.1758 \times 10^3$</td>
<td>$0.4355 \times 10^3$</td>
</tr>
<tr>
<td>1.00</td>
<td>2.00</td>
<td>$0.3353 \times 10^4$</td>
<td>$0.1209 \times 10^4$</td>
</tr>
<tr>
<td>2.00</td>
<td>2.00</td>
<td>$0.4669 \times 10^3$</td>
<td>$0.2191 \times 10^3$</td>
</tr>
</tbody>
</table>

Large uncertainties are also found in the calculation of the bottom production cross section at the LHC. The largest uncertainty is due to unknown higher order effects, and it is traditionally quantified by estimating the scale dependence of the cross section when the renormalization and factorization scales are varied by a factor of 2 above and below their central value, which is usually taken equal to the heavy quark mass. Since this uncertainty is due to a limitation in our current theoretical knowledge, it is hard to overcome. Other sources of uncertainty are related to theoretical and experimental errors in the parameters that enter the perturbative calculation: the value of the strong coupling constant, the heavy quark mass, and the parton density functions.

We present here a benchmark study of $b$ total cross sections at the LHC, using the FMNR package for heavy flavour cross sections [5] [8] (the code for this package is available upon request to the authors). In the study we consider

- The dependence of the total cross section on the choice of the factorization and renormalization scales. We will use the values $\mu = m_b, 2m_b, m_b/2$.
- The dependence on the parton density parametrization. We will use the sets MRST [13], MRST($g_1$), MRST($g_1$), MRST($\alpha_S g_1$) and MRST($\alpha_S g_1$). The first set is used as reference set. MRST($g_1$) and MRST($g_1$) have extreme gluon densities, MRST($\alpha_S g_1$)-MRST($\alpha_S g_1$) have extreme values of the strong coupling constant: $\Lambda_5 = 220$ MeV for MRST, 164 MeV for MRST($\alpha_S g_1$), 280 MeV for MRST($\alpha_S g_1$). Cross section values obtained with the CTEQ4 [14] set are very similar to the MRST set. We have preferred to use the MRST sets because they gave us the possibility to perform a study of sensitivity to $\Lambda$ and to variations in the gluon density.
- The dependence on the $b$ quark mass: $m_b = 4.75 \pm 0.25$ GeV.

Factorization and renormalization scale dependence of the total cross section at $\sqrt{s} = 14$ TeV is reported in table 1, where we have used the MRST parton densities, with $\Lambda_5 = 220$ MeV, and we have fixed the $b$ mass at the value $m_b = 4.75$ GeV. Notice that:

- If we keep $\mu_F = \mu_R$, the full cross section variation is small (467 to 512 $\mu b$).
- The largest cross section corresponds to large $\mu_F$ and small $\mu_R$
- The smallest cross section corresponds to small $\mu_F$ and large $\mu_R$

This is understood since, at small $x$, the gluon density $g(x)$ grows with the scale, and $\alpha_S$ decreases with the scale.

The dependence on the choice of parton density parametrization is shown in table 2. As one can see, the sensitivity to the variation of the gluon density is small. Apparently, the constraints from HERA data are strong enough in the $x$ region where most of the $b$ production takes place. The dependence upon the strong coupling constant is instead larger, and can increase the upper limit of the cross section by about 40%. The last two sets have $\Lambda_5 = 164$ and 288 MeV respectively, corresponding to $\alpha_S(M_Z) = 0.1125$ and 0.1225, which is a reasonably large range.
Table 2: Parton density dependence of total cross sections (in \( \mu b \)).

<table>
<thead>
<tr>
<th></th>
<th>central</th>
<th>lowest</th>
<th>highest</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRST</td>
<td>0.4960 (10^3)</td>
<td>0.1758 (10^3)</td>
<td>0.9538 (10^3)</td>
</tr>
<tr>
<td>MRST((gl))</td>
<td>0.4868 (10^3)</td>
<td>0.1727 (10^3)</td>
<td>0.9337 (10^3)</td>
</tr>
<tr>
<td>MRST((gl))</td>
<td>0.4992 (10^3)</td>
<td>0.1751 (10^3)</td>
<td>0.9610 (10^3)</td>
</tr>
<tr>
<td>MRST((\alpha_s))</td>
<td>0.4487 (10^3)</td>
<td>0.1799 (10^3)</td>
<td>0.7878 (10^3)</td>
</tr>
<tr>
<td>MRST((\alpha_s))</td>
<td>0.6001 (10^3)</td>
<td>0.1894 (10^3)</td>
<td>0.1267 (10^3)</td>
</tr>
</tbody>
</table>

Mass uncertainties are quite important, especially if \(m_b\) is allowed to take very small values. This can be seen from table 3. We see that lowering the \(b\) mass from 4.5 down to 4 GeV raises the upper limit of the cross section by about 50%. It is however unlikely that such small values are viable. A rough view of the status of the bottom mass determination is given in fig. 2, which we obtained by taking the various determinations of the \(\overline{\text{MS}}\) bottom mass from the Particle Data Book, and rescale them by a factor of \((1+0.09+0.06)\), to account for the two-loop correction needed to translate the \(\overline{\text{MS}}\) mass into the pole mass. As one can see, not all determinations are consistent among each other. A critical review of all determinations is beyond the scope of this workshop. We should however point out that recent progress has been made in the bottom mass determination. The reader can find a summary of these issues and further references in ref. [15]. It is argued there that the bottom mass is determined with higher precision in processes where it is probed at short distances, like in the \(\overline{T}\) mass, or via sum rules applied to the bottom vector current spectral function in \(e^+e^-\) annihilation. The mass extracted in this way can be

Table 3: Mass dependence of total cross sections (in \( \mu b \)).

<table>
<thead>
<tr>
<th>(m_b) (GeV)</th>
<th>central</th>
<th>lowest</th>
<th>highest</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>0.7957 (10^3)</td>
<td>0.2336 (10^3)</td>
<td>0.1706 (10^3)</td>
</tr>
<tr>
<td>4.5</td>
<td>0.5789 (10^3)</td>
<td>0.1945 (10^3)</td>
<td>0.1138 (10^3)</td>
</tr>
<tr>
<td>5</td>
<td>0.4313 (10^3)</td>
<td>0.1609 (10^3)</td>
<td>0.8087 (10^3)</td>
</tr>
</tbody>
</table>

Fig. 2: Different determinations of the \(b\) quark pole mass.
reliably related to the so called \( \overline{\text{MS}} \) mass. The relation of the \( \overline{\text{MS}} \) mass to the pole mass is instead not so precise, because the perturbative expansion that relates the two quantities is not convergent. In ref. [15] a preferred value of \( \bar{m}_b(m_b) = 4.23 \pm 0.08 \) is given, where \( \bar{m}_b(m_b) \) is the \( \overline{\text{MS}} \) bottom mass at the scale of the bottom mass itself. The corresponding pole mass, obtained using the newly computed 3-loop relation between the \( \overline{\text{MS}} \) and pole mass [16] [17], is \( 4.98 \pm 0.09 \) GeV. If one wanted to account for the uncertainties due to the lack of convergence of the perturbative expansion, the range obtained in this way should be enlarged by some amount, of the order of 100 MeV. The question arises whether it would be possible to eliminate this uncertainty by expressing the hadroproduction cross section in terms of the \( \overline{\text{MS}} \) mass. In our view, the answer is most likely no, since the bottom hadroproduction cross section does not have the same inclusive character as the sum rules applied to the \( e^+ e^- \) bottom spectral function.

In the present work we thus used the traditional range \( 4.5 \) GeV < \( m_b < 5 \) GeV for the bottom pole mass in the hadroproduction process, keeping in mind that recent determinations seem to favour the upper region of this range. The sensitivity of the cross section to the bottom mass in this range is at most of \( \pm 10\% \), and it becomes much smaller if transverse momentum cuts are applied. Thus, as far as the LHC is concerned, this uncertainty is not very important.

The largest uncertainty in the cross section comes from the scale uncertainty, which is a (rather arbitrary) method to assess the possible impact of unknown higher order corrections. In the following we report a brief discussion of the origin of these large corrections. Radiative corrections for the total cross section are usually parametrized as follows. The total cross sections \( \sigma_{ij} \) for the various parton subprocesses (\( q\bar{q}, gg, gg \)) have a perturbative expansions given by

\[
\sigma_{ij} = \frac{\alpha_s^2(\mu)}{m^2} \left[ f^{(0)}_{ij}(\rho) + 4\pi\alpha_s \left( f^{(1)}_{ij}(\rho) + \bar{f}^{(1)}(\rho) \log \frac{\mu^2}{m^2} \right) \right],
\]

where \( \rho = 4m_b^2/\hat{s} \) and \( \hat{s} \) is the squared partonic center-of-mass energy. The functions \( f^{(0,1)}_{ij} \) for the \( q\bar{q} \) and \( gg \) subprocesses are displayed in fig. 3. Notice the behaviour near threshold

![Fig. 3: Partonic cross section for the \( q\bar{q} \) and \( gg \) subprocesses.](image)

\[
f^{(1)}_{q\bar{q}} \to \frac{f^{(0)}_{q\bar{q}}(\rho)}{8\pi^2} \left[ -\frac{\pi^2}{6\beta} + \frac{16}{3} \ln^2 (8\beta^2) - \frac{82}{3} \ln (8\beta^2) \right]
\]

\[
f^{(1)}_{gg} \to \frac{f^{(0)}_{gg}(\rho)}{8\pi^2} \left[ \frac{11\pi^2}{42\beta} + 12 \ln^2 (8\beta^2) - \frac{366}{7} \ln (8\beta^2) \right]
\]
due to Coulomb $1/\beta$ singularities and to Sudakov double logarithms. Near threshold, these terms may require special treatment, such as resummation to all orders. Notice also the constant asymptotic behaviour of $f^{(1)}_{gg}$, which may cause problems far above threshold.

Plotting the cross section as a function of the partonic energy $\tilde{s}$ may help to understand the origin of large corrections. We find that radiative corrections are large near the production threshold. This problem becomes more and more severe as we approach the production threshold. Thus, it is more important for production of $b$ at fixed target energies, or for production of $t\bar{t}$ pairs at colliders. Techniques to resum these large corrections to all orders of perturbation theory, at the NLO level are available [18], but it is found that little improvement is achieved for the bottom cross section at collider energies. Large corrections are also found far above threshold. This effect is bound to become more and more pronounced in the high energy limit. In order to reduce the scale uncertainties coming from these corrections, one should resum them at the next-to-leading order level. This problem has been discussed in the literature, so far, only at the leading logarithmic level [19] [20] [21] [22]. At the time of the completion of this workshop, no further progress has been achieved in this field.

In fig. 5 we present a study of the scale dependence of the total cross section as a function of $\rho$. We find a large scale dependence near threshold, due to both renormalization and factorization scale variation, and a large scale dependence far from threshold. Here, the renormalization scale dependence plays a dominant role. Renormalization scale variations are mainly due to the large variation of the coupling constant in the $O(\alpha_s^3)$ terms. Where radiative corrections are small, a reasonable scale compensation takes place. Thus, both the threshold and the high energy regions, where corrections are large, are strongly affected. Factorization scale variation has a strong impact on threshold corrections, while in the high energy region we observe some compensation. In fact, the cross section near threshold increases with $\mu_F$ near threshold, while above threshold the $\mu_F = m$ value is above both the $\mu_F = m/2$ and the $\mu_F = 2m$ curves, indicating the presence of some sort of compensation. As of now, it appears therefore that a better understanding of the high energy region will not strongly reduce the scale uncertainty, although it might, of course, improve our confidence in the error band we quote.

The study given here deals with total cross sections. It should be repeated with appropriate rapidity cuts, since this may reduce large effects due to the high energy limit. In general, we may expect that the
cross section with rapidity and transverse momentum cuts may have smaller error bars than the total. It is particularly interesting to investigate directly cross sections for muons originating from $B$ decays, since muons are often used as trigger objects for $B$ physics. We have performed this study using a simple implementation of the $B$ semileptonic decay in the FMNR program, that will be described in more details in the following subsections. The results are shown in table 4. The same results are also reported in fig. 6, since several features become more apparent there. First of all, we point out that, as expected, there is a considerable reduction in the scale dependence in these muon rates. This is mostly due to the presence of cuts in the transverse momentum of the muon, that increases the total transverse
the price of introducing a sensitivity to the fragmentation function parameter. We considered as realistic values of reduced, since they imply higher quark momenta. The reduction in the scale uncertainty is obtained at the smallest values are achieved for the highest momentum cuts. A non perturbative fragmentation function of the Peterson form was also included in the calculation, with ε parameter taking the values 0 (i.e., no fragmentation), 0.002 and 0.006. More details on its implementation are given in the following subsections. Observe that for softer fragmentation functions (i.e. larger ε parameter) the uncertainty is reduced, since they imply higher quark momenta. The reduction in the scale uncertainty is obtained at the price of introducing a sensitivity to the fragmentation function parameter. We considered as realistic values of ε between 0.002 and 0.006. The corresponding variation of the cross section is not large. The impact of an intrinsic transverse momentum of the incoming partons (see the following subsections) is also studied. We have chosen the unrealistically large value \( \langle k_T \rangle = 4 \) GeV just to show that its effect is in all cases not a dramatic one.

### Table 4: Cross sections (in \( \mu b \)) for \( b \rightarrow \mu + X \) production, with a muon, or both, satisfying appropriate cuts. Only muons coming directly from \( B \) decays are included here. The calculation was performed using the CTQ4M parton densities. The upper number are the maximum, and the lower number the minimum of the values obtained by varying the scales in the usual way. The corresponding total cross sections are 165 to 864 \( \mu b \). The \( B \rightarrow \mu \) branching fraction was taken equal to 10.5%. Different values for the ε parameter of the Peterson fragmentation function are assumed. The last two column show the impact of a rather large intrinsic transverse momentum of the incoming partons.

<table>
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<th>ε (GeV)</th>
<th>0.002</th>
<th>0.006</th>
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<th>0.006</th>
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<td>( \langle k_T \rangle ) (GeV)</td>
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<td>0</td>
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<td>4</td>
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<tr>
<td>A: ( b \bar{b} \rightarrow \mu (</td>
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<td>&lt; 2.4, p_T \geq 6) )</td>
<td>3.3</td>
<td>2.41</td>
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<td>1.06</td>
<td>0.81</td>
<td>0.72</td>
<td>1.06</td>
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<tr>
<td>B: ( b \bar{b} \rightarrow \mu (p_T &gt; 6) \mu (p_T &gt; 3) )</td>
<td>0.76</td>
<td>0.52</td>
<td>0.45</td>
<td>0.67</td>
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<td>0.304</td>
<td>0.219</td>
<td>0.19</td>
<td>0.252</td>
</tr>
<tr>
<td>C: ( b \bar{b} \rightarrow \mu (p_T &gt; 6) \epsilon (p_T &gt; 2) )</td>
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<td>0.43</td>
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</tr>
<tr>
<td>D: ( b \bar{b} \rightarrow \mu (p_T &gt; 7,</td>
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<td>&lt; 2.4) )</td>
<td>2.26</td>
<td>1.62</td>
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<tr>
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<td>0.78</td>
<td>0.58</td>
<td>0.5</td>
<td>0.73</td>
</tr>
<tr>
<td>E: ( b \bar{b} \rightarrow \mu (p_T &gt; 7,</td>
<td>\eta</td>
<td>&lt; 2.4) \mu (p_T &gt; 4.5, 0 &lt;</td>
<td>\eta</td>
<td>&lt; 1.5) )</td>
</tr>
<tr>
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<td>0.0087</td>
<td>0.0105</td>
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<td>F: ( b \bar{b} \rightarrow \mu (p_T &gt; 7,</td>
<td>\eta</td>
<td>&lt; 2.4) \mu (p_T &gt; 3.6, 1.5 &lt;</td>
<td>\eta</td>
<td>&lt; 2) )</td>
</tr>
<tr>
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<td>0.0045</td>
<td>0.0032</td>
<td>0.0026</td>
<td>0.0035</td>
</tr>
<tr>
<td>G: ( b \bar{b} \rightarrow \mu (p_T &gt; 7,</td>
<td>\eta</td>
<td>&lt; 2.4) \mu (p_T &gt; 2.6, 2 &lt;</td>
<td>\eta</td>
<td>&lt; 2.4) )</td>
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<td>H: ( b \bar{b} \rightarrow \mu (p_T &gt; 1, 2 &lt;</td>
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<td>&lt; 6) )</td>
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<td>18.8</td>
</tr>
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</tr>
<tr>
<td>I: ( b \bar{b} \rightarrow \mu (p_T &gt; 2, 2 &lt;</td>
<td>\eta</td>
<td>&lt; 6) )</td>
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<td>2.94</td>
<td>2.65</td>
<td>2.51</td>
<td>3.11</td>
</tr>
</tbody>
</table>

energy that characterizes the cross section. Thus, while the ratio of the upper to the lower limit of the cross section is above a factor of 5 in the total rate, it is between a factor of 2 and 4 in the muon rates. The smallest values are achieved for the highest momentum cuts. A non perturbative fragmentation function of the Peterson form was also included in the calculation, with ε parameter taking the values 0 (i.e., no fragmentation), 0.002 and 0.006. More details on its implementation are given in the following subsections. Observe that for softer fragmentation functions (i.e. larger ε parameter) the uncertainty is reduced, since they imply higher quark momenta. The reduction in the scale uncertainty is obtained at the price of introducing a sensitivity to the fragmentation function parameter. We considered as realistic values of ε between 0.002 and 0.006. The corresponding variation of the cross section is not large. The impact of an intrinsic transverse momentum of the incoming partons (see the following subsections) is also studied. We have chosen the unrealistically large value \( \langle k_T \rangle = 4 \) GeV just to show that its effect is in all cases not a dramatic one.

#### 2.2 Transverse momentum spectrum

##### 2.2.1 Benchmark single-inclusive distributions

The fixed-order, NLO result for single-inclusive \( b \) production has several limitations in different regions of the phase space. In particular, one should be aware of the high-energy limit problem when \( p_T \) is small compared to the incoming energy, of the logarithms of \( m_b/p_T \) for high transverse momenta, and of further problems when approaching the threshold region. All these issues will be discussed in some detail in the next Sections. However, the fixed-order calculation at NLO provides a useful starting point
for estimating the differential cross section. At this time, it is probably not useful to perform a cross section study with different sets of parton densities, and for different values of the $b$ mass. We limit ourselves to the MRST set, and we only study the scale dependence of the cross section. We do not include, at this stage, fragmentation effects, which, as shown in the following Sections, can be easily accounted for. In tables 5-8 we collect the results of this study. The central values we obtained are also plotted in figs. 7 and 8, so that the wide kinematic range of heavy flavour production can be appreciated by a glance. More detailed rapidity distributions at low momenta are shown in fig. 9. First of all, we see that the differential cross section spans many orders of magnitude. At a luminosity of $10^{34}$ cm$^{-2}$ sec$^{-1}$ each $\mu$b of cross section corresponds to $10^4$ events per second, or (roughly) $10^{11}$ events per year. Thus, at the level of $10^{-11}$ in the plot there should be one event per year per bin of $p_T$ and $y$. The $p_T$ spectrum

Fig. 7: Differential cross section for heavy flavour production vs. $p_T$, for different rapidities.

Fig. 8: Differential cross section for heavy flavour production vs. $y$ for different $p_T$ values, as given in tables 5-8.
starts to drop fast for $p_T$ larger than the heavy quark mass, dropping even faster as the threshold region is approached. The rapidity distributions have the typical shape of a wide plateau, dropping at the edge of the phase space, and becoming narrower for larger transverse momenta. At the LHC the gluon fusion production mechanism is dominant, as can be seen in fig. 10. There one can see that the quark-antiquark annihilation component is below the gluon fusion component by more than one order of magnitude in the $p_T$ range considered, while the quark-gluon term becomes more important at larger $p_T$. We remind the reader that the cross section for $qg \rightarrow b + X$ is not included in the NLO calculation. One may thus worry about a loss of accuracy in the result, since the quark-quark luminosity at the LHC are by far the largest for high transverse momenta $b$ production. This problem, however, is dealt with appropriately in the
resummation formalism for high $p_T$ heavy flavour production, where a quark-quark fusion contribution does indeed appear.
Table 5: Differential cross section $d\sigma/dp_T^2 dy$ for single inclusive $b$ production at the LHC, for $p_T$ from 1 to 80 GeV and $y$ from 0 to 4. The table was computed with the MRST parton density set, for $m_b = 4.75$ GeV. The central value was obtained with the factorization and renormalization scale set to $\sqrt{p_T^2 + m^2}$. The upper and lower values give the maximal variation when varying the scales independently by a factor of 2 above and below the central value. Cross sections are in $\mu b$; each element in a row should be multiplied by the common scale factor in the left column.

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<tr>
<td>$10^{-1}$×</td>
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</tr>
<tr>
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Table 6: As in table 5, for $y$ from 4.5 to 6.5.

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Table 7: As in table 5, for $p_T$ from 100 to 300.

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Table 8: As in table 5, for \( p_T \) from 320 to 500. In the entries with a * the cross section is too small to be computed reliably.

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2.22 Understanding Tevatron data

It is well known that Tevatron data for the integrated transverse momentum spectrum in $b$ production are systematically larger than QCD predictions. This problem has been around for a long time, although it has become less severe with time. The present status of this issue is summarized in fig. 11. A similar discrepancy is also observed in UA1 data (see ref. [12] for details).

The theoretical prediction has a considerable uncertainty, which is mainly due to neglected higher-order terms in the perturbative expansion. In our opinion, it is not unlikely that we may have to live with this discrepancy, which is certainly disturbing, but not strong enough to question the validity of perturbative QCD calculations. In other words, the QCD $O(\alpha_s^3)$ corrections for this process are above 100% of the Born term, and thus it is not impossible that higher order terms may give contributions of the same size. Nevertheless, it is useful to look for higher-order perturbative effects and non-perturbative effects that may enhance the cross section.

For values of $p_T$ much larger than the $b$ quark mass, large logarithms of the ratio $p_T/m_b$ arise in the coefficients of the perturbative expansion. Techniques are available to resum this class of logarithms to all orders. In ref. [23] the NLO cross section for the production of a massless parton $i$ (a gluon or a massless quark) has been folded with the NLO fragmentation function for the transition $i\rightarrow b$ [24]. The evolution of the fragmentation functions resums all terms of order $\alpha_s^n \log^n(p_T/m_b)$ and $\alpha_s^{n+1} \log^n(p_T/m_b)$. All the dependence on the $b$-quark mass lies in the boundary conditions for the fragmentation function. The result is then matched with the full NLO cross section, which contains the exact dependence on $m_b$ up to order $\alpha_s^3$, in a way that avoids double counting. Corrections to the result of ref. [23] are either of order $\alpha_s^4 \log^i(p_T/m_b)$, with $i \leq 2$, or of order $\alpha_s^4$ times positive powers of $m_b/\sqrt{p_T^2 + m_b^2}$.

Figures 12-13 show the differential and integrated $b$-quark $p_T$ distribution obtained in the fragmentation function approach of ref. [23], compared to the standard fixed-order NLO result. It should be noted that for high transverse momenta the scale dependence is significantly reduced with respect to the

![Fig. 11: The integrated $p_T$ distribution for single $b$ production measured at the Tevatron, and the corresponding QCD prediction.](image-url)
fixed-order calculation. Furthermore, it can be seen from fig. 13 that, for $10 \lesssim p_T \lesssim 30$ GeV, the result of the fragmentation-function approach lies slightly above the fixed-order NLO calculation. This has been interpreted in ref. [23] as an evidence for large, positive higher order corrections. Unfortunately, their effect is not easy to quantify. These higher order terms are in fact computed in a massless approximation, and thus fail at low transverse momenta. In figs. 12-13 these terms are suppressed by a factor that becomes smaller and smaller at low $p_T$. A more detailed discussion of this point can be found in

![Graph 1](image1.png)

**Fig. 12**: Single-inclusive $p_T$ distribution for $b$ production at the Tevatron energy: pure QCD and resummed results.

![Graph 2](image2.png)

**Fig. 13**: Integrated $p_T$ distribution for $b$ production at the Tevatron energy: pure QCD and resummed results.

the original reference. Here, we simply conclude that some evidence for large higher order terms in the intermediate transverse momentum region is present, although difficult to quantify.

Finally, notice that the overall effect of the inclusion of higher-order logarithms is a steepening of the $p_T$ spectrum. This is quite natural, since multiple radiation is accounted for in the resummation procedure.
It has been argued that an intrinsic transverse momentum for the incoming partons may explain the discrepancy observed at the Tevatron. In fact, large values (up to 4 GeV) of the average transverse momentum of the incoming partons have been invoked to explain direct photon production data [25]. Such large values, much larger than typical QCD scales, are clearly incompatible with the usual application of perturbative QCD. Thus, evidence for such a large intrinsic transverse momentum cannot be claimed on the basis of a single observable. In other words, we would need evidence from several observables, all leading to a similar value of the intrinsic $k_T$, before we accept such a flaw in the usual perturbative QCD description. Nevertheless, in the following we will perform the exercise of applying very large intrinsic transverse momenta to the heavy flavour production process. This procedure will lead to an increase in the $b$ transverse momentum spectrum. We will also show, however, that other variables, very sensitive to an intrinsic transverse momentum, that should be strongly affected, do not show any evidence of that.

There are several possible ways to implement the presence of a non-zero transverse momentum of the colliding partons, and the choice is, to a large extent, arbitrary. We implemented it in the FMNR code in the following way. We call $\vec{p}_T(Q\bar{Q})$ the total transverse momentum of the pair. For each event, in the longitudinal centre-of-mass frame of the heavy-quark pair, we boost the $Q\bar{Q}$ system to rest. We then perform a transverse boost, which gives the pair a transverse momentum equal to $\vec{p}_T(Q\bar{Q}) + \vec{k}_T(1) + \vec{k}_T(2)$; $\vec{k}_T(1)$ and $\vec{k}_T(2)$ are the transverse momenta of the incoming partons, which are chosen randomly, with their moduli distributed according to

$$\frac{1}{N} \frac{dN}{dk_T^2} = \frac{1}{\langle k_T^2 \rangle} \exp\left(-\frac{k_T^2}{\langle k_T^2 \rangle}\right).$$

(2)

The reader can find more details in ref. [12].

In fig. 14 we show the effect of an intrinsic $k_T$ generated in this way, with the (unphysically large) choice $\langle k_T \rangle = 4$ GeV (in fig. 14, the sensitivity to the $\epsilon_b$ parameter in the fragmentation function is also shown; fragmentation will be discussed in more detail in the next subsection.) We see that, for $p_T^{min} < 20$ GeV, the $k_T$ effect is sizeable, even in the presence of fragmentation, provided we allow for unphysically large intrinsic $k_T$.

It is fair to ask whether such large values are compatible with other observables. There is a particular class of observables that are particularly sensitive to the intrinsic transverse momentum. One example
is the azimuthal distance $\Delta \phi$ between the directions of the produced $b$ and $\bar{b}$. The $\Delta \phi$ distribution is trivial at leading order: $b$ and $\bar{b}$ are emitted back-to-back, and therefore

$$\frac{d\sigma}{d\Delta \phi} \propto \delta(\phi - \pi).$$  \hspace{1cm} (3)

An intrinsic $k_T$ of the colliding partons has the effect of smearing the $\delta$ function. For $\langle k_T \rangle = 4$ GeV the effect is quite dramatic, as can be seen in fig. 15. Is such an important effect consistent with the observed azimuthal correlations? The CDF and D0 collaborations have measured the azimuthal correlation of muon pairs produced in $b$ decays. In order to compare with these data sets, we have implemented in the FMNR code the semileptonic decay of $b$ quarks. We have assumed that the muon energy is distributed according to the prediction of the spectator model [26] with massless leptons. We have also checked that the muon energy distribution given by PYTHIA leads to similar results. Our results are shown in figs. 16 and 17, where CDF and D0 data are superimposed to the perturbative QCD prediction, with and without an intrinsic $k_T$ with $\langle k_T \rangle = 4$ GeV. Tevatron data do not seem to favour such a large intrinsic transverse momentum. The measured distributions are more peaked at $\Delta \phi = \pi$ than the theoretical curve with $\langle k_T \rangle = 4$ GeV. The effect of Peterson fragmentation is also shown in both cases. We thus conclude that the data does not seem to favour large $k_T$ effects.

2.23 Single-inclusive distributions and correlations at the LHC

In this subsection, we will follow the pragmatic assumption that the discrepancy observed at the Tevatron may either be attributed to a problem in the overall normalization of the cross section, or to the presence of effects, either perturbative or not, that distort the spectrum. We will continue to model these effects as fragmentation effects and intrinsic transverse momentum effects, and see if the LHC can distinguish among the two. In fig. 18 we plot the $b$ cross section with a transverse momentum cut. From the figure it is quite clear that the effects of fragmentation and the effects of an intrinsic transverse momentum kick manifest themselves in quite a different way. In particular, at $p_T > 20$ GeV even the effect of a very large transverse momentum kick is small, while fragmentation has a strong impact. On the other hand, the transverse momentum kick increases the cross section in the intermediate $p_T$ region, with a maximum around 7 GeV. The $p_T$ coverage offered by the combined LHC experiments will allow an effective
Fig. 16: CDF results on azimuthal correlations compared with the perturbative calculation, with and without intrinsic $k_T$.

Fig. 17: D0 results on azimuthal correlations compared with the perturbative calculation, with and without intrinsic $k_T$.

discrimination of the two kinds of effects. For completeness, we also show in fig. 19 a comparison of the fixed-order calculation of the single-inclusive spectrum, using the fixed-order calculation in two different schemes for the light flavour, and the matched-resummed result. As in the Tevatron case, the band obtained with the resummation procedure is much narrower at large transverse momentum. The corresponding uncertainty does not include fragmentation function uncertainties, that will be discussed in more detail further on.

As an example of what could be discriminated at the LHC using correlations, we present in fig. 20 the azimuthal correlation of the muons coming from semileptonic $B$ decays, using typical cuts that are implemented in the LHC experiments triggers for $B$ studies. The curves are obtained with different values of the $\epsilon_b$ parameter for the fragmentation function, and with or without a very large intrinsic
Fig. 18: Cross section with a transverse momentum cut at the LHC.

Fig. 19: Differential cross section for production at the LHC. The bands are obtained by varying independently the renormalization and factorization scales by a factor of 2 above and below the central value, which is the transverse mass. As one can expect, the \( \epsilon_b \) parameter affects only the total rate in this case, while the primordial transverse momentum has a considerable effect on the shape of the distribution. This example shows that, even with very simple experimental setup, at the LHC it will be possible to test important features of the differential distributions.

2.3 Fragmentation function formalism

In analogy with the case of charm production, the agreement between theory and data improves if one does not include any fragmentation effects. It is then natural to ask whether the fragmentation functions commonly used in these calculations are appropriate. Following the LEP measurements, fragmentation functions have appeared to be harder than previously thought. It will be interesting to see whether SLD new data [27] will help in clarifying this issue.

The effect of a non-perturbative fragmentation function on the \( p_T \) spectrum is easily quantified if
one assumes a steeply-falling transverse momentum distribution for the produced $b$ quark

$$\frac{d\sigma}{dp_T} = A p_T^{-M}, \quad (4)$$

The corresponding distribution for the hadron is

$$\frac{d\sigma_{\text{had}}}{dp_T} = A \int \tilde{p}_T^{-M} \delta(p_T - z\tilde{p}_T) D(z) \, dz \, d\tilde{p}_T = A p_T^{-M} \int_0^1 dzz^{-M-1} D(z). \quad (5)$$

We can see that the hadron spectrum is proportional to the quark spectrum times the $M$th moment of the fragmentation function $D(z)$. Thus, the larger the moment, the larger the enhancement of the spectrum.

In practice, the value of $M$ will be slightly dependent upon $p_T$. We thus define a $p_T$ dependent $M$ value

$$\frac{d\log(\sigma(p_T > p_T^{\text{cut}}))}{d\log p_T^{\text{cut}}} = -M(p_T^{\text{cut}}) + 1 \quad (6)$$

and

$$\sigma_{\text{had}}(p_T > p_T^{\text{cut}}) = \sigma(p_T > p_T^{\text{cut}}) \times \int_0^1 dzz^{M(p_T^{\text{cut}})-1} D(z). \quad (7)$$

This gives an excellent approximation to the effect of the fragmentation function, as can be seen from fig. 21.

Since the second moment of the fragmentation function is well constrained by $e^+e^-$ data, it is sensible to ask for what shapes of the fragmentation function, for fixed $\langle z \rangle$, one gets the highest value for $\langle z^{M-1} \rangle$. We convinced ourselves that the maximum is achieved by the functional form

$$D(z) = A\delta(z) + B\delta(1-z) \quad (8)$$

which gives

$$\langle z \rangle = \frac{B}{A+B}; \quad \langle z^{M-1} \rangle = \frac{B}{A+B}. \quad (9)$$
This is however not very realistic: somehow, we expect a fragmentation function which is concentrated at high values of $z$, and has a tail at small $z$. We convinced ourselves that, if we impose the further constraint that $D(z)$ should be monotonically increasing, one gets instead the functional form

$$ D(z) = A + B \delta(1 - z), $$

which gives

$$ \langle z \rangle = \frac{A/2 + B}{A + B}; \quad \langle z^{M-1} \rangle = \frac{A/M + B}{A + B}. $$

We computed numerically the $M^{th}$ moments of the Peterson form,

$$ D(z) \propto \frac{1}{z \left(1 - \frac{1}{z} - \frac{\epsilon}{1-\epsilon}\right)^2} $$

of the form

$$ D(z) \propto z^\alpha (1 - z)^\beta $$

for $\beta = 1$ (Kartvelishvili), for which

$$ \langle z^{M-1} \rangle = \frac{\Gamma(\alpha + M) \Gamma(\alpha + \beta + 2)}{\Gamma(\alpha + 1) \Gamma(\alpha + \beta + M + 1)}. $$

of the form of Collins and Spiller

$$ D(z) \propto \frac{(\frac{1-z}{\epsilon} + \frac{(2-z)\epsilon}{1-\epsilon}) (1 + z^2)}{(1 - \frac{1}{z} - \frac{\epsilon}{1-\epsilon})^2} $$

and of the form in eq. (10), at fixed values of $\langle z \rangle$ corresponding to the choices $\epsilon_b = 0.002$ and $0.006$ in the Peterson form. We found that the $p_T$ distribution at the Tevatron, for $p_T$ in the range 10 to 100 GeV, behaves like $p_T^{-M}$, with $M$ around 5. Therefore, we present in tables 9 and 10 values of the 4th,
Table 9: Values of the 4th, 5th and 6th moment, at fixed $\langle z \rangle$ (corresponding to $\epsilon_b = 0.002$ in the Peterson form), for different forms of the fragmentation function.

<table>
<thead>
<tr>
<th>Form</th>
<th>$\langle z \rangle = 0.879$</th>
<th>$M = 4$</th>
<th>$M = 5$</th>
<th>$M = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peterson</td>
<td>0.711</td>
<td>0.649</td>
<td>0.595</td>
<td></td>
</tr>
<tr>
<td>Kartvelishvili</td>
<td>0.694</td>
<td>0.622</td>
<td>0.562</td>
<td></td>
</tr>
<tr>
<td>Collins-Spiller</td>
<td>0.729</td>
<td>0.677</td>
<td>0.633</td>
<td></td>
</tr>
<tr>
<td>Maximal (eq. (10))</td>
<td>0.818</td>
<td>0.806</td>
<td>0.798</td>
<td></td>
</tr>
</tbody>
</table>

Table 10: Values of the 4th, 5th and 6th moment, at fixed $\langle z \rangle$ (corresponding to $\epsilon_b = 0.006$ in the peterson form), for different forms of the fragmentation function.

<table>
<thead>
<tr>
<th>Form</th>
<th>$\langle z \rangle = 0.828$</th>
<th>$M = 4$</th>
<th>$M = 5$</th>
<th>$M = 6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peterson</td>
<td>0.611</td>
<td>0.535</td>
<td>0.474</td>
<td></td>
</tr>
<tr>
<td>Kartvelishvili</td>
<td>0.594</td>
<td>0.513</td>
<td>0.447</td>
<td></td>
</tr>
<tr>
<td>Collins-Spiller</td>
<td>0.626</td>
<td>0.559</td>
<td>0.505</td>
<td></td>
</tr>
<tr>
<td>Maximal (eq. (10))</td>
<td>0.742</td>
<td>0.724</td>
<td>0.713</td>
<td></td>
</tr>
</tbody>
</table>

5th and 6th moments of the above-mentioned fragmentation functions. We thus find that keeping the second moment fixed the variation of the hadronic $p_T$ distribution obtained by varying the shape of the fragmentation function among commonly used models is between 5% and 13% for both values of $\epsilon_b$. It thus seems difficult to enhance the transverse momentum distribution by suitable choices of the form of the fragmentation function. With the extreme choice of eq. (10), one gets at most a variation of 50% for the largest value of $\epsilon_b$ and $M$. It would be interesting to see if such an extreme choice is compatible with $e^+e^-$ fragmentation function measurements.

3. A STUDY OF HEAVY QUARK NON-PERTURBATIVE FRAGMENTATION IN HERWIG

In this Section we present the results of a phenomenological study of the non-perturbative hadronization of $b$-quarks. According to the standard QCD picture, distributions for an observable hadron $H$ can be computed by convoluting the short-distance cross section $\hat{\sigma}(p)$ with a fragmentation function $D_{H}^{(h)}(z)$ that describes the way in which the heavy quark $h$ hadronizes into $H$:

$$d\sigma_H(p) = \int dz \; D_{H}^{(h)}(z) \; d\hat{\sigma}(p/z).$$ (16)

The precise definition of $D_{H}^{(h)}(z)$ depends on how much of the heavy quark evolution after its production is absorbed into the perturbative part $\hat{\sigma}(p)$, and how much is assigned to the non-perturbative component parameterised by $D_{H}^{(h)}(z)$. Since perturbation theory (PT) is well defined for a massive quark, the standard prescription is to absorb into $\sigma(p)$ not only the hard matrix elements, but also the perturbative part of the fragmentation function, defined by the evolution in $Q^2$ down to a scale equal to the heavy quark mass $m_h$. $D_{H}^{(h)}(z)$ will therefore account for the transition of an “on-shell” quark $h$ into the hadron $H$.

The assumptions built into eq. (16) are that $D_{H}^{(h)}(z)$ depends neither on the type of hard process, nor on the scale at which $h$ was produced. Under these assumptions, $D_{H}^{(h)}(z)$ can be extracted from data in one given reaction (typically, $e^+e^-$), and eventually used to predict the cross section in some other reaction ($p\bar{p}$, DIS and so on).

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QCD factorization theorems indicate that this universality of \( D^{(h)}_H (z) \) holds in the asymptotic limit, and up to corrections of order \( m_h / Q \), \( Q \) being the scale of the hard process. The size of these corrections cannot be calculated, today, in any rigorous way. A possible approach to this problem is to turn to the phenomenological models of hadronization implemented in QCD-based parton-shower Monte Carlo (PSMC) codes. In PSMC the full final-state kinematical configuration is available at both the parton and hadron levels. Therefore, it is possible to “measure” \( D^{(h)}_H (z) \) using eq. (16), both \( d\sigma_H \) and \( d\hat{\sigma} \) being known. In the present section, we carry out this program using the PSMC HERWIG [28]. HERWIG evolves quarks according to perturbative QCD down to small scales. The quarks are paired up at the end of the evolution into colour singlet clusters, which are then decayed to the physical hadrons using phenomenological models of hadronization implemented in QCD-based parton-shower Monte Carlo. The study of the heavy quark hadronisation process in HERWIG will allow us to test the universality assumption, and to measure the size of possible deviations.

We should stress that, at this moment, our conclusions are only relevant for the hadronization model implemented in HERWIG; other PSMC’s, which treat the hadronization process differently (for example, by adopting a string model), may well lead to different conclusions.

In order to precisely define our procedure for extracting \( D^{(h)}_H (z) \), we need to consider in more details the way in which HERWIG generates events. Regardless of the type of initial-state particles, we can distinguish the following steps.

- **Hard subprocess**: at this stage, the PSMC generates the kinematics for the basic \( 2 \rightarrow 2 \) hard reaction. We denote the momentum of the \( b \)-quark (or antiquark) as \( p_{b}^{\text{hard}} \).
- **Parton shower**: the partons resulting from the hard subprocess undergo successive branchings, until their virtuality is smaller than a fixed cutoff value. We denote the momentum of the \( b \)-quark at the end of this phase as \( p_{b}^{\text{PS}} \).
- **Gluon splitting and cluster formation**: the gluons present at the end of the shower are decayed into light-quark pairs. Colour-singlet, two-body clusters are formed, according to colour parenthood and closeness in the phase-space. If there exist one or more cluster whose mass is too large (relative to a given threshold), part of the cluster rest energy is transformed into new \( q\bar{q} \) pairs, and new clusters are defined. In this process, energy-momentum is redistributed among the cluster elements, and the momentum of the \( b \)-quark can therefore be modified with respect to \( p_{b}^{\text{PS}} \). The momentum of the \( b \) quark after completion of the clustering process will be denoted by \( p_{b}^{\text{final}} \).
- **Cluster decay and hadron formation**: the clusters decay into observable hadrons, according to the flavour and to tabulated mass spectra. We therefore obtain \( b \)-flavoured hadrons, whose momentum we denote as \( p_{B} \).

The hard subprocess and parton shower stages are based on perturbative QCD. Thus, we identify the predictions given by HERWIG at the end of the parton shower with the cross section \( \hat{\sigma} \) that appears in eq. (16). On the other hand, the gluon splitting and cluster decay stages do not contain QCD information, as they are performed according to a phenomenological model. The \( g \rightarrow q\bar{q} \) splitting and the decay kinematics are induced by simple phase-space considerations. We thus identify these stages as the long-distance, non perturbative part of the process, which gives rise to \( D^{(h)}_H (z) \). We therefore determine the fragmentation function by comparing the results for \( p_{b}^{\text{PS}} \) and \( p_{B} \), defining, on an event-by-event basis, the following variables:

\[
\begin{align*}
    z_1 &= \frac{\hat{p}_B \cdot \hat{p}_{b}^{\text{PS}}}{|\hat{p}_{b}^{\text{PS}}|^2}, \\
    z_2 &= \frac{E_B + \hat{p}_B \cdot \hat{p}_{b}^{\text{PS}}}{E_{b}^{\text{PS}} + |\hat{p}_{b}^{\text{PS}}|},
\end{align*}
\]

where \( \hat{p}_{b}^{\text{PS}} = \frac{\vec{p}_{b}^{\text{PS}}}{|\vec{p}_{b}^{\text{PS}}|} \). Our conclusions will apply to both \( z_1 \) and \( z_2 \); thus, we will collectively denote them by \( z \). In hadronic collisions, the momenta and energies have to be substituted by transverse momenta and transverse energies respectively. Our results are summarized in table 11; we considered \( e^+e^- \) collisions at \( \sqrt{S} = 91.2 \) GeV and \( p\bar{p} \) collisions at \( \sqrt{S} = 1.8 \) TeV. In the table, we present four of
Table 11: Normalized Mellin moments of the $b$-quark non-perturbative fragmentation function. Results are given for the case of $e^+e^-$ collisions at $\sqrt{S} = 91.2$ GeV and for $p\bar{p}$ collisions at $\sqrt{S} = 1.8$ TeV. All numbers have a statistical accuracy of $\pm 0.01$.

<table>
<thead>
<tr>
<th></th>
<th>$e^+e^-$</th>
<th>$p\bar{p}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\mu_1$</td>
<td>$\mu_2$</td>
</tr>
<tr>
<td>$</td>
<td>\vec{p}_b^{ps}</td>
<td>&lt; 5$ GeV</td>
</tr>
<tr>
<td>$10 &lt;</td>
<td>\vec{p}_b^{ps}</td>
<td>&lt; 15$ GeV</td>
</tr>
<tr>
<td>$20 &lt;</td>
<td>\vec{p}_b^{ps}</td>
<td>&lt; 25$ GeV</td>
</tr>
<tr>
<td>$30 &lt;</td>
<td>\vec{p}_b^{ps}</td>
<td>&lt; 35$ GeV</td>
</tr>
</tbody>
</table>

The (normalized) Mellin moments of the $z$ distribution, defined as follows:

$$\mu_n = \int dz \, z^n \, D_{H}^{(b)}(z) / \int dz \, D_{H}^{(b)}(z).$$

(18)

Usually, $0 \leq z \leq 1$. In the present case, as we will see, we can also have $z > 1$; thus, in eq. (18) the range of integration coincide with the support of $D_{H}^{(b)}(z)$. The Mellin moments appearing in table 11 have been evaluated by considering bins in $|\vec{p}_b^{ps}|$ (in the case of hadronic collisions, the momentum is the transverse one). In $e^+e^-$ collisions larger (smaller) values of $|\vec{p}_b^{ps}|$ correspond to less (more) energy lost to gluons. In hadronic collisions larger (smaller) values of $|\vec{p}_b^{ps}|$ are more likely to correspond to larger (smaller) values of the hard process momentum before evolution. In either case, dependence of $D_{H}^{(b)}(z)$ on $|\vec{p}_b^{ps}|$ signals therefore a departure from universality.

By inspection of the table, we see that $D_{H}^{(b)}(z)$ is scale-independent to a very good extent (the situation appears to be slightly better in the case of $e^+e^-$ collisions), except for the very low $p_b^{ps}$ region; this is what we should expect, since in that region the factorization theorem on which eq. (16) is based is bound to fail. On the other hand, there seems to emerge a clear difference between the fragmentation functions extracted from $e^+e^-$ and $p\bar{p}$ “data”, the latter being substantially softer than the former. The first moment, which is the average value of the fragmentation variable, changes by about 10%. This variation can change the rate of predicted $b$-hadrons in hadronic collisions by almost 50%.

This suggests that transporting to hadronic collisions the non-perturbative fragmentation functions obtained by fitting $e^+e^-$ data may not be correct. Of course, a much more detailed investigation on the subject is required before reaching a firm conclusion; however, this simple exercise of ours shows that universality should not be taken for granted.

We now concentrate on the separate role played in the fragmentation process by the purely perturbative evolution and by the non-perturbative gluon-splitting phase, before the cluster formation and decay take place. We shall confine ourselves to the case of $e^+e^-$ collisions. The variables relevant to our study are the following:

1. Energy fraction retained during the perturbative evolution:

$$z_{ps} = \frac{2 \, |\vec{p}_b^{ps}|}{\sqrt{S}} = \frac{|\vec{p}_b^{ps}|}{|\vec{p}_b^{hard}|}$$

(19)

where $\sqrt{S}$ is the $e^+e^-$ CM energy.

2. Energy fraction retained during the gluon-splitting:

$$z_{gsp} = \frac{|\vec{p}_b^{gsp}|}{|\vec{p}_b^{ps}|}.$$ 

(20)
Fig. 22: Fragmentation functions for $b$ (left) and $c$ (right) quarks, produced in $e^+e^-$ collisions at $\sqrt{s} = 91.2$ GeV.

3. Energy fraction left after the perturbative evolution and the gluon-splitting:

$$z = z_{ps} \times z_{glsp} = \frac{2 |\frac{z_{glsp}^}{\vec{p}_b}|}{\sqrt{S}} = \frac{|\frac{z_{glsp}^}{\vec{p}_b}|}{|\vec{p}_b|^2},$$

(21)

The left panel in fig. 22 shows the three distributions for $b$ quarks at $\sqrt{S} = 91.2$ GeV. The solid histogram represents the distribution of $z_{ps}$. The distribution has the shape of a Gribov-Lipatov, with no indication of a Sudakov turn-over at large $z_{ps}$. The dotted line is the distribution in $z$. A strong deformation of the purely perturbative curve is clearly seen. The dashed line corresponds to the $z_{glsp}$ distribution. This is part of what the MC treats as a non-perturbative component of the fragmentation process. The peak of the dashed histogram at $z_{glsp} = 1$ corresponds to events where the cluster containing the heavy quark does not need to be further split, while the tail corresponds to events where the invariant mass of the heavy-quark cluster is too large, and additional light-quark pairs have to be produced by hand. Notice that almost as much energy is lost during this non-perturbative phase, as is lost during the perturbative evolution.

For comparison, we show the same set of curves for the evolution of the charm quark (right panel of fig. 22). Notice that while the effect of the perturbative evolution is to soften the quark spectrum relative to the $b$-quark case, the amount of energy lost due to gluon splitting is similar ($\langle z_{glsp}^c \rangle = 0.82$, as opposed to $\langle z_{glsp}^b \rangle = 0.85$). This is bizarre, since one expects the non-perturbative part to scale with $1/m_h$. The same result is found for the fragmentation of the $s$ quark (left panel of fig. 23). Here $\langle z_{glsp}^s \rangle$ is 0.81. Again, a violation of the expected $1/m_h$ scaling.

Things improve for the top quark, whose distributions for $\sqrt{S} = 2$ TeV are shown on the right panel of fig. 23. The gluon-splitting part has only a minor impact on the overall spectrum of the top quark.

We are a bit bothered by the dominant role played by the gluon-splitting phase. By comparison, the next step in the evolution, namely the cluster formation and decay, plays only a minor role, as will be shown next. We would have anticipated that the cluster formation and decay should be the place where most of the non-perturbative physics should show up. This suggests that the thresholds for the perturbative evolution in the MC should be lowered, so that the impact of the non-perturbative gluon splitting phase is reduced, and purely perturbative Sudakov effects can manifest themselves.

We now turn again to the non-perturbative part of the fragmentation function. The most striking feature, that cannot be inferred from the simple study of Mellin moments as done in table 11, is the presence of a double peak in the high-$z$ region (see the left panel of fig. 24). A first peak (which we will call peak A) is seen at $z$ values around 0.97. A second peak (peak B) is at $z = 1.01$ (we have a $z$-bin size
Fig. 23: Fragmentation functions for $s$ (left) and $t$ (right) quarks, produced in $e^+e^-$ collisions at $\sqrt{s} = 91.2$ (left) and 2000 (right) GeV.

Fig. 24: $B$-meson non-perturbative fragmentation function in $e^+e^-$ at $\sqrt{s} = 91.2$ GeV, for $30 \text{ GeV} < |p_b| < 35 \text{ GeV}$, CLDIR=1, CLSMR=0. See the text for details.

The structure of the double peak is strongly influenced by the value taken by the two input parameters CLDIR and CLSMR. If the default is used (CLDIR=1, CLSMR=0), the double peak is observed (see the plot on the left of fig.24). On the other hand, by setting CLSMR≠0, the $z$ distributions display a single peak (broader that the previous ones) at about $z = 0.97$. For small CLSMR values and large $b$ momenta, a second peak at $z = 1.01$ tends to re-appear, although smaller than observed before.

The double peak disappears also if one chooses CLDIR=0, as in older HERWIG versions. In this case, the $z$ distributions peak at about $z = 0.9$, this peak being much broader than those obtained of 0.02. We verified that the events contributing to the peak $B$ do not have $z = 1 + \epsilon$, i.e., the peak is not due to a roundoff error). The latter peak is higher than the former.
with CLDIR=1, regardless of the value of CLSMR.

- We then set CLDIR=1 and CLSMR=0. For any given $B$ meson, we looked for the parent cluster $C = \{b_Cq\}$, and for the parent bottom quark, $b_P$ (the parent quark is defined as in the HERWIG routine HWCHAD). We observed what follows.
  - Plotting the $z$ distributions for the events with $\vec{p}_{b_C} \neq \vec{p}_{b_P}$ (i.e. events where the original cluster was split), we see a single peak, at the same $z$ value as for the peak $A$ (solid line, plot on the right of fig.24).
  - The $z$ distributions for events such that $\vec{p}_{b_C} = \vec{p}_{b_P}$ display again a double peak. The two peaks are at the same $z$ values as peaks $A$ and $B$, the latter one being by far dominant.
  - Selecting only events with $\vec{p}_{b_C} = \vec{p}_{b_P}$, we found that the peak at the position of $B$ corresponds to those clusters decaying into a $B$ meson and a $\pi$, while the peak at the position of $A$ is relevant for all the other two-body decays (dotted and dashed lines respectively, plot on the right of fig.24).

Overall, notice also that the amount of energy retained after the gluon-splitting phase is of the same size as that retained at the end of the full hadronization process, indicating that cluster formation and decay have a minor impact on the total amount of energy lost during the non-perturbative part of the evolution.

We were also able to reproduce the previous findings with a very simple model. Given a momentum for a quark $b$, we generate randomly the momentum for a light quark $q$, to be combined with $b$ into a cluster, which eventually decays into a $B$ meson and a particle of given mass $m_P$. The momentum of the quark $q$ is allowed to have a (small) transverse momentum with respect to the direction of the quark $b$. After evaluating the cluster mass, we performed the decay in the rest frame of the cluster, either in an isotropic manner (thus mimicking the choice CLDIR=0), or by letting the momentum of the meson $B$ to be parallel to that of the quark $b$ (which corresponds to CLDIR=1 and CLSMR=0). In the latter case, depending upon the value of $m_P$, we got a peak for $z < 1$ (if $m_P > m_q$) or $z > 1$ (if $m_P < m_q$).

In conclusion, the $z$ distributions we find when using HERWIG seem not to contain a lot of dynamical information, the most important features being those implemented in the cluster-decay routine. If the decay is not smeared out (CLDIR=0), we get a structure which is very difficult to reconcile with the idea of fragmentation we have from QCD. After smearing, the distribution still has a $z > 1$ tail which will be extremely difficult to fit with a function vanishing for $z \to 1$. This problem is related to the fact that the mass of the lightest quarks in the MC is 320 MeV, that is much larger than the pion mass. We performed a test by reducing the light quark masses to 20 MeV, and increasing the shower cutoff VQCUT in such a way as to maintain the default value of the effective infrared threshold. The double peak structure, as expected, disappeared. It remains to be seen, however, whether such a small value of the quark masses is, more generally, acceptable.

4. A STUDY OF THE $b\bar{b}$ PRODUCTION MECHANISM IN PHYTIA

4.1 Introduction

In this section, we present a study on $b\bar{b}$ production performed within the CMS collaboration using the Monte Carlo package PHOENIX as an event generator. In particular, we investigate the influence of the cut-off on the hard interaction transverse momentum $P_t$ on the production of $b\bar{b}$ events.

In Monte Carlo programs, $b\bar{b}$ pairs in hadron collisions are produced by the mechanisms of gluon fusion, gluon splitting and flavour excitation. All of them give contributions of the same order to the total cross section, but they give rise to different kinematical configurations of the final state.

There are two ways to generate $b\bar{b}$ events in PHOENIX:

- Using a steering card MSEL=5, a gluon fusion mechanism ($gg \to b\bar{b}$) is mainly simulated. Each event contains at least one $b\bar{b}$ pair.

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Using MSEL=1, all QCD $2 \rightarrow 2$ processes are simulated. In this case, all production mechanisms contribute to the $b\bar{b}$ production, but the probability to find a $b\bar{b}$ pair in the event is less than 1\%.

About one million events have been simulated in CMS with MSEL=1, in order to have a sample with all $b\bar{b}$ production mechanisms and default PYTHIA cut-off, not to introduce any bias in the kinematics. The selection efficiency of triggered events out of this sample is quite low. In order to have higher signal statistics, in some cases one can use kinematical cuts which are different from the PYTHIA default. [29].

4.2 $b\bar{b}$ production

Two samples have been prepared to investigate the influence of the $\hat{P}_t$ cut on the production of $b\bar{b}$ events. Both of them have been generated using MSEL=1 and contain events with only one $b\bar{b}$ pair. Only events with $\hat{P}_t \geq 10$ GeV have been selected in the samples. The first sample (SAMPLE A) has been generated with the default $\hat{P}_t$ cut of 1 GeV and only events with $\hat{P}_t \geq 10$ GeV were selected. The second sample (SAMPLE B) has been generated with $\hat{P}_t \geq 10$ GeV. In both samples the following processes contribute:

\[ gg \rightarrow q\bar{q} \]  
\[ gg \rightarrow gg \]  
\[ gg \rightarrow gg \]  
\[ \]  

$b\bar{b}$ pair is produced by gluon splitting $g \rightarrow b\bar{b}$ in initial or final state shower evolution (processes (22) to (24)) or in the hard interaction (process (22)). For both samples A and B, we have computed the $b\bar{b}$ production cross section

\[ \sigma_{b\bar{b}}^{tot} = \frac{N_{b\bar{b}}}{N_{tot}} \sigma_{b\bar{b}}^{tot}, \]  

where $N_{b\bar{b}}$ is the number of $b\bar{b}$ events with $\hat{P}_t \geq 10$ GeV, $N_{tot}$ is the total number of generated events, and $\sigma_{b\bar{b}}^{tot}$ is the total cross section (given by PYTHIA). We find that

- for sample A, $\sigma_{b\bar{b}}^{tot} = 150 \mu$b. The gluon fusion contribution is about 20 $\mu$b, while the gluon splitting contributions are $\sim 30 \mu$b and $\sim 100 \mu$b for processes (23) and (24) respectively;
- for sample B, $\sigma_{b\bar{b}}^{tot}=257 \mu$b. Gluon fusion and gluon splitting contributions are at the same level as in sample A. In this case, however, there are also contributions from the processes $bg \rightarrow bg$ and $bg \rightarrow bq$ of about 110 $\mu$b. In the following we will call these contributions flavour excitation.

Figure 25 illustrates the difference in the $b\bar{b}$ production cross sections due to the additional contribution of the flavour excitation mechanism in sample B. The effect has the following explanation. When the default $\hat{P}_t$ cut-off is used, PYTHIA generates processes in the low energy approximation, i.e. there are no heavy quarks inside the parton distribution. This approach changes if one uses a different $\hat{P}_t$ cut-off: the parton distributions in this case include also $b$ and $c$ quarks. As a consequence, samples A and B are different in two respects: values of the cross sections, and set of production mechanisms. The difference in the cross section is not very important, because the results are usually normalized to the total $b\bar{b}$ cross section of 500 $\mu$b. On the other hand, the different production mechanisms could be more dangerous, as they can lead to different kinematical distributions, and therefore affect the efficiencies of physical selection.

4.3 Kinematics

The main kinematical parameters which define the signature of a $b\bar{b}$ event are the transverse momenta and pseudorapidities of the $b$ quarks, and the angular distance $\Delta \phi$ between their directions in the transverse plane. The first two parameters have similar distributions in both samples. The $\Delta \phi$ distribution is shown in fig. 26 for the three different mechanisms. For what concerns gluon splitting, the distribution is slightly peaked at small $\Delta \phi$. The angle between the two $b$-quarks produced by the gluon-fusion mechanism has a peak at $\Delta \phi \sim \pi$, as expected, since in the process $gg \rightarrow b\bar{b}$ the $b$-quarks are produced back-to-back in
the transverse plane. The last distribution corresponds to the flavour excitation production mechanism, for which the back-to-back topology is preferred. We can conclude that the total $\Delta \phi$ distributions of sample A and sample B are slightly different. Some care should be taken about this, as it could affect the estimated efficiency of selection cuts.

4.4 Pythia 6.125

We have studied the same problem using the new 6.125 version Pythia. We have generated two new samples A and B with Pythia 6.125, and we have found the following results:

- Sample A: the $b\bar{b}$ production cross section is $\sigma^{tot}_{b\bar{b}}=230 \mu b$. Gluon fusion contributes $\sim 50 \mu b$ and gluon splitting gives $40 \mu b$ and $140 \mu b$ via processes (23) and (24) respectively.
- Sample B: $b\bar{b}$ production cross section is $\sigma^{tot}_{b\bar{b}}=210 \mu b$, which is similar to the value of sample A. In this case, however, gluon fusion and gluon splitting contributions are decreased by about a factor 2. The contribution from the flavour excitation is about 100 $\mu b$.

Even if the total cross section is in good agreement between the two samples, it is clear that the way Pythia 6.125 generates $b\bar{b}$ pairs depends on the $P_t$ cut-off. Contrary to Pythia 5.75, in the new version gluon splitting and fusion contributions are different in the two samples.
Fig. 26: Angle between the two $b$ quarks in the transverse plane: the upper one is gluon splitting, the middle is gluon fusion, the last is related to flavour excitation. In the plots the values are not normalized.

### 4.5 Interpretation

Many of the features of PYTHIA illustrated in this section are easily explained\(^4\). It turns out that PYTHIA treats differently processes with a low and high $p_T^{\text{min}}$. The limit is related to the scale of multiple interactions, which is fixed to 2 GeV in the older versions, and was made energy dependent in PYTHIA 6, being 3.2 GeV at the LHC energy. When $p_T^{\text{min}}$ is above this scale, the hard process is selected according to conventional matrix elements. Below this scale, the hardest interaction is instead taken from the naive jet cross section multiplied by a “Sudakov style” form factor, that represents the probability that higher $p_T$ interactions did not take place in the rest of the event. Since this procedure implies the computation of all parton-parton scattering processes, the choice was made to exclude from it the incoming $b$ and $c$ components, to save time in the computation. This feature is no longer considered useful in modern times, the computers being much faster. Thus, in PYTHIA 6.138, also the $b$ and $c$ processes will be

\(^4\)T. Sjöstrand and E. Norrbin, private communication.
implemented in the low $p_T$ mode.

The difference in the total cross section in PYTHIA 5.7 and 6.1 have a physical origin, since 6.1 uses newer parton distributions that, according to HERA data, are more singular in the small $x$ region.

As of now, no explanation is given why gluon fusion and gluon splitting contributions should drop by a factor of 2 when going from sample A to B in PYTHIA 6.125; further studies will be required.

The authors of PYTHIA recommend the following procedure for the generation of $b$ events. Parton fusion and flavour excitation can be generated separately; the relevant massive matrix elements are used for parton fusion, and one can go to the limit $p_T \rightarrow 0$ with this process. Gluon splitting cannot be generated separately: all hard processes must be generated, excluding parton fusion and flavour excitation, and one should look for the heavy flavour. Multiple interactions are there switched off, in order to avoid a double-counting of the jet cross section. This is adequate for the study of the $b$ production properties, but clearly does not fully represent the structure of the underlying event. In future PYTHIA versions, when flavour excitation is included in the minimum bias machinery with multiple interaction, this latter should offer an almost equivalent alternative, but still without the correct mass treatment of the parton fusion process near threshold. Other limitations still remain from complex problems related to the treatment of beam remnants; therefore, flavour excitation is only enabled for the hardest interaction in the multiple-interaction scenario.

A sample of commented code is included below. By using different flags (MEKIND=0,1,2) three samples will be generated: parton fusion, flavour excitation and gluon splitting.

```plaintext
INTEGER KFINTMP(-40:40)
C... Multiple interactions switched off
MSTP(81)=0
PARP(81)=0.D0
PARP(82)=0.D0
C... Maximum virtuality in ISR is PARP(67)*Q**2
PARP(67)=1.D0
C... Choose heavy quark (bottom=5, charm=4)
MASSIVE=5
C... Helper variable
HQMASS=PMAS(MASSIVE,1)
C... Choose the kind of heavy quark production:
C... MEKIND is a local variable set to 0, 1 or 2
IF (MEKIND==0) THEN ! Massive matrix elements
  MSEL=MASSIVE
ELSE IF (MEKIND==1) THEN ! Flavour excitation
  MSEL=1
  CKIN(3)=HQMASS
  CKIN(5)=CKIN(3)
ELSE IF (MEKIND==2) THEN ! Gluon splitting (ISR, FSR)
  MSEL=1
  CKIN(3)=HQMASS
  CKIN(5)=CKIN(3)
END IF
C... More restrictive cuts can be put here.
C... Example, 100 events in total.
NEVENTS=100
C *** EVENT LOOP ***
  IF (MEKIND==1) NEVENTS=NEVENTS/2
```
C.... Loop over incoming partons
   DO ISIDE=1,2
      IF (MEKIND/=1.AND.ISIDE==1) THEN
         GOTO 100
      ELSE IF (MEKIND==1) THEN
         C... Only for flavour excitation:
         C... Make backup copy of KFIN array
            DO IKF=-40,40
               KFINTMP(IKF)=KFIN(ISIDE,IKF)
            END DO
         C... Remove all incoming partons:
            KFIN(ISIDE,IKF)=0
      END IF
   DO IEV=1,NEVENTS
      C... Generate an event
      CALL PYEVNT
      C... For gluon splitting, remove events with HQ in the hard interaction
      C... to avoid double counting:
      IF (MEKIND==2) THEN
         DO I=5,8
            IF (ABS(K(I,2))==MASSIVE) GOTO 50
         END DO
      END IF
   C... Analysis...
   50   END DO
   C... Print statistics
   CALL PYSTAT(1)
   C... Restore KFIN matrix:
   IF (MEKIND==1.AND.ISIDE==1) THEN
      DO IKF=-40,40
         KFIN(ISIDE,IKF)=KFINTMP(IKF)
      END DO
   END IF
   100 END IF

5. ASYMMETRIES

5.1 Introduction

Sizeable leading particle asymmetries between e.g. \(D^-\) and \(D^+\) have been observed in several fixed target experiments [30]. It is of interest to investigate to what extent these phenomena translate to bottom production and higher energies. No previous experiment has observed asymmetries for bottom hadrons due to limited statistics or other experimental obstacles. Bottom asymmetries are in general expected to be smaller than for charm because of the larger bottom mass, but there is no reason why they should be absent. In the fixed target experiment HERA-B, bottom asymmetries could very well be large [31] even at central rapidities, but the conclusion of the present study is that asymmetries at the LHC are likely to be small. In the following we study possible asymmetries between \(B\) and \(\bar{B}\) hadrons at the LHC within

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\(^{5}\)Section coordinators: E. Norrbin and R. Vogt
the Lund string fragmentation model [32] and the intrinsic heavy quark model [33].

In the string fragmentation model [34], the perturbatively produced heavy quarks are colour connected to the beam remnants. This gives rise to beam-drag effects where the heavy hadron can be produced at larger rapidities than the heavy quark. The extreme case in this direction is the collapse of a small string, containing a heavy quark and a light beam remnant valence quark of the proton, into a single hadron. This gives rise to flavour correlations which are observed as asymmetries. Thus, in the string model, there can be coalescence between a perturbatively produced bottom quark and a light quark in the beam remnant producing a leading bottom hadron.

There is also the possibility to have coalescence between the light valence quarks and bottom quarks already present in the proton, because the wave function of the proton can fluctuate into Fock configurations containing a $b\bar{b}$ pair, such as $|uudb\bar{b}\bar{b}\rangle$. In these states, two or more gluons are attached to the bottom quarks, reducing the amplitude by $\mathcal{O}(\alpha_s^2)$ relative to parton fusion [35]. The longest-lived fluctuations in states with invariant mass $M$ have a lifetime of $\mathcal{O}(2P_{lab}/M^2)$ in the target rest frame, where $P_{lab}$ is the projectile momenta. Since the comoving bottom and valence quarks have the same rapidity in these states, the heavy quarks carry a large fraction of the projectile momentum and can thus readily combine to produce bottom hadrons with large longitudinal momenta. Such a mechanism can then dominate the hadroproduction rate at large $x_T$. This is the underlying assumption of the intrinsic heavy quark model [33], in which the wave function fluctuations are initially far off shell. However, they materialize as heavy hadrons when light spectator quarks in the projectile Fock state interact with the target [36].

In both models the coalescence probability is largest at small relative rapidity and rather low transverse momentum where the invariant mass of the $Q\bar{Q}$ system is small, enhancing the binding amplitude. One exception is at very large $p_T$, where the collapse of a scattered valence quark with a $T$ quark from the parton shower is also possible, giving a further (small) source of leading particle asymmetries in the string model.

5.2 Lund String Fragmentation

Before describing the Lund string fragmentation model, some words on the perturbative heavy quark production mechanisms included in the Monte Carlo event generator PYTHIA[37] used in this study is in order. We study pp events with one hard interaction because events with no hard interaction are not expected to produce heavy flavours and events with more than one hard interaction — multiple interactions — are beyond the scope of this initial study and presumably would not influence the asymmetries. After the hard interaction is generated, parton showers are added, both to the initial (ISR) and final (FSR) state. The branchings in the shower are taken to be of lower virtualities than the hard interaction introducing a virtuality (or time) ordering in the event. This approach gives rise to several heavy quark production mechanisms, which we will call pair creation, flavour excitation and gluon splitting. The names may be somewhat misleading since all three classes create pairs at $zv$ vertices, but it is in line with the colloquial nomenclature. The three classes are characterized as follows.

**Pair creation** The hard subprocess is one of the two LO parton fusion processes $gg\rightarrow Q\bar{Q}$ or $q\bar{q}\rightarrow Q\bar{Q}$. Parton showers do not modify the production cross sections, but only shift kinematics. For instance, in the LO description, the $Q$ and $\bar{Q}$ have to emerge back-to-back in azimuth in order to conserve momentum, while the parton shower allows a net recoil to be taken by one or several further partons.

**Flavour excitation** A heavy flavour from the parton distribution of one beam particle is put on mass shell by scattering against a parton of the other beam, i.e. $Q_1\rightarrow Q_1$ or $Q_g\rightarrow Q_g$. When the $Q$ is not a valence flavour, it must come from a branching $g\rightarrow Q\bar{Q}$ of the parton-distribution evolution. In most current sets of parton-distribution functions, heavy-flavour distributions are assumed to vanish for virtuality scales $Q^2 < m_Q^2$. The hard scattering must therefore have a virtuality above $m_Q^2$. When the initial-state shower is reconstructed backwards [38], the $g\rightarrow Q\bar{Q}$ branching will
be encountered, provided that $Q_0$, the lower cutoff of the shower, obeys $Q_0^2 < m_Q^2$. Effectively the processes therefore become at least $gQ \rightarrow QgQ$ or $gg \rightarrow Q\bar{Q}g$, with the possibility of further emissions. In principle, such final states could also be obtained in the above pair-creation case, but the requirement that the hard scattering must be more virtual than the showers avoids double counting.

**Gluon splitting** $A_g \rightarrow Q\bar{Q}$ branching occurs in the initial- or final-state shower but no heavy flavours are produced in the hard scattering. Here the dominant $Q\bar{Q}$ source is gluons in the final-state showers since time-like gluons emitted in the initial state are restricted to a smaller maximum virtuality. Except at high energies, most initial state gluon splittings instead result in flavour excitation, already covered above. An ambiguity of terminology exists with initial-state evolution chains where a gluon first branches to $Q\bar{Q}$ and the $Q$ later emits another gluon that enters the hard scattering. From an ideological point of view, this is flavour excitation, since it is related to the evolution of the heavy-flavour parton distribution. From a practical point of view, however, we choose to classify it as gluon splitting, since the hard scattering does not contain any heavy flavours.

In summary, the three classes above are then characterized by having 2, 1 or 0, respectively, heavy flavours in the final state of the LO hard subprocess. Another way to proceed is to add next-to-leading order (NLO) perturbative processes, i.e the $\mathcal{O}(\alpha_s^3)$ corrections to the parton fusion [3] [4]. However, with our currently available set of calculational tools, the NLO approach is not so well suited for exclusive Monte Carlo studies where hadronization is added to the partonic picture.

Flavour excitation and gluon splitting give significant contributions to the total $b$ cross section at LHC energies and thus must be considered when this is of interest, see the following. However, NLO calculations probably do a better job on the total $b$ cross section itself (while, for the lighter $c$ quark, production in parton showers is so large that the NLO cross sections are more questionable). The shapes of single heavy quark spectra are not altered as much as the correlations between $Q$ and $\bar{Q}$ when extra production channels are added. Similar observations have been made when comparing NLO to LO calculations [3] [5]. Likewise, asymmetries between single heavy quarks are also not changed much by adding further production channels, so for simplicity we consider only the pair creation process here.

After an event has been generated at the parton level we add fragmentation to obtain a hadronic final state. We use the Lund string fragmentation model. Its effects on charm production were described in [32]. Here we only summarize the main points.

In the string model, confinement is implemented by spanning strings between the outgoing partons. These strings correspond to a Lorentz-invariant description of a linear confinement potential with string tension $\kappa \approx 1$ GeV/fm. Each string piece has a colour charge at one end and its anticolour at the other. The double colour charge of the gluon corresponds to it being attached to two string pieces, while a quark is only attached to one. A diquark is considered as being in a colour antitriplet representation, and thus behaves (in this respect) like an antiquark. Then each string contains a colour triplet endpoint, a number (possibly zero) of intermediate gluons and a colour antitriplet end. An event will normally contain several separate strings, especially at high energies where $g \rightarrow q\bar{q}$ splittings occur frequently in the parton shower.

The string topology can be derived from the colour flow of the hard process with some ambiguity arising from colour-suppressed terms. Consider e.g. the LO process $gg \rightarrow b\bar{b}$ where two distinct colour topologies are possible. Representing the proton remnant by a $u$ quark and a $ud$ diquark (alternatively $d$ plus $uu$), one possibility is to have the three strings $b \rightarrow u\bar{d}$, $b \rightarrow \bar{u}$ and $u \rightarrow u\bar{d}$, fig. 27, and the other is identical except the $b$ is instead connected to the $ud$ diquark of the other proton because the initial state is symmetric.

Once the string topology has been determined, the Lund string fragmentation model [34] can be applied to describe the nonperturbative hadronization. To first approximation, we assume that the hadronization of each colour singlet subsystem, i.e. string, can be considered separately from that of all the other subsystems. Presupposing that the fragmentation mechanism is universal, i.e. process-
Fig. 27: Example of a string configuration in a pp collision. (a) Graph of the process, with brackets denoting the final colour singlet subsystems. (b) Corresponding momentum space picture, with dashed lines denoting the strings.

independent, the good description of $e^+e^-$ annihilation data should carry over. The main difference between $e^+e^-$ and hadron–hadron events is that the latter contain beam remnants which are colour-connected with the hard-scattering partons.

Depending on the invariant mass of a string, practical considerations lead us to distinguish the following three hadronization prescriptions:

**Normal string fragmentation** In the ideal situation, each string has a large invariant mass. Then the standard iterative fragmentation scheme, for which the assumption of a continuum of phase-space states is essential, works well. The average multiplicity of hadrons produced from a string increases linearly with the string ‘length’, which means logarithmically with the string mass. In practice, this approach can be used for all strings above some cutoff mass of a few GeV.

**Cluster decay** If a string is produced with a small invariant mass, perhaps only a single two-body final state is kinematically accessible. In this case the standard iterative Lund scheme is not applicable. We call such a low-mass string a cluster and consider its decay separately. When kinematically possible, a $Q\bar{Q}$ cluster will decay into one heavy and one light hadron by the production of a light $q\bar{q}$ pair in the colour force field between the two cluster endpoints with the new quark flavour selected according to the same rules as in normal string fragmentation. The new $Q$ cluster end or the new $q\bar{q}$ pair may also denote a diquark. In the latest version of PYTHIA, anisotropic decay of a cluster has been introduced, where the mass dependence of the anisotropy has been matched to string fragmentation.

**Cluster collapse** This is the extreme case of cluster decay, where the string mass is so small that the cluster cannot decay into two hadrons. It is then assumed to collapse directly into a single hadron which inherits the flavour contents of the string endpoints. The original continuum of string/cluster masses is replaced by a discrete set of hadron masses, mainly $B$ and $B^*$ (or the corresponding baryon states). This mechanism plays a special rôle since it allows flavour asymmetries favouring hadron species that can inherit some of the beam-remnant flavour contents. Energy and momentum is not conserved in the collapse so that some energy-momentum has to be taken from, or transferred to, the rest of the event. In the new version, a scheme has been introduced where energy and momentum are shuffled locally in an event.

We assume that the nonperturbative hadronization process does not change the perturbatively calculated total rate of bottom production. By local duality arguments [39], we further presume that the rate of cluster collapse can be obtained from the calculated rate of low-mass strings. In the process $e^+e^-\rightarrow c\bar{c}$ local duality suggests that the sum of the $J/\psi$ and $\psi'$ cross sections approximately equal the perturbative $c\bar{c}$ production cross section in the mass interval below the $D\bar{D}$-threshold. Similar arguments have also been proposed for $\tau$ decay to hadrons [40] and shown to be accurate. In the current case, the presence
of other strings in the event also allows soft-gluon exchanges to modify parton momenta as required to obtain the correct hadron masses. Traditional factorization of short- and long-distance physics would then also preserve the total bottom cross section. Local duality and factorization, however, do not specify how to conserve the overall energy and momentum of an event when a continuum of $\bar{b}d$ masses is to be replaced by a discrete $B^0$. In practice, however, the different possible hadronization mechanisms do not affect asymmetries much. The fraction of the string-mass distribution below the two particle threshold effectively determines the total rate of cluster collapse and therefore the asymmetry.

The cluster collapse rate depends on several model parameters. The most important ones are listed here with the PYTHIA parameter values that we have used. The PYTHIA parameters are included in the new default parameter set in PYTHIA 6.135 and later versions.

- **Quark masses** The quark masses affect the threshold of the string-mass distribution. Changing the quark mass shifts the string-mass threshold relative to the fixed mass of the lightest two-body hadronic final state of the cluster. Smaller quark masses imply larger below-threshold production and an increased asymmetry. The new default masses are $\text{PMAS}(1) = m_u = \text{PMAS}(2) = m_d = 0.33D0$, $\text{PMAS}(3) = m_s = 0.5D0$, $\text{PMAS}(4) = m_c = 1.5D0$ and $\text{PMAS}(5) = m_b = 4.8D0$.

- **Width of the primordial $k_\perp$ distribution.** If the incoming partons are given small $p_\perp$ kicks in the initial state, asymmetries can appear at larger $p_\perp$ since the beam remnants are given compensating $p_\perp$ kicks, thus allowing collapses at larger $p_\perp$. The new parameters are $\text{PARP}(91) = 1.D0$ and $\text{PARP}(93) = 5.D0$.

- **Beam remnant distribution functions (BRDF).** When a gluon is picked out of the proton, the rest of the proton forms a beam remnant consisting, to first approximation, of a quark and a diquark. How the remaining energy and momentum should be split between these two is not known from first principles. We therefore use different parameterizations of the splitting function and check the resulting variations. We find significant differences only at large rapidities where an uneven energy-momentum splitting tend to shift bottom quarks connected to a beam remnant diquark more in the direction of the beam remnant, hence giving rise to asymmetries at very large rapidities. We use an intermediate scenario in this study, given by $\text{MSTP}(92) = 3$.

- **Threshold behaviour between cluster decay and collapse.** Consider a $b \bar{t}$ cluster with an invariant mass at, or slightly above, the two particle threshold. Should this cluster decay to two hadrons or collapse into one? In one extreme point of view, a $B\pi$ pair should always be formed when above this threshold, and never a single $B$. In another extreme, the two-body fraction would gradually increase at a succession of thresholds: $B\pi, B^*\pi, B\rho, B^*\rho$, etc., where the relative probability for each channel is given by the standard flavour and spin mixture in string fragmentation. In our current default model, we have chosen to steer a middle course by allowing two attempts ($\text{MSTJ}(17) = 2$) to find a possible pair of hadrons. Thus a fraction of events may collapse to a single resonance also above the $B\pi$ threshold, but $B\pi$ is effectively weighted up. If a large number of attempts had been allowed (this can be varied using the free parameter $\text{MSTJ}(17)$), collapse would only become possible for cluster masses below the $B\pi$ threshold.

The colour connection between the produced heavy quarks and the beam remnants in the string model gives rise to an effect called beam remnant drag. In an independent fragmentation scenario the light cone energy momentum of the quark is simply scaled by some factor picked from a fragmentation function. Thus, on average the rapidity is conserved in the fragmentation process. This is not necessarily so in string fragmentation, where both string ends contribute to the four-momentum of the produced heavy hadron. If the other end of the string is a beam remnant, the hadron will be shifted in rapidity in the direction of the beam remnant resulting in an increase in $|y|$. This beam-drag is shown qualitatively in fig. 28, where the rapidity shift is shown as a function of rapidity and transverse momentum. This shift is not directly accessible experimentally, only indirectly as a discrepancy between the shape of perturbatively calculated quark distributions and the data.
Fig. 28: (a) Average rapidity shift $\langle \Delta y \rangle$ as a function of $y$ for some different $p_T$ cuts. (b) Average rapidity shift $\langle |\Delta y| \rangle$ in the direction of the “other end of the string” that the bottom quark is connected to, i.e. ignoring the sign of the shift.

5.3 Intrinsic Heavy Quarks

The wavefunction of a hadron in QCD can be represented as a superposition of Fock state fluctuations, e.g. $|nV\rangle$, $|nVg\rangle$, $|nVQ\bar{Q}\rangle$, ... components where $nV = uud$ for a proton. When the projectile scatters in the target, the coherence of the Fock components is broken and the fluctuations can hadronize either by uncorrelated fragmentation as for leading twist production or coalescence with spectator quarks in the wavefunction [33] [36]. The intrinsic heavy quark Fock components are generated by virtual interactions such as $gg \to Q\bar{Q}$ where the gluons couple to two or more more projectile valence quarks. Intrinsc $Q\bar{Q}$ Fock states are dominated by configurations with equal rapidity constituents so that, unlike sea quarks generated from a single parton, the intrinsic heavy quarks carry a large fraction of the parent momentum [33].

The frame-independent probability distribution of an $n$-particle $b\bar{b}$ Fock state is

$$\frac{dP^n_{b\bar{b}}}{dx_i \cdots dx_n} = N_n \frac{\delta(1 - \sum_{i=1}^n x_i)}{(m_i^2 - \sum_{i=1}^n (\hat{m}_i^2 / x_i))^2},$$

where $\hat{m}_i^2 = k_{1,i}^2 + m_i^2$ is the effective transverse mass of the $i^{th}$ particle and $x_i$ is the light-cone momentum fraction. The probability, $P^n_{b\bar{b}}$, is normalized by $N_n$ and $n = 5$ for baryon production from the $|nVb\bar{b}\rangle$ configuration. The delta function conserves longitudinal momentum. The dominant Fock configurations are closest to the light-cone energy shell and therefore the invariant mass, $M^2 = \sum_i \hat{m}_i^2 / x_i$, is minimized. Assuming $\langle \hat{k}_{1,i}^2 \rangle$ is proportional to the square of the constituent quark mass, we choose $\hat{m}_q = 0.45$ GeV, $\hat{m}_s = 0.71$ GeV, and $\hat{m}_b = 5$ GeV [41] [42].

The $x_F$ distribution for a single bottom hadron produced from an $n$-particle intrinsic bottom state can be related to $P^n_{b\bar{b}}$ and the inelastic pp cross section by

$$\frac{\sigma_{pp}^H}{dx_F} = \frac{dP^n_{b\bar{b}}}{dx_F} \delta_{pp} \frac{\mu^2}{4\hat{m}_b^2} \alpha_s(M_{pp}^2).$$

The probability distribution is the sum of all contributions from the $|nVb\bar{b}\rangle$ and the $|nVb\bar{q}\bar{q}\rangle$ configurations with $q = u, d,$ and $s$ and includes uncorrelated fragmentation and coalescence, as described below, when appropriate [43]. The factor of $\mu^2 / 4\hat{m}_b^2$ arises from the soft interaction which breaks the coherence of the Fock state. We take $\mu^2 \sim 0.1$ GeV$^2$ [44]. The intrinsic charm probability, $P^{c}_{1c} = 0.31\%$, was determined from analyses of the EMC charm structure function data [45]. The intrinsic bottom probability is scaled from the intrinsic charm probability by the square of the transverse masses, $P^{b}_{1b} = P^{c}_{1c}(\hat{m}_c / \hat{m}_b)^2$. 
The intrinsic bottom cross section is reduced relative to the intrinsic charm cross section by a factor of \(\alpha_s^4(M_{b\bar{b}})/\alpha_s^4(M_{c\bar{c}})\) [46]. Taking these factors into account, we obtain \(\sigma_{ib}^5(pN) \approx 7\) nb at 14 TeV.

There are two ways of producing bottom hadrons from intrinsic \(b\bar{b}\) states. The first is by uncorrelated fragmentation. If we assume that the \(b\) quark fragments into a \(B\) meson, the \(B\) distribution is

\[
\frac{dP_{nF}}{dx_B} = \int dz \prod_{i=1}^n dx_i \frac{dP_{ib}}{dx_1 \ldots dx_n} \frac{D_{B/b}(z)}{z} \delta(x_B - z x_b),
\]

These distributions are assumed for all intrinsic bottom production by uncorrelated fragmentation with \(D_{H/b}(z) = \delta(z - 1)\). At low \(p_{\perp}\), this approximation should not be too bad, as seen in fixed target production [42].

If the projectile has the corresponding valence quarks, the bottom quark can also hadronize by coalescence with the valence spectators. The coalescence distributions are specific for the individual bottom hadrons. It is reasonable to assume that the intrinsic bottom Fock states are fragile and can easily materialize into bottom hadrons in high-energy, low momentum transfer reactions through coalescence. The coalescence contribution to bottom hadron production is

\[
\frac{dP_{nc}}{dx_H} = \int dx_H \prod_{i=1}^n dx_i \frac{dP_{ib}}{dx_1 \ldots dx_n} \delta(x_H - x_{H_1} - \ldots - x_{H_{nv}}),
\]

where the coalescence function is simply a delta function combining the momentum fractions of the quarks in the Fock state configuration that make up the valence quarks of the final-state hadron.

Not all bottom hadrons can be produced from the minimal intrinsic bottom Fock state configuration, \(|n_{Vb}\rangle\). However, coalescences can also occur within higher fluctuations of the intrinsic bottom Fock state. For example, in the proton, the \(B^-\) and \(\Xi_b^0\) can be produced by coalescence from \(|n_{Vb}\rangle\) and \(|n_{Vb}\rangle\) configurations. These higher Fock state probabilities can be obtained using earlier results on \(\psi\psi\) pair production [47] [48]. If all the measured \(\psi\psi\) pairs [49] arise from \(|n_{Vc\bar{c}}\rangle\) configurations, \(P_{cc} \approx 4.4\% \ P_{bc} [48] [50]\). It was found that the probability of a \(|n_{Vc\bar{c}}\rangle\) state was then \(P_{bc} = (\bar{m}_c/m_b)^2 P_{cc}\) [47]. If we then assume \(P_{1bq} = (\bar{m}_c/m_b)^2 P_{cc}\), we find

\[
P_{1bq} \approx \left(\frac{\bar{m}_c}{m_b}\right)^2 \left(\frac{\bar{m}_c}{m_q}\right)^2 P_{cc},
\]

leading to \(P_{1b} = P_{1bd} \approx 70.4\% \ P_{1b}\) and \(P_{1bs} \approx 28.5\% \ P_{1b}\). To go to still higher configurations, one can make similar assumptions. However, as more partons are included in the Fock state, the coalescence distributions soften and approach the fragmentation distributions, eventually producing bottom hadrons with less momentum than uncorrelated fragmentation from the minimal \(b\bar{b}\) state if a sufficient number of \(c\bar{c}\) pairs are included. There is then no longer any advantage to introducing more light quark pairs into the configuration—the relative probability will decrease while the potential gain in momentum is not significant. Therefore, we consider production by fragmentation and coalescence from the minimal state and the next higher states with \(u\bar{u}, d\bar{d}\) and \(s\bar{s}\) pairs.

The probability distributions entering Eq. (27) for \(B^0\) and \(\bar{B}^0\) are

\[
\frac{dP_{B^0}}{dx_F} = \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{8} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F}. \tag{31}
\]

\[
\frac{dP_{B^-}}{dx_F} = \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F} + \frac{1}{8} \frac{dP_{ib}}{dx_F} + \frac{1}{10} \frac{dP_{ib}}{dx_F}. \tag{32}
\]

See Ref. [43] for more details and the probability distributions of other bottom hadrons.
5.4 Model predictions

In this section we present some results from both models. Figure 29 shows the asymmetry between $B^0$ and $\bar{B}^0$ as a function of $y$ for several $p_\perp$ cuts in the string model. The asymmetry is essentially zero for central rapidities and increases slowly with rapidity. When the kinematical limit is approached, the asymmetry changes sign for small $p_\perp$ because of the drag-effect since $b$-quarks are often connected to diquarks from the proton beam remnant, fig. 27, thus producing $\bar{B}^0$ hadrons which are shifted more in rapidity than $B^0$. Cluster collapse, on the other hand, tend to enhance the production of leading particles (in this case $B^0$) so the two mechanisms give rise to asymmetries with different signs. Collapse is the main effect at small rapidities while eventually at very large $y$, the drag effect dominates.

![Graph](image)

Fig. 29: The asymmetry, $A = \frac{\sigma(B^0) - \sigma(\bar{B}^0)}{\sigma(B^0) + \sigma(\bar{B}^0)}$, as a function of rapidity for different $p_\perp$ cuts: (a) $p_\perp < 5, 10$ GeV and (b) $p_\perp > 5, 10$ GeV using parameter set 1 as described in the text.

In Table 12 we study the parameter dependence of the asymmetry by looking at the integrated asymmetry for different kinematical regions using three different parameter sets:

- **Set 1** is the new default as presented in section 5.2.
- **Set 2** The same as Set 1 except it uses simple counting rules in the beam remnant splitting, i.e. each quark get on average one third of the beam remnant energy-momentum.
- **Set 3** The old parameter set, before fitting to fixed-target data, is included as a reference. This set is characterized by current algebra masses, lower intrinsic $k_\perp$, and an uneven sharing of beam remnant energy-momentum.

We see that in the central region the asymmetry is generally very small whereas for forward (but not extremely forward) rapidities and moderate $p_\perp$ the asymmetry is around 1–2%. In the very forward region at small $p_\perp$, drag asymmetry dominates which can be seen from the change in sign of the asymmetry. The asymmetry is fairly stable under moderate variations in the parameters even though the difference between the old and new parameter sets (Set 1 and 3) are large in the central region. Set 1 typically gives rise to smaller asymmetries.

The cross sections for all intrinsic bottom hadrons are given as a function of $x_F$ in fig. 30. The bottom baryon distributions are shown in fig. 30(a). The $\Lambda_b^0 (\Sigma_b^0)$ distributions are the largest and most forward peaked of all the distributions. The $\Sigma_b^-$ is the smallest and the softest, similar to that of the bottom-strange mesons and baryons shown in fig. 30(b). The different coalescence probabilities assumed for hadrons from the $|uuudbb\bar{q}\bar{q}|$ configuration have little real effect on the shape of the cross section, dominated by independent fragmentation. Of the $B$ mesons shown in fig. 30(c), the $B^+$ and $B^0$ cross sections are the largest since both can be produced from the 5 particle configuration. The $B^-$ and $\bar{B}^0$ distributions are virtually identical. We note that the $x_F$ distributions of other bottom hadrons not
Table 12: Parameter dependence of the asymmetry in the string model. The statistical error in the last digit is shown in parenthesis (95% confidence).

| Parameters | \(|y| < 2.5, p_\perp > 5 \text{ GeV}\) | \(3 < |y| < 5, p_\perp > 5 \text{ GeV}\) | \(|y| > 3, p_\perp < 5 \text{ GeV}\) |
|------------|---------------------------------|---------------------------------|---------------------------------|
| Set 1      | 0.003(1)                        | 0.015(2)                        | −0.008(1)                       |
| Set 2      | −0.000(2)                       | 0.009(3)                        | −0.005(2)                       |
| Set 3      | 0.013(2)                        | 0.020(3)                        | −0.018(2)                       |

included in the figure would be similar to the bottom-strange hadrons since they would be produced by fragmentation only.

Fig. 30: Predictions for bottom hadron production are given for pp collisions at 14 TeV. The bottom baryon distributions are given in (a) for \(\Lambda_0^0 = \Sigma_0^0\) (dot-dashed), \(\Sigma_0^+\) (dashed), and \(\Sigma_0^-\) (solid). The bottom-strange distributions are shown in (b) for \(\Xi_0^0\) (solid), \(\Xi_0^-\) (dashed), \(\Xi'_0\) (dot-dashed), and \(\Xi''_0\) (dotted). In (c), the B meson distributions are given: \(B^+\) (solid), \(B^-\) (dashed), \(B^0\) (dot-dashed), and \(B^*\) (dotted). The \(B^-\) and \(\bar{B}^0\) distributions are virtually identical.

The \(x_F\) distribution for final-state hadron \(H\) is the sum of the leading-twist fusion and intrinsic bottom components,

\[
\frac{d\sigma_{H,N}^{TW}}{dx_F} = \frac{d\sigma_{H}^{TW}}{dx_F} + \frac{d\sigma_{H}^{IB}}{dx_F}.
\] (33)

The intrinsic bottom cross sections from Section 5.3 are combined with a leading twist calculation using independent fragmentation where drag effects are not included. The leading twist results have been
smoothed and extrapolated to large $x_F$ to facilitate a comparison with the intrinsic bottom calculation. The resulting total $B^0$ and $\bar{B}^0$ distributions are shown in fig. 31, along with the corresponding asymmetry. Note that since the intrinsic heavy quark $p_\perp$ distributions are more steeply falling than the leading twist, we only consider $p_\perp < 5$ GeV. The distributions are drawn to emphasize the high $x_F$ region where the distributions differ. The asymmetry is $\sim 0.1$ at $x_F \sim 0.25$, corresponding to $y \sim 6.5$. Therefore, intrinsic bottom should not be a significant source of asymmetries.

Fig. 31: (a) Leading-twist predictions for $B^0$ (solid) and $\bar{B}^0$ (dashed) using independent fragmentation. Model predictions for $B^0$ (dot-dashed) and $\bar{B}^0$ (dotted) distributions from Eq. (33). (b) The asymmetry between $B^0$ and $\bar{B}^0$, the dot-dashed and dotted curves in (a), is also given.

5.5 Summary
To summarize, we have studied possible production asymmetries between $b$ and $\bar{b}$ hadrons, especially $B^0$ and $\bar{B}^0$, as predicted by the Lund string fragmentation model and the intrinsic heavy quark model. We find negligible asymmetries for central rapidities and large $p_\perp$ (in general, less than 1%). For some especially favoured kinematical ranges such as $y > 3$ and $5 < p_\perp < 10$ GeV the collapse asymmetry could be as high as 1–2%. Intrinsic bottom becomes important only for $x_F > 0.25$ and $p_\perp < 5$ GeV, corresponding to $y > 6.5$.

6. Quarkonium Production
The production of charmonium and bottomonium states at high-energy colliders has been the subject of considerable interest during the past few years. New experimental results from $p\bar{p}$, $e^+e^-$, and $e^-e^+$ colliders have become available, some of which revealed dramatic shortcomings of earlier quarkonium production models. In theory, progress has been made on the factorization between the short distance physics of heavy-quark creation and the long-distance physics of bound state formation. The colour-singlet model [51] [52] has been superseded by a consistent and rigorous framework, based on the use of non-relativistic QCD (NRQCD) [53], an effective field theory that includes the so-called colour-octet mechanisms. On the other hand, the colour evaporation model [54] [55] [56] of the early days of quarkonium physics has been revived [57] [58] [59] [60]. However, despite the recent theoretical and experimental developments the range of applicability of the different approaches is still subject to debate, as is the quantitative verification of factorization. Because the quarkonium mass is still not very large with respect to the QCD scale, in particular for the charmonium system, non-factorizable corrections [61] [62] [63] may not be

\[ ^5 \text{Thanks to J. Klay at UC Davis for extending the curves to large } x_F. \]

\[ ^6 \text{Section coordinators: M. Krämer, F. Maltoni, M.A. Sanchis-Lozano} \]
suppressed enough, if the quarkonium is not part of an isolated jet, and the expansions in NRQCD may not converge very well. In this situation a global analysis of various processes is mandatory in order to assess the importance of different quarkonium production mechanisms, as well as the limitations of a particular theoretical framework (for reviews on different quarkonium production processes see e.g. [64] [65] [66].) By the time the LHC starts operating, new experimental data from the Tevatron and HERA as well as theoretical progress, e.g. in the calculation of higher-order corrections, will have significantly improved the present picture and will allow more precise predictions than what is possible at present. In the following, we will therefore focus on the general phenomenological implications of the NRQCD approach for quarkonium production at the LHC, rather than aiming at a detailed and comprehensive numerical analysis. Based on the information provided by the present Tevatron data we will derive predictions for observables crucial for future LHC analyses, such as differential cross sections and quark onium polarization.

In the NRQCD approach, the cross section for producing a quarkonium state $H$ at a hadron collider can be expressed as a sum of terms, each of which factors into a short-distance coefficient and a long-distance matrix element:

$$d\sigma(pp/p\bar{p}\rightarrow H + X) = \sum_n d\hat{\sigma}(pp/p\bar{p}\rightarrow Q\bar{Q}[n] + x) \langle\mathcal{O}^H[n]\rangle,$$

(34)

where $n$ denotes the colour, spin and angular momentum state of an intermediate $Q\bar{Q}$ pair. The short-distance cross section $d\hat{\sigma}$ can be calculated perturbatively in the strong coupling $\alpha_s$. The NRQCD matrix elements $\langle\mathcal{O}^H[n]\rangle$ (see [53] for their definition) are related to the non-perturbative transition probabilities from the $Q\bar{Q}$ state $n$ into the quarkonium $H$. They scale with a definite power of the intrinsic heavy-quark velocity $v$ [67]. ($v^2 \sim 0.3$ for charmonium and $v^2 \sim 0.1$ for bottomonium.) The general expression (34) is thus a double expansion in powers of $\alpha_s$ and $v$.

The NRQCD formalism implies that so-called colour-octet processes associated with higher Fock state components of the quarkonium wave function must contribute to the cross section. Heavy quark pairs that are produced at short distances in a colour-octet state can evolve into a physical quarkonium through radiation of soft gluons at late times in the production process, when the quark pair has already expanded to the quarkonium size. Such a possibility is ignored in the colour-singlet model, where only those heavy quark pairs that are produced in the dominant Fock state (i.e. in a colour-singlet state and with the spin and angular momentum quantum numbers of the meson) are assumed to form a physical quarkonium. The most profound theoretical evidence that the colour-singlet model is incomplete comes from the presence of infrared divergences in the production cross sections and decay rates of $P$-wave states. Within the NRQCD approach, this problem finds its natural solution since the infrared singularities are factored into a colour-octet operator matrix element [68]. While colour-octet contributions are needed for a consistent description of $P$-wave quarkonia, they are phenomenologically even more important for $S$-wave states like $J/\psi$ or $\Upsilon$. According to the velocity scaling rules, colour-octet matrix elements for the production of $S$-wave quarkonia are suppressed by a factor $v^4$ compared to the leading colour-singlet contributions. However, as discussed in some detail below, colour-octet processes can become significant if the short-distance cross section for producing $Q\bar{Q}$ in a colour-octet state is enhanced.

The production of $S$-wave charmonium in $p\bar{p}$ collisions at the Tevatron has attracted considerable attention and has stimulated much of the recent theoretical development in quarkonium physics. The CDF collaboration has measured cross sections for the production of $J/\psi$ and $\psi(2S)$ states not coming from $B$ or radiative $\chi$ decays, for a wide range of transverse momenta $5\text{ GeV} \lesssim p_T(\psi) \lesssim 20\text{ GeV}$ [69] [70]. Surprisingly, the experimental cross sections were found to be orders of magnitudes larger than the theoretical expectation based on the leading-order colour-singlet model [71] [72]. This result is particularly striking because the data extends out to large transverse momenta where the theoretical analysis is rather clean. The shortcoming of the colour-singlet model can be understood by examining a typical
(a) leading-order colour-singlet: \( g + g \rightarrow c\bar{c}^{[3]} S_{1}^{(1)} + g \)

\[
J/\psi \\
\text{+ ...} \quad \sim \alpha_{s}^{3} \frac{(2m_{c})^{2}}{p_{t}^{4}}
\]

(b) colour-singlet fragmentation: \( g + g \rightarrow [c\bar{c}^{[3]} S_{1}^{(1)} + gg] + g \)

\[
\text{+ ...} \quad \sim \alpha_{s}^{5} \frac{1}{p_{t}^{4}}
\]

(c) colour-octet fragmentation: \( g + g \rightarrow c\bar{c}^{[3]} S_{1}^{(8)} + g \)

\[
\text{+ ...} \quad \sim \alpha_{s}^{3} \frac{1}{p_{t}} v^{4}
\]

(d) colour-octet t-channel gluon exchange: \( g + g \rightarrow c\bar{c}^{[1]} S_{0}^{(8)} + \frac{3 P_{f}^{(8)}}{p_{t}^{4}} + g \)

\[
\text{+ ...} \quad \sim \alpha_{s}^{3} \frac{(2m_{c})^{2}}{p_{t}^{4}} v^{4}
\]

Fig. 32: Generic diagrams for \( J/\psi \) production in hadron-hadron collisions via colour-singlet and colour-octet channels.

Feynman diagram contributing to the leading-order parton cross section, fig. 32(a). At large transverse momentum, the two internal quark propagators are off-shell by \( \sim p_{t}^{2} \) so that the parton differential cross section scales like \( d\sigma / dp_{t}^{2} \sim 1/p_{t}^{8} \), as indicated in the figure. On the other hand, when \( p_{t} \gg 2m_{c} \) the quarkonium mass can be considered small and the inclusive charmonium cross section is expected to scale like any other single-particle inclusive cross section \( \sim 1/p_{t}^{4} \). The dominant production mechanism for charmonium at sufficiently large \( p_{t} \) must thus be via fragmentation [73], the production of a parton.
with large $p_t$ which subsequently decays into charmonium and other partons. A typical fragmentation contribution to colour-singlet $J/\psi$ production is shown in fig. 32(b). While the fragmentation contributions are of higher order in $\alpha_s$ compared to the fusion process fig. 32(a), they are enhanced by a power $p_t^4/(2m_c)^4$ at large $p_t$ and can thus overtake the fusion contribution at $p_t \gg 2m_c$. When colour-singlet fragmentation is included, the $p_t$ dependence of the theoretical prediction is in agreement with the Tevatron data but the normalization is still underestimated by about an order of magnitude [74] [75] [76], indicating that an additional fragmentation contribution is still missing. It is now generally believed that gluon fragmentation into colour-octet $^3S_1$ charm quark pairs [77] [78], as shown in fig. 32(c), is the dominant source of $J/\psi$ and $\psi(2S)$ at large $p_t$ at the Tevatron. The probability of forming a $J/\psi$ particle from a pointlike $c\bar{c}$ pair in a colour-octet $^3S_1$ state is given by the NRQCD matrix element $\langle O^{J/\psi}[^3S_1^{(8)}] \rangle$ which is suppressed by $v^4$ relative to the non-perturbative factor of the leading colour-singlet term. However, this suppression is overcompensated for by the gain in two powers of $\alpha_s/\pi$ in the short-distance cross section for producing colour-octet $^3S_1$ charm quark pairs as compared to colour-singlet fragmentation. At $O(v^4)$ in the velocity expansion, two additional colour-octet channels have to be included, fig. 32(d), which do not have a fragmentation interpretation at order $\alpha_s^2$ but which become significant at moderate $p_t \sim 2m_c$ [79] [80]. The importance of the $^1S_0^{(8)}$ and $^3P_J^{(8)}$ contributions cannot be estimated from naive power counting in $\alpha_s$ and $v$ alone, but rather follows from the dominance of $t$-channel gluon exchange, forbidden in the leading-order colour-singlet cross section.

The different contributions to the $J/\psi$ transverse momentum distribution are compared to the CDF data [70] in fig. 33. As mentioned above, the colour-singlet model at lowest order in $\alpha_s$ fails dramati-

![Graph](image-url)
The analysis of the CDF data alone, although very encouraging, does not strictly prove the phenomenological relevance of colour-octet contributions because free parameters have to be introduced to fit the data. However, if factorization holds the non-perturbative matrix elements, Table 13, perfectly consistent with the $v^4$ suppression expected from the velocity scaling rules. Similar conclusions can be drawn for $\psi(2S)$ production at the Tevatron.

The inclusion of higher-order QCD corrections is required to reduce the theoretical uncertainty and to allow a more precise prediction of the LHC cross sections. Next-to-leading order (NLO) calculations for quarkonium production at hadron colliders are presently available only for total cross sections [86] [87]. Significant higher-order corrections to differential distributions are expected from the strong renormalization and factorization scale dependence of the leading-order results [85]. Moreover, the NLO colour-singlet cross section includes processes like $g + g \rightarrow Q\bar{Q} \{ ^3S_1^{(1)} \}$ which are dominated by $t$-channel gluon exchange and scale as $\sim \alpha_s^3(2m_Q)^2/p_t^6$. At $p_t \gg 2m_Q$ their contribution is enhanced with respect to the the leading-order cross section, fig. 32(a), which scales as $\sim \alpha_s^2(2m_Q)^4/p_t^8$. This is
Fig. 34: Cross sections for $J/\psi$ and $\psi(2S)$ production in $pp \to \psi + X$ at the LHC ($\sqrt{s} = 14$ TeV, pseudorapidity cut $|\eta| < 2.5$). Parameter specifications as in fig. 33. The leading logarithms $(\alpha_s \ln p_T^2 / (2m_c)^2)^n$ have been summed by solving the Altarelli-Parisi evolution equations for the gluon fragmentation function. The error bands include the statistical errors in the extraction of the NRQCD matrix elements [Table 13] only.

Born out by preliminary studies [88] which include part of the NLO hadroproduction cross section and by the complete calculation of NLO corrections to the related process of quarkonium photoproduction [89]. The NLO colour-singlet cross section may be comparable in size to the colour-octet $1S_0$ and $3P_J$ processes, which scale as $\sim \alpha_s^3 \ln^4 (2m_Q)^2 / p_T^4$ (see fig. 32(d)), and affect the determination of the corresponding NRQCD matrix elements from the Tevatron data. A full NLO analysis is however needed before quantitative conclusions can be drawn.

Another source of potentially large higher-order corrections is the multiple emission of soft or almost collinear gluons from the initial state partons. These corrections, as well as effects related to intrinsic transverse momentum, are expected to modify the shape of the transverse momentum distribution predominantly at relatively low values of $p_T \lesssim 2m_Q$. Initial state radiation can be partially summed in perturbation theory [90], but so far only total cross sections have been considered in the literature [91]. An estimate of the effect on the transverse momentum distribution should be provided by phenomenological models where a Gaussian $k_t$-smearing is added to the initial state partons. The result of these calculations not only depends on the average $\langle k_t \rangle$, which enters as a free parameter, but also on the details of how the smearing is implemented. Moreover, a lower cut-off has to be provided which regulates the divergences at $p_T = 0$. Using the NLO calculation for the total cross section [87], one can obtain the rough estimate that perturbative Sudakov effects should be confined below $p_T \sim 1 - 2$ GeV for both charmonium and bottomonium production at Tevatron energies. Qualitatively, the inclusion of $k_t$-smearing leads to an enhancement of the short distance cross section at small $p_T$, which results in smaller values
for the fits of the $\langle O^0_S[1S_0] \rangle$ and $\langle O^0_S[3P_0] \rangle$ NRQCD matrix elements [92] [88]. The actual size of the effect, however, turns out to be very different for the two models studied in the literature.

An alternative approach to treat the effect of initial state radiation is by means of Monte Carlo event generators which include multiple gluon emission in the parton shower approximation. Comprehensive phenomenological analyses have been carried out for charmonium production at the Tevatron and at the LHC [93] [94] [95] using the event generator PYTHIA [37] supplemented by the leading colour-octet processes [93]. The inclusion of initial state radiation as implemented in PYTHIA leads to an enhancement of the short-distance cross section. The size of the effect is significantly larger than for the Gaussian $k_t$-smearing mentioned above, and it extents out to large $p_t$. Consequently, the $\langle O^0_S[1S_0] \rangle$ and $\langle O^0_S[3P_0] \rangle$ NRQCD matrix elements estimated from the Monte Carlo analysis of the Tevatron cross sections are significantly lower than the ones listed in Table 13 (see [93] [94] for details). Figure 35 shows the individual contributions to the direct $J/\psi$ cross section at the LHC as estimated with the PYTHIA Monte Carlo [94]. Note that for consistency the curves are based on the NRQCD matrix ele-

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**Fig. 35**: Cross sections for $J/\psi$ production in $pp \rightarrow J/\psi + X$ at the LHC ($\sqrt{s} = 14$ TeV, rapidity cut $|y| < 2.5$) obtained from a Monte Carlo event generator [94]. CTEQ2L parton distribution functions [99]; (i) dotted line: colour-singlet, (ii) dashed line: colour-octet $^1S_0 + ^3P_J$, (iii) dot-dashed line: $^3S_1^{(8)}$, (iv) solid line: all contributions. NRQCD matrix elements as specified in [94].
leading-order fit listed in Table 13. One observes that the final prediction is consistent with the result presented in fig. 34 within errors. The extrapolation of the Tevatron fits to LHC energies seems rather insensitive to the details of the underlying theoretical description, and different approaches yield similar predictions for the LHC cross sections as long as the appropriate NRQCD matrix elements are used. The Monte Carlo implementation [93] should therefore represent a convenient and reliable tool for the experimental simulation of quarkonium production processes at the LHC.

A crucial test of the NRQCD approach to charmonium production at hadron colliders is the analysis of $J/\psi$ and $\psi(2S)$ polarization at large transverse momentum. Recall that at large $p_t$, $\psi$ production should be dominated by gluon fragmentation into a colour-octet $3S_1$ charmonium state, fig. 32(c). When $p_t \gg 2m_c$ the fragmenting gluon is effectively on-shell and transverse. The intermediate $c\bar{c}$ pair in the colour-octet $3S_1$ state inherits the gluon’s transverse polarization and so does the charmonium, because the emission of soft gluons during hadronization does not flip the heavy quark spin at leading order in the velocity expansion. Consequently, at large transverse momentum one should observe transversely polarized $J/\psi$ and $\psi(2S)$ [100]. The polarization can be measured through the angular distribution in the decay $\psi \rightarrow l^+l^-$, given by $d\Gamma/d\cos\theta \propto 1 + \alpha \cos^2\theta$, where $\theta$ denotes the angle between the lepton three-momentum in the $\psi$ rest frame and the $\psi$ three-momentum in the lab frame. Pure transverse polarization implies $\alpha = 1$. Corrections to this asymptotic limit due to spin-symmetry breaking and higher order fragmentation contributions have been estimated to be small [101]. The dominant source of depolarization comes from the colour-octet fusion diagrams, fig. 32(d), which are important at moderate $p_t$. Still, at $O(p^4)$ in the velocity expansion, the polar angle asymmetry $\alpha$ can be unambiguously calculated within NRQCD [84] [102] in terms of the three non-perturbative matrix elements [Table 13] that have been determined from the unpolarized cross section. In fig. 36 we display the theoretical prediction for $\alpha$ in $\psi(2S)$ production at the Tevatron as function of the $\psi(2S)$ transverse momentum. No transverse polarization is expected at $p_t \sim 5$ GeV, but the angular distribution is predicted to change drastically as $p_t$ increases. A preliminary measurement from CDF [103] does not support this prediction, but the experimental errors are too large to draw definite conclusions. A similar picture emerges from the analysis of $J/\psi$ polarization [104], where, however, the theoretical analysis is complicated by the fact that the data sample still includes $J/\psi$ that have not been produced directly but come from decays of higher excited states [105].

Polarization measurements are crucial to discriminate the NRQCD approach from the colour evaporation model, where the cross section for a specific charmonium state is given as a universal fraction of the inclusive $c\bar{c}$ production cross section integrated up to the open charm threshold. In general, the assumption of a single universal long-distance factor is too restrictive. It implies a universal $\sigma(\chi_c)/\sigma(J/\psi)$ ratio, which is not supported by the comparison of charmonium production in hadron-hadron and photon-hadron collisions. Still, since the colour evaporation model allows colour-octet charm quark pairs from gluon fragmentation to hadronize into charmonium, it can describe the $p_t$ distribution of the Tevatron data [57] [58] [59] [60]. In contrast to the NRQCD approach, however, the colour evaporation model predicts charmonium to be produced unpolarized. The model assumes un suppressed gluon emission from the $c\bar{c}$ pair during hadronization which randomizes spin and colour. This assumption is clearly wrong in the heavy quark limit where spin symmetry is at work and soft gluon emission does not flip the heavy quark spin. Nonetheless, since the charm quark mass is not very large with respect to the QCD scale, the applicability of heavy quark spin symmetry to charmonium physics has to be tested by confronting the NRQCD polarization signature with experimental data.

To definitely resolve the issue of charmonium polarization, a high-statistics measurement extending out to large transverse momentum will be necessary. Such a measurement can be carried out at the LHC, where one expects a polarization pattern similar to that predicted for the Tevatron, see fig. 37. The absence of a substantial fraction of transverse polarization in $\psi$ production at large $p_t$ would represent a serious problem for the application of the NRQCD factorization approach to the charmonium system and might indicate that the charm quark mass is not large enough for a nonrelativistic approach to work.
The application of NRQCD should be on safer grounds for the bottomonium system. As $v^2 \sim 0.1$ for bottomonium, higher-order terms in the velocity expansion (in particular colour-octet contributions) are expected to be less relevant than in the case of charmonium. Cross sections for the production of $\Upsilon$ states have been measured at the Tevatron in the region $p_t \lesssim 20$ GeV [106, 107, 108]. The leading-order colour-singlet model predictions underestimate the data, the discrepancy being, however, much less significant than in the case of charmonium. Given the large theoretical uncertainties in the cross section calculation, in particular at small $p_t \lesssim M_{\Upsilon}$, the need for colour-octet contribution is not yet as firmly established as for charmonium production. The inclusion of both next-to-leading order corrections and the summation of soft gluon radiation is required to obtain a realistic description of the $\Upsilon$ cross section in the $p_t$-range probed by present data. Such calculations have not yet been performed, and we have therefore not attempted a systematic fit [79, 80] of the bottomonium NRQCD matrix elements. Our predictions for the $\Upsilon$ cross section at the LHC, figs. 38,39, are based on a simple choice of the non-perturbative input parameters [Table 14] which is consistent with the present experimental information from the Tevatron. The cross sections should thus not be regarded as firm predictions of NRQCD but rather as order-of-magnitude estimates. The expected theoretical progress and more experimental information will allow a more precise prediction in the near future.

The impact of initial state gluon radiation on the $\Upsilon$ cross sections at the Tevatron has been estimated by adding a Gaussian $k_t$-smearing as discussed previously in the context of charmonium production. An average $\langle k_t \rangle \sim 3$ GeV and a $K$-factor $\sim 3$ are found to bring the leading-order colour-singlet
polar angle asymmetry $\alpha$ in $\psi(2S) \rightarrow \mu^+ \mu^-$ at the LHC

Fig. 37: Polar angle asymmetry $\alpha$ for $\psi(2S)$ production in $pp \rightarrow \psi(2S)(\rightarrow \mu^+ \mu^-) + X$ at the LHC as a function of $p_T$. Parameter specifications as in fig. 36.

Table 14: NRQCD matrix elements for bottomonium production. The colour-singlet matrix elements are taken from the potential model calculation of [82] [83]. The colour-octet matrix elements have been determined from the CDF data for $p_T > 8$ GeV [107], where $\langle O^H_{1 S_0} \rangle = \langle O^H_{1 S_0} \rangle = \langle O^H_{1 S_0} \rangle / m_b^2$ has been assumed for simplicity. Parameters: CTEQ5L parton distribution functions [81], renormalization and factorization scale $\mu = (p_T^2 + 4m_b^2)^{1/2}$ and $m_b = 4.88$ GeV.

<table>
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<th>$H$</th>
<th>$\langle O^H \rangle$</th>
<th>$\langle O^H_{1 S_0} \rangle$</th>
<th>$\langle O^H_{1 S_1} \rangle$</th>
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</tbody>
</table>

cross section in line with the experimental $\Upsilon(1S, 2S)$ data at $p_T \lesssim M_T$ [109]. Similar results have been obtained within a Monte Carlo analysis [110], leading to significantly lower fit values for the colour-octet NRQCD matrix elements than those determined from a leading-order calculation [Table 14]. Moreover, the Monte Carlo results imply that no feeddown from $\chi$ states produced through colour-octet $3S_1 b \bar{b}$ states is needed to describe the inclusive $\Upsilon$ cross section, in contrast to what is found at leading-order. The calculation of next-to-leading order corrections and a systematic treatment of soft gluon radiation within perturbation theory are required to resolve these issues. Figure 40 shows the inclusive $\Upsilon(1S)$
Fig. 38: Cross sections for $\Upsilon(1S)$ production in $pp \rightarrow \Upsilon(1S) + X$ at the LHC ($\sqrt{s} = 14$ TeV, pseudorapidity cut $|\eta| < 2.5$). Parameters: CTEQ5L parton distribution functions [81], factorization and renormalization scale $\mu = \sqrt{p_t^2 + 4m_b^2}$, $m_b = 4.88$ GeV. The leading logarithms $(\alpha_s \ln p_t^2/(2m_b^2))^n$ have been summed by solving the Altarelli-Parisi evolution equations for the gluon fragmentation function. NRQCD matrix elements as specified in Table 14. The error band is obtained by varying the colour-octet matrix elements between half and twice their central value for illustration.

cross section at the LHC as obtained from the Monte Carlo calculation [110]. The curves are based on the NRQCD matrix elements extracted from the Monte Carlo analysis of the Tevatron data [110]. As in the case of charmonium production, one observes that the final LHC prediction is consistent with the leading-order result presented in fig. 38 within errors.

Let us finally present the polarization pattern predicted for direct $\Upsilon(1S)$ production at the LHC, fig. 41, based on the NRQCD matrix elements of Table 14. Higher-order corrections to the gluon fragmentation function [101] [84] will lead to a small reduction of the transverse polarization at large $p_t$ and should be included once precise data become available. If the charmonium mass is indeed not large enough for a nonrelativistic expansion to be reliable, the onset of transverse $\Upsilon$ polarization at $p_t \gg M_\Upsilon$ may become the single most crucial test of the NRQCD factorization approach.

In summary, we have discussed some of the phenomenological implications of the NRQCD approach for quarkonium production at the LHC and presented ‘state-of-the-art’ predictions for $\psi$ and $\Upsilon$ differential cross sections and polarization.\footnote{Other processes that have been studied in the literature include quarkonium production in association with photons [111] [112] or electroweak bosons [113], as well as $\eta$ and $P$-wave quarkonium production [114] [115].} Among the theoretical issues that need to be addressed in the future are the calculation of higher-order QCD corrections, the summation of higher-order terms in the velocity expansion and quantitative insights in the effect of higher-twist contributions. Besides a global analysis of different production processes and observables at various colliders, quarkonium physics at the LHC will play a crucial role to assess the importance of colour-octet processes and to conclusively test the applicability of non-relativistic QCD and heavy-quark spin symmetry to the charmonium and
7. PROSPECTS FOR $b$ PRODUCTION MEASUREMENTS AT THE LHC

Of the existing and currently proposed accelerator facilities, the LHC will yield the largest rate of $b$ quarks. A well defined program for $b$ production investigations, and the development of dedicated detection strategies optimised for ATLAS, CMS and LHCb, are required for the successful exploitation of the rich LHC potential. After an introduction summarising the main physics motivations, we review the detector and trigger features relevant for $b$ production in the LHC experiments. The kinematic ranges accessible to the three experiments are then described. Theoretical motivations and possible measurement methods are presented for single $b$ quark properties, correlations in $b$ production, multiple heavy flavour production, polarization, and charge asymmetry effects in $B$-hadron production in $pp$ interactions. Based on earlier performance studies, the potential for these measurements is estimated and some preliminary results are presented. We conclude with a summary of the present status of the preparations for $b$-production studies.

7.1 Introduction

While many LHC studies have been devoted to $B$-decays, $b$ production has not yet been directly addressed. Even though $b$ decay investigations will provide some information on the production, at the discussions of this workshop it became clear that they are not sufficient to cover all aspects of production.

Heavy quark production in high energy hadronic collisions is important for the study of Quantum Chromodynamics (QCD). Nowadays, QCD is recognized as a well established and solid theory. If
disagreements between the theoretical predictions and the experimental data are found, they will suggest the lack of understanding only of a particular production mechanism. In many cases these disagreements may be attributed to a too slow convergence of the perturbation series. In other cases, there may be important contributions from nonperturbative effects. Strictly speaking, the production measurements are not going to test the principles of QCD, but rather to outline the boundaries, where the predictions of perturbation theory provide an adequate description and exhaust all the visible effects. In this context, it will be certainly useful to test as many different processes as possible.

We present below some examples of such processes and observables, which can potentially be studied in the LHC experiments. Besides testing QCD, there exist other motivations to understand production properties; for instance, as a control of the systematics in CP violation. Double $b$-pair production is also a background in some channels of Higgs detection for LHC [116]. Measurements of the $b$ production by ATLAS and CMS in the initial years of low luminosity running will also be used to optimise the trigger selections at high luminosity for rare $B$ decays.

7.2 Detector and trigger characteristics relevant for $b$-production

The ATLAS, CMS and LHCb detectors and triggers are described in detail elsewhere [117]. Even though the signal-to-noise ratio for $b$ events is higher at LHC than at lower energy hadron machines, only about 1% of the non diffractive inelastic collisions will produce $b$-quark pairs. Events with $B$ hadrons can be
Fig. 41: Polar angle asymmetry $\alpha$ for direct $\Upsilon(1S)$ production in $pp \rightarrow \Upsilon(1S)(\rightarrow \mu^+ \mu^-) + X$ at the LHC as a function of $p_T$. NRQCD matrix elements as specified in Table 14 other parameters as in Figure 38. The error band reflects the limiting cases that either $\langle \Omega \rangle$ or $\langle Q \rangle$ is set to zero in the linear combination extracted from the data.

distinguished from other inelastic $pp$ interactions by the presence of leptons, of secondary vertices and particles with high $p_T$. Each of the three experiments will have several levels of triggers to efficiently select the interesting events containing $B$ hadrons while maintaining manageable trigger rates. The information from the muon detectors and the electromagnetic and hadronic calorimeters will be used by the lowest level trigger in all the three experiments. In LHCb the lowest level trigger performs a pile-up veto followed by soft cuts on first level trigger objects like muons ($p_T > 1$ GeV), electrons ($E_T > 2.1$ GeV) or hadron clusters ($E_T > 2.4$ GeV) reducing the trigger rate to 1 MHz. The more time-consuming operations, like vertex reconstruction and using information from the RICH for particle identification, will be performed by the higher level triggers. The final event rate expected from LHCb is $\sim 200$ Hz.

ATLAS and CMS are central detectors for high $p_T$ physics designed to operate at high luminosities. The low-level trigger objects have higher $p_T$ limits than in LHCb: single muons $p_T > 6(7)$ GeV in ATLAS(CMS) or dimuon triggers with a minimal $p_T$ of each muon in the interval $(3 - 6)$ GeV in ATLAS and $(2 - 4)$ GeV in CMS [118] [119]. However, thanks to the higher luminosity, despite the higher $p_T$ thresholds, they will have statistics comparable to LHCb in many exclusive channels. Simulations done on both experiments have demonstrated that at a luminosity of $10^{33} cm^{-2}s^{-1}$, in spite of 2-3 pileup events on the average accompanying the $b$ event in the same bunch crossing, $B$-decays can be triggered on and further cleanly separated from background in off-line reconstruction [120] [121].

### 7.3 Kinematic ranges

The central detectors ATLAS and CMS will cover the pseudorapidity region $|\eta| < 2.5$; the more forward LHCb is optimised for $1.8 < \eta < 4.9$. The overlap between the experiments is less than a unit of pseu-
dorapidity, in the region $1.8 < \eta < 2.5$. The low transverse momentum cutoffs in each experiment are limited mainly by the admissible low-level trigger rates. In the statistically dominant channels, ATLAS and CMS will be efficient for $B$-hadrons with $p_T \gtrsim 10$ GeV and LHCb for $p_T > 2$ GeV. The domains of the Bjorken $x$ variable for different values of the $b$ quark transverse momentum $p_T$ are given in fig. 42 for two situations: when both the $b$ and $\bar{b}$ are in a fiducial volume of a detector; and when only one of them is there. It is clear that in all three LHC experiments the sampled range of $x$ is contained within the region already covered by HERA [122]. For comparison, the analogous distribution is calculated for CDF conditions (fig. 43).

![Fig. 42: Bjorken $x$ region of LHCb for different values of the $b$ $p_T$ for the situation when one of the quarks is in the detector volume (a), both $b$ and $\bar{b}$ are in the detector volume (b). In (c) and (d), analogous distributions are given for ATLAS/CMS.](image)

**7.4 Single $b$ quark production**

**7.41 Theoretical motivations**

The inclusive differential cross section $d\sigma/dp_T d\eta$, where $p_T$ and $\eta$ are the transverse momentum and the pseudorapidity of the $b$ or $\bar{b}$ quarks, provide the basic information on $b$ production. As discussed in the previous section, next-to-leading order (NLO) calculations give a cross section lower than CDF and D0 data by a factor of $\sim 2.4$ [12]. However, the shape of the $p_T$ distribution is well reproduced by LO+NLO predictions, by a semihard model of the BFKL type [123] and also by PYTHIA [124]. In the region of high $p_T$, the effects of higher order contributions are taken into account by means of the resummation technique [125] [23] [126]. In ref. [23] LO+NLO contributions are included together with
the resummation of all terms of order $\alpha_s^k \ln^k(p_T/m_0)$ and $\alpha_s^{k+} \ln^k(p_T/m_0)$. These contributions change the shape of the $p_T$ spectrum. Thus measurements of high $p_T$ single $b$-spectra may be considered as a dedicated test for the QCD resummation technique.

### 7.42 Measurement possibilities

Experiments can measure the doubly differential cross section $d\sigma/dp_T d\eta$, where $p_T$ and $\eta$ are the transverse momentum and the pseudorapidity of a $B$ hadron, or of a jet associated with a $B$ hadron, or only one of the decay products of a $B$ hadron (for example $J/\psi$ or $\mu$). From these experimentally measured quantities, the $d\sigma/dp_T d\eta$ of the parent $b$-quark can be extracted, using appropriate models of hadronization and decay.

The determination of the absolute value of the cross section is also important. Three independent measurements (ATLAS, CMS and LHCb) can be done at the same energy. The determination of the absolute cross sections is always difficult, since it requires a precise understanding of the luminosity, of the trigger and reconstruction efficiency and of the background contributions. Several techniques of luminosity measurement are under study. It appears that precisions of $\sim 3\%$ could be achieved [127]. The overlap in the detection phase space of ATLAS, CMS and LHCb, in the region $1.8 < \eta < 2.5$ and $p_T > 10$ GeV, can be used for cross-checks.

### 7.43 Exclusive channels

From trigger and offline studies and the present experience with CDF it is known that the three LHC experiments can provide high statistics samples of some exclusive $B$-decay channels cleanly separated from the background. The statistically dominant channels are those containing $J/\psi \rightarrow \mu^+\mu^-$ ($B_d \rightarrow J/\psi K^0$, $B_d \rightarrow J/\psi K^*$, $B^\pm \rightarrow J/\psi K^\pm$ and $B^0 \rightarrow J/\psi \phi$), which are also needed for CP violation studies. Moreover, LHCb will cleanly separate large statistics of purely hadronic exclusive decays, where the dominant ones are $B_d \rightarrow D^{*-}\pi^+$ and $B_d \rightarrow D^{*-}a_1^+$. With these processes one can cover the differential $p_T$ cross section measurements starting approximately from $p_T > 10$ GeV for ATLAS and CMS and $p_T > 2$ GeV for LHCb respectively. The numbers of these events after three years of run at luminosities of $10^{33} cm^{-2}s^{-1}$ for ATLAS and CMS and five years at $2 \cdot 10^{32} cm^{-2}s^{-1}$ for LHCb, are shown in fig. 44 as a function of a minimal transverse momentum of the $B$-hadron $p_T$. 

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Fig. 43: Bjorken $x$ region of CDF for different values of $b p_T$ for the situation when one of the quarks is in the detector volume (a), both $b$ and $\bar{b}$ quarks are in the detector volume (b).
### 7.44 Inclusive $b \to J/\psi X$ channels

The inclusive channels $b \to J/\psi X$ can be used to extend the available statistics for production measurements to high transverse momenta (fig. 44).

A preliminary study from CMS [128] shows that for $p_T^{J/\psi} \sim 300$ GeV, which corresponds to $p_T^b \sim 550$ GeV, a b-tagging efficiency of $\sim 50\%$ can be achieved with a $J/\psi$ mass and decay length reconstruction. This will give a signal-to-noise ratio of $\sim 2.5$ taking into account the prediction for prompt $J/\psi$ production of ref. [93].

In ATLAS, a study has been done [129] for events $b \to J/\psi(\mu\mu)X$ in which the $p_T$ of the $b$ quark was chosen larger than 50 GeV. In particular, it was shown that the mass resolution of the $J/\psi$ will not be degraded due to events in which a signal reconstructed in the muon system is wrongly associated to a non-muon track in the inner detector.

![Fig. 44](image.png)

Fig. 44: Number of triggered and reconstructed $B$-hadrons as function of a lower cut on the $B$-hadron transverse momentum $p_T$. Figure (a) shows the ATLAS expectations for inclusive and exclusive $B$-hadrons decays to $J/\psi$ after 3 years, with an integrated luminosity of $30 fb^{-1}$. The full line corresponds to inclusive events, the dashed line to the sum of all exclusive channels. Fig. (b) shows the LHCb expectations for exclusive $B$-hadrons decays after 5 years. The solid line is for all statistically dominant channels, the dashed line shows only the channels with a $c\bar{p}$ in the final state.

### 7.45 Inclusive $b$-jet production

Another method for $b$ production studies discussed at the workshop was based on inclusive $b$-jet reconstruction. In both ATLAS and CMS this technique was developed for the Higgs search [130] [121]. The $b$-jet cross section is expected to be a small fraction (close to or larger than $2\%$ for jets with $E_T$ larger than about 20 GeV) of the single-jet cross section [131] [132]. If this method is to be used for single $b$ quark production, it will require prescaling of the trigger for the lower $p_T$ region or a cut on very high transverse momenta ($p_T > 150$ GeV), to reduce the huge rate of non-$b$ QCD background [133].

Figure 45 shows the preliminary results of the CMS $b$-tagging efficiency and mistagging probability for high $E_T$ jets using the technique described in [134]. The study demonstrates that for tagging efficiencies of $35\% - 55\%$ the mistagging probability is better than $2\%$ up to $E_T \sim 200$ GeV. Beyond that, the $b$-tagging efficiency and mistagging probability deteriorate significantly. The algorithm will be further optimised, possibly including lepton identification.

The method of $b$ cross section determination based on inclusive $b$-jet identification will be heavily dependent on the precise understanding of the non-$b$-jet rejection factors. Further feasibility studies on this method are necessary.
Fig. 45: The results of CMS study of $b$-tagging efficiency and mistagging probability as a function of the jet $E_T$. The squares represent the results of the phase-1 tracker, the circles those of the phase-2 tracker.

7.5 Correlations in $b$ production

7.51 Theoretical motivations

As discussed in Section 2, the overall normalisation of the production cross section, as well as the normalisation of the inclusive $b$ spectra, remain uncertain within a factor $\sim 2$ because of inherent theoretical uncertainties. Therefore the measurement of these values does not provide a stringent test of NLO contributions. The expected correlations between the $b$ and the $\bar{b}$ quarks can be computed in leading and next-to-leading order [5]. The shapes of two-particle distributions are sensitive to the NLO contribution, and thus can be used for these tests. In particular, distributions in the following quantities involving both the $b$ and $\bar{b}$ quark can be considered: the relative azimuthal distance $\Delta \phi(b\bar{b}) < 1$, the pair invariant mass, the pair transverse momentum and the pair rapidity.

7.52 Measurement possibilities

The choice of the decay channels is driven by the requirement that the acceptance should not vanish when the $b$ and the $\bar{b}$ are close in phase space. The goal is to avoid isolation cuts in both trigger and offline algorithms requiring a large separation between the decay products of the $B$ and of the $\bar{B}$. The processes under consideration are based on the reconstruction of a $J/\psi$ originating from the displaced vertex of a $B$-hadron, and of an additional lepton coming from the semileptonic decay of the associated $\bar{B}$ hadron. For example, in the ATLAS experiment, for an integrated luminosity of $30 \, fb^{-1}$, approximately $5 \times 10^5$ such events are expected, with the exclusively reconstructed $B$-decays containing the $J/\psi$ (Table 15). CDF and D0 measurements showed that $b\bar{b}$ pairs are mostly produced back-to-back [135]. However the region most sensitive to differences between the models is $\Delta \phi(b\bar{b}) < 1$ rad, where only $\sim 14\%$ of
the events are expected [124]. The statistics may possibly be increased using the semi-inclusive decays $b\bar{b} \rightarrow J/\psi X$ accompanied by a lepton (Table 15). As an example we quote recent studies in ATLAS [129], performed using simulated events with $B_d \rightarrow J/\psi K^0$, $J/\psi \rightarrow \mu^+ \mu^-$. They indicate that the signal events can indeed be reconstructed in cases when the difference of azimuthal angles between the $J/\psi$ and the other muon is small (Fig. 46). It is important to note that no selection cuts requiring model dependent corrections were necessary.

The study can be extended to events with $J/\psi \rightarrow \mu^+ \mu^-$ accompanied by an electron and for $J/\psi \rightarrow e^+ e^-$ combined with a muon or an electron. Using all these combinations of leptons will allow the measurement of the same variables by different detectors, leading to an improved control of systematic errors.

![Graph showing generated and reconstructed events](image)

Fig. 46: Reconstruction of $b\bar{b} \rightarrow J/\psi(\mu\mu)X + \mu$ using combined Muon-Inner detector off-line reconstruction in ATLAS.

<table>
<thead>
<tr>
<th>Inclusive channels</th>
<th>Statistics</th>
<th>Exclusive channels with the same lepton content</th>
<th>Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b\bar{b} \rightarrow J/\psi(\mu\mu)X + \mu$</td>
<td>$2.8 \cdot 10^6$</td>
<td>$b\bar{b} \rightarrow \text{had}J/\psi \rightarrow \mu^+ \mu^- + \mu$</td>
<td>$2.1 \cdot 10^5$</td>
</tr>
<tr>
<td>$b\bar{b} \rightarrow J/\psi(\mu\mu)X + e$</td>
<td>$3.6 \cdot 10^6$</td>
<td>$b\bar{b} \rightarrow \text{had}J/\psi \rightarrow \mu^+ \mu^- + e$</td>
<td>$2.1 \cdot 10^5$</td>
</tr>
<tr>
<td>$b\bar{b} \rightarrow J/\psi(ee)X + \mu$</td>
<td>$0.6 \cdot 10^6$</td>
<td>$b\bar{b} \rightarrow \text{had}J/\psi \rightarrow e^+ e^- + \mu$</td>
<td>$0.9 \cdot 10^5$</td>
</tr>
</tbody>
</table>
7.6 Multiple heavy quark pair production

7.6.1 Theoretical motivations

At present, only the leading order calculation, $O(\alpha_s^k)$, is available [136] for the $b\bar{b}b\bar{b}$ production cross section. The effects of higher order corrections can only be estimated using the event generator PYTHIA 5.7. Since the predictions of PYTHIA appear to be about a factor of 10 above the leading order analytical calculations [137], further theoretical studies are needed.

7.6.2 Measurement possibilities

Events with four $b$ quarks can be identified in several ways, the most appropriate one depending upon the context. As a background to Higgs search, the requirement is four $b$ jets in the fiducial volume. For the purpose of testing QCD predictions on double $b$ production, it may be sufficient to reconstruct events with three $b$ quarks in the fiducial volume. For three $b$ quarks with $p_T > 10$ GeV and $|\eta| < 2.5$, PYTHIA gives a cross section of 140 nb, which corresponds to 140 events produced per second. Despite this large number, it will be necessary to define features allowing on-line selection of these events in the presence of huge non-$b$ and single $b$ backgrounds.

As a source of an incorrect tag in CP violation measurements, the relevant $b\bar{b}b\bar{b}$ events are those with two $b$ hadrons, produced with the same flavour charge, identified in the fiducial volume, while two other $B$-hadrons, produced with the opposite flavour, are not detected. A direct measurement of this case could use reconstructed charged mesons or baryons, which are self-tagging. However the expected statistics of these events is insufficient. In fact, double $b$ production is expected to be only a minor source of wrong tags [138]. Techniques exist to determine the wrong-tag rate from all processes regardless of its origin, which does not need to be identified [138].

Similar to the case of double $b$ production, the production of doubly heavy hadrons, such as the $B_c + b + c$, $B_c + b + c$, etc., refers to an $O(\alpha_s^k)$ lowest order QCD process, and also provides a test of perturbative QCD calculations. The question of higher order QCD contributions and, probably, non-perturbative contributions is still open [139]. The total production rate in this case will not be indicative enough to establish the role of different production mechanisms, and the measurement should instead concentrate on the specific event topologies. In particular, various correlations between particles carrying charm and bottom may be of importance.

Measurement possibilities are under investigation for the channel $B_c^{(*)} \to J/\psi \pi$ [140]. A list of possible semileptonic and nonleptonic $B_c$ decays can be found for instance in [141]. The decays $B_c^{(*)} \to J/\psi \mu\nu$, $B_c^{(*)} \to J/\psi p^+$, $B_c^{(*)} \to J/\psi K^{*+}$, $B_c^{(*)} \to J/\psi D_s^+$ are other potentially interesting modes.

7.7 Other measurements

$B$ hadrons with non-zero spin can be polarized perpendicular to their production plane. Polarization measurements of $b$ hadrons produced in nucleon fragmentation could clarify the problems of different polarization models [142] that failed to reproduce the existing data on strange hyperon production [143]. In particular, information about the quark mass dependence of polarization effects could be obtained. For symmetry reasons, in $pp$ collisions this polarization vanishes at zero rapidity, so that the expected observed polarization in ATLAS and CMS will be smaller than in the more forward LHCb. Using the method of helicity analyses of cascade decay $\Lambda_b^0 \to \Lambda^0 J/\psi$ the $\Lambda_b^0$ polarization can be measured in ATLAS with a precision better than 0.016 [144]. Another approach to $\Lambda_b^0$ polarisation measurement, using the same decay channel can be found in [145].

In proton-proton collisions a charge production asymmetry of $b$ hadrons is expected. The asymmetry is defined as the difference of production probabilities of a $B$ hadron and its antiparticle. From the theoretical point of view, the asymmetries can provide information on the effects of soft dynamics
during the fragmentation and hadronization (i.e., on the soft interactions between the produced $b$ quark and the remnants of the disrupted proton). The relevant physical effects are expected to be unimportant [47] [146] [30] in the central rapidity region covered by ATLAS and CMS. In the more forward region of LHCb the asymmetry may rise to a few percent. A detailed theoretical discussion of this issues are given in Section 5.

Any production asymmetry is always measured in the presence of a CP violation asymmetry originating from $B$-hadron decays. In some cases these two effects are expected to be of the same order. This is the case, for instance, in the channels $B_d \to J/\psi K^*(K^+\pi^-)$, $B^+ \to J/\psi K^+$ and $\Lambda^0_b \to \Lambda^0 J/\psi$, which are expected to have a small CP violation ($< 1\%$). A way of estimating the relative size of these two effects may be based on the fact that the production asymmetry varies with the transverse momentum and the rapidity of produced $b$-quark, while the decay asymmetry should remain the same. Measurements of such small effects will require good understanding of the possible instrumental detection asymmetries.

### 7.8 Conclusions
The properties of $b$ production at the LHC can be measured by the three experiments, which are complementary in phase space. The small overlap region will allow a cross check on the cross section normalization. The kinematic conditions are such that Bjorken $x$ values sampled in $b$-production are above $10^{-6}$, a region lower than at the Tevatron, but already covered by HERA. Differential cross section measurements using exclusive $B$-hadron decays will be most important at small $p_T$ values. At high $p_T$ values and for correlations and multiple heavy flavour production measurements, the statistics can be increased by semi-inclusive $B$ decays containing $J/\psi$. Possible methods using $b$-jets require further study, to control the non-$b$ QCD background. The enormous LHC statistics will also allow to study the production polarization and charge production asymmetries.

### 8. TUNING OF MULTIPLE INTERACTIONS GENERATED BY PYTHIA

#### 8.1 Introduction
The track multiplicity distribution as well as the transverse momentum distribution of charged particles in proton-proton interactions (the so-called minimum bias events) affect the performances of the low level triggers and the detector occupancy of the LHCb experiment [147]. They should therefore be modelled reliably in Monte Carlo programs. In particular, at LHC energies, multiple interactions play an important role, and should not be neglected.

In Section 8.2 we examine the multiple interaction models available in PYTHIA [37] to describe the event structure in hadron-hadron collisions. In Section 8.3 we select a compilation of homogeneous data at different energies suitable to tune the multiple interactions parameters of PYTHIA; the tuning procedure is presented in Section 8.4. We use the phenomenological extrapolations at LHC energy in order to get the predictions for the track multiplicity and the transverse momentum distributions in minimum bias and $b\bar{b}$ events; these are reported in Section 8.5.

#### 8.2 Multiple interaction models
The multiple interactions scenario is needed to describe the multiplicity observables at hadron colliders [148] and is also supported by direct observation [149] [150] [151]. The basic assumption is that several parton-parton interactions can occur within a single hadron-hadron collision. Four different models are available in PYTHIA. The main parameter of these models, $P_{tr_{\text{min}}}$, is the minimum transverse momentum of the parton-parton collisions; it effectively controls the average number of parton-parton interactions and hence the average track multiplicity. The differences between the four models, which mainly affect the shape of the multiplicity distribution, are the following:

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10Section coordinators: P. Bartalini, O. Schneider.
- Model 1 (default in PYTHIA)
  All the hadron collisions are equivalent (as opposed to model 3 and 4 below) and all the parton-parton interactions are independent; the $P_{T_{\text{min}}}$ parameter represents an abrupt cut-off.

- Model 2
  Same as Model 1 but with a continuous turn-off of the cross section at $P_{T_{\text{min}}}$.

- Model 3
  Same as Model 2, but hadronic matter in the colliding hadrons is distributed according to a Gaussian shape, and a varying impact parameter between the two hadrons is assumed.

- Model 4
  Same as Model 3 but the matter distribution is described by two concentric Gaussian distributions.

The varying impact parameter models (Models 3 and 4) were developed [148] to fit the UA5 data [152]. A recent study performed by the CDF collaboration [153] concludes that a varying impact parameter model (Model 3) is also preferred to describe the underlying tracks in $b$ events produced at the Tevatron.

In the absence of published results on multiplicity distributions in minimum bias events at the Tevatron, we compare again the predictions of Model 1 and Model 3 with the UA5 data, using the final charged multiplicity distribution from the full $p\bar{p}$ data sample collected by UA5 at $\sqrt{s} = 546$ GeV [154], a recent version of PYTHIA$^{11}$, and a modern set of parton distribution functions, CTEQ4L [14], tuned on both HERA and Tevatron data. For this comparison, the main multiple interactions parameter ($P_{T_{\text{min}}}$) is tuned in each model to reproduce the mean value of the measured charged multiplicity distribution in not single diffractive events$^{12}$ ($\langle N_{\text{ch}} \rangle = 29.4 \pm 0.3 \pm 0.9$). We obtain $P_{T_{\text{min}}} = 1.63 \pm 0.02$ for Model 1 and $P_{T_{\text{min}}} = 1.97 \pm 0.03$ for Model 3. The shapes of the multiplicity distributions are compared in fig. 47. It is clear that Model 3 is preferred over Model 1 to describe the UA5 data. In particular the shape of the tail at high multiplicities is reproduced much better by Model 3. The UA5 results are corrected for the lower efficiency expected on double diffractive events. Therefore the simulated samples include the generation of all kind of non single-diffractive events. The uncertainty in the diffractive cross sections relative to the partonic ones can affect the observed discrepancies between the data and the PYTHIA predictions in the low multiplicity region$^{13}$.

### 8.3 Mean charged multiplicity at $\eta = 0$

In order to produce realistic PYTHIA predictions for the multiplicity observables in the LHC environment, it is necessary to take into account the energy dependence of the $P_{T_{\text{min}}}$ parameter. Unfortunately there are not many published data concerning the charged multiplicity distribution in minimum bias events at hadron colliders. On the other hand there are some data available relative to the average charged multiplicity in non single-diffractive events, in particular for the central pseudo-rapidity region. Therefore, to study the energy dependence of the $P_{T_{\text{min}}}$ parameter at generator level, we consider an homogeneous sample of corrected average charged multiplicity measurements at six different center-of-mass energies ($\sqrt{s} = 50$, 200, 546, 630, 900 and 1800 GeV) in the pseudo-rapidity region \(|\eta| < 0.25\) [156] [157]. The energy dependence of $dN_{\text{ch}}/d\eta$ at $\eta = 0$ is shown in fig. 48a together with the fit of a quadratic function of $\ln(s)$ proposed in Reference [157]; using this fit to extrapolate at LHC energy would predict $dN_{\text{ch}}/d\eta = 6.11 \pm 0.29$ at $\eta = 0$.

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$^{11}$Version 6.125 [155] was used for all the studies reported here.

$^{12}$In this paper we define as “non single-diffractive event” any inelastic hadron-hadron interaction that cannot be regarded as a single diffractive event; in the framework of the PYTHIA hadronic interactions, the “non single-diffractive” sample includes the $2 \rightarrow 2$ partonic processes and the double diffractive hadron-hadron interactions.

$^{13}$The $p\bar{p}$ cross sections predicted by PYTHIA at $\sqrt{s} = 546$ GeV are 30.7 mb for the partonic processes and 5.3 mb for the double diffractive processes.
Fig. 47: Charged multiplicity distribution for non single-diffractive events in \( \sqrt{s} = 546 \) GeV as measured by UA5 [154] compared with PYTHIA predictions using the CTEQ4L parton distribution functions and either Model 1 (solid) or 3 (dashed) for multiple interactions. In each case the \( P_{r_{\min}} \) parameter has been tuned to reproduce the mean multiplicity measured in the data.

8.4 Tuning of the multiple interaction parameter \( P_{r_{\min}} \)

The average charged multiplicity measurements performed on non single-diffractive data in \( \bar{p}p \) collisions and described in Section 8.3 are used to tune the main multiple interaction parameter in PYTHIA, \( P_{r_{\min}} \). We generate non single-diffractive events. At each value of \( \sqrt{s} \), the \( P_{r_{\min}} \) parameter is adjusted to reproduce the average multiplicity measured in the data. The uncertainty on the tuned value of \( P_{r_{\min}} \) reflects the uncertainty on the data. However, the tuned parameters depend on other aspects of the PYTHIA simulation: in particular the effects of various choices for the multiple interaction model and the parton distribution functions are investigated. For simplicity, the results of these studies are shown only for some representative settings:

- as an example of pre-HERA parton distribution functions we consider the CTEQ2L [99] set used by default in PYTHIA versions 5.7, but recently retracted by their authors;
- as an example of post-HERA parton distribution functions we consider the CTEQ4L [14] and GRV94L [158] sets, the latter being the new default in PYTHIA versions 6.1.

This study is restricted to Models 1 and 3 for multiple interactions (see Section 8.2).

The results of the tuning procedure are shown in fig. 48b: in each case, \( P_{r_{\min}} \) appears to be monotonically increasing as a function of \( \sqrt{s} \). This dependence is much more pronounced for the post-HERA parton distribution functions regardless of the choice for the multiple interactions model.

It was shown in Reference [159] that the post-HERA parton distribution functions imply an energy-dependent \( P_T \) cut-off. This is heuristically motivated by the Lipatov-like dependence of the gluonic parton distribution function in the small-\( x \) limit:

\[
xg(x, Q^2) \to \text{constant} \times x^{-\epsilon} \quad \text{for} \quad x \to 0
\]

with \( \epsilon \approx 0.08 \), while the pre-HERA parton distribution functions give a reduced charge screening effect and consequently a less sensitive running of the \( P_T \) cut-off. This is heuristically motivated by the Regge-like dependence of the gluonic parton distribution function in the small\( -x \) limit:

\[
xg(x, Q^2) \to \text{constant} \quad \text{for} \quad x \to 0.
\]
Fig. 48: a) $dN_{ch}/d\eta$ at $\eta = 0$ as a function of $\sqrt{s}$. The solid curves represent a phenomenological fit performed by CDF [157] with the formula $dN_{ch}/d\eta(s) = (0.023 \pm 0.008) \ln^2(s) - (0.25 \pm 0.19) \ln(s) + (2.5 \pm 1.0)$. The two dashed curves represent the 1 sigma variations of the fitted parameters.

b) $\sqrt{s}$-dependence of the $P_{T_{\text{min}}}$ parameter for various parton distribution functions and multiple interactions models. The points with error bars are the results of the tuning procedure on the data. The curves are the results of the fits through the points assuming the functional form of Equation 37 and are characterized by the parameters given in Table 16.

Table 16: Results of the fits describing the exponential running of the PYTHIA multiple interactions parameter $P_{T_{\text{min}}}$ for different parton distribution functions and multiple interactions models.

<table>
<thead>
<tr>
<th>Mult. int. model</th>
<th>PDF</th>
<th>$P_{T_{\text{min}}}^{\text{LHC}}$ (GeV/c)</th>
<th>$\epsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>CTEQ2L</td>
<td>$1.99 \pm 0.11$</td>
<td>$0.048 \pm 0.006$</td>
</tr>
<tr>
<td>3</td>
<td>GRV94L</td>
<td>$4.06 \pm 0.24$</td>
<td>$0.103 \pm 0.006$</td>
</tr>
<tr>
<td>3</td>
<td>CTEQ4L</td>
<td>$3.47 \pm 0.17$</td>
<td>$0.087 \pm 0.005$</td>
</tr>
<tr>
<td>1</td>
<td>CTEQ4L</td>
<td>$3.12 \pm 0.16$</td>
<td>$0.100 \pm 0.005$</td>
</tr>
</tbody>
</table>

In order to extrapolate $P_{T_{\text{min}}}$ at LHC energy, one needs to find a reasonable function to fit the tuned $P_{T_{\text{min}}}$ values as a function of $\sqrt{s}$ for the different parton distribution functions and multiple interactions models; a four degree of freedom fit is performed using the following exponential form, inspired by the recent implementations added in PYTHIA since version 6.120 [155]:

$$P_{T_{\text{min}}}^{\sqrt{s}} = P_{T_{\text{min}}}^{\text{LHC}} \left(\frac{\sqrt{s}}{14 \text{ TeV}}\right)^{2\epsilon}. \quad (37)$$

The fitted functions are superimposed on fig. 48b and the results obtained for the fitted parameters $\epsilon$ and $P_{T_{\text{min}}}^{\text{LHC}}$ are given in Table 16. This quantitative analysis demonstrates that the power law expressed in Equation 37 holds for values of $\epsilon$ between $\simeq 0.08$ and $\simeq 0.10$ if post-HERA parton distribution functions are used, and for somewhat smaller values of $\epsilon$ ($\simeq 0.05$) for the pre-HERA parton distribution functions.
8.5 **PYTHIA predictions at LHC energy**

Figures 49a and b show multiplicity and pseudorapidity distributions for charged particles predicted by PYTHIA at LHC with CTEQ4L and Model 3 for multiple interactions. The value of \( P_{\text{T}_{\text{min}}} \) at LHC is obtained by an extrapolation

\[
P_{\text{T}_{\text{min}}}^{\text{LHC}} = P_{\text{T}_{\text{min}}}^{\text{Tevatron}} \left( \frac{14 \text{ TeV}}{1.8 \text{ TeV}} \right)^{2\epsilon}
\]

(38)

where \( P_{\text{T}_{\text{min}}}^{\text{Tevatron}} \) is the \( P_{\text{T}_{\text{min}}} \) value tuned at the Tevatron energy of 1.8 TeV. For the parameter \( \epsilon \), we adopt the results given in Table 16. It is important to note that the predictions \( dN_{\text{ch}} / d\eta = 6.30 \pm 0.42 \) (for \( \epsilon = 0.087 \pm 0.005 \)) at \( \eta = 0 \) are consistent with the phenomenological fit displayed in fig. 48a \( (dN_{\text{ch}} / d\eta = 6.11 \pm 0.29) \).

In order to demonstrate the importance of the correct \( P_{\text{T}_{\text{min}}} \) extrapolation, figs. 49a and b also show results obtained by assuming \( P_{\text{T}_{\text{min}}}^{\text{LHC}} = P_{\text{T}_{\text{min}}}^{\text{Tevatron}} \), i.e. \( \epsilon = 0 \) not supported by the data as demonstrated in Section 8.4. The multiplicity distribution has a tail at high multiplicities and \( dN_{\text{ch}} / d\eta \) at \( \eta = 0 \) is not consistent with that obtained from the phenomenological fit.

![Fig. 49: PYTHIA predictions at LHC energy, using the CTEQ4L parton distribution functions and Model 3 for multiple interactions with \( P_{\text{T}_{\text{min}}} \) given by Equation 38: the solid and dashed distributions correspond to the central value and \( \pm 1 \sigma \) uncertainties of the fitted value \( \epsilon = 0.087 \pm 0.005 \). The dotted histogram is obtained with \( \epsilon = 0 \), i.e. using the \( P_{\text{T}_{\text{min}}} \) value tuned on Tevatron data and ignoring its energy dependence. a) Charged track multiplicity in the entire \( 4\pi \) solid angle. b) Average charged track multiplicity per unit pseudorapidity, as a function of pseudorapidity.](image-url)

Figure 50 shows the same distributions as fig. 49, but for the CTEQ2L parton distribution functions. It is interesting to note that, once the extrapolation of \( P_{\text{T}_{\text{min}}} \) is properly done, there is no large difference between the multiplicity and pseudorapidity distributions obtained with different structure functions.

Figure 51a-d compare the PYTHIA predictions for the multiplicity and transverse momentum distributions in the LHCb angular acceptance (1.8 < \( \eta < 4.9 \)) between minimum bias and \( b\bar{b} \) events\(^{14}\). These predictions are obtained with CTEQ4L, multiple interactions Model 3 and the proper \( P_{\text{T}_{\text{min}}} \) extrapolation. They show clear differences between minimum bias and \( b\bar{b} \) events, in particular higher average multiplicity and transverse momentum for \( b\bar{b} \) events.

\(^{14}\)The \( b\bar{b} \) events are selected among the minimum bias events.
PYTHIA predictions with multiple interactions Model 1 for the multiplicity and transverse momentum distributions are shown in fig. 52 for minimum bias and $b\bar{b}$ events. Compared to the results obtained with Model 3, less significant differences between minimum bias and $b\bar{b}$ events is observed. In Section 8.2 we have stressed that a varying impact parameter model for multiple interactions (i.e. Model 3) is needed to describe the charged track multiplicity in hadron-hadron interactions. There are arguments in favour of adopting a multiple interactions model with varying impact parameter to describe the heavy flavour production at hadron colliders [160], though there are no experimental data at low transverse momentum and high pseudorapidity (i.e. in the LHCb acceptance region). A more detailed discussion on the effect of multiparton interactions in $b\bar{b}$ events at LHCb can be found in Reference [161].

8.6 Conclusions

Comparisons between PYTHIA and experimental data demonstrate that, in order to reproduce the charged track multiplicity spectrum, a varying impact parameter model has to be adopted.

The varying impact parameter models predict sensitive differences in multiplicity and $P_T$ distribution between light and heavy flavour events.

The running of the $P_T$ cut-off parameter in PYTHIA multiple interactions is mandatory. Predictions made at LHC energy with a fixed $P_T$ cut-off tuned at lower energies overestimate the multiplicity observables. Taking into account the running of the $P_T$ parameter is even more important if post-HERA parton distribution functions are used.
Fig. 51: PYTHIA predictions for charged tracks in the LHCb acceptance using the CTEQ4L parton distribution functions and Model 3 for multiple interactions with proper $P_T; \mu$: the normalized predictions for $b\bar{b}$ events (dashed curve) and minimum bias events (solid curve) are superimposed. a) Charged track multiplicity distribution. b) Transverse momentum distribution of charged tracks. c) Distribution of the maximum transverse momentum of charged tracks. d) Rejection efficiency as a function of a traverse momentum cut (an event is rejected if all the charged tracks have a transverse momentum below the cut).

References

Fig. 52: PYTHIA predictions for charged tracks in the LHCb acceptance using the CTEQ4L parton distribution functions and Model 1 for multiple interactions with proper $P_{T_{mis}}$; the normalized predictions for $b\bar{b}$ events (dashed curve) and minimum bias events (solid curve) are superimposed. a) Charged track multiplicity distribution. b) Transverse momentum distribution of charged tracks.

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1. THEORETICAL INTRODUCTION

The exploration of physics with $b$ flavoured hadrons offers a very fertile testing ground for the Standard Model (SM) description of electroweak interactions. One of the key problems to be studied is the phenomenon of CP violation, which, although already discovered in 1964 by Christenson, Cronin, Fitch and Turlay in the neutral kaon system [1], is still one of the experimentally least constrained phenomena. Another main topic is the study of rare $b$ decays induced by flavour changing neutral current (FCNC) transitions $b \to s, d$, which are loop-suppressed in the SM and thus very sensitive to new-physics effects.

During the last few years, B physics has received a lot of attention, both from theorists and experimentalists, and we are presently at the beginning of the B factory era in particle physics. The BaBar (SLAC), BELLE (KEK) and HERA-B (DESY) detectors have already seen their first events, and CLEO-III (Cornell), CDF-II and D0-II (Fermilab) will start taking data in the near future (see [2] for a recent experimental overview). Although the physics potential of these experiments is very promising, it may well be that the “definite” answer in the search for new physics in B decays will be left for second-generation B experiments at hadron machines. In the following, we will give an overview of the B physics potential of the LHC experiments ATLAS, CMS and LHCb, with the main focus on SM physics.

1.1 CP Violation in the B System

Among the most interesting aspects and unsolved mysteries of modern particle physics is the violation of CP symmetry. Studies of CP violation are particularly exciting, as they may open a window to the physics beyond the SM. There are many interesting ways to explore CP violation, for instance through certain rare $K$ or $D$ meson decays (a very recent comprehensive description of all aspects of CP symmetry and its violation can be found in Ref. [3]). However, for testing the SM description of CP violation in a quantitative way, the B system appears to be most promising [4, 5, 6].

1.11 The SM Description of CP Violation

Within the framework of the SM, CP violation is closely related to the Cabibbo–Kobayashi–Maskawa (CKM) matrix [7, 8], connecting the electroweak eigenstates $(d', s', b')$ of the down, strange and bottom quarks with their mass eigenstates $(d, s, b)$ through the following unitary transformation:

$$
\begin{pmatrix}
  d' \\
  s' \\
  b'
\end{pmatrix} =
\begin{pmatrix}
  V_{ud} & V_{us} & V_{ub} \\
  V_{cd} & V_{cs} & V_{cb} \\
  V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix} \equiv \hat{V}_{\text{CKM}} \cdot 
\begin{pmatrix}
  d \\
  s \\
  b
\end{pmatrix}.
$$

(1)

\footnote{Section coordinators: P. Ball and R. Fleischer.}
The elements of the CKM matrix describe charged-current couplings, as can be easily seen by expressing the non-leptonic charged-current interaction Lagrangian in terms of the electroweak eigenstates (1):

\[
\mathcal{L}_{\text{int}}^{\text{CC}} = -\frac{g_2}{\sqrt{2}} \left( \bar{u}_{L_i} \gamma^\mu \hat{V}_{\text{CKM}} \begin{pmatrix} d_L \\ s_L \\ b_L \end{pmatrix} W^\mu_{\mu} + \text{h.c.}, \right.
\]

where the gauge coupling \( g_2 \) is related to the gauge group \( \text{SU}(2) \) and the \( W^\mu_{\mu} \) fields describe the charged \( W \) bosons.

In the case of three generations, three generalized Cabibbo-type angles [7] and a single complex phase [8] are needed in order to parametrize the CKM matrix. This complex phase allows one to accommodate CP violation in the SM, as was pointed out by Kobayashi and Maskawa in 1973 [8]. A closer look shows that CP-violating observables are proportional to the following combination of CKM matrix elements [9]:

\[
J_{\text{CP}} = \pm \text{Im} \left( V_{ik} V_{jl} V_{il}^* V_{jk}^* \right) \quad (i \neq j, l \neq k),
\]

which represents a measure of the “strength” of CP violation in the SM. Since \( J_{\text{CP}} = \mathcal{O}(10^{-5}) \), CP violation is a small effect. However, in scenarios of new physics [10], typically several additional complex couplings are present, leading to new sources of CP violation.

As far as phenomenological applications are concerned, the following parametrization of the CKM matrix, the “Wolfenstein parametrization” [11], which corresponds to a phenomenological expansion in powers of the small quantity \( \lambda \equiv |V_{us}| = \sin \theta_C \approx 0.22 \), turns out to be very useful:

\[
\hat{V}_{\text{CKM}} = \begin{pmatrix}
1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho - i \eta) \\
-\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^3 (1 - \rho - i \eta) \\
A \lambda^3 (1 - \rho - i \eta) & -A \lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4).
\]

The terms of \( \mathcal{O}(\lambda^4) \) can be taken into account systematically [12], and will play an important rôle below.

1.12 The Unitarity Triangle(s) of the CKM Matrix

Concerning tests of the CKM picture of CP violation, the central targets are the unitarity triangles of the CKM matrix. The unitarity of the CKM matrix, which is described by

\[
\hat{V}_{\text{CKM}} \cdot \hat{V}_{\text{CKM}}^\dagger = \hat{1} = \hat{V}_{\text{CKM}} \cdot \hat{V}_{\text{CKM}}^\dagger,
\]

leads to a set of 12 equations, consisting of 6 normalization and 6 orthogonality relations. The latter can be represented as 6 triangles in the complex plane, which all have the same area [13]. However, in only two of them, all three sides are of comparable magnitude \( \mathcal{O}(\lambda^3) \), while in the remaining ones, one side is suppressed relative to the others by \( \mathcal{O}(\lambda^2) \) or \( \mathcal{O}(\lambda^4) \). The orthogonality relations describing the non-squashed triangles are given as follows:

\[
\begin{align*}
V_{ud} V_{cb}^* + V_{cd} V_{tb}^* + V_{td} V_{db}^* &= 0 \\
V_{us} V_{td}^* + V_{as} V_{s}^* + V_{ab} V_{tb}^* &= 0.
\end{align*}
\]

The two non-squashed triangles agree at leading order in the Wolfenstein expansion, i.e. at \( \mathcal{O}(\lambda^3) \), so that we actually have to deal with a single triangle at this order, which is usually referred to as “the” unitarity triangle of the CKM matrix [14]. However, in the LHC era, the experimental accuracy will be so tremendous that we will have to take into account the next-to-leading order terms of the Wolfenstein expansion, and distinguish between the unitarity triangles described by (6) and (7), which are illustrated in Fig. 1. Here, \( \overline{\varphi} \) and \( \overline{\eta} \) are related to the Wolfenstein parameters \( \rho \) and \( \eta \) through [12]

\[
\overline{\varphi} \equiv \left( 1 - \lambda^2/2 \right) \rho, \quad \overline{\eta} \equiv \left( 1 - \lambda^2/2 \right) \eta.
\]
Fig. 1: The two non-squashed unitarity triangles of the CKM matrix: (a) and (b) correspond to the orthogonality relations (6) and (7), respectively.

Note the angles of the triangles, in particular those designated by $\alpha$, $\beta$, $\gamma$ and $\delta\gamma$. These will be referred to frequently throughout this report. The sides $R_b$ and $R_t$ of the unitarity triangle shown in Fig. 1(a) are given as follows:

$$R_b = \left(1 - \frac{\chi^2}{2}\right) \frac{1}{\lambda} \left| \frac{V_{ub}}{V_{cb}} \right| = \sqrt{\beta^2 + \eta^2} = 0.41 \pm 0.07,$$

$$R_t = \frac{1}{\lambda} \left| \frac{V_{td}}{V_{cb}} \right| = \sqrt{(1 - \eta^2)^2 + \eta^2} = \mathcal{O}(1).$$

1.13 Non-Leptonic $B$ Decays and Low-Energy Effective Hamiltonians

With respect to testing the SM description of CP violation, the major rôle is played by non-leptonic $B$ decays, which can be divided into three decay classes: decays receiving both “tree” and “penguin” contributions, pure “tree” decays, and pure “penguin” decays. There are two types of penguin topologies: gluonic (QCD) and electroweak (EW) penguins, which are related to strong and electroweak interactions, respectively. Because of the large top-quark mass, also the latter operators play an important rôle in several processes [15, 16, 17].

In order to analyse non-leptonic $B$ decays theoretically, one uses low-energy effective Hamiltonians, which are calculated by making use of the operator product expansion, yielding transition matrix elements of the following structure:

$$\langle f | \mathcal{H}_{\text{eff}} | \bar{v} \rangle \propto \sum_k C_k(\mu) \langle f | Q_k(\mu) | \bar{v} \rangle.$$  \hspace{1cm} (11)

The operator product expansion allows one to separate the short-distance contributions to this transition amplitude from the long-distance ones, which are described by perturbative Wilson coefficient functions $C_k(\mu)$ and non-perturbative hadronic matrix elements $\langle f | Q_k(\mu) | \bar{v} \rangle$, respectively. As usual, $\mu$ denotes an appropriate renormalization scale.

In the case of $|\Delta B| = 1$, $\Delta C = \Delta U = 0$ transitions, which will be of particular interest for the exploration of CP violation in the $B$ system, we have

$$\mathcal{H}_{\text{eff}} = \mathcal{H}_{\text{eff}}(\Delta B = -1) + \mathcal{H}_{\text{eff}}(\Delta B = -1)^\dagger,$$

where

$$\mathcal{H}_{\text{eff}}(\Delta B = -1) = \frac{G_F}{\sqrt{2}} \left[ \sum_{j=a,c} V_{jq}^* \bar{V}_{jb} \left\{ \sum_{k=1}^2 Q_k^{jq} C_k(\mu) + \sum_{k=3}^{10} Q_k^{q} C_k(\mu) \right\} \right].$$  \hspace{1cm} (13)

Here $\mu = \mathcal{O}(m_b)$ is a renormalization scale, the $Q_k^{q}$ are four-quark operators, the label $q \in \{d, s\}$ corresponds to $b \to d$ and $b \to s$ transitions, and $k$ distinguishes between current–current ($k \in \{1, 2\}$),
QCD \((k \in \{3, \ldots, 6\}\) and EW \((k \in \{7, \ldots, 10\}\) penguin operators. The calculation of such low-energy effective Hamiltonians has been reviewed in [18], where the four-quark operators \(Q_k^{ij}\) are given explicitly, and where also numerical values for their Wilson coefficient functions can be found.

1.14 \(B\!-\!\bar{B}\) Mixing

The eigenstates of flavour, \(B_q = (\bar{b}q)\) and \(\bar{B}_q = (b\bar{q})\) \((q = d, s)\), degenerate in pure QCD, mix on account of weak interactions. The quantum mechanics of the two-state system, with basis \([[1], [2]] \equiv \{|B_q\}, |\bar{B}_q]\} , is described by a complex, \(2 \times 2\) Hamiltonian matrix

\[
H = M - \frac{i}{2} \Gamma = \begin{pmatrix}
M & M_{12} \\
M_{12}^* & M
\end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma & \Gamma_{12} \\
\Gamma^*_{12} & \Gamma
\end{pmatrix}
\]  

(14)

with Hermitian matrices \(M\) and \(\Gamma\). The off-diagonal elements in (14) arise from \(\Delta B = 2\) flavour-changing transitions with virtual \((M_{12})\) or real intermediate states \((\Gamma_{12})\), in the latter case corresponding to decay channels common to \(B\) and \(\bar{B}\).

Diagonalizing (14), one obtains the physical eigenstates \(B_H\) (‘heavy’), \(B_L\) (‘light’) and the corresponding eigenvalues \(M_{H,L} \equiv -\frac{1}{2} \Gamma_{H,L}\). The mass and width differences read

\[
\Delta M_q \equiv M^{(q)}_H - M^{(q)}_L = 2|M^{(q)}_{12}|, \quad \Delta \Gamma_q \equiv \Gamma^{(q)}_H - \Gamma^{(q)}_L = \frac{2 \text{Re}(M^{(q)*}_{12} \Gamma^{(q)}_{12})}{|M^{(q)}_{12}|}.
\]  

(15)

\(\Delta M\) is positive by definition, \(\Delta \Gamma\) is defined in such a way that a negative value\(^2\) is obtained in the SM for the case of \(B_s\), where a sizeable width difference is expected. In the SM, the off-diagonal elements \(M_{12}\) and \(\Gamma_{12}\) inducing \(B\) mixing are described by the box diagrams in Fig. 2.

Detailed numerical results will be given Sec. 7.; here we only summarize a few important general characteristics of \(\Delta B = 2\) second-order weak processes. The relative size of the various contributions is controlled by CKM quantities and quark masses. With \(\lambda_i^{(q)} = V_{iq}^* V_{ib}\), and denoting the magnitude in powers of the Wolfenstein parameter \(\lambda\), we have \(\lambda^{(d)} \sim \lambda^{(d)}_c \sim \lambda^{(d)}_t \sim \lambda^3\) for \(B_d\), and \(\lambda^{(s)}_t \sim \lambda^{(s)}_c \sim \lambda^2\) for \(B_s\). Because the box amplitude strongly grows with large \((\gg m_b)\) internal quark masses \(m_i\), proportional to \(m_i^2\) for \(m_i \gg M_W\), it is clear, considering the above CKM hierarchy, that the top-quark contribution completely dominates the *dispersive part* \(M_{12}\). The remaining contributions \((i = u, c)\) are safely negligible for both the \(B_d\) and \(B_s\) system. Since \(m_t, M_W \gg m_b\), \(M_{12}\) is described by an effectively local interaction already at scales far above \(m_b\). External mass scales can thus be neglected and the resulting \(\Delta B = 2\) effective Hamiltonian is governed by a single operator. It acquires the simple form

\[
\mathcal{H}^{\Delta B = 2}_{\text{eff}} = (V_{tq}^* V_{tb})^2 C(x_t) (\bar{q} \bar{b})_{V-A} (\bar{q} \bar{b})_{V-A}
\]  

(16)

with \(C\) the short-distance Wilson-coefficient and \(x_t = m_t^2/m_W^2\), whence \(M_{12}\) is obtained as

\[
M_{12} = \frac{1}{2M_B} \langle B | \mathcal{H}^{\Delta B = 2}_{\text{eff}} | \bar{B} \rangle.
\]  

(17)

\(^2\)Note that also the opposite sign convention for \(\Delta \Gamma\) is used in the literature.
For the absorptive part $\Gamma_{12}$ the situation is more complicated. First of all, the top contribution, dominant for $M_{12}$, cannot contribute to $\Gamma_{12}$, since, by kinematics, top quarks are forbidden as on-shell final states in $B$ decays. $\Gamma_{12}$ is then determined by the (absorptive parts of) box diagrams with up and charm quarks. Both up and charm are important for $B_d$ because $\lambda_u^{(d)} \sim \lambda_c^{(d)}$. In the case of $B_s$, the up-quark sector is negligible ($\lambda_u^{(s)} \ll \lambda_c^{(s)}$). In calculating $\Gamma_{12}$, the heavy $W$ boson lines can be contracted to form two local $\Delta B = 1$ four-quark interactions (Fig. 2b). By contrast, $u$ and $c$ are lighter than the relevant scale of the process ($\sim m_b$) and cannot be integrated out, unlike the top quark in $M_{12}$. Consequently, $\Gamma_{12}$ is given as the matrix element of a non-local (or ‘bi-local’) product of two local $\Delta B = 1$ Hamiltonian operators $H_{\text{eff}}^{\Delta B=1}$, the usual effective weak Hamiltonian describing $B$ decays:

$$
\Gamma_{12} = \frac{1}{2M_B} \langle B | \text{Im} i \int d^4x T H_{\text{eff}}^{\Delta B=1}(x) H_{\text{eff}}^{\Delta B=1}(0) | \bar{B} \rangle .
$$

(18)

To lowest order in the strong interaction, (18) corresponds to the absorptive part (Im) of the diagram in Fig. 2(b). Taking the absorptive part inside the formal expression (18), the $T$-product is transformed into an ordinary product of the two factors $H_{\text{eff}}^{\Delta B=1}$. Inserting a complete set of hadronic final states $f$ gives

$$
\Gamma_{12}^{(\text{hadron})} = \sum_f \langle B | H_{\text{eff}}^{\Delta B=1} | f \rangle \langle f | H_{\text{eff}}^{\Delta B=1} | \bar{B} \rangle ,
$$

(19)

where one recognizes the usual expression for a decay rate, generalized here to the off-diagonal entry $\Gamma_{12}$. This connection, which allows one to write $\Gamma_{12}$ in (18) as the absorptive part of the $\bar{B} \rightarrow B$ forward scattering amplitude, is known as the optical theorem. $\Gamma_{12}^{(\text{hadron})}$ does, however, escape direct calculation, which instead starts from (18): taking advantage of the large momentum $\sim m_b \gg \Lambda_{\text{QCD}}$ flowing through the internal $u$ and $c$ quark lines of the box diagram, one expands the operator product into a series of local operators [19]:

$$
\Gamma_{12}^{(\text{quark})} = \frac{1}{2M_B} \langle B | \text{Im} i \int d^4x T H_{\text{eff}}^{\Delta B=1}(x) H_{\text{eff}}^{\Delta B=1}(0) | \bar{B} \rangle = \frac{1}{2M_B} \sum_n \frac{C_n}{m_b} \langle B | Q_{n}^{\Delta B=2} | \bar{B} \rangle .
$$

(20)

The identification of the exact $\Gamma_{12}^{(\text{hadron})}$ with the approximation $\Gamma_{12}^{(\text{quark})}$ based on the heavy quark expansion is equivalent to the assumption of local quark-hadron duality (‘local’ in this context refers to the fact that the large energy scale $m_b$ is, in practice, a fixed number, rather than a variable allowing for the consideration of some (‘global’) averaging procedure). When viewed as a function of $m_b$, $\Gamma_{12}^{(\text{hadron})}$ is expected to include terms of the form $\exp(-m_b/\Lambda_{\text{QCD}})$, which oscillates and exponentially suppressed terms are related to the opening of new decay channels as $m_b$ is increased. They are however completely missed in $\Gamma_{12}^{(\text{quark})}$ to any finite order in the heavy quark expansion, which is just a power series in $\Lambda_{\text{QCD}}/m_b$. Of course, for asymptotically large $m_b/\Lambda_{\text{QCD}} \rightarrow \infty$, these terms vanish much faster than power corrections. In any case $\Gamma_{12}^{(\text{quark})} \rightarrow \Gamma_{12}^{(\text{hadron})}$ in the strict limit $m_b \rightarrow \infty$. Nevertheless, for realistic values of $m_b$, those terms may introduce a deviation of $\Gamma_{12}^{(\text{quark})}$ from the correct $\Gamma_{12}^{(\text{hadron})}$ (beyond the omission of higher power corrections). This error is referred to as a violation of local duality. Theoretical knowledge from first principles about duality violating contributions is so far rather limited. Interesting general discussions and further information can be found in Refs. [20, 21, 22].

1.15 CP Violation in Neutral B Decays into CP Eigenstates

The description of CP violation in terms of weak phases becomes particularly simple for decays of neutral $B_q$ mesons ($q \in \{d, s\}$) into CP self-conjugate final states $|f\rangle$, satisfying the relation

$$
(CP) |f\rangle = \pm |f\rangle .
$$

(21)

In this case, the corresponding time-dependent CP asymmetry can be expressed as

$$
A_{\text{CP}}(t) = \frac{\Gamma(B_0^0(t) \rightarrow f) - \Gamma(B_0^0(t) \rightarrow \bar{f})}{\Gamma(B_0^0(t) \rightarrow f) + \Gamma(B_0^0(t) \rightarrow \bar{f})}.
$$
where $\Delta M_q \equiv M_{H}^{(q)} - M_{L}^{(q)}$ denotes the mass difference between the $B_q$ mass eigenstates, and $\Gamma_{H,L}^{(q)}$ are the corresponding decay widths, with

$$\Gamma_q \equiv \frac{\Gamma_{H}^{(q)} + \Gamma_{L}^{(q)}}{2}.$$  \hspace{1cm} (23)

In Eq. (22), we have separated the “direct” from the “mixing-induced” CP-violating contributions, which are described by

$$A_{\text{dir}}^{\text{CP}}(B_q \rightarrow f) \equiv \frac{1 - |\xi_f^{(q)}|^2}{1 + |\xi_f^{(q)}|^2} \quad \text{and} \quad A_{\text{mix}}^{\text{CP}}(B_q \rightarrow f) \equiv \frac{2 \text{Im} \{\xi_f^{(q)}\}}{1 + |\xi_f^{(q)}|^2},$$  \hspace{1cm} (24)

respectively. Here direct CP violation refers to CP-violating effects arising directly in the corresponding decay amplitudes, whereas mixing-induced CP violation is due to interference effects between $B_q^0 - \bar{B}_q^0$ mixing and decay processes. Whereas the width difference $\Delta \Gamma_q$ is negligible in the $B_d$ system, it may be sizeable in the $B_s$ system [23, 24], thereby providing the observable

$$A_{\Delta \Gamma}(B_q \rightarrow f) \equiv \frac{2 \text{Re} \{\xi_f^{(q)}\}}{1 + |\xi_f^{(q)}|^2},$$  \hspace{1cm} (25)

which is not independent of $A_{\text{dir}}^{\text{CP}}(B_q \rightarrow f)$ and $A_{\text{mix}}^{\text{CP}}(B_q \rightarrow f)$:

$$\left[A_{\text{dir}}^{\text{CP}}(B_s \rightarrow f)\right]^2 + \left[A_{\text{mix}}^{\text{CP}}(B_s \rightarrow f)\right]^2 + \left[A_{\Delta \Gamma}(B_s \rightarrow f)\right]^2 = 1.$$  \hspace{1cm} (26)

Essentially all the information needed to evaluate the CP asymmetry (22) is included in the following quantity:

$$\xi_f^{(q)} = e^{-i\phi_q} \sum_{j=a,c} \frac{V_{jr}V_{j\bar{b}}^*(f | Q_r^{j(0)} | B_q^0)}{\sum_{j=a,c} V_{jr}^*V_{j\bar{b}}(f | Q_r^{j(0)} | B_q^0)},$$  \hspace{1cm} (27)

where

$$Q_r^{j(0)} \equiv \sum_{k=1}^{2} Q_r^{kF} C_k(\mu) + \sum_{k=3}^{10} Q_r^{kF} C_k(\mu),$$  \hspace{1cm} (28)

$r \in \{d, s\}$ distinguishes between $\bar{b} \rightarrow \bar{d}$ and $\bar{b} \rightarrow \bar{s}$ transitions, and

$$\phi_q = \begin{cases} +2\beta & (q = d) \\ -2\delta_\gamma & (q = s) \end{cases}$$  \hspace{1cm} (29)

is the weak $B_q^0 - \bar{B}_q^0$ mixing phase which is related to the phase of $M_{12}$, Eq. (17). In general, the observable $\xi_f^{(q)}$ suffers from hadronic uncertainties, which are introduced by the hadronic matrix elements in Eq. (27). However, if the decay $B_q \rightarrow f$ is dominated by a single CKM amplitude, the corresponding matrix elements cancel, and $\xi_f^{(q)}$ takes the simple form

$$\xi_f^{(q)} = \mp \exp \left[- i \left(\phi_q - \phi_{D_f}^{(q)}\right)\right].$$  \hspace{1cm} (30)
where $\phi_D^{(f)}$ is a weak decay phase, which is given as follows:

$$
\phi_D^{(f)} = \begin{cases} 
-2\gamma & \text{for dominant } \bar{b} \to \bar{u} u \bar{\tau} \text{ CKM amplitudes,} \\
0 & \text{for dominant } \bar{b} \to \bar{e} c \bar{\tau} \text{ CKM amplitudes.}
\end{cases}
$$

(31)

This simple formalism has several interesting applications, probably the most important one is the extraction of the CKM angle $\beta$ from CP-violating effects in the “gold-plated” mode $B_d \to J/\psi K_S$. In addition to the CP-violating effects in neutral B decays into CP eigenstates discussed above, also certain modes into non-CP eigenstates, for example $B_d \to D^{(*)}\pm \pi^\pm$ and $B_s \to D_s^{(*)} K^\pm$, play an outstanding role to extract CKM phases. These decays will be discussed in more detail in Secs. 3.4 and 3.5.

1.17 CP Violation in Charged B Decays

The mass difference is an important difference between the $B_d$ and $B_s$ systems: where “PhSp” denotes an appropriate, straightforwardly calculable phase-space factor. Interestingly, there are no rapid oscillatory terms present in this expression. Although it should be no problem to resolve these $B^0_s-B_s^0$ oscillations at the LHC, studies of such untagged rates, which allow the extraction of the observable $A_{\Delta\Gamma}$ introduced in (25) as

$$
A_{\Delta\Gamma} = \frac{R_H - R_L}{R_H + R_L},
$$

(33)

are interesting in terms of efficiency, acceptance and purity.

1.16 The “El Dorado” for the LHC: the $B_s$ System

The $e^+e^-$ B factories operating at the $\Upsilon(4S)$ resonance will not be in a position to explore the $B_s$ system. Since it is, moreover, very desirable to have large data samples available to study $B_s$ decays, they are of particular interest for hadron machines and were one of the central targets of this workshop. There are important differences between the $B_d$ and $B_s$ systems:

- Within the SM, a large $B^0_s-B_s^0$ mixing parameter $x_s \equiv \Delta M_{s}/\Gamma_s = O(20)$ is expected, whereas the mixing phase $\phi_s = -2\lambda^2_H$ is expected to be very small.
- There may be a sizeable width difference $\Delta\Gamma_{s}/\Gamma_s = O(15\%)$, whereas $\Delta\Gamma_d$ is negligible.

The mass difference $\Delta M_{s}$ plays an important role to constrain the apex of the unitarity triangle shown in Fig. 1(a), and the non-vanishing width difference $\Delta\Gamma_s$ may allow studies of CP-violating effects in “untagged” $B_s$ rates, [25]–[28], which are defined as follows:

$$
\Gamma_s[f(t)] \equiv \Gamma(B^0_s(t) \to f) + \Gamma(B^{0*}_s(t) \to f) = \text{PhSp} \times \left[ R_H e^{-\Gamma_{s}^{(1)} t} + R_L e^{-\Gamma_{s}^{(2)} t} \right],
$$

(32)

where “PhSp” denotes an appropriate, straightforwardly calculable phase-space factor. Interestingly, there are no rapid oscillatory $\Delta M_{s} t$ terms present in this expression. Although it should be no problem to resolve these $B^0_s-B_s^0$ oscillations at the LHC, studies of such untagged rates, which allow the extraction of the observable $A_{\Delta\Gamma}$ introduced in (25) as

$$
A_{\Delta\Gamma} = \frac{R_H - R_L}{R_H + R_L},
$$

(33)

are interesting in terms of efficiency, acceptance and purity.

1.17 CP Violation in Charged B Decays

Since there are no mixing effects present in the charged B meson system, non-vanishing CP asymmetries of type

$$
A_{\text{CP}}(B^+ \to \bar{f}) \equiv \frac{\Gamma(B^+ \to \bar{f}) - \Gamma(B^- \to f)}{\Gamma(B^+ \to \bar{f}) + \Gamma(B^- \to f)}
$$

(34)

would give us unambiguous evidence for “direct” CP violation in the B system, which has recently been demonstrated in the kaon system by the new experimental results of the KTeV (Fermilab) and NA48 (CERN) collaborations for $\text{Re}(\epsilon'/\epsilon)$ [29].

The CP asymmetries (34) arise from the interference between decay amplitudes with both different CP-violating weak and different CP-conserving strong phases. In the SM, the weak phases are related to the phases of the CKM matrix elements, whereas the strong phases are induced by final-state-interaction (FSI) processes. In general, the strong phases introduce severe theoretical uncertainties into the calculation of $A_{\text{CP}}(B^+ \to \bar{f})$, thereby destroying the clean relation to the CP-violating weak phases. However, there is an important tool to overcome these problems, which is provided by amplitude relations between certain non-leptonic B decays. There are two types of such relations:
• Exact relations, which involve $B \rightarrow DK$ decays (pioneered by Gronau and Wyler [30]).
• Approximate relations, which rely on the flavour-symmetries of strong interactions and certain plausible dynamical assumptions, and involve $B \rightarrow \pi K, \pi\pi, K\overline{K}$ decays (pioneered by Gronau, Hernández, London and Rosner [31, 32]).

Unfortunately, the $B \rightarrow DK$ approach, which allows a theoretically clean determination of $\gamma$, makes use of certain amplitude-triangles that are expected to be rather squashed. Moreover, there are additional experimental problems [33], so that this approach is very challenging from a practical point of view. The flavour-symmetry relations between the $B \rightarrow \pi K, \pi\pi, K\overline{K}$ decay amplitudes have received considerable attention in the literature during the last couple of years and led to interesting strategies to probe the CKM angle $\gamma$.

1.18 Outline of the CP Violation Part

The outline of the part of this chapter dealing with aspects related to CP violation and the determination of the angles of the unitarity triangles is as follows: after an overview of the experimental aspects in Sec. 2., we have a closer look at the benchmark modes to explore CP violation in Sec. 3., where we will discuss the extraction of $\beta$ from the “gold-plated” decay $B_d \rightarrow J/\psi K_S$, the prospects to probe $\alpha$ with $B_d \rightarrow \pi^+\pi^-$ and $B \rightarrow \rho\pi$ modes, as well as extractions of $\gamma$ from $B_d \rightarrow D^{\mp}\pi^\mp$ and $B_s \rightarrow D_s^\pm K^\mp$ decays. Finally, we will also give a discussion of $\gamma$ determinations from $B \rightarrow DK$ modes.

Section 4. is devoted to a detailed analysis of another CP benchmark mode, $B_s \rightarrow J/\psi \phi$, which is particularly promising for the LHC experiments because of its favourable experimental signature and its rich physics potential, allowing one to extract the $B_s^0 - \overline{B}_s^0$ mixing parameters $\Delta M_s$ and $\Delta \Gamma_s$, as well as the corresponding CP-violating weak mixing phase $\phi_s$. Since the CP-violating effects in $B_s \rightarrow J/\psi \phi$ are tiny in the SM, this channel offers an important tool to search for new physics.

In Sec. 5., we focus on strategies to extract CKM phases that were not considered for the LHC experiments so far, and on new methods, which were developed during this workshop [34]. We discuss extractions of the angle $\gamma$ from $B \rightarrow \pi K$ decays, which received a lot of attention in the literature during the last couple of years. Moreover, we discuss extractions of $\gamma$ that are provided by $B_{s(d)} \rightarrow J/\psi K_S$ and $B_{d(s)} \rightarrow D_{d(s)}^{\pm}D_{d(s)}^{-}$ decays, and a simultaneous determination of $\beta$ and $\gamma$ from a combined analysis of the decays $B_d \rightarrow \pi^+\pi^-$ and $B_s \rightarrow K^+K^-$. Systematic error considerations in CP measurements are discussed in Sec. 6., and the reach for the $B_s^0 - \overline{B}_s^0$ mixing parameters $\Delta M_s$ and $\Delta \Gamma_s$ is presented in Sec. 7..

1.2 Rare B Decays

By rare B decays, one commonly understands heavily Cabibbo-suppressed $b \rightarrow u$ transitions or flavour-changing neutral currents (FCNC) $b \rightarrow s$ or $b \rightarrow d$ that in the SM are forbidden at tree-level. Rare decays are an important testing ground of the SM and offer a strategy in the search for new physics complementary to that of direct searches by probing the indirect effects of new interactions in higher order processes. Assuming the validity of the SM, rare FCNC decays allow the measurement of the CKM matrix elements $|V_{ub}|$ and $|V_{td}|$ and thus complement their determination from $B^0 - \overline{B}^0$ mixing. Any significant deviation between these two determinations would hint at new physics. With the large statistics available at the LHC, also decay spectra will be accessible, which will allow a direct measurement of virtual new physics effects: in some contrast to the investigation of CP violation, we are in the lucky situation that the impact of new physics on FCNC processes can be defined in a model-independent way\(^3\): at quark-level, $b \rightarrow q, q = (d, s)$, transitions can be described in terms of an effective Hamiltonian obtained by integrating out

\(^3\)Barring the possibility that new physics induces new operators not present in the SM, like e.g. in a left-right symmetric model.
virtual effects of heavy particles (top quark and $W$ boson in the SM):

$$\mathcal{H}_{\text{eff}}(b \to q) = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{tq}^\ast \sum_{i=1}^{10} C_i(\mu) O_i(\mu).$$  \hspace{1cm} (35)

The relevant operators will be specified in Sec. 8.; here we would like to stress that the short-distance coefficients $C_i(\mu)$ encode both perturbative QCD evolution between the hadronic scale $\mu \sim O(m_b)$ and the scale of heavy particles $M_H$ and information on the physics at that scale itself, contained in $C_i(M_H)$. A measurement of these coefficients that significantly deviates from the SM expectation thus would constitute immediate and unambiguous evidence for new physics beyond the SM.

In this report, we concentrate on decays that have a favourable experimental signature at the LHC and for which experimental studies exist at the time of writing: the exclusive decays $B_{d,s} \to \mu^+\mu^-$, $B_d \to K^*\gamma$ and $B_d \to K^{*\pm}\mu^\mp$. Although it is generally believed that theoretical uncertainties due to non-perturbative QCD effects are larger for exclusive than for inclusive decays, the experimental environment of a hadronic machine renders it exceedingly difficult to perform inclusive measurements. There has, however, been recent progress in the calculation of exclusive hadronic matrix elements [35], which narrows down the theoretical uncertainty, and as we shall elaborate on in Sec. 8., one can define experimental observables in which a large fraction of theoretical uncertainties cancels.

1.3 Other B Physics Topics

The B physics potential of the LHC is by far not exhausted by the programme sketched above. Possible further lines of investigation include physics with $b$ flavoured baryons (lifetime measurements, spectra, decay dynamics etc.), physics of $b$ flavoured mesons other than $B_{u,d,s}$ (radial and orbital excitations, $B_{c}$), and the study of purely leptonic or semileptonic decays, $B_q \to ev$, $B_q \to Mev$, where $M$ stands for a meson. From the theory point of view, one major topic whose relevance goes beyond the LHC is the calculation of non-leptonic decay amplitudes from first principles: whereas the discussion in Secs. 3 to 5 promotes a very pragmatic approach which aims at eliminating (“controlling”) the effects of strong interactions by measuring a large number of observables that are related by certain approximate symmetry principles, it remains a challenge for theory to provide predictions for non-leptonic decay amplitudes, both in factorization approximation and beyond.

Only a limited number of such topics were discussed during the workshop, and so we restrict ourselves to the presentation of selected aspects and review the present status of the theory of non-leptonic decays in Sec. 9., relevant for the prediction of decay rates in general and the extraction of weak phases from CP asymmetries in theoretically “dirty” channels in particular; in Sec. 10, we give an overview of the physics opportunities and predicted decay rates in $B_c$ decays.

2. EXPERIMENTAL OVERVIEW

The LHC will represent a unique opportunity for B physics studies. At a centre-of-mass energy of $\sqrt{s} = 14$ TeV the production cross-section for $b\bar{b}$ pairs will be very high. While current theoretical predictions of the absolute value are rather uncertain, it is expected that it will be about a factor of five higher than the one obtainable at the Tevatron, running at $\sqrt{s} = 2$ TeV. Naturally, therefore, B physics has been an important consideration in the optimization of the LHC experimental programme. The two multi-purpose experiments, ATLAS [36, 37] and CMS [38], have the capabilities to realize a rich and competitive programme and a dedicated experiment, LHCb [39], will have the sole task of exploiting as wide a range of B physics topics as possible.

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4Section coordinator: G.F. Tartarelli.
2.1 Introduction
The ATLAS and CMS detectors (see Fig. 3) have been designed primarily to search for new particles, such as the Higgs boson. The detectors therefore should be able to operate at the highest LHC luminosity and be sensitive to the highest mass scale. However, specific features required for B hadron reconstruction have been accommodated in the design. Both experiments have also put strong emphasis on ‘b tagging’ (discrimination between b jets and jets from light quarks, which is used in a variety of physics analyses), but this is not discussed in this chapter.

Both the ATLAS and CMS detectors cover the central region of the pp interaction point and have forward-backward and azimuthal symmetry. Inside a super-conducting solenoid (generating a 2 T magnetic field in ATLAS and a 4 T field in CMS, parallel to the beam line), a multi-layer tracking system (ATLAS [40], CMS [41]) covering the $|\eta| < 2.5$ region is located. The system has higher granularity detector layers at small radii (silicon pixel and micro-strip detectors) for good impact parameter resolution and track separation and extends to large radii to improve the transverse momentum resolution (in ATLAS the tracking system has also additional electron/pion separation as explained Sec. 2.5). In both experiments, the tracking system is surrounded by electromagnetic and hadronic calorimeters (ATLAS [42], CMS [43]) which extend up to about $|\eta| = 5.0$. Finally, outside the calorimeters there are high-precision muon chambers (in the region $|\eta| < 2.7$ in ATLAS [44] and $|\eta| < 2.4$ in CMS [45]) and muon trigger chambers in a smaller pseudo-rapidity range ($|\eta| < 2.4$ in both ATLAS and CMS).

The LHCb detector is a single-arm spectrometer covering the forward region of the pp interactions. A schematic view is shown in Fig. 4. The detector covers the angular region from 10 mrad up to 300 mrad in the horizontal plane (the bending plane) and from 10 mrad up to 250 mrad in the vertical plane (the non-bending plane), corresponding to the approximate range $2.1 < \eta < 5.3$ in terms of pseudo-rapidity. Starting from the interaction point, it consists of a silicon vertex detector, a RICH detector and a tracking system; the tracking system is followed by a second RICH detector, electromagnetic and hadron calorimeters and by muon detectors. The vertex detector, which is located inside the beam pipe, also includes a pile-up veto counter to reject events with multiple pp interactions. The tracking system is partly included in a dipole magnet field with a maximum value of 1.1 T in the vertical direction. The calorimetry system extends from 30 mrad to 300 (250) mrad in the horizontal (vertical) direction. Muon coverage is assured in the angular range 25 (15) mrad to 294 (245) mrad in the horizontal (vertical) direction.

2.2 Luminosity
The LHC is being built to run at a design luminosity of $10^{34} \text{ cm}^{-2} \text{s}^{-1}$ to maximize the potential for discovering new, heavy particles. From the point of view of B hadron reconstruction, multiple interactions and pile-up effects in the detectors are a complication both at trigger level and in the reconstruction of relatively low-$p_T$ particles. Moreover, the high luminosity will deteriorate the performance (both in terms of radiation damage and occupancy) of the innermost tracking layer when the reconstruction of the B meson vertex position is needed.

It is expected, however, that the LHC will reach design luminosity only gradually in time, starting at $10^{33} \text{ cm}^{-2} \text{s}^{-1}$ and taking three years to reach $10^{34} \text{ cm}^{-2} \text{s}^{-1}$. ATLAS and CMS will take advantage of this so-called low-luminosity period in order to carry out most of their B physics programme. At this luminosity, each crossing will have an average of 2 to 3 pile-up events in the tracking detectors which, however, have been shown not to affect significantly the detector performances. It is under current investigation if it is possible to continue certain studies at higher luminosity; for some critical channels, like very rare decays (see Sec. 8.), this has been already demonstrated to be feasible (both at trigger and reconstruction level).

In order to have a clean environment, well suited to B physics, the luminosity at LHCb will be locally controlled to have a mean value of $2 \times 10^{32} \text{ cm}^{-2} \text{s}^{-1}$, even when the machine is operating at
Fig. 3: Pictorial 3D-views of the two central multi-purpose LHC detectors: ATLAS (left) and CMS (right).

Fig. 4: Schematic 2D view of the LHCb detector in the bending plane. The interaction point is at $z = 0$. 
design luminosity. This value is chosen to optimize the number of single interaction bunch crossings, which will make up ~ 75\% of crossings within an interaction, and to ensure that radiation damage and occupancy problems are not too severe.

In this report we will present estimates of the $b$ physics potential of the three experiments at various integrated luminosities. When a simple comparison is needed, the results will be normalized to one year of running: this corresponds to $2 \times 10^3$ pb$^{-1}$ for LHCb and to $10^4$ pb$^{-1}$ for ATLAS and CMS running at low luminosity. More often the full potential of each experiment is presented: here we take 5 years of running for LHCb and 3 years at low luminosity for ATLAS and CMS (unless the study can be extended into the high-luminosity running period). Whenever possible, the results of the three experiments have been statistically combined to estimate the ultimate LHC potential.

2.3 Monte Carlo Generators, Simulation Methods and Assumed Cross-Sections

For the performance studies presented in this chapter, large samples of B hadron events have been produced using the PYTHIA 5.7/JETSET 7.4 [46] event generator. In the ATLAS Monte Carlo (MC), flavour-creation, flavour-excitation and gluon-splitting production processes were included. In CMS, flavour-creation and gluon-splitting were included (see also discussion in the "Bottom production" Chapter of this report [47]). The LHCb MC production was based on flavour-creation and flavour-excitation processes, with additional samples including gluon-splitting. The CTEQ2L [48] set of parton-distribution functions has been used. The Peterson function (with $e_b = 0.007$) has been used to fragment $b$ quarks to B hadrons. The other PYTHIA physics parameters have been set to their default values. The agreement between PYTHIA predictions and theoretical calculations is discussed elsewhere in this report.

The response of the detectors to the generated particles is simulated with programs based on the GEANT [49] package. Then the event is reconstructed in the sub-detectors relevant to each particular analysis; event reconstruction includes full pattern recognition in the tracking detectors, vertexing and particle identification (muons and electrons and $q$ separation, if available).

The procedure detailed above is called full simulation and has been used for the majority of the analyses presented. In some cases, a fast simulation which does not use GEANT, but a simple parametrization of the detector response has been used.

The results have been normalized assuming a total inelastic cross-section of 80 mb and a $b\overline{b}$ cross-section of 500 $\mu$b.

2.4 Proper Time Resolution

Different detector layouts used by the three experiments lead to differences in the impact parameter and proper decay-time resolutions.

In LHCb the impact parameter is measured in the $R-z$ plane: the resolution increases with transverse momentum and reaches an asymptotic value of about $40 \mu m$ already for tracks with transverse momenta $p_T > 3$ GeV [39]. Particles coming from B decays are mostly above this threshold and so LHCb can achieve a proper time resolution (for fully reconstructed exclusive decays) of about 0.031 ps (see Fig.5).

The ATLAS and CMS experiments measure precisely the projection of the track impact parameter in the $R-\phi$ plane [37, 38]. The plateau value (for high-$p_T$ tracks) of the transverse impact parameter resolution is about $11 \mu m$ (for comparison, the asymptotic value for the impact parameter in the $R-z$ plane is about $90 \mu m$); however, most of the tracks from B decays concentrate in the low-$p_T$ region where the resolution degrades due to multiple scattering. The proper time resolutions in ATLAS and CMS for typical fully reconstructed B decays are characterized by a Gaussian distribution with a width of about 0.060 ps (see Fig.5).

The proper time resolution estimates summarized in this section refer either to the $B_d^0 \rightarrow J/\psi\phi$
decay analysis discussed in Sec. 4, or to the $B^0_{s} \rightarrow J/\psi K^0_{S}$ sample (see Sec. 3.1). Slightly different values are estimated according to the B decay channel under study.

### 2.5 Particle Identification

Particle identification is a very important tool in many B physics channels. In particular, $\pi/K$ separation plays a key rôle in hadronic B decays (see Secs. 3. and 5.), allowing the separation of the decays of interest from similar, and indeed identical, topologies that would otherwise have overlapping (and in some cases overwhelming) spectra. Moreover, $\pi/K$ separation is crucial for one of the techniques (kaon tagging) used to identify the flavour of the $b$ hadron at production (see Sec. 2.7 for a short review of flavour tagging methods).

For this purpose, the LHCb detector has a dedicated system composed of two RICH detectors. The first system, RICH1, located upstream of the magnet, uses silica aerogel and $C_4F_{10}$ as radiators: this detector is intended to identify low-momentum particles over the full angular acceptance. The RICH2 detector, which uses $C_4F_{10}$, is located downstream of the magnet and covers a smaller solid angle. The purpose of this detector is to complement RICH1 by covering the high-end of the momentum spectrum. The performance of LHCb’s RICH is shown in Fig. 8. Pions and kaons can be cleanly separated with a significance of more than $10 \sigma$ in most of the momentum range $0 < p < 150 \text{ GeV}$. Efficiencies and purities are expected to be in excess of 90%.

In the absence of dedicated detectors for particle identification, ATLAS and CMS have studied other methods for obtaining some level of pion/kaon separation, although with reduced performance. The CMS silicon tracker has analogue read-out electronics so that the pulse-height information is preserved and can be used to estimate $dE/dx$. Preliminary results have been obtained [50] using a full GEANT simulation of the CMS tracker system described in [41]. This study estimates the asymptotic performance of the detector: a number of effects that can influence the $dE/dx$ resolution have not been simulated and will be the subject of future investigations when test-beam data will be available. The estimated $\pi/K$ separation, shown in Fig. 7 as a function of the particle momentum, has been used to obtain some of the CMS results presented in Sec. 3..

The ATLAS outer tracking system, which uses drift tubes (or straws) to provide an average of 36 hits per track, has electron/pion separation capability. The space between the straws is filled with radiator material, and transition-radiation photons, created by crossing electrons, are detected by using a Xenon-based gas mixture in the straws and a double-threshold read-out electronics. This detector can provide some $\pi/K$ separation using $dE/dx$, although the pulse-height is not measured [37]. Information about the deposited energy is extracted from the offset and accuracy of the measured drift distance, the fraction of high-threshold hits and the fraction of missing low-threshold hits. A preliminary study has
concluded that, by combining all this information, a $\pi/K$ separation of $0.8\sigma$ for tracks with $p_T \sim 5$ GeV can be obtained. The expected performance of this method is shown in Fig. 6. The separation is not good enough to identify individual pions and kaons, but can be used on a statistical basis. A more recent study, incorporating some changes to the readout format of the straw-tracker data, which provide a measurement of time-over-threshold for low-threshold hits, improves the separation significantly.

2.6 Triggers

Triggering is the key-issue for B physics studies at the LHC. Careful trigger-strategies are needed to extract interesting channels from inelastic collisions and the trigger-strategies used by ATLAS [51] and CMS [52] will be different from those used by LHCb, whose trigger is entirely dedicated to B decays. For robustness and flexibility, all three experiments will use multi-level trigger systems with the ATLAS and CMS triggers being divided into three and the LHCb trigger into four levels.

The lowest trigger level of ATLAS [53] and CMS [54], called level-1, which operates at the 40 MHz machine bunch-crossing frequency, uses reduced-granularity data from the muon trigger chambers and from the calorimeters. B physics is accommodated in these triggers by pushing the lepton transverse-momentum thresholds down to the minimum possible, still keeping the output trigger rate compatible with the acceptance rate of the next trigger level, level-2. In ATLAS this is achieved by requiring a single muon with $p_T > 6$ GeV in $|\eta| < 2.4$. The possibility of using a level-1 dimuon trigger with $\eta$-dependent thresholds is under study as a means of increasing statistics. However, all ATLAS studies reported in this chapter have been obtained requiring at least one muon with $p_T > 6$ GeV. In CMS, lower transverse momentum thresholds can be achieved, by adding to the single lepton trigger ($p_T > 7$ GeV for muons and $p_T > 12$ GeV for electrons) also double-lepton triggers ($\mu\mu, \mu e$ and $ee$) with thresholds which vary with pseudo-rapidity and can go down to 2 or 4 GeV for the two-muon case and to 5 GeV for the two-electron case.

The lowest trigger level in LHCb, called level-0, works at 40 MHz and is based on the identification of single leptons, hadrons and photons with high-$p_T$ in calorimeters and muon chambers. Because of the forward geometry, and high output rate, the ‘high’-$p_T$ threshold can be as low as 1 GeV. The hadron trigger allows the collection of large event samples in rare decay channels without leptons. The level-0 trigger is combined with the pile-up veto to reject bunch crossings likely to contain more than one $pp$ interaction. After the pile-up veto, the rate is reduced to about 9 MHz already, so that the high-$p_T$ trigger has to provide only an additional reduction factor of about 10 to match the design level-0 output rate of about 1 MHz. The allocation of bandwidth between the trigger components and the assignment of
thresholds is adjustable to match running conditions and physics requirements. At present the nominal thresholds for the single particle triggers are 1 GeV for muons, 2.3 GeV for electrons, 2.4 GeV for hadrons and 4 GeV for photons.

In ATLAS, the level-2 trigger [55] uses full-granularity data from the muon system, the calorimeters and from the tracking system. The level-2 trigger will confirm and refine the level-1 information and then look for specific final states according to the physics channel to be studied. Fast algorithms will be used to reconstruct tracks in the tracking system to allow \( p_T \) and mass-cuts. The second-muon trigger threshold will be set to \( p_T = 3 \) GeV. The dimuon trigger covers both some rare B decays and channels with \( J/\psi \)’s in the final state. Triggers with \( J/\psi \to ee \), with the \( p_T \) threshold on the two electrons as low as 0.5 GeV, will also be available. Hadronic triggers will be available for selected channels. The maximum total level-2 output rate is limited to about 1 kHz. CMS will follow a similar strategy.

In LHCb, the next trigger-level after level-0, called level-1, uses information from the vertex detector. This trigger is meant to complement the level-0 information by exploiting the displacement of \( b \) decay vertices. The vertex trigger will first reconstruct the event primary vertex and then look for track pairs with significant impact parameters with respect to the primary vertex, which are close in space. This signature provides high efficiency in all B decay modes. The total output rate is about 40 kHz. Successively, the level-2 trigger will refine the vertex trigger by adding momentum information to the tracks forming the secondary vertices and reduce the data rate to 5 kHz.

In all three experiments, the final trigger decision will be taken by a level-3 trigger which feeds full event data from all detectors to an offline-like algorithm to reconstruct specific final states. Selected events will be stored for offline analysis.

The trigger performance of the experiments will be summarized elsewhere in this report, for certain important decay modes. It will become clear that the enormous rate of B production at the LHC can indeed be properly exploited.

### 2.7 Flavour Tagging

An important issue of many CP-violation and \( B^0 \) mixing studies is the determination of the flavour of a \( b \) hadron at production. The LHC experiments have already successfully investigated several tagging strategies, but the studies are not yet completed (ATLAS [56], CMS [57], LHCb [39]).

Tagging algorithms can be divided into two broad categories: **Opposite Side** (OS) and **Same Side** (SS) algorithms, according to whether one studies the \( b \) or the \( \bar{b} \) quark in the event. The nomenclature, OS and SS, used to distinguish between the \( b \) and \( \bar{b} \) quarks, is used for historical reasons (it is derived from the LEP experiments), but it does not imply that the two quarks be produced in separate hemispheres. Indeed, for the LHCb experiment there is no other side and both the \( b \) and the \( \bar{b} \) quarks are produced predominantly in the same forward-cone. Moreover, for the LHC experiments, the importance of the gluon-splitting mechanism for producing \( b\bar{b} \) pairs implies that the two quarks be not always on opposite sides. We will thus include in the OS category all algorithms that try to deduce the initial flavour of the B meson under study by identifying the flavour of the other \( b \) hadron in the event. In the SS category we include all algorithms that look directly at the particles accompanying the B meson which has decayed in the channel under investigation (also called **signal** B in the following).

It can be shown that the statistical error of an asymmetry measurement is inversely proportional to the quantity \((1 - 2\omega)\sqrt{\epsilon N}\), where \( N \) is the total (untagged) number of events, \( \epsilon \) is the tagging efficiency and \( \omega \) is the wrong-tag fraction. For this reason, tagger-cuts are chosen in order to maximize the **quality factor** \( Q = \epsilon D^2 \), where \( D = 1 - 2\omega \) is called tagger-dilution. Approximate numbers for efficiencies and dilutions for the algorithms described below are listed in Tab. 1. Further developments and cut optimization might be needed to improve the performance of the tagging algorithms already studied and to bring all of them at the same level of understanding. The majority of the presented results refers to the \( B^0_d \to J/\psi K^0_S \) sample (see Sec. 3.1). Variations from sample to sample have been observed. Because
of this and because of differences in the simulation details, trigger-selections and analysis-cuts, a direct comparison between tagger potentials (and experimental performance) is not straightforward.

2.71 Opposite Side Tagging

The OS techniques which have been studied up to now by the LHC experiments are: lepton (muon or electron) tagging, kaon tagging (LHCb only) and jet-charge tagging.

In the lepton-tagging method, one looks for a lepton in the event coming from the semileptonic decay of the other $b$ quark in the event: $b \rightarrow l$. This method has a low efficiency (due to the relatively low $b$ semileptonic branching ratio of about 10%), but good purity, which is further enhanced by the fact that all three experiments use leptons also for triggering. The main contributions to the mistag rate are due to flavour mixing of the neutral B mesons and to cascade decays $b \rightarrow c \rightarrow l$. Wrong tags from cascade decays can be reduced by increasing the lepton $p_T$ threshold. It has also been shown [56] that the mistag rate increases with increasing $p_T$ of the signal $B$ for a fixed lepton-tag transverse-momentum threshold. For the studies presented in this chapter, the threshold has been set to 5 GeV for both electrons and muons in the ATLAS analysis, to 2 (2.5) GeV for muons (electrons) in CMS and to 1.5 GeV for both muons and electrons in LHCb.

Kaon tagging exploits the decay chain $b \rightarrow c \rightarrow s$ to identify the flavour of the $b$ quark from the charge of the kaon produced in the cascade decay. This method can be only used by LHCb as it requires the particle identification capability of the RICH detector. Candidate kaons are searched for down to a $p_T$ of 0.4 GeV and are required to have impact parameter significance incompatible with the reconstructed primary vertex at the 3σ level. For kaon tagging (as well as for lepton tagging), if more than one candidate survives all cuts, the one with the highest $p_T$ is chosen.

Jet-charge tagging deduces the flavour of the other $b$ quark in the event by looking at the total charge of the tracks which belong to the $b$ fragmentation. At LEP, where this algorithm was first developed, the identification of the opposite-side jet in $Z \rightarrow b\bar{b}$ events was almost straightforward. At the LHC, the other $b$ jet may escape the detector-acceptance and can be identified only by dedicated jet-clustering algorithms. These algorithms are usually based on track clustering possibly seeded by displaced tracks. Once the jet has been found, the jet total charge, $Q_{jet}$, is defined by an average of the tracks’ charge in the cluster, weighted by a function of their momenta. The right (wrong) sign events are then defined as those with $Q_{jet} > +c$ ($Q_{jet} < -c$), where $c$ is a tunable cut. Although investigated in the past, OS jet charge is not used in the ATLAS analyses presented in this report. The LHCb numbers for this tagging method, which are calculated for events where no other type of tag has been found, are preliminary and are not used for the results presented in this chapter.

2.72 Same Side Tagging

The SS techniques presented in this section exploits production and fragmentation properties of the B meson to deduce its flavour. These techniques are not affected by mistags due to mixing. Moreover, as they apply to the same B meson whose decay is under investigation, there is no loss of efficiency due to the identification of the other $b$ jet in the event.

During the process of a $\bar{b}$ quark fragmentation to produce a $B^0_d$ meson, pions which are charge-correlated to the flavour of the B meson, can be produced by two mechanisms [58]. The $\bar{b}$ quark can pick up a $d$ quark from the quark sea to form a $B^0_d$, thus making available a $\bar{d}$ quark to form a $\pi^+$. Another mechanism proceeds through production of orbitally excited states of $B$ mesons, called $B^{**}$, which then decay to $B^0_d$: $B^{**} \rightarrow B^{(*)0}\pi^+$. If a $B^{*0}$ is produced, it decays radiatively as $B^{*0} \rightarrow B^0\gamma$.

The B–$\pi$ correlation method, studied by ATLAS, exploits these correlations by searching for low-$p_T$ pions, compatible with coming from the primary vertex, in proximity of the decayed B meson. Tracks belonging to the B decay products are excluded and what it is called pion is actually a generic charged track, as no $\pi/K$ separation is used. In this method, both production mechanisms described
Table 1: Efficiencies ($\epsilon$) and dilutions ($D$) for the flavour-tagging algorithms described in the text. The shorthand “n/a” (not available) means that one tagger either cannot be used or has not yet been fully studied by a particular experiment. The LHCb numbers for lepton and kaon tagging refer to the combined algorithm described in the text.

<table>
<thead>
<tr>
<th>Method</th>
<th>Lepton Tag</th>
<th>Kaon Tag</th>
<th>Jet Charge</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATLAS</td>
<td>$\epsilon$</td>
<td>$D$</td>
<td>$\epsilon$</td>
</tr>
<tr>
<td></td>
<td>$0.016$</td>
<td>$0.46$</td>
<td>$0.027$</td>
</tr>
<tr>
<td>CMS</td>
<td>$0.025$</td>
<td>$0.52$</td>
<td>$0.034$</td>
</tr>
<tr>
<td>LHCb</td>
<td>$0.40$</td>
<td>$0.40$</td>
<td>$0.60$</td>
</tr>
</tbody>
</table>

Table 2: Combined tagging efficiencies from CMS MC. The last column shows the combined efficiency of algorithms A and B when the overlap has been subtracted: $\epsilon(A \cup B) = \epsilon(A) + \epsilon(B) - \epsilon(A \cap B)$.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>$\epsilon(A)$</th>
<th>$\epsilon(B)$</th>
<th>$\epsilon(A) + \epsilon(B)$</th>
<th>$\epsilon(A \cup B)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton Tag</td>
<td>$B^{**}$</td>
<td>0.06</td>
<td>0.215</td>
<td>0.275</td>
<td>0.26</td>
</tr>
<tr>
<td>Lepton Tag</td>
<td>SS Jet Charge</td>
<td>0.06</td>
<td>0.5</td>
<td>0.56</td>
<td>0.53</td>
</tr>
<tr>
<td>Lepton Tag</td>
<td>OS Jet Charge</td>
<td>0.06</td>
<td>0.7</td>
<td>0.76</td>
<td>0.72</td>
</tr>
<tr>
<td>$B^{**}$</td>
<td>SS Jet Charge</td>
<td>0.215</td>
<td>0.5</td>
<td>0.715</td>
<td>0.56</td>
</tr>
<tr>
<td>$B^{**}$</td>
<td>OS Jet Charge</td>
<td>0.215</td>
<td>0.7</td>
<td>0.915</td>
<td>0.76</td>
</tr>
<tr>
<td>SS Jet Charge</td>
<td>OS Jet Charge</td>
<td>0.5</td>
<td>0.7</td>
<td>1.2</td>
<td>0.845</td>
</tr>
</tbody>
</table>

above contribute correlated pions and no attempt is made to separate these two contributions.

The CMS experiment prefers to concentrate on the explicit reconstruction of the $B^{**}$ resonance ($B^{**}$ method). In the MC, these resonances have been modelled according to [59]. In this method, pions with $p_T > 1$ GeV are combined with a $B^0_{d}$ to give a $B^{**}$ meson with a mass between 5.6 and 5.9 GeV. As above, the charge sign of the associated pion gives the tag. No attempt is made to reconstruct the low-$p_T$ photon which is present when a $B^{*0}$ is produced in the cascade and to resolve the different peaks which superimpose in the $B^{**}$ mass spectrum. It would also be possible to study the mistag rate from the data itself by looking at the side-bands of the mass resonance, so that one need not rely only on the MC modelling of the process.

Similar to the B–π correlation method, the $B^0_{s}$ tagging method, which is under investigation by LHCb, consists in looking for a primary kaon in the vicinity of the $B^0_{s}$ meson. Efficiency and dilution for this tagger, which is not used for the results presented in this chapter, are preliminary.

In a different approach, it is possible to use jet-charge tagging also on the same side. In this case, similarly to the OS jet-charge tagging, the jet charge is a weighted average of the charge of the tracks in the jet, but the tracks belonging to the B meson decay products are excluded from the sum. The weights are functions of the momentum of the track and are often written in the form $w(p)^k$, where $w(p)$ can be chosen as the transverse momentum, the projection of the momentum along the B direction or a more complicated function of them. The parameter $k$ controls the relative weights of soft and hard tracks in the total charge.

2.73 Combined Tagging

The best tagging strategy would combine all taggers, weighted by their dilutions, simultaneously on both sides on an event-by-event basis. This requires, however, a full understanding of tagger correlations. The
This section deals with the use of benchmark B decays to explore CP violation and to extract the angles of the unitarity triangles. By ‘benchmark’ we mean modes that are well established in the literature. Some, but by no means all, of these channels will be probed by other experiments before the LHC starts to operate. To be specific, we will discuss the extraction of $\gamma$ from $B_d \to J/\psi K_S$, the prospects to probe $\alpha$ with $B_d \to \pi^+ \pi^-$ and $B \to \rho \pi$ modes, as well as extractions of $\gamma$ from $B_d \to D_s^{*\pm} \pi^\mp$ and $B_s \to D_s^{*\pm} K^\mp$ decays. Finally, we will also give a discussion of the determination of $\gamma$ from $B \to DK$ decays. Since $B_s \to J/\psi \phi$ – another benchmark CP mode – is of particular interest for the LHC, we have devoted a separate section to the discussion of the physics potential of this “gold-plated” mode for the LHC experiments: Sec. 4.

### 3. BENCHMARK CP MODES

This section deals with the use of benchmark B decays to explore CP violation and to extract the angles of the unitarity triangles. By ‘benchmark’ we mean modes that are well established in the literature. Some, but by no means all, of these channels will be probed by other experiments before the LHC starts to operate. To be specific, we will discuss the extraction of $\beta$ from mixing-induced CP violation in the “gold-plated” decay $B_d \to J/\psi K_S$, the prospects to probe $\alpha$ with $B_d \to \pi^+ \pi^-$ and $B \to \rho \pi$ modes, as well as extractions of $\gamma$ from $B_d \to D_s^{*\pm} \pi^\mp$ and $B_s \to D_s^{*\pm} K^\mp$ decays. Finally, we will also give a discussion of the determination of $\gamma$ from $B \to DK$ decays. Since $B_s \to J/\psi \phi$ – another benchmark CP mode – is of particular interest for the LHC, we have devoted a separate section to the discussion of the physics potential of this “gold-plated” mode for the LHC experiments: Sec. 4.

#### 3.1 Extracting $\beta$ from $B_d \to J/\psi K_S$

Probably the most important application of the formalism discussed in Sec. 1.15 is the decay $B_d \to J/\psi K_S$ [60], which is a transition into a CP eigenstate with eigenvalue $-1$ and originates from $b \to c \bar{c} \bar{s} \bar{s}$ quark-level decays.

---

3.11 Theoretical Aspects

In the case of $B_d \to J/\psi K_S$, we have to consider both current–current, i.e. tree-diagram-like, and penguin contributions, as depicted in Fig. 9. The corresponding transition amplitude can be written as follows [61]:

$$A(B_d^0 \to J/\psi K_S) = \lambda_u^{(s)} \left( A_{cc}^{e'} + A_{pen}^{e'} \right) + \lambda_u^{(s)} A_{pen}^{u'} + \lambda_t^{(s)} A_{pen}^{t'},$$

where $A_{cc}^{e'}$ denotes the current–current contributions, i.e. the “tree” processes in Fig. 9, and the amplitudes $A_{pen}^{e'}$ describe the contributions from penguin topologies with internal $q$ quarks ($q \in \{u, c, t\}$). These penguin amplitudes take into account both QCD and electroweak penguin contributions. The primes in (1) remind us that we are dealing with a $\bar{b} \to \bar{s}$ transition, and the $\lambda_q^{(s)} \equiv V_{qs} V_{qb}^*$ are CKM factors. Making use of the unitarity of the CKM matrix and applying the generalized Wolfenstein parametrization, including non-leading terms in $\lambda$, we obtain

$$A(B_d^0 \to J/\psi K_S) = \left( 1 - \frac{\lambda^2}{2} \right) A' \left[ 1 + \left( \frac{\lambda^2}{1 - \lambda^2} \right) a' e^{i\theta'} e^{i\gamma} \right],$$

where

$$A' \equiv \lambda^2 A \left( A_{cc}^{e'} + A_{pen}^{e'} \right) \text{ and } a' e^{i\theta'} \equiv R_b \left( \frac{A_{pen}^{u'}}{A_{cc}^{e'} + A_{pen}^{e'}} \right),$$

with $A_{pen}^{u'} \equiv A_{pen}^{e'} - A_{pen}^{t'}$. The quantity $A_{pen}^{u'}$ is defined in analogy to $A_{pen}^{e'}$, and the CKM factor $A$ is given as follows:

$$A \equiv \frac{1}{\lambda^2} |V_{cb}| = 0.81 \pm 0.06;$$

the definition of $R_b = 0.41 \pm 0.07$ can be found in (9).

It is very difficult to calculate the “penguin” parameter $a' e^{i\theta'}$, which introduces the CP-violating phase factor $e^{i\gamma}$ into the $B_d^0 \to J/\psi K_S$ decay amplitude and represents – sloppily speaking – the ratio of the penguin to tree contributions. However, this parameter, and therefore also $e^{i\gamma}$, enters in (2) in a doubly Cabibbo-suppressed way. Consequently, to a very good approximation, $B_d^0 \to J/\psi K_S$ is dominated by only one CKM amplitude, so that, from (24) and (30):

$$A_{\text{mix}}^{\text{CP}}(B_d \to J/\psi K_S) = + \sin[-(\phi_d - 0)] = - \sin(2\beta).$$

Since Eq. (30) applies with excellent accuracy to $B_d \to J/\psi K_S$, as penguins enter essentially with the same weak phase as the leading tree contribution, it is referred to as the “gold-plated” mode to determine the $B_d^0$-$\bar{B_d}^0$ mixing phase [60]. Strictly speaking, mixing-induced CP violation in $B_d \to J/\psi K_S$ probes $\sin(\phi_d + \phi_K)$, where $\phi_K$ is related to the CP-violating weak $K^0\bar{K}^0$ mixing phase. Similar modifications must also be performed for other final-state configurations containing $K_S$ or $K_L$ mesons. However, $\phi_K$ is negligible in the SM, and – owing to the small value of the CP-violating parameter $\varepsilon_K$ of the neutral kaon system – can only be affected by very contrived models of new physics [62].

First measurements of $\sin(2\beta)$ from the CP asymmetry (5) have recently been reported by the OPAL, CDF and ALEPH collaborations [63]:

$$\sin(2\beta) = \begin{cases} 3.2^{+1.8}_{-2.0} \pm 0.5 \quad \text{(OPAL Collaboration)} \\ 0.79^{+0.41}_{-0.44} \quad \text{(CDF Collaboration)} \\ 0.93^{+0.64+0.36}_{-0.88-0.24} \quad \text{(ALEPH Collaboration).} \end{cases}$$

Although the experimental uncertainties are very large, it is interesting to note that these results favour the SM expectation of a positive value of $\sin(2\beta)$. In the B factory era, an experimental uncertainty of $\Delta \sin(2\beta)|_{\text{exp}} = 0.05$ appears to be achievable, whereas the experimental uncertainty at the LHC is expected to be one order of magnitude smaller, as discussed on page 22.
In addition to (5), one more important implication of the SM is

\[ A_{\text{CP}}^{\text{dir}}(B_d \rightarrow J/\psi K_S) \approx 0 \approx A_{\text{CP}}(B^+ \rightarrow J/\psi K^+), \]  

which is interesting for the search of new physics. An observation of these direct CP asymmetries at the level of 10% would be a strong indication for physics beyond the SM.

In view of the tremendous experimental accuracy that can be achieved in the LHC era, it is an important issue to investigate the theoretical accuracy of (5) and (7), which is a very challenging theoretical task. An interesting channel in this respect is \( B_s \rightarrow J/\psi K_S \) [61], allowing one to control the (presumably very small) penguin uncertainties in the determination of \( \beta \) from CP-violating effects in \( B_d \rightarrow J/\psi K_S \), and to extract the angle \( \gamma \). We shall come back to this strategy in Sec. 5.2.

### 3.12 Experimental Studies

As well as being theoretically "gold-plated", the decay \( B_d^0 \rightarrow J/\psi K_S^0 \), with \( J/\psi \rightarrow \mu^+\mu^- \) or \( J/\psi \rightarrow e^+e^- \) is experimentally clean, and can be reconstructed with relatively low background. The \( B_d^0 \rightarrow J/\psi K_S^0 \) branching ratio is measured to be \((8.9 \pm 1.2) \times 10^{-4}\) [64], yielding a visible branching ratio \( B(B_d^0 \rightarrow J/\psi [\rightarrow \mu^+\mu^- \text{ or } e^+e^-] K_S^0 [\rightarrow \pi^+\pi^-]) \) of \(1.8 \times 10^{-5}\). For a complete account of each of the analyses described below, see Refs. [56, 57, 39].

### Selection

In each experiment, the event samples were generated using PYTHIA and the full detector response was simulated using the GEANT program. For the ATLAS analysis, electron and muon identification efficiencies are parametrized as a function of \( p_T \) and \( \eta \) using separate samples of fully simulated calorimeter and muon chamber data and then applied to the \( B_d^0 \rightarrow J/\psi K_S^0 \) sample.

The trigger strategies for the three experiments are summarized in Sec. 2.6: here the triggers relevant for the \( B_d^0 \rightarrow J/\psi K_S^0 \) analysis are briefly recalled. In the ATLAS analysis, a single muon with \( p_T > 6 \text{ GeV} \) and \( |\eta| < 2.4 \) is required at level-1. To increase statistics, a dimuon trigger (with \( \eta \)-dependent thresholds) is under study. At level-2, the trigger requires either a second muon with \( p_T > 3 \text{ GeV} \) and an electron with \( p_T > 5 \text{ GeV} \) or an \( e^+e^- \) pair, with electron \( p_T \) thresholds of 0.5 GeV. In CMS, the following level-1 triggers are available: one muon with \( p_T > 7 \text{ GeV} \), two muons with \( p_T > 2 \) or \( 4 \text{ GeV} \) (depending on \( \eta \)), one electron with \( p_T > 12 \text{ GeV} \), two electrons with \( p_T > 5 \text{ GeV} \) or an electron-muon pair with \( p_T(e) > 4.5 \text{ GeV} \) and \( p_T(\mu) > 2 \) or \( 4 \text{ GeV} \).

The first step in reconstructing \( B_d^0 \rightarrow J/\psi K_S^0 \) decays is the selection of oppositely charged lepton pairs originating from a common vertex and with a mass close to the \( J/\psi \) mass. Next, \( K_S^0 \) candidates are selected and combined with the \( J/\psi \) candidates to form \( B_d^0 \) candidates. In ATLAS, the same lepton trigger \( p_T \)-cuts are applied in the offline selection. In CMS and LHCb, no offline-cuts are applied to the lepton-\( p_T \) after pattern-recognition.
For the $J/\psi \rightarrow e^+e^-$ selection, both ATLAS and CMS use an asymmetric window for the reconstructed $J/\psi$ mass in order to account for bremsstrahlung energy-losses which produce a long tail at small invariant masses. Cuts on the $J/\psi$ decay-length remove the prompt-$J/\psi$ background. In LHCb, to guarantee a good vertex resolution, the tracks are required to have hits in the vertex detector.

In ATLAS and CMS, the $K^0_S$ candidates are reconstructed from all oppositely charged track pairs originating from a common vertex and with a mass close to that of the kaon. In LHCb, the charged tracks are required to be identified as pions in the RICH system. To reduce combinatorial background, the $K^0_S$ candidate vertices are required to be well separated from the primary vertex. The leptons and pions from the surviving $J/\psi$ and $K^0_S$ candidates are then used to reconstruct candidate $B^0_d \rightarrow J/\psi K^0_S$ decays using a three-dimensional kinematic fit to the four tracks and applying vertex and mass constraints on both the lepton-lepton and $\pi^+\pi^-$ system. Finally, the fully reconstructed $B^0_d$ is required to point to the reconstructed primary vertex. Table 3 gives the (untagged) event yields at various stages of the selection procedure. The final $B^0_d$ mass and proper time resolutions are shown in Tab. 4.

In each experiment, the dominant source of background arises from the combination of a true $J/\psi$ from B decay and any other $K^0_S$ within the event, which can originate from fragmentation or from other B decays or be a fake $K^0_S$. In LHCb, thanks to the $\pi/K$ separation available in the RICH, the fake $K^0_S$ contribution is reduced and the only significant background is due to real $J/\psi$ from B combined with a real $K^0_S$. However, this background is rather large due to the large number of $K^0_S$ mesons from fragmentation produced in the forward direction within the LHCb acceptance. ATLAS used fast simulation programs (after careful comparison with full simulation results) to generate large samples of all backgrounds, whereas LHCb used smaller samples of fully simulated events and extrapolated to higher statistics. CMS used a combination of the two approaches. The signal/background ratios obtained in the three experiments after all offline selection-cuts and before any flavour tagging (except for the ATLAS $J/\psi \rightarrow e^+e^-$ sample, where the flavour is tagged automatically by the level-1 trigger muon) are summarized in Table 3. Figure 10 shows examples for $B^0_d$ mass peaks (signal and background) after all offline-cuts: the background levels are low in all cases.
Table 5: Tagging efficiencies and dilution factors for each of the tagging methods used by the three collaborations in the $B_d^0 \rightarrow J/\psi K_S^0$ analysis. Numbers before and after the slash (/) are for the $J/\psi \rightarrow \mu^+\mu^-$ and $J/\psi \rightarrow e^+e^-$ samples, respectively. ATLAS uses $B^-\pi$ and CMS uses $B^+\pi$ tagging (see Sec. 2.7). The shorthand “n/a” means not available or not applied in this analysis by a particular experiment.

<table>
<thead>
<tr>
<th>Tagging method</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>efficiency</td>
<td>dilution</td>
<td>efficiency</td>
</tr>
<tr>
<td>electron</td>
<td>0.012/–</td>
<td>0.46/–</td>
<td>0.024/0.035</td>
</tr>
<tr>
<td>muon</td>
<td>0.025/1.</td>
<td>0.52/0.57</td>
<td>0.033/0.035</td>
</tr>
<tr>
<td>$B^-\pi$ (or $B^+\pi$)</td>
<td>0.82/0.80</td>
<td>0.16/0.14</td>
<td>0.21</td>
</tr>
<tr>
<td>jet charge (SS)</td>
<td>0.64/0.71</td>
<td>0.17/0.12</td>
<td>0.50</td>
</tr>
<tr>
<td>jet charge (OS)</td>
<td>n/a</td>
<td>n/a</td>
<td>0.70</td>
</tr>
<tr>
<td>lepton and kaon</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
</tr>
</tbody>
</table>

**Tagging**

Some of the flavour tagging strategies introduced in Sec. 2.7 have been studied in particular detail for the $B_d^0 \rightarrow J/\psi K_S^0$ channel. All three experiments use the lepton from the semileptonic decay of the other $b$ hadron in the event (the opposite side $b$) to tag the flavour of the $B_d^0$ at production. In the ATLAS $J/\psi \rightarrow e^+e^-$ sample, the level-1 trigger muon provides a 100% efficient tag. Using the $\pi$–K separation provided by its RICH detector, LHCb can also use kaons to tag the flavour of the opposite-side $b$ quark.

In addition to the lepton tag, ATLAS and CMS studied jet-charge tagging (both on the opposite and same side) and $B^-\pi$ correlation tagging. The same-side jet-charge tags and the $B^-\pi$ tags are highly correlated. For this reason, ATLAS chose to use only the higher purity $B^-\pi$ tag. It has not yet been demonstrated that the same side $B^-\pi$ tag method will work in LHCb, since the track densities encountered there are large.

All three experiments plan to combine all tagging information in each event in order to obtain optimal statistical precision. The efficiencies and mistag rates of all tagging methods are shown in Tab. 5. For the LHCb study, the overall tagging efficiency and dilution of the combined lepton and kaon tagging method (see Sec. 2.73) have been used.

**Sensitivity to $\beta$**

The CKM parameter $\beta$ is extracted from a fit to the measured time-dependent asymmetry with a function of the form:

$$A_{CP}(B_d^0 \rightarrow J/\psi K_S^0) = D \sin(2\beta) \sin \Delta m t,$$

where $D$ is the overall dilution factor due to both tagging and background. Here any direct CP violation is neglected, and the only free parameter in the fit is $\sin 2\beta$. The background is assumed to have no asymmetry. Figure 11 shows an example of a fit to the LHCb time-dependent CP asymmetry distribution after one year of data taking.

Table 6 summarizes the sensitivity of the three experiments to $\sin 2\beta$ using the different tagging methods studied by each experiment. The ATLAS lepton-tagged events have been removed from the $B^-\pi$ tagged sample to yield two statistically independent samples. The four separate CMS results are statistically correlated. However, to obtain the final precision for $\sin 2\beta$, the analysis was performed using only those events not tagged by another method, as explained in Sec. 2.73.

All experiments estimate a statistical error on $\sin 2\beta$ which is independent of the input value for $\beta$. Combining the statistical precision achievable after 3 years of running at ATLAS and CMS with 5 years of running at LHCb, a total statistical precision for $\sin 2\beta$ of 0.005 can be obtained. This precision is one order of magnitude better than the expected statistical precision at the $e^+e^-$ B factories. With this sensitivity, the experiments can also probe the direct CP violating contribution $A_{CP}^{\text{dir}}(B_d^0 \rightarrow J/\psi K_S^0)$ to the asymmetry. Fitting an additional term to account for such a contribution degrades the precision on $\sin 2\beta$ by $\sim 30\%$ and gives a similarly small uncertainty on $A_{CP}^{\text{dir}}(B_d^0 \rightarrow J/\psi K_S^0)$. 


Fig. 11: Example for a LHCb fit of the time-dependent asymmetry Eq. (8) with one year’s data.

### Table 6: Sensitivity to $\sin 2\beta$ after one year of data taking at the LHC. For the ATLAS $J/\psi \to \mu^+\mu^-$ sample, lepton tags have been removed from the $B^-\pi$ tagged sample. The four partial CMS results are correlated, but in the total sensitivity overlaps have been subtracted. The shorthand “n/a” means not available or not applied in this analysis by a particular experiment.

<table>
<thead>
<tr>
<th>Tagging method</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lepton</td>
<td>0.039</td>
<td>0.031</td>
<td>n/a</td>
</tr>
<tr>
<td>$B^-\pi$</td>
<td>0.026</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>SS Jet charge</td>
<td>n/a</td>
<td>0.021</td>
<td>n/a</td>
</tr>
<tr>
<td>OS Jet charge</td>
<td>n/a</td>
<td>0.023</td>
<td>n/a</td>
</tr>
<tr>
<td>Lepton and kaon</td>
<td>n/a</td>
<td>n/a</td>
<td>0.023</td>
</tr>
</tbody>
</table>

Total        | 0.017 | 0.015 | 0.021

### Systematic Uncertainties

In order not to compromise the excellent statistical precision obtainable for the determination of $\sin 2\beta$, a similar or better control of the systematic uncertainties must be achieved.

A detailed discussion of systematic errors on CP-violation measurements and strategies for their control are presented in Sec. 6. As theoretical uncertainties are expected to be very small, the main contribution to the systematic error comes from the initial-state production-asymmetry and from experimental factors. The latter ones include tagging uncertainties and uncertainties from background.

ATLAS have performed a preliminary estimate of such uncertainties using $B^+ \to J/\psi(\mu\mu)K^+$ and $B^0_d \to J/\psi(\mu\mu)K^{*0}$ control samples [56]. It is estimated that for a statistical error of $\sin 2\beta = 0.010$ (stat.), achievable after 3 years running, a corresponding systematic error of $\sin 2\beta = 0.005$ (sys.), coming from the limited size of the control channels, can be obtained.

#### 3.2 Probing $\alpha$ with $B_d \to \pi^+\pi^-$

Another benchmark CP mode is $B_d \to \pi^+\pi^-$, which allows one to probe the CKM angle $\alpha$. Unfortunately, penguin topologies render the interpretation of the CP-violating $B_d \to \pi^+\pi^-$ observables in terms of $\alpha$ difficult.
3.21 Theoretical Aspects

The decay $B_d^0 \rightarrow \pi^+ \pi^-$ is described by the Feynman diagrams shown in Fig. 12, and in analogy to (1), the corresponding decay amplitude can be expressed as

$$A(B_d^0 \rightarrow \pi^+ \pi^-) = \lambda_u^{(d)} (A_u^{u} + A_{\text{pen}}^{u}) + \lambda_c^{(d)} A_{\text{pen}}^{c} + \lambda_t^{(d)} A_{\text{pen}}^{t}. \quad (9)$$

If this mode did not comprise penguin contributions, its mixing-induced CP asymmetry would allow a measurement of $\sin 2\alpha$, in complete analogy to $B_d \rightarrow J/\psi K_S$:

$$A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-) = -\sin[-(2\beta + 2\gamma)] = -\sin 2\alpha. \quad (10)$$

However, this relation is strongly affected by penguin effects, which were analysed by many authors [65, 66]. Various methods for controlling the corresponding hadronic uncertainties have been proposed; unfortunately, these strategies are usually rather challenging from an experimental point of view.

The best-known approach was proposed by Gronau and London [67]. It makes use of the SU(2) isospin relation

$$\sqrt{2} A(B^+ \rightarrow \pi^+ \pi^0) = A(B_d^0 \rightarrow \pi^+ \pi^-) + \sqrt{2} A(B_d^0 \rightarrow \pi^0 \pi^0), \quad (11)$$

and of its CP-conjugate, which form two triangles in the complex plane. The sides of these triangles can be determined through the corresponding branching ratios, while their relative orientation can be fixed by measuring the CP-violating observable $A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-)$. Following these lines, it is in principle possible to take into account the QCD penguin effects in the extraction of $\alpha$. It should be noted that electroweak penguins cannot be controlled with the help of this isospin strategy. However, their effect is expected to be rather small, and – as was pointed out recently [68, 69] – can be included through additional theory input. Unfortunately, the Gronau–London approach suffers from an experimental problem, since the measurement of $B(B_d \rightarrow \pi^0 \pi^0)$, which is expected to be of $O(10^{-6})$ or smaller, is very difficult. However, upper bounds on the CP-averaged $B_d \rightarrow \pi^0 \pi^0$ branching ratio may already be useful to put upper bounds on the QCD penguin uncertainty that affects the determination of $\alpha$ [66, 70].

Alternative methods for controlling penguin uncertainties are very desirable. One of them is provided by $B \rightarrow \rho \pi$ modes [71, 72], and will be discussed in more detail in the following subsection. As we shall see in Sec. 5.4, another interesting strategy is to use the CP-violating observables of $B_s \rightarrow K^+ K^-$ together with those of $B_d \rightarrow \pi^+ \pi^-$, which allows a simultaneous determination of $\beta$ and $\gamma$ without any assumptions about penguin topologies.

The observation of $B_d \rightarrow \pi^+ \pi^-$ was announced by the CLEO collaboration in the summer of 1999 [73], with a branching ratio of

$$B(B_d \rightarrow \pi^+ \pi^-) = \left(0.47^{+0.18}_{-0.15} \pm 0.13\right) \times 10^{-5}. \quad (12)$$
Other CLEO results on $B \to \pi K$ modes indicate that QCD penguins play in fact an important rôle and definitely do affect the extraction of $\alpha$ from $B_d \to \pi^+ \pi^-$ [74]. In order to discuss penguin effects in a quantitative way, we use once more the unitarity of the CKM matrix, and rewrite (9) as follows:

$$A(B_d^0 \to \pi^+ \pi^-) = e^{i\gamma} T + e^{-i\beta} P,$$

where the complex quantities

$$T \equiv -|\lambda^d_u|^2 \left[ A^u_{\text{cc}} + A^u_{\text{pen}} - A^u_{\text{pen}} \right], \quad P \equiv -|\lambda^d_d|^2 \left[ A^d_{\text{pen}} - A^d_{\text{pen}} \right],$$

(14)
denote the $B_d^0 \to \pi^+ \pi^-$ “tree” and “penguin” amplitudes, respectively. The CP-conjugate amplitude can be obtained straightforwardly from (13) by replacing $\beta$ by $-\beta$ and $\gamma$ by $-\gamma$. For the following considerations, also the CP-conserving strong phase $\delta \equiv \text{Arg}(PT^*)$ plays an important rôle. Since the $B_d^0 \to B_d^0$ mixing phase is given by $2\beta$ in the SM, the unitarity relation $\alpha + \beta + \gamma = 180^\circ$ allows one to express the CP-violating observables $A_{\text{CP}}^\text{dir}(B_d^0 \to \pi^+ \pi^-)$ and $A_{\text{CP}}^\text{mix}(B_d^0 \to \pi^+ \pi^-)$ as functions of the CKM angle $\alpha$, and the hadronic parameters $|P|/|T|$ and $\delta$. Consequently, we have at our disposal two observables that depend on three “unknowns”. Eliminating the CP-conserving strong phase $\delta$, one obtains [66]:

$$A_{\text{CP}}^\text{mix}(B_d^0 \to \pi^+ \pi^-) = -\sqrt{1 - A_{\text{CP}}^\text{dir}^2} \sin 2\alpha_{\text{eff}},$$

(15)

where

$$\cos(2\alpha - 2\alpha_{\text{eff}}) = \frac{1}{\sqrt{1 - A_{\text{CP}}^\text{dir}^2}} \left[ 1 - \left( 1 - \sqrt{1 - A_{\text{CP}}^\text{dir}^2} \right) \frac{|P|}{|T|}^2 \right],$$

(16)

with $2\alpha_{\text{eff}} \equiv \text{Arg} \left[ -\epsilon^{(q)}_{\pi^+ \pi^-} \right]$. $\epsilon^{(q)}$ was defined in Eq. (27). The quantity $2\alpha_{\text{eff}}$ reduces to $2\alpha$ if penguin topologies are neglected. Once the time-dependent CP-asymmetry (22) has been measured, Eqs. (15) and (16) allow one to fix contours in the $|P|/|T|, 2\alpha$ plane. This plot constitutes a model-independent representation of the experimental data in terms of the SM parameters. In order to simplify the following experimental discussion, we keep only leading order terms in $|P|/|T|$, which yields [75]

$$A_{\text{CP}}^\text{dir}(B_d \to \pi^+ \pi^-) = 2 \left| \frac{P}{T} \right| \sin \delta \sin \alpha + \mathcal{O}(\left| \frac{|P|}{|T|} \right|^2),$$

$$A_{\text{CP}}^\text{mix}(B_d \to \pi^+ \pi^-) = -\sin(2\alpha) - 2 \left| \frac{P}{T} \right| \cos \delta \cos(2\alpha) \sin \alpha + \mathcal{O}(\left| \frac{|P|}{|T|} \right|^2),$$

(17)

and leave the analysis of the exact results given in [66] for further studies. Unfortunately, a theoretically reliable prediction for the “penguin” to “tree” ratio $|P|/|T|$, which would allow the extraction of $\alpha$, is very challenging. An interesting new approach in this context was recently proposed in Ref. [76]. We shall come back to it in Sec. 9.. Let us finally note that any QCD-based approach to calculate $|P|/|T|$ requires also knowledge of $|V_{td}/V_{ub}|$. This input can be avoided, if all CP-violating weak phases are expressed in terms of the Wolfenstein parameters $\mathbf{f}$ and $\overline{\mathbf{f}}$, allowing one to fix contours in the $\overline{\mathbf{f}}$-$\mathbf{f}$ plane [66].

### 3.22 Experimental Studies

Low branching ratio and lack of any sub-mass constraints make the reconstruction of $B_d^0 \to \pi^+ \pi^-$ a very demanding task. Isolating the signal from other two-body topologies, like $B_d^0 \to K^{\pm} \pi^{\mp}$, $B_s^0 \to K^{+} K^{-}$, $B_s^0 \to K^{\pm} \pi^\mp$, $\Lambda_b \to p \pi^-$ and $\Lambda_b \to p K^-$, poses additional problems. Despite these challenges, extensive simulation studies have demonstrated the substantial potential of the LHC experiments in this mode. Following recent measurements [73], these studies have assumed branching ratios of $0.5 \times 10^{-5}$ for $B_d^0 \to \pi^+ \pi^-$ and $B_s^0 \to K^{\pm} \pi^{\mp}$, $1.9 \times 10^{-5}$ for $B_d^0 \to K^\pm \pi^\mp$ and $B_s^0 \to K^+ K^-$ and $8 \times 10^{-5}$ for $\Lambda_b \to p \pi^-$ and $\Lambda_b \to p K^-$. Note that much of the following discussion is also relevant for the topics dealt with in Secs. 5.1 and 5.4.
### Selection

The expected event-yields passing the early trigger levels are shown in Tab. 7. In this mode LHCb in particular benefits from the high efficiency of its hadron trigger. For ATLAS and CMS, the triggering muon will be used to flavour-tag the events, whereas for LHCb lepton and kaon tags will be used.

The higher level trigger and reconstruction-cuts are optimized to fight combinatorial background from other $b\bar{b}$ events and select genuine two-body B decays. In these, the requirements on the secondary vertex are the most powerful, but isolation and kinematic cuts also play a rôle. The details of the selection are discussed in Refs. [37, 38, 39]. The event yields after two-body selection are shown in Tab. 7.

In order to reject non-$\pi^+\pi^-$ two-body background, LHCb exploits its powerful RICH system, demanding that both tracks be identified as a pion or lighter particle. This requirement and a window of ±30 MeV/c² around the $B^0_d$ mass reduce the contamination by such decays to 7%. As explained in Sec. 2., CMS will achieve a certain level of $\pi$–K separation from the dE/dx information available from the tracker, and the numbers and fit results presented here rely on this assumption, although Tabs. 7 and 8 contain alternative numbers for a hadron-blind selection. The requirements that both particles have an ionization within the expected pion energy loss, and an invariant mass within of the nominal $B^0_d$ mass, are expected to yield a final contamination of 40%. ATLAS chooses to make no further cuts, but rather exploit the remaining discriminant information in a multi-parameter fit, in particular the limited dE/dx information discussed in Sec. 2.. At present, only ATLAS and the CMS hadron-blind analysis have considered the background contribution from $\Lambda_b$ decays.

In addition to two-body contamination, there will be some residual combinatorial background passing the final cuts, which is expected to be dominated by events with a false vertex being faked by two unrelated high impact parameter tracks. The low branching ratio of the signal-process makes it very difficult to estimate the level of combinatorial background. ATLAS and CMS have used a combination of fast and full simulation techniques, whereas LHCb has extrapolated from a large sample of fully GEANT-simulated events. All experiments conclude that the combinatorial background should be smaller than the signal.

Some attributes of the final selected samples are given in Tab. 8, and examples for mass peaks are

### Table 7: Event yields in $B^0_d \rightarrow \pi^+\pi^-$ at various stages of the selection procedure for one year’s operation. The final yields are for flavour-tagged events (an alternative yield is given for CMS, in brackets, for a selection assuming no dE/dx information).

<table>
<thead>
<tr>
<th>Selection stage</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>First trigger level</td>
<td>46k</td>
<td>52k</td>
<td>149.9k</td>
</tr>
<tr>
<td>Second trigger level</td>
<td>4.2k</td>
<td>4.3k</td>
<td>67.5k</td>
</tr>
<tr>
<td>Two-body selection</td>
<td>2.3k</td>
<td>1.6k</td>
<td>14.5k</td>
</tr>
<tr>
<td>$\pi^+\pi^-$ selection</td>
<td>2.3k</td>
<td>0.9k</td>
<td>4.9k</td>
</tr>
<tr>
<td></td>
<td>(2.6k)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 8: Attributes of the $B^0_d \rightarrow \pi^+\pi^-$ samples for the three experiments (an alternative signal/two-body background number is given for CMS, in brackets, assuming no dE/dx information). Note that ATLAS performs a fit to all events passing its two-body selection; the background levels shown here are for illustration, imposing a 1σ mass window.

<table>
<thead>
<tr>
<th></th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass resolution [MeV/c²]</td>
<td>70</td>
<td>27</td>
<td>17</td>
</tr>
<tr>
<td>Proper time resolution [ps]</td>
<td>0.065</td>
<td>0.060</td>
<td>0.04</td>
</tr>
<tr>
<td>Signal / two-body background</td>
<td>0.19</td>
<td>1.6</td>
<td>15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.33)</td>
<td></td>
</tr>
<tr>
<td>Signal / other background</td>
<td>1.6</td>
<td>5</td>
<td>&gt;1</td>
</tr>
<tr>
<td>Tagging dilution</td>
<td>0.56</td>
<td>0.56</td>
<td>0.40</td>
</tr>
</tbody>
</table>
shown in Fig. 13.

**Fitting the CP Asymmetry**

Assuming the performance figures presented above, the experiments have used MC techniques to estimate their expected sensitivity to the CP asymmetries $A^{\text{mix}}_{\pi^+\pi^-}$ and $A^{\text{dir}}_{\pi^+\pi^-}$ from time-dependent fits, where these are defined in the usual manner:

$$A_{\text{CP}}(B^0_d \to \pi^+\pi^-)(t) = A^{\text{dir}}_{\pi^+\pi^-} \cos \Delta mt + A^{\text{mix}}_{\pi^+\pi^-} \sin \Delta mt.$$  

(18)

For the present study, LHCb has considered two-parameter fits of $A^{\text{mix}}_{\pi^+\pi^-}$ and $A^{\text{dir}}_{\pi^+\pi^-}$ to $B^0_d \to \pi^+\pi^-$ candidates passing tight cuts. Any CP asymmetry in the background has been neglected, assuming that these effects can be controlled with sufficient precision through a study of separate samples isolated by the RICH system. The CMS sensitivity with the dE/dx selection has been evaluated, also assuming any background-asymmetry to be known. The uncertainties obtainable with one year’s statistics are shown in Tab. 9: they are found to be independent of the values of the true CP asymmetries, symmetric and Gaussian. The low frequency of the oscillations means that there is a significant correlation between $A^{\text{mix}}_{\pi^+\pi^-}$ and $A^{\text{dir}}_{\pi^+\pi^-}$.

ATLAS have developed a sophisticated method for extracting the $B^0_d \to \pi^+\pi^-$ asymmetries, whereby they are determined from an unbinned maximum-likelihood fit, simultaneously with the asymmetries of the other two-body classes. Considering the allowed $\pi\pi$, $\pi K$ and $KK$ modes, $\Lambda_0 \to p\pi^-$, $pK^-\pi^+$ decays and the combinatorial background give nine coefficients. The likelihood of a given decay-hypothesis is computed using the event fraction, the proper time, the invariant mass of the two tracks under the hypothesis, the measured specific ionization and the flavour at production and decay-time. It is assumed that the branching ratios will be known with fractional errors of 5% and a possible CP asymmetry of the background is neglected. The uncertainties on the $B^0_d \to \pi^+\pi^-$ coefficients with one year’s statistics are shown in Tab. 9. Without the 0.8 $\sigma$ $\pi/K$ separation provided by the ionization information, the sensitivity is about 20% worse.

**Sensitivity to $\alpha$**

Present studies to estimate the combined LHC precision for $\alpha$ rely on the sensitivities given in Tab. 9 and Eqs. (17); they are presently extended to include the full expression (16). The simpler expression gives rise to ‘singularities’ in the precision for $\alpha$ for certain parameter values [75] which are not likely to occur with the full treatment.
<table>
<thead>
<tr>
<th></th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{\pi^+\pi^-}^{\text{dir}}$</td>
<td>0.16</td>
<td>0.11</td>
<td>0.09</td>
</tr>
<tr>
<td>$A_{\pi^+\pi^-}^{\text{mix}}$</td>
<td>0.21</td>
<td>0.14</td>
<td>0.07</td>
</tr>
<tr>
<td>Correlation coefficient</td>
<td>-0.25</td>
<td>-0.51</td>
<td>-0.49</td>
</tr>
</tbody>
</table>

Table 9: Expected sensitivities for the $B^0_d \to \pi^+\pi^-$ CP asymmetry coefficients $A_{\pi^+\pi^-}^{\text{dir}}$ and $A_{\pi^+\pi^-}^{\text{mix}}$ with one year of data, and correlation between the fitted parameters (the CMS numbers assume a selection exploiting $dE/dx$ information).

Simulated measurements have shown that the sensitivities to the CP asymmetry coefficients quoted in Tab. 9, estimated from the $\chi^2$ parabolic approximation, describe correctly the spread of experimental results. Also, the sensitivities do not depend on the actual values of the asymmetries, so that the numbers in Tab. 9 with correlations are sufficient to summarize the experimental precision of the measurements.

In contrast, the sensitivity to the parameters $\alpha$ and $\delta$ depends on the chosen set of parameters $\alpha$, $\delta$, $|P/T|$ and on the theoretical uncertainty of $|P/T|$, so that the sensitivity to $\alpha$ can only be given for specific scenarios. Also, Eqs. (17) entail a four-fold discrete ambiguity in $\alpha$. Here sensitivities are given under the assumption that this ambiguity can be correctly resolved.

Figure 14(a) shows the expected sensitivity to $\alpha$ as a function of $\alpha$ and $\delta$ for a given $|P/T| = 0.2 \pm 0.02$, after extended LHC running (3 years of low luminosity running of ATLAS and CMS combined with 5 years of LHCb). The sensitivity is around $2\alpha$ in the larger part of the plane, except around the lines corresponding to $\delta = 90^\circ$ and $270^\circ$ and $\alpha = 45^\circ$ and $135^\circ$. For these values of $\delta$ and $\alpha$, the leading-order term in $|P/T|$ of the mixing-induced CP asymmetry $A_{\text{CP}}^{\text{mix}}(B_d \to \pi^+\pi^-)$ vanishes, as can be seen in (17).

Figure 14(b) shows the expected sensitivity to $\alpha$ as a function of $\alpha$ for the fixed values $\delta = 30^\circ$, $|P/T| = 0.2$ and different values of the uncertainty on $|P/T|$ and the integrated luminosities. It appears that for values of $\alpha$ around $90^\circ$, the sensitivity to $\alpha$ is already limited after one year if the uncertainty on $|P/T|$ is not better than 10%. The effect of the uncertainty on $|P/T|$ is less dramatic for values of $\alpha$ around $0^\circ$ or $180^\circ$, which are disfavoured by current SM fits.
3.23 Conclusions

At the LHC it should be possible to measure the $B^0_d \to \pi^+\pi^-$ CP-violating observables with high precision. Interpreting these observables in terms of the angle $\alpha$, however, requires external information on the strength of the penguin contributions. This information has to be rather precise if one is to fully exploit LHC’s powerful reach. Although exact conclusions depend on the particular parameter-set, it appears more promising to analyse the observables of $B^0_d \to \pi^+\pi^-$ and other two-body decays in the context of the approach discussed in Sec. 5.4.

3.3 Extracting $\alpha$ from $B \to \rho\pi$ Modes

3.3.1 Theoretical Introduction

The analysis of the decays $B_d \to \rho^{\pm}\pi^{\mp}$ allows, in principle, the extraction of $\alpha$ [77]. However, the simplest approach, where the $\rho$ meson is considered as stable particle, is plagued by both high order discrete ambiguities and penguin pollution, like in $B^0_d \to \pi^+\pi^-$. To solve either problem, Snyder and Quinn [72] proposed a full three-body analysis of the decay $B^0_d \to \pi^+\pi^-\pi^0$ in the $\rho$ resonance region, taking into account interference effects between vector mesons of different charges. The knowledge of the strong decay $\rho \to \pi\pi$, parametrized as a Breit-Wigner amplitude, allows the extraction of all parameters that describe both the tree and penguin contributions to $B_d \to \rho\pi$, including $\alpha$, from a multi-dimensional likelihood-fit.

The two-body $B_d \to \rho\pi$ amplitudes can be written as:

$$A^{\pm\mp}(B^0_d \to \rho^{\pm}\pi^{\mp}) = e^{-i\alpha}T^{\pm\mp} + P^{\pm\mp}, \quad A^{00}(B^0_d \to \rho^0\pi^0) = e^{-i\alpha}T^{00} + P^{00}. \quad (19)$$

The CP-conjugate amplitudes $\overline{A}^{ij} \equiv A(B_{d}^{-}\to \rho^{i}\pi^{j})$ are obtained by changing the sign of the weak phases. The full three-body $B_d \to \pi^+\pi^-\pi^0$ amplitude takes the form:

$$A(B_d \to \pi^+\pi^-\pi^0) = A^+f_+ + A^-f_- + A^{00}f_0, \quad (20)$$

when $\rho$-dominance is assumed. Here $f_i$ stands for the Breit-Wigner amplitude for the decay of the $\rho^i$, and is a function of the two independent variables of the three-pion Dalitz-plot, which are chosen as the invariant masses $s^{\pm} = (p_{\pi^\pm} + p_{\pi^0})^2$. The Breit-Wigner parametrization is not unique; in the following we take:

$$f_+ \propto \frac{\cos \theta^*}{s^+ - m_{\rho}^2 + im_{\rho} \Gamma_{\rho}}, \quad (21)$$

where $\theta^*$ is the helicity angle of the $\rho$ decay which is given in terms of $(s^+, s^-)$ by the standard formulae. This dependence enhances the number of events in the corners of the Dalitz-plot, where interferences are maximal.

The time-dependent analysis of the event distribution in the Dalitz-plot allows one to extract $|A(B^0_d \to \pi^+\pi^-\pi^0)|$, $|\overline{A}(B_{d}^{0} \to \pi^+\pi^-\pi^0)|$ and $\text{Im}[\frac{2}{m_{\rho}}A\overline{A}^*]$ as functions of $(s^+, s^-)$. Using (20) and (21), it is straightforward to show that the magnitudes and the relative phases of the two-body amplitudes $A^{ij}$ and $\overline{A}^{ij}$ can be obtained [72]; this amounts to determining 11 independent parameters, taking into account that one overall phase is irrelevant, and including the overall normalization. In addition, assuming isospin symmetry and neglecting electroweak penguins, the relation [71]

$$P^{00} = -\frac{1}{2}(P^{+-} + P^{-+}) \quad (22)$$

allows a further reduction in the number of independent parameters that describe $A^{ij}$ and $\overline{A}^{ij}$. These parameters can be chosen as $\alpha$ and the complex amplitudes $T^{+-}$, $T^{00}$, $P^{+-}$ and $P^{-+}$. It is important to note that $A^{ij}$ and $\overline{A}^{ij}$ are determined without discrete ambiguity in the general case, such that both $\cos 2\alpha$ and $\sin 2\alpha$ (and thus $\alpha$ in $[0, \pi]$) are accessible [72]. This resolves in particular the ambiguity between $\alpha$ and $\pi/2 - \alpha$. 

3.32 Experimental Studies

Selection

The LHCb collaboration has performed full simulation studies on the selection of the $B^+_d \rightarrow \pi^+\pi^-\pi^0$ channel. The charged pions are reconstructed in the tracking devices and are identified in the RICH detectors. At present, only $\pi^0$s built from two resolved photons are used in the analysis. Figure 15(a) shows the two-photon invariant-mass in $B^+_d \rightarrow \pi^+\pi^-\pi^0$ events, for photons with energy above 2 GeV. The resolution of the $\pi^0$ mass varies between 5 and 7 MeV, depending on the $\pi^0$ production angle. The overall efficiency for $\pi^0$ reconstruction is 25%, with a signal to combinatorial background ratio of approximately 1. The measured $\pi^0$ mass is used in further $B^+_d$ mass reconstruction.

The background comes from combinatorials and from inclusive $b\bar{b}$ events. For its suppression, the following qualitative selection-cuts have been applied:

- a pre-selection for charged pions and photons which required the momentum or energy to exceed a value depending on the polar angle of the candidate. For charged pions, the momentum-cut varied between 1 and 2 GeV and for photons the energy-cut varied between 2 and 6 GeV;
- selection of signal-like events based on a discriminant variable built from kinematic variables of $\pi$, $\rho$ and $B^0_d$;
- selection based on the reconstructed secondary vertex for a $\pi^+\pi^-$ combination;
- Dalitz-plot cuts to eliminate low energy $\pi^0$ combinatorial background due to particles from the primary vertex.

These selection criteria result in a combinatorial background-suppression factor of the order of $10^7$ and give an acceptance for triggered and tagged events of 1%. Figure 15(b) shows the expected $\pi^+\pi^-\pi^0$ invariant mass distribution after one year of data taking. The measured $B^0_d$ width is 50 MeV/c$^2$. The annual event-yields for triggered, fully reconstructed and tagged events are given in Tab. 10.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$B^0 \rightarrow \rho^+\pi^-$</th>
<th>$B^0 \rightarrow \rho^-\pi^+$</th>
<th>$B^0 \rightarrow \rho^0\pi^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>$44 \times 10^{-6}$</td>
<td>$10 \times 10^{-6}$</td>
<td>$1 \times 10^{-6}$</td>
</tr>
<tr>
<td>Event Yield</td>
<td>1000</td>
<td>200</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 10: Annual event-yields for $B \rightarrow \rho\pi$ decays. The branching fractions are crude estimates used in BABAR’s study of these decays [6].

Figure 16 shows the Dalitz-plot for the $B^0_d \rightarrow \pi^+\pi^-\pi^0$ channel after acceptance-cuts. Helicity effects enhance the population in the interference regions, in particular in the most critical $\rho^+\rho^0$ regions, where the sensitivity to the $\alpha$ parameter is highest. The $\rho^+\rho^-$ interference region is not accessible due to the dominance of combinatorial background in the corresponding area of the Dalitz-space.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>0.9, 1.35 or 1.95 radians</td>
</tr>
<tr>
<td>( T^{+} )</td>
<td>1.00</td>
</tr>
<tr>
<td>( T^{-} )</td>
<td>0.47</td>
</tr>
<tr>
<td>( T^{00} )</td>
<td>0.14</td>
</tr>
<tr>
<td>( P^{++} )</td>
<td>(-0.20 , e^{-0.5i})</td>
</tr>
<tr>
<td>( P^{+-} )</td>
<td>0.15 , e^{2.0i}</td>
</tr>
</tbody>
</table>

Table 11: The three values of \( \alpha \) and the amplitudes used in the generation of the studied samples.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1 year</th>
<th>5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha )</td>
<td>( \langle \alpha \rangle )</td>
<td>( \langle \sigma_{\alpha} \rangle )</td>
</tr>
<tr>
<td>( B )</td>
<td>51.6</td>
<td>4.9</td>
</tr>
<tr>
<td>( B )</td>
<td>77.3</td>
<td>2.5</td>
</tr>
<tr>
<td>( B )</td>
<td>111.7</td>
<td>4.3</td>
</tr>
</tbody>
</table>

Table 12: The mean fitted values of \( \alpha \), \( \langle \alpha \rangle \), and the mean error on \( \alpha \), \( \langle \sigma_{\alpha} \rangle \), for samples approximating 1 or 5 years data taking for LHCb at \( \alpha = 0.90, 1.35 \) and 1.95 radians (51.6°, 77.3° and 111.7°).

### Sensitivity to \( \alpha \)

A stand-alone simulation which introduces the weak phase \( \alpha \) as well as the relative tree and penguin amplitudes was used to generate events for the fitting studies. Cuts in the Dalitz-space have been made to eliminate the \( \psi^{+} - \psi^{0} \) interference region. Furthermore, cuts are applied to the invariant mass of a \( \rho \) candidate to select only resonant decays. However, the full LHCb acceptance has not yet been simulated and backgrounds have not been considered.

The amplitudes used for these studies contain a large penguin contribution and are identical to those studied by Babar [6]. Their values are given Tab. 11. Samples of \( 10^5 \) events were generated for each value of \( \alpha \). An unbinned maximum-likelihood fit was used to extract the parameters. The form of the used likelihood is:

\[
-2 \ln \mathcal{L} = -2 \sum_{i=1}^{N_{B_d^{0}}} \ln \left( \frac{|A(s_i^+, s_i^-, t_i; \alpha)|^2}{\mathcal{N}(\alpha)} \right) - 2 \sum_{j=1}^{N_{\overline{B}_d^{0}}} \ln \left( \frac{|\overline{A}(s_j^+, s_j^-, t_j; \alpha)|^2}{\mathcal{N}(\alpha)} \right),
\]

where \( N_{B_d^{0}} \) and \( N_{\overline{B}_d^{0}} \) are the number of \( B_d^{0} \) and \( \overline{B}_d^{0} \) events, respectively, and \( \mathcal{N} \) is the normalization. It is given by \( \langle |A|^2 + |\overline{A}|^2 \rangle \), integrated over the Dalitz-plot acceptance, and was calculated numerically using a sub-sample of 20000 simulated events. The fit was performed on 75 sub-samples of 1000 events, to simulate approximately 1 year data taking, and 15 samples of 5000 events to simulate 5 years data taking. The mean fitted value of \( \alpha \) and the mean error are given in Tab. 12. The error varies with the true value of \( \alpha \) as expected [72], and the fitted values are unbiased for \( \alpha = 0.9 \) and 1.35 radians. The bias of \( \sim 0.15 \) radians for \( \alpha = 1.95 \) radians was not observed when fits were made to samples where no Dalitz-plot selection was made. Therefore, this bias appears to be related to the exclusion of the \( \rho^{+} - \rho^{0} \) interference region and needs further investigation. Correction for this bias will be required to extract \( \alpha \) from the final data sample and will introduce systematic uncertainties which may be of a magnitude similar to the statistical precision.

In Fig. 17 an example of a likelihood-scan curve is given for 1000 fitted events generated with \( \alpha = 1.35 \) radians. The fake mirror solution at \( \frac{\pi}{2} - \alpha \) gives a local minimum in the likelihood-curve. The difference in the likelihood, expressed as \( \chi^2 = -2 \ln \mathcal{L} \), between the true and the mirror solution for the 75 one year data samples is displayed in Fig. 18(a). In approximately 10% of all cases, the mirror solution is the global minimum or is separated by less than 1 \( \sigma \) from the true solution. The same quantity for the 15 five year data samples is shown in Fig. 18(b). The mirror and true solution minima are now well separated.

### 3.33 Conclusions

From the theoretical point of view, the main advantage of the isospin analysis of the decay \( B_d \rightarrow \pi^+ \pi^- \pi^0 \) in the \( \rho \)-dominance assumption, with respect to its analogue in the two-pion channel, is the
Fig. 17: Example of a likelihood-scan curve for 1000 fitted LHCb events, generated with $\alpha = 1.35$. $\alpha$ was fixed to 40 values between 0 and $\pi$ radians and the negative likelihood was minimized with respect to the other 8 parameters.

Fig. 18: The difference in $-2\ln L$ between the true and mirror solution minima.

**Figures (a) and (b):**

- **(a) LHCb 1 year data samples**
  - Mean: 10.25
  - RMS: 7.374

- **(b) LHCb 5 year data samples**
  - Mean: 54.33
  - RMS: 16.11

**determination of the penguin amplitudes and the resolution of discrete ambiguities.** From the experimental side, it benefits from larger branching ratios [73] and from interference, which entails that the sensitivity of the analysis is directly proportional to the colour-suppressed channel $B \to \rho^0\pi^0$. This can be compared to the Gronau-London branching ratio construction [67] in $B_d \to \pi\pi$, which has a sensitivity proportional to the amplitude squared of $B \to \rho^0\pi^0$.

Preliminary studies for LHCb have shown that $B^0_{\text{d}} \to \pi^+\pi^+\pi^0$ events can be reconstructed and selected in sufficient numbers, so that an unambiguous value for $\alpha$ can be extracted without the problems that afflict the $B^0_{\text{d}} \to \pi^+\pi^-$ channel. It should be stressed that the fitting studies are preliminary and also optimistic in the sense that the exact LHCb acceptance has not been used and backgrounds have not been included. Also, the observed biases are likely to introduce significant systematic uncertainties. Furthermore, several important issues remain to be considered, which already have been studied in the specific context of $e^+e^-B$ factories [78, 79, 80]. One may cite, among others, various points: the influence of higher resonances ($\rho', \rho^0$). . .), the influence of the exact parametrization of the Breit-Wigner amplitude, the existence of bounds on the penguin-induced error on $\alpha$, when the $\rho^0\pi^0$ channel is too scarce to achieve the full analysis, and the rôle of electroweak penguins. All these issues will be further investigated in the future.

There are also some topics, yet to be investigated, which should enhance the precision on $\alpha$: the determination of the branching fractions from $e^+e^-$ experiments provide additional constraints on the fit and the untagged sample can be used to determine parameters other than $\alpha$. It is to be expected that after several years of data taking at $e^+e^-$ experiments and/or at the LHC era, the above issues will be much
better understood.

### 3.4 Extracting 2\(\beta + \gamma\) from \(B_d \rightarrow D^{(*)\pm}\pi^\mp\) decays

So far, we have put a strong emphasis on neutral B decays into final CP eigenstates. However, in order to extract CKM phases, there are also interesting decays of \(B_{d*,}\) mesons into final states that are not eigenstates of the CP operator. An important example is provided by the decays \(B_d \rightarrow D^{(*)\pm}\pi^\mp\), which receive only contributions from tree-diagram-like topologies, and are the topic of this subsection.

#### 3.4.1 Theoretical Aspects

As can be seen in Fig. 19, \(B_d\) and \(\bar{B}_d\) mesons may both decay into \(D^{(*)\pm}\pi^\mp\), thereby leading to interference effects between \(B_d\)–\(\bar{B}_d\) mixing and decay processes. Consequently, the time-dependent decay rates for initially, i.e. at time \(t = 0\), present \(B_d\)–\(\bar{B}_d\) mesons decaying into the final state \(f \equiv D^{(*)\pm}\pi^\mp\) allow one to determine the observable [17]

\[
\xi_f^{(d)} = \frac{e^{-i\phi_d} A(B_d^0 \rightarrow f)}{A(B_d^0 \rightarrow f)} = \frac{1}{\lambda^2 R_b} \left( 1 - \frac{M_f}{M_f} \right)
\]

whereas those corresponding to \(\bar{f} \equiv D^{(*)-}\pi^+\) allow one to extract

\[
\xi_{\bar{f}}^{(d)} = \frac{e^{-i\phi_d} A(\bar{B}_d^0 \rightarrow \bar{f})}{A(\bar{B}_d^0 \rightarrow \bar{f})} = \frac{\lambda^2 R_b}{1 - \lambda^2} \left( 1 - \frac{M_f}{M_f} \right)
\]

Here, \(R_b\) is the usual CKM factor (see (4)), and

\[
\overline{M}_f \equiv \left\langle f \left| \mathcal{O}_1(\mu) C_1(\mu) + \mathcal{O}_2(\mu) C_2(\mu) \right| \overline{B}_d^0 \rightangle, \quad M_f \equiv \left\langle f \left| O_1(\mu) C_1(\mu) + O_2(\mu) C_2(\mu) \right| \overline{B}_d^0 \rightangle
\]

are hadronic matrix elements of the following current–current operators:

\[
\mathcal{O}_1 = (\bar{d}_\alpha u_\beta)_{V-A} (\bar{c}_\beta b_\alpha)_{V-A}, \quad \mathcal{O}_2 = (\bar{d}_\alpha u_\alpha)_{V-A} (\bar{c}_\beta b_\beta)_{V-A}, \quad O_1 = (\bar{d}_\alpha c_\beta)_{V-A} (\bar{q}_\beta b_\alpha)_{V-A}, \quad O_2 = (\bar{d}_\alpha c_\alpha)_{V-A} (\bar{q}_\beta b_\beta)_{V-A},
\]

where \(\alpha\) and \(\beta\) denote colour indices, and \(V-A\) refers to the Lorentz structures \(\gamma_\mu (1 - \gamma_5)\). The observables \(\xi_f^{(d)}\) and \(\xi_{\bar{f}}^{(d)}\) allow a theoretically clean extraction of the weak phase \(\phi_d + \gamma\) [81], as the hadronic matrix elements \(\overline{M}_f\) and \(M_f\) cancel in the following combination:

\[
\xi_f^{(d)} \times \xi_{\bar{f}}^{(d)} = e^{-2i(\phi_d + \gamma)}.
\]

Since the \(B_d\)–\(\bar{B}_d\) mixing phase \(\phi_d\), i.e. \(2\beta\), can be determined rather straightforwardly with the help of the “gold-plated” mode \(B_d \rightarrow J/\psi K_S\) (see Sec. 3.1), one may extract the CKM angle \(\gamma\) from (27). As
the $\bar{b} \to \bar{u}$ quark-level transition in Fig. 19 is doubly Cabibbo-suppressed by $\lambda^2 R_b \approx 0.02$ with respect to the $b \to c$ transition, the interference effects are tiny. However, the branching ratios are large, i.e. of order $10^{-3}$, and the $D^{(*)+} \pi^-$ states can be reconstructed with a good efficiency and modest background. Consequently, $B_d \to D^{(*)+} \pi^-$ decays offer an interesting strategy to determine $\gamma$, as we will discuss in the following.

### 3.42 Experimental Studies

LHCb have investigated the potential of measuring $\gamma$ through $B_d^0 \to D^{(*)+} \pi^-$ with the $D^*$ decaying strongly to a $D^0$ meson. As interference effects are tiny, a very large data sample is necessary to extract $\gamma$ with an interesting precision. Two methods have been studied: first a conventional exclusive reconstruction with $\overline{D}^0 \to K^+ \pi^-$ and second a partial reconstruction approach in order to boost statistics. The reconstruction study has also been extended to $B_d^0 \to D^{(*)+} a_1^\pm$ decays, but such events have not yet been considered for the extraction of CKM phases.

#### Exclusive Reconstruction

Loose RICH criteria were used to select the candidate $\overline{D}^0$ decay products. To identify $D^{*-}$, the difference between the reconstructed $D^{*-}$ and $\overline{D}^0$ masses was required to lie within a $3 \text{ MeV}$ wide window around its nominal value of $144 \text{ MeV}$, just above the pion mass. Figure 20(a) shows the signal-peak ($\sigma = 1 \text{ MeV}$) with the background superimposed in arbitrary units. The usual $B^0$ cuts (high $p_T$ and detached vertex) were applied to the pion coming from the $B^0$. The final $B^0$ mass peak has a width of $13 \text{ MeV}$. Selecting events within a window of $\pm 30 \text{ MeV}$ results in $84k$ selected events (triggered & tagged) per year with a S/B of $\sim 12$.

#### Partial Reconstruction

Instead of reconstructing the full decay chain, one can obtain all necessary information from the pion coming directly from the $B^0$ (the ‘fast pion’, $\pi_f$) and the pion coming from the $D^-$ (the ‘slow pion’, $\pi_s$). As shown below, one can reconstruct the full $B^0$ momentum from the momenta of $\pi_f$ and $\pi_s$ and the direction of the $B^0$. This direction can be inferred from the position of the primary vertex and the decay vertex of the $B^0$, the latter being defined by the crossing point of fast and slow pion.

To reconstruct the $D^*$ (and then $B^0$) momentum from this limited information, we use the fact that, if the $\pi_s$ momentum is known, the possible $D^*$ momenta are restricted to lie on a two-dimensional surface. This surface is shown schematically in Fig. 21(a). Kinematics allows two possible solutions, but in practice they lie very close, and it suffices to approximate the true solution by the minimal distance between the slow pion and the $B^0$ vector as shown in Fig. 21(b). In order to suppress background, a probability distribution is cut on, which exploits the allowed ranges and expected correlations between the parameters in this reconstruction.

To further reduce background, one can use a cut similar to that on the mass difference between the $D^{*-}$ and the $\overline{D}^0$ as applied in the exclusive case. Instead of fully reconstructing the $\overline{D}^0$, one tries to identify two charged decay products of the $\overline{D}^0 (X^+, Y^-)$ and cuts on the difference $\Delta m$ between the pseudo masses:

$$\Delta m = M(X^+ Y^-) - M(X^+ Y^-).$$

(28)

$\Delta m$ would be the mass difference between the $D^*$ and the $D^0$ if $X^+$ and $Y^-$ were the only decay products of the $D^0$. In general, though, there will be some missing momentum. Fortunately the missing momentum cancels to some extent in Eq. (28), so that even for the partially reconstructed $\overline{D}^0$ this remains a powerful cut as shown in Fig. 20(b).

After all cuts, $260k$ reconstructed, triggered and tagged events per year are expected inside the mass window of $\pm 200 \text{ MeV}$ with a S/B $\sim 3$. The reconstruction returns a mass peak of width $200 \text{ MeV}$.
Fig. 20: The difference between reconstructed $D^{*-}$ and $D^0$ masses for the exclusive and inclusive reconstruction. The background is superimposed with arbitrary normalization. For the exclusive reconstruction, $\Delta m \in [143.5 \text{ MeV}, 146.5 \text{ MeV}]$, for the inclusive reconstruction, $\Delta m \in [144 \text{ MeV}, 160 \text{ MeV}].$

$$B^0_d \rightarrow D^{*+} a_1^{\pm}$$

The same inclusive analysis was performed for the channel $B^0_d \rightarrow D^{*+} a_1^{\pm}$, with $a_1^{\pm} \rightarrow \rho^0 \pi^{\pm}$, which has a branching ratio that is about three times larger than that of $B^0_d \rightarrow D^{*-} \pi^{\pm}$. As expected, the efficiency for this channel is lower, as more particles need to be reconstructed, while the mass resolution is slightly improved ($\sigma \approx 180$ MeV), due to better reconstruction of the $B^0$ decay vertex from 4 instead of only 2 particles. 360k reconstructed, triggered and tagged events are expected within a $\pm 200$ MeV mass window per year, with a S/B of $\sim 4$.

The yield in all analyses is summarized in Tab. 13, with a total that assumes negligible correlations between the selections.

**Sensitivity to $\gamma$**

For $B^0_d \rightarrow D^{*+} \pi^{\pm}$ decays the parameters $\xi_f^{(d)}$ and $\xi_j^{(d)}$ can in principle be completely determined by fitting the two time-dependent asymmetries

$$A_{D^{*-}}(\tau) = \frac{\Gamma_\tau (B^0_d \rightarrow D^{*-} \pi^+) - \Gamma_\tau (\overline{B^0_d} \rightarrow D^{*-} \pi^+)}{\Gamma_\tau (B^0_d \rightarrow D^{*-} \pi^+) + \Gamma_\tau (\overline{B^0_d} \rightarrow D^{*-} \pi^+)}$$

$$= \frac{1 - |\xi_j^{(d)}|^2 \cos(\Delta m \tau) - 2 |\xi_j^{(d)}| \sin(-\phi_d + \gamma + \Delta S) \sin(\Delta m \tau)}{1 + |\xi_j^{(d)}|^2},$$

(29)
Table 13: Expected S/B and yields in reconstructed, triggered and tagged events in one year of LHCb data taking.

<table>
<thead>
<tr>
<th>Channel</th>
<th>S/B</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_d \to D^{*+}\pi^\pm$ (excl.)</td>
<td>12</td>
<td>83k</td>
</tr>
<tr>
<td>$B^0_d \to D^{*+}\pi^\pm$ (incl.)</td>
<td>3</td>
<td>260k</td>
</tr>
<tr>
<td>$B^0_d \to D^{*+}a_1^\pm$ (incl.)</td>
<td>4</td>
<td>360k</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td>703k</td>
</tr>
</tbody>
</table>

\[
A_{D^+}(\tau) = \frac{\Gamma_\tau(B^0_d \to D^{*+}\pi^-) - \Gamma_\tau(B^0_d \to D^{*+}\pi^-)}{\Gamma_\tau(B^0_d \to D^{*+}\pi^-) + \Gamma_\tau(B^0_d \to D^{*+}\pi^-)}
\]

\[
= \frac{(1 - |\xi_f^{(d)}|^2) \cos(\Delta m \tau) - 2 |\xi_f^{(d)}| \sin((\phi_d + \gamma) + \Delta S) \sin(\Delta m \tau)}{1 + |\xi_f^{(d)}|^2}, \quad (30)
\]

where $\Delta S$ is a possible strong phase shift entering $\xi_f^{(d)}$ via $M_f/M_T$.

Acceptance-effects cancel in each of the two asymmetries. In practice, as the interference effect is so tiny, $|\xi_f^{(d)}| = 1/|\xi_f^{(d)}|$ needs to be constrained. Therefore, fits have been performed assuming this parameter be known with a relative precision of $\epsilon_f^{(d)}$. This uncertainty translates directly into a relative uncertainty on $\sin(\Delta S \pm \{\phi_d + \gamma\})$. Throughout, a plausible true value of $|\xi_f^{(d)}| = 0.016$ has been assumed; the final resolution on $\gamma$ turns out to be directly proportional to this value (if $\epsilon_f^{(d)} = 0$, i.e. $\sigma_\gamma \propto 1/|\xi_f^{(d)}|$).

Using a stand-alone MC simulation and feeding it with the parameters, event yields (340k) and S/B ratios ($\sim 3$) for $B^0_d \to D^{*+}\pi^\pm$ as discussed above, the statistical error on $\sin(\Delta S \pm \{\phi_d + \gamma\})$ is found to be (for $\epsilon_f^{(d)} = 0$):

\[
\sigma_{\sin} = \frac{0.26}{\sqrt{\text{no. of years}}}, \quad (31)
\]

independent of the input values for $(\phi_d + \gamma)$ and $\Delta S$. Translating this into $\gamma-\Delta S$ space, the resolution now does depend on the input values; an uncertainty in $|\xi_f^{(d)}|$ also introduces a dependence on $\sin(\Delta S \pm \{\phi_d + \gamma\})$. Figure 22 shows the error on $(\phi_d + \gamma)$ as a function of $(\phi_d + \gamma)$ for $\Delta S = 0$, after 1 and after 5 years of LHCb data taking, for the cases that $|\xi_f^{(d)}|$ is known exactly (broken lines) and that the uncertainty in $|\xi_f^{(d)}|$ is 10% (solid lines). Assuming that $\phi_d$ can be fixed with negligible uncertainty from $B^0_d \to J/\psi K^0_S$ decays, this error will apply to $\gamma$ itself.

Presumably the large yield in $B^0_d \to D^{*+}a_1^\pm$ events can also be exploited to obtain additional sensitivity to $\gamma$. However the presence of two spin-1 particles in the decay complicates the extraction, and in order to disentangle the final-state configurations, an angular analysis has to be performed (see [27] for the discussion of an analogous problem). This study has not yet been performed.

3.43 Conclusions

We have seen that the large statistics at the LHC offers the possibility of measuring $\gamma$ with very interesting precision from $B^0_d \to D^{*+}\pi(a_1)\pm$ decays, despite the expected smallness of interference effects.

3.5 Extracting $\gamma - 2\delta\gamma$ from $B_s \to D^\pm K^\mp$ Decays

3.51 Theoretical Aspects

The decays $B_s \to D^\pm K^\mp$ receive only contributions from tree-diagram-like topologies and are the $B_s$ counterparts of the $B_d \to D^{(*)\pm}\pi^\mp$ modes discussed in Sec. 3.4. They probe the CKM combination
\( \gamma - 2\delta \gamma \) instead of \( \gamma + 2\beta \) in a theoretically clean way \cite{82}. Since one decay path in \( B_s^0 \to B_s^{0*} \to D_s^{\pm} K^- \) is only suppressed by \( R_b \approx 0.41 \), and not doubly Cabibbo-suppressed by \( \lambda^2 R_b \), as in the case of \( B_d \to D^{(*)} \pi^\mp \), the interference effects in \( B_s \to D_s^{\pm} K^\mp \) are much larger. A similar strategy to determine \( \gamma - 2\delta \gamma \) is also provided by the colour-suppressed decays \( B_s \to D \phi \) \cite{83}. In Ref. \cite{25}, untagged data samples of these decays were considered to extract CKM phases, and angular distributions of untagged decays of the kind \( B_s \to D^{(*)} K^{\mp} \), \( B_s \to D^{(*)} \phi \) were considered in \cite{27}.

### 3.52 Experimental Studies

LHCb have investigated the expected event yields in \( B_s \to D_s^{\pm} K^\mp \) and the resulting sensitivity to \( \gamma - 2\delta \gamma \) \cite{39}. The selection of this mode is experimentally challenging, as \( B_s \to D_s^{\pm} \pi^\mp \) events which come with a 20 times larger branching ratio need to be rejected. Figure 23 shows the event sample before and after including the information from the RICH detector. It can be seen that with the RICH’s \( \pi-K \) discrimination the \( B_s \to D_s^{\pm} \pi^\mp \) contamination can be adequately suppressed. 2.4k reconstructed and tagged events are expected in one year, with a low background.

The CKM phase \( \gamma - 2\delta \gamma \) can be determined from a fit to such a sample, in a manner directly analogous to that described in Sec. 3.4. Here, however, the intrinsic sensitivity is higher as the interference effects are larger. As always, the precision on the CKM phase depends on the value of the input parameters, which include \( \Delta \Gamma_s/\Gamma_s \) and \( \Delta m_s \). After one year’s operation, the precision is typically \( 8^0 \) (mean) \( \pm 2^0 \) (rms) for scenarios with \( \Delta m_s = 15 \) ps\(^{-1}\), and degrades to \( \sim 12^0 \) for \( \Delta m_s = 45 \) ps\(^{-1}\). Full tables can be found in \cite{39}. Assuming that \( 2\delta \gamma \) can be constrained from \( B_s \to J/\psi \phi \) decays, \( B_s \to D_s^{\pm} K^\mp \) will provide a very clean and competitive measurement of the angle \( \gamma \).

### 3.6 Extracting \( \gamma \) from \( B \to DK \) Decays

During the recent years, relations among amplitudes of non-leptonic B decays have been very popular to develop strategies for extracting the angles of the unitarity triangles, in particular \( \gamma \). The prototype of this approach involves charged \( B_{\pm} \to DK_{\pm} \) decays \cite{30}.

\(^6\)The CP-violating weak \( B_s^0 \to B_s^{0*} \) mixing phase \( \phi_s = -2\delta \gamma \) can be extracted with the help of the decay \( B_s \to J/\psi \phi \), see Sec. 4.
Fig. 24: Feynman diagrams contributing to the decays $B^+ \rightarrow D^0 K^+$ and $B^+ \rightarrow D^0 K^+$.

Fig. 25: Triangle relations between charged $B^\pm \rightarrow D K^\pm$ decay amplitudes.

3.61 Theoretical Aspects

The decays $B^+ \rightarrow D^0 K^+$ and $B^+ \rightarrow D^0 K^+$, which are pure “tree” decays, as can be seen in Fig. 24, provide an interesting strategy to extract $\gamma$, if we make in addition use of the transition $B^+ \rightarrow D^0 K^+$. Here, $D^0_+$ denotes the CP eigenstate of the neutral $D$ meson system with CP eigenvalue $+1$, given by

$$\left| D^0_+ \right> = \frac{1}{\sqrt{2}} \left( \left| D^0 \right> + \left| D^0 \right> \right),$$

and leads to the following amplitude relations:

$$\sqrt{2} A(B^+ \rightarrow D^0_+ K^+) = A(B^+ \rightarrow D^0 K^+) + A(B^+ \rightarrow D^0 K^+),$$

$$\sqrt{2} A(B^- \rightarrow D^0_+ K^-) = A(B^- \rightarrow D^0 K^-) + A(B^- \rightarrow D^0 K^-).$$

Fig. 25: Triangle relations between charged $B^\pm \rightarrow D K^\pm$ decay amplitudes.

Since we are dealing with pure “tree” decays that are caused by $\bar{b} \rightarrow \bar{c} u \bar{s}, \bar{u} c \bar{s}$ quark-level transitions, we have

$$\gamma = A(B^+ \rightarrow D^0 K^+) = A(B^- \rightarrow D^0 K^-) \times e^{2i\gamma},$$

allowing a theoretically clean determination of $\gamma$ with the help of the triangle construction shown in Fig. 25. Unfortunately, we have to deal with rather squashed triangles, since $A(B^+ \rightarrow D^0 K^+)$ is colour-suppressed with respect to $A \equiv A(B^+ \rightarrow D^0 K^+)$:

$$\left| \frac{a}{A} \right| = \frac{|\alpha|}{|\alpha|} \approx \frac{1}{\lambda |V_{ub}|} \times \frac{a_2}{a_1} \approx 0.41 \times \frac{a_2}{a_1} \approx 0.1,$$

where $a_1$ and $a_2$ are the usual phenomenological colour factors.

In 1998, the CLEO collaboration has reported the observation of $B^+ \rightarrow D^0 K^+$ [84]:

$$B(B^+ \rightarrow D^0 K^+) = (0.257 \pm 0.065 \pm 0.032) \times 10^{-5}.$$  

Using arguments based on “colour suppression”, we expect

$$B(B^+ \rightarrow D^0 K^+) \approx 10^{-2} \times B(B^+ \rightarrow D^0 K^+).$$

While the branching ratio $B(B^+ \rightarrow D^0 K^+)$ can be measured using conventional methods, the measurement of $B(B^+ \rightarrow D^0 K^+)$ suffers from considerable experimental problems [33]:
• If the branching ratio of \( B^+ \to D^0 K^+ \) is measured through hadronic decays of the \( D^0 \) meson, e.g. through \( B^+ \to D^0 \to K^- \pi^+ \pi^- \pi^+ K^+ \), we have large interference effects of \( \mathcal{O}(1) \) with the decay chain \( B^+ \to D^0 \to K^- \pi^+ \pi^- \pi^+ K^+ \) (note that the \( D^0 \) decay is doubly Cabibbo-suppressed).

• All possible hadronic tags of the \( D^0 \) in \( B^+ \to D^0 K^+ \) will be affected by such interference effects.

• Such problems can in principle be avoided by using semi-leptonic tags \( D^0 \to l^+ \nu_l X_s \). However, here there will be large backgrounds due to \( B^+ \to l^+ \nu_l X_c \), which may be difficult to control.

Moreover, decays of neutral \( D \) mesons into CP eigenstates, such as \( D_+^0 \to \pi^+ \pi^- K^+ K^- \), are experimentally challenging. Consequently, the original method proposed by Gronau and Wyler [30] will unfortunately be very difficult in practice. A variant of this approach was proposed by Atwood, Dunietz and Soni in [33]. In order to overcome the problems discussed above, the following decay chains can be considered:

\[
B^+ \to \overline{D}^0 \to f_i K^+, \quad B^+ \to D^0 \to f_i K^+, \quad (40)
\]

where \( f_i \) denotes doubly Cabibbo-suppressed (Cabibbo-favoured) non-CP modes of the \( \overline{D}^0 \) (\( D^0 \)), for instance, \( f_i = K^- \pi^+ \), \( K^- \pi^+ \pi^0 \). In order to extract \( \gamma \), at least two different final states \( f_i \) have to be considered. In this method, one makes use of the large interference effects, which spoil the hadronic tag of the \( D^0 \) in the original Gronau–Wyler method. In contrast to the case of \( B^+ \to D^0_+ K^+ \) discussed above, here both contributing decay amplitudes should be of comparable size, thereby leading to potentially large CP-violating effects. Furthermore, the branching ratio \( B(B^+ \to D^0 K^+) \), which is difficult to measure, is not required, but can rather be determined as a by-product. Unfortunately, this approach is also challenging, since many channels are involved, with total branching ratios of \( \mathcal{O}(10^{-7}) \) or even smaller. An accurate determination of the relevant \( D \) branching ratios \( B(D^0 \to f_i) \) and \( B(\overline{D}^0 \to f_i) \) is also essential for this method.

Fig. 26: Feynman diagrams contributing to the decays \( B^0_d \to \overline{D}^0 K^{*0} \) and \( B^0_d \to D^0 K^{*0} \).

So far, we have only considered charged \( B^\pm \to DK^\pm \) decays. However, also neutral decays of the kind \( B^0_d \to DK^{*0} \), which are shown in Fig. 26, allow one to extract \( \gamma \) [85]. As these modes are “self-tagging” through \( K^{*0} \to K^+ \pi^- \), no time-dependent measurements are required in this case. If we make again use of the CP eigenstate \( D_+^0 \) of the neutral \( D \) meson system, we obtain the following amplitude relations:

\[
\sqrt{2} A(B^0_d \to D^0_+ K^{*0}) = A(B^0_d \to \overline{D}^0 K^{*0}) + A(B^0_d \to D^0 K^{*0}),
\]

\[
\sqrt{2} A(B^0_d \to D^0_+ K^{*0}) = A(B^0_d \to D^0 K^{*0}) + A(B^0_d \to \overline{D}^0 K^{*0}).
\]

Moreover, we have

\[
b \equiv A(B^0_d \to D^0 K^{*0}) = A(B^0_d \to \overline{D}^0 K^{*0}) \times e^{2i\gamma}, \quad (43)
\]

\[
B \equiv A(B^0_d \to \overline{D}^0 K^{*0}) = A(B^0_d \to D^0 K^{*0}), \quad (44)
\]
allowing one to extract $\gamma$ from the triangle construction shown in Fig. 27, which is completely analogous to the $B^{\pm} \to DK^{\pm}$ case. However, there is an important difference, which is due to the fact that both decays $B^0_d \to D^0 K^{*0}$ and $B^0_d \to D^0 K^{*0}$ are “colour-suppressed”, as can be seen in Fig. 26:

$$\frac{|A(B^0_d \to D^0 K^{*0})|}{|A(B^0_d \to D^0 K^{*0})|} \approx \frac{1}{\lambda |V_{ub}|} \frac{a_2}{a_2} \approx 0.41.$$ (45)

Consequently, the triangles are expected to be not as squashed as for $B^{\pm} \to DK^{\pm}$. The corresponding branching ratios are expected to be of $\mathcal{O}(10^{-5})$. However, the detection of the neutral $D$ meson CP eigenstate $D_+^*$ also poses considerable difficulties.

![Fig. 27: Triangle relations between neutral $B_d \to DK^*$ decay amplitudes.](image)

### 3.62 Experimental Studies

Both ATLAS [37] and LHCb [39] have investigated the possibility of determining $\gamma$ through amplitude relations in the family of $B^0_d \to D K^{*0}$ decays. Both experiments have demonstrated that it will be possible to reconstruct samples of such events, with LHCb in particular benefiting from its hadron trigger. However, with the branching ratios that have been assumed, the yields are still low, with only a few 10’s of events expected in the $D_1 K^{*0}$ and $D_1 \bar{K}^{*0}$ modes. At this level, several years are required to integrate sufficient statistics for a meaningful measurement. The experiments will continue to investigate this, and associated $B \to DK$ measurements, and search for possible improvement.

### 4. THE “GOLD-PLATED” DECAY $B_s \to J/\psi \phi^7$

The decay $B^0_s \to J/\psi \phi$ shown in Fig. 28 is the $B_s$ counterpart to the “gold-plated” mode $B_d \to J/\psi K_S$ and is particularly interesting because of its rich physics potential. A complete analysis of this decay appears feasible at the LHC, because of the large statistics and good proper time resolution of the experiments.

![Fig. 28: Feynman diagrams contributing to $B^0_s \to J/\psi \phi$. The dashed lines represent a colour-singlet exchange.](image)

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$^7$Section coordinators: R. Fleischer and M. Smizanska.
4.1 Theoretical Aspects

In the case of $B^0_s \to J/\psi \phi$, the final state is an admixture of different CP eigenstates. In order to disentangle them, an angular analysis of the decay products of $B_s \to J/\psi (1^+ - ) \phi (K^+ K^-)$ has to be performed \cite{87, 88}. In addition to interesting strategies to extract the $B^0_s \to B^0_s$ mixing parameters $\Delta G_s$ and $\Delta M_s$, we may also probe the weak mixing phase $\phi_s = -2\delta \gamma = -2\lambda^2 \eta$, thereby allowing one to measure the Wolfenstein parameter $\eta$ \cite{26,88}. A particularly interesting feature of $B^0_s \to J/\psi \phi$ decays is that they exhibit tiny CP-violating effects within the SM. Consequently, they represent a sensitive probe for CP-violating contributions from physics beyond the SM \cite{62, 89}. Since new-physics contributions have to compete with SM tree-diagram-like topologies, the natural place for any manifestation of new physics is in CP asymmetries induced by $B_s$ mixing. Illustrations of the new-physics effects in $B^0_s \to J/\psi \phi$ for specific scenarios of new physics can be found in \cite{90, 91} and are discussed in more detail below.

4.11 General Structure of the Decay Probability Functions

For an initially, i.e. at time $t = 0$, present $B^0_s$ meson, the time-dependent angular distribution of the decay chain $B_s \to J/\psi (1^+ - ) \phi (K^+ K^-)$ can be written generically as follows:

$$f(\Theta', \Theta'', \chi; t) = \sum_k \mathcal{O}^{(k)}(t) g^{(k)}(\Theta', \Theta'', \chi),$$

where we have denoted the angles describing the kinematics of the decay products of $J/\psi \to l^+ l^-$ and $\phi \to K^+ K^-$ by $\Theta'$, $\Theta''$ and $\chi$. The functions $\mathcal{O}^{(k)}(t)$ describe the time-evolution of the angular distribution (1), and can be expressed in terms of real or imaginary parts of bilinear combinations of decay amplitudes. In the case of decays into two vector mesons, such as $B^0_s \to J/\psi \phi$, it is convenient to introduce linear polarization amplitudes $A_0(t)$, $A_\parallel(t)$ and $A_\perp(t)$ \cite{92}. Whereas $A_\perp(t)$ describes a CP-odd final-state configuration, both $A_0(t)$ and $A_\parallel(t)$ correspond to CP-even final-state configurations, i.e. to the CP eigenvalues $-1$ and $+1$, respectively. The $\mathcal{O}^{(k)}(t)$ of the corresponding angular distribution are given by

$$|A_f(t)|^2 \quad \text{with} \quad f \in \{0, ||, \perp\},$$

as well as by the interference terms

$$\text{Re}\{A_0^*(t)A_\parallel(t)\} \quad \text{and} \quad \text{Im}\{A_f^*(t)A_\perp(t)\} \quad \text{with} \quad f \in \{0, ||\}.$$ \hspace{1cm} (3)

These quantities are governed by

$$\xi^{(s)}_{\psi \phi} \propto e^{-i\phi_s} \left[ \frac{\lambda^{(s)*}_u A_{nt}^{\perp} + \lambda^{(s)*}_c (A_{cc}^{\perp} + A_{ct}^{\perp})}{\lambda^{(s)}_u A_{nt}^{\perp} + \lambda^{(s)}_c (A_{cc}^{\perp} + A_{ct}^{\perp})} \right],$$

where we have used the unitarity of the CKM matrix, the $\lambda^{(s)}_q$ are given by $V_{qs}V^{*}_{qb}$, and $A_{nt}^{\perp}$ and $A_{ct}^{\perp}$ denote the differences of penguin topologies with internal up- and top-quark and charm- and top-quark exchanges, respectively. The $A_{nt}^{\perp}$ pieces are strongly CKM-suppressed by $|\lambda^{(s)}_u / \lambda^{(s)}_c| \approx 0.02$; the penguin amplitudes are suppressed even further because of their loop and colour structure. Yet, the “current–current” amplitudes are “colour-suppressed”, and we may well have

$$\frac{|\lambda^{(s)}_u A_{nt}^{\perp}|}{|\lambda^{(s)}_c (A_{cc}^{\perp} + A_{ct}^{\perp})|} = \mathcal{O}(10^{-3}),$$

yielding

$$\xi^{(s)}_{\psi \phi} \propto e^{-i\phi_s} \left[ 1 - 2i \sin \gamma \times \mathcal{O}(10^{-3}) \right].$$

(6)
Since $\phi_s$ is of $\mathcal{O}(0.03)$ in the SM, there may well be hadronic uncertainties as large as $\mathcal{O}(10\%)$ in the extraction of $\phi_s$. These hadronic uncertainties, which are an important issue for the LHC, can be controlled with the help of the decay $B_d \to J/\psi \rho^0$ [93]. Moreover, the angular distribution of this decay allows one to determine both $\sin \phi_d$ and $\cos \phi_d$, i.e. to fix $\phi_d$ unambiguously, and to extract $\gamma$, if penguin effects in $B_d \to J/\psi \rho^0$ are sizeable. An unambiguous determination of the $B_d^0 - \bar{B}_d^0$ mixing phase $\phi_d$ is also possible by combining the $B_d^+ \to J/\psi \phi$ observables with those of the decay $B_d \to J/\psi (l^+ l^-) K^{*0}(\pi^0 K_S^0)$ [94]; other alternatives can be found in [95]. For simplicity, we assume $\xi_{00}^{(s)} \propto e^{-i\phi_s}$ in the following discussion, i.e. that the $B_d^0 \to J/\psi \phi$ decay amplitudes do not involve a CP-violating weak phase, which implies vanishing direct CP violation; the question of the hadronic uncertainties affecting (6) is left for further studies.

### 4.12 Time-Evolution of the Decay Probability Functions

For our considerations, the time-evolution of the decay probability functions specified in (2) and (3) plays a central rôle. In the case of (2), we obtain (see also [89])

\[
|A_0(t)|^2 = \frac{|A_0(0)|^2}{2} \left[ (1 + \cos \phi_s) e^{-\Gamma_0^{(0)} t} + (1 - \cos \phi_s) e^{-\Gamma_0^{(0)} t} + 2 e^{-\Gamma_0^{(0)} t} \sin(\Delta M_s t) \sin \phi_s \right],
\]

(7)

\[
|A_{\parallel}(t)|^2 = \frac{|A_{\parallel}(0)|^2}{2} \left[ (1 + \cos \phi_s) e^{-\Gamma_{\parallel} t} + (1 - \cos \phi_s) e^{-\Gamma_{\parallel} t} + 2 e^{-\Gamma_{\parallel} t} \sin(\Delta M_s t) \sin \phi_s \right],
\]

(8)

\[
|A_{\perp}(t)|^2 = \frac{|A_{\perp}(0)|^2}{2} \left[ (1 - \cos \phi_s) e^{-\Gamma_0^{(0)} t} + (1 + \cos \phi_s) e^{-\Gamma_0^{(0)} t} - 2 e^{-\Gamma_0^{(0)} t} \sin(\Delta M_s t) \sin \phi_s \right],
\]

(9)

whereas we have in the case of the interference terms (3):

\[
\text{Re}\{A_0^* (t) A_{\parallel} (t)\} = \frac{1}{2} |A_0(0)| |A_{\parallel}(0)| \cos(\delta_2 - \delta_1) \\
\times \left[ (1 + \cos \phi_s) e^{-\Gamma_{\parallel} t} + (1 - \cos \phi_s) e^{-\Gamma_{\parallel} t} + 2 e^{-\Gamma_{\parallel} t} \sin(\Delta M_s t) \sin \phi_s \right]
\]

(10)

\[
\text{Im}\{A_0^* (t) A_{\parallel} (t)\} = |A_{\parallel}(0)| |A_{\perp}(0)| \left[ e^{-\Gamma_{\parallel} t} \{ \sin \delta_1 \cos(\Delta M_s t) - \cos \delta_1 \sin(\Delta M_s t) \cos \phi_s \} \\
- \frac{1}{2} \left( e^{-\Gamma_0^{(0)} t} - e^{-\Gamma_{\parallel} t} \right) \cos \delta_1 \sin \phi_s \right],
\]

(11)

\[
\text{Im}\{A_0^* (t) A_{\perp} (t)\} = |A_0(0)| |A_{\perp}(0)| \left[ e^{-\Gamma_0^{(0)} t} \{ \sin \delta_2 \cos(\Delta M_s t) - \cos \delta_2 \sin(\Delta M_s t) \cos \phi_s \} \\
- \frac{1}{2} \left( e^{-\Gamma_0^{(0)} t} - e^{-\Gamma_{\parallel} t} \right) \cos \delta_2 \sin \phi_s \right].
\]

(12)

Here the CP-conserving strong phases $\delta_1$ and $\delta_2$ are defined as follows [88]:

\[
\delta_1 \equiv \text{arg}\left\{ A_{\parallel}(0) A_{\perp}^*(0) \right\}, \quad \delta_2 \equiv \text{arg}\left\{ A_0(0) A_{\perp}^*(0) \right\}.
\]

(13)

The time-evolutions (7)–(12) generalize those given in [88] to the case of a sizeable $B_d^0 - \bar{B}_d^0$ mixing phase $\phi_s$, thereby allowing one to include also new-physics effects [89]; an even more generalized formalism, taking into account also penguin contributions, can be found in [93]. It should be noted that new physics is expected to manifest itself only in the decay probability functions $\mathcal{O}^{(k)}(t)$ and that the form of the $g^{(k)}(\Theta', \Theta'', \chi)$ is not affected.

Since the meson content of the $J/\psi \phi$ final states is independent of the flavour of the initial meson, $B_d^0$ or $\bar{B}_d^0$, we may use the same angles $\Theta', \Theta''$ and $\chi$ to describe the kinematics of the decay products of the CP-conjugate transition $\bar{B}_d^0 \to J/\psi \phi$. Consequently, we have

\[
\mathcal{f}(\Theta', \Theta'', \chi; t) = \sum_k \mathcal{O}^{(k)}(t) g^{(k)}(\Theta', \Theta'', \chi).
\]

(14)
The full angular distribution of $B_s^0 \to J/\psi \phi$ involves three physical angles. The convention used is as follows (see Fig. 29): the $z'$ ($z''$)-axis is defined to be the direction of $p_{J/\psi}$ ($p_\phi$) in the rest frame of the $B_s^0$. Let the $x'$ ($x''$)-axis be any arbitrarily fixed direction in the plane normal to the $z'$ ($z''$) axis. The $y'$ ($y''$)-axis is then fixed uniquely. Let $(\Theta', \varphi')$ specify the direction of the $\ell^+$ in the $J/\psi$ rest frame, and let $(\Theta'', \varphi'')$ be the direction of the $K^+$ in the $\phi$ rest frame. Since the orientation of the $x'$ and $x''$ axes is a matter of convention, only the combination $\chi \equiv \varphi' + \varphi''$ of the two azimuthal angles is physical. The full angular distribution in terms of the three physical angles $(\Theta', \Theta'', \chi)$ (normalized such that $\Gamma = |A_0(t)|^2 + |A_1(t)|^2 + |A_\perp(t)|^2$) is given by

$$W^+(\Omega, t) = \frac{d^3 \Gamma}{d \cos \Theta' d \cos \Theta'' d \chi} = \frac{9}{64 \pi} \left\{ 4 |A_0(t)|^2 \sin^2 \Theta' \cos^2 \Theta'' \right\}$$
The SM expectation (12). The angular distribution of the SB–LR model favours a small CP asymmetry in de viate from the SM expectation of very small CP-violating effects. In the case of the latter modes, the B sector asymmetries of which are decays into CP eigenstates with CP eigenvalue 0%, which is in agreement at the 2σ level with the CDF measurement 0.73 ± 0.21 [99].

4.14 An Illustration of New-Physics Effects

As we have already noted, a very important feature of $B_s \rightarrow J/\psi \phi$ decays is that they represent a sensitive probe for CP-violating contributions to $B_s^0$-$\bar{B}_s^0$ mixing from physics beyond the SM. Let us illustrate these effects in this subsection, where we shall follow closely Ref. [91], for a particular scenario of new physics, the symmetrical $SU_L(2) \times SU_R(2) \times U(1)$ model with spontaneous CP violation (SB–LR) [96, 97]. Needless to note that there are also other scenarios for physics beyond the SM which are interesting in this respect, for example models allowing mixing to a new isosinglet down quark, as in Eq. [90].

In a recent paper [98], the SB–LR model has been investigated in the light of current experimental constraints from $K$- and B decay observables. In a large region of parameter space, the model mainly affects neutral-meson mixing, but does not introduce sizeable “direct” CP violation. The sensitive observables constraining the model are thus the meson mass difference in the kaon sector $\Delta M_K$, those in the B sector $\Delta M_d$, $\Delta M_s$, the “indirect” CP-violating parameter $\epsilon_K$ of the neutral kaon system, and the mixing-induced CP asymmetry $A_{CP}^{mix}(B_d \rightarrow J/\psi K_S)$. In particular, it was found that, for a set of fixed CKM parameters and quark masses, the model predicts a small value for $|A_{CP}^{mix}(B_d \rightarrow J/\psi K_S)|$ below 10%, which is in agreement at the 2σ level with the CDF measurement 0.73 ± 0.21 [99].

As was pointed out in [91], the SB–LR model predicts also values for the mixing-induced CP asymmetries of $B_s \rightarrow J/\psi \phi$ and similar modes, such as $B_s \rightarrow D_s^+ D_s^-$ and $J/\psi \eta(\prime)$, that largely deviate from the SM expectation of very small CP-violating effects. In the case of the latter modes, which are decays into CP eigenstates with CP eigenvalue +1, we simply have

$$A_{CP}^{mix}(B_s \rightarrow f) = \sin \phi_s, \quad \phi_s \equiv \phi_{s}^{SM} + \phi_{s}^{NP} = -2\lambda^2 \eta + \phi_{s}^{NP},$$

with $\phi_{s}^{NP}$ originating from new physics. In Fig. 30(a), we show the allowed region for $A_{CP}^{mix}(B_s \rightarrow f) = \sin \phi_s$ and $A_{CP}^{mix}(B_d \rightarrow J/\psi K_S)$ in the SB–LR model; the corresponding direct CP asymmetries remain very small, since new contributions to the decay amplitudes are strongly suppressed. The figure illustrates nicely that CP asymmetries as large as $O(40\%)$ may arise in the $B_s$ channels, whereas the SB–LR model favours a small CP asymmetry in $B_d \rightarrow J/\psi K_S$.

In order to simplify the discussion of $B_s \rightarrow J/\psi \phi$, let us consider the CP asymmetry

$$A_{CP}(B_s(t) \rightarrow J/\psi \phi) \equiv \frac{\Gamma(t) - \bar{\Gamma}(t)}{\Gamma(t) + \bar{\Gamma}(t)} = \frac{1 - D}{F_1(t) + DF_2(t)} \sin(\Delta M_s t) \sin \phi_s,$$

where $\Gamma(t)$ and $\bar{\Gamma}(t)$ denote the time-dependent rates for decays of initially, i.e. at $t = 0$, present $B_s^0$- and $\bar{B}_s^0$ mesons into $J/\psi \phi$ final states, respectively. The remaining quantities are defined as

$$D \equiv \frac{|A_{\perp}(0)|^2}{|A_{\parallel}(0)|^2 + |A_{\parallel}(0)|^2}, \quad \text{and} \quad F_\pm(t) \equiv \frac{1}{2} \left[ (1 \pm \cos \phi_s) e^{+\Delta \Gamma_{\phi} t/2} + (1 \mp \cos \phi_s) e^{-\Delta \Gamma_{\phi} t/2} \right].$$
Note that we have $F_+(t) = F_-(t) = 1$ for a negligible width difference $\Delta \Gamma_s$. Obviously, the advantage of the “integrated” observable (23) is that it can be measured without performing an angular analysis. The disadvantage is of course that it also depends on the hadronic quantity $D$, which precludes a theoretically clean extraction of $\phi_s$ from (23). However, this feature does not limit the power of this CP asymmetry to search for indications of new physics, which would be provided by a measured sizeable value of (23).

Model calculations of $D$, making use of the factorization-hypothesis, typically give $D = 0.1 \ldots 0.5$ [88], which is also in agreement with a recent analysis of the $B_s \to J/\psi \phi$ polarization amplitudes performed by the CDF collaboration [86]. In order to extract $\phi_s$ from CP-violating effects in the decay $B_s \to J/\psi \phi$ in a theoretically clean way, an angular analysis has to be performed, as is discussed in detail above.

Although the $B_s^0 - \bar{B}_s^0$ oscillations are very rapid, it should be possible to resolve them at the LHC (see Sec. 7.). The first extremal value of the time-dependent CP asymmetry (23), corresponding to $\Delta M_s t = \pi/2$, is given to a very good approximation by

$$A_{\text{CP}}(B_s \to J/\psi \phi) = \left(1 - \frac{D}{1 + D}\right) \sin \phi_s,$$

which would also fix the magnitude of (23) in the case of a negligible width difference $\Delta \Gamma_s$. In Fig. 30(b), we show the prediction of the SB–LR model for (25) as a function of the hadronic parameter $D$. For a value of $D = 0.3$, this CP asymmetry may be as large as $-25\%$. The dilution through the hadronic parameter $D$ is not effective in the case of the CP-violating observables of the $B_s \to J/\psi [-\to t^+ t^-] \phi [-\to K^+ K^-]$ angular distribution, which allow one to probe $\sin \phi_s$ directly (see Sec. 4.12). Predictions for other $B_s$ decays in the SB–LR model have been discussed in [100].

Let us finally note that new physics affects also the $B_s^0 - \bar{B}_s^0$ mass and width differences. In the latter case, we have [101]

$$\Delta \Gamma_s = \Delta \Gamma_s^{\text{SM}} \cos \phi_s,$$

where $\Delta \Gamma_s^{\text{SM}} = \mathcal{O}(-15\%)$ is the SM width difference [23, 24]. In Fig. 30(c), we show the correlation between $\Delta M_s$ and $\Delta \Gamma_s$ in the SB–LR model. The reduction of $\Delta \Gamma_s$ through new-physics effects, which is described by (26), is fortunately not very effective in this case, whereas the mass difference $\Delta M_s$ may be reduced significantly.

### 4.2 Experimental Studies

The prospective performance of ATLAS, CMS and LHCb in analysing $B_s^0 \to J/\psi (\mu^+ \mu^-) \phi (K^+ K^-)$ has been studied in [37, 103, 104, 102].

#### 4.21 Expected Data Characteristics

Despite different strategies, all three experiments expect a large number of events in this channel. With present studies the highest yield is expected in CMS, where a dimuon trigger is used. At higher trigger-
level the identification of two muons is essential for \( J/\psi \rightarrow \mu^+\mu^- \) online selection in all three experiments. The reconstruction is completed in tracking and vertex detectors by fitting muon candidate trajectories into a common vertex. For reconstructing \( \phi \) mesons, pairs of oppositely charged particles are fitted into a common vertex and their invariant mass is calculated assuming they are kaons. In LHCb, the RICHes are used to separate charged K mesons from \( \pi \) mesons, allowing a reduction of the backgrounds to \( B_s^0 \rightarrow J/\psi \phi \). As explained in Sec. 2.5, there is a limited possibility of charged hadron identification in both ATLAS and CMS; however this has not been exploited in the present studies. The stronger solenoidal field in CMS leads to better \( \mu \) invariant mass resolution and lower \( \beta \) background than in ATLAS. For this channel, the most significant difference between the performance of the three experiments lies in the superior proper time resolution of LHCb. The expected data and background-characteristics in \( B_s^0 \rightarrow J/\psi (\mu^+\mu^-)\phi(K^+K^-) \), as presented in the workshop, are summarized in Tab. 14. It is possible that the inclusion of low-threshold dimuon-triggers may also boost the final event yields in ATLAS and LHCb, as has been demonstrated to be the case in CMS.

Flavour tagging is important to properly explore the physics of \( B_s^0 \rightarrow J/\psi \phi \) decays. This study considers only lepton and charged K mesons tags for LHCb and a jet charge method for ATLAS and CMS (see Sec. 2.7). CMS are presently extending their study to include other tags. The efficiencies and the wrong tag fractions in this channel are summarized in Tab. 14.

The studies presented here do not exhaust the whole potential of the three experiments. Future studies can be extended in trigger and offline selections as well as in tagging methods.

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<tr>
<th>Parameter</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Event yields</td>
<td>300,000</td>
<td>600,000</td>
<td>370,000</td>
</tr>
<tr>
<td>Proper time resolution</td>
<td>0.063 ps</td>
<td>0.063 ps</td>
<td>0.031 ps</td>
</tr>
<tr>
<td>Background</td>
<td>~ 15%</td>
<td>~ 10%</td>
<td>~ 3%</td>
</tr>
<tr>
<td>( B_s^0 \rightarrow J/\psi K^*, J/\psi K^+\pi^- )</td>
<td>dominated by</td>
<td>dominated by</td>
<td>combinatorial</td>
</tr>
<tr>
<td>Tagging</td>
<td>jet charge tag</td>
<td>jet charge tag</td>
<td>lepton tag + charged K tag</td>
</tr>
<tr>
<td>( \epsilon \sim 63% )</td>
<td>( \epsilon \sim 32% )</td>
<td>( \epsilon \sim 40% )</td>
<td></td>
</tr>
<tr>
<td>wrong 38%</td>
<td>wrong 33%</td>
<td>wrong 30%</td>
<td></td>
</tr>
</tbody>
</table>

Table 14: Summary of performance parameters for \( B_s^0 \rightarrow J/\psi (\mu^+\mu^-)\phi(K^+K^-) \). The proper time resolutions have been determined by a single Gaussian fit. The event yields assume 3 years operation for ATLAS & CMS, and 5 years for LHCb.

4.22 Modelling \( B_s^0 \rightarrow J/\psi \phi \) Decays

The distribution (21) contains eight unknown independent parameters. These are the amplitudes \( |A_1(0)|, |A_\perp(0)| \), the relative strong phases \( \delta_1 \) and \( \delta_2 \), the decay rate difference, \( \Delta \Gamma_s = \Gamma_H - \Gamma_L \), and mean decay rate \( \Gamma_s = (\Gamma_H + \Gamma_L)/2 \) of the mass eigenstates \( B_s^0_H \) and \( B_s^0_L \), their mass difference \( \Delta M_s = m_s/\Gamma_s \) and the weak phase \( \phi_s \). These parameters can be determined from the measured three decay angles and lifetimes. In the workshop two strategies were studied: the method of moments approach [104] and a maximum-likelihood fit.

In the method of moments approach [88], the terms bilinear in \( A \) in (21) are determined from the data using an appropriate set of weighting functions, which separate out the terms with different angular dependences. The question of information-content loss in the angular moments analysis was investigated in [105]. For the results presented in this report, the likelihood-approach is adopted.
4.23 Parameter Determination and Estimate of Precision

The expected experimental precision is not sufficient to allow a simultaneous determination of eight unknown parameters. Besides the limited statistics, the correlations between different parameters pose a problem: while in (21) the eight parameters are independent, simulations with the maximum-likelihood approach showed that in the experimental data some of the parameters have obvious correlations. There is a strong correlation between the two phases $\delta_1$ and $\delta_2$, which precludes a simultaneous measurement of both of them with this method. Also a second pair of parameters, $\Delta M_s$ and the weak phase $\phi_s$, shows certain correlations that depend on the values of $\Delta M_s$ and the time resolution. Consequently, the reduced set of parameters: $\Delta \Gamma_s$, $\Gamma_s$, $|A_{||}(0)|$, $|A_{\perp}(0)|$ and $\phi_s$ were determined in the fit and the other parameters were fixed. For the strong phases the values $\delta_1 = 0$ and $\delta_2 = \pi$ were used as suggested in Ref. [106]. $\Delta M_s$ is assumed to be determined from other channels, for instance $B_s^0 \rightarrow D_s \pi$, although it should be stressed that also $B_s^0 \rightarrow J/\psi \phi$ is a very suitable channel for such a measurement.

The choice of input values of the unknown parameters, both fixed and free, based on the experimental results [107, 86, 64] and theoretical considerations [88, 106, 108] is summarized in Tab. 15.

The main results of the study are summarized in Tab. 16 for each experiment. With this method, the rate difference $\Delta \Gamma_s$ can be determined with a relative statistical error which for LHCb, CMS and ATLAS varies between 8 and 12% for $\Delta \Gamma_s / \Gamma_s = 0.15$, Fig. 31(a). The differences between the experiments are

Note that this suggestion is based on the factorization approximation and not expected to hold once non-factorizable contributions are taken into account.

| Parameter | $|A_{||}(0)|^2$ | $|A_{\perp}(0)|^2$ | $\delta_1$ | $\delta_2$ | $\Delta \Gamma_s$ | $1/\Gamma_s$ | $x_s$ | $\phi_s$ |
|-----------|----------------|-----------------|-------------|-------------|----------------|-------------|--------|---------|
| Value     | 0.64           | 0.14            | 0           | $\pi$       | 0.15 $\times \Gamma_s$ | 1.54 ps     | 20–40  | 0.04–0.8 |

Table 15: Input values of theory parameters used in simulating $B_s^0 \rightarrow J/\psi \phi$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \Gamma_s$</td>
<td>12%</td>
<td>8%</td>
<td>9%</td>
</tr>
<tr>
<td>$\Gamma_s$</td>
<td>0.7%</td>
<td>0.5%</td>
<td>0.6%</td>
</tr>
<tr>
<td>$A_{</td>
<td></td>
<td>}$</td>
<td>0.8%</td>
</tr>
<tr>
<td>$A_{\perp}$</td>
<td>3%</td>
<td>2%</td>
<td>2%</td>
</tr>
<tr>
<td>$\phi_s$ ($x_s = 20$)</td>
<td>0.03</td>
<td>0.014</td>
<td>0.02</td>
</tr>
<tr>
<td>$\phi_s$ ($x_s = 40$)</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 16: Expected statistical uncertainties on $B_s^0 \rightarrow J/\psi \phi$ parameters for each experiment under the assumptions given in the text. Apart from $\phi_s$, the errors are relative.
Fig. 31: Expected relative error on $\Delta \Gamma_s$ from $B_s^0 \rightarrow J/\psi \phi$. (a) Estimate of the relative error of $\Delta \Gamma_s$ as a function of signal-statistics for several values of background. The background is expected to be between 4% and 15%. (b) Relative error of $\Delta \Gamma_s$ as a function of the relative precision of the lifetime measurement $\delta t / t$ for two values of $\Delta \Gamma_s / \Gamma_s$. LHCb expects $\delta t / t = 2.2\%$, ATLAS/CMS 4.4%. A background of 15% and statistics of 300,000 events is assumed.

Fig. 32: The $x_s - \phi_s$ region allowed in the SM, the left-right symmetric model with spontaneous CP violation (NP-LR) and the iso-singlet down quark mixing model (NP-DQ). Also shown is the region of experimental sensitivity of ATLAS and CMS, corresponding to 63 fs, and of LHCb with 31 fs. The NP–LR allowed region appears smaller than that of the SM, because it does not include all theory uncertainties.
small, mainly because the error is not sensitive to the different proper time precision of each experiment, Fig. 31(b). The statistical errors of $\Gamma_s$, $|A_u(0)|$ and $|A_\perp(0)|$ are typically a few percent. The precision to which the weak phase $\phi_s$ can be measured depends strongly on the proper time resolution and $x_s$ (Fig. 32). There is a sensitivity to the SM range of $\phi_s$, and a clear potential for probing models of new physics, such as for instance the left-right symmetric model [91] or the isosinglet down quark model [90]. If penguin contributions are non-negligible, the number of parameters will increase, which will necessitate simultaneous analyses of $B^0_s \to J/\psi\phi$ and SU(3) related channels as indicated earlier in the theoretical discussion. The combined LHC sensitivity to these parameters will be even better, but this study has not yet been performed.

Studies with the method of moments approach gave results broadly in agreement with the likelihood-fits, but with certain differences which are yet to be resolved. In particular, the moments analysis indicated that the strong phases can be extracted simultaneously with the other parameters through the separation of different angular terms [104]. Future work will resolve these issues.

4.3 Conclusions

A rich variety of physics can be studied through the decay $B^0_s \to \pi^\pm K^0$ and all three LHC experiments will be able to perform powerful and interesting measurements. More work is encouraged to further extend the experimental potential, in particular by improving the sensitivity to the weak mixing phase $\phi_s$, and to establish the optimum approach for analysing the data.

5. New Strategies to Extract CKM Phases

In addition to the refined studies of the usual benchmark CP modes described above, an important goal of the workshop was to explore strategies for the extraction of CKM phases that had not been considered for ATLAS, CMS and LHCb before, and to search for new strategies. In this section, we will discuss extractions of $\gamma$ from $B \to \pi K$ decays, which received a lot of attention in the literature over the last couple of years [109], and new techniques [34, 61, 93, 110], which were developed during this workshop and make use of certain U-spin related B decays, where all down and strange quarks are interchanged with each other [111]. For the “prehistory” of the use of U-spin arguments to relate non-leptonic B decays, the reader is referred to [111]–[116].

5.1 Extracting $\gamma$ from $B \to \pi K$ Decays

In order to obtain direct information on $\gamma$, $B \to \pi K$ decays are very interesting. These modes are not just an “unwanted” background for $B \to \pi\pi$, but have a very interesting physics potential. Experimental

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*Section coordinators: R. Fleischer and G. Wilkinson.*
data for these modes start to become available: since 1997, when the first results on $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ were reported by the CLEO collaboration, there were several updated results for CP-averaged $B \to \pi K$ branching ratios at the $10^{-5}$ level [117]. Interestingly, these CP-averaged branching ratios may already lead to highly non-trivial constraints on $\gamma$ [118, 119]. Unfortunately, the present experimental uncertainties are too large to decide how effective these bounds actually are. The new results of the $e^+e^-$ B factories will certainly improve this situation, so that we should have a much better picture by the time the LHC starts running. In 1999, also the first preliminary results for CP-violating asymmetries in charmless hadronic B meson decays were reported by the CLEO collaboration [117], which do not yet indicate CP violation in such transitions. So far, to probe $\gamma$, the following three combinations of $B \to \pi K$ decays were considered in the literature: $B^\pm \to \pi^\pm K$ and $B_d \to \pi^\mp K^\pm$ [118, 120, 121], $B^\pm \to \pi^\pm K$ and $B^\mp \to \pi^0 K^\pm$ [31, 119, 122, 68], as well as the combination of the neutral decays $B_d \to \pi^0 K_S$ and $B_d \to \pi^\mp K^\pm$ [68]. Since the first combination does not involve a neutral pion, it is particularly promising for the LHC from an experimental point of view, although the other two combinations would have certain advantages from a theoretical point of view. The experimental feasibility studies therefore put a strong emphasis on that approach. Let us note, before having a closer look at this strategy, that $B \to \pi K$ decays play not only an important rôle to probe $\gamma$, but also to obtain insights into the world of electroweak penguins. This interesting aspect is discussed in more detail in Refs. [17, 120, 68, 123].

5.11 The $B^\pm \to \pi^\pm K, B_d \to \pi^\mp K^\pm$ Strategy

Within the framework of the SM, the decays $B^+ \to \pi^+ K^0$ and $B_d^0 \to \pi^- K^+$ receive contributions from Feynman diagrams of the type shown in Figs. 33 and 34, respectively. Because of the tiny ratio $|V_{us}V_{ub}^*|/|V_{ts}V_{tb}^*| \approx 0.02$, the QCD penguins play the dominant rôle in these decays, despite their loop-suppression. Using the SU(2) isospin symmetry of strong interactions to relate QCD penguin topologies, we may derive the following amplitude relations [114]:

$$A(B^+ \to \pi^+ K^0) \equiv P, \quad A(B_d^0 \to \pi^- K^+) = \left[ P + T + P_{ew}^C \right],$$

where

$$T \equiv |T|e^{i\delta_T}e^{i\gamma} \quad \text{and} \quad P_{ew}^C \equiv - |P_{ew}^C|e^{i\delta_{ew}^C}$$

are due to tree-diagram-like topologies and EW penguins, respectively. The label “C” reminds us that only “colour-suppressed” EW penguin topologies contribute to $P_{ew}^C$. Making use of the unitarity of the CKM matrix and applying the generalized Wolfenstein parametrization, including non-leading terms in $\lambda$, we obtain [114]

$$A(B^+ \to \pi^+ K^0) = - \left( 1 - \frac{\lambda^2}{2} \right) \lambda^2 A \left[ 1 + \rho e^{i\theta}e^{i\gamma} \right] \mathcal{P}_{\text{le}},$$

and

$$A(B_d^0 \to \pi^- K^+) = \left[ P + T + P_{ew}^C \right] - \left( 1 - \frac{\lambda^2}{2} \right) \lambda^2 A \left[ 1 + \rho e^{i\theta}e^{i\gamma} \right] \mathcal{P}_{\text{le}}.$$

Fig. 34: Feynman diagrams contributing to $B_d^0 \to \pi^- K^+$. 
where

$$\rho e^{i\theta} = \left( \frac{\lambda^2 R_b}{1 - \lambda^2} \right) \left[ 1 - \left( \frac{P_{ac}}{P_{tc}} \right) \right].$$  \hspace{1cm} (4)

Here $P_{tc} \equiv |P_{tc}|e^{i\delta_{tc}}$ and $P_{ac}$ describe differences of penguin topologies with internal top- and charm-quark and up- and charm-quark exchanges, respectively, and $A$ is due to the annihilation topology in Fig. 33. It is important to note that $\rho$ is strongly CKM-suppressed by $\lambda^2 R_b \approx 0.02$. For the parametrization of $B^\pm \rightarrow \pi^\pm K$ and $B_d \rightarrow \pi^\mp K^\pm$ observables, it is convenient to introduce

$$r \equiv \frac{|T|}{\sqrt{|P|^2}}, \quad \epsilon_C \equiv \frac{|P_{cw}^C|}{\sqrt{|P|^2}} \quad \text{with} \quad \langle |P|^2 \rangle \equiv \frac{1}{2} \left( |P|^2 + |\bar{P}|^2 \right),$$  \hspace{1cm} (5)

as well as the strong phase differences

$$\delta \equiv \delta_T - \delta_{tc}, \quad \Delta_C \equiv \delta_{cw}^C - \delta_{tc}.$$  \hspace{1cm} (6)

In addition to the ratio

$$R \equiv \frac{B(B_d^0 \rightarrow \pi^- K^+) + B(B_d^0 \rightarrow \pi^+ K^-)}{B(B^+ \rightarrow \pi^+ K^0) + B(B^- \rightarrow \pi^- K^0)},$$  \hspace{1cm} (7)

of CP-averaged branching ratios, also the “pseudo-asymmetry”

$$A_0 \equiv \frac{B(B_d^0 \rightarrow \pi^- K^+) - B(B_d^0 \rightarrow \pi^+ K^-)}{B(B^+ \rightarrow \pi^+ K^0) + B(B^- \rightarrow \pi^- K^0)}$$  \hspace{1cm} (8)

plays an important rôle in probing $\gamma$. Here, we have neglected tiny phase-space effects, which can, however, be taken into account in a straightforward way (see [118]). Explicit expressions for $R$ and $A_0$ in terms of the parameters specified above are given in [114]. Using the presently available experimental results from the CLEO collaboration [117], we obtain

$$R = 1.0 \pm 0.3, \quad A_0 = 0.04 \pm 0.18.$$  \hspace{1cm} (9)

The pseudo-asymmetry $A_0$ allows one to eliminate the strong phase $\delta$ in the expression for $R$, and to fix contours in the $\gamma - r$ plane [114]. These contours, which are illustrated in Fig. 35, correspond to the mathematical implementation of a simple triangle construction [120], which is related to the amplitude relation (1), and is shown in Fig. 36. In order to determine $\gamma$, the quantity $r$, i.e. the magnitude of the “tree” amplitude $T$, has to be fixed. At this stage, a certain model dependence enters. An approximate way to fix this amplitude is to neglect “colour-suppressed” current–current operator contributions.
Fig. 36: Triangle construction to determine \( \gamma \) from the \( B_d \to \pi^\pm K^\mp \), \( B^\pm \to \pi^\pm K \) system in the case of \( \rho = \epsilon_C = 0 \). Here we have \( A \equiv A(B_d^0 \to \pi^- K^+) \) and \( \overline{A} \equiv A(B_d^+ \to \pi^+ K^-) \); note that \( \rho = 0 \) implies \( P = \overline{P} \equiv A(B^- \to \pi^- K^0) \).

To \( B^+ \to \pi^+ \pi^0 \), and to use SU(3) flavour-symmetry to relate the “colour-allowed” current–current amplitude of that decay to \( T \):

\[
|T| \approx \lambda \frac{f_K}{f_\pi} |A(B^+ \to \pi^+ \pi^0)|.
\]

(10)

Another approach to obtain information on \( |T| \) is to use “factorization” \[124\], leading to

\[
|T|_{\text{fact}} = \frac{G_F}{\sqrt{2}} \lambda |V_{ab}| a_1 \left( M_{B_d^0}^2 - M_{\pi}^2 \right) f_K F_{B^+} \left( M_{K^*}^2, 0^+ \right),
\]

where \( F_{B^+} \) is a quark–current form factor and \( a_1 \approx 1 \) the usual phenomenological colour factor. Using the form factor \( F_{B^+} \left( M_{K^*}^2, 0^+ \right) = 0.3 \), as obtained e.g. from QCD sum rules on the light-cone \[125, 126\], one finds

\[
|T|_{\text{fact}} = a_1 \times \left[ \frac{|V_{ab}|}{3.2 \times 10^{-3}} \right] \times 7.8 \times 10^{-9} \, \text{GeV}.
\]

(12)

As was pointed out in \[121\], also semileptonic \( B^0 \to \pi^- l^+ \nu_l \) decays may play an important rôle to fix \( |T| \) with the help of arguments based on “factorization”. Using (11), one finds \[118\]

\[
\tau_{\text{fact}} = 0.18 \times a_1 \times \left[ \frac{|V_{ab}|}{3.2 \times 10^{-3}} \right] \sqrt{\frac{1.8 \times 10^{-5}}{B(B^\pm \to \pi^\pm K)}} \times \left[ \frac{\tau_{B_u}}{1.6 \, \text{ps}} \right].
\]

(13)

Making use of such arguments based on “factorization”, present data give \( \tau = 0.18 \pm 0.05 \). Although the factorization-hypothesis \[124\] may work reasonably well for “colour-allowed” tree-diagram-like topologies \[127\], \( T \) may be shifted from its “factorized” value, as the properly defined amplitude \( T \) does not only receive contributions from such “tree” topologies, but also from penguin and annihilation processes \[114, 113\], which are strongly related to rescattering processes \[113, 128, 129\]. In an interesting recent paper by Beneke, Buchalla, Neubert and Sachrajda \[76\], it was pointed out that there is a heavy-quark expansion for non-leptonic \( B \) decays into two light mesons, and that non-factorizable corrections, as well as rescattering processes, are suppressed by \( \Lambda_{\text{QCD}}/m_b \). This approach may turn out to be useful to fix the parameter \( r \), which is required in order to determine \( \gamma \) from \( B_d \to \pi^\mp K^\pm \), \( B^\pm \to \pi^\mp K \) decays.

Interestingly, it is possible to derive bounds on \( \gamma \) that do not depend on \( r \) at all \[118\]. To this end, we eliminate \( \delta \) in \( \tilde{R} \) through \( A_0 \). If we now treat \( r \) as a “free” variable, we find that \( \tilde{R} \) takes the minimal value \[114\]

\[
R_{\text{min}} = \kappa \sin^2 \gamma + \frac{1}{\kappa} \left( \frac{A_0}{2 \sin \gamma} \right)^2 \geq \kappa \sin^2 \gamma,
\]

(14)

where

\[
\kappa = \frac{1}{w^2} \left[ 1 + 2 \left( \epsilon_C \cdot w \right) \cos \Delta_C + \left( \epsilon_C \cdot w \right)^2 \right] \text{ with } w = \sqrt{1 + 2 \rho \cos \theta \cos \gamma + \rho^2}.
\]

(15)

The inequality in (14) arises if we keep both \( r \) and \( \delta \) as free parameters \[118\]. \( R_{\text{min}} \) restricts the allowed range for \( \gamma \), since values of \( \gamma \) implying \( R_{\text{exp}} < R_{\text{min}} \) are excluded. In particular, \( A_0 \neq 0 \) would allow one to exclude a certain range of \( \gamma \) around \( 0^\circ \) or \( 180^\circ \), whereas a measured value of \( R < 1 \) would exclude a certain range around \( 90^\circ \), which would be of great phenomenological importance. The first
results reported by CLEO in 1997 gave \( R = 0.65 \pm 0.40 \) and led to great excitement, whereas the most recent update is the one given in (9). If the parameter \( r \) is fixed, significantly stronger constraints on \( \gamma \) can be obtained from the observable \( R \) \cite{68, 69}. In particular, these constraints require only \( R \neq 1 \) and are also effective for \( R > 1 \).

The theoretical accuracy of the strategies to probe \( \gamma \) through the \( B^\pm \to \pi^\pm K \), \( B_d \to \pi^+ K^\pm \) system is limited both by rescattering processes of the kind \( B^+ \to \{ \pi^0 K^+, \pi^0 K^{*+}, \ldots \} \to \pi^+ K^0 \) \cite{128, 129}, which are illustrated in Fig. 37, and by the “colour-suppressed” EW penguin contributions described by the amplitude \( P^C_{\text{ew}} \) \cite{121, 129}. In (14), these effects are described by the parameter \( \kappa \). If they are neglected, we have \( \kappa \to 1 \). The rescattering effects – it cannot be excluded that they may lead to values of \( \rho \) as large as \( O(0.1) \) – can be controlled in the contours in the \( \gamma - r \) plane and can be included in the constraints on \( \gamma \) related to (14) through experimental data on \( B^\pm \to K^\pm K \) decays, which are the U-spin counterparts of \( B^\pm \to \pi^\pm K \) \cite{114, 115}. Another important indicator for large rescattering effects are the \( B_d \to K^+ K^- \) modes, for which there already exist strong experimental constraints \cite{130}.

An improved description of the EW penguins is possible if we use the general expressions for the corresponding four-quark operators and perform appropriate Fierz transformations \cite{114, 120, 129}. Following these lines, we obtain

\[
q_C e^{i\omega_C} \equiv \frac{e_C}{r} e^{i(\Delta_C - \delta)} = 0.66 \times \left[ \frac{0.41}{R_b} \right] \times a_C e^{i\omega_C}, \tag{16}
\]

where \( a_C e^{i\omega_C} = a_2^{\text{eff}}/a_1^{\text{eff}} \) is the ratio of certain generalized “colour factors”. Experimental data on \( B \to D^{(*)} \pi \) decays imply \( a_2/a_1 = O(0.25) \). A first step to fix the hadronic parameter \( a_C e^{i\omega_C} \) experimentally is provided by the mode \( B^+ \to \pi^+ K^0 \) \cite{114}; interesting constraints were derived in \cite{69}. For a detailed discussion of the impact of rescattering and EW penguin effects on the strategies to probe \( \gamma \) with \( B^\pm \to \pi^\pm K \) and \( B_d \to \pi^+ K^\pm \) decays, the reader is referred to \cite{114, 115, 68, 131}. In order to control these hadronic uncertainties – in addition to the full experimental picture of all \( B \to \pi K, K \overline{\pi} \) decays – also the theoretical approach for dealing with non-leptonic B decays into two light mesons developed recently in Ref. \cite{76} may play an important rôle.

5.12 The Charged \( B^\pm \to \pi^\pm K, B^\pm \to \pi^0 K^\pm \) Strategy

Several years ago, Gronau, Rosner and London proposed an SU(3) strategy to determine \( \gamma \) from the charged decays \( B^\pm \to \pi^\pm K, \pi^0 K^\pm, \pi^0 \pi^\pm \) \cite{31}. However, as was pointed out by Deshpande and He \cite{132}, this elegant approach is unfortunately spoiled by EW penguins \cite{133}, which play an important rôle in several non-leptonic B meson decays because of the large top-quark mass \cite{15, 16}. Recently, this approach was resurrected by Neubert and Rosner \cite{119, 122}, who pointed out that in this case the EW penguin contributions can be controlled by using only the general expressions for the corresponding four-quark operators, appropriate Fierz transformations, and the SU(3) flavour-symmetry of strong interactions (see also \cite{120}).

In the case of \( B^+ \to \pi^+ K^0, \pi^0 K^+ \), SU(2) isospin symmetry implies

\[
A(B^+ \to \pi^+ K^0) + \sqrt{2} A(B^+ \to \pi^0 K^+) = - \left[ (T + C) + P_{\text{ew}} \right]. \tag{17}
\]
The phase structure of this relation is completely analogous to $B^+ \rightarrow \pi^+ K^0$, $B_d^0 \rightarrow \pi^- K^+$, as can be seen by comparing with (1) and (2):
\[ T + C = \left| T + C \right| e^{i\delta + \gamma} e^{i\gamma}, \quad P_{\text{ew}} = - |P_{\text{ew}}| e^{i\delta_{\text{ew}}} . \quad (18) \]
In order to probe $\gamma$, it is useful to introduce the following observables [68]:
\[ R_c \equiv 2 \left[ \frac{B(B^+ \rightarrow \pi^+ K^0) + B(B^- \rightarrow \pi^0 K^-)}{B(B^+ \rightarrow \pi^+ K^0) + B(B^- \rightarrow \pi^- K^0)} \right], \quad (19) \]
\[ A_0^c \equiv 2 \left[ \frac{B(B^+ \rightarrow \pi^0 K^+) - B(B^- \rightarrow \pi^0 K^-)}{B(B^+ \rightarrow \pi^+ K^0) + B(B^- \rightarrow \pi^- K^0)} \right], \quad (20) \]
which correspond to $R$ and $A_0$; general expressions can be obtained from those for $R$ and $A_0$ by the following replacements:
\[ r \rightarrow r_c \equiv \frac{|T + C|}{\sqrt{\langle |P|^2 \rangle}}, \quad \delta \rightarrow \delta_c \equiv \delta_{T + C} - \delta_{r_c}, \quad P_{\text{ew}}^c \rightarrow P_{\text{ew}}. \quad (21) \]
Using the presently available experimental results from the CLEO collaboration [117], one finds
\[ R_c = 1.3 \pm 0.5, \quad A_0^c = 0.35 \pm 0.34. \quad (22) \]
The observables $R_c$ and $A_0^c$ allow one to fix contours in the $\gamma - r_c$ plane, in complete analogy to the $B^+ \rightarrow \pi^+ K$, $B_d \rightarrow \pi^+ K^\pm$ strategy. However, the charged $B \rightarrow \pi \bar{K}$ approach has certain advantages from a theoretical point of view:
- SU(3) flavour-symmetry allows one to fix the parameter $r_c \propto |T + C|$ as follows [31]:
\[ T + C \approx - \sqrt{2} \frac{V_{us}}{V_{ud}} \frac{f_K}{f_{\pi}} A(B^+ \rightarrow \pi^+ \pi^-), \quad (23) \]
where $r_c$ thus determined is – in contrast to $r$ – not affected by rescattering effects; present data give $r_c = 0.21 \pm 0.06$. The factor $f_K / f_{\pi}$ takes into account factorizable SU(3) breaking.
- In the strict SU(3) limit, we have [119]
\[ q e^{i\omega} \equiv \left| \frac{P_{\text{ew}}}{T + C} \right| e^{i(\delta_{\text{ew}} - \delta_{r + C})} = 0.66 \times \left[ \frac{0.41}{R_0} \right], \quad (24) \]
which does – in contrast to (16) – not involve a hadronic parameter. Taking into account factorizable SU(3) breaking and using present data gives $q = 0.63 \pm 0.15$.

The contours in the $\gamma - r_c$ plane may be affected – in analogy to the $B^\pm \rightarrow \pi^\pm K$, $B_d \rightarrow \pi^\pm K^\pm$ case – by rescattering effects [68]. They can be taken into account with the help of additional experimental data [114, 115, 134], and if we use the observable
\[ B_0^c \equiv A_0^c - \left[ \frac{B(B^+ \rightarrow \pi^+ K^0) - B(B^- \rightarrow \pi^- K^0)}{B(B^+ \rightarrow \pi^+ K^0) + B(B^- \rightarrow \pi^- K^0)} \right], \quad (25) \]
instead of $A_0^c$, the terms of $O(\rho)$, which describe the rescattering effects, are suppressed by $r_c$ [131]. The major theoretical advantage of the $B^+ \rightarrow \pi^+ K^0$, $\pi^0 K^+$ strategy with respect to $B^\pm \rightarrow \pi^\pm K$, $B_d \rightarrow \pi^\pm K^\pm$ is that $r_c$ and $P_{\text{ew}} / (T + C)$ can be fixed by using only SU(3) arguments, i.e. no additional dynamical arguments have to be employed. Consequently, the theoretical accuracy is mainly limited by non-factorizable SU(3) breaking effects. The approach developed recently in [76] may help to reduce these uncertainties.
Let us finally note that the observable $R_c$ may also imply interesting constraints on $\gamma$ [119]. These bounds, which are conceptually quite similar to [118], are related to the extremal values of $R_c$ that arise if we keep only the strong phase $\delta_c$ as an “unknown” free parameter. As the resulting general expression is rather complicated [68, 131], let us expand it in $r_c$ [119]. If we keep only the leading-order terms and make use of the SU(3) relation (24), we obtain

$$R^{\text{ext}}_{c| \delta_c} = 1 \pm 2 r_c | \cos \gamma - q |.$$  

Interestingly, there are no terms of $\mathcal{O}(\rho)$ present in this expression, i.e., rescattering effects do not enter at this level [119, 122]. However, FSI processes may still have a sizeable impact on the associated bounds on $\gamma$. Several strategies to control these uncertainties were considered in the recent literature [68, 131, 134], and also the approach of Ref. [76] may shed light on these issues.

Unfortunately, the neutral pions appearing in $B^\pm \rightarrow \pi^0 K^\pm$ render the charged approach challenging from the experimental point of view. The strategy using the neutral decays $B_d \rightarrow \pi^0 K_S$ and $B_d \rightarrow \pi^\pm K^\mp$ to extract $\gamma$, which was proposed in [68], is even worse in this respect, and we will not discuss it here in more detail, although it would have an interesting theoretical advantage concerning the impact of rescattering effects.

### 5.13 Some Remarks about New Physics

Since $B^0_{q} \rightarrow B_{q}^{-}$ mixing ($q \in \{d, s\}$) is a “rare” flavour-changing neutral-current (FCNC) process, it is very likely that it is significantly affected by new physics, which may act upon the mixing parameters $\Delta M_q$ and $\Delta \Gamma_q$ as well as on the CP-violating mixing phase $\phi_q$. Important examples for such scenarios of new physics are non-minimal SUSY models, left–right-symmetric models, models with extended Higgs sectors, four generations, or $Z$-mediated FCNCs [10]. Since $B_d \rightarrow J/\psi K_S$ and $B_s \rightarrow J/\psi \phi$ – the benchmark modes to measure $\phi_d$ and $\phi_s$ – are governed by current–current, i.e. “tree”, processes, new physics is expected to affect their decay amplitudes only in a minor way. Consequently, these modes still measure $\phi_d$ and $\phi_s$.

In the clean strategies to measure $\gamma$ with the help of pure “tree” decays, such as $B \rightarrow D K$, $B_d \rightarrow D^{(*)} \pm \pi^\mp$ or $B_s \rightarrow D_{s}^\pm K^\mp$, new physics is also expected to play a very minor rôle. These strategies therefore provide a “reference” value for $\gamma$. Since, on the other hand, the $B \rightarrow \pi K$ strategies to determine $\gamma$ rely on the interference between tree and penguin contributions, discrepancies with the “reference” value for $\gamma$ may well show up in the presence of new physics [135, 136]. If we are lucky, we may even get immediate indications for new physics from $B \rightarrow \pi K$ decays [137], as the SM predicts interesting correlations between the corresponding observables as shown in Figs. 38 and 39. Here the dotted regions correspond to the CLEO results that were reported in 1999 [117].

If future measurements should yield results significantly outside the allowed regions shown in these figures, we would have an indication for new physics. On the other hand, if we should find values
lying inside these regions, this would not automatically imply a confirmation of the SM. In this case, we would be in a position to extract a value for $\gamma$ by following the strategies described above, which may well lead to discrepancies with the “reference” values for $\gamma$ that are implied by the theoretically clean “tree” strategies, or with the usual fits of the unitarity triangle. In a recent paper [136], several specific models were employed to explore the impact of new physics on $B \rightarrow \pi K$ decays. For example, in models with an extra $Z'$ boson or in SUSY models with broken $R$-parity, the resulting electroweak penguin coefficients can be much larger than in the SM, since they arise already at tree level.

Interestingly, the present experimental range coincides perfectly with the SM region in Fig. 38. This feature should be compared with the situation in Fig. 39. Unfortunately, the present experimental uncertainties are too large to speculate on new-physics effects. However, the experimental situation should improve considerably in the years before the start of the LHC. The strategies discussed in the following subsections are also well suited to search for new physics.

5.14 Experimental Studies

Preliminary studies for the determination of $\gamma$ using the $K\pi$ decay modes of B mesons have been performed for the LHCb experiment. As explained above, $\gamma$ may be determined using a number of strategies that involve the final states $K^{+}\pi^{-}$, $K^{0}\pi^{+}$, $K^{+}\pi^{0}$ and $K^{0}\pi^{0}$. Experimentally it is easiest to reconstruct final states which contain charged particles and have reconstructible decay vertices. Clearly, therefore, the strategy involving $K^{+}\pi^{-}$ and $K^{0}\pi^{+}$ final states provides the cleanest experimental channel and has been studied first. Future work will involve a study of the feasibility of reconstructing the $K^{+}\pi^{0}$ mode. A clean reconstruction of the $K^{0}\pi^{0}$ mode is unlikely to be possible at LHCb.

The experimental values to be determined are the ratios $R$ and $A_0$ given in (7) and (8) with differing final states in numerator and denominator. This means that the ratio of trigger and reconstruction efficiencies must be known for these final states. This is in contrast to most CP violation measurements where these quantities cancel and will be an additional source of systematic error which has yet to be investigated.

The principal features of the $K\pi$ decays used for reconstruction are well separated vertices and large impact parameters. The particle identification provided by the RICH detectors is vital for the $K^{+}\pi^{-}$ mode and very helpful for the $K^{0}\pi^{+}$ mode. The overall trigger efficiencies for the two channels are around 30% each, relative to events decaying in the acceptance. The net trigger and reconstruction efficiency is about 0.02 for the $K^{+}\pi^{-}$ channel and 0.01 for $K^{0}\pi^{+}$. The difference is mainly due to the detector acceptance. With the latest CLEO branching ratio measurements of $(18.2 \pm 5) \times 10^{-6}$ for $K^{0}\pi^{+}$ and $(18.8 \pm 3) \times 10^{-6}$ for $K^{+}\pi^{-}$ [73], these efficiencies result in about 90,000 events in the $K^{0}\pi^{+}$ and 175,000 events in the $K^{+}\pi^{-}$ channel per year. These numbers are rather preliminary since the background studies are still in an early stage, and it may prove necessary to tighten the reconstruction cuts.

Translating these numbers into final CP sensitivities is however not trivial. The measured values of the ratios $R$ and $A_0$ define contours in the $r$–$\gamma$ plane such as those in Fig. 35. A value for $\gamma$ can only be extracted once $r$ is known whose value must be determined from theory. Experimental results indicating large rescattering effects which would imply large errors in $r$ are, for example, large CP violation in the $B^{+} \rightarrow K^{0}\pi^{+}$ channel or branching ratios for $B^{+} \rightarrow K^{+}K^{0}$ and $B^{0} \rightarrow K^{+}K^{-}$, which are larger than expected. The precise value of $r$ will have a large effect on the expected errors. There is also a four-fold ambiguity for the value of $\gamma$. Figure 40 illustrates the errors that might be expected assuming a value for $r$ of $0.18 \pm 10\%$, for two of the possible solutions. For one of these solutions the error is $\sim 2^\circ$, whereas for the other it is $\sim 7^\circ$. These uncertainties are mirrored in the remaining two solutions.

In summary, from this preliminary study, it is expected that LHCb will be able to provide determinations of the ratios $A_0$ and $R$ for the strategy involving $K^{0}\pi^{+}$ and $K^{+}\pi^{-}$ final states with errors around 3%. As explained above this cannot simply be translated into a CP sensitivity. Work on these promising
decays is still underway and will be extended to include a study of the K⁺π⁰ channel.

5.2 Extracting γ from Bₐ(d) → J/ψ Kₛ Decays

As we have already discussed in Sec. 3.1, the “gold-plated” mode B_d → J/ψ Kₛ plays an outstanding rôle in the determination of the B_d–B_d̄ mixing phase 𝛼_d, i.e. of the CKM angle β. In this subsection, we will have a closer look at the decay Bₐ → J/ψ Kₛ [61] (see also [111]), which is related to B_d → J/ψ Kₛ by interchanging all down and strange quarks (see Fig. 9), and may allow an interesting extraction of the CKM angle γ.

5.21 Theoretical Aspects

In analogy to (2), the Bₐ → J/ψ Kₛ decay amplitude can be expressed as follows:

\[ A(Bₐ^0 → J/ψ Kₛ) = λ A \left[ 1 - a e^{iθ} e^{iγ} \right], \]

where

\[ A ≡ λ² A \left( A^c + A^{ct} \right) \quad \text{and} \quad a e^{iθ} ≡ R_b \left( \frac{A_{\text{pen}}^{nt}}{A^c + A^{ct}} \right) \]

and correspond to (3). It should be emphasized that (2) and (27) rely only on the unitarity of the CKM matrix. In particular, these SM parametrizations of the Bₐ^0 → J/ψ Kₛ decay amplitudes also take into account FSI effects, which can be considered as long-distance penguin topologies with internal up- and charm-quark exchanges [113].

Comparing (2) with (27), we observe that the “penguin parameter” \( a' e^{iθ'} \) is doubly Cabibbo-suppressed in the B_d → J/ψ Kₛ decay amplitude (2), whereas \( a e^{iθ} \) enters (27) in a Cabibbo-allowed
way. Consequently, there may be sizeable CP-violating effects in $B_s \to J/\psi K_S$, which provide two independent observables, $\mathcal{A}_{\text{dir}}^{\Delta \Gamma}(B_s \to J/\psi K_S)$ and $\mathcal{A}_{\text{mix}}^{\Delta \Gamma}(B_s \to J/\psi K_S)$, depending on the three “unknowns” $\alpha$, $\theta$ and $\gamma$, as well as on the $B^0_s-K^0_s$ mixing phase $\phi_s$. Consequently, in order to determine these “unknowns”, we need an additional observable, which is provided by

$$H \equiv \left( \frac{1 - \lambda^2}{\lambda^2} \right) \left( \frac{|\mathcal{A}'|}{|\mathcal{A}|} \right)^2 \frac{\langle \Gamma(B_s \to J/\psi K_S) \rangle}{\langle \Gamma(B_d \to J/\psi K_S) \rangle},$$

(29)

where the CP-averaged decay rates $\langle \Gamma(B_s \to J/\psi K_S) \rangle$ and $\langle \Gamma(B_d \to J/\psi K_S) \rangle$ can be determined from the “untagged” rates introduced in (32) through

$$\langle \Gamma(B_q \to f) \rangle \equiv \frac{\Gamma_q[f(0)]}{2}.$$

(30)

In (29), we have neglected tiny phase-space effects, which can be included straightforwardly [61].

Since the U-spin flavour-symmetry of strong interactions implies

$$|\mathcal{A}'| = |\mathcal{A}| \quad \text{and} \quad \alpha' = \alpha, \quad \theta' = \theta,$$

(31)

we can determine $\alpha$, $\theta$ and $\gamma$ as a function of the $B^0_s-K^0_s$ mixing phase $\phi_s$ by combining $H$ with $\mathcal{A}_{\text{dir}}^{\Delta \Gamma}(B_s \to J/\psi K_S)$ and $\mathcal{A}_{\text{mix}}^{\Delta \Gamma}(B_s \to J/\psi K_S)$ or $\mathcal{A}_{\Delta \Gamma}(B_s \to J/\psi K_S)$. In contrast to certain isospin relations, electroweak penguins do not lead to any problems in these U-spin relations. As we have already noted, the $B^0_s-K^0_s$ mixing phase $\phi_s = -2\delta \gamma$ is expected to be negligible in the SM. It can be probed with the help of $B_s \to J/\psi \phi$, Sec. 4. Strictly speaking, in the case of $B_s \to J/\psi K_S$, we have $\phi_s \to -2\delta \gamma - \phi_K$, where $\phi_K$ is related to the $K^0-\bar{K}^0$ mixing phase and is negligible in the SM (see also the comment in Sec. 3.1). Since the value of the CP-violating parameter $\varepsilon_K$ of the neutral kaon system is very small, $\phi_K$ can only be affected by very contrived models of new physics [62].

Interestingly, the strategy to extract $\gamma$ from $B_{s(d)} \to J/\psi K_S$ does not require a non-trivial CP-conserving strong phase $\theta$. However, its experimental feasibility depends strongly on the value of the quantity $a$ introduced in (28). It is very difficult to estimate $a$ theoretically. In contrast to the “usual” QCD penguin topologies, the QCD penguins contributing to $B_{s(d)} \to J/\psi K_S$ require a colour-singlet exchange, as indicated in Fig. 9 through the dashed lines, and are “Zweig-suppressed”. Such a comment does not apply to the electroweak penguins, which contribute in “colour-allowed” form. The current–current amplitude $A_{cc}^c$ is due to “colour-suppressed” topologies, and the ratio $A_{\text{penguin}}^c/(A_{cc}^c + A_{\text{penguin}}^c)$, which governs $a$, may be sizeable. It is interesting to note that the measured branching ratio

$$B(B_d^0 \to J/\psi K^0) = 2B(B_d^0 \to J/\psi K^0) = (8.9 \pm 1.2) \times 10^{-4}$$

[64] probes only the combination $A' \propto (A_{cc}^c + A_{\text{penguin}}^c)$ of current–current and penguin amplitudes, and obviously does not allow their separation. It would be very important to have a better theoretical understanding of the quantity $a e^{i\theta}$. However, such analyses are beyond the scope of this workshop, and are left for further studies. Let us note that the measured $B_d^0 \to J/\psi K_S$ branching ratio implies, if we use U-spin arguments, a $B_s \to J/\psi K_S$ branching ratio at the level of $2 \times 10^{-5}$.

The general formalism to extract $\gamma$ from $B_{s(d)} \to J/\psi K_S$ decays can be found in [61]. Although the corresponding formulae are quite complicated, the basic idea is very simple: if $\phi_s$ is used as input, the CP-violating asymmetries $\mathcal{A}_{\text{dir}}^{\Delta \Gamma}(B_s \to J/\psi K_S)$ and $\mathcal{A}_{\text{mix}}^{\Delta \Gamma}(B_s \to J/\psi K_S)$ allow one to fix a contour in the $\gamma-a$ plane in a theoretically clean way. Another contour can be fixed with the help of the U-spin relations (31) by combining the observable $H$ with $\mathcal{A}_{\Delta \Gamma}^{\text{mix}}(B_s \to J/\psi K_S)$. Alternatively, we may combine $H$ with $\mathcal{A}_{\Delta \Gamma}(B_s \to J/\psi K_S)$ to fix a third contour in the $\gamma-a$ plane. The intersection of these contours then gives $\gamma$ and $a$. The general formulae simplify considerably, if we keep only terms linear in $a$. Within this approximation, we obtain

$$\tan \gamma \approx \sin \phi_s + \mathcal{A}_{\text{mix}}^{\Delta \Gamma}(B_s \to J/\psi K_S).$$

(32)
Let us illustrate this approach by considering a simple example. Assuming a negligible $B^0 - \bar{B}^0$ mixing phase, i.e. $\phi_s = 0$, and $\gamma = 76^\circ$, which lies within the presently allowed “indirect” range for this angle, as well as $a = a' = 0.2$ and $\theta = \theta' = 3\phi$, we obtain the following $B_{s(d)} \rightarrow J/\psi K_S$ observables:

$$
\begin{align*}
A_{CP}^{\Delta_{CP}}(B_s \rightarrow J/\psi K_S) &= 0.20, \\
A_{CP}^{\Delta_{CP}}(B_s \rightarrow J/\psi K_S) &= 0.33, \\
A_{\Delta \Gamma}(B_s \rightarrow J/\psi K_S) &= 0.92, \\
H &= 0.95.
\end{align*}
$$

The corresponding contours in the $\gamma - a$ plane are shown in Fig. 41. Interestingly, in the case of these contours, we would not have to deal with “physical” discrete ambiguities for $\gamma$, since values of $a$ larger than 1 would simply appear unrealistic. If it should become possible to measure $A_{\Delta \Gamma}$ with the help of the widths difference $\Delta \Gamma$, the dotted line could be fixed. In this example, the approximate expression (32) yields $\gamma \approx 82^\circ$, which deviates from the “true” value of $\gamma = 76^\circ$ by only 8%. It is also interesting to note that we have $A_{CP}^{\Delta_{CP}}(B_d \rightarrow J/\psi K_S) = -0.98\%$ in our example.

An important by-product of the strategy described above is that the quantities $a'$ and $\theta'$ allow one to take into account the penguin contributions in the determination of $\phi_d$ from $B_d \rightarrow J/\psi K_S$, which are presumably very small because of the strong Cabibbo suppression in (2). However, as we have already noted in Sec. 3.1, these uncertainties are an important issue for the LHC because of the tremendously small experimental uncertainty for the CP-violating $B_d \rightarrow J/\psi K_S$ observables. Using (31), we obtain an interesting relation between the direct CP asymmetries arising in the modes $B_d \rightarrow J/\psi K_S$ and $B_s \rightarrow J/\psi K_S$ and their CP-averaged rates:

$$
\frac{A_{CP}^{\Delta_{CP}}(B_d \rightarrow J/\psi K_S)}{A_{CP}^{\Delta_{CP}}(B_s \rightarrow J/\psi K_S)} \approx \frac{B(B_s \rightarrow J/\psi K_S)}{B(B_d \rightarrow J/\psi K_S)}.
$$

Let us note that an analogous relation holds also between the CP-violating asymmetries in the decays $B^{\pm} \rightarrow \pi^{\pm} K$ and $B^{\pm} \rightarrow K^{\pm} K$ [113, 114].

Before turning to the experimental feasibility studies, let us say a few words on the SU(3) breaking corrections. Whereas the solid curves in Fig. 41 are theoretically clean, the dot-dashed and dotted lines are affected by U-spin breaking corrections. Because of the suppression of $a'e^{i\theta'}$ in (2) through $\lambda^2$, these contours are essentially unaffected by possible corrections to (31), and rely predominantly on the U-spin relation $|A'| = |A|$. In the “factorization” approximation, we have

$$
\frac{|A'|}{|A|}_{\text{fact}} = \frac{F_{BYK0}^\gamma(M_{J/\psi}^2, 1^-)}{F_{BYK0}^\gamma(M_{J/\psi}^2, 1^-)}.
$$
where the form factors $F_{B_{d}K_{0}}(M_{J/\psi}^{2}; 1^{-})$ and $F_{B_{d}K_{0}^{*}}(M_{J/\psi}^{2}; 1^{-})$ parametrize the quark–current matrix elements $\langle K^{0}|(\bar{b}s)_{V-A}|B_{d}^{0}\rangle$ and $\langle K^{0}|(\bar{b}d)_{V-A}|B_{s}^{0}\rangle$, respectively [106]. We are not aware of quantitative studies of (35), which could be performed, for instance, with the help of sum rule or lattice techniques. In the light-cone sum-rule approach, sizeable SU(3) breaking effects were found for $B_{d,s} \rightarrow K^{*}$ form factors [35]. It should be emphasized that also non-factorizable corrections, which are not included in (35), may play an important rôle. We are optimistic that SU(3) breaking will be under better control by the time the $B_{s} \rightarrow J/\psi K_{S}$ measurements can be performed in practice.

5.22 Experimental Studies

Both CMS and LHCb have performed preliminary studies of the feasibility to extract the CKM angle $\gamma$ from a measurement of the time-dependent CP asymmetry in the decay $B_{s} \rightarrow J/\psi K_{S}$. From these, and the results presented in Sec. 3.1, the potential of ATLAS may also be gauged.

The $B_{s} \rightarrow J/\psi K_{S}$ branching ratio is expected to be at the level of $2.0 \times 10^{-5}$, see Sec. 5.21, compared to $(4.45 \pm 0.6) \times 10^{-4}$ [64] for $B_{d} \rightarrow J/\psi K_{S}$. The $B_{s}$ production rate is $30\%$ of the $B_{d}^{0}$ rate. Assuming the same selection procedure as used in the $B_{d} \rightarrow J/\psi K_{S}$ analysis, the $B_{s} \rightarrow J/\psi K_{S}$ event yield will therefore be $1/74$ that of the $B_{d}^{0}$ yield. Experimentally the isolation of these events is challenging, due to the large combinatorial background and the close $B_{d}^{0}$ peak, only $90$ MeV/$c^{2}$ away.

CMS have developed a selection tailored to $B_{s} \rightarrow J/\psi K_{S}$ decays. The combinatorial background can be heavily suppressed with a lower $p_{T}$-cut of $1.5$ GeV$/c$ on the pions from the $K_{S}^{0}$ decays, A S/B ratio of $\approx 0.5$ can be achieved, with an event yield of 4100 events per year. The mass resolution is better than $<20$ MeV/$c^{2}$ and thus sufficient to separate the events from $B_{d}^{0}$ decays. The reconstructed mass peaks can be seen in Fig. 42(a).

LHCb have not yet investigated cuts specific to $B_{s} \rightarrow J/\psi K_{S}$. As can be seen from Fig. 42(b), the standard $B_{d} \rightarrow J/\psi K_{S}$ selection results in a combinatorial background which is an order of magnitude above the $B_{s} \rightarrow J/\psi K_{S}$ signal. Further work will improve the selection to suppress this contamination. The invariant mass resolution is better than $10$ MeV/$c^{2}$, so that the $B_{d}^{0}$ and $B_{s}^{0}$ peaks are cleanly separated.

These studies indicate that a measurement of the CP asymmetry in $B_{s} \rightarrow J/\psi K_{S}$ is feasible at the LHC, so that $\gamma$ can be extracted from that decay. For the parameter-set considered in Sec. 5.21, CMS estimate that a precision of $\sim 9^{\circ}$ is achievable in $3$ years operation.

![Fig. 42: $B_{d} \rightarrow J/\psi(\rightarrow \mu^{+}\mu^{-}) K_{S}$ and $B_{s} \rightarrow J/\psi(\rightarrow \mu^{+}\mu^{-}) K_{S}$ mass peaks.](image)
5.3 Extracting $\gamma$ from $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ Decays

Usually, $B_d \to D_d^+ D_d^-$ decays appear in the literature as a tool to probe the $B_{d(s)}^0 \to \overline{B}_{d(s)}^0$ mixing phase $\phi_d$ [4, 5, 6]. In fact, if penguins played a negligible rôle in these modes, $\phi_d = 2\beta$ could be determined from the corresponding mixing-induced CP-violating effects. However, penguin topologies, which contain also important contributions from FSI effects, may well be sizeable, although it is very difficult to calculate them in a reliable way. The strategy discussed in this subsection makes use of the penguin topologies [61], allowing one to determine $\gamma$, if the overall $B_d \to D_d^+ D_d^-$ normalization is fixed through the CP-averaged, i.e. “untagged” $B_s \to D_s^+ D_s^-$ rate, and if the $B_{d(s)}^0 \to \overline{B}_{d(s)}^0$ mixing phase $\phi_d$ is determined separately, for instance with the help of the “gold-plated” decay $B_d \to J/\psi K_S$. It should be emphasized that no $\Delta M_{s,d}$ oscillations have to be resolved to measure the untagged $B_s \to D_s^+ D_s^-$ rate.

5.3.1 Theoretical Aspects

The decays $B_{d(s)}^0 \to D_{d(s)}^+ D_{d(s)}^-$ are transitions into a CP eigenstate with eigenvalue $+1$ and originate from $\overline{b} \to c d s (\bar{s})$ quark-level decays. We have to deal with both current–current and penguin contributions, as can be seen in Fig. 43. In analogy to (2) and (27), the corresponding transition amplitudes can be written as follows:

$$A(B_s^0 \to D_s^+ D_s^-) = \left(1 - \frac{\lambda^2}{2}\right) \tilde{A}' \left[1 + \left(\frac{\lambda^2}{1 - \lambda^2}\right) \tilde{a} e^{i\tilde{\theta}} e^{i\gamma}\right],$$

$$A(B_d^0 \to D_d^+ D_d^-) = -\lambda \tilde{A} \left[1 - \tilde{a} e^{i\tilde{\theta}} e^{i\gamma}\right],$$

where the quantities $\tilde{A}$, $\tilde{A}'$ and $\tilde{a} e^{i\tilde{\theta}}$, $\tilde{a}' e^{i\tilde{\theta}'}$ take the same form as for $B_{s(d)} \to J/\psi K_S$. In contrast to the decays $B_{s(d)} \to J/\psi K_S$, there are “colour-allowed” current–current contributions to $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$, as well as contributions from “exchange” topologies, and the QCD penguins do not require a colour-singlet exchange, i.e. they are not “Zweig-suppressed”.

Since the phase structures of the $B_{d(s)}^0 \to D_{d(s)}^+ D_{d(s)}^-$ and $B_{d(s)}^0 \to D_{d(s)}^+ D_{d(s)}^-$ decay amplitudes are completely analogous to those of $B_s^0 \to J/\psi K_S$ and $B_d^0 \to J/\psi K_S$, respectively, the approach discussed in the previous subsection can be applied after a straightforward replacements of variables. Neglecting tiny phase-space effects, which can be taken into account straightforwardly (see [61]), we have

$$\tilde{H} = \left(1 - \frac{\lambda^2}{\lambda^2}\right) \left(\frac{|\tilde{A}'|}{|\tilde{A}|}\right)^2 \frac{\langle\Gamma(B_d \to D_d^+ D_d^-)\rangle}{\langle\Gamma(B_s \to D_s^+ D_s^-)\rangle},$$

where the CP-averaged rates can be determined with the help of (30). The $B_{d(s)} \to D_{d(s)}^+ D_{d(s)}^-$ counter-
part to (32) takes the following form:

$$\tan \gamma \approx \frac{\sin \phi_d - A_{CP}^{\text{mix}}(B_d \to D^+_d \bar{D}^0_d)}{(1 - H) \cos \phi_d}, \quad (39)$$

where the different sign of the mixing-induced CP asymmetry results from the different CP eigenvalues of the $B_d \to D^+_d \bar{D}^0_d$ and $B_s \to J/\psi K_S$ final states.

Let us illustrate the strategy to determine $\gamma$, again by considering a simple example. Assuming $\tilde{a} = \tilde{a}' = 0.1$, $\theta = \theta' = 210^\circ$, $\gamma = 76^\circ$ and a $B_d^0$-$\bar{B}_d^0$ mixing phase of $\phi_d = 2\beta = 53^\circ$, we obtain the following observables:

$$A_{CP}^{\text{mix}}(B_d \to D^+_d \bar{D}^0_d) = -0.092, \quad A_{CP}^{\text{mix}}(B_d \to D^+_d \bar{D}^0_d) = 0.88 \quad \text{and} \quad H = 1.05. \quad (40)$$

In this case, studies of CP violation in $B_d \to J/\psi K_S$ would yield $\sin(2\beta) = 0.8$, which is the central value of the most recent CDF analysis [63], implying $2\beta = 53^\circ$ or $2\beta = 180^\circ - 53^\circ = 127^\circ$. This twofold ambiguity can be resolved experimentally, for example, by combining $B_s \to J/\psi \phi$ with $B_d \to J/\psi \rho^0$ [93] (for alternatives, see [95]), as noted in Sec. 4.. In this example, we obtain the contours in the $\gamma-\tilde{a}$ plane shown in Fig. 44. Since values of $\tilde{a} = O(1)$ appear unrealistic, we would obtain a single “physical” solution of $76^\circ$ in this case. The approximate expression (39) gives $\gamma \approx 70^\circ$.

As in the $B_s(d) \to J/\psi K_S$ case, only the contours involving the observable $H$, i.e. the dot-dashed lines in Fig. 44, are affected by SU(3) breaking corrections, which are essentially due to the U-spin breaking corrections to $|\tilde{A}| = |\tilde{A}|$. Within the “factorization” approximation, we have

$$\frac{|\tilde{A}'|}{|\tilde{A}|_{\text{fact}}} \approx \left( \frac{M_{B_s} - M_{D_s}}{M_{B_d} - M_{D_d}} \right) \frac{f_{D_s} \xi_s(w_s) + f_{D_d} \xi_d(w_d)}{f_{D_s} \xi_s(w_d) + f_{D_d} \xi_d(w_d)}, \quad (41)$$

where the restrictions from heavy-quark effective theory on the $B_q \to D_q$ form factors have been taken into account by introducing appropriate Isgur–Wise functions $\xi_q(w_q)$ with $w_q = M_{B_q}/(2M_{D_q})$ [138]. Studies of the light-quark dependence of the Isgur–Wise function were performed within heavy-meson chiral perturbation theory, indicating an enhancement of $\xi_s/\xi_d$ at the level of 5% [139]. Applying the same formalism to $f_{D_s}/f_{D}$ gives values at the 1.2 level [140], which is of the same order of magnitude as the results of recent lattice calculations [141]. Further studies are needed to get a better picture of the SU(3) breaking corrections to the ratio $|\tilde{A}'|/|\tilde{A}|$. Since “factorization” may work reasonably well for $B_q \to D^+_q D^-_q$, the leading corrections are expected to be due to (41).
The experimental feasibility of the strategy to extract $\gamma$ from $B_{d(s)} \to D_{d(s)}^{+} D_{d(s)}^{-}$ decays depends strongly on the size of the penguin parameter $\bar{a}$, which is difficult to predict theoretically. The branching ratio for $B_{d}^{0} \to D_{d}^{+} D_{d}^{-}$ is expected to be around $4 \times 10^{-4}$ level [138]; the one for $B_{s}^{0} \to D_{s}^{+} D_{s}^{-}$ is enhanced by $1/\lambda^2 \approx 20$, and correspondingly is expected at the $8 \times 10^{-3}$ level.

5.32 Experimental Studies

LHCb have conducted a preliminary feasibility study of this analysis, considering the modes where the $D$ decays to $K\pi\pi$ and the $D_s$ to $K\bar{K}\pi$. For the $B_s \to D_s D_s$ decay, only the total rate is required, which is advantageous experimentally as it is neither necessary to resolve the rapid oscillations, nor does flavour tagging reduce the already suppressed yield in $B_s$ events. The observables $A_{\rm CP}^{\text{mix}}$ and $A_{\rm CP}^{\text{dir}}$ are extracted from a fit to the time-dependent CP asymmetry for $B \to DD$ decays. For this channel it is therefore necessary to obtain the decay-time of the event and to flavour tag the decays. This analysis thus relies on all the fortes of the LHCb detector, namely the specialized trigger, the particle identification capability and the precise vertexing.

The final states for both decays consist of six hadrons, so that the hadron trigger is vital and must be efficient for the low values of $p_T$, which result from the large final-state multiplicity. The vertex trigger is particularly efficient for these channels as there are two vertices containing three tracks ($D$ vertices) to be triggered on in each decay. The particle identification information from the RICH detectors is important for background-suppression and to eliminate reflections from $KK\pi$ to $K\pi\pi$ and vice versa.

The analysis is at a preliminary level and still underway, but the first results look promising. The trigger efficiencies for both channels are found to be around 25% for events decaying within the acceptance. The reconstruction relies principally on requiring well separated secondary vertices, appropriate invariant masses and $p_T$-cuts. Reconstruction efficiencies for the B and $B_s$ of about 30% have been found. Using product branching ratios $B(B \to X) \times B(X \to Y)$ of $3.6 \times 10^{-5}$ for $B \to DD$ and $3.2 \times 10^{-4}$ for $B_s \to D_s D_s$ gives about $3 \times 10^5$ events per year for $B \to DD$, after flavour tagging, and $1.9 \times 10^5$ events per year for $B_s \to D_s D_s$. These estimates have been obtained by studying signal MC simulations only. A study of the effect of backgrounds is currently underway. The errors achievable on $\gamma$ depend on the specific values of $\gamma$ and $\beta$. For $\gamma = 75^\circ$ and $\beta = 50^\circ$, an error of about 1° is expected. It should be emphasized that these numbers are preliminary, but it seems that the potential of LHCb in this promising channel is good.

5.4 A Simultaneous Determination of $\beta$ and $\gamma$ from $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$

In this subsection, we combine the CP-violating observables of the decay $B_d \to \pi^+\pi^-$ with those of the transition $B_s \to K^+K^-$, which is the U-spin counterpart of $B_d \to \pi^+\pi^-$. Following these lines, a simultaneous determination of $\phi_B = 2 \beta$ and $\gamma$ becomes possible [110]. This approach is not affected by any penguin topologies – it rather makes use of them – and does not rely on certain “plausible” dynamical or model-dependent assumptions. Moreover, FSI effects, which led to considerable attention in the recent literature in the context of the determination of $\gamma$ from $B \to \pi K$ decays (see Sec. 5.1), do not lead to any problems, and the theoretical accuracy is only limited by U-spin breaking effects. This strategy, which is furthermore very promising with regard to the search for indications of new physics [137], is conceptually quite similar to the extractions of $\gamma$ with the help of the decays $B_{s(d)} \to J/\psi K_S$ and $B_{d(s)} \to D_{d(s)}^{+} D_{d(s)}^{-}$ discussed in Secs. 5.2 and 5.3, respectively (see also [111]).

5.41 Theoretical Aspects

As can be seen from Fig. 12, $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ are related to each other by interchanging all down and strange quarks, i.e. they are U-spin counterparts. If we make use of the unitarity of the CKM matrix and apply the generalized Wolfenstein parametrization, including non-leading terms in $\lambda$,
the $B^0_d \rightarrow \pi^+ \pi^-$ decay amplitude can be expressed as follows \cite{110}:

$$A(B^0_d \rightarrow \pi^+ \pi^-) = e^{i\gamma} C \left[ 1 - d e^{i\theta} e^{-i\gamma} \right], \quad (42)$$

where

$$C \equiv \lambda^2 A R_b \left( A_{\text{cc}}^u + A_{\text{pen}}^{ut} \right), \quad d e^{i\theta} \equiv \frac{1}{R_b} \left( \frac{A_{\text{pen}}^{ct}}{A_{\text{cc}}^u + A_{\text{pen}}^{ut}} \right) \quad (43)$$

with $A_{\text{pen}}^{ut} \equiv A_{\text{pen}}^{u} - A_{\text{pen}}^{t}$. In analogy to (42), we obtain for the $B^0_s \rightarrow K^+ K^-$ decay amplitude

$$A(B^0_s \rightarrow K^+ K^-) = e^{i\gamma} C' \left[ 1 + \left( 1 - \frac{\lambda^2}{\lambda^2} \right) d' e^{i\theta'} e^{-i\gamma} \right], \quad (44)$$

where

$$C' \equiv \left( \frac{\lambda^2 A R_b}{1 - \lambda^2/2} \right) \left( A_{\text{cc}}^{u'} + A_{\text{pen}}^{u't} \right) \quad \text{and} \quad d' e^{i\theta'} \equiv \frac{1}{R_b} \left( \frac{A_{\text{pen}}^{ct'}}{A_{\text{cc}}^{u'} + A_{\text{pen}}^{u't}} \right) \quad (45)$$

correspond to (43). The general expressions for the $B_d \rightarrow \pi^+ \pi^-$ and $B_s \rightarrow K^+ K^-$ observables \cite{24} and (25) in terms of the parameters specified above can be found in \cite{110}.

Since $B_d \rightarrow \pi^+ \pi^-$ and $B_s \rightarrow K^+ K^-$ are related to each other by interchanging all down and strange quarks, the U-spin flavour-symmetry of strong interactions implies

$$d' = d \quad \text{and} \quad \theta' = \theta. \quad (46)$$

If we assume that the $B^0_s - \overline{B^0_s}$ mixing phase $\phi_s$ is negligible, or that it is fixed through $B_s \rightarrow J/\psi \phi$, the four CP-violating observables provided by $B_d \rightarrow \pi^+ \pi^-$ and $B_s \rightarrow K^+ K^-$ depend – in the strict U-spin limit – on the four “unknowns” $d, \theta, \phi_d = 2\beta$ and $\gamma$. We have therefore sufficient observables at our disposal to extract these quantities simultaneously. In order to determine $\gamma$, it suffices to consider $A_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+ K^-)$ and the direct CP asymmetries $A_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+ K^-), A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-)$. If we make use, in addition, of $A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-), \phi_d$ can be determined as well. The full formulae needed to implement this approach can be found in \cite{110}.

The use of the U-spin flavour-symmetry to extract $\gamma$ can be minimized, if we use not only $\phi_s$, but also the $B^0_s - \overline{B^0_s}$ mixing phase $\phi_d$ as an input. Then, the CP-violating observables $A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-), A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-)$ and $A_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+ K^-)$ allow one to fix contours in the $\gamma - d$ and $\gamma - d'$ planes in a theoretically clean way. In order to extract $\gamma$ and the hadronic parameters $d, \theta, \theta'$ with the help of these contours, the U-spin relation $d' = d$ is sufficient. Let us illustrate this approach for a specific example:

$$A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-) = +24\%, \quad A_{\text{CP}}^{\text{mix}}(B_d \rightarrow \pi^+ \pi^-) = +4.4\%, \quad A_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+ K^-) = -17\%, \quad A_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+ K^-) = -28\%, \quad (47)$$

corresponding to the input parameters $d = d' = 0.3, \theta = \theta' = 210^\circ, \phi_s = 0, \phi_d = 53^\circ$ and $\gamma = 76^\circ$. In Fig. 45, the corresponding contours in the $\gamma - d$ and $\gamma - d'$ planes are represented by the solid and dot-dashed lines, respectively. Their intersection yields a twofold solution for $\gamma$, given by $51^\circ$ and our input value of $76^\circ$. The dotted line is related to

$$K \equiv - \left( \frac{1 - \lambda^2}{\lambda^2} \right) \left[ \frac{A_{\text{CP}}^{\text{dir}}(B_d \rightarrow \pi^+ \pi^-)}{A_{\text{CP}}^{\text{dir}}(B_s \rightarrow K^+ K^-)} \right], \quad (48)$$

which can be combined with the mixing-induced CP asymmetry $A_{\text{CP}}^{\text{mix}}(B_s \rightarrow K^+ K^-)$ through the U-spin relation (46) to fix another contour in the $\gamma - d$ plane. Combining all contours in Fig. 45 with one another, we obtain a single solution for $\gamma$ in this example, which is given by the “true” value of $76^\circ$. 
It should be emphasized that the theoretical accuracy of $\gamma$ and of the hadronic parameters $d$, $\theta$ and $\theta'$ is only limited by U-spin breaking effects. In particular, it is not affected by any FSI or penguin effects. A first consistency check is provided by $\theta = \theta'$. Moreover, we may determine the normalization factors $C$ and $C'$ of the $B_d^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ decay amplitudes (see (42) and (44)) with the help of the corresponding CP-averaged branching ratios. Comparing them with the “factorized” result

$$\left| \frac{C'}{C} \right| = \frac{f_K}{f_{\pi}} \frac{F_{B_s K}(M_{K^0}^2; 0^+)}{F_{B_d \pi}(M_{\pi^+}^2; 0^+)} \left( \frac{M_{B_s}^2 - M_{K^0}^2}{M_{B_d}^2 - M_{\pi^+}^2} \right), \quad (49)$$

we have another interesting probe for U-spin breaking effects. Interestingly, the relation

$$d'e^{i\theta'} = d e^{i\theta} \quad (50)$$

is not affected by U-spin breaking corrections within a certain model-dependent approach (a modernized version [15, 142] of the “Bander–Silverman–Soni mechanism” [143]), which relies on the factorization approximation to estimate the relevant hadronic matrix elements [110]. Although this approach appears rather simple and may be affected by non-factorizable effects, it strengthens our confidence into the U-spin relations used for the extraction of $\beta$ and $\gamma$ from the decays $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$. Further theoretical studies along the lines of Ref. [76] to investigate the U-spin breaking effects in the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ system would be very interesting. In order to obtain further experimental insights, the $B_d \to \rho^+\rho^-$, $B_s \to K^{*+}K^{*-}$ system would be of particular interest, allowing one to determine $\gamma$ together with the mixing phases $\phi_d$ and $\phi_s$, and tests of several interesting U-spin relations [93].

Since penguin processes play an important rôle in the decays $B_s \to K^+K^-$ and $B_d \to \pi^+\pi^-$, they may well be affected by new physics – which likewise applies to the determination of $\gamma$, where furthermore the unitarity of the CKM matrix is employed. Interestingly, the SM implies a rather restricted region in the space of the CP-violating observables of the $B_s \to K^+K^-$, $B_d \to \pi^+\pi^-$ system [137], which is shown in Fig. 46. A future measurement of observables lying significantly outside of the allowed region shown in this figure would be an indication for new physics. Such a discrepancy could either be due to CP-violating new-physics contributions to $B_s^0$–$\bar{B}_s^0$ mixing, or to $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ decay amplitudes. The former case would also be indicated simultaneously by large CP-violating effects in the mode $B_s \to J/\psi \phi$, which would allow us to extract the $B_s^0$–$\bar{B}_s^0$ mixing phase $\phi_s$ (see Sec. 4.). A discrepancy between the measured $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ observables and the region

![Fig. 45: The contours in the $\gamma$–$d^{'1/2}$ planes fixed through the CP-violating $B_d \to \pi^+\pi^-$ and $B_s \to K^+K^-$ observables for a specific example discussed in the text.](image-url)
corresponding to the value of $\phi_s$ from $B_s \to J/\psi \phi$ would then signal new-physics contributions to the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ decay amplitudes. On the other hand, if $B_s \to J/\psi \phi$ should exhibit negligible CP-violating effects, any discrepancy between the $B_d \to \pi^+\pi^-$, $B_s \to K^+K^-$ observables and the volume shown in Fig. 46 would indicate new-physics contributions to the corresponding decay amplitudes. If, however, the observables should lie within the region predicted by the SM, we can extract a value for the CKM angle $\gamma$ by following the strategy discussed above, which may well be in disagreement with those implied by theoretically clean strategies making use of pure “tree” decays, thereby also indicating the presence of new physics.

5.42 Experimental Studies

It was demonstrated in Sec. 3.22 that the LHC experiments can expect large event yields in the two-body decay $B_s^0 \to \pi^+\pi^-$. With an appropriately modified selection, similarly high statistics can be accumulated in $B_s^0 \to \pi^+\pi^-$. The excellent proper time resolution of the experiments then allows $B_s^0$ oscillations to be distinguished, and the CP asymmetry coefficients to be measured. By using the relations presented above, the $B_s^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$ observables can be used to cleanly extract CP phases, most interestingly the angle $\gamma$. The potential of this approach has been investigated by all three experiments.

Event Yields and Asymmetry Sensitivity

Apart from the final requirements on the best particle-hypothesis and the invariant mass of the two candidate tracks, the CMS and LHCb isolation of $B_s^0 \to K^+K^-$ events is identical to the $B_d \to \pi^+\pi^-$ selection described in Sec. 3.22. After flavour tagging, LHCb expects an annual yield of 4.6 events, with a contamination from other two-body modes of 15%. The equivalent numbers for CMS are 960 and 540 respectively, assuming the dE/dx based selection. As explained previously, ATLAS favours an approach where the asymmetry of all selected two-body events is fitted simultaneously. In this sample, $1.4k B_s^0 \to K^+K^-$ events are expected within the one $\sigma$ mass window.

The precision of the fitted $B_s^0 \to K^+K^-$ CP parameters $A_{K^+K^-}^{\text{mix}}$ and $A_{K^+K^-}^{\text{dir}}$ depends not only on the event yields, but also on the value of $\Delta m_s$, which governs the rapidity of the $B_s^0 - \overline{B_s^0}$ oscillations. Table 17 shows the precision expected for three different values of $\Delta m_s$ after an extended period of running. The uncertainties for one year's running scale in the expected statistical manner, except that
ATLAS and CMS retain no sensitivity for $\Delta m_s = 30 \text{ ps}^{-1}$ with the smaller data-set.

### Sensitivity to the CP Violating Phases

The sensitivity to which $\gamma$ can be determined has been studied by all three experiments, assuming that the $B_s^0 \to K^+K^-$ and $B_d^0 \to \pi^+\pi^-$ asymmetries be known to the precision given in Tabs. 17 and 9, respectively. With the scenario given in the previous subsection ($d = d' = 0.3$, $\theta = \theta' = 210^\circ$, $\phi_s = 0$, $\phi_d = 53^\circ$, $\gamma = 76^\circ$ and $\Delta m_s = 15 \text{ ps}^{-1}$, $\Delta \Gamma_s = 0$ and assuming an uncertainty of 1% on $\sin(2\beta) = \sin(\phi_d)$), the sensitivity after one year at LHC is $\sigma_\gamma = 3.7^\circ$, if the constraints $d = d'$ and $\theta = \theta'$ are applied. It improves to $\sigma_\gamma = 1.9^\circ$ after 5 years. Table 18 shows how these uncertainties increase with $\Delta m_s$. In the considered range of parameters, the sensitivity is clearly impressive.

To give an indication on how the sensitivity depends on the scenario, Fig. 47 shows the ultimate sensitivity for 5 years of LHC, in the scenario given above but as a function of the true value of $\gamma$ and $\theta = \theta'$. For most values of $\gamma$ and $\theta$, the sensitivity to $\gamma$ is better than $4^\circ$, except in regions around $\gamma = 90^\circ$ and $\gamma = 20^\circ$. The sensitivity depends significantly on the assumed value of $d = d'$: it decreases (increases) by a factor of two if $d = d' \approx 0$ ($d = d' = 0.5$).

The number of the degrees of freedom is large enough to allow one to relax one of the two constraints $d = d'$ and $\theta = \theta'$. This approximately doubles the uncertainty on $\gamma$, but allow one to check the U-spin flavour-symmetry relations $d = d'$ and $\theta = \theta'$. Figure 47 shows that a typical precision of $15^\circ$ on $\theta - \theta'$ and 0.1 on $d - d'$ can be reached, but in regions that are largely disjoint in $\theta$. These numbers also indicate the level to which U-spin symmetry must hold in order to improve the estimate of $\gamma$ without biasing it.

### 5.5 Conclusions

Thanks to their high yield in two-body decays and good proper time resolution, the LHC experiments are well suited to performing a combined analysis of $B_s^0 \to \pi^+\pi^-$ and $B_s^0 \to K^+K^-$. This analysis offers a powerful and precise way to determine the angle $\gamma$ in a manner sensitive to new-physics contributions.

#### 6. SYSTEMATIC ERROR CONSIDERATIONS IN CP MEASUREMENTS

##### 6.1 Introduction

The excellent statistical precision expected in many CP-violation measurements at the LHC demands that there be good control of systematic uncertainties. The challenges posed by hadronic effects in interpreting certain observables are discussed elsewhere in this chapter; here, biases from experimental factors and initial state asymmetries will be considered, and possible control strategies examined.

---

(a) Sensitivity to $\gamma$ for 5 years of LHC, with the constraints $\phi_s = 0$, $d = d'$ and $\theta = \theta'$ assuming an uncertainty of 1% on $\sin(2\beta)$, and with input values $d = d' = 0.3$, and $\phi_d = 55^\circ$, $\Delta m_s = 15 \text{ ps}^{-1}$ and $\Delta \Gamma_s = 0$. The contour lines correspond to sensitivities of $2^\circ$ (solid), $4^\circ$ (dashed), $6^\circ$ (dotted) and $8^\circ$ (dotted-dashed).

(b) Sensitivity to $\theta - \theta'$ for the same fit as in (a) except the relaxed $\theta = \theta'$ constraint. The contour lines correspond to sensitivities of $10^\circ$ (solid), $15^\circ$ (dashed), $20^\circ$ (dotted) and $50^\circ$ (dotted-dashed).

(c) Sensitivity to $d - d'$ for the same fit as in (a) except the relaxed $d = d'$ constraint. The contour lines correspond to sensitivities of $0.05$ (solid), $0.1$ (dashed), $0.2$ (dotted) and $0.4$ (dotted-dashed).

Fig. 47: Sensitivity of fits to the LHC combined $B_d^0 \to \pi^+ \pi^-$ and $B_s^0 \to K^+ K^-$ samples.
6.2 Sources and Categories of Systematic Bias

CP measurements require the reconstruction of a final state, and frequently the tagging of the initial state flavour. Time-dependent rates, or branching ratios, are then combined into asymmetries from which CKM phases can be extracted. These measurements are inherently robust, in that to first order experimental unknowns will cancel or can be assumed to be the same for all processes under consideration. However, certain charge- and flavour-dependent effects may exist, which can indeed bias the measurement:

- **Production asymmetries**
  As explained in the Chapter on $b$ production [47], the initial fraction of $b$ and $\bar{b}$ hadrons at the LHC is not expected to be identical. A production asymmetry will exist, and this asymmetry will vary as a function of rapidity and $p_T$, reaching values of several percent. Furthermore, this asymmetry can be different for each hadron species. In this section, the fractions of $B^0$, $\bar{B}^0$, $B^+_s$, $\bar{B}^0_s$, $B^+$ and $B^-$ mesons per event are denoted by $f_0$, $\bar{f}_0$, $f_s$, $\bar{f}_s$, $f_+$ and $f_-$. 

- **Tagging efficiency**
  All methods of flavour tagging rely on measuring the charge of one or more selected tracks. If the track reconstruction efficiency, or particle assignment (for lepton or kaon tags), has a charge-dependence, then a difference in the tagging efficiency for $b$ and $\bar{b}$ hadrons will result. Such a dependence is certainly possible, for instance in LHCb where positive and negative tracks are preferentially swept by the dipole to different areas of the detector. Furthermore, an asymmetric tagging efficiency can develop from effects such as a difference in interaction cross-sections for $K^+$ and $K^-$. The tagging efficiency for $B$ and $\bar{B}$ mesons will be denoted by $\epsilon$ and $\bar{\epsilon}$.

- **Mistag rate**
  Assuming a flavour tag has been performed, the probability of that tag being correct can also have a flavour-dependence. For instance in a lepton tag, different reconstructed momentum spectra for $\ell^+$ and $\ell^-$ are conceivable. These will result not only in different efficiencies, but also in different purities for the two samples. The mistag rates for $B$ and $\bar{B}$ mesons will be represented by $\omega$ and $\bar{\omega}$.

- **Final state acceptance**
  Clearly, in any measurement where different final states are being compared, the relative trigger and reconstruction efficiencies can be different. However, even if the asymmetry involves a single topology in the final state, the efficiency may differ for the charge-conjugate case, for the same charge acceptance reasons as explained above. Background is obviously an additional source of possible bias, and will require careful attention. However, this is a problem common to most physics measurements, and therefore is not considered here.

These effects will have different consequences for each category of measurement. The present discussion focuses on measurements involving decays into CP eigenstates, such as $B^+_d \rightarrow J/\psi K^0_S$. Here the observed asymmetry $A^{\text{obs}}(t)$ is constructed from the number of flavour-tagged $B^0$ and $\bar{B}^0$ decays into $J/\psi K^0_S$, as a function of proper time. Allowing for the factors considered above, $A^{\text{obs}}(t)$ is related to the true decay distributions $R^{\text{true}}_{B^0 \rightarrow J/\psi K^0_S}$ as follows:

$$A^{\text{obs}}(t) = \frac{(1 - 2\omega) R^{\text{true}}_{B^+_d \rightarrow J/\psi K^0_S}(t) - \frac{\bar{\omega}}{\epsilon} (1 - 2\bar{\omega}) R^{\text{true}}_{\bar{B}^0_d \rightarrow J/\psi K^0_S}(t)}{R^{\text{true}}_{B^+_d \rightarrow J/\psi K^0_S}(t) + \frac{\bar{\omega}}{\epsilon} R^{\text{true}}_{\bar{B}^0_d \rightarrow J/\psi K^0_S}(t)}.$$  

(1)

Assuming that the flavour-dependent effects in tagging and production are small, $A^{\text{obs}}(t)$ is related to the true physics asymmetry

$$A^{\text{phy}}(t) = \frac{R^{\text{true}}_{B^+_d \rightarrow J/\psi K^0_S}(t) - R^{\text{true}}_{\bar{B}^0_d \rightarrow J/\psi K^0_S}(t)}{R^{\text{true}}_{B^+_d \rightarrow J/\psi K^0_S}(t) + R^{\text{true}}_{\bar{B}^0_d \rightarrow J/\psi K^0_S}(t)}.$$
as follows:

\[
A^{\text{obs}}(t) \approx (1 - 2\omega) \left[ A^{\text{phy}}(t) - \frac{1}{2} \left( \frac{f_0}{f_\pi} - 1 \right) \left( 1 - A^{\text{phy}}(t)^2 \right) - \left( \frac{\omega - \overline{\omega}}{1 - 2\omega} \right) (1 - A^{\text{phy}}(t)) \right]. \tag{2}
\]

In the absence of production or tagging asymmetries, this reduces to the well known expression \(A^{\text{obs}}(t) = (1 - 2\omega) A^{\text{phy}}(t)\). Even here, therefore, the extraction of \(A^{\text{phy}}(t)\) requires that the mistag rate \(\omega\) be known. In the more general case it is also necessary to know \(f_0/f_\pi, \tau/\epsilon\) and \(\omega - \overline{\omega}\). Note because there is only a single final state involved, there is no dependence on any acceptance. In the following, we consider strategies to determine the tagging and production factors.

### 6.3 Use of Control Channels

#### 6.31 Introduction and Event Yields

Several channels are useful for controlling systematic biases of the type considered above. Three which are discussed here are \(B^\pm \to J/\psi K^\pm\), \(B^0_{q\bar{q}} \to J/\psi K^{0*}\) and \(B^0_{q\bar{q}} \to D^\pm\pi\). The LHC experiments expect significant event yields in these modes, as is shown in Tab. 19, with background-levels well under control. Sample invariant mass distributions for \(B^\pm \to J/\psi K^\pm\) and \(B^0_{q\bar{q}} \to J/\psi K^{0*}\) are shown in Fig. 48.

Here an approach is presented which shows how any flavour-dependent tagging-efffects and production asymmetries may be determined from these channels alone. This is to demonstrate the power of the available constraints. In practice it is envisaged that a combination of these channels, MC, and detailed detector cross-checks will be used. An example of the latter is the intention of LHCb to take data-sets with swapped dipole polarity, thereby constraining any charge-acceptance systematics.

#### 6.32 \(B^\pm \to J/\psi K^\pm\)

By reconstructing and flavour tagging \(B^\pm \to J/\psi K^\pm\) decays, the tagging efficiencies and mistag rates \(\epsilon, \overline{\epsilon}, \omega\) and \(\overline{\omega}\) may be directly measured. The expected event yields enable this to be done with annual
relative precision of a few $10^{-3}$ per experiment, which is certainly adequate for the CP asymmetry measurements. These factors can be determined in bins of tag-method, trigger-category, $p, p_T$ and rapidity, in order to account for correlations.

Comparing the number of untagged $J/\psi K^+$ and $J/\psi K^-$ events gives sensitivity to the $B^+/B^-$ production fractions $f^+/f^-$. However, what is generally of interest are the $B^0_d$ and $B^0_s$ quantities, $f^+_d/f^-_d$ and $f^+_s/f^-_s$. More importantly, any observed asymmetry may well receive contributions from direct CP violation and detector effects, and the decoupling of these factors will be very difficult. This motivates the use of other control channels.

6.3.3 $B^0_d \rightarrow J/\psi K^{0*}$

The final state of the family of modes $B^0, \overline{B^0} \rightarrow J/\psi K^{0*}$ is flavour specific to the meson at decay, therefore enabling these events to be used in a similar manner to $B^\pm \rightarrow J/\psi K^\pm$. However, the oscillation of the mesons before decay provides additional observables which may be usefully exploited.

Consider the four decay rates $R^{B^0, \overline{B^0} \rightarrow B^0, \overline{B^0}}(t)$ of genuine $B^0$ and $\overline{B^0}$ mesons into reconstructed $B^0$ and $\overline{B^0}$ final states:

$$
R^{B^0 \rightarrow B^0}(t) \propto f_0 |A|^2 a(t) (1 + \cos \Delta m t) e^{-\Gamma t}; \quad R^{\overline{B^0} \rightarrow B^0}(t) \propto \overline{f_0} |\overline{A}|^2 \overline{a}(t) (1 + \cos \Delta m t) e^{-\overline{\Gamma}t};
$$

$$
R^{B^0 \rightarrow \overline{B^0}}(t) \propto f_0 |A|^2 \overline{a}(t) (1 - \cos \Delta m t) e^{-\Gamma t}; \quad R^{\overline{B^0} \rightarrow B^0}(t) \propto \overline{f_0} |\overline{A}|^2 a(t) (1 - \cos \Delta m t) e^{-\overline{\Gamma}t},
$$

where $|A|$ and $|\overline{A}|$ represent the absolute rates of the decays, which may be different because of direct CP violation, and $a(t)$ and $\overline{a}(t)$ are acceptance factors for the two final states. Then the observed untagged decay distribution into $B^0$ events, $R^{X \rightarrow B^0}(t)$, is:

$$
R^{X \rightarrow B^0}(t) = |A|^2 a(t) \left( f_0 + \overline{f_0} \right) \left[ 1 + \frac{f_0 - \overline{f_0}}{f_0 + \overline{f_0}} \cos \Delta m t \right] e^{-\Gamma t}, \quad (3)
$$

with the conjugated expression for $R^{X \rightarrow \overline{B^0}}(t)$. Therefore evidence of any oscillation term in the untagged rates signifies an initial state production asymmetry, independent of CP violation and detector effects. Fitting this term enables the ratio $f_0/\overline{f_0}$ to be determined.

Information on the flavour-dependence of the tagging efficiency can also be obtained. The observed decay distribution for $B^0$ mesons of initial state flavour tagged $B^0$ and $\overline{B^0}$ events is $R^{X_{\text{tag}} \rightarrow B^0}(t)$, where:

$$
R^{X_{\text{tag}} \rightarrow B^0}(t) = |A|^2 a(t) \left( f_0 \epsilon + \overline{f_0} \overline{\epsilon} \right) \left[ 1 + \frac{f_0 \epsilon - \overline{f_0} \overline{\epsilon}}{f_0 \epsilon + \overline{f_0} \overline{\epsilon}} \cos \Delta m t \right] e^{-\Gamma t}. \quad (4)
$$

Thus here, and in the charge conjugated case, fitting an oscillation amplitude to the decay distribution enables the ratio $\overline{f_0} \overline{\epsilon} / f_0 \epsilon$ to be determined.

Finally, there are four decay distributions for initial state tagged $B^0$, $\overline{B^0}$ mesons decaying as $B^0$, $\overline{B^0}$, denoted by $R^{B^0, \overline{B^0}_{\text{tag}} \rightarrow B^0, \overline{B^0}}(t)$ with

$$
R^{B^0_{\text{tag}} \rightarrow B^0}(t) = |A|^2 a(t) \left( f_0 \epsilon (1 - \omega) + \overline{f_0} \overline{\epsilon} \omega \right) \left[ 1 + \frac{f_0 \epsilon (1 - \omega) - \overline{f_0} \overline{\epsilon} \omega}{f_0 \epsilon (1 - \omega) + \overline{f_0} \overline{\epsilon} \omega} \cos \Delta m t \right] e^{-\Gamma t}. \quad (5)
$$

Fitting the oscillation amplitude for $R^{B^0_{\text{tag}} \rightarrow B^0}(t)$ and $R^{B^0_{\text{tag}} \rightarrow \overline{B^0}}(t)$ and using the previous results enables $\omega/(1 - \omega)$ to be determined. The other two distributions do the same for $\omega/(1 - \omega)$. From these results $\omega$ and $\overline{\omega}$ can be fixed.

These expressions show how the necessary correction factors can be extracted from data. However, the arguments presented so far do not account for any proper time-dependence in the acceptance, which
is certainly not realistic. If the time-dependence is identical for \( a(t) \) and \( \sigma(t) \), then the extractions are still possible, as it will cancel in the ratios of say, \( R^X - B^0 \) and \( R^{\overline{X}} - B^0 \).

A still more general approach is possible, which dispenses with any assumption on the proper time and flavour-dependence of the acceptance. Consider the ratio

\[
\frac{R^{B^0_{u\bar{u}}-B^0}(t)/R^{\overline{B}^{0}_{u\bar{u}}-B^0}(t)}{R^{B^0_{u\bar{u}}-B^0}(t)/R^{\overline{B}^{0}_{u\bar{u}}-B^0}(t)} = \frac{\left[1 + \frac{1}{2} \eta \cos \Delta m t\right]}{\left[1 - \frac{1}{2} \eta \cos \Delta m t\right]} \cdot \frac{\left[1 + \frac{1}{2} \eta \cos \Delta m t\right]}{\left[1 - \frac{1}{2} \eta \cos \Delta m t\right]},
\]

where \( \eta \) is given as \( \omega f_0 / \sigma(1 - \overline{\omega} f_0) \), and \( \overline{\sigma} \) is the conjugated expression. These factors may be simultaneously fitted and combined with the \( B^\pm \to J/\psi K^\pm \) results to extract \( f_0 / f_0 \). Alternatively, they may be used directly to extract \( \sin 2\beta \) from the \( B^0_d \to J/\psi K^0 \) decay rates. Rather than constructing the conventional CP asymmetry, the ratio of the \( B^0 \) tagged and \( B^{\overline{0}} \) tagged decays may be formed:

\[
\frac{R^{B^0_{u\bar{u}}-J/\psi K^0}(t)}{R^{\overline{B}^{0}_{u\bar{u}}-J/\psi K^0}(t)} = K \left[\frac{1 - \left(\frac{1}{2} \eta \right) \sin 2\beta \sin \Delta m t}{1 + \left(\frac{1}{2} \eta \right) \sin 2\beta \sin \Delta m t}\right],
\]

where \( K \) is a normalization factor and \( \eta, \overline{\sigma} \) are the factors determined from (6). With this method, \( \sin 2\beta \) can be cleanly determined, although the need to also fit \( K \) reduces the statistical precision with respect to the conventional approach.

### 6.34 \( B^0_s \to D_s \pi \)

In controlling tagging systematics in \( B^0_s \) measurements, the values of \( \epsilon, \overline{\epsilon}, \omega \) and \( \overline{\omega} \) measured in the \( B^0_d \) channels may be used. However, constraints are required on the production ratio \( f_s / \overline{f}_s \). Here it is impracticable to use \( J/\psi K \) channels, as these are suppressed with respect to the \( B^0_d \) case. Rather it is preferable to use the decay \( B^0_s \to D_s \pi \), where no CP violation is expected. Attention must be given to detector acceptance effects in the final state, but it should prove possible to control these to the level required by the precision of \( B^0_s \) measurements.

### 6.4 Application to the \( B^0_d \to J/\psi K^0_S \) Sample

To give a quantitative impression of the precision expected from the control channels, Tab. 20 shows the results of an ATLAS study into the expected uncertainties after 3 years operation on the \( B^0_d \to B^{\overline{0}}_d \) production asymmetry, \( (f_0 - \overline{f}_0)/(f_0 + \overline{f}_0) \), and the tagging dilution, \( D = 1 - 2\omega \). \( D \) has been evaluated separately for lepton tagging and \( B-\pi \) correlation tagging (see Sec. 2.7) [56]. Uncertainties have been calculated with both the \( B^\pm \to J/\psi K^\pm \) and the \( B^0_d \to J/\psi K^0 \) samples. The study has been done in the context of the \( B^0_d \to J/\psi K^0_S \) analysis (leading to the estimate of the systematic uncertainty on the \( \sin 2\beta \) measurement given in Sec. 3.1), but the results are more general. The errors are small compared to the expected statistical uncertainty of \( \sin 2\beta \).

<table>
<thead>
<tr>
<th>Measurement</th>
<th>( B^\pm \to J/\psi K^\pm )</th>
<th>( B^0_d \to J/\psi K^0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta[(f_0 - \overline{f}_0)/(f_0 + \overline{f}_0)] )</td>
<td>0.05%</td>
<td>0.07%</td>
</tr>
<tr>
<td>( \delta D/D ) (Lepton Tagging)</td>
<td>0.0038</td>
<td>0.0047</td>
</tr>
<tr>
<td>( \delta D/D ) (B–\pi Tagging)</td>
<td>0.0030</td>
<td>0.0039</td>
</tr>
</tbody>
</table>
6.5 Other Measurements and Conclusions
The discussion so far has focused on $B^0_d \rightarrow J/\psi K^0_d$, since this is a very important measurement, with an excellent statistical precision expected. However there are other classes of measurement planned for the LHC:

- **Asymmetries involving decays to non-CP eigenstates**
  Measurements such as the determination of $\gamma$ from $B^0_d \rightarrow \pi^-\pi^+$ involve the comparison of four different decay rates, as explained in Sec. 3.42. Although there are two final states which may have different acceptances, due to detector-charge effects, the asymmetries which are formed to extract the physics unknowns do not compare these states. Therefore charge acceptance effects will not bias the measurement. Information on tagging factors and production asymmetries is obtained from the usual control channels.

- **Branching ratio comparisons**
  Methods such as the $B^0_d \rightarrow \pi K$ strategies to determine $\gamma$, described in Sec. 5.14, rely on the comparison of several branching ratios. Here it is necessary to know well the relative reconstruction efficiencies, in particular the contribution of the trigger. Although challenging, this should prove possible at a level which will be adequate alongside the statistical and theoretical uncertainties.

It can be concluded that there is no a priori reason why tagging related biases, production asymmetries or detector effects should prevent the experiments from properly exploiting the enormous $B$ statistics at the LHC.

7. $B^0\mathit{\rightarrow B}^\mathit{\bar{B}}$ MIXING

The physics of $B^0\mathit{\rightarrow B}^\mathit{\bar{B}}$ mixing is of prime importance for the study of flavour dynamics. Today, the experimental information on $B_d$ and $B_s$ mixing, i.e. the mass differences $\Delta M_d$ and $\Delta M_s$, implies already significant constraints on the unitarity triangle. A precise measurement of $\Delta M_s$, for which only a lower limit exists so far, will be an invaluable piece of information on the flavour sector of either the SM or its possible extension. Even if $\Delta M_s$ is measured before, LHC’s $B$ physics capabilities are likely to remain indispensable to fully exploit the potential of $B^0\mathit{\rightarrow B}^\mathit{\bar{B}}$ mixing. In addition to $\Delta M_s$, also the lifetime difference $\Delta \Gamma_s$ provides us with interesting opportunities. The measurement of this quantity is likewise very difficult and will be a suitable goal for the LHC $B$ physics programme.

The main theory input needed is, on the one hand, perturbative QCD corrections and, on the other hand, hadronic matrix elements of four-quark operators, schematically

$$\langle B_q | (q\Gamma \bar{b}) | \bar{B}_q \rangle,$$

where $\Gamma$, $\Gamma'$ stand for the relevant combinations of Dirac matrices and $q \in \{s,d\}$. Whereas the perturbative terms are known to NLO in QCD [144, 23], hadronic matrix elements can be obtained from first principles using lattice QCD and we start this section by an overview of the relevant lattice results. We then discuss specifically the mass and width difference $\Delta M_q$ and $\Delta \Gamma_q$ of the $B_q$ system and give predictions for the expected ranges of $\Delta M_s$ and $\Delta \Gamma_s$ in the SM. The section concludes with experimental considerations on the measurement of $B^0_s$ oscillations at the LHC.

The numerical results presented in this section are obtained using the following input parameters:

$$m_b = 4.8 \text{ GeV}, \quad \bar{m}_b(m_b) = 4.4 \text{ GeV}, \quad \bar{m}_s(m_b) = 0.1 \text{ GeV}, \quad \bar{m}_t(m_t) = 167 \text{ GeV}, \quad (1)$$

$$M_B = 5.28 \text{ GeV}, \quad M_{B_s} = 5.37 \text{ GeV}, \quad B(B_s \rightarrow Xe\nu) = 0.104,$$

and the two-loop expression for $\alpha_s$ with $\Lambda^{(5)}_{\overline{MS}} = 225 \text{ MeV}$. Above, $m_b$ is the pole mass and the barred masses refer to the $\overline{\text{MS}}$ scheme.

---

7.1 Hadronic Matrix Elements from Lattice Calculations

The matrix elements relevant for B mixing are

$$\langle B_q | \bar{q}b_{V-A}(\bar{q}b)_{V-A} | \bar{B}_q \rangle = \frac{8}{3} B_{Bq}(\mu) \bar{f}_{Bq} M_{Bq}^2,$$

(2)

$$\langle B_s | \bar{q}b_{S+P}(\bar{q}b)_{S+P} | \bar{B}_s \rangle = -\frac{5}{3} \frac{M_{Bs}^2 B_S(\mu)}{(\bar{m}_b(\mu) + \bar{m}_s(\mu))^2} \bar{f}_{Bs} M_{Bs}^2,$$

(3)

$$\langle 0 | \bar{q} \gamma_{\mu} \gamma_5 b | \bar{B}_q \rangle = i \bar{f}_{Bq} p_{\mu},$$

(4)

which are parametrized in terms of the leptonic decay constants $f_{Bq}$ and the B-parameters $B_{(B_B, S)}(\mu)$. Instead of the scale- and scheme-dependent parameter $B_{B_B}$, one usually introduces the renormalization-group invariant parameter $\tilde{B}_{Bq}$, which to NLO in QCD is given by [144, 18]

$$\tilde{B}_{Bq}^{\text{NLO}} = B_{Bq}(\mu)[\alpha_s(\mu)]^{-6/23} \left[ 1 + \frac{\alpha_s(\mu)}{4\pi} J_5 \right], \quad J_5 = \frac{5165}{3174} \quad \text{(NDR scheme).}$$

(5)

While the matrix elements (2) and (3) can be determined as such on the lattice, the dimensionless quantities $B_{Bq}$ and $M_{Bq}^2 B_S/(\bar{m}_b + \bar{m}_s)^2$ are obtained from ratios of Euclidean correlation functions in which many statistical and systematic uncertainties are expected to cancel. Thus, it is advantageous to get the matrix elements from an independent determination of the above quantities and $f_{Bq}$, combined with the experimental value of $M_{Bq}$.

Because the $b$ quark with mass $m_b \sim 5$ GeV has a Compton-wavelength that is not large compared to typical (quenched) lattice spacings, $a \sim (2 - 4)$ GeV$^{-1}$, it cannot be simulated directly as a relativistic quark on present day lattices. This has led to a variety of approaches for studying hadrons composed of a heavy quark and light degrees of freedom. In the relativistic approach, calculations are performed with a discretization of the relativistic Dirac action, for heavy quarks with masses around that of the charm and extrapolated in mass up to $m_b$, using heavy quark effective theory as a guide. There are also effective theory approaches, in which QCD is expanded in inverse powers of the $b$ quark mass. Of these, there is the static-quark approach, in which the heavy quark is treated as an infinite-mass, spin-1/2, static source of colour; a variant of this approach, in which a number of leading $1/m_b$ corrections to the static limit are included in the action, goes under the name of non-relativistic QCD or NRQCD. Finally, there is a hybrid approach in which results, calculated at $m_b$ with a relativistic action, are given a non-relativistic interpretation. While we favour the relativistic approach, which does not suffer from the typical ills of effective theories (operator proliferation and power divergences when higher-order corrections are taken into account), the different approaches should be viewed as complementary and any significant disagreement amongst them should be understood.

An important source of uncertainty in many present day lattice calculations is the quenched approximation ($N_f = 0$), in which the feedback of quarks on the gauge fields is neglected. More and more, though, groups are doing away with this approximation and are performing full QCD calculations with two flavours of sea quarks ($N_f = 2$), usually with masses around that of the strange quark. Even then, there is some way to go to reach our physical world where there are $N_f = 3$ light sea quarks: the two very light up and down quarks, and the more massive strange quark.

Because this is not the place for a full-fledged review, we will only very rarely quote individual results and rather give summary numbers, which are meant to reflect the present state of lattice calculations. The results taken into account are those obtained as of January 2000, most of which are referenced in one of the reviews in Ref. [145].

7.11 Leptonic Decay Constants

Lattice calculations of the leptonic decay constants $f_{Bq}$ have a long history and results obtained in the quenched approximation with the different approaches to heavy quarks described above are gradually
converging. The dominant systematic errors (quenching aside) depend on the approach used, but they are typically of the order of 10%.

In the past year or two, a number of groups have begun studying the effect of unquenching on decay constants by performing $N_f = 2$ calculations with a variety of approaches to heavy quarks. While these calculations are still in rather early stages, and should therefore be given time to mature, they nevertheless suggest an $\mathcal{O}(10-20\%)$ increase in $f_{B_q}$, $f_{B_s}/f_B$, however, appears to change very little, indicating that theoretical uncertainties, including the effects of quenching, cancel in such SU(3)-breaking ratios. Because systematic errors depend on the approach and parameters used, it is difficult to combine systematically results from different groups. We therefore choose to give, in Tab. 21, summary numbers for the quenched and unquenched decay constants which are meant to reflect the present situation.

Because a final number is needed for phenomenological purposes, we provide the following summary of the summaries, taking into account the fact that the unquenched results are still rather preliminary and correspond to $N_f = 2$:

\[
 f_B = (200 \pm 40) \text{ MeV}, \quad f_{B_s} = (230 \pm 40) \text{ MeV} \quad \text{and} \quad \frac{f_{B_s}}{f_B} = 1.15 \pm 0.07 .
\]  

These are the values of the decay constants to be used for numerical estimates in the subsequent subsections. The errors will certainly come down significantly once the unquenched calculations mature.

### 7.12 $B$-Parameters for $\Delta M$

The lattice calculation of these $B$-parameters is less mature than that of leptonic decay constants. Nonetheless, there have been a number of calculations over the years.

Agreement amongst calculations using the relativistic approach is good, and recent work at different values of the lattice spacing [146, 141] indicates that discretization errors are small in this approach. Agreement with the NRQCD calculation of Ref. [147] is less good. However, in matching the lattice results to $\overline{\text{MS}}$, the authors use the one-loop static instead of NRQCD coefficients, thereby inducing large systematic uncertainties. Thus, until the NRQCD results are finalized, we choose to use the relativistic results to establish our summary numbers for $B$-parameters. In any case, all methods predict that $B_{B_s}/B_B$ is very close to one.

An effect that has not yet been addressed in $B$-parameter calculations is the error associated with the quenched approximation: there exist no unquenched calculations of $B_{B_q}$ to date. However, because these parameters correspond to ratios of rather similar matrix elements, their errors are expected to be smaller than those of decay constants.

Compiling the relativistic results, we give for the $B$-parameters:

\[
 B_{B_q}(m_b) = 0.91 \pm 0.06, \quad B_{B_q}^{\text{nlo}} = 1.40 \pm 0.09 \quad \text{and} \quad \frac{B_{B_s}}{B_B} = 1.00(3) ,
\]

where we do not distinguish $q = d$ from $q = s$. The renormalization group invariant parameter $\hat{B}_{B_q}^{\text{nlo}}$ is obtained from $B_{B_q}(m_b)$ using (5) with the input parameters of (1).
The theoretical determination $\Delta M_s/\Delta M_d$ requires calculation of the non-perturbative parameter $R_{sd}$ (or $\xi$), defined as

$$
\frac{\Delta M_s}{\Delta M_d} = \left| \frac{V_{ts}}{V_{td}} \right|^2 R_{sd} = \left| \frac{V_{ts}}{V_{td}} \right|^2 \left( \frac{M_{B_s}}{M_{B_d}} \right) \xi^2.
$$

(8)

While there are at least two possible ways of obtaining $R_{sd}$ from the lattice, the most accurate and most reliable, at present, is via:

$$
R_{sd} = \left( \frac{M_{B_s}}{M_B} \right) \left( \frac{f_{B_s}}{f_B} \right)^2 \left( \frac{B_{B_s}}{B_B} \right),
$$

(9)

with $(f_{B_s}/f_B)$ and $(B_{B_s}/B_B)$ determined on the lattice and $(M_{B_s}/M_B)$ measured experimentally. The different approaches have been explored using relativistic quarks by two groups [146, 141].

Because the results obtained by these groups are fully compatible with the value of $R_{sd}$ obtained using the results (6) and (7), we quote the latter value as our summary number:

$$
R_{sd} = 1.35(17) \quad \text{or} \quad \xi = \sqrt{R_{sd} \left( \frac{M_B}{M_{B_s}} \right)} = 1.15(7).
$$

(10)

7.13 \text{ B-Parameter for } \Delta \Gamma_s

No complete calculation of $m^2_{B_s}B_S/(\bar{m}_b + \bar{m}_s)^2$ in (3) exists to date. There has been one calculation performed within the relativistic approach, but with only a single heavy quark whose mass is close to that of the charm [148]. There is also an NRQCD calculation, but where the matching of the lattice to $\overline{\text{MS}}$ is performed using the one-loop static instead of NRQCD coefficients [24]. Both are quenched.

The two results are, respectively:

$$
\frac{M_{B_s}^2B_S(m_b)}{(\bar{m}_b(m_b) + \bar{m}_s(m_b))^2} = 1.07(1) \quad \text{and} \quad 1.54(3)(24),
$$

(11)

where the first was obtained from [148] using the conversion of [23] and the masses in (1). Both these numbers should be considered preliminary, though the second does include an estimate of systematic errors. So, for the moment, we take

$$
\frac{M_{B_s}^2B_S(m_b)}{(\bar{m}_b(m_b) + \bar{m}_s(m_b))^2} = 1.4(4).
$$

(12)

The near future, however, should bring new results.

7.2 \text{ The Mass Difference } \Delta M

In the SM the $B_q$ mass difference, calculated from box diagrams with virtual top exchange, is given by

$$
\Delta M_q = \frac{G_F^2M_W^2}{6\pi^2} \eta_B S_0(x_t) M_{B_q} \bar{B}_{B_q} f_{B_q}^2 |V_{tq}|^2.
$$

(13)

Here $S_0(x_t)$, where $x_t = \bar{m}_t^2/M_W^2$, is the top-quark mass dependent Inami-Lim function for $B\to\bar{B}$ mixing. To an accuracy of better than 1%, $S_0(x_t) \simeq 0.784x_t^{0.76}$. $\eta_B$ is a correction factor describing short-distance QCD effects. It has been calculated at next-to-leading order in [144]. With the definition of $\bar{B}_{B_q}$ in (5), and employing the running mass $\bar{m}_t(m_t)$ in $S_0(x_t)$, the numerical value is $\eta_B = 0.55$ (with negligible uncertainty). Note that $\eta_B$, being a short-distance quantity, is independent of the flavour content of the B meson: it is identical for $B_d$ and $B_s$. The dependence on the light-quark flavour $q = d, s$ belongs to the non-perturbative, long-distance effects, which are isolated in the matrix element (2) [144, 18].
Experimentally, $\Delta M_q$ can be measured from flavour oscillations of neutral $B_q$ mesons. The current world average is given by [149]

$$\Delta M_d = (0.476 \pm 0.016) \text{ ps}^{-1}, \quad \Delta M_s > 14.3 \text{ ps}^{-1} \ @ 95\% \ CL. \quad (14)$$

The measurement of $\Delta M_d$ can be used to constrain $|V_{td}|$ via (13). While the short-distance quantity $\eta_B S_b(x_t)$ is known very precisely, large uncertainties are still present in the hadronic matrix element $B_{B_d} f^2_{B_d}$. Numerically,

$$|V_{td}| = 7.36 \times 10^{-3} \left[ \frac{167 \text{ GeV}}{m_t(m_t)} \right]^{0.76} \left[ \frac{237 \text{ MeV}}{f_{B_d} \sqrt{\frac{N_{\text{had}}}}{B_d}} \right]^{0.5} \Delta M_d^{0.5}. \quad (15)$$

The theoretical uncertainties are reduced considerably in the ratio $\Delta M_s/\Delta M_d$, as given in (8). With the results (14), an upper limit on $|V_{td}/V_{ts}|$ can be inferred from (8). This limit already represents a very interesting CKM constraint, which disfavors negative values of the Wolfenstein parameter $g$. A future precision measurement of $\Delta M_s$ will be a crucial input for the phenomenology of quark mixing. Using $|V_{td}/V_{ts}| > 0.17$ [5] and Eqs. (8), (10), (14), we find a SM prediction of

$$\Delta M_s = (14.3 - 26) \text{ ps}^{-1}. \quad (16)$$

### 7.3 The Width Difference $\Delta \Gamma$

$(\Delta \Gamma/\Gamma)_{B_s}$ is expected to be one of the largest rate differences in the $b$ hadron sector,$^{12}$ with typical size of (10–20)% [19, 108]. The measurement of a substantial $(\Delta \Gamma/\Gamma)_{B_s}$ would open new possibilities for CP violation studies with untagged $B_s$ mesons [25, 27, 26]. Numerically, one has, using NLO coefficients [23]:

$$\left( \frac{|\Delta \Gamma|}{\Gamma} \right)_{B_s} = \left( \frac{f_{B_s}}{230 \text{ MeV}} \right)^2 \left[ 0.007 B(m_b) + 0.132 \frac{M^2_{B_s} B_S(m_b)}{(m_b(m_b) + m_s(m_b))^2} - 0.078 \right] = 0.11(7)$$

with the B-parameters as discussed in Sec. 7.1. Note that the B-parameters are to be taken in the NDR scheme as defined in [23]. The last term in (17), −0.078, represents $1/m_b$ corrections [108] and has a relative uncertainty of at least 20%. An additional 30% scale-ambiguity from perturbation theory has not been displayed in (17).

### 7.4 Measurement of $B^0_s$ Oscillations

The probability density to observe an initial $B^0_s$ meson decaying as a $\bar{B}^0_s$ meson at time $t$ after its creation is given by:

$$P_{B^0_s \to \bar{B}^0_s}(t) = \frac{\Gamma^2_s - (\Delta \Gamma_s/2)^2}{2\Gamma_s} e^{-\Gamma_s t} \left[ \cosh \frac{\Delta \Gamma_s t}{2} + \mu \cos(\Delta M_s t) \right], \quad (18)$$

where $\mu = -1$, $\Delta \Gamma_s = \Gamma_H - \Gamma_L$ and $\Gamma_s = (\Gamma_H + \Gamma_L)/2$. If the initial $B^0_s$ meson decays as a $B^0_s$ at time $t$, the probability density $P_{B^0_s \to B^0_s}$ is given by the above expression with $\mu = +1$. Experimentally, $\Delta M_s$ can be determined by measuring the following time-dependent asymmetry:

$$A(t) = \frac{P_{B^0_s \to \bar{B}^0_s}(t) - P_{B^0_s \to \bar{B}^0_s}}{P_{B^0_s \to \bar{B}^0_s} + P_{B^0_s \to \bar{B}^0_s}} = \frac{\cos(\Delta M_s t)}{\cosh \frac{\Delta \Gamma_s t}{2}}. \quad (19)$$

$^{12}$The width difference in the $B_d$ system is Cabibbo suppressed. We thus only consider the $B_s$ sector.
The mass difference $\Delta M_s$ is $2\pi$ times the oscillation frequency. Within the SM, one has, using the formulae of [23] and the matrix elements of Sec. 7.1,\(^{13}\) suppressing a 30% renormalization-scale uncertainty,

$$\frac{|\Delta \Gamma_s|}{\Delta M_s} = (4.3 \pm 2.0) \times 10^{-3}, \quad \text{(20)}$$

which is independent of uncertainties due to CKM matrix elements. It has mainly hadronic uncertainties which are expected to decrease in the future. Therefore, within the SM, $\Delta M_s$ can in principle be inferred from a direct measurement of $\Delta \Gamma_s$, although with a large error. Small values of $\Delta \Gamma_s$ and large values of $\Delta M_s$ are difficult to measure. However, Eq. (20) implies that the smaller $\Delta \Gamma_s$ is, the easier it should be to measure $\Delta M_s$, and, inversely, the larger $\Delta M_s$ is, the easier it should be to measure $\Delta \Gamma_s$.

The effect of $\Delta \Gamma_s$ being non-zero is to damp the $B_s^0$ oscillations with a time-dependent factor. Figure 49 shows the proper time distributions of $B_s^0 \to D_s^- \pi^+$ candidates generated with two different values of $\Delta \Gamma_s$ [39]. The curves display the result of a maximum-likelihood fit to the total sample. The damping of the $B_s^0$ oscillations due to $\Delta \Gamma_s/\Gamma_s$ is not significant at the expected value of 16%, but could be important if $\Delta \Gamma_s$ turns out to be unexpectedly large. The $B_s^0$ decay-width difference can be obtained by fitting proper time distributions of untagged samples of events simultaneously for the mean

\(^{13}\)Note that according to the sign convention used in this report, (15), $\Delta \Gamma_s$ is negative in the SM.
\[ B_s^0 \] lifetime \( \tau_{B_s} = 1/\Gamma_s \) and \( \Delta \Gamma_s/\Gamma_s \). All three experiments will use their \( B_s^0 \to J/\psi \phi \) events for this measurement as described in Sec. 4.2. In addition, LHCb will have an untagged sample of \( B_s^0 \to D^- \pi^+ \) events thanks to their low-level hadronic triggers. LHCb expect to directly observe and measure \( \Delta \Gamma_s/\Gamma_s \) after one year of data-taking with their untagged \( B_s \to D^- \pi^+ \) sample, if \( \Delta \Gamma_s/\Gamma_s \) is at least 20% [39].

The B meson flavour at production and decay-time and the \( B_s^0 \) proper time with good resolution are the ingredients needed to measure \( \Delta M_s \). The best channels to make this measurement are \( B_s^0 \) decays to exclusive, flavour specific states like \( B_s^0 \to D^- \pi^+ \). The flavour of the \( B_s^0 \) at its decay is unambiguously tagged by the sign of the \( D^- \). The \( B_s^0 \) flavour at production can be determined from the sign of the decay product(s) of the other \( b \) hadron in the event. The factors which affect the sensitivity of an experiment to measure \( \Delta M_s \) are the wrong tag fraction, \( \omega_{tag} \), the presence of background and the proper time resolution, \( \sigma_t \). The corresponding dilution factors for the time-dependent asymmetry are \( D_{tag} = 1 - 2\omega_{tag} \), \( D_{bg} \approx N_{signal}/(N_{signal} + N_{bkg}) \) and \( D_{time} \approx \exp(-(\Delta M_s \sigma_t)^2 / 2) \). Here, \( N_{signal} \) and \( N_{bkg} \) are the number of signal- and background-events, respectively. The measured asymmetry is given by

\[ A_{meas}(t) = A(t) \cdot D_{tag} \cdot D_{bg} \cdot D_{time}. \]  

The amplitude fit method [150] has been used to determine the experimental reach for a \( \Delta M_s \) measurement from the time-dependent asymmetry. In this method, \( \cos (\Delta M_s t) \) is multiplied by an amplitude parameter \( A \). The value of the parameter and its error \( \sigma_A \) are determined for each \( \Delta M_s \) value by a maximum-likelihood fit. For a measurement of \( \Delta M_s \) in a region well inside the sensitivity of an experiment, the standard maximum-likelihood method is foreseen.

ATLAS [37], CMS [151, 152] and LHCb [39] have determined their sensitivities to \( \Delta M_s \) using events generated by PYTHIA [46] and then passed through detailed detector simulation. Table 22 summarizes the channels used, assumptions, performance and results of the three analyses. All three have used \( B_s^0 \to D^- \pi^+ \) and ATLAS also used \( B_s^0 \to D^- a_1^+ \) followed by \( a_1^+ \to \rho^0 \pi^+ \). The \( D^- \) is reconstructed via its decay into \( \phi \pi^- \) followed by \( \phi \to K^+K^- \) by all three experiments and also \( D^- \to K^0K^- \) followed by \( K^+\pi^- \) by CMS. CMS has assumed a 50% efficiency of the higher level triggers for calculating the final yield of reconstructed \( B_s^0 \) mesons. ATLAS also reconstructed \( D^- \to K^0K^- \), but did not include it in their final analysis since after applying the cuts needed to obtain a reasonable rate of the level-2 trigger, the addition of this mode did not improve their limit.

\( D^- \) decay modes other than \( \phi \pi^- \) contributing to the \( K^+K^-\pi^- \) final state will also be reconstructed by LHCb; for the yield presented in Tab. 22, an effective \( D^- \to K^+K^-\pi^- \) branching ratio of 4% is assumed, with the same efficiency and purity as for \( D^- \to \phi \pi^- \). For flavour tagging at production, ATLAS and CMS have used the trigger muon, which primarily comes from the semileptonic decay of the other \( b \) hadron in the event. LHCb use identified muons, electrons and kaons from the decay of the other \( b \) hadron. Other tagging techniques will be developed in the future.

Figures 50 and 51 from ATLAS illustrate the sensitivity of \( \Delta M_s \) measurements as a function of the integrated luminosity and the signal-content of the sample. 1000 experiments were performed at each \( \Delta M_s \) point and a \( \Delta M_s \) value was considered “reachable” if 95% of the experiments gave a value within \( 2\sigma \) of the input value. CMS and LHCb have defined two kinds of reaches — one for a measurement and the other one for 95% CL exclusion. Figure 52 shows the result for \( \Delta M_s \) reach from CMS using the amplitude method. The amplitude, \( A \), together with its error, \( \sigma_A \), is shown for different \( x_s \), where \( x_s \equiv \Delta M_s/\Gamma_s \). \( x_s \) values below the intersection point of the 1.645 \( \sigma_A \) curve and the line \( A = 1 \) are excluded at 95% CL. CMS determined their reach by a method similar to that used by ATLAS, but an experiment was considered “successful” if the \( x_s \) value corresponded to the highest peak in the amplitude spectrum and was in the vicinity of \( x_s^{true} \) within the natural width (± 1.5 in \( x_s \)) of the amplitude distribution. The two methods yielded the same results. Figure 53 shows the statistical significance \( S = 1/\sigma_A \) of the \( B_s^0 \) oscillation signal as a function of \( \Delta M_s \) from LHCb. The LHCb reach for \( \Delta M_s \) quoted in Tab. 22 is for \( S = 5 \) (5\( \sigma \) measurement) and \( S = 1.645 \) (95% CL exclusion). According to these studies, \( \Delta M_s \) can be measured up to 30 ps\(^{-1} \) (ATLAS), 26 ps\(^{-1} \) (CMS) and 48 ps\(^{-1} \).
(LHCb) with one year of data. The addition of more channels is likely to improve the reach. Thus, each of the three experiments will be able to fully explore the $\Delta M_s$ range allowed in the SM, Eq. (16), after one year of data-taking. In addition, the likely precision on $\Delta M_s$ will be such that the extraction of $|V_{ts}/V_{td}|^2$ will be limited by the theoretical uncertainty on $R_{sd}$ (see expressions (8) to (10)).

8. RARE DECAYS$^{14}$

Flavour-changing neutral current decays involving $b \to s$ or $b \to d$ transitions occur only at loop-level in the SM, come with small exclusive branching ratios $\sim O(10^{-5})$ or smaller and thus provide an excellent probe of indirect effects of new physics and information on the masses and couplings of the virtual SM or beyond-the-SM particles participating. Within the SM, these decays are sensitive to the CKM matrix elements $|V_{ts}|$ and $|V_{td}|$, respectively; a measurement of these parameters or their ratio would be complementary to their determination from B mixing, discussed in Sec. 7..

The effective field theory for $b \to s(d)$ transitions is universal for all the channels discussed here. Due to space-restrictions, we cannot review all important features of that effective theory; for a quick overview we refer to Chapter 9 of the BaBar Physics Book [6], where also references to more detailed reviews can be found. Here we simply state that the effective Hamiltonian governing rare decays can be obtained from the SM Hamiltonian by performing an operator product expansion yielding

$$\mathcal{H}_{\text{eff}}^{q} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{tq}^{\ast} \sum_{i=1}^{11} C_i(\mu)\mathcal{O}_i^{q}(\mu),$$  \hspace{1cm} (1)

where the $\mathcal{O}_i^{q}$ are local renormalized operators and $V_{tb}V_{tq}^{\ast}$ are CKM matrix elements with $q = s, d$. The Wilson-coefficients $C_i$ can be calculated in perturbation theory and encode the relevant short-distance physics, in particular any potential new-physics effects. The renormalization-scale $\mu$ can be viewed as separating the long- and short-distance regimes. For calculating decay rates with the help of (1), the value of $\mu$ has to be chosen as $\mu \sim m_b$ in a truncated perturbative expansion. The Hamiltonian (1) is suitable to describe physics in the SM as well as in a number of its extensions, for instance the minimal SUSY model. The operator basis in (1) is, however, not always complete, and in some models, for instance those exhibiting left-right symmetry, new physics also shows up in the form of new operators. This proviso should be kept in mind when analysing rare B decays for new-physics effects by measuring Wilson-coefficients.

At present, the following channels have been evaluated for LHCb, CMS and ATLAS:

- purely muonic $B_{da,s}^0 \to \mu^+\mu^-$ (all experiments);
- the radiative decay $B_d^0 \to K^*\gamma$ (LHCb only);
- semimuonic decays $B_{d}^0 \to \rho^0\mu^+\mu^-$, $B_{d}^0 \to K^*0\mu^+\mu^-$, $B_{s}^0 \to \phi0\mu^+\mu^-$ (all experiments).

As a reflection of this rather preliminary status of rare B decay studies for the LHC, we confine this section’s discussion to channels most of which are in principle accessible at $e^+e^-$ B factories and can also be studied at the Tevatron. This applies in particular to the radiative decay $B \to K^*\gamma$ that has already been measured at CLEO [153] and for which at the time of the first physics runs at the LHC rather accurate measurements should be available. The situation is different for $B \to \mu^+\mu^-$, which will be seen before the start of the LHC only if it is enhanced drastically, i.e. by orders of magnitude, by new-physics effects. Also the measurement of the spectra of $B \to K^*\mu^+\mu^-$ will be reserved to the LHC, although the decay itself should be seen at the B factories before. In general, and in contrast to the exploration of CP violation, the main impact of the LHC on the study of rare decays will be to provide radically increased statistics rather than opening new, alternative channels.

$^{14}$Section coordinators: P. Ball and F. Rizatdinova.
Fig. 49: Fraction of events tagged as having oscillated as a function of proper time for two different values of \( \Delta \Gamma_s / \Gamma_s \), for \( \Delta M_s = 10 \text{ ps}^{-1} \) [39]. The curves display the result of the maximum-likelihood fit to the total sample.

Fig. 50: \( \Delta M_s \) reach of ATLAS as a function of the integrated luminosity for various proper time resolutions \( \sigma_t \).

Fig. 51: \( \Delta M_s \) reach of ATLAS as a function of the signal content of the sample for nominal proper time resolution and integrated luminosities of 10 fb\(^{-1}\) and 30 fb\(^{-1}\).

Fig. 52: The amplitude with its error, \( \sigma_A \), for an input value of \( x_s = 30 \) from CMS.

Fig. 53: Statistical significance of the \( B_s^0 \) oscillation signal as a function of \( \Delta M_s \). The band delimited by the two curves reflects the 1\( \sigma \) statistical uncertainty on the proper-time resolution of \( \sigma_t = (43 \pm 2) \text{ fs} \).
8.1 $B^0 \rightarrow \mu^+ \mu^-$

This decay is an experimental favourite thanks to its unique signature and at the same time a challenge, as its SM branching ratio is of order $10^{-9}$. The motivation for measuring this decay lies mainly in its rôle as indicator for possible new physics which might significantly enhance the branching ratio. The present experimental bounds from Tevatron are in the $10^{-6}$ range.

8.11 Theoretical Framework

The purely muonic neutral B decays are described by only three operators [154]:

$$O'_P = (\bar{q} \gamma_5 b)(\bar{\mu} \gamma_5 \mu), \quad O'_{P'} = (\bar{q} \gamma_5 b)(\bar{\mu} \mu), \quad O'_A = (\bar{q} \gamma^\alpha \gamma_5 b)(\bar{\mu} \gamma_\alpha \gamma_5 \mu),$$

with $q = s, d$. In the SM, these transitions proceed through electroweak penguin diagrams with $Z$ and $H^0$ exchange as well as $W$ box diagrams. Introducing dimensionless Wilson-coefficients $C^q_{P, A}$, the branching ratio is given by

$$B(B_q \rightarrow \mu^+ \mu^-) = \frac{G_F^2}{8\pi} \frac{\tau_B}{m_B} f_B^2 m_B^3 \left( 1 - \frac{4m_\mu^2}{m_B^2} \right) \left\{ C_P^q - \frac{2m_\mu}{m_B} C_A^q \right\}^2 + \left( 1 - \frac{4m_\mu^2}{m_B^2} \right) \left| C_{P'}^q \right|^2. \quad (2)$$

In the SM, the coefficients $C_P$ arise from penguin diagrams with physical and unphysical neutral scalar exchange and are suppressed by a factor $(m_b/m_W)^2$ [155]. The decay rate is then determined solely by the coefficient

$$C_{A, SM}^q = \frac{\alpha V_{tb} V_{tq}^*}{\sqrt{8\pi} \sin^2 \theta_w} Y(x_t), \quad (3)$$

where $x_t \equiv m_t^2/m_W^2$, $\sin^2 \theta_w$ is the weak mixing angle and the function $Y(x)$ is at leading order in QCD given by [156]

$$Y(x) = \frac{x}{8} \left( \frac{x - 4}{x - 1} + \frac{3x}{(x - 1)^2} \ln x \right). \quad (4)$$

The SM branching fractions are then given by (with $f_{B_q}$ from (6), $|V_{tq}|$ from (15) and $m_t = 167$ GeV)

$$B(B_d \rightarrow \mu^+ \mu^-) = (1.0 \pm 0.5) \times 10^{-10} \left[ \frac{f_{B_d}}{200 \text{ MeV}} \right]^2 \left[ \frac{m_t(m_t)}{167 \text{ GeV}} \right]^{3.12} \left[ \frac{|V_{td}|}{0.0074} \right]^2 \left( \frac{\tau_{B_d}}{1.56 \text{ ps}} \right), \quad (5)$$

$$B(B_s \rightarrow \mu^+ \mu^-) = (3.7 \pm 1.0) \times 10^{-9} \left[ \frac{f_{B_s}}{230 \text{ MeV}} \right]^2 \left[ \frac{m_t(m_t)}{167 \text{ GeV}} \right]^{3.12} \left[ \frac{|V_{ts}|}{0.040} \right]^2 \left( \frac{\tau_{B_s}}{1.54 \text{ ps}} \right). \quad (6)$$

Due to these tiny SM branching ratios and the favourable experimental signature, these decay processes are ideal candidates for new physics to be observed, for example flavour-changing neutral Higgses. New-physics scenarios have been investigated e.g. in Refs. [155, 157].

8.12 Experimental Considerations

Purely muonic B decays, so-called "self-triggering" channels, have a clear signature that can be used at level-1 trigger in all LHC experiments. Only muon identification is necessary. The expected numbers of events quoted in the following refer to the SM branching ratios $B(B^0_s \rightarrow \mu^+ \mu^-) = (3.5 \pm 1.0) \times 10^{-9}$ and $B(B^0_d \rightarrow \mu^+ \mu^-) = 1.5 \times 10^{-10}$, i.e. the "optimistic" end of the theory prediction (5).

The CMS collaboration has performed a detailed study of the observability of $B^0 \rightarrow \mu^+ \mu^-$ [158] at both low and high luminosity, implementing the complete pattern recognition and track reconstruction procedure. Both the gluon-fusion and the gluon-splitting production mechanisms are included and yield comparable contributions. CMS has tuned the experimental selection criteria to optimize the signal-to-background ratio as follows:
1. Only muon pairs satisfying the requirement \(0.4 < \Delta R_{\mu\mu} < 1.2\) were considered as candidates for \(B_s^0 \rightarrow \mu^+ \mu^-\); the transverse momentum of the muon pair must be larger than 12 GeV and \(p_T\) of either muon be larger than 4.3 GeV.

2. The effective mass of the dimuon pair was required to be within a 80 MeV mass window around the nominal \(B_s^0\) mass. Only 1.1% of background combinations are retained after this mass-cut.

3. The third set of cuts is based on the secondary vertex reconstruction: the distance between \(B_s^0\) and primary vertex in the transverse plane is required to be larger than 12\(\sigma_{\text{vtx}}\), about 820 \(\mu m\), where \(\sigma_{\text{vtx}}\) is the vertex resolution. The angle \(\alpha\) between the line joining primary and secondary vertex and transverse momentum vector was required to satisfy \(\cos\alpha > 0.9997\). The absolute error of the secondary vertex reconstruction was required to be less than 80 \(\mu m\). The distance between the two muons, \(d_2\), had to be smaller than 50 \(\mu m\) and the ratio \(d_2/\sigma(d_2)\) smaller than 2.

4. Isolation of the dimuon pair in the tracker was required, i.e. no charged particles with \(p_T > 0.9\) GeV must be found in the cone \(R < 0.5 \times \Delta R_{\mu\mu} + 0.4\) around the dimuon momentum direction. The isolation requirement is important for suppressing the background induced by gluon-splitting. About 50% of the signal- and 3% of the background-events passed through the isolation-cut in the tracker. An additional factor 2.3 of background-suppression was obtained by requiring isolation of the dimuon pair in the calorimeters, i.e. the transverse energy in the electromagnetic and hadron calorimeters was required to be less than 4 GeV in the same tracker cone.

After applying these cuts, the number of expected events detected by CMS after 3 years running at low luminosity is 21 with less than 3 background-events at 90% C.L., assuming the SM branching ratio. CMS will observe this channel even after 1 year running at low luminosity. Taking into account the production ratio \(B_s^0/B_s^0 = 0.40/0.11\) and the expected SM branching ratio (5), CMS also expects, for three years running at low luminosity, to find \(2.2 \pm 1.1\) \(B_s \rightarrow \mu^+ \mu^-\) events with again essentially no background.

LHCb’s sensitivity to the decay \(B_s \rightarrow \mu^+ \mu^-\) has been studied using fully GEANT generated samples of both signal- and background-events. Good quality tracks are combined into a vertex if they are identified as muon tracks with high confidence level and are within 50 \(\mu m\) in space. The secondary vertex must also satisfy quality criteria and be well displaced from the primary vertex. The impact parameter of the reconstructed \(B_s\) candidate is required to be smaller than 35 \(\mu m\) and a mass window of 20 MeV around the nominal \(B_s\) mass is applied. After all those selection-cuts 11 signal-events per year are expected. Since the initial background-sample was very small compared to the number of events in one year of LHCb operation, pions which are a direct product of B decays were allowed to make pairs with muons, “faking” the background-signature, in order to increase the statistics of the sample. Using this procedure, it was possible to estimate the rejection power of the cuts in the impact parameter and the mass of the \(B_s\) candidate, assuming they are uncorrelated and that the mass distribution in a mass window of 200 MeV around the nominal value is flat. The expected background-yield in one year is 3.3 events. Studies with high statistics samples of full GEANT simulation are under way, in order to make the background-estimate more precise. Hence LHCb will observe the decay \(B_s \rightarrow \mu^+ \mu^-\) within 1 year of running.

The ATLAS collaboration has made a detailed study of the decay mode \(B_s^0 \rightarrow \mu^+ \mu^-\), using fully simulated samples [37]. To suppress the combinatorial background, cuts on the quality of vertex reconstruction and on the decay length of the reconstructed B meson were applied. Further background-reduction was obtained by imposing cuts on the angle between the line joining primary and secondary vertex and the transverse momentum vector and on the isolation of the dimuon pair formed in the decay of the B meson. The mass resolution obtained after all selection-cuts is \(\sigma(M) = 68\) MeV. The mass window \(\frac{2\sigma}{1\sigma}\) was taken for estimating the number of signal- and background-events. After applying cuts, the number of expected events detected by ATLAS after 3 years running at low luminosity, assuming the SM branching fraction, is 27 with 93 background-events. For \(B_s^0 \rightarrow \mu^+ \mu^-\), one can expect 4 signal-events with 93 background-events.

Hence, all three experiments will be able to measure the SM branching fraction of \(B_s^0 \rightarrow \mu^+ \mu^-\).
The numbers of events expected by the three collaborations after 3 years’ data collection are given in Tab. 23.

Both ATLAS and CMS are planning to continue the study of purely muonic decays at high luminosity $10^{34}$ cm$^{-2}$ s$^{-1}$. This is made possible by the low dimuon trigger rate which is expected to be around 30 Hz in ATLAS. In both experiments, the number of minimum bias events accepted together with the triggered events is expected to be 10 times larger than at the LHC run at low luminosity. The CMS collaboration estimated the possibility to detect the purely muonic decay using a high luminosity pixel configuration that leads to degradation of the vertex resolution. The ATLAS collaboration assumed that the geometry of the inner detector will be the same as at low luminosity (no degradation in vertex and $p_T$ resolution is expected compared to the low luminosity results). The same analysis-cuts as at low luminosity were applied to the signal- and background-events by both collaborations. The resulting numbers of events expected by the ATLAS and CMS collaborations after 1 year running at high luminosity are given in Tab. 24, assuming the SM branching fraction. The decay $B^0_s \to \mu^+\mu^-$ can clearly be observed after 1 year running at high luminosity by both collaborations. Concerning $B^0_{d} \to \mu^+\mu^-$, the sensitivity of ATLAS to the branching ratio will be at the level of $\pm 10^{-5}$, i.e. roughly a factor 3 above the SM prediction. High luminosity measurements of the purely muonic decays would significantly improve the data to be obtained at low luminosity.

### 8.2 $B \to K^*\gamma$

In this subsection we discuss the specifics of the radiative FCNC transition $B \to K^*\gamma$ relevant for the LHC, concentrating on non-perturbative QCD effects. For the treatment of perturbative issues, in particular the reduction of renormalization-scale dependence and remaining uncertainties, we refer to [159, 160].

#### 8.21 Theoretical Framework

The theoretical description of the $B \to K^*\gamma$ decay is quite involved with regard to both long- and short-distance contributions. In terms of the effective Hamiltonian (1), the decay amplitude can be written as

$$A(B \to K^*\gamma) = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \langle K^*\gamma | C_7 O_7 + i e_\mu \sum_{i \neq 7} C_i \int d^4 x \epsilon^{\mu \nu \rho \sigma} T \{j_{\mu m}^{\text{em}} (x) O_i (0) \} | B \rangle,$$

(7)

where $j_{\mu m}^{\text{em}}$ is the electromagnetic current and $e_\mu$ the polarization vector of the photon. $O_7$ is the only operator containing the photon field at tree-level:

$$O_7 = \frac{e}{16\pi^2 m_b \bar{s}_\mu \bar{\nu}_\mu R_b F^{\mu \nu}},$$

(8)

with $R = (1 + \gamma_5)/2$. Other operators, the second term in (7), contribute mainly closed fermion loops. The first complication is now that the first term in (7) depends on the regularization- and renormalization-scheme. For this reason, one usually introduces a scheme-independent linear combination of coefficients, called “effective coefficient” (see [160] and references therein):

$$C_7^{\text{eff}}(\mu) = C_7(\mu) + \sum_{i=3}^{6} y_i C_i(\mu),$$

(9)
where the numerical coefficients $y_i$ are given in [160].

The current-current operators

\begin{align}
O_1 = (\bar{s}\gamma^\mu L b)(\bar{c}\gamma^\mu L c), \quad O_2 = (\bar{s}\gamma^\mu L c)(\bar{c}\gamma^\mu L b)
\end{align}

give vanishing contribution to the perturbative $b \to s\gamma$ amplitude at one loop. Thus, to leading logarithmic accuracy (LLA) in QCD and neglecting long-distance contributions from $O_{1,2}$ to the $b\bar{s}\gamma X$ Green’s functions, the $\bar{B} \to \bar{K}^*\gamma$ amplitude is given by

\begin{align}
A^{\text{LLA}}_{O_1}(\bar{B} \to \bar{K}^*\gamma) &= -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* C_7^{(0)\text{eff}} \langle \bar{K}^*\gamma | O_7 | \bar{B} \rangle.
\end{align}

Here, $C_7^{(0)\text{eff}}$ denotes the leading logarithmic approximation to $C_7^{\text{eff}}$. The above expression is, however, not the end of the story, as the second term in (7) also contains long-distance contributions. Some of them can be viewed as the effect of virtual intermediate resonances $\bar{B} \to \bar{K}^*V^* \to \bar{K}^*\gamma$. The main effect comes from $c\bar{c}$ resonances and is contributed by the operators $O_1$ and $O_2$ in (7). It is governed by the virtuality of $V^*$, which, for a real photon, is just $-1/m_{V^*}^2 \sim -1/4m_c^2$. The presence of such power-suppressed terms $\sim 1/m_{V^*}^2$ has first been derived for inclusive decays in Ref. [161] in a framework based on operator product expansion. The first, and to date only, study for exclusive decays was done in [162]. Technically, one performs an operator product expansion of the correlation function in (7), with a soft non-perturbative gluon being attached to the charm-loop, resulting in terms being parametrically suppressed by inverse powers of the charm quark mass. As pointed out in [163], although the power increases for additional soft gluons, it is possible that contributions of additional external hard gluons could remove the power-suppression. This question is also relevant for inclusive decays and deserves further study.

After inclusion of the power-suppressed terms $\sim 1/m_c^2$, the $\bar{B} \to \bar{K}^*\gamma$ amplitude reads

\begin{align}
A^{\text{LLA}}(\bar{B} \to \bar{K}^*\gamma) &= -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \langle \bar{K}^*\gamma | C_7^{(0)\text{eff}} O_7 + \frac{1}{4m_c^2} C_2^{(0)} O_F | \bar{B} \rangle.
\end{align}

Here, $O_F$ is the effective quark-quark-gluon operator obtained in [162], which describes the leading non-perturbative corrections. The two hadronic matrix elements can be described in terms of three form factors, $T_1$, $L$, and $\tilde{L}$:

\begin{align}
\langle \bar{K}^*(p)\gamma | \bar{s}\sigma_{\mu\nu}q^\nu | \bar{B}(p_B) \rangle &= i\epsilon_{\mu\nu\rho\sigma} \epsilon^{\ast\mu}_{\ast\nu} \epsilon^{\ast\rho}_{\ast\sigma} p_B^\rho p_B^\sigma 2T_1(0),
\end{align}

\begin{align}
\langle \bar{K}^*(p)| O_F | \bar{B}(p_B) \rangle &= \frac{e}{36\pi^2} \left[ L(0) \epsilon_{\mu\nu\rho\sigma} \epsilon^{\ast\mu}_{\ast\nu} \epsilon^{\ast\rho}_{\ast\sigma} p_B^\rho p_B^\sigma 
\right.

\left. + i\tilde{L}(0) \left\{ (\epsilon^{\ast\mu}_{K\ast}\epsilon^{\ast\nu}_{p_B})(\epsilon^{\ast\rho}_{p_B}) - \frac{1}{2} (\epsilon^{\ast\mu}_{K\ast}\epsilon^{\ast\nu}_{p_B})(m_B^2 - m_{K\ast}^2) \right\} \right].
\end{align}

The calculation of the above form factors requires genuinely non-perturbative input. Available methods include, but do not exhaust, lattice calculations and QCD sum rules. Again, a discussion of the respective strengths and weaknesses of these approaches is beyond the scope of this report. Let it suffice to say that – at least at present – lattice cannot reach the point $(p_B - p)^2 = 0$ relevant for $B \to K^*\gamma$, and that QCD sum rules on the light-cone predict [35]

\begin{align}
T_1(0) = 0.38 \pm 20\%.
\end{align}

at the renormalization scale $\mu = 4.8$ GeV. For the other two form factors, QCD sum rules predict [162]

\begin{align}
L(0) = (0.55 \pm 0.10) \text{ GeV}^3, \quad \tilde{L}(0) = (0.7 \pm 0.1) \text{ GeV}^3.
\end{align}
Numerically, these corrections increase the decay rate by about 5 to 10%. After their inclusion, one obtains

\[
B(B \rightarrow K^*\gamma) = \frac{\alpha}{32\pi^4} G_F^2 |V_{tb}V_{tb}^*|^2 \left| C_{7}^{(0)\text{eff}} \right|^2 m_b \left( \frac{m_B^2 - m_{K^*}^2}{m_b^2} \right)^3 \left| T_1(0) \right|^2 \\
\times \left( 1 - \frac{1}{18m_c^2} \frac{C_{2}^{(0)\text{eff}}}{C_{7}^{(0)\text{eff}}} \frac{1}{m_b} \frac{L(0) + \bar{L}(0)}{T_1(0)} \right) \\
= 4.4 \times 10^{-5} (1 + 8\%)
\]

for the central values of the QCD sum rule results, which agrees with the experimental measurement.

Let us close this subsection with a few remarks on the decay $B \rightarrow \rho \gamma$. Although at first glance it might seem that its structure is the same as that of $B \rightarrow K^*\gamma$, this is actually not the case. There are additional long-distance contributions to $B \rightarrow \rho \gamma$, which are CKM-suppressed for $B \rightarrow K^*\gamma$ and have been neglected in the previous discussion; these contributions comprise

- weak annihilation mediated by $O_{1,2}^u$ with non-perturbative photon emission from light quarks; these contributions are discussed in [164] and found to be of order 10% at the amplitude level;
- effects of virtual $u\bar{u}$ resonances ($\rho$, $\omega$, . . .); they are often said to be small, but actually have not been studied yet in a genuinely non-perturbative framework, so that statements about their smallness lack proper justification.

For the above reasons it is, at present, premature to aim at an accurate determination of $|V_{ts}|/|V_{td}|$ from a measurement of $B(B \rightarrow \rho \gamma)$ and $B(B \rightarrow K^*\gamma)$. A very recent discussion of long-distance effects in $B \rightarrow V \gamma$ decays can also be found in Ref. [165].

### 8.22 Experimental Considerations

The radiative decay $B_d^0 \rightarrow K^{*0}\gamma$ has been studied by the LHCb collaboration at both the particle and the full-simulation level [166]. The event selection and reconstruction can be summarized as follows:

- selection: $X^+X^-\gamma$ combinations; tracks are consistent with $K^-$- and $\pi^+$-hypotheses; $|M(K^-\pi^+)-M(K^{*0})| < 55$ MeV; cluster in the electromagnetic calorimeter with $E_T > 4$ GeV;
- geometrical cuts: $\chi^2 < 9$ of secondary vertex fit; $|\Delta[Z]| > 1.5$ mm between primary and secondary vertex; impact parameters of both tracks > 400$\mu$m; the angle between the momentum vector and the line joining primary and secondary vertex smaller than 0.1 rad; the angle $\theta$ between $B_d^0$ and $K^*$ in the $K^{*0}$ rest frame $|\cos \theta| < 0.6$;
- $p_T > 4$ GeV of reconstructed $B_d^0$.

The mass resolution obtained at the particle-level study is 67 MeV. The mass window taken for estimates is 200 MeV around the nominal $B_d^0$ mass. Assuming $B(B_d^0 \rightarrow K^{*0}\gamma) = (4.9 \pm 2.0) \times 10^{-5}$, the expected number of signal-events after 1 year running is 26000, with $S/B \sim 1$. This will be sufficient to measure the branching fraction with high accuracy. The expected accuracy in the CP asymmetry measurement is $\delta_{CP} = 0.01$. The SM predicts a CP asymmetry of order 1%.

### 8.3 $B \rightarrow K^*\mu^+\mu^-$

Like with $B \rightarrow K^*\gamma$, we can only review the essentials and put emphasis on recent developments in theory and the specifics for the LHC experiments. A slightly more detailed discussion and relevant references can be found in the BaBar physics book [6]. The current state-of-the-art of perturbation theory is summarized in Ref. [167]. The motivation for studying this decay is either, assuming the SM to be correct, the measurement of the CKM matrix element $|V_{ts}|$, or the search for manifestations of new physics in non-standard values of the Wilson-coefficients. A very suitable observable for the latter purpose is the forward-backward asymmetry which is independent of CKM matrix elements and, due to
extremely small event numbers, only accessible at the LHC. Of all the rare decay channels discussed in this section, \( B \to K^* \mu^+ \mu^- \) is definitely the one whose detailed study is only possible at the LHC and which has the potential for high impact both on SM physics and beyond.

### 8.31 Theoretical Framework

The presentation in this section follows closely Ref. [168]; for other relevant recent papers treating \( B \to K^* \mu^+ \mu^- \), see [169].

At the quark-level, the effective Hamiltonian (1) leads to the following decay amplitude:

\[
A(b \to s \mu^+ \mu^-) = \frac{G_F \alpha}{\sqrt{2\pi}} V_{ts} V_{tb} \left\{ C_9^{\text{eff}}(s) \left[ \bar{s} \gamma_\alpha L b \right] \left[ \bar{\mu} \gamma^\alpha \mu \right] + C_{10} \left[ \bar{s} \gamma_\alpha L b \right] \left[ \bar{\mu} \gamma^\alpha \gamma_5 \mu \right] \right. \\
-2m_q C_7^{\text{eff}} \left[ \bar{s} \gamma_\alpha \gamma^\nu \frac{q^\nu}{s} \bar{R} b \right] \left[ \bar{\mu} \gamma^\alpha \mu \right] \right\}. 
\] (17)

Here, \( L/R = (1 \mp \gamma_5)/2 \), \( s = q^2 \), \( q = p_+ + p_- \), where \( p_\pm \) are the four-momenta of the leptons. We neglect the strange quark mass, but keep the leptons massive. Already the free quark decay amplitude \( A(b \to s \mu^+ \mu^-) \) contains certain long-distance effects which usually are absorbed into a redefinition of the Wilson-coefficient \( C_9 \). To be specific, we define, for exclusive decays, the momentum-dependent effective coefficient of the operator \( \mathcal{O}_9 = \epsilon^2/(16\pi^2) (\bar{s} \gamma_\alpha L b) (\bar{\mu} \gamma^\alpha \mu) \) as

\[
C_9^{\text{eff}}(s) = C_9 + Y(s), 
\] (18)

where \( Y(s) \) stands for matrix elements of four-quark operators. Formulas can be found in [170]. The prominent contribution to \( Y(s) \) comes from the \( c \bar{c} \) resonances \( J/\psi, \psi^\prime, \psi^\prime \prime \), which show up as peaks in the dimuon spectrum, but are irrelevant for the short-distance physics one is interested in. Note that the effective coefficient depends on the process being considered and is, in particular, not the same for exclusive and inclusive decays: in the latter ones, also virtual and bremsstrahlung corrections to \( \langle \mu^+ \mu^- s | \mathcal{O}_9 | b \rangle \), usually denoted by \( \omega(s) \), are included, whereas for exclusive decays, they are contained in the hadronic matrix elements to be defined below.

For \( s \) far below the \( c \bar{c} \) threshold, perturbation theory, augmented by non-perturbative power-corrections in \( 1/m_c^2 \), is expected to yield a reliable estimate for long-distance effects in \( C_9^{\text{eff}} \). In contrast to inclusive decays, however, the corresponding \( 1/m_c^2 \) terms have not yet been worked out for exclusive decays. To date, one has to rely on phenomenological prescriptions for incorporating non-perturbative contributions to \( Y(s) \) [171]. The resulting uncertainties on \( C_9^{\text{eff}} \) and on various distributions in inclusive decays have been worked out in Refs. [170, 167] to which we refer for a detailed discussion.

Other long-distance corrections, specific for the exclusive decay \( B \to K^* \mu^+ \mu^- \), are described in terms of matrix elements of the quark operators in (17) between meson states and can be parametrized in terms of form factors. Denoting by \( \epsilon_\mu \) the polarization vector of the \( K^* \) vector meson, we define

\[
\langle K^*(p) | (V - A)_\mu | B(p_B) \rangle = -i \epsilon^*_\mu (m_B + m_{K^*}) A_1(s) + i (p_B + p) \mu (\epsilon^*_p p_B) - \frac{A_2(s)}{m_B + m_{K^*}} \epsilon_\mu (p_B) \frac{2m_{K^*}}{s} (A_3(s) - A_0(s)) + \epsilon_\mu \epsilon_\nu \epsilon^*_\nu (p_B B^\sigma) \frac{2Y(s)}{m_B + m_{K^*}}, \\
\langle K^*(p) | \bar{s} \gamma_\mu \gamma^\nu (1 + \gamma_5) b | B(p_B) \rangle = i \epsilon_\mu \epsilon_\nu \epsilon^*_\nu (p_B B^\sigma) \frac{2T_1(s)}{m_B + m_{K^*}} \epsilon_\nu (p_B) \left( \epsilon^*_p p_B (p_B + p) \mu \right) + T_2(s) \left\{ \epsilon_\mu (m_B^2 - m_{K^*}^2) - (\epsilon^*_p p_B (p_B + p) \mu) \right\} + T_3(s) (\epsilon^*_p p_B) \right\} 
\] (19)

with

\[
A_3(s) = \frac{m_B + m_{K^*}}{2m_{K^*}} A_1(s) - \frac{m_B - m_{K^*}}{2m_{K^*}} A_2(s), \quad A_0(0) = A_3(0), \quad T_1(0) = T_2(0).
\]
The form factors $T_i$ are renormalization-scale dependent. All signs are defined in such a way as to render the form factors positive. The physical range in $s$ extends from $s_{\text{min}} = 0$ to $s_{\text{max}} = (m_B - m_{K^*})^2$. As described in the last subsection for the $B \rightarrow K^*\gamma$ form factor $T_1$, the above form factors are essentially non-perturbative. Lacking results from lattice calculations, we quote the form factors as calculated from QCD sum rules on the light-cone [126, 35], in the parametrization suggested in [168], where also a discussion of the theoretical uncertainties can be found. The form factors can be parametrized as

$$F(s) = F(0) \exp(c_1 \hat{s} + c_2 \hat{s}^2)$$  \hspace{1cm} (20)

with $\hat{s} = s/m_B^2$. The central values of the parameters $c_i$ are given in Tab. 25.

Let us now turn to the various decay distributions relevant for the phenomenological analysis. For lack of space, we cannot give detailed expressions for decay amplitudes and spectra in terms of the hadronic matrix elements (19); they can be found in [168]. Besides the total branching fraction and the spectrum in the dimuon mass, it is in particular the forward-backward asymmetry that is interesting for phenomenology. It is defined as

$$A_{FB}(s) = \frac{1}{d\Gamma/ds} \left( \int_0^1 d(\cos \theta) \frac{d^2\Gamma}{ds d\cos \theta} - \int_{-1}^0 d(\cos \theta) \frac{d^2\Gamma}{ds d\cos \theta} \right),$$  \hspace{1cm} (21)

where $\theta$ is the angle between the momenta of the B meson and the $\mu^+$ in the dilepton centre-of-mass system. The asymmetry is governed by

$$A_{FB} \propto C_{10} \left[ \text{Re} \left( C_9^{\text{eff}} V(s) A_1(s) + \frac{\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \{ V(s) T_2(s) (1 - \hat{m}_{K^*}) + A_1(s) T_1(s) (1 + \hat{m}_{K^*}) \} \right) \right].$$  \hspace{1cm} (22)

In the SM, $A_{FB}$ exhibits a zero at $s = s_0$, given by

$$\text{Re} \left( C_9^{\text{eff}}(s_0) \right) = - \frac{\hat{m}_b}{\hat{s}} C_7^{\text{eff}} \left\{ \frac{T_2(s_0)}{A_1(s_0)} (1 - \hat{m}_{K^*}) + \frac{T_1(s_0)}{V(s_0)} (1 + \hat{m}_{K^*}) \right\}. $$  \hspace{1cm} (23)

The forward-backward asymmetry has a zero if and only if

$$\text{sign}(C_7^{\text{eff}} \text{Re} C_9^{\text{eff}}) = -1.$$  \hspace{1cm} (24)

It is interesting to observe that in the Large Energy Effective Theory (LEET) [172], both ratios of the form factors appearing in Eq. (23) have essentially no hadronic uncertainty, i.e. all dependence on intrinsically non-perturbative quantities cancels, and one has simply

$$\frac{T_2(s)}{A_1(s)} = \frac{1 + \hat{m}_{K^*}}{1 + \hat{m}_{K^*}^2 - \hat{s}} \left( 1 - \frac{\hat{s}}{1 - \hat{m}_{K^*}^2} \right), \quad \frac{T_1(s)}{V(s)} = \frac{1}{1 + \hat{m}_{K^*}}.$$  \hspace{1cm} (25)

These relations are fulfilled by QCD sum rules on the light-cone to 2% accuracy, which indicates that corrections to the LEET limit are extremely small. In that limit, one thus has a particularly simple form for the equation determining $s_0$, namely

$$\text{Re}(C_9^{\text{eff}}(s_0)) = -2 \frac{\hat{m}_b}{s_0} C_7^{\text{eff}} \frac{1 - \hat{s}_0}{1 + \hat{m}_{K^*}^2 - \hat{s}_0}.$$  \hspace{1cm} (26)

<table>
<thead>
<tr>
<th>$F(0)$</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_0$</th>
<th>$V$</th>
<th>$T_1$</th>
<th>$T_2$</th>
<th>$T_3$</th>
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<td>0.337</td>
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<td>1.172</td>
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<td>1.482</td>
<td>1.519</td>
<td>0.517</td>
<td>1.129</td>
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<tr>
<td>$c_2$</td>
<td>0.258</td>
<td>0.567</td>
<td>0.710</td>
<td>1.015</td>
<td>1.030</td>
<td>0.426</td>
<td>1.128</td>
</tr>
</tbody>
</table>

Table 25: Central values of parameters for the parametrization (20) of the $B \rightarrow K^*$ form factors. Renormalization scale for $T_i$ is $\mu = m_B$. 

Thus, the precision of the zero of the forward-backward asymmetry in $B \to K^* \mu^+ \mu^-$ is determined essentially by the precision of the ratio of the effective coefficients and $m_b$ and is largely independent of hadronic uncertainties. The insensitivity of $s_0$ to the decay form factors in $B \to K^* \mu^+ \mu^-$ is a remarkable result, which has first been observed in [173] by scanning over a number of form factor models. LEET puts this observation on theoretically more rigorous grounds, although (25), and consequently (26) are expected to be modified by hard perturbative QCD corrections. In the SM, one finds $s_0 \approx 2.9 \text{ GeV}^2$ at $\mu = 4.8 \text{ GeV}$. From Eq. (23) it also follows that there is no zero below the $c\bar{c}$ resonances if both $C_{\text{eff}}$ and $C_{\text{eff}}^T$ have the same sign as predicted in some beyond-the-SM models. Thus, condition (24) provides a discrimination between the SM and certain models with new physics. Due to space limitations we cannot discuss in detail the possible impact of particular beyond-the-SM scenarios on the decay distributions introduced above. To illustrate the fact that large effects are indeed possible, we show, in Figs. 54 and 55, the results for the dimuon spectrum and the forward-backward asymmetry obtained in [168] for several SUSY-extensions of the SM.

Note that the above formulas and considerations cannot immediately be applied to the decay $B \to \rho \mu^+ \mu^-$, whose measurement could, in principle, together with that of $B \to K^* \mu^+ \mu^-$, be used to determine the ratio of CKM matrix elements $|V_{ts}/V_{td}|$, as an alternative to the determination from B mixing. The problem lies in new contributions to $C_{\text{eff}}^T$ originating from light-quark loops and associated with the presence of low-lying resonances, for instance $\rho$ and $\omega$, in the dimuon spectrum. These contributions are CKM-suppressed in $B \to K^* \mu^+ \mu^-$, so that the corresponding uncertainties can be neglected, but they are unsuppressed in $B \to \rho \mu^+ \mu^-$ decays. The problematic part in that is that the theory tools that allow one to treat $c\bar{c}$ resonance contributions to $B \to K^* \mu^+ \mu^-$ are not applicable any more: perturbation theory does only work in the unphysical region $s < 0$, and an operator-product expansion which would indicate potential power-suppressed terms also fails. No satisfactory solution to that problem is presently available.

Finally, we note that the analysis of $B_s \to \phi \mu^+ \mu^-$ parallels exactly that of $B_d \to K^* \mu^+ \mu^-$; the corresponding form factors can be found in Ref. [35]. Also semimuonic decays with a pseudoscalar meson in the final state, e.g. $B_d \to K \mu^+ \mu^-$ and $B_d \to \pi \mu^+ \mu^-$, are, from a theoretical point of view, viable sources for information on short-distance physics and CKM matrix elements. Their experimental detection is, however, extremely difficult due to the overwhelming combinatorial background; no experimental feasibility studies exist to date.

8.32 Experimental Considerations

As with $B \to \mu^+ \mu^-$, the semimuonic decays $B_t^0 \to K^* \mu^+ \mu^-$ are ”self-triggering” channels thanks to the presence of two muons with high $p_T$ in the final state. Particle identification helps decisively in sepa-
and ISGW2, were implemented into PYTHIA and the final numbers of expected events after trigger-cuts for the case of the phase-space decay, GI and ISGW2 parametrizations. It was found that the matrix were evaluated for these two samples of signal-events. The dimuon mass distribution is shown in Fig. 56 of the recent developments in the theoretical calculation of hadronic matrix elements as discussed in hadronic uncertainties, see e.g. [175], these papers tend to underestimate the uncertainty associated with observing these three channels are shown in Fig. 57.

The ATLAS collaboration has studied the decays $B_d^0 \rightarrow \rho^0 \mu^+\mu^-$, $B_d^0 \rightarrow K^{*0} \mu^+\mu^-$ and $B_s^0 \rightarrow \phi^0 \mu^+\mu^-$. All these channels were fully simulated and reconstructed in the inner detector. As possible background, the following reactions have been considered: $B_d^0$ meson decays to $J/\psi K_S^0$, $\omega^0 \mu^+\mu^-$, reflection of $B_d^0 \rightarrow \rho^0 \mu^+\mu^-$ and $B_s^0 \rightarrow K^{*0} \mu^+\mu^-$ to other signal-channels; $B_s^0$ meson decays to $K^{*0}(\phi) \mu^+\mu^-$, semimuonic decays of one of the $b$ quarks and semimuonic decays of both $b$ quarks. An additional minimum bias of 2.4 events in the precision tracker and 3.2 events in the transition radiation tracker were taken into account when studying the signal and background. The expected results for observing these three channels are shown in Fig. 57.

Assuming the SM to be valid, the measurement of the branching fractions of the decays $B_d^0 \rightarrow \rho^0 \mu^+\mu^-$ and $B_d^0 \rightarrow K^{*0} \mu^+\mu^-$ gives, in principle, the possibility to extract the ratio of the CKM elements $|V_{td}|/|V_{ts}|$ using the following equation:

$$\frac{N(B_d^0 \rightarrow \rho^0 \mu^+\mu^-)}{N(B_d^0 \rightarrow K^{*0} \mu^+\mu^-)} = k_d |V_{td}|^2/|V_{ts}|^2.$$

(27)

The quantity $k_d$ depends on form factors and Wilson-coefficients and also on the experimental cuts. Although there exist claims in the literature that, with proper cuts, $k_d$ may be calculated with small hadronic uncertainties, see e.g. [175], these papers tend to underestimate the uncertainty associated with the impact on $c\bar{c}$ resonances on the spectrum (for $B_d^0 \rightarrow \rho^0 \mu^+\mu^-$, there are also $u\bar{u}$ resonances whose contributions are often completely ignored). Our present knowledge of these long-distance effects in rating the final-state hadrons. All three experiments assume the branching ratio $B(B_d^0 \rightarrow K^{*0} \mu^+\mu^-) = 1.5 \times 10^{-6}$ for estimating the number of events to be observed.

ATLAS have investigated form factor effects on the detection of $B_d^0 \rightarrow K^{*0} \mu^+\mu^-$; details of the analysis can be found in [174]. Two different parametrizations of the hadronic matrix elements (19), GI and ISGW2, were implemented into PYTHIA and the final numbers of expected events after trigger-cuts were evaluated for these two samples of signal-events. The dimuon mass distribution is shown in Fig. 56 for the case of the phase-space decay, GI and ISGW2 parametrizations. It was found that the matrix elements practically do not change the inclusive parameters of the muons and the $K^{*0}$ meson, which is important for triggering these events. They do, however, strongly influence the spectrum in the dimuon mass and the forward-backward asymmetry. Although quark model calculations of form factors like GI and ISGW2 may serve as rough guidelines for first estimates, they do not reflect the modern state-of-the-art of theoretical calculations. For this reason, it is important to extend existing studies, taking advantage of the recent developments in the theoretical calculation of hadronic matrix elements as discussed in Sec. 8.31, and in particular to use only such model calculations that reproduce the model-independent results for certain form factor ratios like (25).
Table 26: Expected precision for asymmetry measurements at ATLAS and LHCb, for 3 and 1 years running, respectively, at low luminosity and assuming SM branching ratios; the experimental numbers rely on [176] and the theoretical predictions on the form factors in the GI parametrization and MSSM parameters as discussed in [177]. The kinematic limits are given by

\[
\delta_{\text{min}} = \frac{4m_{\mu}^2}{m_B^2} \quad \text{and} \quad \delta_{\text{max}} = \frac{(m_B - m_{K^*})^2}{m_B^2}.
\]

The precision for asymmetry measurements in three different \(s\) intervals was estimated by ATLAS. The data are presented in Tab. 26, together with asymmetry values in the SM and one exemplary SUSY model, integrated over the corresponding intervals in \(s = s/m_B^2\). The expected accuracy of the asymmetry measurement with the ATLAS detector will be sufficient to distinguish between the SM and some of its extensions. It should, however, be stressed that new-physics effects do not change \(A_{FB}\) dramatically as compared to the SM.

LHCb has also performed an analysis of \(B_d^0 \rightarrow \bar{K}^{*0}\mu^+\mu^-\). The matrix elements reproducing the correct dimuon mass distribution were implemented into PYTHIA. The detector response for both signal- and background-events was simulated and the charged particles were reconstructed in the detector. LHCb expects to observe 4500 \(B_d^0 \rightarrow 
abla K^{*0}\mu^+\mu^-\) events per year. For background-studies, the following reactions were simulated with PYTHIA: \(B_d^0 \rightarrow K^{+}\mu^+\mu^-\), \(B_d^0 \rightarrow J/\psi(K^{*0}, K_0^0, \phi, K^+)\), with the subsequent decay of \(J/\psi\) into two muons, inclusive \(B \rightarrow 4\pi, b \rightarrow \mu X, \bar{b} \rightarrow \mu X\) and \(B \rightarrow \mu D(\mu X)X\). The total number of background-events is estimated to be 280. The large signal-statistics with very low backgrounds gives a nice possibility to study this channel in detail. LHCb also evaluated the sensitivity of \(A_{FB}\) measurements. The results are shown in the Tab. 26. Promising results were obtained by LHCb for measuring the position of the zero of \(A_{FB}\), \(s_0\) with \(A_{FB}(s_0) = 0\). As discussed in the previous subsection, the position of the zero is proportional to the ratio of two Wilson-coefficients, \(C_9^{\text{eff}}/C_7^{\text{eff}}\), with only small hadronic uncertainties from form factors. Note, however, that it is the effective Wilson-coefficients that determine \(s_0\) and that these coefficients encode both short-distance SM and – potentially – new-physics effects and long-distance QCD effects, which latter ones do come with a certain hadronic uncertainty that to date has not been investigated in sufficient detail. LHCb simulated the expected measurements of the asymmetry, see Fig. 58, and made a linear fit of the "experimental points". It is shown that \(s_0\) can be measured with 25% accuracy, which leads to a 4% error in extracting the ratio \(C_9^{\text{eff}}/C_7^{\text{eff}}\).

The CMS collaboration studied three rare B meson decay channels, \(B_d^0 \rightarrow \bar{K}^{*0}\mu^+\mu^-\), \(B_d^0 \rightarrow
Fig. 58: LHCb fit of the FB-asymmetry $A_{FB}$ for $B \rightarrow K^+\mu^+\mu^-$ around the zero $\delta_0$ with $A_{FB}(\delta_0) = 0$. Squares denote generated and dots reconstructed data (one year statistics). The linear fit of reconstructed data intersects at $0.138 \pm 0.035$ (only statistical error).

<table>
<thead>
<tr>
<th>Channel</th>
<th>$B$</th>
<th>ATLAS</th>
<th>CMS</th>
<th>LHCb</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B^0_d \rightarrow \rho^0 \mu^+\mu^-$</td>
<td>$10^{-4}$</td>
<td>222</td>
<td>950</td>
<td>not yet estimated</td>
</tr>
<tr>
<td>$B^0_d \rightarrow K^{*0} \mu^+\mu^-$</td>
<td>$1.5 \times 10^{-6}$</td>
<td>1995</td>
<td>290</td>
<td>12600</td>
</tr>
<tr>
<td>$B^0_s \rightarrow \phi^0 \mu^+\mu^-$</td>
<td>$10^{-6}$</td>
<td>411</td>
<td>140</td>
<td>3600</td>
</tr>
</tbody>
</table>

Table 27: Expected signal- and background-statistics for rare semimuonic decays, for 3 years’ running of ATLAS and CMS at low luminosity and 5 years’ of LHCb. The CMS simulation was done at the particle level only.

The numbers of signal- and background-events expected by ATLAS, CMS and LHCb are given in Tab. 27.

8.4 Inclusive Decays

The inclusive decay mode $B \rightarrow X_s \gamma$ has received much attention in connection with its measurement at CLEO, $B(B \rightarrow X_s \gamma) = (3.15 \pm 0.35 \pm 0.32 \pm 0.26) \times 10^{-4}$ [178], which should become much more accurate with data being taken at the $e^+e^-$ B factories. A state-of-the-art review on inclusive decays can be found in the corresponding chapter of the BaBar physics book [6]. The experimental environment of a hadronic machine makes it very hard to measure inclusive decays. Nevertheless, the D0 collaboration at Fermilab was able to set a 90% CL bound $B(B \rightarrow X_s \mu^+\mu^-) < 3.2 \times 10^{-4}$ [179], which should be compared to the corresponding CLEO [180] result of $5.8 \times 10^{-5}$ and the SM expectation of $6 \times 10^{-6}$. In the D0 analysis, no displaced vertex was required for the muon pair, contrary to the CDF analysis of the exclusive $B \rightarrow K^+\mu^+\mu^-$ mode [181], where a sensitivity of order $10^{-6}$ has been reached. It is an interesting question to ask whether LHC could improve the D0 result (e.g. by requiring a displaced vertex) and whether it could possibly reach the sensitivity required for measuring the branching ratios of $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \mu^+\mu^-$ as predicted in the SM.

The theoretical advantage of the inclusive decays $B \rightarrow X_s \gamma$ and $B \rightarrow X_s \mu^+\mu^-$ over particular exclusive channels lies in the fact that non-perturbative contributions to the inclusive modes can be
calculated in a model-independent way with the help of the Operator Product Expansion (OPE) within the Heavy Quark Effective Theory (HQET) [182]. Actually, this statement is true only at the leading order in hard strong interactions (i.e. in $\alpha_s(m_t)/\pi$) and only after imposing certain kinematical cuts (see e.g. [167, 163]). Even with these restrictions, the accuracy of theoretical predictions for the inclusive branching ratios is expected to be better than in the exclusive case.

The theoretical analysis of $\bar{B} \to X_s\gamma$ proceeds along the same lines as in the $\bar{B} \to K^*\gamma$ case, up to Eq. (12), where $K^*$ has to be replaced by any $S = -1$ hadronic state $X_s$. Then, the modulus squared of the amplitude is taken, and a sum over all the states $X_s$ is performed. The obtained sum can be related via the optical theorem to the imaginary part of the $\bar{B}\gamma \to \bar{B}\gamma$ elastic scattering amplitude, analogously to what is done in the analysis of $\bar{B} \to X_{u,c,e}d$ [183]. After OPE and calculating matrix elements of several local operators between $\bar{B}$ meson states at rest, one finds that the “subtracted” branching ratio

$$B(\bar{B} \to X_s\gamma)^{\text{subtracted}} \equiv B(\bar{B} \to X_s\gamma)_{E_r > E_{\text{cut}}} - B(\bar{B} \to X_{\text{no charm}}^{(1)}\gamma) \times B(\psi \to X_{\text{no charm}}^{(2)}\gamma)$$

(29)

is given in terms of the purely perturbative $b$ quark decay width, up to small non-perturbative corrections

$$\frac{\Gamma(\bar{B} \to X_s\gamma)^{\text{subtracted}}}{\Gamma(\bar{B} \to X_{c,e}d)} \simeq \frac{\Gamma(b \to X_s\gamma)^{\text{perturbative NLO}}}{\Gamma(b \to X_{c,e}d)^{\text{perturbative NLO}}} \times \left[ 1 + (O(\Lambda^2/m_b^2) \simeq 1\%) + (O(\Lambda^2/m_b^2) \simeq 3\%) \right].$$

(30)

The normalization to the semileptonic rate has been used here to cancel uncertainties due to $m_b^\beta$, CKM-angles and some of the non-perturbative corrections. One has to keep in mind that (30) becomes a bad approximation for $E_{\text{cut}} \ll 1$ GeV, and that non-perturbative corrections grow dramatically when $E_{\text{cut}} > 2$ GeV. Moreover, non-perturbative effects arising at $O(\alpha_s(m_b))$ are not included in (30). Estimating the size of such non-perturbative effects requires further study, see Ref. [163]. For $E_{\text{cut}} = 1$ GeV, Eq. (30) gives

$$B(\bar{B} \to X_s\gamma)^{\text{subtracted}} = (3.29 \pm 0.33) \times 10^{-4},$$

(31)

where the dominant uncertainties originate from the uncalculated $O(\alpha_s^2)$ effects and from the ratio $m_c/m_b$ in the semileptonic decay (around 7% each).

The calculation of $\bar{B} \to X_s\mu^+\mu^-$ for small dimuon invariant mass is conceptually analogous to $\bar{B} \to X_s\gamma$, but technically more complicated, because more operators become important. Here, we shall quote only the numerical estimate [167]

$$B(\bar{B} \to X_s\mu^+\mu^-)_{s \in [0.05, 0.25] m_b^2} = (1.46 \pm 0.19) \times 10^{-6},$$

(32)

where only the error from $\mu$-dependence of the perturbative amplitude is included.

### 8.5 Conclusions

The LHC experiments will be able to make precise measurements of rare radiative, semimuonic and muonic B decays. ATLAS and CMS will measure rare decays in the central $\eta$ region, which will be complementary to the data to be taken by LHCb. A first assessment of LHC’s potential to measure rare B decays has shown that it will be possible to

- observe $B_s^0 \to \mu^+\mu^-$, measure its branching ratio, which is of order $10^{-9}$ in the SM, and perform a high sensitivity search for $B_{d}^0 \to \mu^+\mu^-$;
- measure the branching ratio and decay characteristics of $B_{d}^0 \to K^{*0}\gamma$ at LHCb;
- measure the branching ratios of $B_{d}^0 \to \phi^0\mu^+\mu^-$, $B_{d}^0 \to \rho^0\mu^+\mu^-$ and $B_{d}^0 \to K^{*0}\mu^+\mu^-$ and study the dynamics of these decays;
- measure the FB-asymmetry in $B_{d}^0 \to K^{*0}\mu^+\mu^-$, which allows the distinction between the SM and a large class of SUSY models.
Studying rare muonic decays at high luminosity with the ATLAS and CMS detectors would significantly improve the results that can be obtained at low luminosity.

Open questions to be discussed in the future include:

- assessment of the combined performance of LHC experiments on rare muonic and semimuonic decays;
- studies of CP asymmetries in rare semileptonic B decays at LHC;
- evaluation of the potential of ATLAS, CMS and LHCb to measure inclusive $B_{d,s}^0 \to X\mu^+\mu^-$ branching ratios;
- detection of rare decays with a $\tau$ in the final state;
- feasibility study for measuring semimuonic decays with a pseudoscalar meson in the final state, e.g. $B_d \to \pi\mu^+\mu^-$, $B_d \to K\mu^+\mu^-$. 

From the theory point of view, the most urgent question left open is the precise assessment of long-distance effects both in the radiative B decays $B \to (K^*, \rho)\gamma$ and in the semimuonic ones, encoded in the effective Wilson-coefficient $C_{\text{eff}}$; the lack of knowledge of these effects limits the precision with which CKM matrix elements and short-distance coefficients can be extracted from semimuonic decays. Other tasks remaining are the improvement of form factor calculations, for instance from lattice, and the parametrization of form factors in a form that includes as much known information on the positions of poles and cuts as possible. Also, the possible size of CP asymmetries in semimuonic decays deserves further study; only few papers treat that subject, see e.g. [184].

9. THEORETICAL DESCRIPTION OF NON-LEPTONIC DECAYS

Exclusive non-leptonic B decays form an important part of LHC’s B physics programme and at the same time pose a big challenge for theory. In the standard approach using an effective weak Hamiltonian, non-leptonic decay amplitudes are reduced to products of short-distance Wilson-coefficients and hadronic matrix elements. The calculation of the latter ones requires genuine knowledge of non-perturbative QCD and is often done in the so-called factorization approximation, where a matrix element over typically a four-quark operator is “factorized” into a product of matrix elements over current operators, which are much easier to calculate:

$$\langle J/\psi K_S \mid (\bar{c}\gamma_\mu c)(\bar{s}\gamma^\mu b) \mid B \rangle \to \langle J/\psi \mid (\bar{c}\gamma_\mu c) \mid 0 \rangle \times \langle K_S \mid (\bar{s}\gamma^\mu b) \mid B \rangle.$$ 

The factorization approximation is, of course, not exact and the assessment of “non-factorizable contributions”, including FSI phases, is a fundamental problem of strong interactions, which affects both the extraction of weak phases from CP asymmetries, like $A_{\text{CP}}(B \to \pi\pi)$, and the determination of CKM angles or new physics from rare decays. Whereas in Secs. 3 to 5 a pragmatic approach has been presented which aims at constraining strong-interaction effects from experiment, it remains a big challenge for theory to predict these effects from first principles. For this reason we devote a separate section to review several ansätze for solving or rather approaching the problem, although it is to be admitted that a complete solution is still far beyond our power. In three subsections we discuss the calculation of non-factorizable contributions to $B \to J/\psi K^{(*)}$ from QCD sum rules on the light-cone [185, 186], a method for obtaining information on the strong phase in $B \to \pi\pi$ from dispersion relations [187] and, finally, an approach that applies the methods developed for hard exclusive QCD reactions to certain B decays in the heavy quark limit $m_b \to \infty$ [76]. We would like to stress, however, that the problem of how to calculate non-factorizable contributions and, in particular, FSI phases, is very challenging indeed and that a lot of theory work remains to be done. We thus can present, instead of a coherent picture, only facettes, albeit scintillating ones.

---

9.1 Non-Factorizable Contributions to $B \to J/\psi K^{(*)}$

The non-factorizable contributions to the amplitudes of $B \to J/\psi K^{(*)}$ decays have recently been estimated [185, 186] using operator product expansion (OPE) and QCD light-cone sum rules. In this subsection, we outline the main results of this study.

With the effective Hamiltonian (1), the matrix element of $B \to J/\psi K^{(*)}$ has the following form:

$$
\langle K^{(*)} J/\psi \mid H_{\text{eff}}^s \mid B \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left( C_1(\mu) + \frac{C_2(\mu)}{3} \right) \langle K^{(*)} J/\psi \mid O_1^{s}(\mu) \mid B \rangle 
+ \frac{1}{2} C_2(\mu) \langle K^{(*)} J/\psi \mid \tilde{O}_1^{s}(\mu) \mid B \rangle ,
$$

(1)

The explicit form of the four-quark operators $O_1^{s}$ is given in (10). The operator

$$
\tilde{O}_1^{s} = (\bar{c} \Gamma^\rho \lambda^a \frac{1}{2} c) (\bar{s} \Gamma^\rho \lambda^a \frac{1}{2} b)
$$

with $\Gamma^\rho = \gamma^\rho(1 - \gamma_5)$ originates from the Fierz rearrangement of $O_2^{s}$. In the factorization approximation, the matrix elements of $\tilde{O}_1^{s}$ vanish and the matrix elements of $O_1^{s}$ are split into the product

$$
\langle K^{(*)} J/\psi \mid O_1^{s}(\mu) \mid B \rangle = \frac{1}{4} \langle J/\psi \mid \bar{c} \Gamma^\rho c \mid 0 \rangle \langle K^{(*)} \mid \bar{s} \Gamma^\rho b \mid B \rangle ,
$$

(2)

involving similar matrix elements of quark currents: $\langle 0 \mid \bar{c} \Gamma^\rho c \mid J/\psi(p) \rangle = f_{\psi} m_{\psi} \epsilon^\rho_{\psi}$ and

$$
\langle K(p) \mid \bar{s} \Gamma^\rho b \mid B(p_B) \rangle = f_+ (s)(p_{Bp} + p_p) + f_- (s) q_{p}.
$$

(3)

The form factor decomposition of the matrix element $\langle K^{(*)}(p) \mid \bar{s} \Gamma^\rho b \mid B(p_B) \rangle$ can be found in (19). In the above, $q = p_B - p$, $s = q^2$, $f_{\psi}$ is the $J/\psi$ decay constant, $\epsilon_{\psi}$, $\epsilon_{K^*}$ are the polarization vectors of $J/\psi$ and $K^*$, respectively, and $f_{\pm}$ are the form factors for $B \to K$. For the numerical analysis we use the form factors as calculated from QCD sum rules on the light-cone [188, 35, 126, 168].

The short-distance coefficients $C_{1,2}(\mu)$ and the matrix elements entering (1) are scale-dependent, whereas the decay constants and form factors determining the right-hand side of (2) are physical scale-independent quantities. Therefore, factorization can at best be an approximation valid at one particular scale. In fact, in both $B \to J/\psi K$ and $B \to J/\psi K^*$, factorization does not work at $\mu = \mathcal{O}(m_b)$ and is unable to reproduce both partial widths and their ratio as can be seen from Tab. 28. Factorization in these channels has to be generalized by replacing the short-distance coefficient $C_1(\mu) + C_2(\mu)/3$ by effective coefficients $a_2$ which are supposed to be scale-independent and incorporate possible non-factorizable effects. The most general decomposition of the matrix elements in (1) includes one effective coefficient for $B \to J/\psi K$ and three for $B \to J/\psi K^*$ (one for each partial wave):
Table 28: \(B \to J/\psi K^{(*)}\) decay characteristics calculated in naive factorization approximation, neglecting non-factorizable contributions and taking \(C_{1,2}(\mu)\) from [6] in NLO at (a) \(\mu = m_b\), (b) \(\mu = m_b/2\) and compared with experiment. The intervals of theoretical predictions reflect the uncertainties in the \(B \to K\) and \(B \to K^*\) form factors taken from [168].

<table>
<thead>
<tr>
<th>Decay Parameter</th>
<th>(a) (\pm 0.2)</th>
<th>(b) (\pm 0.09)</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Gamma(B \to J/\psi K) ) ((\text{in } 10^8 \text{ sec}^{-1}))</td>
<td>1.0 (\div 1.5)</td>
<td>0.15 (\div 0.2)</td>
<td>(5.8 \pm 0.8 ) ([B^0]) [64] (6.1 \pm 0.6 ) ([B^+]) [64]</td>
</tr>
<tr>
<td>(\Gamma(B \to J/\psi K^*) ) ((\text{in } 10^8 \text{ sec}^{-1}))</td>
<td>3.9 (\div 6.0)</td>
<td>0.6 (\div 0.9)</td>
<td>(9.7 \pm 1.1 ) ([B^0]) [64] (9.0 \pm 1.6 ) ([B^+]) [64]</td>
</tr>
<tr>
<td>(\Gamma(B \to J/\psi K^*)/(\Gamma(B \to J/\psi K)) (P_L = \Gamma_L/\Gamma)</td>
<td>2.6 (\div 6.2)</td>
<td>0.475 (\div 0.465)</td>
<td>1.45 (\pm 0.26) [107] (0.52 \pm 0.08) [107] (0.65 \pm 0.11) [86, 107]</td>
</tr>
</tbody>
</table>

Now we turn to describing how these coefficients can be estimated theoretically. The main non-factorizable contributions to \(a_2\) come from the matrix elements of \(\bar{O}_1\), which are parametrized in the form [185]

\[
\langle K J/\psi \mid \bar{O}_1^2(\mu) \mid B \rangle = 2 f_\psi m_\psi \bar{f}(\mu) (\epsilon^*_\psi \cdot p),
\]

\[
\langle K^* J/\psi \mid \bar{O}_1^2(\mu) \mid B \rangle = m_\psi f_\psi \epsilon^*_\psi \left[ -\frac{i}{m_B + m_{K^*}} \epsilon^*_{K^*} \mu \bar{A}_1(s) \right.
+ \frac{i}{m_B + m_{K^*}} \epsilon^*_{K^*} (p_B + p) \rho \bar{A}_2(s)
+ \left. 2 \frac{\epsilon_{\mu \alpha \beta} \epsilon^*_{K^*} \epsilon_{K^*} \rho^\beta q^\alpha}{m_B + m_{K^*}} \bar{V}(s) \right].
\]

\(a_{2}^{B\psi K}\) can be expressed as

\[
a_{2}^{B\psi K} = C_1(\mu) + \frac{C_2(\mu)}{3} + 2C_2(\mu) \frac{\bar{f}(\mu)}{f_+(m_\psi^2)} + \ldots
\]

and similar expressions for \(a_{2,1}^{B\psi K^*}\), \(a_{2,2}^{B\psi K^*}\) and \(a_{2,V}^{B\psi K^*}\) with the ratios \(\bar{A}_1/A_1(m_\psi^2), \bar{A}_2/A_2(m_\psi^2)\) and \(\bar{V}/V(m_\psi^2)\), respectively, replacing \(\bar{f}(\mu)/f_+(m_\psi^2)\). In the above, the ellipses denote neglected non-factorizable contributions of \(O_2^2\), which are supposed to be subdominant. The non-factorizable amplitudes \(\bar{f}_{B\psi K}, \bar{A}_{1,2}\) and \(\bar{V}\) have been estimated in Ref. [186] following the approach suggested in Ref. [189] and using OPE. In this report we do not have the space to explain the method in detail, but simply state the results. At the current level of accuracy, one predicts the following ranges of non-factorizable amplitudes:

\[
\bar{f}(\mu_0) = -(0.06 \pm 0.02), \quad \bar{A}_1(\mu_0) = 0.0050 \pm 0.0025, \quad \bar{A}_2(\mu_0) = -(0.002 \pm 0.001), \quad \bar{V}(\mu_0) = -(0.09 \pm 0.04).
\]

These estimates reveal substantial non-universality in absolute values and difference in signs of the non-factorizable amplitudes. Although the ratios of these amplitudes to form factors, e.g. \(\bar{f}(\mu_0)/f_+(m_\psi^2)\) \(\approx 0.1\) are small, they have a strong impact on the coefficients \(a_2\) because of a strong cancellation in \(C_1(\mu_0) + C_2(\mu_0)/3 \approx 0.055, \mu_0 = 2m_c = 2.6\text{ GeV}\) (which is numerically close to \(m_b/2\)) being the relevant scale in the process. From (8) and the corresponding relations for the other \(a_2\), we obtain:

\[
a_{2}^{B\psi K} = -(0.09 \div 0.23), \quad a_{2,1}^{B\psi K^*} = 0.07 \div 0.09, \quad a_{2,2}^{B\psi K^*} = 0.04 \div 0.05, \quad a_{2,V}^{B\psi K^*} = -(0.05 \div 0.26),
\]
where an additional $\pm(10 \div 20) \%$ uncertainty from the form factors should be added. Although in comparison with the experimental numbers for $|a_2^{B\psi K}|$ and $|a_2^{B\psi K^*}|$, the estimates (11) fall somewhat short, the gap between naive factorization at $\mu = m_t/2$ and experiment is narrowed considerably. Note also that the sum rule estimates for $a_2^{B\psi K}$ and $a_2^{B\psi K^*}$ yield negative sign for these two coefficients in contradiction to the global fit of the factorized decay amplitudes to the data [138], yielding a universal positive $a_2$. For $a_2^{B\psi K^*}$ and $a_2^{B\psi K}$, the estimates in (11) are not very conclusive in view of the large experimental uncertainties of these two coefficients. Clearly, further improvements in the sum rules are needed to achieve more accurate estimates. Nevertheless, the above calculation has demonstrated that for future theoretical studies of exclusive non-leptonic decays of heavy mesons QCD sum rule techniques provide new ways to go beyond factorization.

9.2 Dispersion Relations for B Non-Leptonic Decays into Light Pseudoscalar Mesons

Rescattering effects in non-leptonic B decays into light pseudoscalar mesons were investigated in [187] by the method of dispersion relations in terms of the external masses. Defining the weak decay amplitude $A_{B \rightarrow P_1 P_2} = A(m_B^2, m_1^2, m_2^2)$, where $P_1, P_2$ are pseudoscalar mesons, one can show [187] that the weak amplitude satisfies the following dispersion representation:

$$ A(m_B^2, m_1^2, m_2^2) = A^{(0)}(m_B^2, m_1^2, m_2^2) + \frac{1}{\pi} \int_0^\infty dz \frac{\text{Disc} A(m_B^2, z, m_2^2)}{z - m_1^2 - i\epsilon}.$$ (12)

The first term in this representation is the amplitude in the factorization limit, while in the second term the dispersion variable is the mass squared of the meson which does not contain the spectator quark. The representation (12) allows one to recover the amplitude in the factorization approximation when the strong rescattering is switched off, which is a reasonable consistency condition. As shown in [187], in the two-particle approximation (12) can be written as

$$ A_{B \rightarrow P_1 P_2} = A^{(0)}_{B \rightarrow P_1 P_2} + \frac{1}{2} \sum_{\{P_3 P_4\}} \Gamma_{P_3 P_4; P_1 P_2} \tilde{A}_{B \rightarrow P_3 P_4} + \frac{1}{2} \sum_{\{P_3 P_1\}} \Gamma_{P_3 P_1; P_2 P_4} A_{B \rightarrow P_3 P_4}.$$ (13)

In this relation $A^{(0)}_{B \rightarrow P_1 P_2}$ is the amplitude in the factorization limit, $\tilde{A}_{B \rightarrow P_3 P_4}$ is obtained from $A_{B \rightarrow P_3 P_4}$ by changing the sign of the strong phases, the coefficients $\Gamma_{P_3 P_4; P_1 P_2}$ are computed as dispersive integrals

$$ \Gamma_{P_3 P_4; P_1 P_2} = \frac{1}{\pi} \int_0^\infty dz \frac{C_{P_3 P_4; P_1 P_2}(z)}{z - m_1^2 - i\epsilon},$$ (14)

and $\Gamma_{P_3 P_4; P_1 P_2}$ are defined as in (14), with the numerator $C$ replaced by $C^*$, where

$$ C_{P_3 P_4; P_1 P_2}(z) = \frac{1}{2} \int \frac{d^3k_3}{(2\pi)^3 2\omega_3} \frac{d^3k_4}{(2\pi)^3 2\omega_4} (2\pi)^4 \delta^{(4)}(p - k_3 - k_4) \mathcal{M}_{P_3 P_4 \rightarrow P_1 P_2}(s, t).$$ (15)

The strong amplitudes $\mathcal{M}_{P_3 P_4 \rightarrow P_1 P_2}(s, t)$ entering this expression are evaluated for an off-shell meson $P_1$ of mass squared equal to $z$, at the c.m. energy squared $s = m_B^2$, which is high enough to justify the application of Regge-theory. A detailed calculation [187] takes into account both the $t$-channel trajectories describing the scattering at small angles and the $u$-channel trajectories describing the scattering at large angles.

Let us apply the dispersive formalism to the decay $B^0 \rightarrow \pi^+ \pi^-$, taking as intermediate states in the dispersion relation (13) the pseudoscalar mesons $\pi^+ \pi^-, \pi^0 \pi^0, K^+ K^-, K^0 \bar{K}^0, \eta_8 \eta_8, \eta_1 \eta_1$ and $\eta_1 \eta_8$. Assuming SU(3) flavour-symmetry and keeping only the contribution of the dominant quark topologies,
the dispersion relation (13) becomes an algebraic equation involving tree and penguin amplitudes, $A_T$ and $A_P$. With the Regge parameters discussed in [187], this relation can be written as

$$A(B^0 \to \pi^+\pi^-)/A_T = e^{i\gamma} + R e^{i\delta} e^{-i\beta}$$

$$= \frac{e^{-i\delta_T}}{A_T} \left( A_T^{(0)} e^{i\gamma} + A_P^{(0)} e^{-i\beta} - \left( 0.01 + 1.27 i \right) + \left( 0.75 - 1.01 i \right) e^{-2i\delta_T} \right) e^{i\gamma}$$

$$+ R \left[ \left( 1.97 + 2.64 i \right) e^{i\delta} - \left( 1.79 - 2.00 i \right) e^{-i\delta} e^{-2i\delta_T} \right] e^{-i\beta}. \quad (16)$$

Here $A_T^{(0)}$ and $A_P^{(0)}$ are the amplitudes in the factorization approximation, $R = |A_P/A_T|$ and $\delta = \delta_P - \delta_T$, $\delta_T (\delta_P)$ being the strong phase of $A_T (A_P)$, respectively. It is seen that the weak angles appear in the combination $\gamma + \beta = \pi - \alpha$. Solving the complex equation (16) for $R$ and $\alpha$, we derive their expressions as functions of $\delta_T$ and $\delta$. The evaluation of these expressions requires also the knowledge of the ratios $A_P/A_T$ and $A_B/A_T$. In Fig. 59 we represent $R$ and $\alpha$ as functions of the phase difference $\delta$, for two values of $\delta_T$, using as input $A_P^{(0)}/A_T^{(0)} = 0.08$ and $A_B^{(0)}/A_T \approx 0.9$ [76]. Values of the ratio $R$ less than one are obtained for both $\delta_T = 0$ and $\delta_T = \pi/12$. The dominant contribution is given by the elastic channel, more precisely by the Pomeron, as is seen in Fig. 59, where the dotted curve shows the ratio $R$ for $\delta_T = \pi/12$, keeping only the contribution of the Pomeron in the Regge amplitudes.

The above results show that the dispersive formalism is consistent with the treatment based on factorization and perturbative QCD in the heavy quark limit presented in Ref. [76], supporting therefore the physical idea of parton-hadron duality. From a practical point of view, the dispersion representations in the external mass provide a set of algebraic equations for on-shell decay amplitudes, leading to non-trivial constraints on the hadronic parameters.

### 9.3 QCD Factorization for Exclusive Non-Leptonic B Decays

The theory of hadronic B decay matrix elements is a crucial basis for precision flavour physics with non-leptonic modes, which is one of the central goals of the B physics programme at the LHC. A new, systematic approach towards this problem, going beyond previous attempts, was recently proposed in [76]. It solves the problem of how to calculate non-factorizable contributions, and in particular FSIs, in the heavy quark limit and constitutes a promising approach, complementary to the one discussed in the preceding sections. In this approach, the statement of QCD factorization in the case of $B \to \pi\pi$, for instance, can be schematically written as

$$A(B \to \pi\pi) = \langle \pi | j_1 | B \rangle \langle \pi | j_2 | 0 \rangle \left[ 1 + \mathcal{O}(\alpha_s) + \mathcal{O} \left( \frac{\Lambda_{QCD}}{m_B} \right) \right]. \quad (17)$$
Up to corrections suppressed by $\Lambda_{QCD}/m_B$ the amplitude is calculable in terms of simpler hadronic objects: It factorizes, to lowest order in $\alpha_s$, into matrix elements of bilinear quark currents ($j_{1,2}$). To higher order in $\alpha_s$, but still to leading order in $\Lambda_{QCD}/m_B$, there are ‘non-factorizable’ corrections, which are however governed by hard-gluon exchange. They are therefore again calculable in terms of few universal hadronic quantities. More explicitly, the matrix elements of four-quark operators $Q_i$ are expressed by the factorization formula

$$
\langle p' | p | Q_i | \bar{B} (p) \rangle = f_{B \to \pi} (q^2) \int_0^1 dx \, T^I_i (x) \Phi_\pi (x) + \int_0^1 d\xi d\eta dy \, T^{II}_i (\xi, x, y) \Phi_B (\xi) \Phi_\pi (x) \Phi_\pi (y),
$$

which is valid up to corrections of relative order $\Lambda_{QCD}/m_b$. Here $f_{B \to \pi} (q^2)$ is a $B \to \pi$ form factor [185, 126] evaluated at $q^2 = m_B^2 \approx 0$, and $\Phi_\pi$ ($\Phi_B$) are leading-twist light-cone distribution amplitudes of the pion (B meson). The $T^{I,II}_i$ denote hard-scattering kernels, which are calculable in perturbation theory. The corresponding diagrams are shown in Fig. 60.

![Fig. 60: Order $\alpha_s$ corrections to the hard scattering kernels $T^I_i$ (first two rows) and $T^{II}_i$ (last row). In the case of $T^I_i$, the spectator quark does not participate in the hard interaction and is not drawn. The two lines directed upwards represent the two quarks forming the emitted pion. $T^I_i$ starts at $O(\alpha_s^0)$, $T^{II}_i$ at $O(\alpha_s^2)$.](image)

This treatment of hadronic B decays is based on the analysis of Feynman diagrams in the heavy quark limit, utilizing consistent power counting to identify the leading contributions. The framework is very similar in spirit to more conventional applications of perturbative QCD in exclusive hadronic processes with a large momentum transfer, as the pion electromagnetic form factor [190, 191, 192]. It may be viewed as a consistent formalization of Bjorken’s colour transparency argument [127]. In addition the method includes, for $B \to \pi\pi$, the hard non-factorizable spectator interactions, penguin contributions and rescattering effects. As a corollary, one finds that strong rescattering phases are either of $O(\alpha_s)$, and calculable, or power suppressed. In any case they vanish therefore in the heavy quark limit. QCD factorization is valid for cases where the emitted particle (the meson created from the vacuum in the weak process, as opposed to the one that absorbs the $b$ quark spectator) is a small size colour-singlet object, e.g. either a fast light meson ($\pi$, $\varrho$, $K$, $K^*$) or a $J/\psi$. For the special case of the ratio $\Gamma(B \to D^{(*)} \pi)/\Gamma(B \to D \pi)$ the perturbative corrections to naive factorization have been evaluated in [193] using a formalism similar to the one described above. Note that factorization cannot be justified in this way if the emitted particle is a heavy-light meson ($D^{(*)}$), which is not a compact object and has strong overlap with the remaining hadronic environment.

### 9.31 Final State Interactions

A general issue in hadronic B decays, with important implications for CP violation, is the question of FSIs. When discussing this problem, we may choose a partonic or a hadronic language. The partonic language can be justified by the dominance of hard rescattering in the heavy quark limit. In this limit the number of physical intermediate states is arbitrarily large. We may then argue on the grounds of parton-hadron duality that their average is described well enough (say, up to $\Lambda_{QCD}/m_b$ corrections) by a partonic
calculation. This is the picture implied by (18). The hadronic language is in principle exact. However, the large number of intermediate states makes it almost impossible to observe systematic cancellations, which usually occur in an inclusive sum of intermediate states.

Consider again the decay of a B meson into two pions. Unitarity implies Im $A(B \rightarrow \pi\pi) \sim \sum_n A(B \rightarrow n)A^*(n \rightarrow \pi\pi)$. The elastic rescattering contribution ($n = \pi\pi$) is related to the $\pi\pi$ scattering amplitude, which exhibits Regge behaviour in the high-energy ($m_b \rightarrow \infty$) limit. Hence the soft, elastic rescattering phase increases slowly in the heavy quark limit [194]. On general grounds, it is rather improbable that elastic rescattering gives an appropriate description at large $m_b$. This expectation is also borne out in the framework of Regge behaviour, see [194], where the importance of inelastic rescattering is emphasized. However, the approach pursued in [194] leaves open the possibility of soft rescattering phases that do not vanish in the heavy quark limit, as well as the possibility of systematic cancellations, for which the Regge language does not provide an appropriate theoretical framework.

Eq. (18) implies that such systematic cancellations do occur in the sum over all intermediate states $n$. It is worth recalling that such cancellations are not uncommon for hard processes. Consider the example of $e^+e^- \rightarrow$ hadrons at large energy $q$. While the production of any hadronic final state occurs on a time-scale of order $1/A_{QCD}$ (and would lead to infrared divergences if we attempted to describe it in perturbation theory), the inclusive cross section given by the sum over all hadronic final states is described very well by a $q\bar{q}$ pair that lives over a short time-scale of order $1/q$. In close analogy, while each particular hadronic intermediate state $n$ cannot be described in terms of partons, the sum over all intermediate states is accurately represented by a $q\bar{q}$ fluctuation of small transverse size of order $1/m_b$, which therefore interacts little with its environment. Note that precisely because the $q\bar{q}$ pair is small, the physical picture of rescattering is very different from elastic $\pi\pi$ scattering – hence the Regge picture is difficult to justify in the heavy quark limit.

As is clear from the discussion, parton-hadron duality is crucial for the validity of (18) beyond perturbative factorization. A quantitative proof of how accurately duality holds is a yet unsolved problem in QCD. Short of a solution, it is worth noting that the same (often implicit) assumption is fundamental to many successful QCD predictions in jet and hadron-hadron physics or heavy quark decays.

### 9.32 QCD Factorization in $B \rightarrow \pi\pi$

Let us finally illustrate one phenomenological application of QCD factorization in the heavy quark limit for $B \rightarrow \pi^+\pi^-$. The $B_d \rightarrow \pi^+\pi^-$ decay amplitude $A$ reads

$$A = i \frac{G_F}{\sqrt{2}} m_B^2 f_+(0) f_- |\lambda_c| \cdot [R_b e^{-i\gamma} (a_4^u(\pi\pi) + a_4^i(\pi\pi) + a_6^u(\pi\pi) r_\chi) - (a_4^c(\pi\pi) + a_6^c(\pi\pi) r_\chi)].$$

Here $R_b$ is the ratio of CKM matrix elements defined in (9), $\gamma$ is the phase of $V_{ub}$, and we will use $|V_{cb}| = 0.039 \pm 0.002$, $|V_{ub}/V_{cb}| = 0.085 \pm 0.020$. We also take $f_+ = 131$ MeV, $f_- = (180 \pm 20)$ MeV, $f_{+}(0) = 0.275 \pm 0.025$, and $\tau(B_d) = 1.56$ ps; $\lambda_c \equiv V_{cb}^\ast V_{ub}$. The contribution of $a_6^c(\pi\pi)$ is multiplied by $r_\chi = 2m_b^2/(m_b (m_c + m_\pi)) \sim (\Lambda_{QCD}/m_b)$. It is thus formally power suppressed, but numerically relevant since $r_\chi \approx 1$. The coefficients $a_i$ are estimated in Tab. 29. We then find for the branching fraction

$$B(B_d \rightarrow \pi^+\pi^-) = 6.5 \times [6.1] \times 10^{-6} \left| e^{-i\gamma} + 0.09 [0.18] e^{i2.7} [6.7] \right|^2.$$
The default values correspond to $a^P_0(\pi\pi) = 0$, the values in brackets use $a^P_0(\pi\pi)$ at leading order. The predictions for the $\pi^+\pi^-$ final state are relatively robust, with errors of the order of $\pm 30\%$ due to the input parameters. The direct CP asymmetry in the $\pi^+\pi^-$ mode is approximately $4\% \times \sin \gamma$.

As a further example, we use the factorization formula to compute the time-dependent, mixing-induced asymmetry in $B_d \to \pi^+\pi^-$ decay,

$$\mathcal{A}(t) = -S \cdot \sin(\Delta M_{B_d} t) + C \cdot \cos(\Delta M_{B_d} t).$$

In the absence of a penguin contribution (defined as the contribution to the amplitude which does not carry the weak phase $\gamma$ in standard phase conventions), $S = \sin 2\alpha$ (where $\alpha$ refers to one of the angles of the CKM unitarity triangle) and $C = 0$. Fig. 61 shows $S$ as a function of $\sin 2\alpha$ with the amplitudes computed according to (18) and (19). The central of the solid lines refers to the heavy quark limit including $\alpha_s$ corrections to naive factorization and including the power-suppressed term $a^P_0 r^\chi$ that is usually also kept in naive factorization. The other two solid lines correspond to dropping this term or multiplying it by a factor of 2. This exercise shows that formally power-suppressed terms can be non-negligible, but it also shows that a measurement of $S$ can be converted into a range for $\sin 2\alpha$ which may already provide a very useful constraint on CP violation.

More work remains to be done. The proof of factorization has to be completed. Power corrections are an important issue, as $m_b$ is not arbitrarily large. There exist ‘chirally enhanced’ corrections $\sim r^\chi$. All such terms can be identified, but they involve non-factorizable soft gluons. The size of these terms has to be estimated to arrive at a realistic phenomenology. If this can be done, one may expect promising constraints and predictions for a large number of non-leptonic two-body final states. We emphasize in particular the experimentally attractive possibility to determine $\sin 2\alpha$ from $B \to \pi^+\pi^-$ decays alone.

10. $B_c$ PHYSICS

The $B_c^+$ meson is the lowest lying bound state of two heavy quarks, $\bar{b}$ and $c$. The QCD dynamics of this state is therefore similar to that of quarkonium systems, such as the $b\bar{b}$ or $c\bar{c}$ families, which are approximately non-relativistic. In contrast to the common quarkonia, however, $B_c$ carries open flavour and the ground state is stable under strong interactions. In fact, $B_c$ is the only hadron combining these features and forming a flavoured, weakly decaying quarkonium. Since the complicated interplay of strong and weak forces is the key problem in the theoretical analysis of weak decays of hadrons, the quarkonium-like $B_c$ provides us with a very interesting special case to study such a general question. Tools for calculation, for example heavy quark expansions, non-relativistic QCD (NRQCD), factorization, which are important in many areas of heavy flavour physics, can be tested in a complementary setting.

The observation of the $B_c$ meson by the CDF Collaboration in the channel $B_c \to J/\psi l\nu$, with

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16 Section coordinators: G. Buchalla, P. Colangelo and F. De Fazio.
The total decay rate of the $B_c$ can be computed starting from a heavy-quark expansion of the transition operator, supplemented by NRQCD. This framework is familiar from the study of ordinary, heavy-light $b$ hadron lifetimes [19], with the basic difference that in the heavy-light case the rôle of NRQCD is played by heavy-quark effective theory (HQET). For $B_c$, the characteristic features of NRQCD result in a particularly intuitive expression for the total rate: \[
\Gamma_{B_c} = \Gamma_b \left(1 - \frac{v_b^2}{2}\right) + \Gamma_c \left(1 - \frac{v_c^2}{2}\right) + \Delta\Gamma_{PI} + \Delta\Gamma_{WA} + \mathcal{O}(v^4). \tag{23}
\]
Eq. (23) is written as an expansion in the heavy quark velocities $v$, complete through order $v^3$. To lowest order, $v^0$, we have $\Gamma_{B_c} = \Gamma_b + \Gamma_c$, the sum of the free decay rates of the heavy quark constituents $b$ and $c$. The first bound state corrections arise at $\mathcal{O}(v^2)$ only and are equivalent to time-dilatation. The effect of binding compels the heavy quarks to move around each other, thus retarding their decay. At $\mathcal{O}(v^3)$ there are two terms. First, a correction from Pauli interference (PI) of the two $c$ quarks in the final state of $(c)b \rightarrow (c)c\bar{c}\sigma$ decay. Second, a contribution from the weak annihilation of the constituents $\bar{b}c$, either into hadrons or into leptons, the latter dominated by $B_c \rightarrow \tau\nu$. A numerical analysis of (23) gives the estimate [197]
\[
\tau_{B_c} = \frac{1}{\Gamma_{B_c}} = (0.4 - 0.7) \text{ ps}, \tag{24}
\]
with a large uncertainty from the heavy quark masses ($m_c$), but in agreement with the measurement (22). The same framework can be used to calculate other inclusive decay properties of $B_c$, for instance the semileptonic branching fraction $B(B_c \rightarrow X\ell\nu)$, which turns out to be $\sim 12\%$. More details and references can be found in [197, 198, 199].

### 10.2 Leptonic and Radiative Leptonic $B_c$ Decays

The purely muonic $B_c$ branching ratio is determined by the decay constant $f_{B_c}$:
\[
B(B_c \rightarrow \mu\nu) = \frac{\tau_{B_c} G_F^2 |V_{cb}|^2 f_{B_c} M_{B_c}^3}{8\pi} \left(1 - \frac{m_\mu^2}{M_{B_c}^2}\right)^2 \left(1 - \frac{m_\mu^2}{M_{B_c}^2}\right)^2 \tau_{B_c} = 6.8 \times 10^{-6} \frac{M_{B_c}}{6.28 \text{ GeV}} \left|\frac{|V_{cb}|}{0.04}\right|^2 \left(\frac{f_{B_c}}{400 \text{ MeV}}\right)^2 \frac{\tau_{B_c}}{0.46 \text{ ps}}. \tag{25}
\]
The value of $f_{B_c}$ has been computed by lattice NRQCD: $f_{B_c} = (420 \pm 13) \text{ MeV}$ [200], QCD Sum Rules: $f_{B_c} = (360 \pm 60) \text{ MeV}$ [201, 202], and various quark models with predictions in the range $f_{B_c} = [430 - 570] \text{ MeV}$ [203].

The photon emission in the radiative muonic decay $B_c \rightarrow \mu\nu\gamma$ removes the helicity suppression of the purely muonic mode. In the non-relativistic limit, the radiative muonic decay width is also determined by $f_{B_c}$ [204]. In this limit one obtains the ratio $\Gamma(B_c \rightarrow \mu\nu\gamma)/\Gamma(B_c \rightarrow \mu\nu) \simeq 0.8$. Corrections to this result within a relativistic quark model have been discussed in [205].
10.3 Semileptonic $B_c$ Decay Modes

The calculation of the matrix elements governing the exclusive semileptonic $B_c$ decay modes has been carried out using QCD sum rules [202, 206] and quark models [207, 208]. The predictions for the various exclusive decay rates are reported (in rather conservative ranges) in Tab. 30, with the conclusion that the semileptonic $B_c$ decay width is dominated by the modes induced by the charm decay.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$\Gamma(10^{-15} \text{ GeV})$</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^+ \rightarrow B_s e^+ \nu$</td>
<td>$11 - 61$</td>
<td>$[8 - 42] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow B_s^* e^+ \nu$</td>
<td>$30 - 79$</td>
<td>$[21 - 55] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow B_d e^+ \nu$</td>
<td>$1 - 4$</td>
<td>$[7 - 28] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow B_d^* e^+ \nu$</td>
<td>$2 - 6$</td>
<td>$[14 - 42] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow \eta_c e^+ \nu$</td>
<td>$2 - 14$</td>
<td>$[14 - 98] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow J/\psi e^+ \nu$</td>
<td>$22 - 35$</td>
<td>$[15 - 24] \times 10^{-3}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow \eta'_c e^+ \nu$</td>
<td>$0.3 - 0.7$</td>
<td>$[2 - 5] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow \psi' e^+ \nu$</td>
<td>$1 - 2$</td>
<td>$[7 - 14] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow D^0 e^+ \nu$</td>
<td>$0.01 - 0.09$</td>
<td>$[7 - 63] \times 10^{-6}$</td>
</tr>
<tr>
<td>$B_c^+ \rightarrow D^0 e^+ \nu$</td>
<td>$0.1 - 0.3$</td>
<td>$[7 - 21] \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Table 30: Semileptonic $B_c^+$ decay widths and branching fractions ($\tau_{B_c} = 0.46$ ps).

The calculation of the semileptonic matrix elements can be put on a firmer theoretical ground taking into account the decoupling of the spin of the heavy quarks of the $B_c$ meson, as well as of the meson produced in the semileptonic decays, i.e. mesons belonging to the $c\bar{c}$ family ($\eta_c, J/\psi$, etc.) and mesons containing a single heavy quark ($B^{(*)}_s, B^{(*)}_d, D^{(*)}$). The decoupling occurs in the heavy quark limit ($m_b, m_c \gg \Lambda_{QCD}$); it produces a symmetry, the heavy quark spin symmetry, allowing to relate, near the zero-recoil point, the form factors governing the $B_c$ decays into a $0^-$ and $1^-$ final meson to a few invariant functions [209]. Examples are the processes $B_c \rightarrow (B_s, B_s^*)\mu\nu$ and $B_c \rightarrow (B_d, B_d^*)\mu\nu$, where the energy released to the final hadronic system is much less than $m_b$, thus leaving the $b$ quark almost unaffected. The final $B_s$ meson ($a = d, s$) keeps the same $B_c$ four-velocity $v$, apart from a small residual momentum $q$. Defining $p_{B_c} = M_{B_c}v$ and $p_{B_a} = M_{B_a}v + q$, one has:

$$
\langle B_a, v, q|V_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{B_a}}[\Omega_1^a v_\mu + a_0 \Omega_2^a q_\mu],
$$

$$
\langle B_a^*, v, q, \epsilon|V_\mu|B_c, v\rangle = -i \sqrt{2M_{B_c}2M_{B_a}^*} a_0 \Omega_2^a \epsilon_{\mu\nu\alpha\beta}e^{\nu\alpha} q^{\beta},
$$

$$
\langle B_a^*, v, q, \epsilon|A_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{B_a}^*}[\Omega_1^{a*} \epsilon^*_{\mu} + a_0 \Omega_2^{a*} \epsilon^* \cdot q \, v_\mu],
$$

i.e. only two form factors are needed to describe the previous transitions. The scale parameter $a_0$ is related to the $B_c$ Bohr radius [209]. For the $B_c$ transitions into a $c\bar{c}$ meson, $B_c \rightarrow (\eta_c, J/\psi)\mu\nu$, spin symmetry implies that the semileptonic matrix elements can be expressed, near the zero-recoil point, in terms of a single form factor:

$$
\langle \eta_c, v, q|V_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{\eta_c}} \Delta v_\mu, \quad \langle J/\psi, v, q, \epsilon|A_\mu|B_c, v\rangle = \sqrt{2M_{B_c}2M_{J/\psi}} \Delta \epsilon^*_\mu.
$$

Model-independent results exist in the heavy-quark limit for $\Delta$ and $\Omega_1^a$ at the zero-recoil point [209]. Additional information on the form factors $\Delta$ and $\Omega_1^a$ is available from quark models [208, 210] and NRQCD sum rules [211]. The related predictions are included in the ranges reported in Tab. 30. Moreover, spin symmetry implies relations between $B_c$ decays to pseudoscalar and vector states, near the non-recoil point, that can be experimentally tested at the LHC [210].
10.4 Non-Leptonic Decay Modes

Two-body non-leptonic decays are of prime importance for the measurement of the $B_c$ mass. In particular, decay modes with a $J/\psi$ in the final state are suitable for an efficient background-rejection.

The non-leptonic $B_c$ decay rates have been computed in the factorization approximation, using various parametrizations of the semileptonic form factors and different prescriptions for the parameters $a_1$ and $a_2$ appearing in the factorized matrix elements [207, 208, 210]. Predictions for the various decay modes, induced by the beauty and charm quark transitions, are reported in Tabs. 31 and 32, respectively, using $M_{B_c} = 6.28$ GeV. For several modes, ranges of values for the branching fractions are reported; they are obtained considering the spread of predictions by different approaches, and suggest the size of the theoretical uncertainty for each decay mode.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$B$</th>
<th>Channel</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^+ \to \eta_c \pi^+$</td>
<td>$[3 - 25] \times 10^{-4}$</td>
<td>$B_c^+ \to \eta_c K^+$</td>
<td>$[2 - 17] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to \eta_c \rho^+$</td>
<td>$[7 - 60] \times 10^{-4}$</td>
<td>$B_c^+ \to \eta_c K^{*+}$</td>
<td>$[4 - 31] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to \eta_c a_1^+$</td>
<td>$9 \times 10^{-4}$</td>
<td>$B_c^+ \to \eta_c K_1^{*+}$</td>
<td>$5 \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi \pi^+$</td>
<td>$[1 - 2] \times 10^{-3}$</td>
<td>$B_c^+ \to J/\psi K^+$</td>
<td>$[7 - 17] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi \rho^+$</td>
<td>$[4 - 7] \times 10^{-3}$</td>
<td>$B_c^+ \to J/\psi K^{*+}$</td>
<td>$[2 - 4] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi a_1^+$</td>
<td>$5 \times 10^{-3}$</td>
<td>$B_c^+ \to J/\psi K_1^{*+}$</td>
<td>$3 \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \to \psi' \pi^+$</td>
<td>$[2 - 3] \times 10^{-3}$</td>
<td>$B_c^+ \to \psi' K^+$</td>
<td>$[1 - 2] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to \psi' \rho^+$</td>
<td>$[5 - 8] \times 10^{-4}$</td>
<td>$B_c^+ \to \psi' K^{*+}$</td>
<td>$[3 - 4] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to \psi' a_1^+$</td>
<td>$6 \times 10^{-4}$</td>
<td>$B_c^+ \to \psi' K_1^{*+}$</td>
<td>$3 \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to D^+ D^0$</td>
<td>$[1 - 12] \times 10^{-5}$</td>
<td>$B_c^+ \to D_c^+ D^0$</td>
<td>$[6 - 62] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \to D^+ D_s^0$</td>
<td>$[1 - 12] \times 10^{-5}$</td>
<td>$B_c^+ \to D_s^+ D^0$</td>
<td>$[6 - 62] \times 10^{-7}$</td>
</tr>
<tr>
<td>$B_c^+ \to D^{*+} \bar{D}^0$</td>
<td>$[8 - 10] \times 10^{-5}$</td>
<td>$B_c^+ \to D_s^{*+} \bar{D}^0$</td>
<td>$[5 - 6] \times 10^{-6}$</td>
</tr>
<tr>
<td>$B_c^+ \to D^{*+} \bar{D}_s^0$</td>
<td>$[1 - 2] \times 10^{-4}$</td>
<td>$B_c^+ \to D_s^{*+} \bar{D}_s^0$</td>
<td>$[8 - 11] \times 10^{-6}$</td>
</tr>
<tr>
<td>$B_c^+ \to \eta_c D_s$</td>
<td>$[5 - 7] \times 10^{-3}$</td>
<td>$B_c^+ \to \eta_c D^+$</td>
<td>$[5 - 8] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to \eta_c D_s^*$</td>
<td>$[4 - 6] \times 10^{-4}$</td>
<td>$B_c^+ \to \eta_c D^*$</td>
<td>$[2 - 6] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi D_s$</td>
<td>$[2 - 3] \times 10^{-3}$</td>
<td>$B_c^+ \to J/\psi D^+$</td>
<td>$[5 - 13] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to J/\psi D_s^*$</td>
<td>$[6 - 12] \times 10^{-3}$</td>
<td>$B_c^+ \to J/\psi D^*$</td>
<td>$[2 - 4] \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 31: Branching fractions of $B_c^+$ non-leptonic decays induced by $b \to c, u$ transitions.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$B$</th>
<th>Channel</th>
<th>$B$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_c^+ \to B_s \pi^+$</td>
<td>$[4 - 17] \times 10^{-2}$</td>
<td>$B_c^+ \to B_s K^+$</td>
<td>$[3 - 12] \times 10^{-3}$</td>
</tr>
<tr>
<td>$B_c^+ \to B_s \rho^+$</td>
<td>$[2 - 7] \times 10^{-2}$</td>
<td>$B_c^+ \to B_s K^{*+}$</td>
<td>$[5 - 9] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^{*+}$</td>
<td>$[3 - 7] \times 10^{-2}$</td>
<td>$B_c^+ \to B_s K^+$</td>
<td>$[2 - 5] \times 10^{-3}$</td>
</tr>
<tr>
<td>$B_c^+ \to B_s \rho^+$</td>
<td>$[14 - 19] \times 10^{-2}$</td>
<td>$B_c^+ \to B_s K^{*+}$</td>
<td>$[2 - 3] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \to B_s \pi^+$</td>
<td>$[2 - 4] \times 10^{-3}$</td>
<td>$B_c^+ \to B_s K^{*+}$</td>
<td>$[4 - 20] \times 10^{-5}$</td>
</tr>
<tr>
<td>$B_c^+ \to B_s \rho^+$</td>
<td>$[2 - 4] \times 10^{-3}$</td>
<td>$B_c^+ \to B_s K^+$</td>
<td>$[1 - 3] \times 10^{-4}$</td>
</tr>
<tr>
<td>$B_c^+ \to B_s^{*+}$</td>
<td>$[1 - 2] \times 10^{-2}$</td>
<td>$B_c^+ \to B_s K^+$</td>
<td>$[4 - 6] \times 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 32: Branching fractions of $B_c^+$ decays induced by $c \to s, d$ transitions.

10.5 $B_c$ Decays induced by FCNC Transitions

Among the rare $B_c$ decay processes that have been discussed in the literature are the radiative decays $B_c \to B_c^* \gamma$ and $B_c \to D_c^* \gamma$, induced at the quark level by the $c \to u \gamma$ and $b \to s \gamma$ transitions, respectively [212]. The interest for the former decay mode is related to the possibility of studying the $c \to u$ electromagnetic penguin transition, which in the charm mesons is overwhelmed by long-distance
contributions. In the case of $B_c$, long- and short-distance contributions have been estimated to be of comparable size, and the branching fraction $B(B_c \to B_s^* \gamma)$ is predicted, in the SM, at the level of $10^{-8}$.

### 10.6 CP Violation in $B_c$ Decays

$B_c$ decays can give information about CP violation and the weak CKM phases. Promising channels are $B_c^+ \to (\overline{c}e)c\bar{D}^0$, in particular the one where the charmonium state is a $J/\psi$, whose decay mode to $\mu^+\mu^-$ can be easily identified. In this case, CP violation is due to the difference between the weak phases of the tree and penguin diagrams contributing to the decay. The CP asymmetry $A(B_c^+ \to J/\psi D^\pm)$ has been estimated: $A(B_c^+ \to J/\psi D^\pm) \approx 4 \times 10^{-3}$ [213]. Interesting channels are also those with a light meson in the final state, e.g. $B_c \to D\rho$ and $B_c \to D\pi$. However, in this case the sizeable rôle played by the annihilation mechanism makes it difficult to predict the decay rates and the CP asymmetries. Decay modes such as $B_c \to D^0S$ can also be considered, although considerable difficulties would be met in the experimental detection of $D_s$ and in the removal of the background from $B_s$ decays.

Finally, the decay $B_c^+ \to B_s^*(\ell^+\nu)$ has been proposed as an interesting source of flavour-tagged $B_s$ mesons for the study of mixing and CP violation in the $B_s$ sector [198].

### 10.7 Experimental Considerations

ATLAS have studied the reconstruction of $B_c$ mesons using the decays $B_c \to J/\psi\pi$ and $B_c \to J/\psi\ell\nu$, with $J/\psi \to \mu^+\mu^-$ (see [37]). For this study, the following branching ratios have been assumed: $B(B_c \to J/\psi\pi) = 0.2 \times 10^{-2}$ and $B(B_c \to J/\psi\mu\nu) = 2 \times 10^{-2}$. It is estimated that after 3 years of running at low luminosity, it will be possible to fully reconstruct 12000 $B_c \to J/\psi\pi$ events and $3 \times 10^6$ events in the $B_c \to J/\psi\mu\nu$ channel. The statistics would allow a very precise determination of the $B_c$ mass and lifetime.

### 10.8 Concluding Remarks

The $B_c$ meson is of particular interest as a unique case to study the impact of QCD dynamics on weak decays. Applications in flavour physics (CKM parameters, rare decays, $B_s$ flavour tagging) have also been considered in the literature. Important theoretical questions that need further attention are the issues of quark-hadron duality for inclusive decays and, for exclusive modes, the importance of corrections to the heavy-quark and non-relativistic limits, as well as corrections to the factorization approximation. The experimental feasibility for various observables needs likewise to be assessed in more detail. The aim of the present section has been to give a flavour of the special opportunities that exist, from a theoretical perspective, in studying the physics of the $B_c$. Some of these are realizable at the LHC, where it will be possible to investigate also the production, spectrum, lifetimes and decays of baryons containing two heavy quarks [214]. It is to be hoped that the results summarized in this section will trigger more detailed experimental studies.

### 11. CONCLUSIONS

The studies presented at and initiated by the workshop have clearly shown that the LHC is very well equipped and prepared to pursue a rigorous $b$ physics programme. The main emphasis in the studies presented here has been on exploring LHC’s potential for measuring CP violating phenomena and, on the theory side, on enabling a meaningful extraction of information on the underlying mechanism on CP violation in the SM. Most of the presented “strategies” aim at extracting the three angles of the unitarity triangle, $\alpha$, $\beta$ and $\gamma$, as well as $\delta\gamma$, in as many different ways as possible; any significant discrepancy between the extracted values or with the known lengths of the sides of the triangle would constitute evidence for new physics. Apart from detailed studies of the $e^+e^-$ B factory “benchmark modes”, also

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the hadron collider “gold-plated” mode $B_s \to J/\psi\phi$ has been studied, and new strategies for measuring $\beta$ and $\gamma$ from $B_s$ decays, which cannot be accessed at $e^+e^-$ B factories, have been developed. We conclude that the three experiments are well prepared to solve the “mystery of CP violation” (p. 1).

Another important goal to be pursued is the measurement of $B$ mixing parameters, and the studies summarized in this report make clear that all three LHC experiments have excellent potential. There is sensitivity in one year’s operation to a mass difference in the $B_s$ system far beyond the SM expectation, and similarly good prospects for a rapid measurement of the width difference.

The second focus of the workshop was the assessment of LHC’s reach in rare decays. The discussion centred on decay modes with the favourable experimental signature of two muons or one photon in the final state. It has been demonstrated that the decay $B_s \to \mu^+\mu^-$ with a SM branching ratio of $\sim 10^{-9}$ can be seen within one year’s running. It has also been shown that decay spectra of semimuonic rare decays like $B \to K^*\mu^+\mu^-$ are accessible, which opens the possibility to extract information on short-distance (new) physics in a theoretically controlled way. LHC’s full potential for rare decays has, however, not yet been fully plumbed, and further studies, in particular about the feasibility of inclusive measurements, are ongoing.

Of the many other possible $b$ physics topics, only a few could be marked out, and we have reported some recent developments in the theoretical description of non-leptonic decays and discussed a few issues in $B_c$ physics. The exploration of other exciting topics, such as physics with $b$, baryons or (non-rare) semileptonic decays, to name only a few, has to await a second round of workshops.

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TOP QUARK PHYSICS

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1. INTRODUCTION

The top quark, when it was finally discovered at Fermilab in 1995 [1, 2, 3], completed the three-generation structure of the Standard Model (SM) and opened up the new field of top quark physics. Viewed as just another SM quark, the top quark appears to be a rather uninteresting species. Produced predominantly, in hadron-hadron collisions, through strong interactions, it decays rapidly without forming hadrons, and almost exclusively through the single mode $t \to W b$. The relevant CKM coupling $V_{tb}$ is already determined by the (three-generation) unitarity of the CKM matrix. Rare decays and CP violation are unmeasurably small in the SM.

Yet the top quark is distinguished by its large mass, about 35 times larger than the mass of the next heavy quark, and intriguingly close to the scale of electroweak (EW) symmetry breaking. This unique property raises a number of interesting questions. Is the top quark mass generated by the Higgs mechanism as the SM predicts and is its mass related to the top-Higgs-Yukawa coupling? Or does it play an even more fundamental role in the EW symmetry breaking mechanism? If there are new particles lighter than the top quark, does the top quark decay into them? Could non-SM physics first manifest itself in non-standard couplings of the top quark which show up as anomalies in top quark production and decays? Top quark physics tries to answer these questions.

Several properties of the top quark have already been examined at the Tevatron. These include studies of the kinematical properties of top production [4], the measurements of the top mass [5, 6], of the top production cross-section [7, 8], the reconstruction of $t\bar{t}$ pairs in the fully hadronic final states [9, 10], the study of $\tau$ decays of the top quark [11], the reconstruction of hadronic decays of the $W$ boson from top decays [12], the search for flavour changing neutral current decays [13], the measurement of the $W$ helicity in top decays [14], and bounds on $t\bar{t}$ spin correlations [15]. Most of these measurements are limited by the small sample of top quarks collected at the Tevatron up to now. The LHC is, in comparison, a top factory, producing about 8 million $t\bar{t}$ pairs per experiment per year at low luminosity (10 fb$^{-1}$/year), and another few million (anti-)tops in EW single (anti-)top quark production. We therefore expect that top quark properties can be examined with significant precision at the LHC. Entirely new measurements can be contemplated on the basis of the large available statistics.

In this chapter we summarize the top physics potential of the LHC experiments. An important aspect of this chapter is to document SM model properties of the top quark against which anomalous behaviour has to be compared. In each section (with the exception of the one devoted to anomalous couplings) we begin by summarizing SM expectations and review the current theoretical status on a particular topic. This is followed by a detailed description of experimental analysis strategies in the context of the ATLAS and CMS experiments. Particular emphasis is given to new simulations carried out in the course of this workshop. In detail, the outline of this chapter is as follows:
In Section 2, we summarize SM precision calculations of the top quark mass relations and of the total top quark width. We then recall the importance of the top quark mass in EW precision measurements. We discuss, in particular, the role of EW precision measurements under the assumption that a SM Higgs boson has been discovered.

Section 3 deals with the $t\bar{t}$ production process: expectations for and measurements of the total cross section, the transverse momentum and $t\bar{t}$ invariant mass distribution are discussed. A separate subsection is devoted to EW radiative corrections to $t\bar{t}$ production, and to radiative corrections in the Minimal Supersymmetric SM (MSSM).

The prospects for an accurate top quark mass measurement are detailed in Section 4. Next to “standard” measurements in the lepton+jets and di-lepton channels, two mass measurements are discussed that make use of the large number of top quarks available at the LHC: the selection of top quarks with large transverse momentum in the lepton+jets channel and the measurement of $\ell J/\psi$ correlations in $t\to\ell J/\psi X$ decays. This decay mode appears to be particularly promising and the systematic uncertainties are analyzed in considerable detail.

Single top quark production through EW interactions provides the only known way to directly measure the CKM matrix element $V_{tb}$ at hadron colliders. It also probes the nature of the top quark charged current. In Section 5, the SM expectations for the three basic single top production mechanisms and their detection are documented, including the possibility to measure the high degree of polarisation in the SM.

The issue of top quark spin is pursued in Section 6. Here we summarize expectations on spin correlations in $t\bar{t}$ production, the construction of observables sensitive to such correlations and the results of a simulation study of di-lepton angular correlations sensitive to spin correlations. Possible non-SM CP violating couplings of the top quark can be revealed through anomalous spin-momentum correlations and are also discussed here.

As mentioned above, the search for anomalous (i.e. non-SM) interactions is one of the main motivations for top quark physics. In Section 7, the sensitivity of the LHC experiments to the following couplings is investigated: $g t\bar{t}$ couplings and anomalous $W t\bar{b}$ couplings in top production, flavour-changing neutral currents (FCNCs) in top production and decay.

Section 8 is devoted to rare top decays. The SM expectations for radiative top decays and FCNC decays are documented. Decay rates large enough to be of interest require physics beyond the SM. The two Higgs Doublet Models, the MSSM and generic anomalous couplings are considered explicitly followed by ATLAS and CMS studies on the expected sensitivity in particular decay channels.

Finally, the measurement of the top quark Yukawa coupling in $t\bar{t}H$ production is considered (Section 9). The SM cross sections are tabulated in the various production channels at the LHC. For the case of a low mass Higgs boson, the results of a realistic study using a simulation of the ATLAS detector are discussed.

The following topics are collected in the appendices: $b$-quark tagging and the calibration of the jet energy scale in top events; the direct measurement of the top quark spin (as opposed to that of a top squark) and of top quark electric charge; the total cross section for production of a fourth generation heavy quark; a compendium of Monte Carlo event generators available for top production and its backgrounds.

The internal ATLAS and CMS notes quoted in the bibliography can be obtained from the collaborations’ web pages [16, 17]. Updated versions of this document, as well as a list of addenda and errata, will be available on the web page of the LHC Workshop top working group [18].
2. TOP QUARK PROPERTIES AND ELECTROWEAK PRECISION MEASUREMENTS

The top quark is, according to the Standard Model (SM), a spin-1/2 and charge-2/3 fermion, transforming as a colour triplet under the group $SU(3)$ of the strong interactions and as the weak-isospin partner of the bottom quark. None of these quantum numbers has been directly measured so far, although a large amount of indirect evidence supports these assignments. The analysis of EW observables in $Z^0$ decays [19] requires the existence of a $T_3 = 1/2$, charge-2/3 fermion, with a mass in the range of 170 GeV, consistent with the direct Tevatron measurements. The measurement of the total cross section at the Tevatron, and its comparison with the theoretical estimates, are consistent with the production of a spin-1/2 and colour-triplet particle. The LHC should provide a direct measurement of the top quantum numbers. We present the results of some studies in this direction in Appendix B.

2.1 Top quark mass and width

In addition to its quantum numbers, the two most fundamental properties of the top quark are its mass $m_t$ and width $\Gamma_t$, defined through the position of the single particle pole $m_t^* = m_t - i\Gamma_t/2$ in the perturbative top quark propagator. In the SM $m_t$ is related to the top Yukawa coupling:

$$y_t(\mu) = 2^{3/4} G_F^{1/2} m_t \left(1 + \delta_t(\mu)\right),$$  \hspace{1cm} (1)

where $\delta_t(\mu)$ accounts for radiative corrections. Besides the top quark pole mass, the top quark $\overline{MS}$ mass $\overline{m}_t(\mu)$ is often used. The definition of $\overline{m}_t(\mu)$ including EW corrections is subtle (see the discussion in [20]). As usually done in the literature, we define the $\overline{MS}$ mass by including only pure QCD corrections:

$$\overline{m}_t(\mu) = m_t \left(1 + \delta_{\overline{QCD}}(\mu)\right)^{-1}.$$  \hspace{1cm} (2)

The conversion factor $\delta_{\overline{QCD}}(\mu)$ is very well known [21]. Defining $\overline{m}_t = \overline{m}_t(\mu)$ and $a_s = \alpha_s^{\overline{MS}}(\overline{m}_t)/\pi$, we have

$$\delta_{\overline{QCD}}(\overline{m}_t) = \frac{4}{3} a_s + 8.2366 a_s^2 + 73.638 a_s^3 + \ldots$$

$$= (4.63 + 0.99 + 0.31 + 0.11^{+0.11}_{-0.11})\% = (6.05^{+0.11}_{-0.11})\%.$$  \hspace{1cm} (3)

This assumes five massless flavours besides the top quark and we use $a_s = 0.03475$ which corresponds to $\alpha_s^{\overline{MS}}(m_Z) = 0.119$ and $\overline{m}_t = 165$ GeV. The error estimate translates into an absolute uncertainty of $\pm 180$ MeV in $m_t - \overline{m}_t$ and uses an estimate of the four-loop contribution. Note that the difference between the two mass definitions, $m_t - \overline{m}_t$, is about 10 GeV. This means that any observable that is supposed to measure a top quark mass with an accuracy of $1-2$ GeV and which is known only at leading order (LO) must come with an explanation for why higher order corrections are small when the observable is expressed in terms of that top quark mass definition that it is supposed to determine accurately.

We will return to this point in Section 4.

The on-shell decay width $\Gamma_t$ is less well known, but the theoretical accuracy (\(< 1\%\)) is more than sufficient compared to the accuracy of foreseeable measurements. The decay through $t \to bW$ is by far dominant and we restrict the discussion to this decay mode. It is useful to quantify the decay width in units of the lowest order decay width with $M_W$ and $m_b$ set to zero and $|V_{tb}|$ set to 1:

$$\Gamma_0 = \frac{G_F m_b^3}{8\pi \sqrt{2}} = 1.76 \text{ GeV}.$$  \hspace{1cm} (4)

Incorporating $M_W$, the leading order result reads

$$\Gamma_0(t \to bW)/|V_{tb}|^2 = \Gamma_0 \left(1 - \frac{3M_W^4}{m_t^4} + 2 \frac{M_W^6}{m_b^6}\right) = 0.885 \Gamma_0 = 1.56 \text{ GeV}.$$  \hspace{1cm} (5)

\(^1\)Section coordinators: M. Beneke, G. Weiglein.
Table 1: Corrections to the top quark width $\Gamma_0$ ($M_W = 0$, lowest order) in units of $\Gamma_0$. The best estimate of $\Gamma(t \to bW)/|V_{tb}|^2$ is obtained by adding all corrections together. Parameters: $\alpha_s = 0.03475$, $M_W = 80.4$ GeV and $m_t = 175$ GeV.

| $M_W \neq 0$ correction at lowest order, see (5) | $-11.5\%$ |
| $\alpha_s$ correction, $M_W = 0$ | $-9.5\%$ |
| $\alpha_s$ correction, $M_W \neq 0$ correction | $+1.8\%$ |
| $\alpha_s^2$ correction, $M_W = 0$ [22, 23] | $-2.0\%$ |
| $\alpha_s^2$ correction, $M_W \neq 0$ correction [23] | $+0.1\%$ |
| EW correction [24] | $+1.7\%$ |

The correction for non-vanishing bottom quark mass is about $-0.2\%$ in units of $\Gamma_0$. Likewise corrections to treating the $W$ boson as a stable particle are negligible. Radiative corrections are known to second order in QCD and to first order in the EW theory. Table 1 summarises the known corrections to the limiting case (4). Putting all effects together we obtain:

$$\Gamma(t \to bW)/|V_{tb}|^2 \approx 0.807 \Gamma_0 = 1.42 \text{ GeV}. \quad (6)$$

The top quark lifetime is small compared to the time scale for hadronisation [25]. For this reason, top-hadron spectroscopy is not expected to be the subject of LHC measurements.

### 2.2 Role of $m_t$ in EW precision physics

The EW precision observables serve as an important tool for testing the theory, as they provide an important consistency test for every model under consideration. By comparing the EW precision data with the predictions (incorporating quantum corrections) within the SM or its extensions, most notably the minimal supersymmetric extension of the Standard Model (MSSM) [26], it is in principle possible to derive indirect constraints on all parameters of the model. The information obtained in this way, for instance, on the mass of the Higgs boson in the SM or on the masses of supersymmetric particles is complementary to the information gained from the direct production of these particles.

In order to derive precise theoretical predictions, two kinds of theoretical uncertainties have to be kept under control: the uncertainties from unknown higher-order corrections, as the predictions are derived only up to a finite order in perturbation theory, and the parametric uncertainties caused by the experimental errors of the input parameters. The top quark mass enters the EW precision observables as an input parameter via quantum effects, i.e. loop corrections. As a distinctive feature, the large numerical value of $m_t$ gives rise to sizable corrections that behave as powers of $m_t$. This is in contrast to the corrections associated with all other particles of the SM. In particular, the dependence on the mass of the Higgs boson is only logarithmic in leading order and therefore much weaker than the dependence on $m_t$. In the MSSM large corrections from SUSY particles are only possible for large splittings in the SUSY spectrum, while the SUSY particles in general decouple for large masses.

The most important $m_t$-dependent contribution to the EW precision observables in the SM and the MSSM enters via the universal parameter $\Delta \rho$ which is proportional to $m_t^2$ [27],

$$\Delta \rho = \left( \frac{\Sigma^Z(0)}{M_Z^2} - \frac{\Sigma^W(0)}{M_W^2} \right)_{t,b} = N_C \frac{\alpha}{16 \pi^2 s_W^2 c_W} \frac{m_t^2}{M_Z^2}, \quad (7)$$

where the limit $m_b \to 0$ has been taken, $s_W$ ($c_W$) is the sin (cos) of the weak mixing angle, and $\Sigma^Z(0)$ and $\Sigma^W(0)$ indicate the transverse parts of the gauge-boson self-energies at zero momentum transfer.

The theoretical prediction for $M_W$ is obtained from the relation between the vector-boson masses
and the Fermi constant,

\[ M_W^2 \left( 1 - \frac{M_W^2}{M_Z^2} \right) = \frac{\pi \alpha}{\sqrt{2} G_F} (1 + \Delta r), \tag{8} \]

where the quantity \( \Delta r \) [28] is derived from muon decay and contains the radiative corrections. At one-loop order, \( \Delta r \) can be written as \( \Delta r = \Delta \alpha - \frac{\alpha^2}{\pi \alpha_W} \Delta \rho + (\Delta r)_{\text{nl}}, \) where \( \Delta \alpha \) contains the large logarithmic contributions from the light fermions, and the non-leading terms are collected in \((\Delta r)_{\text{nl}}\).

The leptonic effective weak mixing angle is determined from the effective couplings of the neutral current at the Z-boson resonance to charged leptons, \( J_{\mu}^{NC} = (\sqrt{2} G_F M_Z^2)^{1/2} \left( g_\nu \gamma_\mu - g_A \gamma_\mu \gamma_5 \right), \) according to

\[ \sin^2 \theta^{\text{lept}}_{\text{eff}} = \frac{1}{4} \left( 1 - \frac{\text{Re}(g_\nu)}{\text{Re}(g_A)} \right). \tag{9} \]

In \( \sin^2 \theta^{\text{lept}}_{\text{eff}} \) the leading \( m_t \)-dependent contributions enter via \( \delta \sin^2 \theta^{\text{lept}}_{\text{eff}} = -(c_W^2 s_W^2)/(c_W^2 - s_W^2) \Delta \rho. \)

The precision observables \( M_W \) and \( \sin^2 \theta^{\text{lept}}_{\text{eff}} \) are currently known with experimental accuracies of 0.05% and 0.07%, respectively [19]. The accuracy in \( M_W \) will be further improved at the LHC by about a factor of three (see the EW chapter of this Yellow Report). Besides the universal correction \( \Delta \rho, \) there is also a non-universal correction proportional to \( m_t^2 \) in the \( Zb\bar{b} \) coupling, which however is less accurately measured experimentally compared to \( M_W \) and \( \sin^2 \theta^{\text{lept}}_{\text{eff}}. \) The strong dependence of the SM radiative corrections to the precision observables on the input value of \( m_t \) made it possible to predict the value of \( m_t \) from the precision measurements prior to its actual experimental discovery, and the predicted value turned out to be in remarkable agreement with the experimental result [5, 6].

Within the MSSM, the mass of the lightest CP-even Higgs boson, \( m_h, \) is a further observable whose theoretical prediction strongly depends on \( m_t. \) While in the SM the Higgs-boson mass is a free parameter, \( m_h \) is calculable from the other SUSY parameters in the MSSM and is bounded to be lighter than \( M_Z \) at the tree level. The dominant one-loop corrections arise from the top and scalar-top sector via terms of the form \( G_F m_t^4 \ln(m_t^2 / m_i^2) \) [29]. As a rule of thumb, a variation of \( m_t \) by 1 GeV, keeping all other parameters fixed, roughly translates into a shift of the predicted value of \( m_h \) by 1 GeV. If the lightest CP-even Higgs boson of the MSSM will be detected at the LHC, its mass will be measurable with an accuracy of about \( \Delta m_h = 0.2 \) GeV [30].

Due to the sensitive dependence of the EW precision observables on the numerical value of \( m_t, \) a high accuracy in the input value of \( m_t \) is very important for stringent consistency tests of a model, for constraints on the model’s parameters (e.g. the Higgs boson mass within the SM), and for a high sensitivity to possible effects of new physics. It should be noted that this calls not only for a high precision in the experimental measurement of the top quark mass, but also for a detailed investigation of how the quantity that is actually determined experimentally is related to the parameter \( m_t \) used as input in higher-order calculations. While these quantities are the same in the simplest approximation, their relation is non-trivial in general due to higher-order contributions and hadronisation effects. A further discussion of this problem, which can be regarded as a systematic uncertainty in the experimental determination of \( m_t, \) is given in Section 4.

### 2.3 Physics gain from improving \( \Delta m_t \) from \( \Delta m_t = 2 \) GeV to \( \Delta m_t = 1 \) GeV

During this workshop the question was investigated of how much information one could gain from the EW precision observables by improving the experimental precision in \( m_t \) from \( \Delta m_t = 2 \) GeV, reachable within the first year of LHC running (see Section 4.2), to \( \Delta m_t = 1 \) GeV, possibly attainable on a longer time scale (see Section 4.6).

In order to analyse this question quantitatively, we have considered the case of the SM and the MSSM and assumed that the Higgs boson has been found at the LHC. For the uncertainty in \( \Delta \alpha_{\text{had}} \) (the
hadronic contribution to the electromagnetic coupling at the scale $M_Z$) we have adopted $\delta(\Delta \alpha_{\text{had}}) = 0.00016$, which corresponds to the “theory driven” analyses of [31].

Concerning the current theoretical prediction for $M_W$ and $\sin^2 \theta^\text{lept}_{\text{eff}}$ in the SM, the theoretical uncertainty from unknown higher-order corrections has been estimated to be about $\Delta M_W = 6$ MeV and $\Delta \sin^2 \theta^\text{lept}_{\text{eff}} = 4 \times 10^{-5}$ [32]. In Table 2 the theoretical uncertainties for $M_W$ and $\sin^2 \theta^\text{lept}_{\text{eff}}$ from unknown higher-order corrections are compared with the parametric uncertainty from the input parameters $\Delta \alpha_{\text{had}}$ and $m_t$ for $\Delta m_t = 2$ GeV as well as $\Delta m_t = 1$ GeV. The parametric uncertainties from the other parameters, supposing that the SM Higgs boson has been found at the LHC in the currently preferred range, are negligible compared to the uncertainties from $\Delta \alpha_{\text{had}}$ and $m_t$. The resulting uncertainties in $M_W$ and $\sin^2 \theta^\text{lept}_{\text{eff}}$ have been obtained using the parameterisation of the results for these quantities given in [33]. As can be seen in the table, for $\Delta m_t = 2$ GeV the parametric uncertainty in $m_t$ gives rise to the largest theoretical uncertainty in both precision observables. While for $\sin^2 \theta^\text{lept}_{\text{eff}}$ the uncertainty induced from the error in $m_t$ is comparable to the one from the error in $\Delta \alpha_{\text{had}}$, for $M_W$ the uncertainty from the error in $m_t$ is twice as big as the one from unknown higher-order corrections and four times as big as the one from the error in $\Delta \alpha_{\text{had}}$. A reduction of the error from $\Delta m_t = 2$ GeV to $\Delta m_t = 1$ GeV will thus mainly improve the precision in the prediction for $M_W$. The uncertainty induced in $M_W$ by $\Delta m_t = 1$ GeV is about the same as the current uncertainty from unknown higher-order corrections. The latter uncertainty can of course be improved by going beyond the present level in the perturbative evaluation of $\Delta r$.

In Fig. 1 the theoretical predictions for $M_W$ and $\sin^2 \theta^\text{lept}_{\text{eff}}$ (see [34] and references therein) are compared with the expected accuracies for these observables at LEP2/Tevatron and at the LHC (for the central values, the current experimental values are taken). The parametric uncertainties corresponding to $\delta(\Delta \alpha_{\text{had}}) = 0.00016$ and $\Delta m_t = 2$ GeV, $\Delta m_t = 1$ GeV are shown for two values of the Higgs boson mass, $m_H = 120$ GeV and $m_H = 200$ GeV, and the present theoretical uncertainty is also indicated (here $m_H$ is varied within $100$ GeV $\leq m_H \leq 400$ GeV and $\Delta m_t = 5.1$ GeV). The figure shows that, assuming that the Higgs boson will be discovered at the LHC, the improved accuracy in $m_t$ and $M_W$ at the LHC will allow a stringent consistency test of the theory. A reduction of the experimental error in $m_t$ from $\Delta m_t = 2$ GeV to $\Delta m_t = 1$ GeV leads to a sizable improvement in the accuracy of the theoretical prediction. In view of the precision tests of the theory a further reduction of the experimental error in $M_W$ and $\sin^2 \theta^\text{lept}_{\text{eff}}$ would clearly be very desirable.

While within the MSSM the improved accuracy in $m_t$ and $M_W$ at the LHC will have a similar impact on the analysis of the precision observables as in the SM, the detection of the mass of the lightest CP-even Higgs boson will provide a further stringent test of the model. The prediction for $m_h$ within the MSSM is particularly sensitive to the parameters in the $t\bar{t}$ sector, while in the region of large $M_A$ and large $\tan \beta$ (giving rise to Higgs masses beyond the reach of LEP2) the dependence on the latter two parameters is relatively mild. A precise measurement of $m_h$ can thus be used to constrain the parameters in the $t\bar{t}$ sector of the MSSM.

In Fig. 2 it is assumed that the mass of the lightest scalar top quark, $m_{t_1}$, is known with high precision, while the mass of the heavier scalar top quark, $m_{t_2}$, and the mixing angle $\theta^t_1$ are treated as free parameters. The Higgs boson mass is assumed to be known with an experimental precision of $\pm 0.5$ GeV and the impact of $\Delta m_t = 2$ GeV and $\Delta m_t = 1$ GeV is shown (the theoretical uncertainty in the Higgs-
Fig. 1: The SM prediction in the \( M_W - \sin^2 \theta_{\text{eff}}^{\text{lep}} \) plane is compared with the expected experimental accuracy at LEP2/Tevatron (\( \Delta M_W = 30 \text{ MeV}, \sin^2 \theta_{\text{eff}}^{\text{lep}} = 1.7 \times 10^{-4} \)) and at the LHC (\( \Delta M_W = 15 \text{ MeV}, \sin^2 \theta_{\text{eff}}^{\text{lep}} = 1.7 \times 10^{-4} \)). The theoretical uncertainties induced by \( \delta(\Delta \alpha_{\text{had}}) = 0.00016 \) and \( \Delta m_t = 2 \text{ GeV} \) (full line) as well as \( \Delta m_t = 1 \text{ GeV} \) (dashed line) are shown for two values of the Higgs boson mass \( m_H \).

Fig. 2: Indirect constraints on the parameters of the scalar top sector of the MSSM from the measurement of \( m_t \) at the LHC. The effect of the experimental error in \( m_t \) is shown for \( \Delta m_t = 2 \text{ GeV} \) and \( \Delta m_t = 1 \text{ GeV} \).
mass prediction from unknown higher-order contributions and the parametric uncertainties besides the ones induced by $m_{\tilde{t}}$, $\theta_\ell$ and $m_t$ have been neglected here). The two bands represent the values of $m_{\tilde{t}}$, $\theta_\ell$ which are compatible with a Higgs-mass prediction of $m_h = 120.5 \pm 0.5$ GeV, where the two-loop result of [35] has been used (the bands corresponding to smaller and larger values of $m_{\tilde{t}}$ are related to smaller and larger values of the off-diagonal entry in the scalar top mixing matrix, respectively). Combining the constraints on the parameters in the scalar top sector obtained in this way with the results of the direct search for the scalar top quarks will allow a sensitive test of the MSSM. As can be seen in the figure, a reduction of $\Delta m_t$ from $\Delta m_t = 2$ GeV to $\Delta m_t = 1$ GeV will lead to a considerable reduction of the allowed parameter space in the $m_{\tilde{t}} - \theta_\ell$ plane.

3. $t\bar{t}$ PRODUCTION AT THE LHC

The determination of the top production characteristics will be one of the first measurements to be carried out with the large statistics available at the LHC. The large top quark mass ensures that top production is a short-distance process, and that the perturbative expansion, given by a series in powers of the small parameter $\alpha_s(m_t) \sim 0.1$, converges rapidly. Because of the large statistics (of the order of $10^7$ top quark pairs produced per year), the measurements and their interpretation will be dominated by experimental and theoretical systematic errors. Statistical uncertainties will be below the percent level for most observables. It will therefore be a severe challenge to reduce experimental and theoretical systematic uncertainties to a comparable level. In addition to providing interesting tests of QCD, accurate studies of the top production and decay mechanisms will be the basis for the evaluation of the intrinsic properties of the top quark and of its EW interactions. An accurate determination of the production cross section, for example, provides an independent indirect determination of $m_t$. Asymmetries in the rapidity distributions of top and antitop quarks [36] are sensitive to the light-quark parton distribution functions of the proton. Anomalies in the total $t\bar{t}$ rate would indicate the presence of non-QCD production channels, to be confirmed by precise studies of the top quark distributions (e.g. $p_T$ and $t\bar{t}$ invariant mass spectra). These would be distorted by the presence of anomalous couplings or $s$-channel resonances expected in several beyond-the-SM (BSM) scenarios. Parity-violating asymmetries (for example in the rapidity distributions of right and left handed top quarks) are sensitive to the top EW couplings, and can be affected by the presence of BSM processes, such as the exchange of supersymmetric particles. As already observed at the Tevatron [5, 6], the structure of the $t\bar{t}$ final state affects the direct determination of $m_t$. Initial and final-state gluon radiation do in fact contribute to the amount of energy carried by the jets produced in the decay of top quarks, and therefore need to be taken into proper account when jets are combined to extract $m_t$. The details of the structure of these jets (e.g. their fragmentation function and their shapes), will also influence the experimental determination of the jet energy scales (important for the extraction of $m_t$), as well as the determination of the efficiency with which $b$-jets will be tagged (important for the measurement of the production cross section).

It is therefore clear that an accurate understanding of the QCD dynamics is required to make full use of the rich statistics of $t\bar{t}$ final states in the study of the SM properties of top quarks, as well as to explore the presence of possible deviations from the SM. In this section we review the current state of the art in predicting the production properties for top quark pairs (for a more detailed review of the theory of heavy quark production, see [37]). The study of single top production will be presented in Section 5.

3.1 Tools for QCD calculations

Full next-to-leading-order (NLO, $\mathcal{O}(\alpha_s^3)$) calculations are available for the following quantities:

1. Total cross sections [38]
3. Double-differential spectra ($m_{t}\bar{t}$, azimuthal correlations $\Delta\Phi$, etc.) [40]

---

All of the above calculations are available in the form of Fortran programs [40, 41], so that kinematical distributions can be evaluated at NLO [42] even in the presence of analysis cuts.

Theoretical progress over the last few years has led to the resummation of Sudakov-type logarithms [43] which appear at all orders in the perturbative expansion for the total cross sections [44, 45]. More recently, the accuracy of these resummations has been extended to the next-to-leading logarithmic (NLL) level [46, 47]. For a review of the theoretical aspects of Sudakov resummation, see the QCD chapter of this report. As will be shown later, while the inclusion of these higher-order terms does not affect significantly the total production rate, it stabilises the theoretical predictions under changes in the renormalisation and factorisation scales, hence improving the predictive power.

Unfortunately, the results of these resummed calculations are not available in a form suitable to implement selection cuts, as they only provide results for total cross-sections, fully integrated over all of phase space. The formalism has been generalised to the case of one-particle inclusive distributions in [48], although no complete numerical analyses have been performed yet.

The corrections of $O(\alpha_s^2)$ to the full production and decay should include the effect of gluon radiation off the quarks produced in the top decay. Interference effects are expected to take place between soft gluons emitted before and after the decay, at least for gluon energies not much larger than the top decay width. While these correlations are not expected to affect the measurement of generic distributions, even small soft-gluon corrections can have an impact on the determination of the top mass. Matrix elements for hard-gluon emission in $t\bar{t}$ production and decay ($p\bar{p} \rightarrow W^+W^-b\bar{b}$, with $t$ and $\bar{t}$ intermediate states) are implemented in a parton-level generator [49]. The one-gluon emission off the light quarks from the $W$ decays was implemented, in the soft-gluon approximation, in the parton-level calculation of [50].

The above results refer to the production of top quarks treated as free, stable partons. Parton-shower Monte Carlo programs are available (HERWIG [51], PYTHIA [52], ISAJET [53]) for a complete description of the final state, including the full development of the perturbative gluon shower from both initial and final states, the decay of the top quarks, and the hadronisation of the final-state partons. These will be reviewed in Appendix D. Recently, $O(\alpha_s)$ matrix element corrections to the decay of the top quark ($t \rightarrow Wb\bar{g}$) have been included in the HERWIG Monte Carlo [54]. The impact of these corrections will be reviewed in Sections 3.3 and 4.62.

3.2 Total $t\bar{t}$ production rates
In this section we collect the current theoretical predictions for cross sections and distributions, providing our best estimates of the systematic uncertainties. The theoretical uncertainties we shall consider include renormalisation ($\mu_R$) and factorisation ($\mu_F$) scale variations, and the choice of parton distribution functions (PDF’s).

We shall explore the first two by varying the scales over the range $\mu_0/2 < \mu < 2\mu_0$, where $\mu = \mu_R = \mu_F$ and

- $\mu_0 = m_t$ for the total cross sections
- $\mu_0 = \sqrt{m_t^2 + p_T^2}$ for single inclusive distributions
- $\mu_0 = \sqrt{m_t^2 + (p_T^{T,t} + p_T^{\bar{T},\bar{t}})^2}/2$ for double inclusive distributions

In the case of PDF’s, we shall consider the latest fits of the CTEQ [55] and of the MRST [56, 57] groups:

- MRST ($\alpha_s(M_Z) = 0.1175$, $\langle k_T \rangle = 0.4$ GeV) (default)
- MRST($g$ $\parallel$) ($\alpha_s = 0.1175$, $\langle k_T \rangle = 0.64$ GeV)
- MRST($g$ $\perp$) ($\alpha_s = 0.1175$, $\langle k_T \rangle = 0$)
- MRST($\alpha_s$ $\parallel$) ($\alpha_s = 0.1125$, $\langle k_T \rangle = 0.4$ GeV)
Fig. 3: $t\bar{t}$ production rates. Left: scale dependence at fixed order (NLO, dashed lines in the lower inset), and at NLO+NLL (solid lines). Right: PDF dependence. See the text for details.

- MRST($\alpha_S \uparrow\uparrow$) ($\alpha_S = 0.1225$, $\langle k_T \rangle = 0.4$ GeV)
- CTEQ5M ($\alpha_S = 0.118$)
- CTEQ5HJ ($\alpha_S = 0.118$, enhanced weight for Tevatron high-$E_T$ jets)
- CTEQ5HQ ($\alpha_S = 0.118$, using the ACOT heavy flavour scheme [58].)

All our numerical results relative to the MRST sets refer to the updated fits provided in [57]. These give total rates which are on average 5% larger than the fits in [56]. The total $t\bar{t}$ production cross section is given in Fig. 3, as a function of the top mass. As a reference set of parameters, we adopt $\mu_0 = m_t$ and MRST. Full NLO+NLL corrections are included. The upper inset shows the dependence of the cross section on the top mass. A fit to the distribution shows that $\Delta \sigma / \sigma \sim 5 \Delta m_t / m_t$. As a result, a 5% measurement of the total cross section is equivalent to a 1% determination of $m_t$ (approximately 2 GeV). As will be shown later on, 2 GeV is a rather safe estimate of the expected experimental accuracy in the determination of $m_t$ (1 GeV being the optimistic ultimate limit). It follows that 5% should be a minimal goal in the overall precision for the measurement of $m_t$. The scale uncertainty of the theoretical predictions is shown in the lower inset of Fig. 3. The dashed lines refer to the NLO scale dependence, which is of the order of $\pm 12\%$. The dotted lines refer to the inclusion of the NLL corrections, according to the results of [47]. The solid lines include the resummation of NLL effects, but assume a different structure of yet higher order (NNLL) corrections, relative to those contained in the reference NLL results (this is indicated by the value of the $A$ parameter equal to 2, see [47] for the details). The scale uncertainty, after inclusion of NLL corrections, is significantly reduced. In the most conservative case of $A = 2$, we have a $\pm 6\%$ variation. A detailed breakdown of the NLO $\mathcal{O}(\alpha_S^2 + \alpha_S^3)$ and higher-order $\mathcal{O}(\alpha_S^3)$ contributions, as a function of the scale and of the value of the parameter $A$, is given in Table 3. A recent study [59] of resummation effects on the total cross section for photo- and hadro-production of quarkonium states indicates that allowing $\mu_R \neq \mu_F$ increases the scale dependence of the NLL resummed cross-sections to almost match the scale dependence of the NLO results [60]. Preliminary results of this study also suggest a similar increase of scale dependence in the case of $t\bar{t}$ production, if $\mu_R$ and $\mu_F$ are varied independently. This dependence can however be reduced by replacing $\mu_R$ with $\mu_T$ as the argument of $\alpha_S$ in the sub-leading coefficients of the resummed exponent [61].

The PDF dependence is shown on the right hand side of Fig. 3, and given in detail for $m_t = 175$ GeV in Table 4. The current uncertainty is at the level of $\pm 10\%$. Notice that the largest deviations from the default set occur for sets using different input values of $\alpha_S(M_Z)$. The difference between the reference sets of the two groups (MRST and CTEQ5M) is at the level of 3%. It is interesting to explore potential correlations between the PDF dependence of top production, and the PDF dependence of other
Table 3: Resummation contributions to the total $t\bar{t}$ cross-sections ($m_t = 175$ GeV) in pb. PDF set MRST.

<table>
<thead>
<tr>
<th>$\mu_r = \mu_f$</th>
<th>NLO</th>
<th>NLL resummed, $A=2$</th>
<th>NLL resummed, $A=0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t/2$</td>
<td>890</td>
<td>-7</td>
<td>-12</td>
</tr>
<tr>
<td>$m_t$</td>
<td>796</td>
<td>29</td>
<td>63</td>
</tr>
<tr>
<td>$2m_t$</td>
<td>705</td>
<td>77</td>
<td>148</td>
</tr>
</tbody>
</table>

Table 4: Total $t\bar{t}$ cross-sections ($m_t = 175$ GeV) in pb. NLO+NLL ($A = 0$).

<table>
<thead>
<tr>
<th>PDF</th>
<th>$\mu = m_t/2$</th>
<th>$\mu = m_t$</th>
<th>$\mu = 2m_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MRST</td>
<td>877</td>
<td>859</td>
<td>853</td>
</tr>
<tr>
<td>MRST $g$</td>
<td>881</td>
<td>862</td>
<td>857</td>
</tr>
<tr>
<td>MRST $g$</td>
<td>876</td>
<td>858</td>
<td>852</td>
</tr>
<tr>
<td>MRST $\alpha_s$</td>
<td>796</td>
<td>781</td>
<td>777</td>
</tr>
<tr>
<td>MRST $\alpha_s$</td>
<td>964</td>
<td>942</td>
<td>934</td>
</tr>
<tr>
<td>CTEQ5M</td>
<td>904</td>
<td>886</td>
<td>881</td>
</tr>
<tr>
<td>CTEQ5HJ</td>
<td>905</td>
<td>886</td>
<td>881</td>
</tr>
</tbody>
</table>

processes induced by initial states with similar parton composition and range in $x$. One such example is given by inclusive jet production. Fig. 4 shows the initial-state fraction of inclusive jet final states (with $|\eta| < 2.5$) as a function of the jet-$E_T$ threshold. For values of $E_T \sim 200$ GeV, 90% of the jets come from processes with at least one gluon in the initial state. This fraction is similar to that present in $t\bar{t}$ production, where 90% of the rate is due to $gg$ collisions. On the right side of Fig. 4 we show the double ratios:

$$\frac{[\sigma(t\bar{t})/\sigma(\text{jet}, E_T > E_T^{\text{min}})]_{PDF}}{[\sigma(t\bar{t})/\sigma(\text{jet}, E_T > E_T^{\text{min}})]_{MRST}}$$

(10)

As the plot shows, there is a strong correlation between the PDF dependences of the two processes. The correlation is maximal for $E_T^{\text{min}} \sim 200$ GeV, as expected, since for this value the flavour composition of the initial states and the range of partonic momentum fractions probed in the two production processes are similar. In the range $180 \lesssim E_T^{\text{min}} \lesssim 260$ GeV the PDF dependence of the ratio $\sigma(t\bar{t})/\sigma(\text{jet}, E_T > E_T^{\text{min}})$ is reduced to a level of $\pm 1\%$, even for those sets for which the absolute top cross-section varies by $\pm 10\%$. The jet cross-sections were calculated [62] using a scale $\mu_{jet}^2 = E_T$. If we vary the scales for $t\bar{t}$ and jet production in a correlated way (i.e. selecting $\mu_{jet}^2 / \mu_0^2 = \mu_T^2 / \mu_0^2$), no significant scale dependence is observed. There is however no a-priori guarantee that the scales should be correlated. Unless this correlation can be proved to exist, use of the inclusive-jet cross section to normalise the $t\bar{t}$ cross-section will therefore leave a residual systematic uncertainty which is no smaller than the scale dependence of the jet cross section. We do not expect this to become any smaller than the PDF dependence in the near future.

Combining in quadrature the scale and PDF dependence of the total $t\bar{t}$ cross section, we are left with an overall 12% theoretical systematic uncertainty, corresponding to a 4 GeV uncertainty on the determination of the top mass from the total cross section.

### 3.3 Kinematical properties of $t\bar{t}$ production

We start from the most inclusive quantity, the top $p_T$ spectrum. The NLO predictions are shown in Fig. 5. Here we also explore the dependence on scale variations and on the choice of PDF. The uncertainties are
\[ \text{Initial state fractions in inclusive jet production} \]

Fig. 4: Left: initial state composition in inclusive jet events, as a function of the jet $E_T$ ($|\eta| < 2.5$). Right: PDF dependence of the top-to-jet cross-section ratio, as a function of the minimum jet $E_T$.

\[ \text{PDF dependence of the top-to-jet cross-section ratio, as a function of the minimum jet } E_T. \]

Fig. 5: Inclusive top $p_T$ spectrum. Left: scale and PDF dependence at NLO. Right: event rates above a given $p_T$ threshold.

\[ \pm 15\% \text{ and } \pm 10\%, \text{ respectively. The reconstruction of top quarks and their momenta, as well as the determination of the reconstruction efficiencies and of the possible biases induced by the experimental selection cuts, depend on the detailed structure of the final state. It is important to verify that inclusive distributions as predicted by the most accurate NLO calculations are faithfully reproduced by the shower Monte Carlo calculations, used for all experimental studies. This is done in Fig. 6, where the NLO calculation is compared to the result of the HERWIG Monte Carlo, after a proper rescaling by an overall constant $K$-factor. The bin-by-bin agreement between the two calculations is at the level of 10%, which should be adequate for a determination of acceptances and efficiencies at the percent level.} \]

Similar results are obtained for the invariant mass distribution of top quark pairs, shown in the plot of Fig. 6. The scale and PDF dependence of the NLO calculation are similar to those found for the inclusive $p_T$ spectrum, and are not reported in the figure.

Contrary to the case of inclusive $p_T$ and $M_\ell$ spectra, other kinematical distributions show large differences when comparing NLO and Monte Carlo results [42]. This is the case of distributions which are trivial at LO, and which are sensitive to Sudakov-like effects, such as the azimuthal correlations or the spectrum of the $t\bar{t}$ pair transverse momentum $p_T^t$. These two distributions are shown in the two plots of Fig. 7. Notice that the scale uncertainty at NLO is larger for these distributions than for previous inclusive
Fig. 6: Comparison of NLO and (rescaled) HERWIG spectra. Left: inclusive top $p_T$ spectrum. Right: inclusive $M_{tt}$ spectrum.

Fig. 7: Left: azimuthal correlation between the $t$ and $\bar{t}$. Right: integrated transverse momentum spectrum of the top quark pair. Continuous lines correspond to the parton-level NLO calculation, for different scale choices; the plots correspond to the result of the HERWIG Monte Carlo.

Quantities. These kinematical quantities are in fact trivial at $O(\alpha_s^2)$ (proportional to $\delta$-functions), and their evaluation at $O(\alpha_s^3)$ is therefore not a true NLO prediction. The regions $p_T^{t} \rightarrow 0$ and $\Delta \phi \rightarrow \pi$ are sensitive to multiple soft-gluon emission, and the differences between the NLO calculation (which only accounts for the emission of one gluon) and the Monte Carlo prediction (which includes the multi-gluon emission) is large. The region $p_T^{\tau} \gg m_t$ is vice-versa sensitive to the emission of individual hard gluons, a process which is more accurately accounted for by the full $O(\alpha_s^3)$ matrix elements included in the NLO calculation than by the Monte Carlo approach. Notice that the average value of $p_T^{t}$ is quite large, above 50 GeV. This is reasonable, as it is of the order of $\alpha_s$ times the average value of the hardness of the process ($\langle M_{tt} \rangle \sim 540$ GeV). It is found that this large transverse momentum is compensated by the emission of a jet recoiling against the top pair, with a smaller fraction of events where the $p_T^{t}$ comes from emission of hard gluons from the final state top quarks. The large-$p_T^{t}$ discrepancy observed in Fig. 7 should be eliminated once the matrix element corrections to top production will be incorporated in HERWIG, along the lines of the work done for Drell-Yan production in [63].

Emission of extra jets is also expected from the evolution of the decay products of the top quarks ($b$'s, as well as the jets from the hadronic $W$ decays). Gluon radiation off the decay products is included
in the shower Monte Carlo calculations. In the case of the latest version of HERWIG (v6.1) [51], the emission of the hardest gluon from the $b$ quarks is evaluated using the exact matrix elements [54]. This improvement, in addition to a few bug fixes, resolve the discrepancies uncovered in [49] between an exact parton level calculation and previous versions of HERWIG. The matrix-element corrections do not alter significantly most of the inclusive jet observables. As examples, we show in Fig. 8 the $\Delta R$ and the jet multiplicity distributions for events where both $W$'s decay leptonically. More details can be found in [64]. Jets are defined using the $k_T$ algorithm [65], with radius parameter $R = 1$. As can be seen, the impact of the exact matrix element corrections is limited, mostly because the extra-jet emission is dominated by initial-state radiation.

The impact on quantities which more directly affect the determination of the top mass remains to be fully evaluated. Given the large rate of high-$E_T$ jet emissions, their proper description will be a fundamental ingredient in the accurate reconstruction of the top quarks from the final state jets, and in the determination of the top quark mass. A complete analysis will only be possible once the matrix element corrections to the $t\bar{t}$ production will be incorporated in the Monte Carlos. Work in this direction is in progress (G. Corcella and M.H. Seymour).

### 3.4 Non-QCD radiative corrections to $t\bar{t}$ production

The production and decay of top quarks at hadron colliders is a promising environment for the detection and study of loop induced SUSY effects: at the parton level there is a large center of mass energy $\sqrt{s}$ available and owing to its large mass, the top quark strongly couples to the (virtual) Higgs bosons, a coupling which is additionally enhanced in SUSY models. Moreover, it might turn out that SUSY loop effects in connection with top and Higgs boson interactions less rapidly decouple than the ones to gauge boson observables.

To fully explore the potential of precision top physics at the LHC and at the Tevatron [66] to detect, discriminate and constrain new physics, the theoretical predictions for top quark observables need to be calculated beyond leading order (LO) in perturbation theory. Here we will concentrate on the effects of non-QCD radiative corrections to the production processes $gg \rightarrow t\bar{t}$ and $q\bar{q} \rightarrow t\bar{t}$, including supersymmetric corrections. When searching for quantum signatures of new physics also the SM loop effects have to be under control. The present SM prediction for $t\bar{t}$ observables includes the QCD corrections as discussed above and the EW one-loop contributions to the QCD $t\bar{t}$ production processes [67, 68, 69]. The latter modify the $g(t\bar{t}g)$ vertex by the virtual presence of the EW gauge bosons and the SM Higgs boson. At the parton level, the EW radiative corrections can enhance the LO cross sections by up to $\approx 30\%$ close to the threshold $\sqrt{s} \approx 2m_t$ when the SM Higgs boson is light and reduce the LO cross sections with increasing $\sqrt{s}$ by up to the same order of magnitude. After convoluting with the parton distribution
functions (PDF’s), however, they only reduce the LO production cross section $\sigma_{pp\to t\bar{t}X}$ at the LHC by a few percent [67]: up to 2.5(1.8)% for the following cuts on the transverse momentum $p_T$ and the pseudo rapidity $\eta$ of the top quark: $p_T > 100(20)$ GeV and $|\eta| < 2.5$.

So far, the studies of loop induced effects of BSM physics in $t\bar{t}$ production at hadron colliders include the following calculations:

**The $\mathcal{O}(\alpha_s)$ corrections** within a general two Higgs doublet model (G2HDM) (=SM with two Higgs doublets but without imposing SUSY constraints) to $q\bar{q}\to t\bar{t}$ [70, 71] and $gg\to t\bar{t}$ [71]. In addition to the contribution of the $W$ and $Z$, the $gt\bar{t}(q\bar{g})$ vertex is modified by the virtual presence of five physical Higgs bosons which appear in any G2HDM after spontaneous symmetry breaking: $H^0$, $h^0$, $A^0$, $H^\pm$. Thus, the G2HDM predictions for $t\bar{t}$ observables depend on their masses and on two mixing angles, $\beta$ and $\alpha$. The G2HDM radiative corrections are especially large for light Higgs bosons and for very small ($<1$) and very large values of $\tan \beta$ due to the enhanced Yukawa-like couplings of the top quark to the (virtual) Higgs bosons. Moreover, there is a possible source for large corrections due to a threshold effect in the renormalised top quark self-energy, i.e. when $m_t \approx M_{H^\pm} + m_b$. In [71] the $s$-channel Higgs exchange diagrams in the gluon fusion subprocess, $gg\to h^0$, $H^0\to t\bar{t}$, had been included. For this workshop we also considered the $gg\to A^0\to t\bar{t}$ contribution [72]. A study of the $s$-channel Higgs exchange diagrams alone, can be found in [73] ($H^0$) and [74, 75] ($H^0$ and $A^0$). They are of particular interest, since they can cause a peak-dip structure in the invariant $t\bar{t}$ mass distribution for heavy Higgs bosons, $M_{H^0, A^0} > 2m_t$, when interfered with the LO QCD $t\bar{t}$ production processes.

**The SUSY EW $\mathcal{O}(\alpha_s)$ corrections** within the MSSM to $q\bar{q}\to t\bar{t}$ [71, 76, 77, 78] and $gg\to t\bar{t}$ [71, 79]. In [71] also the squark loop contribution to the $gg\to h^0$, $H^0$ production process in the $s$ channel Higgs exchange diagrams has been taken into account. The SUSY EW corrections comprise the contributions of the supersymmetric Higgs sector, and the genuine SUSY contributions due to the virtual presence of two charginos $\tilde{\chi}^\pm$, four neutralinos $\tilde{\chi}^0$, two top squarks $\tilde{t}_{L,R}$ and two bottom squarks $\tilde{b}_{L,R}$. The MSSM input parameters can be fixed in such a way that the $t\bar{t}$ observables including MSSM loop corrections depend on a relatively small set of parameters [71]: $\tan \beta$, $M_{A^0}$, $m_{\tilde{t}_1}$, $m_{\tilde{b}_L}$, $\Phi_\tau$, $\mu$, $M_2$, where LR mixing is considered only in the top squark sector, parametrized by the mixing angle $\Phi_\tau$. $m_{\tilde{t}_1}$ and $m_{\tilde{b}_L} = m_{\tilde{b}_R}$ denote the mass of the lighter top squark and the bottom squark, respectively. The effects of the supersymmetric Higgs sector tend to be less pronounced than the ones of the G2HDM: since supersymmetry tightly correlates the parameters of the Higgs potential, the freedom to choose that set of parameters which yield the maximum effect is rather limited. On the other hand, they can be enhanced by the genuine SUSY contribution depending on the choice of the MSSM input parameters. The SUSY EW corrections can become large close to the threshold for the top quark decay $t\to \tilde{t} + \tilde{\chi}^0$. They are enhanced for very small ($<1$) and very large values of $\tan \beta$ and when there exists a light top squark ($m_{\tilde{t}_1} \approx 100$ GeV).

**The SUSY QCD $\mathcal{O}(\alpha_s)$ corrections** to $q\bar{q}\to t\bar{t}$ [78, 80, 81, 82, 83] and $gg\to t\bar{t}$ [84]. So far, there are only results available separately for the $q\bar{q}\to t\bar{t}$ (Tevatron) and the $gg\to t\bar{t}$ (LHC) production processes. The combination of both is work in progress and will be presented in [85]. The SUSY QCD contribution describes the modification of the $g\tilde{t}\tilde{t}(q\bar{g})$ vertex and the gluon vacuum polarisation due to the virtual presence of gluinos and squarks. Thus, additionally to the dependence on squarks masses (and on mixing angles if LR mixing is considered) the SUSY QCD corrections introduce a sensitivity of $t\bar{t}$ observables on the gluino mass $m_{\tilde{g}}$. As expected, the effects are the largest the lighter the gluino and/or the squarks. Again, there are possible enhancements due to threshold effects, for instance close to the anomalous threshold $m^2_{q\bar{q}} = m^2_{\tilde{g}} + m^2_{\tilde{t}_1}$.

The $t\bar{t}$ observables under investigation so far comprise the total $t\bar{t}$ production cross section $\sigma_{t\bar{t}}$, the invariant $t\bar{t}$ mass distribution $d\sigma/dM_{t\bar{t}}$ and parity violating asymmetries $A_{LR}$ in the production of left and right handed top quark pairs. At present, the numerical discussion is concentrated on the impact of BSM quantum effects on $t\bar{t}$ observables in $p\bar{p}\to t\bar{t}X$. A parton level Monte Carlo program for $p\bar{p}\to t\bar{t}\to W^+W^-b\bar{b}\to (f_i f'_i)(f'_j f_j)b\bar{b}$ is presently under construction [72]. This will allow a more
realistic study of the sensitivity of a variety of kinematical distributions to SUSY quantum signatures in the $t\bar{t}$ production processes, for instance by taking into account detector effects.

In the following we give an overview of the present status of BSM quantum effects in $t\bar{t}$ observables at the LHC:

$\sigma_{t\bar{t}}$ : In Table 5 we provide the relative corrections for $\sigma_{t\bar{t}}$ at the LHC for different BSM physics scenarios. They reflect the typical maximum size of the radiative corrections within the models under consideration. As already mentioned there are possible enhancements due to threshold effects, which can yield much larger relative corrections. However, they only arise for very specific choices of the MSSM input parameters. The SUSY EW one-loop corrections always reduce the LO production cross sections and range from SM values, to up to $\approx -5\%$ for heavy squarks and up to $\approx -20\%$ close to $m_t = m_{\tilde{t}_1} + m_{\tilde{\chi}_1^0}$. The SUSY QCD one-loop corrections, however, can either reduce or enhance $\sigma_{t\bar{t}}$. The relative corrections are negative for small $m_{\tilde{g}}$ and increase with decreasing gluino and/or squark masses. They change sign when approaching the threshold for real sparticle production and reach a maximum at $m_{\tilde{g}} \approx 200$ GeV of about $+2\%$ [84]. Again, very large corrections arise in the vicinity of a threshold for real sparticle production, $m_{\tilde{u}} = m_{\tilde{\chi}} + m_{\tilde{t}_1}$. The SUSY EW and QCD one-loop corrections, so far, have only been combined for the $q\bar{q} \rightarrow t\bar{t}$ production process and numerical results are provided for the Tevatron $p\bar{p}$ collider in [78, 83]. To summarise, apart from exceptional regions in the MSSM parameter space, it will be difficult to detect SUSY through loop contributions to the $t\bar{t}$ production rate. If light sparticles exist, they are most likely directly observed first. Then, the comparison of the precisely measured top production rate with the MSSM predictions will test the consistency of the model under consideration at quantum level and might yield additional information on the parameter space, for instance constraints on $\tan \beta$ and $\Phi_t$.

$\frac{d\sigma}{dM_{t\bar{t}}}$ : More promising are the distributions of kinematic variables. Here we will concentrate on the impact of SUSY quantum signatures on the invariant $t\bar{t}$ mass distribution. Results for the effects of EW one-loop corrections within the G2HDM and the MSSM on $d\sigma / dM_{t\bar{t}}$ at the LHC are provided in [71]. So far, the impact of the SUSY QCD one-loop contribution on $d\sigma / dM_{t\bar{t}}$ has only been discussed for the Tevatron $p\bar{p}$ collider [81], where it turned out that they can significantly change the normalisation and distort the shape of $d\sigma / dM_{t\bar{t}}$. As already mentioned, there is the possibility for an interesting peak-dip structure due to a heavy neutral Higgs resonance in $gg \rightarrow t\bar{t}$ within two Higgs doublet models. The potential of the LHC for the observation of such resonances has been studied in [74, 86]. In Section 3.5 the results of an ATLAS analysis of the observability of the $H/A \rightarrow t\bar{t}$ channel for different luminosities are presented. In Fig. 9 we show preliminary results for the invariant $t\bar{t}$ mass distribution to $pp \rightarrow t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow (\nu_e e^+)\bar{b}b$ at the LHC when including MSSM EW one-loop corrections [72]. When $M_{A^0} > 2m_t$ the $gg \rightarrow H^0, A^0 \rightarrow t\bar{t}$ contributions can cause an excess of $t\bar{t}$ events at $M_{t\bar{t}}$ slightly below $M_{A^0}$, when the Higgs bosons are not too heavy, and a dip in the distribution slightly above $M_{t\bar{t}} = M_{A^0}$. For the choice of MSSM parameters used in Fig. 9 the peak vanishes for $M_{A^0} > 400$ GeV and only a deficiency of events survives which decreases rapidly for increasing $M_{A^0}$. These effects can be enhanced when the SUSY QCD contributions are taken into account.

$A_{LR}$ : Parity violating asymmetries in the distribution of left and right-handed top quark pairs at hadron colliders directly probe the parity non-conserving parts of the non-QCD one-loop corrections to the $t\bar{t}$ production processes within the model under consideration and have been studied at the Tevatron [77, 82, 87, 68, 81, 83] and at the LHC [88]. In Fig. 10 we show the left-right asymmetries $A_{LR}$ in

<table>
<thead>
<tr>
<th>$\sigma_{t\bar{t}}^{NLO} - \sigma_{t\bar{t}}^{LO} / \sigma_{t\bar{t}}^{LO}$</th>
<th>$\sigma_{t\bar{t}}$ (M$_H = 100$ GeV)</th>
<th>G2HDM</th>
<th>SUSY EW</th>
<th>SUSY QCD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\leq 4%$</td>
<td>$\leq 4%$</td>
<td>$\leq 10%$</td>
<td>$\leq 4%$</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 9: The LO and NLO invariant mass distributions $d\sigma/dM(t\bar{t})$ (left) and the relative corrections (right) to the reaction $pp \rightarrow t\bar{t}(\nu, e^+) (d\sigma/d\phi)$ at the LHC with $p_T(c, q) > 20$ GeV, $E_T(\nu) > 20$ GeV and $[\eta(c, q)] < 3.2$ for different values of $M_{A^0}$ ($\tan \beta = 1.6$, $m_{t_1} = 160$ GeV, $m_{b_L} = 500$ GeV, $\mu = 120$ GeV and $M_Z = 3|\mu|$). For comparison, the relative correction when only taking into account the EW SM one-loop contribution is also shown. The CTEQ3LO set of PDF’s is used and $m_t = 174$ GeV.

Fig. 10: The left-right asymmetry $A_{L.R}$ in the invariant mass distribution of $t\bar{t}$ mass to $pp \rightarrow t\bar{t}X$ at the LHC with $p_T > 100$ GeV for different values of $\Phi_t$ and $m_{\tilde{t}_L}$ and for two extreme choices of $\tan \beta$: $\tan \beta = 0.7$ (left) and $\tan \beta = 50$ (right) (with $m_{t_1} = 90$ GeV, $M_{H^\pm} = 110$ GeV, $\mu = 120$ GeV and $M_Z = 3|\mu|$).

According to the SM, the top quark decays almost exclusively via $t \rightarrow W b$. The final state topology of $t\bar{t}$ events then depends on the decay modes of the $W$ bosons. In approximately 65.5% of $t\bar{t}$ events,
both W bosons decay hadronically via $W \rightarrow jj$, or at least one W decays via $W \rightarrow \tau \nu$. These events are difficult to extract cleanly above the large QCD multi-jet background, and are for the most part not considered further. Instead, the analyses presented here concentrate on leptonic $t\bar{t}$ events, where at least one of the W bosons decays via $W \rightarrow \ell \nu$ ($\ell = e, \mu$). The lepton plus large $E_T^{miss}$, due to the escaping neutrino(s), provide a large suppression against multi-jet backgrounds. The leptonic events, which account for approximately 34.5% of all $t\bar{t}$ events, can be subdivided into a “single lepton plus jets” sample and a “di-lepton” sample, depending on whether one or both W bosons decay leptonically. As discussed below, the selection cuts and background issues are quite different for the various final state topologies.

An important experimental tool for selecting clean top quark samples is the ability to identify b-jets. Techniques for $b$-tagging, using secondary vertices, semi-leptonic $b$-decays, and other characteristics of $b$-jets, have been extensively studied. Both ATLAS and CMS expect to achieve, for a $b$-tagging efficiency of 60%, a rejection of at least 100 against prompt jets (i.e. jets containing no long-lived particles) at low luminosity. At high luminosity, a rejection factor of around 100 can be obtained with a somewhat reduced $b$-tagging efficiency of typically 50%.

All the results presented in this section are obtained using for the signal the PYTHIA Monte Carlo program. Most background processes have also been generated with PYTHIA, with the exception of $Wb\bar{b}$, which has been produced using the HERWIG implementation [89] of the exact massive matrix-element calculation.

### 3.51 Single lepton plus jets sample

The single lepton plus jets topology, $t\bar{t} \rightarrow WWb\bar{b} \rightarrow (\ell\nu)(jj)b\bar{b}$ arises in $2 \times 2/9 \times 6/9 \approx 29.6\%$ of all $t\bar{t}$ events. One expects, therefore, production of almost 2.5 million single lepton plus jet events for an integrated luminosity of 10 fb$^{-1}$, corresponding to one year of LHC running at $10^{33}$ cm$^{-2}$ s$^{-1}$. The presence of a high $p_T$ isolated lepton provides an efficient trigger. The lepton and the high value of $E_T^{miss}$ give a large suppression of backgrounds from QCD multi-jets and $b\bar{b}$ production.

For the single lepton plus jets sample, it is possible to fully reconstruct the final state. The four-momentum of the missing neutrino can be reconstructed by setting $M^{\nu} = 0$, assigning $E_T^{\nu} = E_T^{miss}$, and calculating $p_T^{\nu}$, with a quadratic ambiguity, by applying the constraint that $M^{\ell\nu} = M_W$.

An analysis by ATLAS [30] examined a typical set of selection cuts. First, the presence of an isolated electron or muon with $p_T > 20$ GeV and $|\eta| < 2.5$ was required, along with a value of $E_T^{miss} > 20$ GeV. At least four jets with $p_T > 20$ GeV were required, where the jets were reconstructed using a fixed cone algorithm with cone size of $\Delta R = 0.7$. After cuts, the major sources of backgrounds were $W$+jet production with $W \rightarrow \ell\nu$ decay, and $Z$+jet events with $Z \rightarrow \ell^+\ell^-$. Potential backgrounds from $WW$, $WZ$, and $ZZ$ gauge boson pair production have also been considered, but are reduced to a negligible level after cuts.

A clean sample of $t\bar{t}$ events was obtained using $b$-tagging. Requiring that at least one of the jets be tagged as a $b$-jet yielded a selection efficiency (not counting branching ratios) of 33.3%. For an integrated luminosity of 10 fb$^{-1}$, this would correspond to a signal of 820,000 $t\bar{t}$ events. The total background, dominated by $W$+jet production, leads to a signal-to-background ratio (S/B) of 18.6. Tighter cuts can be used to select a particularly clean sample. Examples of this will be given in Section 4.

### 3.52 Di-lepton sample

Di-lepton events, where each W decays leptonically, provide a particularly clean sample of $t\bar{t}$ events, although the product of branching ratios is small, $2/9 \times 2/9 \approx 4.9\%$. With this branching ratio, one expects the production of over 400,000 di-lepton events for an integrated luminosity of 10 fb$^{-1}$.

The presence of two high $p_T$ isolated leptons allows these events to be triggered efficiently. Backgrounds arise from Drell-Yan processes associated with jets, $Z \rightarrow \tau^+\tau^-$ associated with jets, $WW$+jets,
and $b\bar{b}$ production. Typical selection criteria [30, 90] require two opposite-sign leptons within $|\eta| < 2.5$, with $p_T > 35$ and 25 GeV respectively, and with $E_{T}^{\text{miss}} > 40$ GeV. For the case of like-flavour leptons ($e^+ e^-$ and $\mu^+ \mu^-$), an additional cut $|M_{\ell\ell} - M_{\ell\ell}^{\text{Z}}| > 10$ GeV was made on the di-lepton mass to remove $Z$ candidates. Requiring, in addition, at least two jets with $p_T > 25$ GeV produced a signal of 80,000 events for 10 fb$^{-1}$, with S/B around 10. Introducing the requirement that at least one jet be tagged as a b-jet reduced the signal to about 58,000 events while improving the purity to S/B $\approx 50$.

3.53 Multi-jet sample

The largest sample of $t\bar{t}$ events consists of the topology $t\bar{t} \rightarrow WWb\bar{b} \rightarrow (jj)(jj)b\bar{b}$. The product of branching ratios of $6/9 \times 6/9 \approx 44.4\%$ implies production of 3.7 million multi-jet events for an integrated luminosity of 10 fb$^{-1}$. However, these events suffer from a very large background from QCD multi-jet events. In addition, the all-jet final state poses difficulties for triggering. For example, the trigger menus examined so far by ATLAS [30] consider multi-jet trigger thresholds only up to four jets, for which a jet luminosity of 10 fb$^{-1}$ is considered. The all-jet final state poses difficulties for triggering. For example, the trigger menus examined so far by ATLAS [30] consider multi-jet trigger thresholds only up to four jets, for which a jet $E_T$ threshold of 55 GeV is applied at low luminosity. Further study is required to determine appropriate thresholds for a six-jet topology.

At the Fermilab Tevatron Collider, both the CDF and D0 collaborations have shown that it is possible to isolate a $t\bar{t}$ signal in this channel. The CDF collaboration has obtained a signal significance over background of better than three standard deviations [9] by applying simple selection cuts and relying on the high $b$-tagging efficiency ($\approx 46\%$). To compensate for the less efficient $b$-tagging, the D0 collaboration has developed a more sophisticated event selection technique [10]. Ten kinematic variables to separate signal and background were used in a neural network, and the output was combined in a second network together with three additional variables designed to best characterise the $t\bar{t}$ events.

ATLAS has made a very preliminary investigation [30, 91] of a simple selection and reconstruction algorithm for attempting to extract the multi-jet $t\bar{t}$ signal from the background. Events were selected by requiring six or more jets with $p_T > 15$ GeV, and with at least two of them tagged as $b$-jets. Jets were required to satisfy $|\eta| < 3$ ($|\eta| < 2.5$ for $b$-jet candidates). In addition, the scalar sum of the transverse momenta of the jets was required to be greater than 200 GeV. The $t\bar{t}$ signal efficiency for these cuts was 19.3\%, while only 0.29\% of the QCD multi-jet events survived. With this selection, and assuming a QCD multi-jet cross-section of $1.4 \times 10^{-3}$ mb for $p_T$ (hard process) $> 100$ GeV, one obtains a signal-to-background ratio S/B $\approx 1/57$.

Reconstruction of the $t\bar{t}$ final state proceeded by first selecting di-jet pairs, from among those jets not tagged as $b$-jets, to form $W \rightarrow jj$ candidates. A $\chi^2_W$ was calculated from the deviations of the two $M_{jj}$ values from the known value of $M_W$. The combination which minimised the value of $\chi^2_W$ was selected, and events with $\chi^2_W > 3.5$ were rejected. For accepted events, the two $W$ candidates were then combined with $b$-tagged jets to form top and anti-top quark candidates, and a $\chi^2_t$ was calculated as the deviation from the condition that the top and anti-top masses are equal. Again, the combination with the lowest $\chi^2_t$ was selected, and events with $\chi^2_t > 7$ were rejected. After this reconstruction procedure and cuts, the value of S/B improved to 1/8 within the mass window 130-200 GeV. Increasing the $p_T$ threshold for jets led to some further improvement; for example, requiring $p_T > 25$ GeV yielded S/B = 1/6.

The isolation of a top signal can be further improved in a number of ways, such as using a multivariate discriminant based on kinematic variables like aplanarity, sphericity or $\Delta R$(jet-jet), or restricting the analysis to a sample of high $p_T$ events. These techniques are undergoing further investigation, but it will be very difficult to reliably extract the signal from the background in this channel. In particular, the multi-jet rates and topologies suffer from very large uncertainties.

3.54 Measurement of the $t\bar{t}$ invariant mass spectrum

As discussed previously, properties of $t\bar{t}$ events provide important probes of both SM and BSM physics. For example, a heavy resonance decaying to $t\bar{t}$ might enhance the cross-section, and might produce a
peak in the $M_{t\bar{t}}$ invariant mass spectrum. Deviations from the SM top quark branching ratios, due for example to a large rate of $t \rightarrow H^+ b$, could lead to an apparent deficit in the $t\bar{t}$ cross-section measured with the assumption that BR($t \rightarrow W b$) $\approx 1$.

Due to the very large samples of top quarks which will be produced at the LHC, measurements of the total cross-section $\sigma(t\bar{t})$ will be limited by the uncertainty of the integrated luminosity determination, which is currently estimated to be 5%-10%. The cross-section relative to some other hard process, such as $Z$ production, should be measured more precisely.

Concerning differential cross-sections, particular attention has thus far been paid by ATLAS [30] to measurement of the $M_{t\bar{t}}$ invariant mass spectrum. A number of theoretical models predict the existence of heavy resonances which decay to $t\bar{t}$. An example within the SM is the Higgs boson, which will decay to $t\bar{t}$ provided the decay is kinematically allowed. However, the strong coupling of the SM Higgs boson to the $W$ and $Z$ implies that the branching ratio to $t\bar{t}$ is never very large. For example, for $M_H = 500$ GeV, the SM Higgs natural width would be 63 GeV, and BR($H \rightarrow t\bar{t}$) $\approx 17\%$. The resulting value of $\sigma \times$BR for $H \rightarrow t\bar{t}$ in the SM is not sufficiently large to see a Higgs peak above the large background from continuum $t\bar{t}$ production. In the case of MSSM, however, if $M_{H,A} > 2m_t$, then BR($H/A \rightarrow t\bar{t}$) $\approx 100\%$ for $\tan \beta \approx 1$. For the case of scalar or pseudo-scalar Higgs resonances, it has been pointed out [73, 74] that interference can occur between the amplitude for the production of the resonance via $gg \rightarrow H/A \rightarrow t\bar{t}$ and the usual gluon fusion process $gg \rightarrow t\bar{t}$. The interference effects become stronger as the Higgs’ mass and width increase, severely complicating attempts to extract a resonance signal.

The possible existence of heavy resonances decaying to $t\bar{t}$ arises in technicolor models [92] as well as other models of strong EW symmetry breaking [93]. Recent variants of technicolor theories, such as Topcolor [94], posit new interactions which are specifically associated with the top quark, and could give rise to heavy particles decaying to $t\bar{t}$. Since $t\bar{t}$ production at the LHC is dominated by $gg$ fusion, colour octet resonances (“colourons”) could also be produced [95].

Because of the large variety of models and their parameters, ATLAS performed a study [30, 96] of the sensitivity to a “generic” narrow resonance decaying to $t\bar{t}$. Events of the single lepton plus jets topology $t\bar{t} \rightarrow WWb\bar{b} \rightarrow (\ell\nu)(jj)b\bar{b}$ were selected by requiring $E_T^{miss} > 20$ GeV, and the presence of an isolated electron or muon with $p_T > 20$ GeV and $|\eta| < 2.5$. In addition, it was required that there were between four and ten jets, each with $p_T > 20$ GeV and $|\eta| < 3.2$. At least one of the jets was required to be tagged as a b-jet. After these cuts, the background to the $t\bar{t}$ resonance search was dominated by continuum $t\bar{t}$ production.

The momentum of the neutrino was reconstructed, as described previously, by setting $M_\nu = 0$, assigning $E_T^{miss} = E_{T^{miss}}$, and calculating $p_T^\nu$ (with a quadratic ambiguity) by applying the constraint that $M_{t\bar{t}} = M_W$. The hadronic $W \rightarrow jj$ decay was reconstructed by selecting pairs of jets from among those not tagged as b-jets. In cases where there were at least two b-tagged jets, candidates for $t \rightarrow W b$ were formed by combining the $W$ candidates with each b-jet. In events with only a single b-tagged jet, this was assigned as one of the b-quarks and each of the still unassigned jets was then considered as a candidate for the other b-quark.

Among the many different possible jet-parton assignments, the combination was chosen that minimised the following $\chi^2$:

$$\chi^2 = (M_{jjb} - m_t)^2 / \sigma^2(M_{jjb}) + (M_{t\bar{b}} - m_t)^2 / \sigma^2(M_{t\bar{b}}) + (M_{jj} - M_W)^2 / \sigma^2(M_{jj})$$

Events were rejected if either $M_{t\bar{b}}$ or $M_{jjb}$ disagreed with the known value of $m_t$ by more than 30 GeV.

For events passing the reconstruction procedure, the measured energies were rescaled, according to their resolution, to give the correct values of $M_W$ and $m_t$ for the appropriate combinations. This procedure improved the resolution of the mass reconstruction of the $t\bar{t}$ pair to $\sigma(M_{tt})/M_{tt} \approx 6.6\%$. As an example, Fig. 11 shows the reconstructed $M_{tt}$ distribution for a narrow resonance of mass 1600 GeV. The width of the Gaussian core is well described by the resolution function described above. The size
Reconstructed $m_{t\bar{t}}$ (GeV)

Fig. 11: Measured $t\bar{t}$ invariant mass distribution for reconstruction of a narrow resonance of mass 1600 GeV decaying to $t\bar{t}$.

The reconstruction efficiency, not including branching ratios, for a resonance of mass 400 GeV, decreasing gradually to about 15% for $M_{t\bar{t}} = 2$ TeV.

For a narrow resonance $X$ decaying to $t\bar{t}$, Fig. 12 shows the required $\sigma \times$ BR($X \rightarrow t\bar{t}$) for discovery of the resonance. The criterion used to define the discovery potential was observation within a $\pm 2\sigma$ mass window of a signal above the $t\bar{t}$ continuum background, where the required signal must have a statistical significance of at least 5$\sigma$ and must contain at least ten events. Results are shown versus $M_X$ for integrated luminosities of 30 fb$^{-1}$ and 300 fb$^{-1}$. For example, with 30 fb$^{-1}$, a 500 GeV resonance could be discovered provided its $\sigma \times$ BR is at least 2560 fb. This value decreases to 830 fb for $M_X = 1$ TeV, and to 160 fb for $M_X = 2$ TeV. The corresponding values for an integrated luminosity of 300 fb$^{-1}$ are 835 fb, 265 fb, and 50 fb for resonances masses $M_X = 500$ GeV, 1 TeV, and 2 TeV, respectively.

Once predictions from models exist for the mass, natural width, and $\sigma \times$ BR for a specific resonance, the results in Fig. 12 can be used to determine the sensitivity and discovery potential for those models. As discussed above, for the case of scalar or pseudo-scalar Higgs resonances, extra care must be taken due to possible interference effects. While such effects are small for the case of a narrow resonance,
they can be significant once the finite widths of heavy resonances are taken into account. For example, ATLAS has performed an analysis [30, 97] of the decays $H/A \to t\bar{t}$ in MSSM with $\tan \beta = 1.5$ and $M_{H,A} > 2m_t$. Assuming the $t\bar{t}$ continuum background is well known, a combined $H + A$ signal would be visible for Higgs masses in the range of about 370 - 450 GeV. However, the interference effects produce an effective suppression of the combined $H + A$ production rates of about 30% for $M_{H,A} = 370$ GeV, increasing to 70% for masses of 450 GeV, essentially eliminating the possibility to extract a signal for higher Higgs masses, and thereby severely limiting the MSSM parameter space for which this channel has discovery potential (see Fig. 13).

4. TOP QUARK MASS

As discussed in Section 2.2 one of the main motivations for top physics at the LHC is an accurate measurement of the top mass. Currently the best Tevatron single-experiment results on $m_t$ are obtained with the lepton plus jets final states. These yield: $m_t = 175.9 \pm 4.8$ (stat.) $\pm 5.3$ (syst.) (CDF) [6] and $173.3 \pm 5.6$ (stat.) $\pm 5.5$ (syst.) (DØ) [5]. The systematic errors in both measurements are largely dominated by the uncertainty on the jet energy scale which amounts to 4.4 GeV and 4 GeV for CDF and DØ, respectively. On the other hand, the systematic errors in the di-lepton channels are somewhat less, but the statistical errors are significantly larger, by a factor of $\gtrsim 2$, as compared to the lepton plus jets final states. Future runs of the Tevatron with an about 20-fold increase in statistics promise a measurement of the top mass with an accuracy of up to $\sim 3$ GeV [98]; in the lepton plus jets channel the error is dominated by the systematics while in the di-lepton channels the limiting factor is still the statistics.

Several studies of the accuracy which can be expected with the LHC experiments have been performed in the past [99]. It is interesting to see whether one can use the large statistics available after a few years of high-luminosity running to push the precision further. In particular, it is interesting to study the ultimate accuracy achievable at a hadronic collider, and the factors that limit this accuracy.

In the following subsections, we begin with general remarks on the top quark mass and a very brief review of the present status of the theoretical understanding of top quark mass measurement in the threshold scan at a future $e^+e^-$ collider. We then present the results of a recent studies of top mass reconstruction at the LHC. The techniques used include the study of the lepton plus jets final states (inclusive, as well as limited to high-$p_T$ top quarks), di-lepton final states (using the di-leptons from the leptonic decay of both $W$'s, as well as samples where the isolated $W$ lepton is paired with a non-isolated

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lepton from the decay of the companion $b$ hadron). A very promising analysis using the $J/\psi$ from the $b$ hadron decay paired with the lepton from the leptonic decay of the $W$ is discussed at the end. The conclusions of these studies indicate that an accuracy of 2 GeV should be achievable with the statistics available after only 1 year of running at low luminosity. An accuracy of 1 GeV accuracy could be achieved after the high luminosity phase.

4.1 General remarks and the top mass measurement in $e^+e^-$ annihilation

Although one speaks of “the” top quark mass, one should keep in mind that the concept of quark mass is convention-dependent. The top quark pole mass definition is often implicit, but in a confining theory it can be useful to choose another convention. This is true even for top quarks when one discusses mass measurements with an accuracy of order of or below the strong interaction scale. Since different mass conventions can differ by 10 GeV (see Section 2.1), the question arises which mass is actually determined to an accuracy of 1-2 GeV by a particular measurement.

The simple answer is that a particular measurement determines those mass parameters accurately in terms of which uncalculated higher order corrections to the matrix elements of the process are small. This in turn may depend on the accuracy one aims at and the order to which the process has already been calculated. To clarify these statements we briefly discuss the top quark mass measurement at a high energy $e^+e^-$ collider.

“The” top quark mass can be measured in $e^+e^-$ collisions by reconstructing top quark decay products in much the same way as at the LHC. In addition, there exists the unique possibility of determining the mass in pair production near threshold. This is considered to be the most accurate method [100] and it appears that an uncertainty of $\delta m_t \approx 0.15$ GeV can be achieved for the top quark $\overline{MS}$ mass with the presently available theoretical input [101]. This is a factor two improvement compared to the accuracy that could be achieved with the same theoretical input if the cross section were parametrised in terms of the top quark pole mass. The fundamental reason for this difference is the fact that the concept of a quark pole mass is intrinsically ambiguous by an amount of order $\Lambda_{QCD}$ [102] and this conclusion remains valid even if the quark decays on average before hadronisation [103]. In the context of perturbation theory this ambiguity translates into sizeable higher order corrections to the matrix elements of a given process renormalized in the pole mass scheme. This makes it preferable to choose another mass convention if large corrections disappear in this way as is the case for the total cross section in $e^+e^-$ annihilation, because the total cross section is less affected by non-perturbative effects than the pole mass itself. Note, however, that despite this preference the position of the threshold is closer to twice the pole mass than twice the $\overline{MS}$ mass, hence a leading order calculation determines the pole mass more naturally. It is possible to introduce intermediate mass renormalizations that are better defined than the pole mass and yet adequate to physical processes in which top quarks are close to mass shell [101, 104]. The conclusion that the top quark pole mass is disfavoured is based on the existence of such mass redefinitions and the existence of accurate theoretical calculations.

The situation with mass determinations at the LHC appears much more complicated, since the mass reconstruction is to a large extent an experimental procedure based on leading order theoretical calculations, which are not sensitive to mass renormalization at all. Furthermore the concept of invariant mass of a top quark decay system is prone to “large” non-perturbative corrections of relative order $\Lambda_{QCD}/m_t$, because the loss or gain of a soft particle changes the invariant mass squared by an amount of order $m_t \Lambda_{QCD}$. The parametric magnitude of non-perturbative corrections is of the same order of magnitude as for the top quark pole mass itself and cannot be decreased by choosing another mass renormalization prescription. For this reason, top mass measurements based on reconstructing $m_t$ from the invariant mass of the decay products of a single top quark should be considered as measurements of the top quark pole mass. From the remarks above it follows that there is a limitation of principle on the accuracy of such measurements. However, under LHC conditions the experimental systematic uncertainty discussed later in this section is the limiting factor in practice. A potential exception is the measure-
Table 6: Efficiencies (in percent) for the inclusive $t\bar{t}$ single lepton plus jets signal and for background processes, as a function of the selection cuts applied. No branching ratios are included in the numbers. The last column gives the equivalent number of events for an integrated luminosity of 10 fb$^{-1}$, and the signal-to-background ratio.

<table>
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<th>$E_T^{miss} &gt; 20$ GeV as before,</th>
<th>$E_T^{miss} &gt; 20$ GeV as before,</th>
<th>events,</th>
<th>per 10 fb$^{-1}$</th>
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<td>$S/B$</td>
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<td></td>
<td></td>
<td>65</td>
<td></td>
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</tbody>
</table>

Note: For the background processes, the HERWIG [51, 89] generator was used for the background process $Wb\bar{b}$.

4.2 $m_t$ in the lepton plus jets channel. Inclusive sample

The inclusive lepton plus jets channel provides a large and clean sample of top quarks for mass reconstruction. Considering only electrons and muons, the branching ratio of this channel is 29.6%. Therefore, one can expect more than 2 millions events for one year of running at low luminosity. ATLAS performed an analysis in that channel using events generated using PYTHIA [52] and the ATLAS detector fast simulation package ATLAST [105]. The top mass is determined using the hadronic part of the decay, as the invariant mass of the three jets coming from the same top: $m_t = m_{jjb}$. The leptonic top decay is used to tag the event with the presence of a high $p_T$ lepton and large $E_T^{miss}$. For the background processes, the HERWIG [51, 89] generator was used for the background process $Wb\bar{b}$.

The following background processes have been considered: $b\bar{b}$, $W + jets$ with $W \rightarrow \ell\nu$, $Z + jets$ with $Z \rightarrow \ell^+\ell^-$, $WW$ with one $W \rightarrow \ell\nu$ and the other $W \rightarrow q\bar{q}$, $WZ$ with $W \rightarrow \ell\nu$ and $Z \rightarrow q\bar{q}$, $ZZ$ with one $Z \rightarrow \ell^+\ell^-$ and $Z \rightarrow q\bar{q}$, and $Wb\bar{b}$ with $W \rightarrow \ell\nu$. Events are selected by requiring an isolated lepton with $p_T > 20$ GeV and $|\eta| < 2.5$, $E_T^{miss} > 20$ GeV, and four jets with $p_T > 40$ GeV and $|\eta| < 2.5$, of which two of them were required to be tagged as $b$-jets. Jets were reconstructed using a fixed cone algorithm with $\Delta R = 0.4$. Although at production level the signal over background is very unfavourable, after the selection cuts and for an integrated luminosity of 10 fb$^{-1}$, 126000 signal events and 1922 background events were kept, yielding a value of $S/B = 65$ (see Table 6).

The reconstruction of the decay $W \rightarrow jj$ is first performed. The invariant mass $m_{jj}$ of all the combinations of jets (with $p_T > 40$ GeV and $|\eta| < 2.5$) that were not tagged as $b$-jets is computed and the jet pair with an invariant mass closest to $m_W$ is selected as the $W$ candidate. Fig. 14 represents the invariant mass distribution of the selected jet pairs. The reconstructed $W$ mass is consistent with the generated value, the mass resolution being 7.8 GeV. Within a window of $\pm 20$ GeV around the $W$ mass, the purity (P) and the overall efficiency (E) of the $W$ reconstruction are respectively P=67% and E=1.7%. Additional pair association criteria, such as requiring the leading jet to be part of the combination, did not improve significantly the purity and have not been considered further in the analysis. $W$ candidates, retained if $|m_{jj} - M_W| < 20$ GeV, have then to be associated with one $b$-tagged jet to reconstruct the decay $t \rightarrow Wb$. To reconstruct the right combination, some association criteria have been tried, such as choosing the $b$-jet furthest from the isolated lepton, the $b$-jet closest to the reconstructed $W$, and choosing
the $jjb$ combination having the highest $p_T$ for the reconstructed top. These various methods gave similar results. Fig. 14 presents the invariant mass distribution of the reconstructed top when the $jjb$ combination having the highest $p_T$ has been used as association criteria. No $M_W$ constraint is applied for the light quark jets. For an integrated luminosity of 10fb$^{-1}$, the total number of reconstructed top is 32000 events, of which 30000 are within a window of $\pm$35 GeV around the generated top mass $m_t=175$ GeV. The total number of combinatorial events is 34000, of which 14000 are within the mass window. The number of background events coming from other processes is negligible. The $m_{jjb}$ distribution fitted by a Gaussian plus a third order polynomial yields a top mass consistent with the generated value of 175 GeV and a top mass resolution of 11.9 GeV. The resulting statistical uncertainty for an integrated luminosity of 10fb$^{-1}$ is $\delta m_t = 0.070$ GeV.

The dependence of the top reconstruction algorithm on the top mass has been checked using several samples of $t\bar{t}$ events generated with different values of $m_t$ ranging from 160 to 190 GeV. The results, shown in Fig. 15, demonstrate a linear dependence of the reconstructed top mass on the generated value: the data points are fitted to a linear function with $\chi^2/ndf = 6.7/8$. The stability of the mass value as a function of the transverse momentum of the reconstructed top ($p_T$(top)) was also checked. As shown in Fig. 15, no significant $p_T$(top) dependence is observed: the data points are fitted to a constant with $\chi^2/ndf = 6.25/5$. For more details of this analysis, see [106].
The results presented above, obtained with a fast simulation package, have been cross-checked with 30000 events passed through the ATLAS GEANT-based full simulation package [107]. In full simulation, in order to save computing time, events have been generated under restrictive conditions at the generator level. The comparison is done by using the same generated events which have been passed through both the fast and full simulation packages. The results, in terms of purity, efficiency and mass resolutions show a reasonable agreement between fast and full simulation. In addition, as it is shown in Fig. 16, the shape and amount of the combinatorial background for the $m_{j\bar{b}}$ distributions are in good agreement between the two types of simulations.

It has to be noted that for this analysis as well as for the other top mass reconstruction studies performed within ATLAS, the jets were calibrated using the ratio $p_T^{\text{parton}}/p_T^{\text{jet}}$ obtained from Monte Carlo samples of di-jet events or $H \rightarrow b\bar{b}$ with $m_H = 100$ GeV. In that aspect this calibration does not include all possible detector effects and corrections. More details can be found in Chapter 20 of [30] and in Appendix A.

4.3 $m_t$ in the lepton plus jets channel. High $p_T$ sample

An interesting possibility at the LHC, thanks to the large $t\bar{t}$ production rate, is the use of special sub-samples, such as events where the top and anti-top quarks have high $p_T$. In this case, they are produced back-to-back in the lab-frame, and the daughters from the two top decays will appear in distinct “hemispheres” of the detector. This topology would greatly reduce the combinatorial background as well as the backgrounds from other processes. Furthermore, the higher average energy of the jets to be reconstructed should reduce the sensitivity to systematic effects due to the jet energy calibration and to effects of gluon radiation. However, in this case a competing effect appears which can limit the resulting precision: as the top $p_T$ increases, the jet overlapping probability increases as well, which again affects the jet calibration. ATLAS performed a preliminary study of this possibility using two different reconstruction methods:

• in the first one an analysis similar to the inclusive case is done, with $m_t$ being reconstructed from the three jets in the one hemisphere ($m_t=m_{jib}$);
• in the second one, $m_t$ is reconstructed summing up the energies in the calorimeter towers in a large cone around the top direction.

In the following paragraphs, highlights of these analyses are discussed.

4.3.1 Jet Analysis

High $p_T$ $t\bar{t}$ events were generated using PYTHIA 5.7 [52] with a $p_T$ cut on the hard scattering process above 200 GeV. The expected cross-section in this case is about 120 pb, or about 14.5% of the total
Fig. 17: Left: invariant mass distribution of the selected di-jet combinations for the high $p_T$ (top) sample. Right: invariant mass distribution of the accepted combinations for the high $p_T$ (top) sample. Both distributions are normalised to an integrated luminosity of $10\text{fb}^{-1}$.

$t\bar{t}$ production cross-section. The selection cuts required the presence of an isolated lepton with $p_T > 30$ GeV and $|\eta| < 2.5$, and $E_T^{\text{miss}} > 30$ GeV. The total transverse energy of the event was required to be greater than 450 GeV. Jets were reconstructed using a cone algorithm with radius $\Delta R=0.4$. The plane perpendicular to the direction of the isolated lepton was used to divide the detector into two hemispheres. Considering only jets with $p_T > 40$ GeV and $|\eta| < 2.5$, the cuts required one $b$-tagged jet in the same hemisphere as the lepton, and three jets, one of which was $b$-tagged, in the opposite hemisphere. Di-jet candidates for the $W \rightarrow jj$ decay were selected among the non-$b$-tagged jets in the hemisphere opposite to the lepton. The resultant $m_{jj}$ invariant mass distribution is shown in Fig. 17 (left). Fitting the six bins around the peak of the mass distribution with a Gaussian, yielded a $W$ mass consistent with the generated value, and a $m_{jj}$ resolution of 7 GeV, in good agreement with that obtained for the inclusive sample. Di-jets with $40$ GeV $< m_{jj} < 120$ GeV were then combined with the $b$-tagged jet from the hemisphere opposite to the lepton to form $t \rightarrow jjb$ candidates. Finally, the high $p_T$ (top) requirement was imposed by requiring $p_T(jjb) > 250$ GeV. With these cuts, the overall signal efficiency was 1.7%, and the background from sources other than $t\bar{t}$ was reduced to a negligible level. The invariant mass distribution of the accepted $jjb$ combinations is shown in Fig. 17 (right). Fitting the six bins around the peak of the mass distribution with a Gaussian, yielded a top mass consistent with the generated value of 175 GeV, and a $m_{jjb}$ mass resolution of 11.8 GeV. For an integrated luminosity of 10 fb$^{-1}$, a sample of 6300 events would be collected in ATLAS, leading to a statistical error of $\delta m_t(\text{stat.}) = \pm 0.25$ GeV, which remains well below the systematic uncertainty. As in the case of the inclusive sample, no strong $p_T$ dependence was observed and the reconstructed mass depends linearly on the Monte Carlo input value.

4.32 Using a large calorimeter cluster

For sufficiently high $p_T$ (top) values, the jets from the top decay are close to each other with a large possibility of overlap. In such a case it might be possible to reconstruct the top mass by collecting all the energy deposited in the calorimeter in a large cone around the top quark direction. Such a technique has the potential to reduce the systematic errors, since it is less sensitive to the calibration of jets and to the intrinsic complexities of effects due to leakage outside the smaller cones, energy sharing between jets, etc. Some results from a preliminary investigation of the potential of this technique are discussed here. More details of the analysis can be found in [30, 108].

Similar event selection criteria as in the previous case were used: an isolated lepton with $p_T > 20$ GeV and $|\eta| < 2.5$, $E_T^{\text{miss}} > 20$ GeV, one $b$-tagged jet (with $\Delta R=0.4$ and $p_T > 20$ GeV) in the lepton hemisphere, and at least 3 jets in the hemisphere opposite to the lepton ($\Delta R=0.2$, $p_T > 20$ GeV) with
Fig. 18: Left: invariant mass distribution of the selected $jjb$ combination, using $\Delta R=0.2$ cones for the high $p_T$ (top) sample, normalised to an integrated luminosity of $10 fb^{-1}$. The shaded area corresponds to the combinations with the correct jet-parton assignments. Right: distance $\Delta R$ between the reconstructed and the parton level direction of the top quark. The dots correspond to the correct $jjb$ combinations.

one of them $b$-tagged. For the accepted events, the two highest $p_T$ non-$b$-tagged jets were combined with the highest $p_T$ $b$-jet candidate in the hemisphere opposite to the lepton to form candidates for the $jjb$ hadronic top decay. The selected $jjb$ combination was required to have $p_T > 150$ GeV and $|\eta| < 2.5$. With these selection criteria, about 13000 events would be expected in the mass window from 145 to 200 GeV, with a purity of 90%, for an integrated luminosity of $10 fb^{-1}$. The reconstructed invariant mass of the $jjb$ combination is shown in Fig. 18 (left). The direction of the top quark was then determined from the jet momenta. Figure 18 (right) shows the distance $\Delta R$ in $(\eta, \phi)$ space between the reconstructed and the true top direction at the parton level, demonstrating good agreement.

A large cone of radius $\Delta R$ was then drawn around the top quark direction, and the top mass was determined by adding the energies of all calorimeter “towers” within the cone. A calorimeter tower has a size of $\delta \eta \times \delta \phi = 0.1 \times 0.1$, combining the information of both the EM and hadronic calorimeters. The invariant mass spectrum is shown in Fig. 19 (left) for a cone size $\Delta R = 1.3$, and exhibits a clean peak at the top quark mass. The fitted value of the reconstructed top mass is shown in Fig. 19 (right), where it displays a strong dependence on the cone size. If initial (ISR) and final (FSR) state radiation in PYTHIA are turned off, the fitted mass remains constant (to within 2%), independently of cone size.

The large dependence of the reconstructed top mass on the cone size can be attributed to the underlying event (UE) contribution. A method was developed to evaluate and subtract the underlying event contribution using the calorimeter towers not associated with the products of the top quark decay. The UE contribution was calculated as the average $E_T$ deposited per calorimeter tower, averaged over those towers which were far away from the reconstructed jets of the event. As expected, the average $E_T$ per calorimeter tower increases as more activity is added, especially in the case of ISR. However, only a rather small dependence is observed on the radius $\Delta R$ used to isolate the towers associated with the hard scattering process. The resulting value of the reconstructed mass ($m_{cone}$), with and without UE subtraction, is also shown in Fig. 19 (right) as a function of the cone radius. As can be seen, after the UE subtraction, the reconstructed top mass is independent of the cone size used. As a cross-check, the mean $E_T$ per cell subtracted was varied by $\pm 10\%$ and the top mass recalculated in each case. As shown superimposed on Fig. 19 (right), these “miscalibrations” lead to a re-emergence of a dependence of $m_T$ on the cone size. While the prescription for the UE subtraction does lead to a top mass which is independent of the cone size, it should be noted that the reconstructed mass is about 15 GeV (or 8.6%) below the nominal value, $m_t = 175$ GeV, implying that a rather large correction is needed.
Fig. 19: Left: reconstructed $t \rightarrow jjb$ mass spectrum obtained using cells in a single cone of size $\Delta R = 1.3$, normalised to an integrated luminosity of 10 fb$^{-1}$. Right: the fitted top mass using cells in a single cone, before and after the underlying event (UE) subtraction and as a function of the cone size.

To investigate if this correction can be extracted from the data without relying on Monte Carlo simulations, the same procedure was applied to a sample of $W$+ jet events generated with a range of $p_T$ comparable to that of the top sample. The $W$ was forced to decay hadronically into jets. The UE contribution was estimated with the same algorithm as described above. The results agreed within 1% with the values determined for the high $p_T$ (top) sample. As in the case of the top events, the reconstructed $W$ mass after UE subtraction is independent of the cone size. The average value of $m_{jj}$ after the UE subtraction is about 8.5 GeV (or 10.6%) below the nominal value of $m_W$. The fractional error on $m_{jj}$, as measured with the $W$+jet sample, was used as a correction factor to $m_{\text{cone}}$ in the high $p_T$ (top) sample. For a cone of radius $\Delta R = 1.3$, the top mass after UE subtraction increases from 159.9 GeV to 176.0 GeV after rescaling. The rescaled values of $m_{\text{cone}}$ are about 1% higher than the generated top mass. This over-correction of $m_t$ using the value of $m_W$ measured with the same method, is mainly due to ISR contributions. If ISR is switched off, the rescaling procedure works to better than 1%.

4.4 Systematic uncertainties on the measurement of $m_t$ in the single lepton plus jets channel

For the analyses presented above within ATLAS, a number of sources of systematic error have been studied using samples of events generated with PYTHIA and simulated mainly with the fast detector package ATLFAST, but also using a relatively large number of fully simulated events in order to cross-check some of the results. The results of these studies are summarised in Fig. 20 and discussed below.

Jet energy scale: The measurement of $m_t$ via reconstruction of $t \rightarrow jjb$ relies on a precise knowledge of the energy calibration for both light quark jets and $b$-jets. The jet energy scale depends on a variety of detector and physics effects, including non-linearities in the calorimeter response, energy lost outside the jet cone (due, for example, to energy swept away by the magnetic field or to gluon radiation at large angles with respect to the original parton), energy losses due to detector effects (cracks, leakage, etc.), and “noise” due to the underlying event. Preliminary studies done in ATLAS indicate that a jet energy scale calibration at the level of 1% for both light quark and $b$-jets would be feasible at the LHC (see discussion in the Appendix A). In the case of the $m_t$ reconstructed from the invariant mass of the three jets ($m_{jjb}$) the $b$-jet energy scale enters directly in the measurement and therefore it must be calibrated from other sources, while the energy of the two light quark jets can be calibrated event-by-event using the $W$ mass constraint. This would work quite well at least for the inclusive sample, where the jets are well separated. In the high $p_T$ case, energy sharing algorithms and corrections for the two jets are needed, and therefore in order to be conservative we assume in the following that no such an event-by-event correction can be made. To estimate the effect of an absolute jet energy scale uncertainty,
**Inclusive sample**

High $P_T$ sample: jet

High $P_T$ sample: cluster

**Light jet E scale**

1% scale error

**b-jet E scale**

1% scale error

**b-quark fragmentation**

$\varepsilon_b=0.005$

**ISR off**

**FSR off**

**Background**

**Calo tower E scale**

1% scale error

**Calibration (re-scaling)**

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Fig. 20: Summary of systematic errors in the $m_T$ measurement. Left: the observed mass shifts for different effects studied. The dashed lines indicate a 1% mass window. Right: the quoted error in the $m_T$ measurement.

Different “miscalibration” coefficients were applied to the measured jet energies. A linear dependence was observed.

**b-quark fragmentation:** The fraction of the original $b$-quark momentum which will appear as visible energy in the reconstruction cone of the corresponding $b$-jet depends on the fragmentation function of the $b$-quark. This function is usually parametrised in PYTHIA in terms of one variable, $\varepsilon_b$, using the Peterson fragmentation function [109]. To estimate the systematic error in $m_T$, the “default” value for $\varepsilon_b$ ($=-0.006$) was varied within its experimental uncertainty (0.0025) [110, 19] and the difference in the reconstructed $m_T$ was taken as the systematic error $\delta m_T$.

**Initial and final state radiation:** The presence of ISR or FSR can impact the measurement of $m_T$. To estimate the systematic error due to these, data samples were generated where ISR or FSR in the PYTHIA generator were switched off. In the case of FSR, a large mass shift was observed for a jet cone of $\Delta R=0.4$. This is reduced as expected when a larger cone is used. Clearly this case is rather pessimistic since the knowledge in both ISR and FSR is typically at the level of 10%. Therefore as a conservative estimate of the resultant systematic errors in $m_T$, 20% of the mass shifts were used.

An alternative approach uses the measured jet multiplicity to search, event-by-event, for the presence of hard gluon radiation. Following the convention for this approach adopted at the Tevatron [5, 6], the mass shift would be defined not by comparing events with radiation switched on and events with radiation switched off, but by the difference, $\Delta m_T$, between the value of $m_T$ determined from events with exactly four jets and that determined from events with more than four jets. The systematic error due to effects of initial and final radiation would then be considered as $\delta m_T = \Delta m_T / \sqrt{12}$. Such a calculation would yield systematic errors of approximately 0.4-1.1 GeV, smaller than the more conservative approach adopted here.

**Background:** Uncertainties in the size and shape of the background, which is dominated by “wrong combinations” in $t\bar{t}$ events, can affect the top mass reconstruction. The resultant systematic uncertainty
on \( m_t \) was estimated by varying the assumptions on the background shape in the fitting procedure. Fits of the \( m_{jjb} \) distribution were performed assuming a Gaussian shape for the signal and either a polynomial or a threshold function for the background. Varying the background function resulted in a systematic error on \( m_t \) of 0.2 GeV. The structure of the UE can affect the top mass reconstruction. However, as discussed above, it is possible to estimate and correct for this effect using data. Given the large statistics available at the LHC, it is assumed that the residual uncertainty from the underlying event will be small compared to the other errors (note that the UE denotes here a minimum bias event, since the impact of ISR has already been accounted for).

For the particular case of the \( m_t \) reconstructed using a large calorimeter cluster, similar procedures were adopted to estimate the the systematic errors. It is important to notice that, as expected, the use of a large cone substantially reduces the effects of FSR and \( b \)-quark fragmentation, each of which gives rise to a systematic error of 0.1 GeV. The uncertainty arising from ISR, which can affect the determination of the UE subtraction, is about 0.1 GeV as well. However, the main uncertainty in this technique comes from the calibration procedure. The calibration with the \( W + \text{jet} \) sample produces a value of \( m_t \) which is about 1\% above the generated value. Furthermore, the \( W \rightarrow jj \) events would suffer from background from QCD multi-jet events. Ongoing studies suggest that one could calibrate using \( W \rightarrow jj \) decays from the high \( p_T \) (top) events themselves, selecting those events in which the \( b \)-tagged jet is far away from the other two jets of the \( W \) decay and then reconstructing the \( W \rightarrow jj \) decay using a single cone of size \( \Delta R=0.8 \). Further study is required to reliably estimate the potential of this calibration procedure, and therefore a conservative systematic uncertainty of 1\% is assigned to it.

### 4.5 \( m_t \) in the di-lepton channel

Di-Lepton events can provide a measurement of the top quark mass complementary to that obtained from the single lepton plus jets mode. The signature of a di-lepton event consists of two isolated high \( p_T \) leptons, high \( E_T^{miss} \) due to the neutrinos, and two jets from the \( b \)-quarks. The measurement of \( m_t \) using di-lepton events is not a direct measurement as in the previous case but it relies on the relation between the kinematical distributions of the top decay products and \( m_t \), and on how they can be reproduced by the Monte Carlo simulation. About 400000 di-lepton \( t\bar{t} \) events are expected to be produced in a data sample corresponding to an integrated luminosity of 10 fb\(^{-1} \). Backgrounds arise from Drell-Yan processes associated with jets, \( Z \rightarrow \tau\tau \) associated with jets, \( WW+ \) jets and \( b\bar{b} \) production.

Of the many possible kinematic variables which could be studied, ATLAS performed a preliminary study using: the mass \( m_{\ell b} \) of the lepton+\( b \)-jet system, the energy of the two highest \( E_T \) jets, and the mass \( m_{\ell \ell} \) of the di-lepton system formed with both leptons originating from the same top decay (i.e. \( t \rightarrow \ell vb \) followed by \( b \rightarrow \ell wc \)). The event selection criteria required two opposite-sign leptons within \(|\eta|<2.5\), with \( p_T > 35 \) and 25 GeV respectively, and with \( E_T^{miss} > 40 \) GeV. Two jets with \( p_T > 25 \) GeV were required in addition. After the selection cuts, 80000 signal events survived, with \( S/B \) around 10.

#### 4.5.1 Top mass measurement using \( m_{\ell b} \)

In this analysis, the value of \( m_t \) was estimated using the expression:

\[
m_t^2 = M_W^2 + 2\langle m_{\ell b}^2 \rangle / [1 - \langle \cos \theta_{\ell b} \rangle]
\]  

(11)

Here, \( \langle m_{\ell b}^2 \rangle \) is the squared mean invariant mass of the lepton and \( b \)-jet from the same top decay. The mean value of \( \langle \cos \theta_{\ell b} \rangle \), the angle between the lepton and the \( b \)-jet in the \( W \) rest frame, can be regarded as an input parameter to be taken from Monte Carlo. To obtain a very clean sample, the two highest \( p_T \) jets were required to be tagged as \( b \)-jets, leaving a total of about 15200 signal events per 10 fb\(^{-1} \). One cannot determine, in general, which lepton should be paired with which \( b \)-jet. The pairing which gave the smaller value of \( \langle m_{\ell b}^2 \rangle \) was chosen, and checking the parton-level information showed that this criterion selected the correct pairing in 85\% of the cases, for a generated top mass of 175 GeV. The mean
The two distributions are normalised to the same area.

value $\langle m_{t\bar{t}}^2 \rangle$ was measured for samples generated with different input top masses $m_t$, and then $m_t$ was calculated from the expression above. For an integrated luminosity of 10 fb$^{-1}$, the expected statistical uncertainty on $m_t$ using this method is ±0.9 GeV. Major sources of systematics include uncertainty on the $b$-quark fragmentation function, which produces a systematic error on $m_t$ of 0.7 GeV if defined as described in Section 4.4. Systematic errors due to the effects of FSR and ISR together are about 1 GeV, while those due to varying the jet energy scale by 1% are 0.6 GeV. Further studies are required to estimate the uncertainties due to the reliance upon the Monte Carlo modelling of the $t\bar{t}$ kinematics.

4.52 Top mass measurement using the energy of the two leading jets

Increased sensitivity could be obtained with a technique which utilises not only the mean, but also the shape of the kinematic distribution. As an example, a study has been made of the sensitivity to $m_t$ obtained by comparing to “template” distributions the energy of the two highest-$E_T$ jets. The template distributions were made by generating PYTHIA samples of $t\bar{t}$ events with different values of $m_t$ in the range 160-190 GeV, in steps of 5 GeV. Figure 21 shows, as an example, the templates obtained for $m_t = 165$ GeV and 175 GeV. For each possible top mass value $m$, a $\chi^2(m)$ was obtained by comparing the kinematical distribution of the simulated data with the templates of mass $m$. The best value for the mass was the value which, for the “data” set, generated with $m_t = 175$ GeV, gave the minimum $\chi^2$. For an integrated luminosity of 10 fb$^{-1}$, the expected statistical sensitivity on $m_t$ corresponds to about ±0.4 GeV. Varying the calorimeter jet energy scale by 1% produced a systematic error on $m_t$ of 1.5 GeV. Other sources of systematic error result from the dependence of the method on the Monte Carlo modelling of the $t\bar{t}$ kinematics, and require further study. As an example, changing the choice of the structure functions used in the Monte Carlo simulation (for example, from CTEQ2L to CTEQ2M or EHQL1) led to differences in the top mass of ±0.7 GeV.

4.53 Top mass measurement using $m_{ll}$ in tri-lepton events

The invariant mass distribution of the two leptons from the same top quark decay (i.e. $t \to \ell\nu b$ followed by $b \to \ell(\nu c)$) is quite sensitive to $m_t$. It has been shown that the mass distribution of lepton pairs from the same top quark decay is much less sensitive to the top quark transverse momentum distribution than that of lepton pairs from different top quarks [99]. Signal events are expected to contain two leptons from the decay of the $W$ bosons produced directly in the top and anti-top quark decays, and one lepton from the $b$-quark decay. In addition to the cuts described above, one non-isolated muon with $p_T > 15$ GeV was required. For an integrated luminosity of 10 fb$^{-1}$, the expected signal would be about 7250 events,
yielding a statistical uncertainty on the measurement of $m_t$ of approximately $\pm 1$ GeV. This technique is insensitive to the jet energy scale. The dominant uncertainties arise from effects of ISR and FSR and from the $b$-quark fragmentation, which sum up to about 1.5 GeV.

4.6 $m_t$ from $t \rightarrow l + J/\psi + X$ decays

An interesting proposal [111] by CMS, explored in detail during the workshop [112], is to take advantage of the large top production rates and exploit the correlation between the top mass and the invariant mass distribution of the system composed of a $J/\psi$ (from the decay of a $b$ hadron) and of the lepton ($\ell = e, \mu$) from the associated $W$ decay (see Fig. 22).

The advantage of using a $J/\psi$ compared to the other studies involving leptons as presented above is twofold: first, the large mass of the $J/\psi$ induces a stronger correlation with the top mass (as will be shown later). Second, the identification of the $J/\psi$ provides a much cleaner signal. In order to uniquely determine the top decay topology one can tag the charge of the $b$ decaying to $J/\psi$ by requiring the other $b$-jet to contain a muon as well. The overall branching ratio is $5.3 \times 10^{-5}$, taking into account the charge conjugate reaction and $W \rightarrow e\nu$ decays. In spite of this strong suppression, we stress that these final states are experimentally very clean and can be exploited even at the highest LHC luminosities. Furthermore, one can also explore other ways to associate the $J/\psi$ with the corresponding isolated lepton – for example by measuring the jet charge of identified $b$’s. One should say that all these methods of top mass determination essentially rely on the Monte-Carlo description of its production and decay. Nonetheless the model, to a large extent, can be verified and tuned to the data.

4.6.1 Analysis

In the following we assume a $t\bar{t}$ production cross-section of 800 pb for $m_t = 175$ GeV. Events are simulated with the PYTHIA5.7 [52] or HERWIG 5.9 [51] event generators. Particle momenta are smeared according to parameterisations obtained from detailed simulation of the CMS detector performance. Four-lepton events are selected by requiring an isolated lepton with $p_T > 15$ GeV and $|\eta| < 2.4$, and three non-isolated, centrally produced muons of $p_T > 4$ GeV and $|\eta| < 2.4$, with the invariant mass of the two of them being consistent with the $J/\psi$ mass. These cuts significantly reduce the external (non-$t\bar{t}$) background, mainly $Wb\bar{b}$ production, which can be further reduced by employing, in addition, two central jets from another $W$. The resulting kinematical acceptance of the selection criteria is 30%; this rather small value is largely due to soft muons from $J/\psi$ and $b$. In one year high luminosity running of LHC, corresponding to an integrated luminosity of 100 fb$^{-1}$, and assuming trigger plus reconstruction efficiency of 0.8, we expect about $10^5 \times 800 \times 5.3 \times 10^{-5} \times 0.3 \times 0.8 = 1000$ events.

An example of the $\ell J/\psi$ mass distribution with the expected background is shown in Fig. 23. The background is internal (from the $t\bar{t}$ production) and is due to the wrong assignment of the $J/\psi$ to the corresponding isolated lepton. These tagging muons of wrong sign are predominantly originating from

PYTHIA results indicate that with the above cuts this source of the background can be kept at a per cent level.
$B^0/\overline{B}^0$ oscillations, $b\to c\to \mu$ transitions, $W(\to e, \tau)\to \mu$ decays, $\pi/K$ decays in flight and amount to $\sim 30\%$ of the signal combinations. The shape of the signal $\ell J/\psi$ events (those with the correct sign of the tagging muon) is consistent with a Gaussian distribution over the entire mass interval up to its kinematical limit of $\sim 175$ GeV. The background shape is approximated by a cubic polynomial. The parameters of this polynomial are determined with “data” made of the wrong combinations of $\ell J/\psi$ with an admixture of signal. In such a way the shape of the background is determined more precisely and in situ. Thus, when the signal distribution is fitted, only the background normalisation factor is left as a free parameter along with the three parameters of a Gaussian. The result of the fit is shown in Fig. 23. We point out that this procedure allows to absorb also the remaining external background (if any) into the background fit function.

![Figure 23: Example of the $\ell J/\psi$ invariant mass spectrum in four-lepton final states. The number of events corresponds to four years running at LHC high luminosity.](image)

![Figure 24: Correlation between $M_{\text{max}}^{\ell J/\psi}$ and the top quark mass in isolated lepton plus $J/\psi$ (solid line) and isolated lepton plus $\mu$-in-jet (dashed line) final states.](image)

As a measure of the top quark mass we use the mean value (position of the maximum of the distribution) of the Gaussian, $M_{\text{max}}^{\ell J/\psi}$. In four years running at LHC with high luminosity the typical errors on this variable, including the uncertainty on the background, are about 0.5 GeV. It is composed of $\lesssim 0.5$ GeV statistical error and $\lesssim 0.15$ GeV systematics contribution due to the uncertainty on the measurement of the background shape.\(^5\)

The measurement of the $M_{\text{max}}^{\ell J/\psi}$ can then be related to the generated top quark mass. An example of the correlation between the $M_{\text{max}}^{\ell J/\psi}$ and $m_t$ is shown in Fig. 24 along with the parameters of a linear fit. For comparison, we also show the corresponding dependence in a more traditional isolated lepton plus $\mu$-in-jet channel. Not surprisingly, the stronger correlation, and thus a better sensitivity to the top mass, is expected in the $\ell J/\psi$ final states as compared to the isolated lepton plus $\mu$-in-jet channel. This is because, in the former case, we pickup a heavy object (the $J/\psi$) which carries a larger fraction of the $b$-jet momentum. The $M_{\text{max}}^{\ell J/\psi}$ measurement error, statistical and systematic, scales as the inverse slope value of the fit, which is a factor of 2 in our case. Hence the statistical error on the top mass in this particular example is $\sim 1$ GeV.

It is appropriate to comment on the ways to obtain a larger event sample. Encouraging results have been obtained in [113] to reconstruct the $b\to J/\psi\to e^+e^-$ decays for low luminosity runs. The extension of these studies for a high luminosity environment is very desirable. Another possibility would be to relax the kinematical requirements. The choice of $p_T$ cut on soft muons is not dictated by the

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\(^5\)The statistical power of the sample can be further improved by exploiting full spectrum, rather than its Gaussian part.
background considerations but by the trigger rates, and is set here to 4 GeV rather arbitrarily. For example, the di-muon trigger with $\eta$-dependent thresholds which is available in CMS for low luminosity runs [114] allows to significantly increase the kinematical acceptance, practically to the limit determined by muon penetration up to the muon chambers. Therefore, the assessment of the trigger rates at high luminosity with lower $p_T$ thresholds and in multi-lepton events clearly deserves a dedicated study.

An even larger event sample can be obtained in three lepton final states, using instead the jet-charge technique to determine the $t\bar{t}$ decay topology instead of the tagging muon. The jet charge is defined as a $p_T$-weighted charge of particles collected in a cone around the $J/\psi$ direction. Obviously, this kind of analysis requires detailed simulations with full pattern recognition which are under way. However, particle level simulations performed with PYTHIA and with realistic assumptions on track reconstruction efficiency give event samples comparable to the muon-tag performance, with about 10 times less integrated luminosity. In any case, through the LHC lifetime, one can collect enough events so that the overall top mass measurement accuracy would not be hampered by the lack of statistics; it would rather be limited by the systematic uncertainties which are tightly linked with the Monte-Carlo tools in use, as will be argued in the following section.

4.62 Systematics

An essential aspect of the current analysis is to understand limitations which would arise from the Monte-Carlo description of the top production and decay. It is important to realize that the observable used in this study enjoys two properties: it is Lorentz invariant an it does not depend on the detailed structure of the jets, but only on the momentum spectrum of the $b$-hadron and of the $J/\psi$ from its decay.

As a result, were it not for distortions of the $J/\psi$ mass distribution induced by acceptance effects and by the presence of an underlying background, the measurement would be entirely insensitive with respect to changes in the top production dynamics, and in the structure of the underlying event. As a result, typical systematics such as those induced by higher-order corrections to the production process, or by the ISR and by the structure of the minimum bias event, are strongly reduced relative to other measurements of $m_t$. This expectation will be shown to be true in the following of this section.

The main limitations to an accurate extraction of the top mass using this technique are expected to come from: i) the knowledge of the fragmentation function of the $b$ hadrons contained in the $b$-jet and, ii) the size of the non-perturbative corrections to the relation between the top quark mass and the $J/\psi$ mass distribution. The $J/\psi$ spectrum in the decay of the $b$-hadrons will be measured with high accuracy in the next generation of $B$-factory experiments. It should be pointed out, however, that the composition of $b$-hadrons measured at the $\Upsilon(4S)$ and in the top decays will not be the same. In this second case, one expects a non-negligible contribution from baryons and from $B_s$ states. The size of the relevant corrections to the inclusive $J/\psi$ spectrum in top decays is not known, and, although expected to be small, it needs to be studied. Additional effects, such as QED corrections to the $W$ leptonic decay, $W$ polarisation and spin correlation effects can all be controlled and included in the theoretical simulations.

The rest of this section presents the results of a detailed study [112] of the systematics, mostly based on PYTHIA.

**Detector resolution:** Here we have considered only Gaussian smearing of particle momenta and the effect on the $M_{\ell\ell}^\text{max}$ measurement uncertainty is negligible. A possible nonlinearity of the detector response can be well controlled with the huge sample of $J/\psi$, $\Upsilon$ and $Z$ leptonic decays that will be available.

**Background:** The uncertainty would be mainly due to an inaccurate measurement of the background shape and the systematics contribution of $\lesssim 0.15$ GeV quoted in previous section would scale down with increasing statistics. For example, already with $\sim 10^4$ events the induced uncertainty is $\lesssim 0.1$ GeV.

**PDF:** Depending on the relative fraction of gluon/quarks versus $x$ in various PDF’s the top production kinematics might be different. No straightforward procedure is available for the moment to evaluate uncertainties due to a particular choice of PDF. We compared results obtained with the default set
As shown in Section 3.3, one does not expect significant uncertainties in the prediction of the \( p_T \) spectrum. However, to see an effect we have artificially altered the top \( p_T \) spectrum by applying a cut at the generator level. We found that even requiring all top quarks to have \( p_T > 100 \) GeV gives rise to only a 1\( \sigma \) change (\( \pm 0.7 \) GeV) in the fitted value of \( m_t \).

Initial state radiation: The \( M_{J/\psi}^{\text{max}} \) value is unchanged even switching off completely the ISR.

Top and \( W \) widths: Kinematical cuts that are usually applied affect the observed Breit-Wigner shape (tails) of decaying particles. Conversely, poor knowledge of the widths may alter the generated \( J/\psi \) mass spectrum depending on the cuts. In our case, only a small change in the \( M_{J/\psi}^{\text{max}} \) value is seen relative to the zero-width approximation.

\( W \) polarisation: A significant shift is found for the isotropic decays of \( W \) when compared to the SM expectation of its \( \sim 70\% \) longitudinal polarisation. In future runs of the Tevatron the \( W \) polarisation will be measured with a \( \sim 2\% \) accuracy [98], and at the LHC this would be further improved, so that it should not introduce additional uncertainties in simulations.

\( t\bar{t} \) spin correlations: A “cross-talk” between \( t \) and \( \bar{t} \) decay products is possible due to experimental cuts. To examine this effect in detail the 2\( \rightarrow \)6 matrix elements have been implemented in PYTHIA preserving the spin correlations [117]. No sizeable difference in the \( M_{J/\psi}^{\text{max}} \) value is seen compared to the default 2\( \rightarrow \)2 matrix elements.

QED bremsstrahlung: Only a small effect is observed when it is switched off. Furthermore, QED radiation is well understood and can be properly simulated.

Final State Radiation: A large shift of \( \sim 7 \) GeV is observed when the FSR is switched off. This is due to the absence of evolution for the \( b \) quark, whose fragmentation function will be unphysically hard. To evaluate the uncertainty we varied the parton virtuality scale \( m_{\text{min}} \), the invariant mass cut-off below which the showering is terminated. A \( \pm 50\% \) variation of it around the default (tuned to data) value of 1 GeV induces an uncertainty of \( \pm 0.1 \) GeV.

\( b \) fragmentation, except FSR: As a default, in PYTHIA we have used the Peterson form for the \( b \)-quark fragmentation function with \( \varepsilon_b = 0.005 \). Variation of this value by \( \pm 10\% \) [118] leads to an uncertainty of \( \pm 0.3 \) GeV. (The \( \pm 10\% \) uncertainty on \( \varepsilon_b \) is inferred from LEP/SLD precision of \( \sim 1\% \) on the average scaled energy of \( B \)-hadrons.) It should be pointed out that recent accurate measurements of the \( b \)-quark fragmentation function [119] are not well fitted by the Peterson form.

The last two items of this list deserve some additional comments. While the separation between the FSR and the non-perturbative fragmentation phases seems unnecessary, and liable to lead to an overestimate of the uncertainty, it is important to remark that our knowledge of the non-perturbative hadronisation comes entirely from the production of \( b \)-hadrons in \( Z^0 \) decays at LEP and SLC. It is important to ensure that the accuracy of both perturbative and non-perturbative effects is known, since the perturbative evolution of \( b \) quarks from \( Z^0 \) and top decays are not the same owing to the different scales involved. An agreement between data and Monte Carlo calculations for the \( b \)-hadron fragmentation function at the \( Z^0 \) does not guarantee a correct estimate of the \( b \)-hadron fragmentation function in top decays.

To be specific, we shall consider here the effects induced by the higher-order matrix element corrections to the radiative top decays \( t \rightarrow bWg \) [54]. These effects cannot be simulated by a change in the virtuality scale \( m_{\text{min}} \) as explored above in the study based on PYTHIA, as they have a different physical origin. The extended phase-space available for gluon emission after inclusion of the matrix-element corrections leads to a softening of the \( b \)-quark, and, as a result, of the \( \ell J/\psi \) spectrum. For simplicity, we study here the invariant mass of the system \( B\ell \). The resulting invariant mass distributions, for \( m_t = 175 \) GeV, with (HERWIG 6.1) and without (HERWIG 6.0) matrix element corrections are shown in Fig. 25. The averages of the two distributions, as a function of the top mass, are given on the right of the figure, and the difference of the averages are given in Table 7. Given the slopes of the correlation...
between $\langle m_{BL} \rangle$ and $m_t$, we see that the corrections due to inclusion of the exact matrix elements are between 1 GeV (for $m_{BL} > 50$ GeV) and 1.5 GeV (for the full sample).

More details of the analysis will be found in [64]. It is also found there that the dependence of $\langle m_{BL} \rangle$ on the hadronic center of mass energy, or on the partonic initial state producing the $t\bar{t}$ pair, is no larger than 100 MeV. We take this as an indication that the effects of non-factorisable non-perturbative corrections (such as those induced by the neutralisation of the colour of the top quark decay products) are much smaller than the 1 GeV accuracy goal on the mass.

A summary of these studies is given in Fig. 26. One sees an impressive stability of the results for reasonable choices of parameters. The expected systematic error in the $M_{jj}^{mag}$ determination is $\lesssim 0.3$ GeV which translates into a systematic error on the top mass of $\delta m_t \lesssim 0.6$ GeV.

In addition to the above studies, we also compared directly the results of HERWIG (v5.9) and PYTHIA. With HERWIG we have tried various tunings from LEP experiments as well as its default settings [51]. They all yield comparable results to each other and to PYTHIA results, and are within $\lesssim 0.5$ GeV. This corresponds to a systematic uncertainty $\delta m_t \lesssim 1$ GeV.

### 4.7 Conclusions for the top mass measurement at the LHC

The very large samples of top quark events which will be accumulated at the LHC lead to a precision measurement of the top quark mass. Different statistically independent channels have been investigated and from the studies so far a precision of better than 2 GeV in each case can be obtained. In particular for the lepton plus jets channel where the $m_t$ is measured directly reconstructing the invariant mass of the $m_{jjb}$ candidates, such a precision can be achieved within a year or running at low luminosity. For the channels involving two or more leptons, data from several years have to be combined to limit the statistical error in the measurement beyond the expected systematic errors.
With the statistical error not being a problem, the emphasis of the work was devoted to estimate the systematic error involved in each method. For each sample, the contributing systematic errors are different, a fact which will allow important cross-checks to be made. The results indicate that a total error below 2 GeV should be feasible. In the case of the lepton plus jet channel the major contribution to the uncertainty is identified in the jet energy scale (in particular for the $b$-jets) and in the knowledge of FSR. When a special sub-sample of high $p_T$ top events is used and the $m_t$ is reconstructed using a large calorimeter cluster the FSR sensitivity is reduced, but further work is required to validate it. For the channels using two or more leptons for the top decay, the major contribution in the systematic error comes from the Monte Carlo and from how well the kinematic observable used for the mass measurement is related to the mass of the top quark.

In $\ell J/\psi$ final states the top mass can be determined with a systematic uncertainty of $\lesssim 1$ GeV. These final states are experimentally very clean and can be exploited even at highest LHC luminosities. The precision would be limited by the theoretical uncertainties which is basically reduced to the one associated with the $t\rightarrow B$ meson transition. This method of top mass determination looks very promising, and a final definition of its ultimate reach will rely on a better understanding of theoretical issues, and on the possibility to minimise the model dependence using the LHC data themselves.

5. SINGLE TOP PRODUCTION

At the LHC, top quarks are mostly produced in pairs, via the strong process $gg\rightarrow t\bar{t}$ (and, to a lesser extent, $q\bar{q}\rightarrow t\bar{t}$). However, there are a significant number of top quarks that are produced singly, via the weak interaction. There are three separate single-top quark production processes of interest at the LHC, which may be characterised by the virtuality of the $W$ boson (of four-momentum $q$) in the process:

- $t$-channel: The dominant process involves a space-like $W$ boson ($q^2 \leq 0$), as shown in Fig. 27(a) [120]. The virtual $W$ boson strikes a $b$ quark in the proton sea, promoting it to a top quark. This

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process is also referred to as $W$-gluon fusion, because the $b$ quark ultimately arises from a gluon splitting to $b\bar{b}$.

- **$s$-channel**: If one rotates the $t$-channel diagram such that the virtual $W$ boson becomes time-like, as shown in Fig. 27(b), one has another process that produces a single top quark [121, 122]. The virtuality of the $W$ boson is $q^2 \geq (m_t + m_b)^2$.

- **Associated production**: A single top quark may also be produced via the weak interaction in association with a real $W$ boson ($q^2 = M_W^2$), as shown in Fig. 27(c) [123, 124]. One of the initial partons is a $b$ quark in the proton sea, as in the $t$-channel process.

The total cross sections for these three single-top quark production processes are listed in Table 8, along with the cross section for the strong production of top quark pairs. The $t$-channel process has the largest cross section; it is nearly one third as large as the cross section for top quark pairs. The $s$-channel process has the smallest cross section, more than an order of magnitude less than the $t$-channel process. The $Wt$ process has a cross section intermediate between these two. We will argue that all three processes are observable at the LHC. The $t$-channel and $s$-channel processes will first be observed at the Fermilab Tevatron [127]; the $Wt$ process will first be seen at the LHC.

There are several reasons for studying the production of single top quarks at the LHC:

- The cross sections for single-top quark processes are proportional to $|V_{tb}|^2$. These processes provide the only known way to directly measure $V_{tb}$ at hadron colliders.

- Single-top quark events are backgrounds to other signals. For example, single-top quark events are backgrounds to some signals for the Higgs boson [128].

- Single top quarks are produced with nearly 100% polarisation, due to the weak interaction [123, 129, 130, 131]. This polarisation serves as a test of the $V-A$ structure of the top quark charged-current weak interaction.

- New physics may be discernible in single-top quark events. New physics can influence single-top quark production by inducing non-SM weak interactions [129, 132, 133, 134, 135], via loop effects [136, 137, 138, 139, 140], or by providing new sources of single-top quark events [133, 137, 141, 142].

In the next three subsections we separately consider the three single-top quark production processes. The subsection after these discusses the polarisation of single top quarks. In the concluding section, we discuss the accuracy with which $V_{tb}$ can be measured in single-top quark events at the LHC.
5.1 t-channel single-top production

5.11 Theory

The largest source of single top quarks at the LHC is via the t-channel process, shown in Fig. 27(a) [120, 123, 125, 129, 143, 144, 145]. A space-like ($q^2 < 0$) $W$ boson strikes a $b$ quark in the proton sea, promoting it to a top quark. As shown in Table 8, the cross section for this process is about one third that of the strong production of top quark pairs. Thus there will be an enormous number of single top quarks produced via the $t$-channel process at the LHC.

It is perhaps surprising that the cross section for the weak production of a single top quark, of order $\alpha_{\text{em}}^2$, is comparable to that of the strong production of top quark pairs, of order $\alpha_s^2$. There are several enhancements to the $t$-channel production of a single top quark that are responsible for this:

- The differential cross section for the $t$-channel process is proportional to $d\sigma/dq^2 \sim 1/(q^2 - M_W^2)^2$, due to the $W$-boson propagator. The total cross section is therefore dominated by the region $|q^2| \leq M_W^2$, and is proportional to $1/M_W^2$. In contrast, the total cross section for the strong production of top quark pairs is proportional to $1/s$, where $s \geq 4m_t^2$ is the parton center-of-mass energy.
- Since only a single top quark is produced, the typical value of the parton momentum fraction $x$ is half that of top quark pair production. Since parton distribution functions scale roughly like $1/x$ at small values of $x$, and there are two parton distribution functions, this leads to an enhancement factor of roughly four.

The fact that the total cross section is dominated by the region $|q^2| \leq M_W^2$ also has the implication that the final-state light quark tends to be emitted at small angles, i.e., high rapidities. This characteristic feature of the signal proves to be useful when isolating it from backgrounds.

The $b$ distribution function in the proton sea arises from the splitting of virtual gluons into nearly-collinear $b\bar{b}$ pairs. Thus it is implicit that there is a $b$ in the final state, which accompanies the top quark and the light quark. The final-state $b$ tends to reside at small $p_T$, so it is usually unobservable.

The total cross section for the $t$-channel production of single top quarks has been calculated at NLO [125, 143]; the result is given in Table 8. A subset of the NLO corrections is shown in Fig. 28(a). This correction arises from an initial gluon which splits into a $b\bar{b}$ pair. If the $b\bar{b}$ pair is nearly collinear, then this process contributes to the generation of the $b$ distribution function, which is already present at leading order; hence, one does not include this kinematic region as a contribution to the NLO correction. This is indicated schematically in Fig. 28(b). Only the contribution where the $b\bar{b}$ pair is non-collinear is a proper NLO correction to the total cross section. The other corrections to this process, due to final-state and virtual gluons, as well as corrections associated with the light quark, are also included in the cross section given in Table 8.

The central value for the cross section is obtained by setting the factorisation scale of the $b$ distribution function equal to $\mu^2 = -q^2 + m_t^2$. The uncertainty in the NLO cross section due to the variation of the factorisation scale between one half and twice its central value is 4%. Due to the similarity with deep-inelastic scattering, the factorisation scale of the light quark is $\mu^2 = -q^2$, and is not varied [125].

Since the $b$ tends to reside at low $p_T$, the dominant final state is $Wb\bar{j}$, where the $Wb$ are the decay products of the top quark, and the jet is at high rapidity. However, the $b$ is at $p_T > 20$ GeV in roughly 40% of the events, in which case the final state is $Wb\bar{b}j$. From a theoretical perspective, the optimal strategy is to isolate both final states and thereby measure the total cross section, which has an uncertainty of only 4% from varying the factorisation scale, as mentioned above. However, the $Wb\bar{b}j$ final state has a large background from $t\bar{t}$, and it has not yet been established by ATLAS or CMS that this signal can be isolated, although the analysis of [145] gives cause for optimism. Thus we focus on the $Wb\bar{j}$ final state,

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Footnotes:

7The formalism for separating the nearly-collinear and non-collinear regions, and for generating the $b$ distribution function, was developed in Refs. [146, 58].

8The factorisation and renormalisation scales are set equal.
Fig. 28: (a) Initial-gluon correction to single-top quark production via the t-channel process (the diagram with the W and gluon lines crossed is not shown); (b) the kinematic region in which the gluon splits to a nearly-collinear b\bar{b} pair (the double line through the \( q \)) propagator indicates that it is nearly on shell) is subtracted from the correction, as it is already included at leading order.

demanding that the \( b \) have \( p_T < p_{T,\text{cut}} \). For \( p_{T,\text{cut}} = 20 \text{ GeV} \), the cross section for this semi-inclusive process is 164 pb, with an uncertainty of 10% from varying the factorisation scale [144], about twice the uncertainty of the total cross section. Work is in progress to calculate the differential cross section \( d\sigma/dp_T \bar{b} \) at NLO with the goal of reducing this uncertainty [147]. It would also be desirable to calculate the total cross section at next-to-next-to-leading order (NNLO).

Additional theoretical uncertainties stem from the top quark mass and the parton distribution functions. An uncertainty in the top quark mass of \( \pm 2 \text{ GeV} \) yields an uncertainty of only \( \pm 0.5 \) in the cross section, which is negligible. This is due to the fact that the cross section scales like \( 1/\mu_r^2 \) rather than \( 1/s \). The uncertainty in the cross section due to the parton distribution functions is estimated in [148] to be 10%. That analysis suggests that the uncertainty can be reduced below this value. Combining all uncertainties in quadrature, we conclude that the total theoretical uncertainty is presently 15% in the \( \bar{b}Wj \) cross section (11% in the total cross section). The discussion above suggests that this can be significantly reduced with further effort.

5.12 Phenomenology

Studies of the t-channel process have been carried out by both ATLAS and CMS. We will first describe the CMS study, and then that of ATLAS.

In order to reject the large \( t\bar{t} \) background in this channel, it is necessary to impose a cut on jet multiplicity. Accurate modelling of jet response and resolution is therefore desirable, and so CMS [149] used a full GEANT calorimeter simulation of the detector. The GEANT simulation also allows a more realistic modelling of the missing-\( p_T \) response of the detector, which is important in understanding the mass resolution which can be obtained on the reconstructed t quark. The detailed calorimeter simulation was combined with a parameterised b-tagging efficiency.

Signal events were generated using PYTHIA 5.72 [52], with \( m_t = 175 \text{ GeV} \) and the CTEQ2L parton distribution functions. Events were preselected at the generator level to have one and only one charged lepton (with \( p_T > 25 \text{ GeV} \) and \( |\eta| < 2.5 \)) and one or two jets (generator-level jets were found using the LUCELL clustering algorithm, which is part of PYTHIA). Generated events were then passed through the parameterised b-tagging and the GEANT detector simulation. The CMS b-tagging performance is taken from a study which used a detailed detector simulation combined with existing CDF data on impact-parameter resolutions. The tagging efficiency for \( p_T > 50 \text{ GeV} \) is typically 50% for b-jets, 10% for c-jets, and 1–2% for light quarks and gluons. These efficiencies fall quite rapidly for lower transverse momenta, and it was assumed no tagging could be performed for \( p_T < 20 \text{ GeV} \) or \( |\eta| > 2.4 \). The generated luminosity corresponded to about 100 pb\(^{-1} \) – only 30 hours of running at \( 10^{33} \text{ cm}^{-2}\text{s}^{-1} \).

The \( t\bar{t} \) and WZ backgrounds were also generated using PYTHIA 5.72. The same pre-selection were applied at the generator level. The W+ jets backgrounds were generated using the VECBOS

\(^9\)The CMS analysis presented below uses \( p_{T,\text{cut}} = 20 \text{ GeV} \); the ATLAS analysis uses \( p_{T,\text{cut}} = 15 \text{ GeV} \).
Fig. 29: Reconstructed top mass for signal plus backgrounds (open histogram) and backgrounds only (shaded). The backgrounds considered are $t\bar{t}$, $W+2$ jets and $W+3$ jets. The vertical scale is events per 6 GeV mass bin per pb$^{-1}$ of luminosity. The generator [150], combined with HERWIG 5.6 [51] to fragment the outgoing partons. $W+2$ jets and $W+3$ jets processes were generated separately. Again, events were preselected to have a charged lepton with $p_T > 25$ GeV and $|\eta| < 2.5$, and to have a (parton-level) $p_T > 15$ GeV for the final-state jets.

Events were then selected which passed the following requirements:

- One and only one isolated lepton ($\ell = e$ or $\mu$) with $p_T > 20$ GeV and $|\eta| < 2.5$. This allows the events to pass a reasonable lepton trigger.
- Missing $p_T > 20$ GeV, and transverse mass (of the lepton and missing $p_T$) $50 < m_T < 100$ GeV. These two requirements select $W\rightarrow\ell\nu$ candidates.
- Exactly two jets with $p_T > 20$ GeV and $|\eta| < 4$. Requiring at least two jets reduces the $W+J$ jets background, while requiring no more than two jets rejects the $t\bar{t}$ background which naively would produce four jets in the final state.
- One jet with $p_T > 20$ GeV and $|\eta| < 2.5$, the other jet with $p_T > 50$ GeV and $2.5 < |\eta| < 4.0$. The requirement that the second jet be at forward rapidities tends to select the desired $t$-channel process.
- Leading jet $p_T < 100$ GeV. This helps to reduce the $t\bar{t}$ background.
- Exactly one $b$-tagged jet (given the $b$-tagging acceptance, this is always the central jet). This requirement again reduces $t\bar{t}$, and of course rejects $W+J$ jets processes with light-quark or gluon jets.
- Invariant mass of the two jets in the $80 - 100$ GeV range. This rejects $WZ$ events with $Z\rightarrow b\bar{b}$.

The single-top signal is then searched for in the invariant mass of the $W$ and the $b$-tagged jet (which should peak at the top quark mass). The mass was reconstructed assuming the solution for the $W$ kinematics which yields the lower $p_T^Z$. (It is possible to use other choices, for example the solution which gives the $Wb$ mass closest to $m_t$. This would result in an apparently better top mass resolution but would also severely bias the background shape; the statistical significance of the signal would not be improved.)

Figure 29 shows the reconstructed mass distribution for signal and background combined. The signal is apparent as an excess over the background (the shaded histogram) around 160 GeV. (Since jet

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10The version of VECBOS used here, and its interface to HERWIG, were developed for use in CDF [151], and were adapted for CMS by R. Vidal.
energy scale corrections have not been applied to the simulated events, the top mass reconstructs to less than its true value.) The signal-to-background ratio in a window of $160 \pm 20$ GeV is 3.5 with a clear peak visible in the $Wb$ invariant-mass distribution. The number of signal events is 66 in 100 pb$^{-1}$, giving a signal efficiency of 1.2% (after the $W \rightarrow \ell \bar{\nu}$ branching ratio). We then find that 10 fb$^{-1}$ would yield 6600 signal events ($S$) and 1900 background ($B$), sufficient for a statistical accuracy on the number of signal events of $\sqrt{S + B}/S = 1.4%$.

The largest background comes from $Wc\bar{s}$ with the charm jet mistagged as a $b$-jet. It would be worthwhile to develop a $b$-tagging algorithm having greater rejection against such mistags, even at the cost of some signal efficiency. The $Wb\bar{b}$ background was found to be a small contribution to the $W + 2$ jets background at the parton level for the selection cuts employed here, and was therefore not explicitly included in the analysis.

The use of the forward jet tag substantially improves the signal-to-background ratio, and allows a clear reconstructed top-mass peak to be seen. However, it does not significantly improve $\sqrt{S + B}/S$ [144]. One could therefore imagine omitting the forward jet requirement if the systematic uncertainty could thereby be reduced.

Compared with earlier studies (for example [144]), this analysis uses more realistic jet and missing-$p_T$ resolutions, and includes initial- and final-state gluon radiation. As a result, the top-mass resolution is worsened; but the resolution found here compares well with the result of a full simulation of single-top production in CDF.

A study of the cross-section measurement for the $t$-channel process was also carried out by ATLAS [152]. Signal events were generated using the ONETOP parton-level Monte Carlo [153] with fragmentation, radiation, and underlying event simulated by PYTHIA 5.72. Backgrounds containing top quarks ($t\bar{t}$ and other single-top production) were also generated using ONETOP, while $W + 2$ jets and $Wb\bar{b}$ backgrounds were generated by HERWIG 5.6.11 These events were processed by the ATLAS parameterised detector simulation assuming a 60% $b$-tagging efficiency for $b$-jets, 10% for $c$-jets, and 1% for light quarks and gluons. The events were then analysed with a view towards separating $t$-channel single top from background and measuring its cross section.

Event selection criteria were divided into two types: pre-selection and selection cuts. The pre-selection criteria were as follows:

- at least one isolated lepton with $p_T > 20$ GeV;
- at least one $b$-tagged jet with $p_T > 50$ GeV;
- at least one other jet with $p_T > 30$ GeV.

These were followed by the selection cuts:

- two and only two jets in the event (a jet has $p_T > 15$ GeV);
- one jet is a central $b$-tagged jet;
- the other jet is a forward ($|\eta| > 2.5$) untagged jet with $p_T > 50$ GeV.

The application of these cuts, and also the requirement of a reconstructed top mass between 150 and 200 GeV, yields the number of events shown in Table 9. The final signal efficiency is 3% and the signal-to-background ratio is 2.4. This implies a statistical precision on the cross-section measurement of $\sqrt{S + B}/S = 0.9%$ with 10 fb$^{-1}$ of data. Introducing other event selection variables (see [30, 154, 155]) it is possible to improve the signal-to-background ratio to nearly 5, but this does not improve the cross-section measurement due to the small remaining signal efficiency.

Both the CMS and ATLAS studies indicate that it will be possible to observe $t$-channel single-top production with a good signal-to-background ratio and a statistical uncertainty in the cross section of less than 2% with 10 fb$^{-1}$. Thus the uncertainty in the extracted value of $V_{tb}$ will almost certainly be dominated by systematic uncertainties, as discussed in the conclusions.

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11The $Wb\bar{b}$ background was generated using the matrix element from [89] interfaced to HERWIG 5.6.
Table 9: Cumulative effect of cuts on $t$-channel signal and backgrounds. The first four rows of this table refer to cumulative efficiencies of various cuts. The last two rows refer to the number of events for $10 \text{ fb}^{-1}$. Only events in which $W \rightarrow \ell \nu$ or $\mu\nu$ are considered in this table. Uncertainties quoted in this table are due entirely to Monte Carlo statistics.

<table>
<thead>
<tr>
<th>cut</th>
<th>$t$-channel eff(%)</th>
<th>$t\bar{t}$ eff(%)</th>
<th>$Wb\bar{b}$ eff(%)</th>
<th>$W+$ jets eff(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-selection</td>
<td>18.5</td>
<td>44.4</td>
<td>2.53</td>
<td>0.66</td>
</tr>
<tr>
<td>njet=2</td>
<td>12.1</td>
<td>0.996</td>
<td>1.55</td>
<td>0.291</td>
</tr>
<tr>
<td>fwd jet $</td>
<td>\eta</td>
<td>&gt; 2.5$</td>
<td>4.15</td>
<td>0.035</td>
</tr>
<tr>
<td>$p_T &gt; 50 \text{ GeV}$</td>
<td>3.00</td>
<td>0.017</td>
<td>0.023</td>
<td>0.016</td>
</tr>
<tr>
<td>events/10 fb$^{-1}$</td>
<td>$5.43 \times 10^5$</td>
<td>$2.40 \times 10^6$</td>
<td>$6.67 \times 10^5$</td>
<td>$4.00 \times 10^7$</td>
</tr>
<tr>
<td>(before cuts)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>events/10 fb$^{-1}$</td>
<td>$16515 \pm 49$</td>
<td>$455 \pm 74$</td>
<td>$155 \pm 17$</td>
<td>$6339 \pm 265$</td>
</tr>
<tr>
<td>(after cuts)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5.2 $s$-channel single-top production

5.21 Theory

The $s$-channel production of single top quarks is shown in Fig. 27(b) [121, 122, 123, 126, 144, 145]. The cross section is much less than that of the $t$-channel process because it scales like $1/s$ rather than $1/M_W^2$. However, the $s$-channel process has the advantage that the quark and antiquark distribution functions are relatively well known, so the uncertainty from the parton distribution functions is small. Furthermore, the parton luminosity can be constrained by measuring the Drell-Yan process $q\bar{q} \rightarrow W^{\pm} \rightarrow \ell \nu$, which has the identical initial state [122, 156].\(^{12}\)

The total cross section for the $s$-channel process has been calculated at NLO [126]; the result is given in Table 8. The factorisation and renormalisation scales are set equal to $\mu^2 = q^2$; varying each, independently, between one-half and twice its central value yields uncertainties in the cross section of $2\%$ from each source. The uncertainty in the cross section from the parton distribution functions is estimated to be $4\%$. The largest single source of uncertainty is the top quark mass; an uncertainty of $2 \text{ GeV}$ yields an uncertainty in the cross section of $5\%$. The relatively large sensitivity of the cross section to the top quark mass is a manifestation of the $1/s$ scaling. Combining all theoretical uncertainties in quadrature yields a total uncertainty in the cross section of $7\%$. This is much less than the present theoretical uncertainty in the $t$-channel cross section.

The Yukawa correction to this process, of order $\alpha_W m_t^2/M_W^2$, is less than one percent [126]. However, this correction could be significant in a two-Higgs-doublet model for low values of $\tan \beta$, in which the Yukawa coupling is enhanced [138].

5.22 Phenomenology

In order to evaluate the potential to separate the $s$-channel signal from its backgrounds, Monte Carlo events have been processed by a fast (parameterised) simulation of an LHC detector. At parton level the signal and the $t\bar{t}$ background were generated by the ONETOP Monte Carlo [153]. Radiation, showering, and the underlying event were added by PYTHIA 5.72 [52]. The $W+$ jets and $Wb\bar{b}$ backgrounds were generated using HERWIG 5.6 [51].\(^{13}\) Table 8 presents the cross sections assumed for the processes

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\(^{12}\)The parton luminosity can only be constrained, not directly measured, with this process. Since the neutrino longitudinal momentum is unknown, the $q^2$ of the virtual $W$ cannot be reconstructed.

\(^{13}\)The $Wb\bar{b}$ background was generated using the matrix element from [89] interfaced to HERWIG 5.6.
Table 10: Cumulative effect of cuts on s-channel signal and backgrounds. The first five rows of this table refer to cumulative efficiencies of various cuts. The last two rows refer to the number of events for 30 fb$^{-1}$. Only events in which $W \rightarrow e \nu$ or $\mu \nu$ are considered in this table. Uncertainties quoted in this table are due entirely to Monte Carlo statistics.

<table>
<thead>
<tr>
<th>cut</th>
<th>s-channel eff(%)</th>
<th>t-channel eff(%)</th>
<th>W$t$ eff(%)</th>
<th>$t\bar{t}$ eff(%)</th>
<th>$Wb\bar{b}$ eff(%)</th>
<th>W+jets eff(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-selection</td>
<td>27.0</td>
<td>18.5</td>
<td>25.5</td>
<td>44.4</td>
<td>2.53</td>
<td>0.667</td>
</tr>
<tr>
<td>njets=2</td>
<td>18.4</td>
<td>12.1</td>
<td>4.03</td>
<td>0.996</td>
<td>1.55</td>
<td>0.291</td>
</tr>
<tr>
<td>nbjet=2 $p_T &gt; 75$ GeV</td>
<td>2.10</td>
<td>0.035</td>
<td>0.018</td>
<td>0.023</td>
<td>0.034</td>
<td>0.0005</td>
</tr>
<tr>
<td>$\sum_{jets} p_T &gt; 175$ GeV</td>
<td>1.92</td>
<td>0.031</td>
<td>0.016</td>
<td>0.021</td>
<td>0.028</td>
<td>0.0005</td>
</tr>
<tr>
<td>$M_{t\bar{t}b}$ 150-200 GeV</td>
<td>1.36</td>
<td>0.023</td>
<td>0.006</td>
<td>0.012</td>
<td>0.0097</td>
<td>0.00014</td>
</tr>
<tr>
<td>events/30 fb$^{-1}$ (before cuts)</td>
<td>$6.66 \times 10^4$</td>
<td>$1.63 \times 10^6$</td>
<td>$4.5 \times 10^6$</td>
<td>$6.9 \times 10^6$</td>
<td>$2.0 \times 10^6$</td>
<td>$1.2 \times 10^8$</td>
</tr>
<tr>
<td>events/30 fb$^{-1}$ (after cuts)</td>
<td>$908 \pm 35$</td>
<td>$375 \pm 13$</td>
<td>$27 \pm 15$</td>
<td>$853 \pm 175$</td>
<td>$194 \pm 34$</td>
<td>$169 \pm 76$</td>
</tr>
</tbody>
</table>

containing top quarks. The cross section for the $W+$ jets background is normalised to that predicted by the VECBOS Monte Carlo [150] and is taken to be 18000 pb. The $Wb\bar{b}$ cross section is taken from [144] to be 300 pb.

From a phenomenological standpoint the most important distinction between the $s$-channel and $t$-channel sources of single top is the presence of a second high-$p_T$ b-jet in the $s$-channel process. As mentioned previously, in $t$-channel events the second b-jet tends to be at low $p_T$ and is often not seen. Therefore, requiring two b-jets above 75 GeV $p_T$ will eliminate most of the $t$-channel background. Requiring two high-$p_T$ b-jets in the event also suppresses the $W+$ jets background relative to the signal.

In addition to suppressing the $t$-channel background it is also necessary, as in other single-top signals, to design cuts to reduce the $W+$ jets and $t\bar{t}$ backgrounds. In order to reduce contamination by $W+$ jets events, the reconstructed top mass in each event must fall within a window about the known top mass (150-200 GeV), and the events must have a total transverse jet momentum$^{15}$ above 175 GeV. Only events containing exactly two jets (both tagged as $b$’s) are kept in order to reduce the $t\bar{t}$ background.

Table 10 presents the cumulative effect of all cuts on the $s$-channel signal and on the backgrounds. Events from $t$-channel single-top production are included in this table as a background to the $s$-channel process. From this table the predicted signal-to-background ratio for the $s$-channel signal is calculated to be 0.56. The results also imply a signal statistical significance ($S/\sqrt{B}$) of 23 with an integrated luminosity of 30 fb$^{-1}$. The statistical precision on the cross section, calculated from $\sqrt{S+B}/S$, is 5.5% with 30 fb$^{-1}$.

This study indicates that, despite the large anticipated background rate, it should be possible to perform a good statistical measurement of the $s$-channel single-top cross section. The accuracy with which $V_{tb}$ can be measured is discussed in the conclusions.

5.3 Associated production

5.3.1 Theory

Single top quarks may also be produced in association with a $W$ boson, as shown in Fig. 27(c) [123, 124, 145]. Like the $t$-channel process, one of the initial partons is a $b$ quark. However, unlike the $t$-channel

$^{14}$This cross section is defined for events containing at least two jets, each with $p_T > 15$ GeV and $|\eta| < 5$.

$^{15}$Scalar sum of the transverse momentum of all jets in the event.
process, this process scales like $1/s$. This, combined with the higher values of $\alpha$ needed to produce both a top quark and a $W$ boson, leads to a cross section for associated production which is about a factor of five less than that of the $t$-channel process, despite the fact that it is of order $\alpha_s \alpha_{\text{W}}$ rather than $\alpha^2_{\text{W}}$.

The total cross section for associated production has been calculated at leading order, with a subset of the NLO corrections included [124, 145]; the result is given in Table 8. This subset is analogous to the initial-gluon correction to the $t$-channel process, discussed previously. The other corrections have not yet been evaluated.\footnote{The analogous calculation for $Wc$ production has been performed in [157].} The initial-gluon correction contains an interesting feature which has no analogue in the $t$-channel process. One of the contributing diagrams to the initial-gluon correction ($gg \rightarrow Wt\bar{b}$) corresponds to $gg \rightarrow t\bar{t}$, followed by $t\rightarrow Wb$. This should not be considered as a correction to associated production, but rather as a background (it is in fact the dominant background, as discussed below). Thus, when evaluating the initial-gluon correction, it is necessary to subtract the contribution in which the $t$ is on shell. This is done properly in [124].

The cross section is evaluated with the common factorisation and renormalisation scales set equal to $\mu^2 = s$. The uncertainty in the cross section due to varying these scales between one half and twice their central value is 15%. This uncertainty would presumably be reduced with a full NLO calculation. The uncertainty in the cross section from the parton distribution functions is estimated to be 10% [148],\footnote{This is the uncertainty in the gluon-gluon luminosity at $\sqrt{s} = \left( m_t + M_W \right) / \sqrt{s} \approx 0.02$, where $\sqrt{s} = 14$ TeV.} although this could be improved with further study. The uncertainty in the cross section due to an uncertainty in the top quark mass of $2$ GeV is 4%, relatively large due to the $1/s$ scaling of the cross section. Combining all theoretical uncertainties in quadrature yields a total uncertainty at present of 18%, the largest of the three single-top processes.

5.32 Phenomenology

The strategy for measuring the cross section for associated production ($Wt$ mode) is similar to that for the $t$-channel process, as they share the same backgrounds. However, the nature of associated production makes it relatively easy to separate from $W+J$ and difficult to separate from $t\bar{t}$ events. This difficulty in removing the $t\bar{t}$ background does not preclude obtaining a precise cross-section measurement in this channel, assuming the rate for $t\bar{t}$ can be well measured at the LHC.

Two studies designed to separate signal from background have been performed using two different final states. The first is a study by ATLAS [30] which attempts to isolate $Wt$ signal events in which one $W$ decays to jets and the other decays to leptons. The second study, which is presented in [124], attempts to isolate signal events in which both $W$’s decay leptonically.

The first study presented here was done by ATLAS using the same event sample described in Section 5.1. Since the presence of a single isolated high-$p_T$ lepton is one of the preconditions of this study, the second $W$ must decay to two jets to be accepted by the event pre-selection. Therefore requiring a two-jet invariant mass within a window around the $W$ mass will serve to eliminate most events that do not contain a second $W$. The two-jet invariant-mass distribution is shown in Fig. 30 and clearly demonstrates the presence of a sharp peak in the associated-production signal and the $t\bar{t}$ background. This effectively leaves $t\bar{t}$ as the only background to $Wt$ events.

In addition to these special distinguishing features of the $Wt$ signal, there are several simple kinematic requirements which can be employed to reduce the $t\bar{t}$ background. By choosing events with exactly three jets and with exactly one of them tagged as a $b$-jet, some rejection of the $t\bar{t}$ background is possible. Some further rejection is obtained by limiting the selection to events with invariant mass less than 300 GeV, where the invariant mass of an event is defined as the invariant mass obtained by adding the four-vectors of all reconstructed jets and charged leptons ($e$ and $\mu$). However, even with these cuts the $t\bar{t}$ background is significantly larger than the $Wt$ signal.

Table 11 presents the cumulative effect of all cuts on the $Wt$ signal and on the $t\bar{t}$ and $W+$ jets
Fig. 30: The normalised two-jet invariant-mass distribution. For each event the two-jet combination with mass closest to the $W$ mass is plotted. This clearly shows a peak in the distribution for $Wt$ and $t\bar{t}$ which is not present for the other backgrounds. The $Wb\bar{b}$ and $t$-channel single-top backgrounds are virtually eliminated by the cuts and so are not included in the table. From this table the predicted signal-to-background ratio for the $Wt$ signal is calculated to be 0.24. After three years of running at low luminosity ($30 \text{ fb}^{-1}$), this implies a signal statistical significance ($S/\sqrt{B}$) of 25 and a statistical error on the $Wt$ cross section $(\sqrt{S + B}/S)$ of 4.4%.

The second study [124] was done at parton level and involved the separation of signal from background in the mode in which both $W$'s decay to leptons. This signal contains two high $p_T$ leptons and only one jet (the $b$-jet produced from the top decay). In this decay channel it was found that, after applying detector acceptance cuts, requiring precisely one $b$-tagged jet with $p_T > 15 \text{ GeV}$ is enough to yield a signal-to-background ratio of nearly unity. Also, the signal efficiency is significantly higher than in the ATLAS analysis, allowing more total signal events to pass the cuts despite the lower branching ratio for this decay mode. The statistical precision on the cross section measured in this analysis is 1.3% with an integrated luminosity of $30 \text{ fb}^{-1}$. The accuracy with which $V_{tb}$ can be extracted is discussed in the conclusions.

5.4 Polarisation in single-top production

5.4.1 Theory

Because single top quarks are produced through the weak interaction, they are highly polarised [123, 129, 130, 131, 144]. In the ultra-relativistic limit, the top quarks are produced in helicity eigenstates with helicity $-1/2$ (the top antiquarks have helicity $+1/2$), because the $V - A$ structure of the weak interaction selects quarks of a definite chirality. However, if the top quarks are not ultra-relativistic, chirality is not the same as helicity. Nevertheless, it was shown in [130] that there is a basis in which the top quark is 100% polarised, regardless of its energy. The top quark spin points along the direction of the $d$-type or $\bar{d}$-type quark in the event, in the top quark rest frame (the $\bar{t}$ spin points opposite this direction). In $t$-channel production, this is the direction of the final-state light quark ($u\bar{d} \rightarrow dl$) or the beam direction ($d\bar{b} \rightarrow \bar{u}t$). In $s$-channel production, this is the beam direction ($u\bar{d} \rightarrow \bar{t}b$). In associated production ($gb \rightarrow Wt$), this is the direction of the $d$ quark (or charged lepton) from the $W$ decay.

We focus our attention on the $t$-channel single-top process for the remainder of this section. The top quark polarisation in the $t$-channel process has been calculated at NLO [131]; the results below are taken from this study. In the case of $t$ production, 80% of the events have the $d$-type quark in the final state. This suggests using the direction of the light-quark jet, as observed in the top quark rest frame, to
Table 11: Cumulative effect of cuts on $Wt$ signal and backgrounds. Pre-selection cuts are defined in the same way as for the ATLAS $t$-channel analysis described earlier in this report. The first five rows of this table refer to cumulative efficiencies of various cuts. The last two rows refer to the number of events for 30 fb$^{-1}$. Only events in which $W \rightarrow e\nu$ or $\mu\nu$ are considered in this table. Uncertainties quoted in this table are due entirely to Monte Carlo statistics.

<table>
<thead>
<tr>
<th>cut</th>
<th>$Wt$ (\text{eff(%)})</th>
<th>$t\bar{t}$ (\text{eff(%)})</th>
<th>$W + \text{jets}$ (\text{eff(%)})</th>
</tr>
</thead>
<tbody>
<tr>
<td>pre-selection</td>
<td>25.5</td>
<td>44.4</td>
<td>0.66</td>
</tr>
<tr>
<td>(n\text{jets}=3), (p_T &gt; 50\text{ GeV})</td>
<td>3.41</td>
<td>4.4</td>
<td>0.030</td>
</tr>
<tr>
<td>(n\bar{\text{b}}\text{jet}=1), (p_T &gt; 50\text{ GeV})</td>
<td>3.32</td>
<td>3.24</td>
<td>0.028</td>
</tr>
<tr>
<td>Invariant Mass (&lt; 300\text{ GeV})</td>
<td>0.55</td>
<td>0.36</td>
<td>0.00051</td>
</tr>
<tr>
<td>$65 &lt; M_{jj} &lt; 95$</td>
<td>0.49</td>
<td>0.14</td>
<td>0.000085</td>
</tr>
</tbody>
</table>

| events/30 fb$^{-1}$ (before cuts) | $5.3 \times 10^5$ | $7.2 \times 10^6$ | $1.2 \times 10^6$ |
| events/30 fb$^{-1}$ (after cuts)   | $2608 \pm 166$  | $10616 \pm 625$  | $102 \pm 59$   |

measure the spin. This has been dubbed the “spectator basis” [130]. The polarisation of the top quark in this basis (defined as $P = (N_{\uparrow} - N_{\downarrow})/(N_{\uparrow} + N_{\downarrow})$) is 0.89. However, the polarisation is increased to nearly 100% when the cuts used in the $t$-channel analysis are imposed. This is because the polarisation is diluted by events in which the $b$ is produced at high $p_T$; but such events are eliminated by the requirement of only two jets.

In the case of $t\bar{t}$ production, 69% of the events have the $d$-type quark in the initial state. This suggests using the beam direction to measure the $t\bar{t}$ spin. However, it turns out that the spectator basis again yields the largest polarisation, $P = -0.87$. This polarisation is increased to $P = -0.96$ when cuts are applied.\footnote{With cuts applied, the polarisation in the so-called “$\eta$-beamline basis” is slightly higher, $P = -0.97$.}

Since the top quark decays via the weak interaction, its spin is analysed by the angular distribution of its decay products. The most sensitive spin analyser in top decay is the charged lepton, which has a (leading order) angular distribution with respect to the top quark spin of

$$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos \theta_e} = \frac{1}{2} (1 + \cos \theta_e)$$ (12)

in the top quark rest frame [158]. Hence the charged lepton tends to point along the direction of the spectator jet.

5.42 Phenomenology

The goal of this analysis is to estimate the sensitivity of ATLAS and CMS to the measurement of the polarisation of the top quarks produced by the $t$-channel single-top process. The $t$-channel process was chosen due to the large statistics available in this channel and the relative ease with which it is separated from its backgrounds. The $t$-channel events produced by the ONETOP generator and passed through PYTHIA and a parameterised detector simulation are analysed to attempt to recover the predicted SM top polarisation in the presence of background and detector effects. Details of the study are presented in [152, 154].
The experimental measurement of the polarisation of the top quark is essentially a measurement of the angular distribution of its decay products in the top quark rest frame. As explained above, the most sensitive angle is between the charged lepton from top decay and the direction of the spectator jet, in the top quark rest frame. In the absence of background or detector effects the angular distribution of the charged lepton is given by

\[ f(\cos \theta_L) = \frac{1}{2}(1 + P \cos \theta_L) \] (13)

where \( P \) is the polarisation of the sample and can range from \(-1\) to \(1\).

Experimentally, in order to measure the angular distribution of the charged lepton in the top quark rest frame, it is necessary to first reconstruct the four-momentum of the top quark. However, the reconstruction of the top four-momentum suffers from an ambiguity due to the unknown longitudinal momentum of the neutrino produced in the top decay. Using the \( W \) and top masses as constraints, one can reconstruct the top four-momentum, but the quality of the reconstruction is degraded by this ambiguity. Once the top four-momentum has been reconstructed, one can determine the direction of the spectator jet and the charged lepton in the top quark rest frame. The angle between these two directions is \( \theta_L \).

In order to extract the value of the top polarisation from the angular distribution, reference event samples were created with 100% alignment with the polarisation axis (spin up, \( P = +1 \)) and with 100% anti-alignment with the polarisation axis (spin down, \( P = -1 \)). These reference distributions were compared to a statistically-independent data set with the predicted SM top quark polarisation. This comparison was done by minimising

\[ \chi^2 = \sum_{(\cos \theta)_i} \frac{(f_{\text{th}}(\cos \theta)_i - f_{\text{d}}(\cos \theta)_i)^2}{\sigma_{\text{th},i}^2 + \sigma_{\text{d},i}^2} \] (14)

where the subscript \( \text{d} \) represents quantities calculated for the data distribution and the subscript \( \text{th} \) refers to the generated reference distribution. The theoretical value \( f_{\text{th}}(\cos \theta_L) \) is calculated via

\[ f_{\text{th}}(\cos \theta_L) = \frac{1}{2}((1 - P)f_D(\cos \theta_L) + (1 + P)f_U(\cos \theta_L)) \] (15)

where \( f_D \) and \( f_U \) refer to the value of the generated theoretical distribution for the 100% spin-down and the 100% spin-up tops, respectively, and \( P \) is the polarisation of the top sample. The procedure returns an estimate of the top polarisation and an error on that estimate. In this way the sensitivity to changes in top polarisation can be quantified.

Moving from the parton-level simulation to a simulation which includes both hadronisation and detector effects is certain to complicate the measurement of the polarisation of the top quark. In addition, the signal could be biased by an event selection designed to eliminate background and will be contaminated by residual background events.

The first histogram in Fig. 31 shows the angular distribution for signal only, at parton-level. The second histogram in Fig. 31 shows the angular distribution of the charged lepton after detector effects have been simulated. In addition to effects associated with detector energy smearing, jet and cluster definitions, etc., this distribution includes the effects of ambiguities in reconstructing the top quark due to the absence of information about the neutrino longitudinal momentum. It does not, however, contain the effects of any event selection in order to separate signal from background. This histogram demonstrates that the effect of hadronisation and detector resolution changes the shape of the angular distribution but still produces a highly asymmetric distribution.

In addition to the effects introduced by the detector resolution, the effect of applying the event-selection criteria can be evaluated by applying them one at a time and observing the change in shape of this distribution. For the purposes of the polarisation analysis the event-selection criteria are:

19The \( W \) mass can be used to calculate the neutrino longitudinal momentum to within a two-fold ambiguity. Of these two solutions the one which produces the best top mass is chosen.
Fig. 31: Angular distribution of charged lepton in top rest frame for various data samples. The histograms progress from left-to-right, top-to-bottom. The first histogram shows the parton-level distribution. The second histogram is after the simulation of detector and reconstruction effects. The final four histograms illustrate the influence of event selection criteria on the angular distribution. The effects of the cuts are cumulative and are the result of adding pre-selection cuts, a jet-multiplicity requirement, a forward jet tag, and a top mass window, respectively.

- Pre-selection (trigger) cuts as in ATLAS $t$-channel analysis described previously;
- number of jets = 2;
- forward jet ($|\eta| > 2.5$) with $p_T > 50$ GeV;
- reconstructed top mass in the range 150–200 GeV.

This set of criteria leads to a signal efficiency of 3.0%, corresponding to more than 16000 events in 10 $fb^{-1}$ of integrated luminosity. Fig. 31 demonstrates the effect of applying these cuts in a cumulative manner. Again the asymmetry of the $t$-channel angular distribution is preserved, though more degradation is clearly evident, in particular near $\cos \theta_\ell = 1$. The degradation is worse at these values of $\cos \theta_\ell$ because the leptons from these events are emitted in the direction opposite to the top boost. This reduces the momentum of the leptons causing more of them to fail $p_T$-based selection criteria.

Since $W+$ jet events dominate the background remaining after cuts, they are taken as the only background in this analysis. Fig. 32 shows the cumulative effect of cuts on the angular distribution of the charged lepton from $W+$ jets events. A peculiar feature of these events is evident in all of these distributions. This is the tendency for events to be grouped near $\cos \theta_\ell = 1$. The events which populate this region tend to be the highest $p_T$ events. This shows that even basic jet and isolated-lepton definitions and pre-selection cuts bias the angular distribution of $W+$ jets events.

When the event-selection criteria described in the previous sections are applied, the signal-to-background ratio (treating $W+$ jets as the only background) is found to be 2.6. Using the methods described earlier it is possible to estimate the polarisation of a mixed sample of $t$-channel signal and $W+$ jets background. The reference distributions for 100% spin-down and 100% spin-up top quarks mixed with background in a ratio of 2.6 are shown in Fig. 33. Also shown is the angular distribution corresponding to a statistically-independent data sample with SM polarisation mixed with background.
in the ratio 2.6. The $\chi^2$ function presented in (14) is minimised to obtain an estimate of the polarisation of the top. To estimate the precision for one year of data-taking, the fit was done with 3456 signal events and 1345 background events, corresponding to 2 fb$^{-1}$ of integrated luminosity ($\sim 1/5$ of a year). For this integrated luminosity the error on the polarisation measurement is 4.0%. Then, assuming the statistics on the reference distributions, $f_D(\cos \theta_t)$ and $f_U(\cos \theta_t)$, will lead to a negligible source of error, this precision improves to 3.5%. Projecting these results to one year of data-taking at low luminosity (10 fb$^{-1}$), assuming that the statistics scale as the square root of the number of events, yields a predicted statistical precision of 1.6% on the measurement of the top polarisation.

5.5 Conclusions on single top production

As mentioned in the introduction, single-top quark production is the only known way to directly measure $V_{tb}$ at a hadron collider. In this section we estimate the accuracy with which $V_{tb}$ can be extracted at the LHC, and discuss what will be required to achieve that accuracy.

There are four sources of uncertainty in the extraction of $|V_{tb}|^2$ from the single-top cross section: theoretical, experimental, statistical, and machine luminosity. As we have seen, the statistical uncertainty with 30 fb$^{-1}$ of integrated luminosity is less than 2% for both the $t$-channel process and associated production, and is 5.5% for the $s$-channel process (3% with 100 fb$^{-1}$). It will be a challenge to reduce the other sources of uncertainty to 5%, so we regard the statistical accuracy as being sufficient in all three processes.

The traditional uncertainty in the machine luminosity is about 5% [159]. It may be possible to reduce the uncertainty below this value using Drell-Yan data, but this relies on accurate knowledge of the quark distribution functions. However, the process $q\bar{q} \to W^+ t\bar{t}$ involves the identical combination

\footnote{Only 10 fb$^{-1}$ are required to achieve this accuracy in the $t$-channel process.}
Fig. 33: The first histogram shows the reference distribution for 100% spin-up top quarks after detector effects and event-selection criteria have been applied and the appropriate level of background has been mixed in. The second histogram shows the reference distribution for 100% spin-down top quarks. The third histogram represents the expected SM distribution for a statistically-independent sample of signal and background.

of parton distribution functions as the $s$-channel process, so it can be used to almost directly measure the relevant parton luminosity, thereby avoiding the need to measure the machine luminosity [156].

The theoretical uncertainty is under the best control in the $s$-channel process. The theoretical uncertainty is dominated by the uncertainty in the top quark mass; an uncertainty of 2 GeV yields an uncertainty of 5%. This is cut in half if the uncertainty in the top mass is reduced to 1 GeV. The small uncertainty due to variation of the factorisation and renormalisation scales can be reduced to a negligible amount by calculating the cross section at NNLO order, which should be possible in the near future. The small uncertainty from the parton distribution functions can be further reduced as described in the previous paragraph; this also obviates the need for a measurement of the machine luminosity.

The theoretical uncertainty in the $t$-channel process is presently dominated by the factorisation-scale dependence and the parton luminosity. Although the scale dependence of the total cross section is small (4%), the uncertainty in the semi-inclusive cross section ($\sigma(p_T\bar{b}) < 20$ GeV) is about 10%. This can be reduced by calculating the $p_T$ spectrum of the $\bar{b}$ at NLO. It may also prove possible to measure the total cross section, although this has yet to be demonstrated. It is therefore plausible that the factorisation-scale dependence will be about 5% once the LHC is operating. It is also likely that the uncertainty from the parton distribution functions will be reduced below its present value of 10%. The parton luminosity could be directly measured using $Wj$ production, which is dominated by $gg\rightarrow Wq$, and therefore involves the identical combination of parton distribution functions as the $t$-channel process. Again, this has the desirable feature of eliminating the need to measure the machine luminosity.

The theoretical uncertainty in the associated-production cross section can be reduced far below its present value of 18%. A full NLO calculation should reduce the factorisation-scale dependence to roughly 5%. It is likely that the uncertainty from the parton distribution functions will also be reduced. Unless it is possible to measure the $gg$ luminosity directly, the uncertainty from the parton distribution functions will be augmented by the uncertainty in the machine luminosity.

As far as experimental systematic uncertainties are concerned, the extraction of a signal cross section requires knowledge of the backgrounds and of the efficiency and acceptance for the signal. These
analyses require hard cuts on both signal and background, and so the processes need to be modelled and understood very well.

For all of these processes, the major backgrounds are $t\bar{t}$ and $W +$ jets. The largest background for the $s$-channel process (where a double $b$-tag is employed) and associated production is $t\bar{t}$. The $t\bar{t}$ process can be isolated in other decay modes and in principle well measured. In the $t$-channel process the biggest background comes from $Wc\bar{c}$ with the charm jet mistagged as a $b$-jet. Obviously it would be worthwhile to develop a $b$-tagging algorithm having greater rejection against such mistags, even at the cost of some signal efficiency, given that the signal rate is large. It may be possible to understand the $W+$jets backgrounds by comparing with a sample of $Z+$jets events after applying similar selections to those used to select the single-top sample in $W+$ jets. The $Z+$charm rate will be suppressed compared to the $W+$ charm rate since the latter is mostly produced from the strange sea, which is bigger than the charm sea; nonetheless, the cross section, kinematics, jet multiplicities and so on can all be compared to our simulations using the $Z+$ jets sample.

The forward jet tag is very effective in enhancing the signal-to-background ratio in the $t$-channel process. This means that jets need to be found with good efficiency up to large rapidities, at least $|\eta| \sim 4$ in the calorimeter. Unfortunately these observations also imply that the background estimate is very sensitive to the Monte Carlo predicting the correct mix of jet flavours and jet rapidities in the $W+$ jets events. (We note that VECBOS generates very few jets in the tagging region, and so far there is no collider data on forward jets in vector-boson events which could verify whether this is correct.) Of course, effort applied to understanding $W+$ heavy-flavour jets backgrounds will pay off in many other searches besides this one, and will be a very worthwhile investment. We also look forward to the results of ongoing efforts to improve the Monte Carlo simulation of vector-boson plus jet production [160]. Requiring exactly two jets (as was done here to reject the $t\bar{t}$ background) also means that we will be very sensitive to our knowledge of jet efficiencies, QCD radiation, etc. The cross-section measurement also requires knowledge of the $b$-tagging efficiency. This should be measurable at the few-percent level using control samples of $t\bar{t}$ events selected with kinematic cuts alone.

As mentioned above, the purely statistical uncertainty in the cross-section measurement will be less than 5%, as will most of the theoretical uncertainties. It will be a considerable challenge to reduce the experimental systematic uncertainty to this level. At the present time, the experimental systematic uncertainty in the $t\bar{t}$ cross section at the Tevatron (which is a similar challenge in many respects, involving jets, $b$-tagging, and background subtraction) is about 19% [10]. This total is made up of many components which are each at the 5% level, so while it will be a lot of work to reduce them, there is no obvious “brick wall” that would prevent this.

Many of these systematic issues can also be addressed by comparing the $t$-channel and $s$-channel single-top processes. It will be a powerful tool to be able to measure $V_{tb}$ in two channels which have different dominant backgrounds, different selection cuts, and a different balance between theoretical and experimental systematic uncertainties.

We are only just now entering the era of precision top physics with Run II at the Tevatron. Single-top production has not yet even been observed. We will learn a great deal over the next few years about how to model top events and their backgrounds, and how to understand the systematic uncertainties. The LHC will undoubtedly benefit from all this experience.

If all sources of uncertainty are kept to the 5% level or less, it should be possible to measure $|V_{tb}|^2$ to 10% or less. We therefore regard the measurement of $V_{tb}$ with an accuracy of 5% or less as an ambitious but attainable goal at the LHC. We have also seen that a measurement of the polarisation of single top quarks produced via the $t$-channel process will be possible with a statistical accuracy of 1.6% with 10 fb$^{-1}$. We have not attempted to estimate the systematic uncertainty in this measurement.
6. \( \bar{t}t \) SPIN CORRELATIONS AND CP VIOLATION\(^{21} \)

For \( \bar{t}t \) production at the LHC quantities associated with the spins of the top and antitop quark will be “good” observables as well. The reason for this is well known. Because of its extremely short lifetime \( \tau_t \) (see Section 2.1) the top quark decays before it can form hadronic bound states. Thus the information on the spin of the top quark does not get diluted. As the spin-flip time is much larger than \( \tau_t \) it is, moreover, very unlikely that the top quark changes its spin-state by emitting gluon(s) via a chromomagnetic dipole transition before it decays. In any case this amplitude is calculable with QCD perturbation theory. Hence by measuring the angular distributions and correlations of the decay products of \( t \) and \( \bar{t} \) the spin-polarisations and the spin-spin correlations that were imprinted upon the \( t\bar{t} \) sample by the production mechanism can be determined and compared with predictions made within the SM or its extensions. Therefore these spin phenomena are an additional important means to study the fundamental interactions involving the top quark.

In this section we are concerned with the production and decay of top-antitop pairs. At the LHC the main \( t\bar{t} \) production process is gluon-gluon fusion, \( q\bar{q} \) annihilation being sub-dominant. As the main SM decay mode of \( t \to W^+b \) we shall consider here the parton reactions

\[
gg, q\bar{q} \to t\bar{t} + X \to b\bar{b} + 4f + X, \tag{16}
\]

where \( f \) denotes either a quark, a charged lepton or a neutrino. If the final state in (16) contains two, one, or no high \( p_T \) charged lepton(s) then we call these reactions, as usual, the di-lepton, single lepton, and non-leptonic \( t\bar{t} \) decay channels, respectively. To lowest order QCD the matrix elements for (16), including the complete \( t\bar{t} \) spin correlations and the effects of the finite top and \( W \) widths, were given in [161, 162]. Spin correlation effects in \( t\bar{t} \) production in hadron collisions were studied within the SM in [162, 163, 164, 165, 166, 167, 168].

In order to discuss the top spin-polarisation and correlation phenomena that are to be expected at the LHC it is useful to employ the narrow-width approximation for the \( t \) and \( \bar{t} \) quarks. Because \( \Gamma_t/m_t \ll 1 \) one can write, to good approximation, the squares of the exact Born matrix elements \( \mathcal{M}^{(\lambda)} \), \( \lambda = gg, q\bar{q}, \) in the form

\[
|\mathcal{M}^{(\lambda)}|^2 \propto \text{Tr} |\rho R^{(\lambda)}\bar{p}| \equiv \rho_{\alpha'\alpha} R^{(\lambda)}_{\alpha\alpha',\beta\beta'} \bar{p}_{\beta'} \bar{p}_{\beta}.
\]

The complete spin information is contained in the (unnormalised) spin density matrices \( R^{(\lambda)} \) for the production of on-shell \( t\bar{t} \) pairs and in the density matrices \( \rho, \bar{p} \) for the decay of polarised \( t \) and \( \bar{t} \) quarks into the above final states. The trace in (17) is to be taken in the \( t \) and \( \bar{t} \) spin spaces. The decay density matrices will be discussed below. The matrix structure of \( R^{(\lambda)} \) is

\[
R^{(\lambda)}_{\alpha\alpha',\beta\beta'} = A^{(\lambda)} \delta_{\alpha\alpha'} \delta_{\beta\beta'} + B^{(\lambda)}_{\alpha\alpha'} \delta_{\beta\beta'} + \bar{B}^{(\lambda)}_{\alpha\alpha'} \delta_{\beta\beta'} + C^{(\lambda)}_{ij} \sigma^i_{\alpha\alpha'} \sigma^j_{\beta\beta'}, \tag{18}
\]

where \( \sigma^i \) are the Pauli matrices. Using rotational invariance the “structure functions” \( B^{(\lambda)}_{\alpha\alpha'} \) and \( C^{(\lambda)}_{ij} \) can be further decomposed. A general discussion of the symmetry properties of these functions is given in [169]. The function \( A^{(\lambda)} \), which determines the \( t\bar{t} \) cross section, is known in QCD at NLO [38]. Because of parity (P) invariance the vectors \( B^{(\lambda)} \), \( \bar{B}^{(\lambda)} \) can have, within QCD, only a component normal to the scattering plane. This component, which amounts to a normal polarisation of the \( t \) quark, \( \mathcal{P}^{\perp}_{\lambda} \), is induced by the absorptive part of the respective scattering amplitude and was computed for the above LHC processes to order \( \alpha_s^3 \) [170]. \( \mathcal{P}^{\perp}_{\lambda} = \mathcal{P}^t \) if CP invariance holds.) The size of the normal polarisation depends on the top quark scattering angle and on the c.m. energy. In the gluon-gluon fusion process \( \mathcal{P}^t \) reaches peak values of about 1.5%. In \( t\bar{t} \) production at the LHC the polarisation of the top quark within the partonic scattering plane, which is P-violating, is small as well within the SM. Therefore the \( t \) and \( \bar{t} \) polarisations in the scattering plane are good observables to search for P-violating non-SM interactions in the reactions (16) – see Section 3.4.

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The $t\bar{t}$ production by the strong interactions leads, on the other hand, to a significant correlation between the $t$ and $\bar{t}$ spins. This correlation is encoded in the functions $C_{ij}^{(\lambda)}$. Using P- and charge-conjugation (C) invariance they have, in the case of a $t\bar{t}$ final state, the structure

$$C_{ij}^{(\lambda)} = c_1^{(\lambda)} \hat{k}_{ij} + c_2^{(\lambda)} \hat{p}_i \hat{p}_j + c_3^{(\lambda)} \hat{k}_{ij} \hat{k}_{kj} + c_4^{(\lambda)} (\hat{k}_{ij} \hat{p}_j + \hat{p}_i \hat{k}_{kj}),$$  \hspace{1cm} (19)

where $\hat{p}$ and $\hat{k}$ are the directions of flight of the initial quark or gluon and of the $t$ quark, respectively, in the parton c.m. frame. So far the functions $c_i^{(\lambda)}$ are known to lowest-order QCD only (see, e.g., [164]). For a $t\bar{t}X$ final state a decomposition similar to (19) can be made.

From (19) one may read off the following set of spin-correlation observables [164]:

$$(\hat{k}_t \cdot s_t)(\hat{k}_\bar{t} \cdot s_\bar{t}),$$  \hspace{1cm} (20)

$$(\hat{p} \cdot s_t)(\hat{p} \cdot s_\bar{t}),$$  \hspace{1cm} (21)

$$s_t \cdot s_\bar{t},$$  \hspace{1cm} (22)

$$(\hat{p} \cdot s_t)(\hat{k}_\bar{t} \cdot s_\bar{t}) + (\hat{p} \cdot s_\bar{t})(\hat{k}_t \cdot s_t),$$  \hspace{1cm} (23)

where $s_t, s_\bar{t}$ are the $t$ and $\bar{t}$ spin operators, respectively. The observables (20), (21), and (23) determine the correlations of different $t, \bar{t}$ spin projections. Eq. (20) corresponds to a correlation of the $t$ and $\bar{t}$ spins in the helicity basis, while (21) correlates the spins projected along the beam line. We note that the “beam-line basis” defined in [166] refers to spin axes being parallel to the left- and right-moving beams in the $t$ and $\bar{t}$ rest frames, respectively. The $t\bar{t}$ spin correlation in this basis is a linear combination of (20), (21), and (23).

A natural question is: what is – assuming only SM interactions – the best spin basis or, equivalently, the best observable for investigating the $t\bar{t}$ spin correlations? For quark-antiquark annihilation, which is the dominant production process at the Tevatron, it turns out that the spin correlation (21) [164, 168] and the correlation in the beam-line basis [166] is stronger than the correlation in the helicity basis. In fact, for $q\bar{q}$ annihilation a spin-quantisation axis was constructed in [167] with respect to which the $t$ and $\bar{t}$ spins are 100% correlated. At the LHC the situation is different. For $gg \rightarrow t\bar{t}$ at threshold conservation of total angular momentum dictates that the $t\bar{t}$ is in a $1S_0$ state. Choosing spin axes parallel to the right- and left-moving beams this means that we have $t_{L}\bar{t}_{L}$ and $t_{R}\bar{t}_{R}$ states at threshold. On the other hand at very high energies helicity conservation leads to the dominant production of unlike helicity pairs $t_{R}\bar{t}_{L}$ and $t_{L}\bar{t}_{R}$. One can show that no spin quantisation axis exists for $gg \rightarrow t\bar{t}$ with respect to which the $t$ and $\bar{t}$ spins are 100% correlated. The helicity basis is a good choice, but one can do better. This is reflected in the above observables. Computing their expectation values and statistical fluctuations one finds [164] that (22) has a higher statistical significance than the helicity correlation (20) which in turn is more sensitive than (21) or the correlation in the beam-line basis.

The spins of the $t$ and $\bar{t}$ quarks are to be inferred from their P-violating weak decays, i.e., from $t \rightarrow bW^+ \rightarrow b\ell^+\nu_\ell$ or $bq\ell^\pm$ and likewise for $\bar{t}$ if only SM interactions are relevant. As already mentioned and used in previous sections, in this case the charged lepton from $W$ decay is the best analyser of the top spin. This is seen by considering the decay distribution of an ensemble of polarised $t$ quarks decaying into a particle $f$ (plus anything) with respect to the angle between the polarisation vector $\xi_t$ of the top quark and the direction of flight $\hat{q}_f$ of the particle $f$ in the $t$ rest frame. This distribution has the generic form

$$\frac{1}{\Gamma} \frac{d\Gamma}{d \cos \theta_f} = \frac{1}{2} (1 + \kappa_f \xi_t \cdot \hat{q}_f),$$  \hspace{1cm} (24)

where the magnitude of the coefficient $\kappa_f$ signifies the spin-analyser quality of $f$. The SM values for some $f$, collected from [171, 172, 173, 174], are given in Table 12. The corresponding $t$ decay density matrix in the $t$ rest frame is read off from (24) to be $\rho_{\alpha'\alpha} = (1 + \kappa_f \sigma \cdot \hat{q}_f)_{\alpha'\alpha}$. The distributions for
the decay of polarised antitop quarks are obtained by replacing $\kappa_f \rightarrow -\kappa_f$ in (24). The order $\alpha_s$ QCD corrections to the decays $t \rightarrow b \ell \ell$ and $t \rightarrow Wb$ of polarised $t$ quarks were computed in [171] and [175], respectively. For $t, \bar{t}$ polarisation observables these corrections are small.

From the above table it is clear that the best way to analyse the $t\bar{t}$ spin correlations is through angular correlations among the two charged leptons $\ell^+\ell^-$ in the di-lepton final state. Using the production and decay density matrices in (17), neglecting the 1-loop induced QCD normal polarisation, and integrating over the azimuthal angles of the charged leptons one obtains the following normalised double distribution, e.g. in the helicity basis

$$\frac{1}{\sigma} d^2\sigma \frac{d^2\sigma}{d\cos\theta_+ d\cos\theta_-} = \frac{1 + C_{\kappa_{\ell^+} \kappa_{\ell^-}} \cos\theta_+ \cos\theta_-}{4}, \quad (25)$$

where $\kappa_{\ell^+} \kappa_{\ell^-} = -1$ and $\theta_+ (\theta_-)$ is the angle between the $t(\bar{t})$ direction in the $t\bar{t}$ c.m. frame and the $\ell^+(\ell^-)$ direction of flight in the $t(\bar{t})$ rest frame. The coefficient $C$, which is the degree of the spin correlation in the helicity basis, results from the $c_i^{(\lambda)}$ in (19) and it is related to [165]:

$$C = \frac{N(t_L \bar{t}_L + t_R \bar{t}_R) - N(t_L \bar{t}_R + t_R \bar{t}_L)}{N(t_L \bar{t}_L + t_R \bar{t}_R) + N(t_L \bar{t}_R + t_R \bar{t}_L)}. \quad (26)$$

For partonic final states and to lowest order in $\alpha_s$ one gets $C = 0.332$ for the LHC. (The number depends somewhat on the parton distributions used. Here and below the set CTEQ4L [116] was used.) The optimum would be to find a spin axis with respect to which $|C| = 1$. But, as stated above, this is not possible for $gg$ fusion. In addition to (25), analogous correlations among $\ell^+$ from $t$ and jets from $\bar{t}$ decay (and vice-versa) in the single lepton channels, and jet-jet correlations in the non-leptonic decay channels should, of course, also be studied. While the spin-analysing power is lower in these cases, one gains in statistics.

From the above example is quite obvious that, for a given $t\bar{t}$ decay channel, the $t\bar{t}$ spin correlation will be most visible when the angular correlations among the $t$ and $\bar{t}$ decay products are exhibited in terms of variables defined in the $t$ and $\bar{t}$ rest frames. An important question is therefore how well the 4-momenta of the $t$ and $\bar{t}$ quarks can be reconstructed experimentally? We briefly discuss the results of a simulation of the single lepton and di-lepton channels [176] which includes hadronisation and detector effects using PYTHIA [52] and the ATLASFAST [105] software packages. The transverse momentum of every reconstructed object like a jet, a charged lepton, or the missing transverse energy of an event has to exceed a certain minimum value $p_T^{min}$. The detector acceptances impose further restrictions on the phase space of the detected objects in pseudo-rapidity.

In the case of the single lepton $t\bar{t}$ decay channels one isolated lepton ($\ell^+$ or $\mu^+$) is required. From the missing transverse energy of the event and the $W$ mass constraint the longitudinal momentum $p_z$ of the neutrino can be determined up to a twofold ambiguity. It turns out that in most cases the lower solution of $p_z$ is the correct one. To complete the event topology, four jets are demanded. Two of them have to be identified as $b$-jets coming from top decay.

The two non-tagged jets are often misidentified due to additional activity in the detector from initial and final state radiation. To suppress the QCD background the invariant mass of the two jets has to lie in a narrow mass window around the known mass of the $W$ boson. After this cut the two-jet system is rescaled to the $W$ mass. Finally there is a twofold ambiguity when the $b$-jets are combined with the
reconstructed $W$ bosons. The combination which yields the lower reconstructed top mass turns out to be the correct one most of the time.

In the case of the di-lepton decay channels two isolated oppositely charged leptons are requested. Moreover two jets have to be detected and tagged as $b$-jets. With the known top and $W$ masses and with the missing transverse energy of the event the unknown 3-momenta of the neutrino and anti-neutrino can be computed using the kinematic constraints of the event. These result in a system of two linear and four quadratic equations. The equations can be solved numerically and usually several solutions arise. Since the experimentally determined momenta do not coincide with the corresponding variables at the parton level the kinematic constraints have to be relaxed somewhat in order to improve the reconstruction efficiency. The algorithm set up in [176] was used to solve these equations. The best solution can be obtained by computing weights from known distributions. Following [176] the highest efficiency was obtained using the weight given by the product of the energy distributions of $\nu_\ell$ and $\bar{\nu}_\ell$ and the $\cos \theta^*_\ell$ distribution in the $t\bar{t}$ c.m. frame.

For the LHC running at low luminosity ($\mathcal{L} = 10^{33}$ cm$^{-2}$ s$^{-1}$), about $4 \times 10^5$ $t\bar{t}$ events per year are expected in the di-lepton decay channels ($\ell = e, \mu$). A further simulation of these channels was performed in order to study the joint distribution (25). PYTHIA 5.7 [52] was used for the event generation, CMSJET [177] for the detector response and the algorithm of [176] for the reconstruction of the $t, \bar{t}$ momenta. The transverse momenta of the two isolated, oppositely charged leptons and of the two jets were required to exceed 20 GeV. The minimal missing transverse energy of the event was chosen to be 40 GeV. A further selection criterion was that each jet provides at least two tracks with a significance of the transverse impact parameter above 3.0 to be tagged as $b$-jet. The processes were simulated in two different ways. First the SM matrix elements of [75] for the reactions (16), which contain the $t\bar{t}$ spin correlations, were implemented into PYTHIA. For comparison these channels were also simulated with the PYTHIA default matrix elements for $gg, q\bar{q} \rightarrow t\bar{t}$ which do not contain spin correlations. In both simulations initial and final state radiation, multiple interactions, and the detector response was included. In Figs. 34, 35 we have plotted the resulting double distributions $d^2N/d\cos \theta_+ d\cos \theta_-$. They have been corrected for the distortions of the phase space due to the cuts. A fit to the distribution Fig. 35 according to (25) yields the correlation coefficient $C = 0.331 \pm 0.023$, in agreement with the value $C = 0.332$ obtained at the parton level without cuts. A fit to Fig. 34 leads to $C = -0.021 \pm 0.022$ consistent with

![Fig. 34: Joint distribution $d^2N/d\cos \theta_+ d\cos \theta_-$ generated with default PYTHIA. The detector response was simulated with CMSJET.](image1)

![Fig. 35: Same distribution as in the figure to the left, but including the SM $t\bar{t}$ spin correlations. The detector response was simulated with CMSJET.](image2)
Systematic errors, for instance due to background processes, e.g., $Z^0 \to \ell^+ \ell^-$ accompanied by two $b$-jets, remain to be investigated.

From these double distributions one may form one- or zero-dimensional projections, for instance asymmetries as considered in [166, 165, 168]. Another approach is to study distributions and expectation values of angular correlation observables which would be zero in the absence of the $t\bar{t}$ spin correlations. A suitable set of observables is obtained by transcribing, for instance, the spin observables given above into correlations involving the directions of flight of those final state particles that are used to analyse the $t$ and $\bar{t}$ spins. As an example we discuss the case of the single lepton channels $t \to bqq^*, \bar{t} \to b\ell^\mp n_{\ell}$.

One may choose to analyse the $t$ spin by the direction of flight $\hat{q}_b^*$ of the $b$-jet in the rest frame of the $t$ quark and the $\bar{t}$ spin by the momentum direction $\hat{q}_{\ell^-}$ of the $\ell^-$ in the laboratory frame. The latter is rather conservative in that no reconstruction of the $\ell$ momentum is necessary. Then (20)-(22) are translated into the observables

$$O_1 = \langle \hat{q}_b^* \cdot \hat{p}_p \rangle \langle \hat{q}_{\ell^-} \cdot \hat{p}_\ell \rangle,$$

$$O_2 = \langle \hat{q}_b^* \cdot \hat{k}_\ell \rangle \langle \hat{q}_{\ell^-} \cdot \hat{k}_\ell \rangle,$$

$$O_3 = \langle \hat{q}_b^* \cdot \hat{k}_\ell \rangle \langle \hat{q}_{\ell^-} \rangle,$$

where $\hat{p}_p$ refers to the beam direction. The pattern of statistical sensitivities of the spin observables (20)-(22) stated above is present also in these angular correlations. Computing the expectation values $\langle O_i \rangle$ and the statistical fluctuations $\Delta O_i$ and those of the observables for the corresponding charge conjugated channels, one gets for the statistical significances of these observables at the parton level [164]: $S_1 \approx 0.007\,\sqrt{N_{6\ell^-}}$, $S_2 \approx 0.025\,\sqrt{N_{6\ell^-}}$, and $S_3 \approx 0.055\,\sqrt{N_{6\ell^-}}$, where $N_{6\ell^-}$ is the number of reconstructed events in the specific single lepton channel. The linear combination

$$O_4 = O_3 - O_1$$

has a still higher sensitivity than $O_1$, namely $S_4 \approx 0.073\,\sqrt{N_{6\ell^-}}$. Even with $10^4$ reconstructed $b\ell^-$ and $\bar{b}\ell^+$ events each one would get a $7.3\sigma$ spin-correlation signal with this observable. The significance of these observables after the inclusion of hadronisation and detector effects remains to be studied.

The results of the above simulations are very encouraging for the prospect of $t$, $\bar{t}$ spin physics. On the theoretical side the NLO QCD corrections to the helicity amplitudes, and to the spin density matrices should be computed in order to improve the precision of the predictions and simulation tools.

If $t\bar{t}$ production and/or decay is affected by non-SM interactions then the correlations above will be changed. One interesting possibility would be the existence of a heavy spin-zero resonance $X_0$ (for instance a heavy (pseudo)scalar Higgs boson as predicted, e.g., by SUSY models or some composite object) that couples strongly to top quarks. For a certain range of masses and couplings to $t\bar{t}$ such an object would be visible in the $t\bar{t}$ invariant mass spectrum [74, 75]. Suppose one will be fortunate and discover such a resonance at the LHC. Then the parity of this state may be inferred from an investigation of $t\bar{t}$ spin correlations. This is illustrated by the following example. As already mentioned above, close to threshold gluon-gluon fusion produces a $t\bar{t}$ pair in a $^1S_0$ state. On the other hand if the pair is produced by the $X_0$ resonance, $gg \to X_0 \to t\bar{t}$, then for a scalar (pseudo-scalar) $X_0$ the $t\bar{t}$ pair is in a $^3P_0$ ($^1S_0$) state and has therefore characteristic spin correlations. Let us evaluate, for instance, the observable (22). Its expectation value at threshold is $\langle s_t \cdot s_{\bar{t}} \rangle = 1/4 \times (-3/4)$ if $t\bar{t}$ is produced by a (pseudo)scalar spin-zero boson, ignoring the $gg \to t\bar{t}$ background. An analysis which includes the interference with the QCD $t\bar{t}$ amplitude shows characteristic differences also away from threshold. By investigating several correlation observables (i.e., employing different spin bases) one can pin down the scalar/pseudo-scalar nature of such a resonance for a range of $X_0$ masses and couplings to top quarks [75].

Another effect of new physics might be the generation of an anomalously large chromomagnetic form factor $\kappa$ (see Section 7.1) in the $t\bar{t}$ production amplitude which would change the spin correlations with respect to the SM predictions [178, 179] (see also [180, 181]). For the LHC with 100 fb$^{-1}$ integrated
luminosity one obtains from a study of asymmetries (that were also used in [179]) at the parton level a statistical sensitivity of $\delta K \approx 0.02$.

The top quark decay modes $t \rightarrow b\ell^+\nu_\ell$, $b\ell^\pm$ might also be affected by non-SM interactions, for instance by right-handed currents or by charged Higgs-boson exchange, and this would alter the angular correlations discussed above as well. A Michel-parameter type analysis of the sensitivity to such effects at the LHC remains to be done.

The large $t\bar{t}$ samples to be collected at the LHC offer, in particular, an excellent opportunity to search for CP-violating interactions beyond the SM in high energy reactions. (The Kobayashi-Maskawa phase induces only tiny effects in $t\bar{t}$ production and decay.) We mention in passing that such interactions are of great interest for attempts to understand the baryon asymmetry of the universe. Many proposals and phenomenological studies of CP symmetry tests in $t\bar{t}$ production and decay at hadron colliders have been made. The following general statements apply [169]: A P- and CP-violating interaction affecting $t\bar{t}$ production induces additional terms in the production density matrices $P^{(A)}$ which generate two types of CP-odd spin-momentum correlations, namely

$$\hat{k}_t \cdot (s_t - s_\bar{t}), \quad (31)$$

and

$$\hat{k}_t \cdot (s_t \times s_\bar{t}), \quad (32)$$

and two analogous correlations where $\hat{k}_t$ is replaced by $\hat{p}$. The longitudinal polarisation asymmetry (31) requires a non-zero CP-violating absorptive part in the respective scattering amplitude. In analogy to the SM spin correlations above, (31) and (32) can also be transcribed into angular correlations among the $t$ and $\bar{t}$ decay products, which may serve as basic CP observables (see below).

As to the modelling of non-SM CP violation two different approaches have been pursued. One is to parameterise the unknown dynamics with form factors or, neglecting possible dependences on kinematic variables, with couplings representing the strength of effective interactions [180, 182, 173, 183, 178, 179, 184, 185], and compute the effects on suitable observables. This yields estimates of the sensitivities to the respective couplings. For instance if $t\bar{t}$ production is affected by a new CP-violating interaction with a characteristic energy scale $\Lambda_{CP} > \sqrt{s}$ then this interaction may effectively generate a chromoelectric dipole moment (CEDM) $d_t$ of the top quark (see Section 7.1). Assuming $10^7$ non-leptonic, $6 \times 10^6$ single lepton, and $10^6$ $t\bar{t}$ di-lepton events, the analysis of [185], using optimal CP observables, comes to the conclusion that a 1$\sigma$ sensitivity of $\delta(Re\,d_t) \approx 5 \times 10^{-20}$ g can be reached at the LHC. A detector-level study of CP violation in $t\bar{t}$ decays with di-lepton final states was performed in [186].

Alternatively one may consider specific extensions of the SM where new CP-violating interactions involving the top quark appear and compute the induced effects in $t\bar{t}$ production and decay, in particular for the reactions (16). We mention two examples. In supersymmetric extensions of the SM, in particular in the minimal one (MSSM), the fermion-sfermion-neutralino interactions contain in general CP-violating phases which originate from SUSY-breaking terms. These phases are unrelated to the Kobayashi-Maskawa phase. The interaction Lagrangian for the top quark coupling to a scalar top $\tilde{t}_{1,2}$ and a gluino $\tilde{G}$ reads in the mass basis

$$\mathcal{L}_{G\tilde{t}} = i\sqrt{2}g_s \sum_{l=1,2} \left(e^{-i\phi_t} T_L \Gamma_l \tilde{G}^\alpha T^\alpha \tilde{t}_1 + e^{+i\phi_t} T_R \Gamma_l \tilde{G}^\alpha T^\alpha \tilde{t}_\bar{t} \right) + h.c., \quad (33)$$

where $g_s$ is the QCD coupling. A priori the phase $\phi_t$ is unrelated to the analogous phases in the light quark sector which are constrained by the experimental upper bound on the electric dipole moment of the neutron. The CP-violating one-loop contributions of (33) to $gg, \bar{q}q \rightarrow t\bar{t}$ were computed in [187, 185]. A non-zero CP effect requires, apart from a non-zero phase $\phi_t$, also non-degeneracy of the masses of $\tilde{t}_{1,2}$. For fixed phase and $\tilde{t}_1 - \tilde{t}_\bar{t}$ mass difference the effect decreases with increasing gluino and scalar top masses. Assuming the same data samples as in the CEDM analysis above, [185] concludes from
used:

Fig. 36: Left: differential expectation value of $Q_1$ as a function of the $t\bar{t}$ invariant mass at $\sqrt{s} = 14$ TeV for reduced Yukawa couplings $a_t = 1$, $\bar{a}_t = -1$, and a Higgs boson mass $m_\phi = 400$ GeV. The dashed line represents the resonant and the solid line the sum of the resonant and non-resonant $\phi$ contributions. Right: same as figure to the left, but for the observable $Q_2$ [86].

a computation of optimal CP observables that a sensitivity $|\phi_t| \gtrsim 0.1$ can be reached at the LHC if the gluino and squark masses do not exceed 400 GeV.

Another striking possibility would be CP violation by an extended scalar sector manifesting itself through the existence of non-degenerate neutral Higgs bosons with undefined CP parity. Higgs sector CP violation can occur already in extensions of the SM by an extra Higgs doublet (see, for instance [188]). It may also be sizable in the MSSM within a certain parameter range [189]. The coupling of such a neutral Higgs boson $\phi$ with undefined CP parity to top quarks reads

$$\mathcal{L}_Y = - (2G_F)^{1/2} m_t (a_t \bar{t} t + \bar{a}_t \bar{t} \gamma_5 t) \phi,$$

where $a_t$ and $\bar{a}_t$ denote the reduced scalar and pseudo-scalar Yukawa couplings, respectively (in the SM $a_t = 1$ and $\bar{a}_t = 0$). The CP-violating effects of (34) on $gg, \bar{q}q \rightarrow t\bar{t}$ were investigated for light $\phi$ in [190] and for $\phi$ bosons of arbitrary mass in [191, 169] (see also [185, 86]). The exchange of $\phi$ bosons induces, at the level of the $t\bar{t}$ states, both types of correlations (32), (31) (the CP asymmetry $\Delta N_{L,R} = [N(t_L \bar{t}_L) - N(t_R \bar{t}_R)] / (all\, t\bar{t})$ considered in [190] corresponds to the longitudinal polarisation asymmetry $\langle \hat{k}_t \cdot (s_\ell - s_{\bar{t}}) \rangle$). If the mass of $\phi$ lies in the vicinity or above $2m_t$ the $s$-channel $\phi$-exchange diagram $gg \rightarrow \phi \rightarrow t\bar{t}$ becomes resonant and is by far the most important $\phi$ contribution.

Simple and highly sensitive observables and asymmetries were investigated for the different $t\bar{t}$ decay channels in [86]. For the di-lepton channels the following transcriptions of (31) and (32) may be used:

$$Q_1 = \hat{k}_t \cdot \hat{q}_+ - \hat{k}_\bar{t} \cdot \hat{q}_-,$$  

$$Q_2 = (\hat{k}_t - \hat{k}_\bar{t}) \cdot (\hat{q}_- \times \hat{q}_+)/2,$$  

where $\hat{k}_t, \hat{k}_\bar{t}$ are here the $t, \bar{t}$ momentum directions in the $t\bar{t}$ c.m. frame and $\hat{q}_+, \hat{q}_-$ are the $\ell^+, \ell^-$ momentum directions in the $t$ and $\bar{t}$ quark rest frames, respectively. Note that $Q_1 = \cos \theta_+ - \cos \theta_-$ where $\theta_{\pm}$ are defined after (25). When taking expectation values of these observables the channels $\ell^+, \ell^-$ with $\ell, \ell' = e, \mu$ are summed over. The sensitivity to the CP-violating product of couplings $\gamma_{CP} = -a_t \bar{a}_t$ of heavy Higgs bosons is significantly increased when expectation values of (35), (36) are taken with respect to bins of the $t\bar{t}$ invariant mass $M_{t\bar{t}}$. Two examples of these “differential expectation values” are shown in Fig. 36. In order to estimate the measurement errors we have used a sample of di-lepton events, obtained from a simulation at the detector level using the same selection criteria as in the
simulation described above, and determined the resulting error on the expectation value of $Q_1$, choosing $M_{\ell\ell}$ bins with a width of 10 GeV. With $2 \times 10^5$ reconstructed di-lepton events in the whole $M_{\ell\ell}$ range we find that the error on $\langle Q_1 \rangle_{M_{\ell\ell}}$ is slightly below 1% for a bin at, say, $M_{\ell\ell} = 400$ GeV. In addition one may employ the following asymmetries which are experimentally more robust than $\langle Q_1 \rangle$:

$$A(Q_i) = \frac{N_{\ell\ell}(Q_i > 0) - N_{\ell\ell}(Q_i < 0)}{N_{\ell\ell}},$$

(37)

where $i = 1, 2$ and $N_{\ell\ell}$ is the number of di-lepton events. From an analysis of these observables and asymmetries and analogous ones for the single lepton channels at the level of partonic final states the conclusion can be drawn [86] that one will be sensitive to $|\gamma_{CP}| \gtrsim 0.1$ at the LHC. This will constitute rather unique CP tests.

7. TOP QUARK ANOMALOUS INTERACTIONS

In the SM the gauge couplings of the top quark are uniquely fixed by the gauge principle, the structure of generations and the requirement of a lowest dimension interaction Lagrangian. Due to the large top mass, top quark physics looks simple in this renormalisable and unitary quantum field theory. Indeed,

- the top quark production cross section is known with a rather good accuracy ($\sim (10 - 15)$ %),
- there are no top hadrons (mesons or baryons),
- the top quark decay is described by pure $(V - A)$ weak interactions,
- only one significant decay channel is present: $t \to bW^+$ (other decay channels are very suppressed by small mixing angles).

This simplicity makes the top quark a unique place to search for new physics beyond the SM. If anomalous top quark couplings exist, they will affect top production and decay at high energies, as well as precisely measured quantities with virtual top quark contributions.

We do not know which type of new physics will be responsible for a future deviation from the SM predictions. However, top quark couplings can be parametrized in a model independent way by an effective Lagrangian. The top quark interactions of dimension 4 can be written (in standard notation [192]):

$$\mathcal{L}_4 = -g_s T^{\gamma \mu} T^a t G^a_{\mu} - \frac{g}{\sqrt{2}} \sum_{q=d,s,b} T^{\gamma \mu} (v^W_{tq} - a^W_{tq} \gamma_5) q W^+_{\mu} - \frac{2}{3} \varepsilon T^{\gamma \mu} t A_{\mu} - \frac{g}{2 \cos \theta_W} \sum_{q=u,c,t} T^{\gamma \mu} (v^Z_{tq} - a^Z_{tq} \gamma_5) q Z_{\mu}$$

(38)

plus the hermitian conjugate operators for the flavour changing terms. $T^a$ are the Gell-Mann matrices satisfying $\text{Tr} (T^a T^b) = \delta^{ab}/2$. Gauge invariance fixes the strong and electromagnetic interactions in (38) and hemiticity implies real diagonal couplings $v^{Z, W}_{tq}$, whereas the non-diagonal ones $v^{W, Z}_{tq}, a^{W, Z}_{tq}$ can be complex in general. Within the SM $v^W_{tq} = a^W_{tq} = V_{tb}/2$, with $V_{tb}$ the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $v^Z_{tq} = 1 - 4 \sin^2 \theta_W$, $a^Z_{tq} = 1/2$, and the non-diagonal $Z$ couplings are equal to zero. Typically modifications of the SM couplings can be traced back to dimension 6 operators in the effective Lagrangian description valid above the EW symmetry breaking scale [193, 194, 195] (see also [196, 132, 197]). Hence, they are in principle of the same order as the other dimension 5 and 6 couplings below the EW scale. However, in specific models the new couplings in Eq. (38) can be large [198]. Moreover, the present experimental limits are relatively weak and these couplings can show up in simple processes and can be measured with much better precision at the LHC.

The dimension 5 couplings to one on-shell gauge boson, after gauge symmetry breaking, have the generic form: [199] :

$$\mathcal{L}_5 = -g_s \sum_{q=u,c,t} \frac{\kappa^{W}_{tq}}{\Lambda} T^{\sigma \mu \nu} T^a (f^0_{Wq} + i h^0_{Wq} \gamma_5) q G^{a}_{\mu \nu} - \frac{g}{\sqrt{2}} \sum_{q=d,s,b} \frac{\kappa^{W}_{tq}}{\Lambda} T^{\sigma \mu \nu} (f^0_{Wq} + i h^0_{Wq} \gamma_5) q W^{+}_{\mu \nu}$$

---

Section coordinators: F. del Aguila, S. Slabospitsky, M. Cobal (ATLAS), E. Boos (CMS).
\[- \epsilon \sum_{q=u,c,t} \frac{\kappa_{1q}}{\Lambda} \overline{l}_\sigma \gamma^\mu (f_{1q}^\gamma + i h_{1q}^\gamma \gamma_5) q A_{\mu} - \frac{g}{2 \cos \theta_W} \sum_{q=u,c,t} \frac{\xi_{2q}^Z}{\Lambda} \overline{l}_\sigma \gamma^\mu (f_{2q}^Z + i h_{2q}^Z \gamma_5) q Z_{\mu} \]

(39)

plus the hermitian conjugate operators for the flavour changing terms. \(G_{\mu \nu}^a\) is \(\partial_\mu G_{\nu}^a - \partial_\nu G_{\mu}^a\) (see, however, below) and similarly for the other gauge bosons. We normalise the couplings by taking \(\Lambda = 1 \text{ TeV}\). \(\kappa\) is real and positive and \(f, h\) are complex numbers satisfying for each term \(|f|^2 + |h|^2 = 1\). As in the dimension 3 case these dimension 5 terms typically result from dimension 6 operators after the EW breaking. They could be large, although they are absent at tree level and receive small corrections in renormalizable theories. At any rate the LHC will improve appreciably their present limits.

There are also dimension 5 terms with two gauge bosons. However, the only ones required by the unbroken gauge symmetry \(SU(3)_C \times U(1)_Q\), and taken into account here, are the strong couplings with two gluons and the EW couplings with a photon and a W boson. They are obtained including also the bilinear term \(g_s f^{abc} G_{\mu}^b G_{\mu}^c\), with \(f^{abc}\) the \(SU(3)_C\) structure constants, in the field strength \(G_{\mu}^a\) in (39) and the bilinear term \(- i e (A_{\mu} W_{\nu}^+ - A_{\nu} W_{\mu}^+)\) in \(W_{\mu \nu}^+\), respectively. We do not consider any other dimension 5 term with two gauge bosons for their size is not constrained by \(SU(3)_C \times U(1)_Q\) and/or they only affect to top quark processes with more complicated final states than those discussed here. We will not elaborate on operators of dimension 6, although the first \(q^2\) corrections to dimension 4 terms could be eventually observed at large hadron colliders [134]. In this section we are not concerned with the effective top couplings to Higgs bosons either.

In what follows we study the LHC potential for measuring or putting bounds on the top quark anomalous interactions in (38), (39) through production processes. Results from top quark decays are presented in Section 8. The \(t\bar{t}\) couplings to gluons are considered first, since they are responsible for \(t\bar{t}\) production. Secondly we discuss the top quark couplings to \(b\bar{W}\). In the SM this coupling is not only responsible for almost 100\% of the top decays but it also leads to an EW single top production mode, as reviewed in Section 5. Finally we deal with the \(t\) flavour changing neutral currents (FCNC). The \(\gamma t\bar{t}\) and \(Zt\bar{t}\) vertices have not been considered here because \(e^+ e^-\) and \(\mu^+ \mu^-\) colliders can give a cleaner environment for their study.

With the exception of the summary Table 23, we will quote limits from the literature without attempting to compare them. In Table 13 we illustrate statistics frequently used and which we will refer to in the text when presenting the bounds. As can be observed, the number of signal events, and the limit estimates, vary appreciably with the choice of statistics. We do correct for the different normalizations of the couplings used in the literature.

<table>
<thead>
<tr>
<th>(B)</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
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<tr>
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<td>9</td>
<td>3.84</td>
<td>0</td>
<td>0</td>
<td>4.74</td>
<td>3.09</td>
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<td>12.57</td>
<td>6.71</td>
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<td>10.83</td>
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<tr>
<td>15</td>
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<td>9.75</td>
<td>11.62</td>
<td>7.59</td>
<td>12.81</td>
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</tbody>
</table>

| 7.1 Probes of anomalous \(gt\bar{t}\) couplings |

The combination \(\frac{\Lambda_{\mu\nu}}{\Lambda} \kappa_{1t} f_{1t}^0\) (see (39)) can be identified with the anomalous chromomagnetic dipole moment of the top quark, which, as is the case of QED, receives one-loop contributions in QCD. Therefore,
its natural size is of the order of $\alpha_s/\pi$. As we observed above, when this coupling is non-zero a direct $gg\ell\ell$ four-point vertex is induced as a result of gauge invariance.

On the other hand the combination $4m_t^2\kappa_{tt}^0h_{tt}^0$ can be identified as the anomalous chromoelectric dipole moment of the top quark. Within the SM this can arise only beyond two loops [201]. On the other hand it can be much larger in many models of CP violation such as multi-Higgs-doublet models and SUSY [202]. Therefore, such a non-vanishing coupling would be a strong indication of BSM physics.

Considering the gluonic terms in (38), (39) for the process of light quark annihilation into $\ell\ell$ one obtains [181, 203]

$$\frac{d\sigma_{gg}}{dt} = \frac{2\pi\alpha_s}{9s^2} \left[ 2 - \beta^2(1 - z^2) - \frac{8m_t}{\lambda} \kappa_t^2 \left( f_{tt}^q + f_{tt}^g \right) + \frac{32m_t^2}{\lambda^2} \left( \kappa_{tt}^0 \right)^2 \left| f_{tt}^q \right|^2 + \frac{4\tilde{s}}{\lambda^2} \left( \kappa_{tt}^0 \right)^2 \beta^2(1 - z^2) \right],$$

(40)

$\tilde{s}$ being the incoming parton total energy squared, $z$ being the cosine of the scattering angle $\theta^*$ in the cms of the incoming partons, and $\beta = \sqrt{1 - 4m_t^2/s}$.

The squared matrix element for $gg$ annihilation is a more complicated expression; we refer to [181, 204] for exact formulas. If the (anomalous) couplings are assumed to be functions (form-factors) of $q^2$ and then corrected by operators of dimension higher than 5, the $gg$ annihilation amplitude would be evaluated at different scales (for the $\ell(\ell)$ and $s$ channels), and an additional violation of the $SU(3)_C$ gauge invariance could be made apparent. For a detailed discussion of this problem see, for example, [181] and references therein.

The effects associated with $\kappa_{tt}^0 f_{tt}^q$ were examined in [181, 205, 206]. As shown in [134] they will be easily distinguishable from the effects of $q^2$ corrections to the strong coupling due to operators of dimension 6, which are relatively straightforward to analyse [195] in $t\bar{t}$ production since the effective coupling would be a simple rescaling of the strength of the ordinary QCD coupling by an additional $q^2$-dependent amount. It was shown in [206] that the high-end tail of the top quark $p_T$ and $M_{t\bar{t}}$ distributions are the observables most sensitive to non-zero values of $\kappa_{tt}^0 f_{tt}^q$, with a reach for $\kappa = \frac{4m_t^2\kappa_{tt}^0 f_{tt}^q}{\lambda}$ as small as $\approx 0.03$. For these values of $\kappa$, only a minor change in the total $t\bar{t}$ rate is expected (see Fig. 37). The effect of a non-zero $\kappa_{tt}^0 h_{tt}^0$ was analysed, in particular, in [204, 207, 172]. It was shown in [204] that information on $\kappa_{tt}^0 h_{tt}^0$ could be obtained by studying the following correlation observables between $\ell^+\ell^-$ lepton pairs produced in $t\bar{t}$ in di-lepton decays:

$$T_{33} = 2(p_T - p_e)_3(p_T \times p_e)_3,$$

$$A_E = E_{\ell^+} - E_{\ell^-},$$

$$Q_{53} = 2(p_T + p_e)_3(p_T - p_e)_3 - \frac{2}{3}(p_T^2 - p_e^2).$$

Table 14 shows the $1\sigma$ sensitivities of these correlations to $Re(d_t)$ and $Im(d_t)$ (where, $d_t = g_s^2 \kappa_{tt}^0 h_{tt}^0$).

Table 14: Attainable $1\sigma$ limits on $Re(d_t)$ and $Im(d_t)$, through $T_{33}$, $A_E$ and $Q_{33}$ for one year of the LHC running at low luminosity ($10\, \text{fb}^{-1}$) [204].

<table>
<thead>
<tr>
<th>Observable</th>
<th>Attainable $1\sigma$ limits</th>
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<tbody>
<tr>
<td>$T_{33}$</td>
<td>$</td>
</tr>
<tr>
<td>$A_E$</td>
<td>$</td>
</tr>
<tr>
<td>$Q_{33}$</td>
<td>$</td>
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Quantitatively, $T_{33}$ and $Q_{33}$ enable us to probe $Re(d_t)$ and $Im(d_t)$ of the order of $10^{-17} g_s\text{cm}$, respectively, and $A_E$ allows us to probe $Im(d_t)$ down to the order of $10^{-18} g_s\text{cm}$ (see [204] for details).

### 7.2 Search for anomalous $Wtb$ couplings

The $Wtb$ vertex structure can be probed and measured using either top pair or single top production processes. The total $t\bar{t}$ rate depends very weakly on the $Wtb$ vertex structure, as top quarks are dom-
In the model independent effective Lagrangian approach [193, 194, 195] there are four independent form factors describing the $Wtb$ vertex (see [195] for details). The effective Lagrangian in the unitary gauge [211, 208, 135] is given in (38), (39). As already mentioned the $(V - A)$ coupling in the SM carries the CKM matrix element $V_{tb}$ which is very close to unity. The value of a $(V + A)$ coupling is already bounded by the CLEO $b \rightarrow s \gamma$ data [212, 213] at a level [195, 213] such that it will be out of reach even at the high energy $\gamma e$ colliders. Since we are looking for small deviations from the SM, in the following $v_{tb}^W$ and $\alpha_{tb}^W$ will be set to $v_{tb}^W = \alpha_{tb}^W = \frac{1}{2}$ and an analysis is presented only for the two ‘magnetic’ anomalous couplings $F_{L2} = \frac{2M_W}{\lambda} \kappa_{tb}^W (-f_{tb}^W - i\lambda_{tb}^W)$, $F_{R2} = \frac{2M_W}{\lambda} \kappa_{tb}^W (-f_{tb}^W + i\lambda_{tb}^W)$. Natural values for the couplings $|F_{L(R)2}|$ are in the region of $\sqrt{\frac{M_W}{m_t}} \sim 0.1$ [196] and do not exceed the unitarity violation bounds for $|F_{L(R)2}| \sim 0.6$ [194].

Calculations of the complete set of diagrams for the two main processes $pp \rightarrow b\bar{b}W$ and $pp \rightarrow b\bar{b}W + \text{jet}$ have been performed [135] for the effective Lagrangian in (38), (39), using the package CompHEP [214]. The calculation includes the single-top signal and the irreducible backgrounds. Appropriate observables and optimal cuts to enhance the single-top signal have been identified through an analysis of singularities of Feynman diagrams and explicit calculations. The known NLO corrections to the single top rate [126, 125] have been included, as well as a simple jet energy smearing. The upper part of Fig. 38 presents the resulting $2 \sigma$ exclusion contour for an integrated luminosity of 100 fb\(^{-1}\), assuming $e, \mu$ and $\tau \rightarrow \ell$ decays of the $W$-boson. The combined selection efficiency in the kinematical region of interest, including the double $b$-tagging, is assumed to be 50%. Figure 38 demonstrates that it will be essential to measure both processes $pp \rightarrow b\bar{b}W$ and $pp \rightarrow b\bar{b}W + \text{jet}$ at the LHC. The allowed region for each single process is a rather large annuli, but the overlapping region is much smaller and allows an improvement of the sensitivity on anomalous couplings of an order of magnitude with respect to the Tevatron. Since the production rate is large, even after strong cuts, expected statistical errors are rather small, and the systematic uncertainties (from luminosity measurements, parton distribution functions, QCD scales, $m_t$, ...) will play an important role. As it is not possible to predict them accurately before the LHC startup,
Fig. 38: Limits on anomalous couplings after optimised cuts from two processes $pp \rightarrow b\bar{b}W$ and $pp \rightarrow b\bar{b}W + \text{jet}$ (upper plot). Dependence of the combined limits on the values of systematic uncertainties (lower plot).

we show here how the results depend on the assumed combined systematic uncertainty. Figure 38 (lower part) shows how the exclusion contours deteriorate when systematic errors of 1% and 5% are included. Note that a systematic error of 10% at the LHC will diminish the sensitivity significantly and the allowed regions will be comparable to those expected at the upgraded Tevatron.

The rate of single top production at LHC is different from the rate of single anti-top production. This asymmetry provides an additional observable at LHC that is not available at the Tevatron and which allows to reduce systematic uncertainties.

The potential of the hadron colliders can be compared to the potential of a next generation $e^+e^-$ linear collider (LC) where the best sensitivity could be obtained in high energy $\gamma e$-collisions [208, 215]. The results of this comparison are shown in Table 15. From the table we see that the upgraded Tevatron will be able to perform the first direct measurements of the structure of the $Wtb$ coupling. The LHC with 5% systematic uncertainties will improve the Tevatron limits considerably, rivalling with the reach of a high-luminosity ($500 \text{ fb}^{-1}$) 500 GeV LC option. The very high energy LC with 500 fb$^{-1}$ luminosity will eventually improve the LHC limits by a factor of three to eight, depending on the coupling under consideration.
Table 15: Uncorrelated limits on anomalous couplings from measurements at different machines.

<table>
<thead>
<tr>
<th></th>
<th>$F_{L2}$</th>
<th>$F_{R2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tevatron ($\Delta_{\text{sys}} \approx 10%$)</td>
<td>$-0.18 \ldots +0.55$</td>
<td>$-0.24 \ldots +0.25$</td>
</tr>
<tr>
<td>LHC ($\Delta_{\text{sys}} \approx 5%$)</td>
<td>$-0.052 \ldots +0.097$</td>
<td>$-0.12 \ldots +0.13$</td>
</tr>
<tr>
<td>$\gamma e$ ($\sqrt{s_{e^+e^-}} = 0.5\text{ TeV}$)</td>
<td>$-0.1 \ldots +0.1$</td>
<td>$-0.1 \ldots +0.1$</td>
</tr>
<tr>
<td>$\gamma e$ ($\sqrt{s_{e^+e^-}} = 2.0\text{ TeV}$)</td>
<td>$-0.008 \ldots +0.035$</td>
<td>$-0.016 \ldots +0.016$</td>
</tr>
</tbody>
</table>

7.3 FCNC in top quark physics

In the previous subsections, we analysed top quark anomalous couplings as small deviations from the ordinary SM interactions ($gl\bar{t}$ and $tWb$ vertices). Here we consider new processes which are absent at tree-level and highly suppressed in the SM, namely the FCNC couplings $tVc$ and $tVu$ ($V = g, \gamma, Z$).

The SM predicts very small rates for such processes [216] (see Table 16). The top quark plays therefore a unique rôle compared to the other quarks, for which the expected FCNC transitions are much larger: the observation of a top quark FCNC interaction would signal the existence of new physics. As an illustration, Table 16 shows predictions for the top quark decay branching ratios evaluated in the two-Higgs doublet model [217], the SUSY models [218], and the SM extension with exotic (vector-like) quarks [198].

Table 16: Branching ratios for FCNC top quark decays as predicted within the SM and in three SM extensions.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$B(t \rightarrow qg)$</td>
<td>$5 \times 10^{-11}$</td>
<td>$\sim 10^{-5}$</td>
<td>$\sim 10^{-4}$</td>
<td>$\sim 5 \times 10^{-4}$</td>
</tr>
<tr>
<td>$B(t \rightarrow q\gamma)$</td>
<td>$5 \times 10^{-13}$</td>
<td>$\sim 10^{-7}$</td>
<td>$\sim 10^{-5}$</td>
<td>$\sim 10^{-5}$</td>
</tr>
<tr>
<td>$B(t \rightarrow qZ)$</td>
<td>$\sim 10^{-13}$</td>
<td>$\sim 10^{-6}$</td>
<td>$\sim 10^{-4}$</td>
<td>$\sim 10^{-2}$</td>
</tr>
</tbody>
</table>

In the effective Lagrangian description of (38), (39) it is straightforward to calculate the top quark decay rates as a function of the top quark FCNC couplings:

$$\Gamma(t \rightarrow qg) = \left(\frac{\kappa_{tq}^g}{\Lambda}\right)^2 \frac{8}{3} \alpha_s m_t^3 \ , \quad \Gamma(t \rightarrow q\gamma) = \left(\frac{\kappa_{tq}^\gamma}{\Lambda}\right)^2 2\alpha m_t^3 ,$$  

(41)

$$\Gamma(t \rightarrow qZ)_\gamma = \left(|v_{tq}^Z|^2 + |a_{tq}^Z|^2\right) \alpha m_t^3 \frac{1}{4M_Z^2 \sin^2 2\theta_W} \left(1 - \frac{M_Z^2}{m_t^2}\right)^2 \left(1 + 2\frac{M_Z^2}{m_t^2}\right) ,$$  

(42)

$$\Gamma(t \rightarrow qZ)_\sigma = \left(\frac{\kappa_{tq}^Z}{\Lambda}\right)^2 \alpha m_t^3 \frac{1}{\sin^2 2\theta_W} \left(1 - \frac{M_Z^2}{m_t^2}\right)^2 \left(2 + \frac{M_Z^2}{m_t^2}\right) .$$  

(43)

For comparison, Table 17 collects the rare top decay rates normalised to $\kappa_{tq}^g = \kappa_{tq}^\gamma = |v_{tq}^Z|^2 + |a_{tq}^Z|^2 = \kappa_{tq}^Z = 1$, and for the SM. We assume $m_t = 175\text{ GeV}$, $\Lambda = 1\text{ TeV}$, $\alpha = \frac{1}{128}$, $\alpha_s = 0.1$ and sum the decays into $q = u, c$. In this ‘extreme’ case with the anomalous couplings equal to one the top can decay into a gluon or a $Z$ boson plus a light quark $q = u, c$ and into the SM mode $b\bar{W}$ at similar rates.

7.3.1 Current Constraints on FCNC in top quark physics

Present constraints on top anomalous couplings are derived from low-energy data, direct searches of top rare decays, deviations from the SM prediction for $t\bar{t}$ production and searches for single top production at LEP2.

**Indirect constraints:** The top anomalous couplings are constrained by the experimental upper bounds on the induced FCNC couplings between light fermions. For example, the $\gamma^u$ term in the $Ztq$...
The CDF collaboration has searched for the decays $J/\psi \to \gamma c(u)$ and $J/\psi \to Z c(u)$ in the reaction $p\bar{p} \to t\bar{t} X$ at $\sqrt{s} = 1.8$ TeV, obtaining the following 95\% CL limits [13]:

$$
\text{BR}(t\to c\gamma) < 3.2\%, \quad \text{BR}(t\to u\gamma) < 3.2\%,
$$

\[
\text{BR}(t\to cZ) + \text{BR}(t\to uZ) < 33\%.
\]

These translate into the bounds on the top anomalous couplings

$$
\kappa^\gamma_{tt} < 0.78, \quad \sqrt{|v_{tt}^Z|^2 + |v_{tt}^Z|^2} < 0.73.
$$

**CDF results**: The CDF collaboration has searched for the decays $t\to \gamma c(u)$ and $t\to Z c(u)$ in the reaction $p\bar{p} \to t\bar{t} X$ at $\sqrt{s} = 1.8$ TeV, obtaining the following 95\% CL limits [13]:

$$
\text{BR}(t\to c\gamma) < 3.2\%, \quad \text{BR}(t\to u\gamma) < 3.2\%,
$$

\[
\text{BR}(t\to cZ) + \text{BR}(t\to uZ) < 33\%.
\]

These translate into the bounds on the top anomalous couplings

$$
\kappa^\gamma_{tt} < 0.78, \quad \sqrt{|v_{tt}^Z|^2 + |v_{tt}^Z|^2} < 0.73.
$$

**t\bar{t} production via FCNC**: Constraints on the vertex $glq$ can be derived from the study of the $t\bar{t}$-pair production cross-section. Imposing that the $t\bar{t}$-pair production cross-section, including the possible effect of anomalous couplings, should not differ from the observed one (assumed in this study to be $\sigma_{t\bar{t}}^{\exp} = 6.7 \pm 1.3$ pb [6]) by more than 2 pb, leads to the constraint [220]:

$$
\frac{\kappa^g_{t\bar{t}}}{\Lambda} \leq 0.47 \text{ TeV}^{-1}.
$$

### Table 17: Top quark decay widths and corresponding branching ratios for the anomalous couplings equal to one and for the SM.

<table>
<thead>
<tr>
<th>FCNC coupling</th>
<th>$W^+b$</th>
<th>$(c + u)\gamma$</th>
<th>$(c + u)Z, (c + u)Z_\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{SM}$ (GeV)</td>
<td>1.56</td>
<td>2.86</td>
<td>0.17</td>
</tr>
<tr>
<td>B</td>
<td>0.20</td>
<td>0.37</td>
<td>0.022</td>
</tr>
<tr>
<td>Constraints on the vertex $\mathbf{g}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Gamma_{SM}$ (GeV)</td>
<td>1.56</td>
<td>$8 \times 10^{-10}$</td>
<td>$3 \times 10^{-6}$</td>
</tr>
<tr>
<td>$B_{SM}$</td>
<td>1</td>
<td>$5 \times 10^{-11}$</td>
<td>$5 \times 10^{-13}$</td>
</tr>
</tbody>
</table>
Table 18: Short summary of the LEP 2 results for $e^+e^- \rightarrow t\bar{q}$.

<table>
<thead>
<tr>
<th>Collab.</th>
<th>$\sqrt{s}$ (GeV)</th>
<th>$L$(pb$^{-1}$)</th>
<th>BR($t \rightarrow Zq$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>189-202 GeV [221]</td>
<td>411</td>
<td>&lt; 22%</td>
</tr>
<tr>
<td>DELPHI</td>
<td>192-202 GeV [223]</td>
<td>233</td>
<td>&lt; 18%</td>
</tr>
</tbody>
</table>

$\text{Br}(t \rightarrow Zq)$, $q=c+u$, DELPHI PRELIMINARY

![Graph of $\text{Br}(t \rightarrow Zq)$ vs $M_t$](image)

Fig. 39: Upper limit on the branching fraction of $t\rightarrow Zq$ obtained from LEP 2 data, evaluated at $\kappa_{tq} = 0$.

**FCNC at LEP 2:** Since 1997, LEP2 has run at cms energies in excess of 180 GeV, making the production of single top quark kinematically possible through the reaction:

$$e^+e^- \rightarrow t\bar{q}, \quad E_{cm} = 192 - 202 \text{ GeV}$$

Two LEP experiments [221, 222, 223] have presented the results of their search for this process. A short summary of these data is given in Table 18. The production cross section is very sensitive to the top quark mass, $\sigma_{tq} \sim (1 - \frac{m_t^2}{m^2})^2$ (see [224] for details). Therefore, the upper limit on the corresponding branching ratio depends from the exact value of $m_t$ as well, as shown in Fig.39. The current constraints on the top quark FCNC processes are summarised in Table 19. Note that the LEP2 limit is slightly better then that given by CDF (49). These constraints should further improve once the data from the highest-energy runs are analysed.

### 7.4 Search for FCNC in top quark production processes

FCNC interactions of top quarks will be probed through anomalous top decays (as discussed in Section 8.), and through anomalous production rates or channels, as discussed in the remainder of this section.
Table 19: Current constraints on top quark FCNC interactions.

<table>
<thead>
<tr>
<th>Process</th>
<th>BR (%)</th>
<th>Constraint</th>
<th>Other FCNC couplings zero</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t \to g q$</td>
<td>$&lt; 17%$</td>
<td>$\kappa_{tgq}^0 &lt; 0.47$</td>
<td></td>
</tr>
<tr>
<td>$t \to \gamma q$</td>
<td>$&lt; 3.2%$</td>
<td>$\kappa_{tgq}^0 &lt; 0.78$</td>
<td></td>
</tr>
<tr>
<td>$t \to Z q$</td>
<td>$&lt; 18%$</td>
<td>$\sqrt{</td>
<td>\kappa_{tgq}^0</td>
</tr>
</tbody>
</table>

Table 20: Upper bounds on the anomalous couplings $\kappa_{tu}$ and $\kappa_{tc}$ from single top production processes. The symbols 2→1 and 2→2 correspond to the reactions quark-gluon fusion, and single top production, respectively [225, 142].

<table>
<thead>
<tr>
<th></th>
<th>Tevatron</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Run 1</td>
<td>Run 2</td>
</tr>
<tr>
<td>$\sqrt{s}$ (TeV)</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td>$\mathcal{L}$ (fb$^{-1}$)</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$\kappa_{tu}^0$ (2→1)</td>
<td>0.058</td>
<td>0.019</td>
</tr>
<tr>
<td>$\kappa_{tu}^0$ (2→2)</td>
<td>0.082</td>
<td>0.026</td>
</tr>
<tr>
<td>$\kappa_{tc}^0$ (2→1)</td>
<td>0.22</td>
<td>0.062</td>
</tr>
<tr>
<td>$\kappa_{tc}^0$ (2→2)</td>
<td>0.31</td>
<td>0.092</td>
</tr>
</tbody>
</table>

7.41 Deviations from SM expectations for $t \bar{t}$ production

As shown in the previous subsection, the FCNC $tgq$-vertex contributes to $gg \to t \bar{t}$ transitions, and to a possible enhancement of the top quark production at large $E_t$ and $M_{t\bar{t}}$. A recent study [220] shows that at the LHC the sensitivity to these couplings is equivalent to that found with the data of Run 1 at the Tevatron:

$$\left( \frac{\kappa_{tgq}^0}{A} \right)_{\text{LHC}} \simeq \left( \frac{\kappa_{tgq}^0}{A} \right)_{\text{FNAL}} \simeq 0.5 \text{ TeV}^{-1}. \quad (53)$$

7.42 ‘Direct’ top quark production (2→1)

The ‘quark–gluon’ fusion process [225] $g + u(c) \to t$ is characterised by the largest cross-section for top quark production through FCNC-interactions assuming equal anomalous couplings. At the LHC, using the CTEQ2L structure functions [115], these cross sections for $\frac{\kappa_{tgq}^0}{A} = 1 \text{ TeV}^{-1}$ are equal to:

$$\sigma(u g \to t) \simeq 4 \times 10^4 \text{ pb} \quad , \quad \sigma(\bar{u} g \to t) \simeq 1 \times 10^4 \text{ pb} \quad , \quad \sigma(c g \to t) \simeq 6 \times 10^3 \text{ pb}. \quad (54)$$

Note that $\sigma(u g \to t)$ is about 50 times larger than the SM $t \bar{t}$ cross section. The major source of background to this is the $W^+$ jet production. The additional background due to single top production, when the associated jets are not observed, should not exceed 20% of the total background and was therefore ignored. To reproduce the experimental conditions, a Gaussian smearing of the energy of the final leptons and quarks was applied (see [225] for details). Cuts on the transverse momentum ($p_T > 25$ GeV), pseudo-rapidity ($|\eta| < 2.0$, $|\eta| < 3.0$), and lepton-jet separation ($\Delta R \geq 0.4$) were applied. A $b$-tagging efficiency of 60% and a mistagging probability of 1% were assumed.

The criterion $S/\sqrt{S + B} \geq 3$ was used to determine the minimum values of anomalous couplings. The couplings $tg$ and $tg$ have been considered separately. The resulting constraints on $\kappa_{tu}$ and $\kappa_{tc}$ are given in Table 20, which also contains the results of an analysis done for the Tevatron.

7.43 Single top quark production (2→2)

Single top quark production in 2→2 processes has been studied as well [142]. There are four different subprocesses, which lead to one top quark in the final state together with one associated jet (see Fig. 40
Fig. 40: 2→2 single top quark production.

and [142] for detailed considerations:

\[ q\bar{q} \rightarrow t\bar{q}, \quad gg \rightarrow t\bar{q}, \quad qq \rightarrow tq, \quad gg \rightarrow tg \]  

(55)

The major background comes from \( W + 2 \) jets and \( W + b\bar{b} \) production, as well as from single top production. In addition to the cuts and tagging rates used in the above analysis of ‘direct’ top production, additional cuts on the reconstructed top mass (145 GeV < \( M_{tW} < 205 \) GeV), on \( p_T > 35 \) GeV, and on jet-jet and lepton-jet separation (\( \Delta R_{jj} > 1.5, \Delta R_{ij} > 1.0 \)) were applied here to improve the signal/background separation. The corresponding limits on anomalous couplings in the top-gluon interaction with \( c \) or \( u \) quarks are given in Table 20.

7.44 \( tZ \) and \( t\gamma \) production

All the anomalous couplings may contribute to the processes \( q g \rightarrow tZ(\gamma) \), and were considered in [226, 227]. The left diagram in Fig. 41 corresponds to the \( Z(\gamma)tq \) coupling, while the right one shows the top-gluon anomalous coupling (the corresponding \( t \)-channel diagrams are not shown). For all the calculations presented here, the MRSA PDF set [228] with \( Q^2 = \hat{s} \) was used. The resulting total cross sections for \( \kappa_{tq}^2 = \sqrt{|v_{tq}^2|^2 + |a_{tq}^2|^2} = 1 \) are [227]:

\[
\begin{align*}
\sigma(u g \rightarrow t \gamma t) &= 73 \text{ pb}, \\
\sigma(c g \rightarrow t \gamma t) &= 10 \text{ pb}, \\
\sigma(u g \rightarrow Z t t) &= 746 \text{ pb}, \\
\sigma(c g \rightarrow Z t t) &= 114 \text{ pb}.
\end{align*}
\]

Different background sources (\( W + \) jets, \( Z + \) jets, \( ZW + \) jets, \( Wb\bar{b} + \) jets, \( tt \), and \( Wt \) production) were considered. The experimental conditions were simulated by a Gaussian smearing of the lepton, photon and jet energies (see [227] for details). Cuts on the transverse momenta, \( p_T(\ell, j, \gamma) > (15, 20, 40) \) GeV, on pseudo-rapidities, \( |\eta_{jj,\ell,\gamma}| < 2.5 \), and on lepton-jet-photon separation (\( \Delta R \geq 0.4 \)) were applied. A \( b \)-tagging efficiency of 60% and a mistagging probability of 1% were assumed. It was found that \( b \)-tagging plays an essential role in tracing the top quark and reducing backgrounds.

It has been shown that the best limits on the top quark FCNC couplings can be obtained from the decay channels \( Zt \rightarrow \ell^+ \ell^- \ell \nu \) and \( \gamma t \rightarrow \ell \nu \) (see [226] and [227] for details). Upper bounds at 95% CL are derived using the FC statistics [200]. Table 21 collects the corresponding limits on eight top anomalous couplings. Like in previous cases the bounds on \( u \) and \( c \) couplings were obtained under the assumption that only one anomalous coupling at a time is non-zero. The analysis was done for both Tevatron and LHC but with different optimized cuts.

7.45 Like-sign \( tt \) (\( \bar{t}\bar{t} \)) pair production

Additional evidence for a FCNC \( glq \) coupling can be sought through the production of like-sign top pairs (see Fig. 42).

\[
p p \rightarrow ttX, \quad p p \rightarrow \bar{t}\bar{t}X
\]

(56)
Table 21: Upper bounds on top anomalous couplings (see (38,39)) from $Zt$ and $\gamma t$ production. We have corrected for the different normalizations used in [226, 227].

<table>
<thead>
<tr>
<th>$\sqrt{s}$ (TeV)</th>
<th>Tevatron</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Run 1</td>
<td>Run 2</td>
</tr>
<tr>
<td>$\mathcal{L}$ (fb$^{-1}$)</td>
<td>1.8</td>
<td>2.0</td>
</tr>
<tr>
<td>$\kappa_{tu}$</td>
<td>0.31</td>
<td>0.057</td>
</tr>
<tr>
<td>$\kappa_{tc}$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\kappa_{\gamma}$</td>
<td>0.86</td>
<td>0.18</td>
</tr>
<tr>
<td>$\kappa_{\gamma}$</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>$\sqrt{</td>
<td>v_{Z_{tu}}^2</td>
<td>+</td>
</tr>
<tr>
<td>$\sqrt{</td>
<td>v_{Z_{tc}}^2</td>
<td>+</td>
</tr>
<tr>
<td>$\kappa_{Z_{tu}}$</td>
<td>1.71</td>
<td>0.43</td>
</tr>
<tr>
<td>$\kappa_{Z_{tc}}$</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

Fig. 42: Diagram describing like-sign top quark pair production

The ATLAS collaboration performed a detailed investigation of this reaction for the case of high luminosity, $\mathcal{L}_{\text{int}} = 100$ fb$^{-1}$ (see [30] and [220] for details). All the three anomalous couplings contribute to this process and the kinematics of the $tt$-pair is almost the same as for the conventional $t\bar{t}$-pair production.

An experimentally clean signature of $tt$ ($t\bar{t}$) production is the production of like-sign high $p_T$ leptons plus two hard $b$-jets. The main sources of background are $qq'\rightarrow t\bar{t}W$ and $qq\rightarrow W^\pm q'W^\pm q'$. The expected cross sections for the signal (with $\kappa_{tiq}^s = \kappa_{tiq}^\gamma = |v_{Z_{tiq}}^2| + |a_{Z_{tiq}}^2| = 1$) and background processes are equal to:

$\sigma(tt) = 1920$ pb,
$\sigma(W^+t\bar{t}) = 0.5$ pb,
$\sigma(W^+q\bar{q}) = 0.5$ pb,
$\sigma(\square) = 64$ pb,
$\sigma(W^-t\bar{t}) = 0.24$ pb,
$\sigma(W^-q\bar{q}) = 0.23$ pb.

CTEQ2L structure functions [115] were used with the evolution parameter $Q^2 = m_t^2$ for the signal and $Q^2 = m_W^2$ for the background calculations. PYTHIA 5.7 [52] was used for the fragmentation and all events were passed through the ATLAS detector simulation. An additional reducible like-sign di-lepton background is due to $t\bar{t}$ events with a $b$ semi-leptonic decay. The initial selection required therefore two like-sign isolated leptons with $p_T > 15$ GeV and $|\eta| < 2.5$ as well as at least two jets with $p_T > 20$ GeV and $|\eta| < 2.5$. In order to get a better signal/background separation jets with $p_T > 40$ GeV (with at least one tagged as a $b$-jet) were required (see [30, 220] for other cuts). The potential reach of this study, using the $S/\sqrt{S+B} \geq 3$ criterion, is given in Table 22.

7.5 Conclusion on $tqV$ anomalous couplings

Table 23 presents a short summary of LHC sensitivities to anomalous FCNC couplings of the top quark. For comparison, we present also the estimates of the corresponding sensitivities at Tevatron. For compare-
Table 22: The limits on anomalous couplings from an improved ATLAS analysis [30, 220] of like-sign top-pair production at the LHC for the case of high luminosity, \( \mathcal{L}_{\text{int}} = 100 \, \text{fb}^{-1} \). The contribution from the \( \sigma^{\mu\nu} \) term in the \( Ztq \) vertex is ignored.

| \( \kappa_{\Sigma}^{\tau} \) | \( \kappa_{\Sigma}^{\ell} \) | \( \kappa_{\Sigma}^{\gamma} \) | \( \sqrt{\left| v_{\Sigma}^{\tau} \right|^2 + \left| a_{\Sigma}^{\tau} \right|^2} \) | \( \sqrt{\left| v_{\Sigma}^{\ell} \right|^2 + \left| a_{\Sigma}^{\ell} \right|^2} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| 0.078           | 0.25            | 0.14            | 0.32            | 0.27            | 0.85            |

Table 23: Summary of the LHC sensitivity to the top quark anomalous couplings \( \kappa_{\Sigma}^{\tau}, \kappa_{\Sigma}^{\ell}, \kappa_{\Sigma}^{\gamma} \) and \( \sqrt{\left| v_{\Sigma}^{\tau} \right|^2 + \left| a_{\Sigma}^{\tau} \right|^2} \). The resulting constraints are presented in terms of ‘branching ratio’, \( \Gamma(t\rightarrow qV)/\Gamma_{SM} = 1.56 \, \text{GeV} \). The results for the Tevatron option are also given (see text for explanation). 2\( \rightarrow \)1, 2\( \rightarrow \)2, \( tV \), and \( tt \) stand for quark-gluon fusion, single top production, \( t + \gamma(Z) \) production, and like-sign top-pair final states, respectively. The ‘decay’, ‘ATLAS’, and ‘CMS’ labels denote the results obtained from the study of top decay channels, documented in Section 8.

<table>
<thead>
<tr>
<th>( \sqrt{s} ) (TeV)</th>
<th>( \mathcal{L} ) (fb(^{-1}))</th>
<th>Tevatron</th>
<th>LHC</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>0.1</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>14</td>
<td>10</td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

| \( t\alpha \gamma \) | \( 7.9 \times 10^{-2} \) | \( 3.5 \times 10^{-3} \) | \( 3.8 \times 10^{-3} \) | \( 6.5 \times 10^{-3} \) | \( 6.5 \times 10^{-6} \) | \( 3.9 \times 10^{-6} \) | \( 4.8 \times 10^{-5} \) | \( 4.8 \times 10^{-5} \) |
| \( t\alpha \) | \( 4.4 \times 10^{-2} \) | \( 3.5 \times 10^{-3} \) | \( 3.0 \times 10^{-3} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) |
| \( t\gamma \) | \( 7.9 \times 10^{-2} \) | \( 3.5 \times 10^{-3} \) | \( 3.0 \times 10^{-3} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) |
| \( t\gamma \) | \( 7.9 \times 10^{-2} \) | \( 3.5 \times 10^{-3} \) | \( 3.0 \times 10^{-3} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) |
| \( t\alpha Z \) | \( 4.5 \times 10^{-1} \) | \( 3.2 \times 10^{-2} \) | \( 5.2 \times 10^{-3} \) | \( 4.8 \times 10^{-4} \) | \( 5.8 \times 10^{-4} \) | \( 5.8 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) |
| \( t\gamma Z \) | \( 4.5 \times 10^{-1} \) | \( 3.2 \times 10^{-2} \) | \( 5.2 \times 10^{-3} \) | \( 4.8 \times 10^{-4} \) | \( 5.8 \times 10^{-4} \) | \( 5.8 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) | \( 1.9 \times 10^{-4} \) |
| \( t\gamma Z \) | \( 4.5 \times 10^{-1} \) | \( 3.2 \times 10^{-2} \) | \( 5.2 \times 10^{-3} \) | \( 1.9 \times 10^{-3} \) | \( 1.9 \times 10^{-3} \) | \( 1.9 \times 10^{-3} \) | \( 1.9 \times 10^{-3} \) | \( 1.9 \times 10^{-3} \) |
pleteness we anticipate and include here the results from rare decays discussed in the next section (see also [219, 229]). To unify the description of the LHC potential to detect top anomalous couplings from production and decay processes, all results in Table 23 are expressed in terms of limits on top decay branching ratios: \( \Gamma(t\rightarrow qV)/\Gamma_{SM} \approx 1.56 \text{ GeV} \). The results were obtained using \( m_t = 175 \text{ GeV} \), \( \alpha_s = 0.1 \), and \( \alpha = 1/128 \). When needed the limits quoted in the table have been rescaled to the different luminosities and to the \( S/\sqrt{S+B} \geq 3 \) criterion by using a simple linear extrapolation of the available bounds (see [30, 230] and Section 8.). The limits on the top anomalous couplings from \( tV \) production in Table 21 were obtained using the FC prescription [200] and have been multiplied by a factor of \( \sqrt{2} \), which roughly relates this prescription with the statistical criterion adopted in Table 23 [226, 227].

At present, only few cases (like-sign top-pair production, \( t\rightarrow qZ \) and \( t\rightarrow q\gamma \) decays, see [30, 230]) were investigated with a more or less realistic detector simulation (ATLFAST and CMSJET). Other investigations were done at the parton level (the final quarks were considered as jets and a simple smearing of lepton, jet and photon energies was applied). Of course, more detailed investigations with a more realistic simulation of the detector response may change these results.

The most promising way to measure the anomalous FCNC top-gluon coupling seems to be the investigation of single top production processes, as the search for \( t\rightarrow gq \) decays would be overwhelmed by background from QCD multi-jet events. At the same time, both top quark production and decay would provide comparable limits on top quark anomalous FCNC interactions with a photon or a Z-boson. In general, the studies shown above indicate that the LHC will improve by a factor of at least 10 the Tevatron sensitivity to top quark FCNC couplings. Of course, the results presented here are not complete, since other new kinds of interactions may lead to the appearance of unusual properties of the top quark. For example, recently proposed theories with large extra-dimensions predict a significant modification of \( t\bar{t} \) pair production (see, for example, [231] and references therein). It was found that the exchange of spin-2 Kaluza-Klein gravitons leads to a modification of the total \( t\bar{t} \) production rate as well as to a noticeable deviation in the \( p_T \) and \( M_T \) distributions with respect to the SM predictions. Naturally, we may expect also the modifications of spin-spin correlations due to graviton exchange.

It has to be stressed that different types of new interactions may affect the same observable quantity. Only a careful investigation of different aspects of top quark physics may provide a partial separation of these interactions.

8. RARE DECAYS OF THE TOP QUARK

The production of \( 10^7 - 10^8 \) top quark pairs per year at LHC will allow to probe the top couplings to both known and new particles involved in possible top decay channels different from the main \( t\rightarrow bW \). Thanks to the large top mass, there are several decays that can be considered, even involving the presence of on-shell heavy vector bosons or heavy new particles in the final states. On a purely statistical basis, one should be able to detect a particular decay channel whenever its branching ratio (BR) is larger than about \( 10^{-6} - 10^{-7} \). In practice, we will see that background problems and systematics will lower this potential by a few orders of magnitude, the precise reduction being dependent of course on the particular signature considered. We will see, that the final detection threshold for each channel will not allow the study of many possible final states predicted in the SM, unless new stronger couplings come into play.

8.1 Standard Model top decays

In this section, we give an overview of the decay channels of the top quark in the framework of the SM. In the SM the decay \( t\rightarrow bW \) is by far the dominant one. The corresponding width has been discussed in Section 2.1. The rates for other decay channels are predicted to be smaller by several orders of magnitude in the SM. The second most likely decays are the Cabibbo-Kobajashi-Maskawa (CKM) non-diagonal
decays $t \to sW$ and $t \to dW$. Assuming $|V_{ts}| \simeq 0.04$ and $|V_{td}| \simeq 0.01$, respectively [192], one gets

$$\text{BR}(t \to sW) \sim 1.6 \times 10^{-3} \quad \text{and} \quad \text{BR}(t \to dW) \sim 1 \times 10^{-4}$$

(57)
in the SM with three families. From now on, for a generic decay channel $X$, we define

$$\text{BR}(t \to X) = \frac{\Gamma(t \to X)}{\Gamma(t \to bW)_{\text{SM}}}.$$ 

(58)

The two-body tree-level decay channels are the only ones that the LHC could detect in the framework of the SM. With the exception of higher-order QED and QCD radiative decays, the next less rare processes have rates no larger than $10^{-6}$.

At tree level, the decay $t \to bWZ$ (Fig. 43) has some peculiar features, since the process occurs near the kinematical threshold ($m_t \sim M_W + M_Z + m_b$) [232, 233, 234, 235]. This fact makes the $W$ and $Z$ finite-width effects crucial in the theoretical prediction of the corresponding width [233]. Because the $W$ and $Z$ are unstable and not observed directly, more than one definition of the $t \to bWZ$ branching ratio is possible. If defined according to

$$\Gamma(t \to bWZ) = \frac{\Gamma(t \to b\mu \nu_\mu \nu_\tau \bar{\nu}_\tau)}{\text{BR}(W \to \mu \nu_\mu) \text{BR}(Z \to \nu_\tau \bar{\nu}_\tau)},$$

(59)

including a consistent treatment of $W$ and $Z$ width effects, the branching ratio is to a very good approximation given by the double resonant set of diagrams (shown in Fig. 43), since the background to the neutrino decay of the $Z$ is negligible. One obtains [235], for $m_t = 175$ GeV,

$$\text{BR}(t \to bWZ) = BR_{\text{res}}(t \to bWZ) = 2.1 \times 10^{-6}.$$ 

(60)

However, the signature $b\mu \nu_\mu \nu_\tau \bar{\nu}_\tau$ is not practical from an experimental point of view. In [233], a first estimate of $\text{BR}(t \to bWZ)$ was given on the basis of the definition

$$\Gamma(t \to bWZ) = \frac{\Gamma(t \to b\mu \nu_\mu e^+ e^-)}{\text{BR}(W \to \mu \nu_\mu) \text{BR}(Z \to e^+ e^-)}.$$ 

(61)
which involves experimentally well-observable decays, but includes contributions to the numerator from $t \rightarrow b W \gamma$ decays (with $\gamma \rightarrow e^+ e^-$) and other “background” diagrams. The estimate for the corresponding branching ratio is

$$BR_{cut}(t \rightarrow b W Z) \approx 6 \times 10^{-7},$$

(62)

for $m_t = 175$ GeV, assuming a minimum cut of $0.8M_Z$ on the $e^+ e^-$-pair invariant mass. This cut tries to cope with the contribution of background graphs where the $e^+ e^-$ pair comes not from a $Z$ boson but from a photon.

If the Higgs boson is light enough, one could also have the decay $t \rightarrow b W H$ (Fig. 43), although the present limits on $m_H$ strongly suppress its rate. For $m_H \gtrsim 100$ GeV, one gets [233]:

$$BR(t \rightarrow b W H) \lesssim 7 \times 10^{-8}.$$ 

(63)

Finally, the decay $t \rightarrow c W W$ is very much suppressed by a GIM factor $m_c^2 M_W$ in the amplitude. One then gets [234]:

$$BR(t \rightarrow c W W) \sim 10^{-13}.$$ 

(64)

One can also consider the radiative three-body decays $t \rightarrow b W g$ and $t \rightarrow b W \gamma$. These channels suffer from infrared divergences and the evaluation of their rate requires a full detector simulation, including for instance the effects of the detector resolution and the jet isolation algorithm. In an idealised situation where the rate is computed in the $t$ rest frame with a minimum cut of 10 GeV on the gluon or photon energies, one finds [236]:

$$BR(t \rightarrow b W g) \simeq 0.3, \quad BR(t \rightarrow b W \gamma) \simeq 3.5 \times 10^{-3}.$$ 

(65)

The FCNC decays $t \rightarrow c g$, $t \rightarrow c \gamma$ and $t \rightarrow c Z$ occur at one loop, and are also GIM suppressed by a factor $m_c^2 M_W$ in the amplitude. Hence, the corresponding rates are very small [249]:

$$BR(t \rightarrow c g) \simeq 5 \times 10^{-11}, \quad BR(t \rightarrow c \gamma) \simeq 5 \times 10^{-13}, \quad BR(t \rightarrow c Z) \simeq 1.3 \times 10^{-13}$$ 

(66)

For a light Higgs boson, one can consider also the FCNC decay $t \rightarrow c H$. A previous evaluation of its rates [249] has now been corrected. For $m_H \simeq 100 (160)$ GeV, one gets [237]:

$$BR(t \rightarrow c H) \simeq 0.9 \times 10^{-13} \ (4 \times 10^{-15}).$$

(67)

To conclude the discussion of rare SM decays of the top quark, we point out here the existence of some studies on semi-exclusive $t$-quark decays where the interaction of quarks among the $t$ decay products may lead to final states with one hadron (meson) recoiling against a jet. In [238] decays with a $\Upsilon$ meson in the final state and decays of the top through an off-shell $W$ with virtual mass $M_W$, near to some resonance $M$, like $\pi^+, \rho^+, K^+, D_s^+$, were considered. An estimate for the latter case is

$$\Gamma(t \rightarrow b M) \approx \frac{G_F^2 m_t^3}{144\pi} f_M^2 |V_{tb}|^2.$$ 

(68)

The typical values of the corresponding branching ratios are too small to be measured:

$$BR(t \rightarrow b \pi) \sim 4 \times 10^{-8}, \quad BR(t \rightarrow b D_s) \sim 2 \times 10^{-7}.$$ 

(69)

In Table 24 we summarize the expected decay rates for the main top decay channels in the SM.

### 8.2 Beyond the Standard Model decays

The fact that a measurement of the top width is not available and that the branching ratio $BR(t \rightarrow b W)$ is a model dependent quantity makes the present experimental constraints on the top decays beyond the SM quite weak. Hence, the possibility of $t$ decays into new massive states with branching fraction of order $BR(t \rightarrow b W)$ is not excluded. Apart from the production of new final states with large branching fractions, we will see that new physics could also give rise to a considerable increase in the rates of many decay channels that in the SM framework are below the threshold of observability at the LHC.
8.21 4th fermion family

Extending the SM with a 4th fermion family can alter considerably a few $t$ decay channels. First of all, when adding a 4th family to the CKM matrix the present constraints on the $|V_{ts}|$ elements are considerably relaxed. In particular, $|V_{ts}|$ and $|V_{td}|$ can grow up to about 0.5 and 0.1, respectively [192]. Correspondingly, assuming $|V_{tb}| \sim 1$ for the sake of normalisation, one can have up to:

$$\text{BR}_4(t \to sW) \sim 0.25 \quad \text{and} \quad \text{BR}_4(t \to dW) \sim 0.01,$$

(70)
to be confronted with the SM expectations in (57).

The presence of a 4th fermion family could also show up in the $t$ direct decay into a heavy $b'$ quark with a relatively small mass ($m_{b'} \sim 100 \text{ GeV}$) [239]. This channel would contribute to the $t \to cW$ decay, with a rate:

$$\text{BR}(t \to W^+ b' (\to W^c)) \sim 10^{-5} \left(10^{-7}\right) \quad \text{at} \quad m_{b'} = 100 \left(300\right) \text{ GeV},$$

(71)
to be confronted with the SM prediction in (64).

8.22 Two Higgs Doublet models (2HDM's)

The possibility that the EW symmetry breaking involves more than one Higgs doublet is well motivated theoretically. In particular, three classes of two Higgs doublet models have been examined in connection with rare top decays, called model I, II and III. The first two are characterised by an ad hoc discrete symmetry which forbids tree-level FCNC [240], that are strongly constrained in the lightest quark sector.

In model I and model II, the up-type quarks and down-type quarks couple to the same scalar doublet and to two different doublets, respectively (the Higgs sector of the MSSM is an example of model II).

In model III [241, 242], the above discrete symmetry is dropped and tree-level FCNC are allowed. In particular, a tree-level coupling $tcH$ is predicted with a coupling constant $\lesssim \sqrt{m_t m_c/v}$ (where $v$ is the Higgs vacuum expectation value).

Since enlarging the Higgs sector automatically implies the presence of charged Higgs bosons in the spectrum, one major prediction of these new frameworks is the decay $t \to b H^+$, possibly with rates competitive with $\text{BR}(t \to bW)$ for $m_{H^+} \lesssim 170 \text{ GeV}$. In the MSSM, one expects $\text{BR}(t \to b H^+) \sim 1$, both at small and large values of $\tan \beta$. The interaction Lagrangian describing the $H^+ t \bar{b}$-vertex in the MSSM is [243]:

$$\mathcal{L}_{Htb} = \frac{g}{\sqrt{2} M_W} H^+ [\bar{t} (m_t \cot \beta P_L + m_b \tan \beta P_R) b + \bar{b} (m_t \cot \beta P_R) \bar{t}] + \text{h.c.},$$

(72)

where $P_{L,R} = 1/2 (1 \mp \gamma_5)$ are the chiral projector operators.

At tree level the corresponding decay widths of $t \to b H^+, H^+ \to \tau \nu$, and $H^+ \to t \bar{b}$ (or, analogously, of $H^+ \to c \bar{s}$) are equal to [243]

$$\Gamma(t \to b H^+) = \frac{g^2}{64 \pi M_W^2 m_t} |V_{tb}|^2 \lambda^{1/2} \left(1, \frac{m_b^2}{m_t^2}, \frac{m_H^2}{m_t^2}\right) \times$$
where $\lambda(a, b, c) = a^2 + b^2 + c^2 - 2(ab + ac + bc)$, and $m_H = m_{H^+}$.

Consequently, if $m_H < m_t - m_b$, one expects $H^+ \to \tau^+\nu$ (favoured for large $\tan\beta$) and/or $H^+ \to c\bar{s}$ (favoured for small $\tan\beta$) to be the dominant decays. Hence, for $\tan\beta > 1$ and $H^+ \to \tau^+\nu$ dominant, one can look for the channel $t \to bH^+$ by studying a possible excess in the $\tau$ lepton signature from the $t$ pair production [244]. On the other hand, if $\tan\beta < 2$ and $m_H > 130$ GeV, the large mass (or coupling) of the $t$-quark causes $\text{BR}(H^+ \to t^+\bar{b} \to W^+b\bar{b})$ to exceed $\text{BR}(H^+ \to c\bar{s})$ (Fig. 44, see [245] for details).

As a consequence, new interesting signatures at LHC such as leptons plus multi-jet channels with four $b$-tags, coming from the gluon-gluon fusion process $gg \to t\bar{b}H^-$, followed by the $H^- \to t\bar{b}$ decay, have been studied [246]. These processes could provide a viable signature over a limited but interesting range of the parameter space.

One should recall however that both $\text{BR}(t \to bH^+)$ and $\text{BR}(H^+ \to W^+b\bar{b})$ are very sensitive to higher-order corrections, which are highly model dependent [247].

In model III, the tree-level FCNC decay $t \to ch$ can occur with branching ratios up to $10^{-2}$ [242]. In [248], the rate for the channel $t \to ch \to cWW(cZZ)$ has been studied. Accordingly, $\text{BR}(t \to cWW)$ can be enhanced by several orders of magnitude with respect to its SM value. In particular, for an on-shell decay with $2M_W \lesssim m_b \lesssim m_t$, one can have up to $\text{BR}(t \to cWW) \sim 10^{-4}$ from this source. The same process was considered in a wider range of models, where the decay $t \to cWW$ can occur not only through a scalar exchange but also through a fermion or vector exchange [239]. In this framework, the fermion exchange too could lead to detectable rates for $t \to cWW$, as in (71).
In 2HDM’s, the prediction for the FCNC decays $t \to eg$, $t \to c\gamma$ and $t \to cZ$ can also be altered. While in models I and II the corresponding branching fractions cannot approach the detectability threshold [249], in model III predicts values up to $\text{BR}(t \to cg) \simeq 10^{-6}$, $\text{BR}(t \to c\gamma) \simeq 10^{-7}$ and $\text{BR}(t \to cZ) \simeq 10^{-6}$ [217].

By further extending the 2HDM’s Higgs sector and including Higgs triplets, one can give rise to a vertex $HWZ$ at tree level in a consistent way [250]. Accordingly, the $t \to bWZ$ decay can be mediated by a charged Higgs (coupled with $m_h$) that can enhance the corresponding branching fraction up to $\text{BR}(t \to bWZ) \sim 10^{-2}$. Large enhancements can also be expected in similar models for the channels $t \to sWZ$ and $t \to dWZ$.

8.23 Minimal Supersymmetric Standard Model (MSSM)

Supersymmetry could affect the $t$ decays in different ways. (Here, we assume the MSSM framework [26], with (or without, when specified) $R$ parity conservation.)

First of all, two-body decays into squarks and gauginos, such as $t \to \tilde{t}_1 \tilde{g}$, $t \to \tilde{b}_1 \tilde{\chi}_1^+$, $t \to \tilde{t}_1 \tilde{\chi}_1^0$, could have branching ratios of order $\text{BR}(t \to bW)$, if allowed by the phase space (see, i.e. [251] for references). QCD corrections to the channel $t \to \tilde{t}_1 \tilde{g}$ have been computed in [252] and were found to increase its width up to values even larger than $\Gamma(t \to bW)$. Three-body $t$ decays in supersymmetric particles were surveyed in [251].

The presence of light top and bottom squarks, charginos and neutralinos in the MSSM spectrum could also give rise to a CP asymmetry of the order $10^{-3}$ in the partial widths for the decays $t \to bW^+$ and $\bar{t} \to \bar{b}W^-$ [140, 253].

Explicit $R$-parity violating interactions [254] could provide new flavour-changing $t$ decays, both at tree level (as in the channels $t \to \tau b$ and $t \to \tau b\tilde{\chi}_1^0$ [255]) and at one loop (as in $t \to c\tau$ [256]), with observable rates. For instance, $\text{BR}(t \to c\tau) \sim 10^{-4} - 10^{-3}$ in particularly favourable cases.

Another sector where supersymmetric particles could produce crucial changes concerns the one-loop FCNC decays $t \to eg$, $t \to c\gamma$, $t \to cZ$ and $t \to cH$, which in the SM are unobservably small. In the MSSM with universal soft breaking the situation is not much affected, while, by relaxing the universality with a large flavour mixing between the 2nd and 3rd family only, one can reach values such as [257, 258]:

$$\text{BR}_{\text{MSSM}}(t \to cg) \sim 10^{-6}, \quad \text{BR}_{\text{MSSM}}(t \to c\gamma) \sim 10^{-8}, \quad \text{BR}_{\text{MSSM}}(t \to cZ) \sim 10^{-8}, \quad (76)$$

which, however, are still not observable. The introduction of baryon number violating couplings in broken $R$-parity models could on the other hand give large enhancements [218], and make some of these channels observable:

$$\text{BR}_F(t \to cg) \sim 10^{-3}, \quad \text{BR}_F(t \to c\gamma) \sim 10^{-5}, \quad \text{BR}_F(t \to cZ) \sim 10^{-4}. \quad (77)$$

A particularly promising channel is the FCNC decay $t \to ch$ in the framework of MSSM, where $h = h^0, H^0, A^0$ is any of the supersymmetric neutral Higgs bosons [259]. By including the leading MSSM contributions to these decays (including gluino-mediated FCNC couplings), one could approach the detectability threshold, especially in the case of the light CP-even Higgs boson, for which one can get up to:

$$\text{BR}_{\text{MSSM}}(t \to ch^0) \sim 10^{-4}. \quad (78)$$

8.24 Anomalous couplings

In the framework of the top anomalous couplings described in Section 7., one can predict large enhancements in different FCNC top decay channels. While the $t \to cg, t \to c\gamma$, and $t \to cZ$ processes are analysed in section 7., here we concentrate on the possible FCNC contributions to the top decays into two gauge
bosons, $t \rightarrow qVV$, where $V$ is either a $W$ or a $Z$ and $q = c, u$:

$$t \rightarrow qW^+Z, \quad t \rightarrow qW^+W^-, \quad t \rightarrow qZZ.$$  \hspace{1cm} (79)

In the SM, the first two decays occur at tree level, while $t \rightarrow qZZ$ proceeds only through loop contributions. We will see that within the present experimental limits on the top anomalous couplings, the rates for these processes can be large with respect to the SM prediction, but are still below the detectability threshold at the LHC.

The FCNC contribution to the first channel in (79), for the anomalous coupling $\kappa_z \approx 0.3$, has a rate of the same order of magnitude as the SM BR($t \rightarrow bWZ$) [260]:

$$\text{BR}_{FCNC}(t \rightarrow cWZ) \sim 10^{-6} \approx \text{BR}_{SM}(t \rightarrow bWZ).$$ \hspace{1cm} (80)

Top anomalous FCNC interactions with both a photon and a $Z$-boson contribute to the second process in eq.(79). Contrary to the SM case this amplitude has no GIM suppression. As a result, the corresponding branching ratio can have almost the same value as that of the $t \rightarrow qWZ$ decay [260]:

$$\text{BR}_{FCNC}(t \rightarrow cW^+W^-) \sim 10^{-7} \gg \text{BR}_{SM}(t \rightarrow cWW).$$ \hspace{1cm} (81)

For the $t \rightarrow qZZ$ decay mode, a coupling $\kappa_z \sim 0.3$ gives a branching ratio much greater than the corresponding SM one ($\lesssim 10^{-13}$ [260]):

$$\text{BR}_{FCNC}(t \rightarrow qZZ) \sim 10^{-8} \gg \text{BR}_{SM}(t \rightarrow qZZ),$$ \hspace{1cm} (82)

but still too small to be detected at LHC.

In summary, the observation of any of these decays at LHC would indicate new physics not connected with the top FCNC interactions (see, for example, [248]).

### 8.3 ATLAS studies of (rare) top quark decays and couplings

In ATLAS various analyses have been performed on top decays, using the PYTHIA Monte-Carlo interfaced to a fast detector simulation (ATLFAST). In the following, the most relevant results are reported.

#### 8.31 $\text{BR}(t \rightarrow bX)$ and measurement of $|V_{tb}|$

The SM prediction $\text{BR}(t \rightarrow W^+b) \approx 1$ can be checked by comparing the number of observed (1 or 2) $b$-tags in a $t\bar{t}$ sample. The first $b$-tag is used to identify the event as a $t\bar{t}$ event, and the second $b$-tag (if seen) is used to determine the fraction of top decays producing a $b$ quark. Within the three-generation SM, and assuming unitarity of the CKM matrix, the ratio of double $b$-tag to single $b$-tag events is given by:

$$R_{2b/1b} = \frac{\text{BR}(t \rightarrow Wb)}{\text{BR}(t \rightarrow Wq)} = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2} = |V_{tb}|^2$$ \hspace{1cm} (83)

The CDF collaboration has used the tagging method in leptonic $t\bar{t}$ events to obtain the result $R_{2b/1b} = 0.99 \pm 0.29$ [261], which translates to a limit of $|V_{tb}| > 0.76$ at the 95% CL assuming three-generation unitarity. If this constraint is relaxed, a lower bound of $|V_{tb}| > 0.048$ at the 95% CL is obtained, implying only that $|V_{tb}|$ is much larger than either $|V_{ts}|$ or $|V_{td}|$.

The LHC will yield a much more precise measurement of $R_{2b/1b}$. For example, $t\bar{t}$ events in the single lepton plus jets mode can be selected by requiring an isolated electron or muon with $p_T > 20$ GeV, $E_T^{miss} > 20$ GeV, and at least four jets with $p_T > 20$ GeV. Requiring that at least one of the jets be tagged as a $b$-jet produces a clean sample of $t\bar{t}$ events, with $S/B = 18.6$, with the remaining background coming mostly from $W$+jet events [30]. Assuming a $b$-tagging efficiency of 60%, a sample of 820 000 single $b$-tagged events would be selected for an integrated luminosity of 10 fb$^{-1}$. Of these,
276,000 would be expected to have a second b-tag, assuming the SM top quark branching ratios. This ATLAS study indicates that the statistical precision achievable would correspond to a relative error of $\delta R_{2b/1b}/R_{2b/1b}$ (stat.) = 0.2% for an integrated luminosity of 10 fb$^{-1}$. The final uncertainty will be dominated by systematic errors due to the uncertainty in the b-tagging efficiency and fake b-tag rates, as well as correlations affecting the efficiency for b-tagging two different jets in the same event. Further study is needed to estimate the size of these systematic uncertainties.

8.32 $BR(t \rightarrow WX)$

The measurement of the ratio of di-lepton to single lepton events in a $t\bar{t}$ sample can be used to determine $BR(t \rightarrow WX)$. In this case, the first lepton tags the $t\bar{t}$ event, and the presence of a second lepton is used to determine the fraction of top quark decays producing an isolated lepton, which can then be related to the presence of a $W$ (or other leptonically decaying states) in the decay. The SM predicts that $R_{2l/1l} = BR(W \rightarrow \ell \nu) \approx 2/9$ where $\ell = (e, \mu)$. Deviations from this prediction could be caused by new physics, for example, the existence of a charged Higgs boson. The dominant $H^+$ decays in such instances are usually considered to be $H^+ \rightarrow \tau \nu$ or $H^+ \rightarrow e\gamma$. In either case, the number of isolated electrons and muons produced in top decay would be reduced, and $R_{2l/1l}$ would be less than the SM prediction.

A study performed by ATLAS [30] shows that with an integrated luminosity of 10 fb$^{-1}$, a clean sample of about 443,000 $t\bar{t}$ events in the single lepton plus jets mode could be selected by requiring an isolated electron or muon with $p_T > 20$ GeV, $E_T^{\text{miss}} > 20$ GeV, and at least two $b$-tagged jets with $p_T > 20$ GeV. To determine $R_{2l/1l}$, one then measures how many of these events have a second isolated electron or muon, again with $p_T > 20$ GeV, and of the opposite sign to the first lepton. Assuming the SM, one would expect a selected sample of about 46,000 di-lepton events with these cuts. Given these numbers, the statistical precision achievable would correspond to a relative error of $\delta R_{2l/1l}/R_{2l/1l}$ (stat.) = 0.5% for an integrated luminosity of 10 fb$^{-1}$. Further study is required to estimate the systematic uncertainty on $R_{2l/1l}$ due to the lepton identification and fake rates.

8.33 Radiative Decays: $t \rightarrow WbZ, t \rightarrow WbH$

The ‘radiative’ top decay $t \rightarrow WbZ$ has been suggested [233] as a sensitive probe of the top quark mass, since the measured value of $m_t$ is close to the threshold for this decay. For the top mass of $(173 \pm 5.2)$ GeV [192], the SM prediction, based on the $Z \rightarrow ee$ signature and a cut $m_{ee} > 0.8 M_Z$ (see Section 8.1), is $BR_{\text{cut}}(t \rightarrow WbZ) = (5.4^{+4.7}_{-2.0}) \times 10^{-7}$ [233]. Thus, within the current uncertainty $\delta m_t \approx 5$ GeV, the predicted branching ratio varies by approximately a factor of three. A measurement of $BR(t \rightarrow WbZ)$ could therefore provide a strong constraint on the value of $m_t$. Similar arguments have been made for the decay $t \rightarrow WbH$, assuming a relatively light SM Higgs boson.

ATLAS has studied the experimental sensitivity to the decay $t \rightarrow WbZ$ [30, 262], with the $Z$ being reconstructed via the leptonic decay $Z \rightarrow ll (\ell = e, \mu)$, and the $W$ through the hadronic decay $W \rightarrow jj$. The efficiency for exclusively reconstructing $t \rightarrow WbZ$ is very low, due to the soft $p_T$ spectrum of the $b$-jet in the $t \rightarrow WbZ$ decay. Instead, a semi-inclusive technique was used, where a $WZ$ pair close to threshold was searched for as evidence of the $t \rightarrow WbZ$ decay. Since the $t \rightarrow WbZ$ decay is so close to threshold, the resolution on $m_{WZ}$ is not significantly degraded with respect to the exclusive measurement. The selection of $Z \rightarrow ll$ candidates required an opposite-sign, same-flavor lepton pair, each lepton having $p_T > 30$ GeV and $|\eta| < 2.5$. The clean $Z \rightarrow ll$ signal allows a wide di-lepton mass window to be taken (60 GeV < $m_{ll}$ < 100 GeV) in order to have very high efficiency. Candidates for $W \rightarrow jj$ decay were formed by requiring at least two jets, each having $p_T > 30$ GeV and $|\eta| < 2.5$, and satisfying 70 GeV < $m_{jj}$ < 90 GeV. The $lljj$ invariant mass resolution was $\sigma[m_{WZ}] = 7.2 \pm 0.4$ GeV, and the signal efficiency was 4.3%.

The dominant backgrounds come from processes with a $Z$ boson in the final state, primarily $Z$+jet production, and to a much lesser extent from $WZ$ and $t\bar{t}$ production. In order to reduce the
$Z$+jet background, an additional cut requiring a third lepton with $p_T > 30$ GeV was made. For the signal process $t \bar{t} \rightarrow (WbZ)(Wb)$, this cut selects events in which the $W$ from the other top decays leptonically. After this selection, and with a cut on $m_W$ of $\pm 10$ GeV around the top mass, the total expected background was reduced to $\approx 1.5$ events (mostly from $WZ$ production) per 10 fb$^{-1}$. Requiring at least five events for signal observation leads to a branching ratio sensitivity of order $10^{-3}$. Since the background has been reduced essentially to zero, the sensitivity should improve approximately linearly with integrated luminosity. However, even with a factor of ten improvement for an integrated luminosity of 100 fb$^{-1}$, the sensitivity would still lie far above the SM expectation of order $10^{-7} - 10^{-6}$.

Given this result, observation of the decay $t \rightarrow WbH$ does not look possible. The current LEP limit on $m_H$ implies that the Higgs is sufficiently heavy that, in the most optimistic scenario that the Higgs mass is just above the current limit, $\text{BR}(t \rightarrow WbH) \lesssim \text{BR}(t \rightarrow WbZ)$. As $m_H$ increases further, $\text{BR}(t \rightarrow WbH)$ drops quickly. Assuming $m_H \approx m_Z$, one would have to search for $t \rightarrow WbH$ using the dominant decay $H \rightarrow b\bar{b}$. The final state suffers much more from background than in the case of $t \rightarrow WbZ$, where the clean $Z \rightarrow \ell^+ \ell^-$ signature is a key element in suppressing background. Although $\text{BR}(H \rightarrow b\bar{b})$ in this $m_H$ range is much larger than $\text{BR}(Z \rightarrow \ell^+ \ell^-$), the large increase in background will more than compensate for the increased signal acceptance, and so one expects the sensitivity to $\text{BR}(t \rightarrow WbH)$ to be worse than for $\text{BR}(t \rightarrow WbZ)$. The decay $t \rightarrow WbH$ has therefore not been studied in further detail.

8.34 $t \rightarrow H^+b$

Limits on the mass of the charged Higgs have been obtained from a number of experiments. An indirect limit obtained from world averages of the $\tau$ branching ratios excludes at 90% CL any charged Higgs with $m_{H^+} < 1.5 \tan \beta$ GeV [263], where $\tan \beta$ is the ratio of the vacuum expectation values of the two Higgs doublets. CLEO indirectly excludes $m_{H^+} < 244$ GeV for $\tan \beta > 50$ at 95% CL, assuming a two-Higgs-doublet extension to the SM [212], while the LEP experiments directly exclude $m_{H^+} < 59.5$ GeV/$c^2$ at 95% CL [264]. Searches at the Tevatron have extended the region of excluded $[m_{H^\pm}, \tan \beta]$ parameter space, particularly at small and large $\tan \beta$, and set a limit on the branching ratio $\text{BR}(t \rightarrow H^+b) < 0.45$ at 95% CL [265]. Run 2 at the Tevatron will be sensitive to branching fractions $\text{BR}(t \rightarrow H^+b) > 11\%$ [266].

ATLAS has performed an analysis of the experimental sensitivity to the $t \rightarrow H^+b$ decay, followed by $H^+ \rightarrow \tau \nu$, in the context of the MSSM [30, 267]. Since the relevant $t \rightarrow H^+b$ branching ratio is proportional to $(m_t^2 \cot^2 \beta + m_h^2 \tan^2 \beta)$ (see (73)), for a given value of $m_{H^\pm}$ the branching ratio for such decays is large at small and at large $\tan \beta$, but has a pronounced minimum at $\tan \beta \sim \sqrt{m_t/m_h} \sim 7.5$. The exact position of this minimum and its depth is sensitive to QCD corrections to the running $b$-quark mass.

In the ATLAS analysis, an isolated high-$p_T$ lepton with $|\eta| < 2.5$ is required to trigger the experiment, which in signal events originates from the semi-leptonic decay of the second top quark. One identified hadronic tau is then required, and at least three jets with $p_T > 20$ GeV and $|\eta| < 2.5$, of which two are required to be tagged as $b$-jets. This reduces the potentially large backgrounds from $W$+jet and $b\bar{b}$ production to a level well below the $t\bar{t}$ signal itself. These cuts enhance the $\tau$-lepton signal from $H^\pm$ decays with respect to that from $W$ decay, and select mostly single-prong $\tau$-decays. After the selection cuts and the $\tau$ identification criteria are applied, $t \rightarrow H^+b$ decays appear as final states with an excess of events with one isolated $\tau$-lepton compared to those with an additional isolated electron or muon.

A signal from charged Higgs-boson production in $t\bar{t}$ decays would be observed for all values of $m_{H^\pm}$ below $m_t - 20$ GeV over most of the $\tan \beta$ range. For moderate values of $\tan \beta$, for which the expected signal rates are lowest, the accessible values of $m_{H^\pm}$ are lower than this value by 20 GeV. The limit on the sensitivity to $\text{BR}(t \rightarrow H^+b)$ is dominated by systematic uncertainties, arising mainly from imperfect knowledge of the $\tau$-lepton efficiency and of the number of fake $\tau$-leptons present in the final sample. These uncertainties are estimated to limit the achievable sensitivity to $\text{BR}(t \rightarrow H^+b) = 3\%$.
For charged Higgs masses below 150 GeV and for low values of $\tan \beta$, the $H^\pm \to cs$ and $H^\pm \to cb$ decay modes are not negligible. In the same mass range, the three-body off-shell decays $H^\pm \to hW^*$, $H^\pm \to AW^*$ and $H^\pm \to bt^* \to bbW$ also have sizeable branching ratios. When the phase-space increases, for 150 GeV $< m_{H^\pm} < 180$ GeV, both the $bbW$ and the $hW^*$ mode could be enhanced with respect to the $\tau\tau$ mode. Decays into the lightest chargino $\tilde{\chi}_1^\pm$ and neutralino $\tilde{\chi}_1^0$ or decays into sleptons would dominate whenever kinematically allowed. For large values of $\tan \beta$ the importance of these SUSY decay modes would be reduced. However, for values as large as $\tan \beta = 50$, the decay $H^\pm \to \tau\tau$ would be enhanced, provided it is kinematically allowed and would lead to $\tau$’s in the final state. Their transverse momentum spectrum is, however, expected to be softer than that of $\tau$’s from the direct $H^\pm \to \tau\tau$ decays.

The $H^\pm \to cs$ decay mode has been considered as a complementary one to the $H^\pm \to \tau\tau$ channel by ATLAS for low values of $\tan \beta$. In the ATLAS analysis, one isolated high $p_T$ lepton with $|\eta| < 2.5$ is required to trigger the experiment, which in signal events originates from the semi-leptonic decay of the second top quark. Two $b$-tagged jets with $p_T > 15$ GeV and $|\eta| < 2.5$ are also required, with no additional $b$-jet. Finally, at least two non-$b$ central jets with $|\eta| < 2.0$ are required for the $H^\pm \to cs$ reconstruction, and no additional jets above 15 GeV in this central region. Evidence for $H^\pm$ is searched for in the two-jet mass distribution. The mass peak from an $H^\pm$ decay can be reconstructed with a resolution of $\sigma = (5 - 8)$ GeV if the mass of the $H^\pm$ is in the range between 110 and 130 GeV. In this mass range, the peak sits on the tail of the reconstructed $W \to jj$ distribution from $t\bar{t}$ background events which decay via a $Wb$ instead of a $H^\pm b$. In the mass range $110 < H^\pm < 130$ GeV, the $H^\pm$ peak can be separated from the dominant $W \to jj$ background, with $S/B \approx 4$-5% and $S/\sqrt{B} \approx 5$. This channel is complementary to the $H^\pm \to \tau\tau$ channel for low $\tan \beta$ values. Whereas the $H^\pm \to \tau\tau$ channel allows only the observation of an excess of events, it is possible to reconstruct a mass peak in the $H^\pm \to cs$ decay mode.

The $H^\pm \to hW^*$, $H^\pm \to AW^*$ and $H^\pm \to bt^* \to bbW$ have not been studied so far by ATLAS. With the expected $b$-tagging efficiency, these multi-jet decay modes are very interesting for a more detailed investigation.

8.35 $t \to Zq$ decay

The sensitivity to the FCNC decay $t \to Zq$ (with $q = u, c$) has been analyzed [268] by searching for a signal in the channel $t\bar{t} \to (Wb)(Zq)$, with the boson being reconstructed via the leptonic decay $Z \to ll$. The selection cuts required a pair of isolated, opposite sign, same flavor leptons (electrons or muons), each with $p_T > 20$ GeV and $|\eta| < 2.5$ and with $|m_H - m_Z| < 6$ GeV. The dominant backgrounds come from $Z + j e t$ and $WZ$ production. Not only cuts were applied on the $Zq$ final state, but also on the $Wb$ decay of the other top quark in the event, to further reduce the background. Two different possible decay chains have been considered: the first (“leptonic mode”) where the $W$ decays leptonically $W \to ll$, and the second (“hadronic mode”) with $W \to jj$. The hadronic $W$ decay signature has a much larger branching fraction, but suffers from larger backgrounds. The search in the leptonic mode required, in addition to the leptons from the $Z$ boson decay, a further lepton with $p_T > 20$ GeV and $|\eta| < 2.5$, $E_T^{miss}$ > 30 GeV, and at least two jets with $p_T > 50$ GeV and $|\eta| < 2.5$. Exactly one of the high $p_T$ jets was required to be tagged as a $b$-jet. The invariant mass spectrum of each $Zq$ combination was then formed from the $Z \to ll$ candidates taken with each of the non-$b$-tagged jets. The $Zq$ invariant mass resolution was 10.1 GeV. Combinations were accepted if $m_{Zq}$ agreed with the known top mass within $\pm 24$ GeV. Assuming an integrated luminosity of 100 fb$^{-1}$, 6.1 signal events survive the cuts with 7 background events. A value of BR($t \to Zq$) as low as $2 \cdot 10^{-4}$ could be discovered at the 5$\sigma$ level.

The search in the hadronic mode required, in addition to the $Z \to ll$ candidate, at least four jets with $p_T > 50$ GeV and $|\eta| < 2.5$. One of the jets was required to be tagged as a $b$-jet. To further reduce the background, the decay $t \to jjb$ was first reconstructed. A pair of jets, among those not tagged as a $b$-jet, was considered a $W$ candidate if $|m_{jj} - M_W| < 16$ GeV. $W$ candidates were then combined with the $b$-jet, and considered as a top candidate if $|m_{jjb} - m_t| < 8$ GeV. For those events with an accepted
$t \rightarrow jjb$ candidate, the invariant mass of the $Z$ candidate with the remaining unassigned high $p_T$ jets was calculated to look for a signal from $t \rightarrow Zq$ decays. Combinations were accepted in case $|m_{Zq} - m_t| < 24$ GeV. Assuming an integrated luminosity of 100 fb$^{-1}$, one would get 0.4 signal events, with 2 background events.

### 8.36 $t \rightarrow \gamma q$ decay

The FCNC decay $t \rightarrow \gamma q$ (with $q = u, c$) can be searched for as a peak in the $M_{\gamma j}$ spectrum in the region of $m_t$. The requirement of a high $p_T$ isolated photon candidate in $t\bar{t} \rightarrow (Wb)(\gamma q)$ events is not sufficient to reduce the QCD multi-jet background to a manageable level. Therefore, the $t \rightarrow Wb$ decay of the other top (anti-) quark in the event was reconstructed using the leptonic $W \rightarrow \ell \nu$ decay mode, and looking for the $t\bar{t} \rightarrow (Wb)(\gamma q) \rightarrow (\ell\nu b)(\gamma q)$ final state. For the event selection, the ATLAS collaboration [30, 262] required the presence of an isolated photon with $p_T > 40$ GeV and $|\eta| < 2.5$, an isolated electron or muon with $p_T > 20$ GeV and $|\eta| < 2.5$, and $E_T^{miss} > 20$ GeV. Exactly 2 jets with $p_T > 20$ GeV were required, in order to reduce the $t\bar{t}$ background. At least one of the jets was required to be tagged as a $b$-jet with $p_T > 30$ GeV and $|\eta| < 2.5$. The $t \rightarrow \ell\nu b$ candidate was first reconstructed. The combination was accepted as a top quark candidate if $m_{\ell\nu b}$ agreed with $m_t$ within $\pm 20$ GeV. For these events the $t \rightarrow \gamma q$ decay was sought by combining the isolated photon with an additional hard jet with $p_T > 40$ GeV and $|\eta| < 2.5$. The invariant mass of the $\gamma j$ system was required to agree with the known value of $m_t$ within $\pm 20$ GeV. The $m_{\gamma j}$ resolution with the cuts described above was 7.7 GeV, and the signal efficiency (not counting branching ratios) was 3.3%, including a $b$-tagging efficiency of 60%. The background (155 events for an integrated luminosity of 100 fb$^{-1}$) is dominated by events with a real $W \rightarrow \ell \nu$ decay and either a real or a fake photon. These processes include $t\bar{t}$, single top production, $W + jets$ and $Wb$ production. The corresponding 5$\sigma$ discovery limit is

$$\text{BR}(t \rightarrow \gamma q) = 1.0 \times 10^{-4}. \quad (84)$$

### 8.37 $t \rightarrow ggq$ decay

The search for a FCNC $tgq$ coupling (with $q = u, c$) through the decay $t \rightarrow ggq$ was analyzed in [229] for the Tevatron. However, as can be seen from Table 23 in Section 7., the sensitivity for such a coupling turns out to be much larger in the $t$ production processes than in the decay $t \rightarrow ggq$, whose signal will be overwhelmed by the QCD background. We refer the reader to Section 7. for a detailed discussion of this point.

### 8.4 CMS studies of FCNC top quark decays and $t \rightarrow H^+b$

The CMS sensitivity to $t \rightarrow \gamma (Z)(u,c)$ decays was studied recently (see [230] for details). The PYTHIA 5.7 [52] generator was used for the signal and background simulations and the detector response was simulated at the fast MC level (CMSJET [177]). For the $t \rightarrow \gamma (u,c)$ signal the exact 2 $\rightarrow$ 5 matrix elements $gg(q\bar{q}) \rightarrow t\bar{t} \rightarrow \gamma u(c) + W^+b(\rightarrow \ell\nu b)$ were calculated and included in PYTHIA. The $t \rightarrow \gamma (Z)(u,c)$ decays would be seen as peaks in the $M_{\gamma(Z)jet}$ spectrum in the region of $m_t$. To separate the signal from the background one has to exploit the presence of the additional top decaying to the $\ell\nu b$ in the same event. The signature with the hadronic decay of the additional top was found to be hopeless.

### 8.41 $t \rightarrow \gamma (u,c)$

In order to separate the $(\gamma q)(\ell\nu b)$ final state from the backgrounds several selection criteria were found to be effective. First, the presence of one isolated photon with $E_T \geq 75$ GeV and $|\eta| \geq 2.5$, one isolated lepton ($\mu, e$) with $E_T \geq 15$ GeV and $|\eta| \geq 2.5$, and at least two jets with $E_T \geq 30$ GeV and $|\eta| \geq 2.4$ is required. One top quark has to be reconstructed from the photon and jet ($M_{\gamma jet} < m_t \pm 15$ GeV), the corresponding jet is not allowed to be $b$-tagged. On the contrary, the jet with maximal $E_T$,
which is not involved in the $(\gamma, \text{jet})$ system has to be $b$-tagged, should have $E_t \geq 50$ GeV and contribute to another reconstructed top quark ($M_{t}\tilde{t} \subset m_t \pm 25$ GeV). There must be no additional jets with $E_t \geq 50$ GeV. The $b$-tagging efficiency was assumed to be 60% for the purity 1%(10%) with respect to the gluon and light quark jets ($c$-quark jets). After this selection, approximately 270 background events dominated by the $t\bar{t}$ and $W +$ jets, including $W\bar{b}b$, survive for the integrated luminosity of 100 fb$^{-1}$, while the signal efficiency is 9.1%. The $S/B$ ratio is about 1 for $\text{BR}(t \rightarrow \gamma (u, c)) = 10^{-4}$ and the $5\sigma$ discovery limit is as low as $3.4 \times 10^{-5}$ for 100 fb$^{-1}$.

8.42 $t \rightarrow Zq$

The $t \rightarrow Zq$ signal was searched for in the $t\bar{t} \rightarrow (\ell\bar{\ell}q)\ell\nu b$ final state. Three isolated leptons with $E_t \geq 15$ GeV and $|\eta| \leq 2.5$, and exactly two jets with $E_t \geq 30$ GeV and $|\eta| \leq 2.5$ are required. The pair of the opposite-sign same-flavour leptons has to be constrained to the $Z$ mass ($M_{\ell\bar{\ell}} \subset M_Z \pm 6$ GeV) and one jet, combined with the reconstructed $Z$, has to form the top system ($M_{t\ell\bar{\ell}} \subset m_t \pm 15$ GeV). This jet is not allowed to be the $b$-jet, but the last "free" jet in the event has to be $b$-tagged. For the integrated luminosity of 100 fb$^{-1}$ just $\sim 9$ background events coming from the $WZ$, $t\bar{t}Z$ and $Z +$ jets processes survive. The signal efficiency is about 6.8% which corresponds, however, only to $\sim 12$ events for $\text{BR}(t \rightarrow Z(u, c)) = 10^{-4}$. The indication is that one can reduce the background rate to the nearly zero level tightening the selection criteria. In particular, requiring in addition $E_t^{miss} \geq 30$ GeV and a harder jet involved in the top ($t\bar{t}q$) system ($E_t \geq 50$ GeV) one can reduce the background to the level of $\sim 0.6$ events still keeping $\sim 3.7\%$ of the signal (6.6 events for $\text{BR}(t \rightarrow Z(u, c)) = 10^{-4}$ and 100 fb$^{-1}$). One can conclude that the $t \rightarrow Z(u, c)$ signal should be very clean but, due to the low signal event rate, only $\sim 3 \times 100$ fb$^{-1}$ of integrated luminosity would allow one to probe $\text{BR}(t \rightarrow Z(u, c))$ as low as $10^{-4}$, provided the present background understanding is correct and the detector performance will not be deteriorated during the long run. The $5\sigma$ reach for 100 fb$^{-1}$ is $\sim 1.9 \times 10^{-4}$.

8.43 $t \rightarrow H^+b$

CMS has investigated the production of the light charged Higgs, $m_{H^\pm} < m_t$, in $t\bar{t}$ events using the decay chain $t\bar{t} \rightarrow H^\pm bWb \rightarrow (\tau^\pm \nu, b) + (\ell\nu b)$ [269]. The $H^\pm \rightarrow \tau\nu$ branching ratio is large $\sim 98\%$ in this mass range for $\tan \beta > 2$ and only slightly dependent on $\tan \beta$. The $t \rightarrow H^\pm b$ branching ratio is large both at high and at low $\tan \beta$ values and has a minimum of $\sim 0.8\%$ around $\tan \beta \sim 6$. Since the Higgs mass cannot be reconstructed in this process the signal can be only inferred from the excess of $\tau$ production over what is expected from the SM $t \rightarrow Wb$, $W^\pm \rightarrow \tau^\pm \nu$ decay.

An isolated lepton with $p_t > 20$ GeV is required to identify the top decay and to trigger the event. The $\tau$’s are searched starting from calorimeter jets with $E_t > 40$ GeV within $|\eta| < 2.4$. For the $\tau$ identification the tracker information is used, requiring one hard isolated charged hadron with $p_t > 30$ GeV within the cone of $\Delta R < 0.1$ inside the calorimeter jet. The algorithm thus selects the one prong $\tau$ decays.

The main backgrounds are due to the $t\bar{t}$ events with $t\bar{t} \rightarrow WbWb \rightarrow (\tau^\pm \nu, b) + (\ell\nu b)$ and $W+$ jet events with $W \rightarrow \tau\nu$. The $t\bar{t}$ background is irreducible, but can be suppressed by exploiting the $\tau$ polarisation effects [270]. Due to the $\tau$ polarisation the charged pion from $\tau \rightarrow \pi^\pm \nu$ decay has a harder $p_t$ spectrum when coming from $H^\pm \rightarrow \tau\nu$ than from $W^\pm \rightarrow \tau\nu$. The decay matrix elements with polarisation [271] were implemented in PYTHIA [52]. Due to the polarisation, the efficiency of the above $\tau$ selection is significantly better for $H^\pm \rightarrow \tau\nu$ ($\sim 19\%$) than for $W^\pm \rightarrow \tau\nu$ ($\sim 6\%$).

The events were required to have at least one $b$-jet with $E_t > 30$ GeV tagged with an impact parameter method [272]. This $b$-tagging suppresses efficiently, by a factor of $\sim 70$, the background from $W+$ jet events. The efficiency for $t\bar{t}$ events is $\sim 35\%$. The expected $5\sigma$ discovery range for 10 fb$^{-1}$ in the MSSM ($m_A, \tan \beta$) parameter space was found to be: $m_A < 110$ GeV for all $\tan \beta$ values and somewhat extended ($m_A \lesssim 140$) for $\tan \beta \lesssim 2$. 
8.5 Conclusions on rare top decays

In the framework of the SM, the top rare decays (that is any channel different from $t\to qW$) are definitely below the threshold for an experimental analysis at LHC. On the other hand, LHC experiments will be able to probe quite a few predictions of possible extensions of the SM.

An extended Higgs sector will be looked for through the tree-level decay $t\to bH^\pm$. ATLAS estimates its sensitivity to this channel in the MSSM, through an excess in the tau lepton signal, to be around $\text{BR}(t\to H^\pm b) = 3\%$ (that is almost 4 times better than what expected from Run 2 at the Tevatron). This would allow to probe all values of $m_{H^\pm}$ below $m_t - 20$ GeV over most of the $\tan \beta$ range. For low $\tan \beta$, the complementary decay mode $H^\pm\to cs\bar{s}$ has been considered. In the mass range $110 < H^\pm < 130$ GeV, the $H^\pm$ peak can be reconstructed and separated from the dominant $W\to jj$ background.

For CMS, using the $\tau$ excess signature, the expected $5\sigma$ discovery range for $10$ fb$^{-1}$ in the MSSM ($m_A, \tan \beta$) parameter space is $m_A < 110$ GeV, for all $\tan \beta$ values, and somewhat extended ($m_A \lesssim 140$), for $\tan \beta \lesssim 2$.

Other interesting signatures like $H^\pm\to hW^*$, $H^\pm\to AW^*$ and $H^\pm\to bt^*\to bbW$ are very promising in particular parameter ranges, but have not yet been thoroughly investigated.

ATLAS has studied its sensitivity to the radiative decay $t\to WbZ$. This has been found to be at most of the order $10^{-4}$, that is insufficient for the study of a SM signal ($\sim 10^{-5}$), but possibly useful for exploring the predictions of some extended Higgs-sector model, for which $\text{BR}(t\to WqZ) \lesssim 10^{-2}$. On the other hand, the radiative Higgs decay $t\to WbH$ seems out of the reach of LHC in any realistic model.

The LHC reach for the FCNC decays $t\to qZ$, $t\to q\gamma$ and $t\to qg$ has also been thoroughly investigated. Apart from the $t\to qg$, which is completely overwhelmed by the hadronic background, both ATLAS and CMS have a sensitivity of about $2 \times 10^{-4}$ to the $t\to qZ$ channel, while the CMS reach for the $t\to q\gamma$ channel is about $3.4 \times 10^{-5}$, that is slightly better than the ATLAS sensitivity ($1.0 \times 10^{-4}$), assuming an integrated luminosity of $100$ fb$^{-1}$. These thresholds could be largely sufficient to detect some manifestation of possible FCNC anomalous couplings in the top sector.

ATLAS has also investigated its sensitivity to a measurement of $|V_{tb}|$ through a determination of the rate $\text{BR}(t\to bX)$, by comparing the number of observed (1 or 2) $b$-tags in a $t\bar{t}$ sample. Within the three-generation SM, the ratio of double $b$-tag to single $b$-tag events is $R_{2b/1b} = |V_{tb}|^2$. LHC will allow a much more precise determination of $R_{2b/1b}$ with respect to the Tevatron (where, presently, one gets $|V_{tb}| > 0.76$ at the 95% CL). On a purely statistical basis, the expected relative error on $R_{2b/1b}$ is $\delta R_{2b/1b}/R_{2b/1b} \text{ (stat.)} = 0.2\%$ for an integrated luminosity of $10$ fb$^{-1}$, that would imply a relative error on $|V_{tb}|$ of about $1/\sqrt{10}$. On the other hand, the final uncertainty will be dominated by systematic errors related to the $b$-tagging. Further study is needed to estimate the size of these systematic uncertainties.

9. ASSOCIATED TOP PRODUCTION

The associated production of a Higgs boson (both SM-like and MSSM) with a top-antitop pair, is one of the most promising reactions to study both top quark and Higgs boson physics at the LHC.

The $pp \to t\bar{t}H$ channel can be used in the difficult search for an intermediate mass Higgs ($m_H \simeq 100 - 130$ GeV), as first proposed in [273]. In this mass region, the associated top production cross section is quite high but still smaller than the leading $gg \to H$ and $qq \to Hqq$ cross sections by two orders and one order of magnitude, respectively. However, since the final state $t\bar{t}H$ signature is extremely distinctive, even such a small signal production rate can become relevant, especially if identifying the Higgs through its dominant $H \to b\bar{b}$ decay becomes realistic, as will be discussed in the following.

Associated $t\bar{t}H$ production will furthermore provide the first direct determination of the top quark Yukawa coupling, allowing to discriminate, for instance, a SM-like Higgs from a more general MSSM Higgs. Processes like $gg \to H$ or $H \to \gamma\gamma$ are also sensitive to the top Yukawa coupling, but only

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through large top loop corrections. Therefore loop contributions from other sources of new physics can pollute the interpretation of the signal as a measurement of the top Yukawa coupling.

In the following we will concentrate on the case of a SM-like Higgs boson, whose top Yukawa coupling \( g_t = 2^{3/4} G_F^{1/2} m_t \) is enhanced with respect to the corresponding MSSM (scalar Higgs) coupling for \( \tan \beta > 2 \), the region allowed by LEP data. Predictions for the MSSM case can be easily obtained by rescaling both the \( t\bar{t}H \) coupling and any other coupling that appears in the decay of the Higgs boson.

The cross section for \( pp \to t\bar{t}H \) at LO in QCD has been known for a long time [274] and has been confirmed independently by many authors. We have recalculated it and found agreement with the literature. Of the two parton level processes (\( q\bar{q} \to t\bar{t}H \) and \( gg \to t\bar{t}H \)), \( gg \to t\bar{t}H \) dominates at the LHC due to the enhanced gluon structure function. The complete gauge invariant set of Feynman diagrams for \( gg \to t\bar{t}H \) is presented in Fig. 46. The corresponding analytical results are too involved to be presented here. The numerical results for a few values of the QCD scale \( \mu \) are given in Table 25, and illustrated in Fig. 45 as functions of \( m_H \), for \( \mu = m_H \). For consistency, we have used the leading order CTEQ4L PDFs [115] as well as the leading order strong coupling constant (for reference, \( \alpha_s^{LO}(\mu = M_Z) = 0.1317 \) for \( \alpha_s^{(5)} = 0.181 \)). The cross section, as expected from a LO calculation, shows a strong scale dependence, as can be see in Table 25, where results for \( \mu = m_H, m_t, m_H + 2m_t \) and \( \sqrt{s} \) are presented. In comparison with \( \mu = 2m_t + m_H \), for \( \mu = m_H \) we have 80-50% higher cross sections, when 100 GeV < \( m_H < 200 \) GeV. Since the choice of the QCD scale at LO is pretty arbitrary, and since we expect NLO QCD corrections to enhance the LO cross section, we decide to use \( \mu = m_H \) in Fig. 45 and in the following presentation. These calculations have been performed independently using the CompHEP software package [275] and MADGRAPH [276]+HELAS [277].

The NLO QCD corrections are expected to enhance the cross section, but their complete evaluation is still missing at the moment. Associated top production is in fact the only Higgs production mode for which the exact NLO QCD corrections have not been calculated yet. The task is very demanding, since it requires the evaluation of several one loop five-point functions for the virtual corrections and the integration over a four-particle final state (three of which massive) for the real corrections.

For large \( m_H \), the cross section for \( t\bar{t}H \) has been calculated including a complete resummation of potentially large logarithms, of order \( \ln(m_H/m_t) \), to all orders in the strong coupling [278]. These effects can almost double the cross section for \( m_H = 1 \) TeV.
Table 25: Leading order cross sections for $t\bar{t}H$ production at the LHC. The individual parton level channels ($q\bar{q} \rightarrow t\bar{t}H$ and $gg \rightarrow t\bar{t}H$) as well as their sum are given for a few values of the renormalization scale $\mu$.

<table>
<thead>
<tr>
<th>$m_H$ [GeV]</th>
<th>$q\bar{q}$ [fb]</th>
<th>$gg$ [fb]</th>
<th>$q\bar{q}+gg$ [fb]</th>
<th>$q\bar{q}+gg$ [fb]</th>
<th>$q\bar{q}+gg$ [fb]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu = m_H$</td>
<td>$\mu = m_H$</td>
<td>$\mu = m_H$</td>
<td>$\mu = m_H$</td>
<td>$\mu = 2m_t + m_H$</td>
<td>$\mu = \sqrt{s}$</td>
</tr>
<tr>
<td>100</td>
<td>348.0</td>
<td>990.0</td>
<td>1340.0</td>
<td>1070.0</td>
<td>765.0</td>
</tr>
<tr>
<td>110</td>
<td>279.0</td>
<td>740.0</td>
<td>1020.0</td>
<td>840.0</td>
<td>596.0</td>
</tr>
<tr>
<td>120</td>
<td>227.0</td>
<td>558.0</td>
<td>785.0</td>
<td>674.0</td>
<td>473.0</td>
</tr>
<tr>
<td>130</td>
<td>186.0</td>
<td>428.0</td>
<td>613.0</td>
<td>542.0</td>
<td>379.0</td>
</tr>
<tr>
<td>140</td>
<td>153.0</td>
<td>334.0</td>
<td>487.0</td>
<td>445.0</td>
<td>308.0</td>
</tr>
<tr>
<td>150</td>
<td>128.0</td>
<td>263.0</td>
<td>391.0</td>
<td>367.0</td>
<td>251.0</td>
</tr>
<tr>
<td>160</td>
<td>107.0</td>
<td>210.0</td>
<td>317.0</td>
<td>306.0</td>
<td>207.0</td>
</tr>
<tr>
<td>170</td>
<td>90.5</td>
<td>169.0</td>
<td>260.0</td>
<td>257.0</td>
<td>173.0</td>
</tr>
<tr>
<td>180</td>
<td>76.8</td>
<td>139.0</td>
<td>216.0</td>
<td>218.0</td>
<td>145.0</td>
</tr>
<tr>
<td>190</td>
<td>65.7</td>
<td>115.0</td>
<td>181.0</td>
<td>187.0</td>
<td>124.0</td>
</tr>
<tr>
<td>200</td>
<td>56.4</td>
<td>97.1</td>
<td>153.0</td>
<td>162.0</td>
<td>106.0</td>
</tr>
<tr>
<td>300</td>
<td>15.0</td>
<td>29.5</td>
<td>44.5</td>
<td>55.7</td>
<td>33.2</td>
</tr>
<tr>
<td>400</td>
<td>5.11</td>
<td>15.6</td>
<td>20.7</td>
<td>29.6</td>
<td>16.2</td>
</tr>
<tr>
<td>500</td>
<td>2.04</td>
<td>9.51</td>
<td>11.5</td>
<td>18.4</td>
<td>9.32</td>
</tr>
<tr>
<td>600</td>
<td>0.909</td>
<td>6.00</td>
<td>6.91</td>
<td>12.1</td>
<td>5.73</td>
</tr>
<tr>
<td>700</td>
<td>0.439</td>
<td>3.86</td>
<td>4.29</td>
<td>8.20</td>
<td>3.63</td>
</tr>
<tr>
<td>800</td>
<td>0.226</td>
<td>2.50</td>
<td>2.72</td>
<td>5.62</td>
<td>2.34</td>
</tr>
<tr>
<td>900</td>
<td>0.122</td>
<td>1.65</td>
<td>1.76</td>
<td>3.90</td>
<td>1.54</td>
</tr>
<tr>
<td>1000</td>
<td>0.0684</td>
<td>1.10</td>
<td>1.16</td>
<td>2.73</td>
<td>1.02</td>
</tr>
</tbody>
</table>

For an intermediate mass Higgs, the $K$ factor ($\sigma_{NLO}/\sigma_{LO}$) has been estimated in the Effective Higgs Approximation (EHA) [279]. The EHA neglects terms of $O(m_H/\sqrt{s})$ and higher and works extremely well for $e^+e^- \rightarrow t\bar{t}H$ already at $\sqrt{s}=1$ TeV. However, it is a much poorer approximation in the $pp \rightarrow t\bar{t}H$ case, since it does not include the $t$-channel emission of a Higgs boson for $gg \rightarrow t\bar{t}H$. Indicatively, at $\sqrt{s} = 14$ TeV, for a SM-like Higgs boson with $m_H \approx 100-130$ GeV, the EHA gives $K \approx 1.2 - 1.5$, with some uncertainty due to scale and PDF dependence. Only the complete knowledge of the NLO level of QCD corrections will allow to reduce the strong scale and PDF dependence of the LO and EHA cross sections. For the following analysis we choose to use the pure LO cross section with no $K$-factor, both due to the uncertainty of the result and for consistency with the corresponding background cross sections. However, one should point out that, due to the choice of a quite low QCD scale ($\mu=m_H$), a sort of effective $K$-factor has been automatically included in our analysis.

In the following subsection we present the analysis and results from the ATLAS collaboration as well as a discussion of the main backgrounds. The analysis mainly focuses on the search and study of an intermediate mass Higgs boson. To introduce the study, it is useful to discuss and qualitatively understand the size of the possible irreducible backgrounds in the $100 < m_H < 140$ GeV mass region.

Given the relatively small number of events that will be available, one should try to consider all possible decay channels of the Higgs boson in the intermediate mass region: $H \rightarrow b\bar{b}$, $\tau^+\tau^-$, $\gamma\gamma$, $WW$ and $ZZ$. The corresponding irreducible backgrounds are: 1) $t\bar{t}bb$, 2) $t\bar{t}\tau\tau$, 3) $t\bar{t}\gamma\gamma$, 4) $t\bar{t}WW$, and 5) $t\bar{t}ZZ$. The number of events expected from signal and background signatures for 1)-5) are presented in Fig. 47. This figure shows the number of signal and background events in each bin of the corresponding invariant mass: $M_{bb}$, $M_{\tau\tau}$, $M_{\gamma\gamma}$, $M_{WW}$ or $M_{ZZ}$. They are obtained multiplying the $t\bar{t}H$ cross section by the respective Higgs boson branching ratios. In order to take into account finite mass resolution effects, we have chosen 10 GeV bins for the $M_{\gamma\gamma}$ distribution and 50 GeV for the others. The presented...
Table 26: Leading-order cross sections for various $t\bar{t}XX$ backgrounds.

<table>
<thead>
<tr>
<th>cuts</th>
<th>$t\bar{t}b\bar{b}$</th>
<th>$t\bar{t}\tau\tau$</th>
<th>$t\bar{t}\gamma\gamma$</th>
<th>$t\bar{t}WW$</th>
<th>$t\bar{t}ZZ$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>\eta_b</td>
<td>&lt; 3$</td>
<td>$E_T^{\tau} &gt; 15$ GeV</td>
<td>$</td>
<td>\eta_{\tau}</td>
</tr>
<tr>
<td>$m_{\bar{b}} &gt; 90$ GeV</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$ [fb] $q\bar{q}$</td>
<td>41.2</td>
<td>2.9</td>
<td>2.73</td>
<td>0.50</td>
<td>1.11</td>
</tr>
<tr>
<td>$gg$</td>
<td>846.</td>
<td>15.7</td>
<td>1.82</td>
<td>1.52</td>
<td>0.567</td>
</tr>
<tr>
<td>$q\bar{q}+gg$</td>
<td>887.</td>
<td>18.6</td>
<td>4.55</td>
<td>2.53</td>
<td>1.68</td>
</tr>
</tbody>
</table>

numbers correspond to 30 fb$^{-1}$ of integrated luminosity. The corresponding total cross sections are given in Table 26.

Cross sections for backgrounds 1)-3) were calculated with the kinematic cuts shown in Table 26, while for processes 4) and 5) no cuts were applied. We have used CTEQ4L PDF and $\mu^2 = M_{XX}$, where $XX$ is $b\bar{b}$, $\tau^+\tau^-$, $\gamma\gamma$, $WW$ or $ZZ$ depending on the channel. One can see that the $t\bar{t}b\bar{b}$ signature has the highest signal (and background) event rate. It has been the object of the study of the ATLAS collaboration and will be discussed in the next section. The $t\bar{t}\gamma\gamma$ channel has also been the subject of [280] where signal as well as reducible and irreducible backgrounds have been studied in details at the parton level. However, one can see that other signatures could also be interesting and helpful in searching for the Higgs boson and measuring the $t\bar{t}H$ Yukawa coupling, and should be taken into account in future studies.

9.1 $t\bar{t}H$: Analyses and Results

The ATLAS collaboration has studied several channels in which the discovery of a SM-like Higgs boson would be possible and obtained a quite complete Higgs discovery potential [30]. One of the most important channels for discovery of a low mass Higgs boson ($100-130$ GeV) is the $t\bar{t}H$, $H \rightarrow b\bar{b}$ channel, in which it is possible to obtain quite large signal significance [281] and also to measure the top-Higgs Yukawa coupling.

The final state of this channel consists of two $W$ bosons and four $b-$jets: two from the decay of the top quarks, and two from the decay of the Higgs boson. In order to trigger signal events, one $W$ boson is required to decay leptonically. The second $W$ boson is reconstructed from the decay to a $q\bar{q}$ pair. This channel could be also investigated with both $W$ bosons decaying leptonically. However, for this signature the total branching ratio is much smaller and, in addition, it is more difficult to reconstruct two neutrino momenta from the measured missing energy.

In the analysis both top quarks are fully reconstructed, and this reduces most of the $W+$jets background. The reconstruction is done using strategies similar to those discussed in Section 3.5 for the kinematic studies of $t\bar{t}$ production. The main backgrounds for this process are:

- the irreducible continuum $t\bar{t}b\bar{b}$ background;
- the irreducible resonant $t\bar{t}Z$ background, which is not very important for this channel as it has a very small cross section;
- the reducible backgrounds which contain jets misidentified as $b$-jets, such as $t\bar{t}jj$, $Wjjjjjj$, $WWb\bar{b}jj$, etc.

After the reconstruction of the two top quarks, it has been found that the most dangerous background is $t\bar{t}b\bar{b}$ (56% of all $t\bar{t}$+jets background). In Table 27 we give $\sigma \times \text{BR}$, where BR represents the product of the branching ratios for $t \rightarrow Wb$, $W_1 \rightarrow \ell\nu$, $W_2 \rightarrow q\bar{q}$, and $H \rightarrow b\bar{b}$. We also give the number of events expected after the reconstruction procedure for 3 years of low luminosity operation ($b$-tagging efficiency $\epsilon_b = 60\%$; probability to mistag $c$-jet as $b$-jet $\epsilon_c = 10\%$; probability to
Fig. 47: Number of events for $t\bar{t}H$ signal (solid line) and background $t\bar{t}\ell\ell$, $t\bar{t}\gamma\gamma$, $t\bar{t}WW$, $t\bar{t}ZZ$ signatures (histogram), as a function of the corresponding invariant masses $M_{XX}$, assuming 30 fb$^{-1}$ of integrated luminosity at $\sqrt{s}=14$ TeV. The bin size is 10 GeV for the $M_{\ell\ell}$ distribution and 50 GeV for the others.

mistag any other jet as $b$-jet $\epsilon_b = 1\%$; $p_T^{\text{jet}} > 15$ GeV; lepton identification efficiency $\epsilon_L = 90\%$; $p_T^{\ell,i} > 20$ GeV), and after one year of high luminosity operation (for high luminosity the $b$-tagging efficiency is degraded to $\epsilon_b = 50\%$ ($\epsilon_c$, $\epsilon_j$ and $\epsilon_L$ remain unchanged), the threshold on jet reconstruction is raised to $p_T > 30$ GeV and the electron $p_T$ threshold is raised to $p_T^e > 30$ GeV. Combined results are also shown.

Figure 48 shows the signal and background shapes for $m_H = 120$ GeV and 100 fb$^{-1}$ of integrated luminosity obtained with combined detector performance (30 fb$^{-1}$ with low luminosity and 70 fb$^{-1}$ with high luminosity). On the other hand, Fig.49 illustrates the signal shape for $m_H = 100$ GeV, as obtained by using the full (GEANT) simulation of the detector. In this figure, the shaded area represents the true signal where both $b$-jets come from the Higgs boson, and the solid line stands for the signal obtained through the method that we described above. The combinatorial background, which comes from taking at least one $b$-jet from a top instead the one from the Higgs, is quite large and the signal purity is at the level of 60% for low luminosity.

For the fast simulation the $m_{b\bar{b}}$ peak mass resolution is $\sigma_{m_{b\bar{b}}} = 19.0$ GeV, while for the full simulation, including the influence of electronic noise and the threshold on cell energy, a resolution $\sigma_{m_{b\bar{b}}} = 20.0$ GeV has been obtained.
Table 27: Cross sections multiplied by branching ratios and numbers of events after all cuts, including the $\pm 30 m_{t\bar{t}}$ mass window cut, for $30 \text{ fb}^{-1}$ (low luminosity detector performance), $100 \text{ fb}^{-1}$ (high luminosity detector performance) and combined $100 \text{ fb}^{-1}$ (30 fb$^{-1}$ with low luminosity and 70 fb$^{-1}$ with high luminosity detector performance) of integrated luminosity.

<table>
<thead>
<tr>
<th>process</th>
<th>$\sigma \times \text{BR}$ (pb)</th>
<th>nr. of reconstructed events</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>low lumi</td>
</tr>
<tr>
<td>$t\bar{t}H, m_H = 120 \text{ GeV}$</td>
<td>0.16</td>
<td>40</td>
</tr>
<tr>
<td>$t\bar{t} + \text{jets}$</td>
<td>87</td>
<td>120</td>
</tr>
<tr>
<td>$Wjjjjjj$</td>
<td>65200</td>
<td>5</td>
</tr>
<tr>
<td>$t\bar{t}Z$</td>
<td>0.02</td>
<td>2</td>
</tr>
<tr>
<td>total background</td>
<td></td>
<td>127</td>
</tr>
<tr>
<td>$S/J$</td>
<td></td>
<td>0.32</td>
</tr>
<tr>
<td>$S/\sqrt{B}$</td>
<td></td>
<td>3.6</td>
</tr>
<tr>
<td>$S_{t\bar{t}\to b\bar{b}}/S_{\text{total}}$</td>
<td></td>
<td>59%</td>
</tr>
<tr>
<td>$\delta_{y_h}/y_h$ (stat.)</td>
<td></td>
<td>16.2%</td>
</tr>
</tbody>
</table>

Similar analyses have been performed for the $t\bar{t}H, H \to \gamma\gamma$ channel. Since the signal rate for this channel is very small, it will not be useful during the low luminosity period. However, thanks to the high purity of the signal, it will be possible to obtain between 4 or 5 signal events above 1 event from $t\bar{t}\gamma\gamma$ background per one year of high luminosity operation [282]. To increase the signal rate, $WH$ and $ZH$ with $H \to \gamma\gamma$ channels have been included into the analysis and 14 signal events above 5 background events ($W\gamma\gamma, Z\gamma\gamma, t\bar{t}\gamma\gamma$ and $b\bar{b}\gamma\gamma$) are expected for one year of high luminosity operation [30].

The statistical uncertainty in the determination of the top-Higgs Yukawa coupling $y_h$ is given in the last row of Table 27. These results assume that the theoretical uncertainty is small, as we expect to be the case by the time the LHC turns on. Many statistical uncertainties of the direct measurement of $y_h$, such as those associated with uncertainties in the integrated luminosity and in the $t\bar{t}$ reconstruction efficiency, could be controlled by comparing the $t\bar{t}H$ rate with the $t\bar{t}$ rate.

To conclude, the $t\bar{t}H, H \to b\bar{b}$ and $H \to \gamma\gamma$ channels are very useful for Higgs boson discovery as well as for the measurement of the top-Higgs Yukawa coupling.

9.11 A closer look at the $t\bar{t}b\bar{b}$ background: CompHEP versus PYTHIA

It is necessary to stress that the correct understanding of the $t\bar{t}b\bar{b}$ background is one of the main points of this study. One can simulate this background using PYTHIA, by generating events of top pair production and emitting $b\bar{b}$ pairs from the gluon splitting after the initial and final state radiation. In order to understand how good or bad this approximation is, one needs to calculate and simulate the complete $t\bar{t}b\bar{b}$ process. We have done this using the CompHEP package [275].

In order to compare CompHEP and PYTHIA on the same footing, one should take into account the effects of the initial and final state radiation in CompHEP. This has been done through a CompHEP-PYTHIA interface [283]. We use parton level events generated by CompHEP and link them to PYTHIA in order to include initial and final state radiation effects as well as hadronization effects.

Table 28 presents parton level CompHEP and PYTHIA cross sections including branching ratios of the $W$-boson decay, for the same choice of structure function (CTEQ4L [115]) and QCD scale ($\mu^2 = m_T^2 + p_T^2$ (average)). We can see a good agreement for the total cross sections between the exact calculation and the gluon splitting approximation.

In Fig. 50 we present the distribution of $b$-jet separation in $t\bar{t}b\bar{b}$ events. One can see a quite good agreement between CompHEP and PYTHIA. Figures 51 and 52 compare the transverse momentum
Fig. 48: Invariant mass distribution of tagged $b$-jet pairs in fully reconstructed $t\bar{t}H$ signal events and background events, obtained using the fast simulation of the ATLAS detector, for $m_H = 120$ GeV and integrated luminosity of 100 fb$^{-1}$ (30 fb$^{-1}$ at low plus 70 fb$^{-1}$ at high luminosity). The points with error bars represent the result of a single experiment and the dashed line represents the background distribution.

Fig. 49: Invariant mass distribution of tagged $b$-jet pairs in fully reconstructed $t\bar{t}H$ signal events, obtained using a full (GEANT) simulation of the ATLAS detector, for $m_H = 100$ GeV and low luminosity performance. The shaded area denotes those events for which the jet assignment in the Higgs boson reconstruction is correct.

Table 28: Results for the $t\bar{t}b\bar{b}$ background, assuming an integrated luminosity $\mathcal{L}_{int} = 30$ fb$^{-1}$: CompHEP (ISR and FSR included) versus PYTHIA (default).

<table>
<thead>
<tr>
<th>Selection</th>
<th>CompHEP</th>
<th>PYTHIA</th>
<th>CompHEP / PYTHIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 $b$-quarks with $p_T &gt; 15$ GeV/$c$ ; $</td>
<td>\eta</td>
<td>&lt; 2.5$</td>
<td>92000 events</td>
</tr>
<tr>
<td>$\Delta R(b,b) &gt; 0.5$</td>
<td>$\sigma = 3.1$ pb</td>
<td>$\sigma = 2.9$ pb</td>
<td></td>
</tr>
<tr>
<td>$b$-quarks not from top decay</td>
<td>54000 events</td>
<td>48900 events</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Table 28: Results for the $t\bar{t}b\bar{b}$ background, assuming an integrated luminosity $\mathcal{L}_{int} = 30$ fb$^{-1}$: CompHEP (ISR and FSR included) versus PYTHIA (default).

<table>
<thead>
<tr>
<th>Selection</th>
<th>CompHEP</th>
<th>PYTHIA</th>
<th>CompHEP / PYTHIA</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 $b$-quarks with $p_T &gt; 15$ GeV/$c$ ; $</td>
<td>\eta</td>
<td>&lt; 2.5$</td>
<td>92000 events</td>
</tr>
<tr>
<td>$\Delta R(b,b) &gt; 0.5$</td>
<td>$\sigma = 3.1$ pb</td>
<td>$\sigma = 2.9$ pb</td>
<td></td>
</tr>
<tr>
<td>$b$-quarks not from top decay</td>
<td>54000 events</td>
<td>48900 events</td>
<td>1.10</td>
</tr>
</tbody>
</table>

distributions of the most energetic $b$-jet and of the least energetic $b$-jet in $t\bar{t}b\bar{b}$ production, as reproduced using PYTHIA and CompHEP respectively. These distributions also confirm that PYTHIA describes well the $t\bar{t}b\bar{b}$ background.

9.2 Summary and conclusions for $t\bar{t}H$ production

The associated production of a Higgs boson with a top-antitop pair is important for the discovery of an intermediate mass Higgs boson ($m_H \approx 100 - 130$ GeV) and provides a direct determination of the top-Higgs Yukawa coupling. From studies of the couplings and of the CP-parity of the Higgs boson [284] it will be possible to discriminate, for instance, a SM-like Higgs boson from a generic MSSM one.

The ATLAS analysis has focused on the $t\bar{t}H$, $H \rightarrow b\bar{b}$ channel for the low luminosity run of the LHC (30 fb$^{-1}$ of integrated luminosity). The results presented in Section 9.1 are very encouraging and indicate that a signal significance of 3.6 as well as a precision of 16% in the determination of the Yukawa coupling can be reached (for $m_H = 100$ GeV). Better results can be obtained from the high luminosity run of the LHC (100 fb$^{-1}$ of integrated luminosity), when also the high purity $t\bar{t}H, H \rightarrow \gamma\gamma$ channel is available.
Fig. 50: Results for $t\bar{t}b\bar{b}$ background, assuming an integrated luminosity $L_{\text{int}} = 4.4$ fb$^{-1}$; CompHEP (ISR and FSR included) versus PYTHIA (default).

Fig. 51: Transverse momentum distribution of the most energetic $b$-jet from $t\bar{t}b\bar{b}$ processes at the LHC: CompHEP (dark-grey histogram) versus PYTHIA (light-grey)

Fig. 52: Transverse momentum distribution of the least energetic $b$-jet from $t\bar{t}b\bar{b}$ processes at the LHC: CompHEP (dark-grey histogram) versus PYTHIA (light-grey)

A  APPENDIX: b-TAGGING AND JET E-SCALE CALIBRATION IN TOP EVENTS$^{25}$

For the reconstruction of the top events and in particular for the precision measurement of the top mass two important aspects in the detector performance have to be considered:

- the $b$-quark jet tagging capabilities and efficiency in top events, and
- the jet energy scale calibration for the light quark jets but in particular for the $b$-jets.

In both experiments ATLAS and CMS several studies have been made on these, highlights of which are presented here. From the preliminary results available so far, there is confidence that the numbers used or implied in the analyses presented in this report are realistic. Needless to say that these are preliminary results and several detailed studies need to be performed with the final detector simulations and the first LHC data.

$^{25}$Section coordinator: I. Efthymiopoulos
A1 b-jet tagging in the top events

ATLAS has done extensive studies for the b-tagging performance using jets from the decay of 100 and 400 GeV Higgs bosons ([30], Chapter 10). In Fig. 53 the rejection factors for the light quark jets versus the b-tagger efficiency and the jet p_T are shown.

Typically in the ATLAS analyses discussed here, and in particular for the fast simulation studies, an overall b-jet tagging efficiency of 60% (50%) for low (high) luminosity of LHC is used. The mis-tagging inefficiencies for the c-jets (or other light quark jets) were 10% (1%) for the p_T range interesting for the top physics. Although most of the studies were done with events from the Higgs decays, the results were verified with the top events themselves and no significant differences were found.

A2 Absolute jet energy scale calibration

Determining the absolute jet energy scale at LHC will be a rather complex issue because it is subject to both physics (initial-final state radiation, fragmentation, underlying event, jet algorithm etc.) and detector (calorimeter response over a wide range of energies and over the full acceptance of the detector, non-linearities at high energies, e/h ratio etc.) effects. All these have to be understood at the level of a fraction of a percent in terms of systematic uncertainties as required for the precision measurements of the top mass.

ATLAS has done an extensive study of the possible in situ jet scale calibration methods using specific data samples available at LHC ([30], Chapter 12). In general, good candidate event classes at LHC will be:

- reconstruction of W → jj decays within the top events themselves [12] to obtain the light quark jet calibration and,
- events containing a Z boson decaying into leptons balanced with one high-p_T jet to cross-check the light quark jet calibration but in addition to calibrate the b-jets and extend the energy reach to the TeV range.

In Fig. 54 the results obtained are shown. As can be seen (left plot) for the case of W → jj events,
once the jet 4-vectors are rescaled using the $M_{W}$ constraint the required 1% uncertainty is reached for jets with $p_T > 70$ GeV up to several hundred GeV. The lower and upper end of this range will depend on how well residual systematic effects can be controlled in the data and the Monte Carlo simulation [285].

The use of the $Z$ + jets sample in LHC is a bit less straightforward than at the Tevatron [286] due to the ISR radiation which produces an additional high-$p_T$ jet which degrades the quality of the $p_T$-balance between the $Z$ boson and the leading jet. In Fig. 54 (right) the variation of the average fractional imbalance between the $p_T$ of the leading jet and the $Z$ boson as a function of the $p_T$ of the jet. Rescaling the jet $p_T$ to satisfy $p_T$ balance with the $Z$ boson and applying tight selection criteria (jet veto and difference in azimuth $\delta \phi$ between the reconstructed $Z$ and the leading jet) the desired goal of $\pm 1\%$ systematic uncertainty on the absolute jet energy scale can be achieved for jets with $p_T > 50$ GeV and up to the TeV range [287].

However, as shown in Fig. 54 (right), it is possible, taking advantage of the large rate and requiring tight event selection criteria, to obtain the required precision for jets with $p_T > 40$ GeV and up to the TeV range.

Clearly more studies are needed, and will be done in the years to come, to understand the limitations of the proposed methods and to devise possible improvements.

B APPENDIX: DIRECT MEASUREMENT OF TOP QUANTUM NUMBERS

B1 Top spin and experimental tests

Evidence to date is circumstantial that the top events analysed in Tevatron experiments are attributable to a spin-1/2 parent. The evidence comes primarily from consistency of the distribution in momentum of the decay products with the pattern expected for the weak decay $t \to b + W$, with $W \to \ell + \nu$ or $W \to$ jets, where the top $t$ is assumed to have spin-1/2.

It is valuable to ask whether more definitive evidence for spin-1/2 might be obtained in future experiments at the Tevatron and LHC. We take one look at this question by studying the differential cross section $d\sigma/dM_{t\bar{t}}$ in the region near production threshold [288]. Here $M_{t\bar{t}}$ is the invariant mass of the $t\bar{t}$ pair. We contrast the behaviour of $t\bar{t}$ production with that expected for production of a pair of spin-0 objects. We are motivated by the fact that in electron-positron annihilation, $e^+ + e^- \rightarrow q + \bar{q}$, there is a dramatic difference in energy dependence of the cross section in the near-threshold region for quark spin assignments of 0 and 1/2.

For definiteness, we compare top quark $t$ and top squark $\tilde{t}$ production since a consistent phenomenology exists for top squark pair production, obviating the need to invent a model of scalar quark production. Moreover, top squark decay may well mimic top quark decay. Indeed, if the chargino $\tilde{\chi}^+$ is lighter than the light top squark, as is true in many models of supersymmetry breaking, the dominant decay of the top squark is $\tilde{t} \rightarrow b + \tilde{\chi}^+$. If there are no sfermions lighter than the chargino, the chargino decays to a $W$ and the lightest neutralino $\tilde{\chi}^0$. In another interesting possible decay mode, the chargino decays into a lepton and slepton, $\tilde{\chi}^+ \rightarrow \ell^+ \tilde{\nu}$. The upshot is that decays of the top squark may be very similar to those of the top quark, but have larger values of missing energy and softer momenta of the visible decay products. A recent study for Run II of the Tevatron [289] concluded that even with 4 fb$^{-1}$ of data at the Tevatron, and including the LEP limits on chargino masses, these decay modes remain open (though constrained) for top squarks with mass close to the top quark mass.

At the energy of the CERN LHC, production of $t\bar{t}$ pairs and of $\tilde{t}\tilde{\bar{t}}$ pairs is dominated by $gg$ subprocess, and the threshold behaviours in the two cases do not differ as much as they do for the $q\bar{q}$ incident channel. In Fig. 55(a), we show the partonic cross sections $\hat{\sigma}(\sqrt{s})$ as functions of the partonic sub-energy $\sqrt{s}$ for the $gg$ channel. In Fig. 55(b), we display the hadronic cross sections for $pp \rightarrow t\bar{t}X$ and $pp \rightarrow \tilde{t}\tilde{\bar{t}}X$ at proton-proton center-of-mass energy 14 TeV as a function of pair mass. We include the relatively small contributions from the $q\bar{q}$ initial state. After convolution with parton densities, the shape of the $t\bar{t}$ pair mass distribution is remarkably similar to that of the $t\bar{t}$ case.

Based on shapes and the normalisation of cross sections, it is difficult to exclude the possibility that some fraction (on the order of 10%) of top squarks with mass close to 165 GeV is present in the current Tevatron $t\bar{t}$ sample. The invariant mass distribution of the produced objects, $M_{t\bar{t}}$, is quite different at the partonic level for the $q\bar{q}$ initial state (dominant at the Tevatron), but much less so for the $gg$ initial state (dominant at the LHC). However, after one folds with the parton distribution functions, the difference in the $q\bar{q}$ channel at the Tevatron is reduced to such an extent that the $M_{t\bar{t}}$ distribution is not an effective means to isolate top squarks from top quarks.

Ironically, the good agreement of the absolute rate for $t\bar{t}$ production with theoretical expectations [45, 47] would seem to be the best evidence now for the spin-1/2 assignment in the current Tevatron sample.
A promising technique to isolate a top squark with mass close to $m_t$ would be a detailed study of the momentum distribution of the top quark decay products (presumably in the top quark rest frame). One could look for evidence of a chargino resonance in the missing transverse energy and charged lepton momentum, or for unusual energy or angular distributions of the decay products owing to the different decay chains. One could also look for deviations from the expected correlation between angular distributions of decay products and the top spin [167].

As a concrete example of an analysis of this type, in Fig. 56 we present the distribution in the invariant mass $X$ of the bottom quark and charged lepton, with $X = (p_b + p_{\ell^+})^2$, where the bottom quark and lepton are decay products of either a top quark with $m_t = 175$ GeV or a top squark $\tilde{t} \rightarrow \tilde{\chi}^+ b \rightarrow W^+ \tilde{\chi}^0 b \rightarrow \ell^+ \nu \tilde{\chi}^0 b$, with $m_{\tilde{t}} = 165$ GeV, $m_{\tilde{\chi}^+} = 130$ GeV, $m_{\tilde{\chi}^0} = 40$ GeV, and $m_b = 5$ GeV. The $X$ distribution is a measure of the degree of polarisation of the $W$ boson in top quark decay [290], and the figure shows that the different dynamics responsible for top squark decay result in a very different distribution, peaked at much lower $X$. The areas under the curves are normalised to the inclusive $t\bar{t}$ and $\tilde{t}\tilde{\ell}$ rates at the LHC.

In this simple demonstration potentially important effects are ignored such as cuts to extract the $t\bar{t}$ signal from its backgrounds, detector resolution and efficiency, and ambiguities in identifying the correct $b$ with the corresponding charged lepton from a single decay. Detailed simulations would be required to determine explicitly how effective this variable would be in extracting a top squark sample from top quark events. Nevertheless, such techniques, combined with the large $t\bar{t}$ samples at the Tevatron Run II and LHC, should prove fruitful in ruling out the possibility of a top squark with mass close to the top quark mass, or alternatively, in discovering a top squark hidden in the top sample.

**B2 Direct Measurement of the Top Quark Electric Charge**

In order to confirm that the electric charge of the top quark is indeed $Q_{\text{top}} = 2/3$, one can either measure the charge of the $b$-jet and $W$ boson, or attempt to directly measure the top quark electromagnetic coupling through photon radiation in

$$pp \rightarrow t\bar{t}\gamma, \quad pp \rightarrow t\bar{t}, \ t \rightarrow Wb\gamma.$$  \hspace{1cm} (85)

Since the process $pp \rightarrow t\bar{t}\gamma$ is dominated by $gg$ fusion at the LHC, one expects that the $t\bar{t}\gamma$ cross section is approximately proportional to $Q_{\text{top}}^2$. For radiative top decays the situation is more complicated because
the photon can also be radiated off the $b$-quark or the $W$ line.

The charge of the $b$-jet can most easily be measured by selecting events where the $b$-quarks are identified through their semi-leptonic decays, $b \to \ell \nu c$ with $\ell = e, \mu$. The small semi-leptonic branching ratio of the $b$-quark ($\text{Br}(b \to \ell \nu c) \approx 10\%)$ and wrong sign leptons originating from $B \to \bar{B}$ mixing are the main problems associated with this method. For a quantitative estimate realistic simulations are needed. Nevertheless, we believe that the enormous number of top quark events produced at the LHC should make it possible to use semi-leptonic $b$-tagging to determine the electric charge of the top quark.

In our analysis, we focus on top charge measurement through the photon radiation processes listed in (85), concentrating on the lepton+jets mode,

$$pp \to \gamma \ell \nu jjb\bar{b},$$

We assume that both $b$-quarks are tagged with a combined efficiency of 40%. Top quark and $W$ decays are treated in the narrow width approximation. Decay correlations are ignored. To simulate detector response, the following transverse momentum, rapidity and separation cuts are imposed:

$$p_T(b) > 15 \text{ GeV}, \quad \left| y(b) \right| < 2, \quad \text{(87)}$$
$$p_T(\ell) > 20 \text{ GeV}, \quad \left| \eta(\ell) \right| < 2.5, \quad \text{(88)}$$
$$p_T(j) > 20 \text{ GeV}, \quad \left| \eta(j) \right| < 2.5, \quad \text{(89)}$$
$$p_T(\gamma) > 30 \text{ GeV}, \quad \left| \eta(\gamma) \right| < 2.5, \quad \text{(90)}$$
$$p_T > 20 \text{ GeV}, \quad \text{all } \Delta R_{b} > 0.4. \quad \text{(91)}$$

In addition, to suppress contributions from radiative $W$ decays, we require that

$$m(jj) > 90 \text{ GeV} \quad \text{and} \quad m_T(\ell\gamma; p_T) > 90 \text{ GeV}, \quad \text{(92)}$$

where $m_T$ is the cluster transverse mass of the $\ell\gamma$ system.

The events passing the cuts listed in (88) – (92) can then be split into three different subsamples:

1. By selecting events which satisfy

$$m(bj\ell \gamma) > 190 \text{ GeV} \quad \text{and} \quad m_T(b\ell\gamma; p_T) > 190 \text{ GeV}, \quad \text{(93)}$$

radiative top quark decays can be suppressed and an almost pure sample of $t\bar{t}\gamma$ events is obtained ("t\bar{t}\gamma cuts").

2. For

$$m_T(b_{1,2} \ell\gamma; p_T) < 190 \text{ GeV} \quad \text{and} \quad m(b_{2,1} jj \gamma) > 190 \text{ GeV}, \quad \text{(94)}$$

the process $pp \to t\bar{t}, t \to W b\gamma, W \to \ell\nu$ dominates ("t\to Wb\gamma, W\to \ell\nu cuts").

3. Requiring

$$m_T(b_{1,2} \ell\gamma; p_T) > 190 \text{ GeV} \quad \text{and} \quad 150 \text{ GeV} < m(b_{2,1} jj \gamma) < 190 \text{ GeV}, \quad \text{(95)}$$

one obtains an event sample where the main contribution originates from the process $pp \to t\bar{t}, t \to W b\gamma, W \to jj$ ("t\to Wb\gamma, W\to jj cuts").

For $m_t = 175 \text{ GeV}, Q_{\text{top}} = 2/3$, and $\int L dt = 100 \text{ fb}^{-1}$, one expects about 2400, 11000 and 9400 events in the regions of phase space corresponding to the three sets of cuts. We have not studied any potential background processes. The main background should originate from $W\gamma+\text{jets}$ production and should be manageable in a way similar to the $W^+\text{jets}$ background for regular $t\bar{t}$ production.

The differential cross section for the photon transverse momentum at the LHC is shown in Fig. 57. Results are shown for $m_t = 175 \text{ GeV}$ and three “top” quark charges: $Q_{\text{top}} = 2/3, Q_{\text{top}} = -4/3$, and
Fig. 57: The differential cross section for the photon transverse momentum in the reaction $pp \rightarrow \gamma t\bar{t}$ at the LHC for three different “top” quark charges.

$Q_{top} = 1/3$. For $Q_{top} = -4/3$, the “top” quark decays into a $W^-$ and a $b$-quark instead of $t \rightarrow W^+ b$. If $Q_{top} = 1/3$, the “$b$”-quark originating from the “top” decay is a (exotic) charge $-2/3$ quark. In the $t\bar{t}\gamma$ region (Eq. (93) and Fig. 57a), the $pp \rightarrow \gamma t\bar{t} b\bar{b}$ cross section for a charge $-4/3$ (“top” quark is uniformly a factor $\approx 3.3$ larger ($\approx 2.3$ smaller) than that for $Q_{top} = 2/3$, reflecting the dominance of the $gg \rightarrow t\bar{t}\gamma$ process for which the cross section scales with $Q_{top}^2$. On the other hand, for the $pp \rightarrow t\bar{t}$, $t \rightarrow Wb\gamma$, $W \rightarrow \ell\nu$ selection cuts (Eq. (94) and Fig. 57b), the cross section for $Q_{top} = -4/3$ is a factor 3 to 5 smaller than that for a charge 2/3 top quark, due to destructive interference effects in the $t \rightarrow Wb\gamma$ matrix element. If $Q_{top} = 1/3$, the interference is positive, and the cross section is about a factor 2.5 larger than for $Q_{top} = 2/3$. The results for the $t \rightarrow Wb\gamma$, $W \rightarrow jj$ selection cuts (95) are similar to those shown in Fig. 57b, and are therefore not shown here. Note that the photon $p_T$ distribution for radiative top decays is much softer than that for $t\bar{t}\gamma$ production.

From our (simplified) calculation we conclude that the large number of double-tagged $\gamma t\bar{t} b\bar{b}$ events, together with the significant changes in the $t\bar{t}\gamma$ and the $t\bar{t}$, $t \rightarrow Wb\gamma$ cross sections should make it possible to accurately determine $Q_{top}$ at the LHC.

C APPENDIX: 4th GENERATION QUARKS

For completeness, we present here results for the total cross section of possible heavy quarks above the top quark mass. The scale and PDF dependences are shown in Fig. 58. The uncertainty due to the choice of scale is comparable to that of the $t\bar{t}$ cross section, although the effects of the higher order corrections are more important at large masses (see Fig. 59). The uncertainty induced by PDF changes becomes very large at large masses, in particular if one considers sets such as CTEQ5HJ which have harder gluons. Notice however that the relative effect due to the resummation corrections depends only very weakly upon the choice of PDF’s (cf. Section 3.2).

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$Q_{top} = 1/3$. For $Q_{top} = -4/3$, the “top” quark decays into a $W^-$ and a $b$-quark instead of $t \rightarrow W^+ b$. If $Q_{top} = 1/3$, the “$b$”-quark originating from the “top” decay is a (exotic) charge $-2/3$ quark. In the $t\bar{t}\gamma$ region (Eq. (93) and Fig. 57a), the $pp \rightarrow \gamma t\bar{t} b\bar{b}$ cross section for a charge $-4/3$ (“top” quark is uniformly a factor $\approx 3.3$ larger ($\approx 2.3$ smaller) than that for $Q_{top} = 2/3$, reflecting the dominance of the $gg \rightarrow t\bar{t}\gamma$ process for which the cross section scales with $Q_{top}^2$. On the other hand, for the $pp \rightarrow t\bar{t}$, $t \rightarrow Wb\gamma$, $W \rightarrow \ell\nu$ selection cuts (Eq. (94) and Fig. 57b), the cross section for $Q_{top} = -4/3$ is a factor 3 to 5 smaller than that for a charge 2/3 top quark, due to destructive interference effects in the $t \rightarrow Wb\gamma$ matrix element. If $Q_{top} = 1/3$, the interference is positive, and the cross section is about a factor 2.5 larger than for $Q_{top} = 2/3$. The results for the $t \rightarrow Wb\gamma$, $W \rightarrow jj$ selection cuts (95) are similar to those shown in Fig. 57b, and are therefore not shown here. Note that the photon $p_T$ distribution for radiative top decays is much softer than that for $t\bar{t}\gamma$ production.

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Fig. 58: Heavy quark total production rates. Left figure: scale dependence at fixed NLO (dashed lines in the lower inset), and at NLO+NLL (solid lines). Right figure: PDF dependence. See the Section 3.2 for details.

Fig. 59: Heavy quark total production rates. Left figure: fractional contribution induced by resummation contributions of order $\mathcal{O}(\alpha_s^\infty)$. Right figure: initial state composition.

D APPENDIX: MONTE CARLO TOOLS

D1 Parton shower Monte Carlos

General purpose Monte Carlo event generators like HERWIG, PYTHIA and ISAJET are essential tools for measuring the top quark cross section, mass and other production and decay properties. They are complementary to the QCD tools described in Section 3.1 since, although they are less reliable for inclusive quantities like the total cross section, they provide a fully exclusive description of individual events at the hadron level. These can be analysed in exactly the same way as experimental data and can be put through full or fast detector simulations to estimate experimental systematics. In certain kinematic regions, particularly the quasi-elastic limit in which accompanying radiation is suppressed, they give more reliable QCD predictions than the available calculations. They include approximate treatments of higher order perturbative effects, hadronisation, secondary decays and underlying events.

The three programs we discuss have the same basic structure, although the precise details vary enormously. Events are generated by starting with the hardest (highest momentum scale) interaction, described by exact QCD (or EW) matrix elements. This is usually only done to leading order so describes

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a 2→2 scattering process. The production of multi-parton final states is described as the emission of additional partons from the incoming and outgoing partons of the hard process. This is simulated by a parton shower algorithm in which the partons evolve downwards in some energy-like scale according to perturbatively-calculable probabilistic distributions. When the evolution scale becomes small the running coupling grows, phase space fills with (mostly soft) partons and perturbation theory breaks down. At this point a model of the non-perturbative physics is needed: the perturbative emission is cutoff by a fixed infrared cutoff and the system of partons is confined into hadrons. Having treated all outgoing partons we are left with the remnants of the incoming protons, stripped of the partons that participated in the hard process. These remnants can interact with each other, to produce additional soft hadrons in the event, known as the underlying event.

Parton shower algorithms are developed by studying the amplitude to emit an additional parton into a given process. This is enhanced in two kinematic regions: collinear, where two massless partons are much closer to each other than any others or where a massless parton is close to the incoming proton direction; and soft, where a gluon is much softer than any other parton. In both cases the enhanced terms are universal, allowing a factorisation of emission by a system of partons from the process that produced them. In the collinear case, this factorisation works at the level of cross sections, so it is not surprising that a probabilistic approach can be set up. In the soft case however, the factorisation theorem is valid at amplitude level and it turns out that in any given configuration, many different amplitudes contribute equally. It therefore seems impossible to avoid quantum mechanical interference and so to set up the evolution in a probabilistic way. The remarkable result though is that, due to coherence between all the coloured partons in an event, the interference is entirely destructive outside angular-ordered regions of phase space. This means that the soft emission can be incorporated into a collinear algorithm, simply by choosing the emission angle as its evolution variable, as is done in HERWIG. The most important effects of coherence can be approximately incorporated by using some other evolution variable, like virtuality, and vetoing non-angular-ordered emission, as is done in PYTHIA. If the colour-coherence is not treated at all, one obtains the wrong energy-dependence of jet properties. Such models, like ISAJET, are completely ruled out by $e^+e^-$ annihilation data. Colour coherence effects are also important in determining the initial conditions for the parton evolution, resulting in physically-measurable inter-jet effects [292], which are also in disagreement with ISAJET.

Since the top quark decays faster than the typical hadronisation time, its width cuts off the parton shower before the infrared cutoff. Its decay then acts as an additional hard process and the resulting bottom quark (and two more partons if the W decays hadronically) continue to evolve. Additional coherence effects mean that radiation from the top quark is suppressed in the forward direction (the dead cone effect), as is radiation in the W direction in the top decay. These effects are again included in HERWIG, partially included in PYTHIA and not included in ISAJET. Since the top quark is coloured, the $b$ quark in its decay is colour-connected to the rest of the event, meaning that its properties are not necessarily the same as in a ‘standard’ $b$ production event. As mentioned in Section 4.6 and as discussed in more detail in [64], such non-universal effects are small.

Although parton showers are reliable for the bulk of emission, which is soft and/or collinear, it is sometimes the rare hard emissions that are most important in determining experimental systematics and biases. Such non-soft non-collinear emission should be well described by NLO perturbation theory, since it is far from all divergences. However, it is not straightforward to combine the advantages of the parton shower and NLO calculation, so it has only been done for a few specific cases. Most notable for hadron-hadron collisions are the Drell-Yan process, for which matrix element corrections are included in both HERWIG and PYTHIA, and top decay, which is included in HERWIG and discussed earlier in Section 4.62 in this report. The corrections to Drell-Yan events are particularly important at high transverse momenta, where the uncorrected algorithms predict far too few events. It is likely that implementing corrections to $t\bar{t}$ pair production would cure the analogous deficit at high $p_T$ seen in Fig. 7.

Hadronisation models describe the confinement of partons into hadrons. Although this process
is not well understood from first principles, it is severely constrained by the excellent data from LEP, SLD and HERA. The string model, used by PYTHIA, and the cluster model, used by HERWIG, both take account of the colour structure of the perturbative phase of evolution, with colour-connected pairs producing non-perturbative singlet structures that decay to hadrons. The biggest difference between these models is in how local these colour-singlet structures are. In the string model they stretch from a quark (or anti-di-quark) via a series of colour-connected gluons to an antiquark (or di-quark). In the cluster model each gluon decays non-perturbatively to a quark-antiquark pair and each resulting quark-antiquark singlet (coming one from each of two colour-connected gluons) decays to hadrons. The independent fragmentation model, used by ISAJET, on the other hand, treats each parton as an independent source of hadrons and is strongly ruled out by \(e^+e^-\) data, for example on inter-jet effects in three-jet events, the so-called string effect. Of the other two models, PYTHIA gives the better description of \(e^+e^-\) data, but HERWIG also gives an adequate description, despite having a lot fewer adjustable parameters.

Models of the underlying event are not strongly constrained by either theoretical understanding or experimental data. Two extreme models are available and the truth is likely to lie between them. In the soft model, used in HERWIG, the collision of the two proton remnants is assumed to be like a minimum bias hadron-hadron collision at the same energy. A simple parametrisation of minimum bias data (from UA5 [293]) is used with little additional physics input. In the mini-jet model, used in PYTHIA and available as an additional package for HERWIG, on the other hand, the remnant-remnant collisions act as a new source of perturbative scattering, which ultimately produce the hadrons of the underlying event. To avoid regions of unstable perturbative predictions and problems with unitarity, a cutoff must be used, \(p_{T,\text{min}} \sim 1\) GeV. Presumably for a complete description, some soft model should describe the physics below \(p_{T,\text{min}}\) such that the results do not depend critically on its value. Unfortunately no such model exists at present. Although the two models give rather similar predictions for average properties of the underlying event, they give very different probabilities for the rare fluctuations that can be most important in determining jet uncertainties. This is an area that needs to be improved before LHC running begins.

### D2 Parton-level Monte Carlos

With few exceptions (e.g. 3 or 4-jet final states in \(e^+e^-\) collisions) multi-jet final states are not accurately described by the shower MC’s described above. This is because emission of several hard and widely separated partons is poorly approximated by the shower evolution algorithms, and exact (although perhaps limited to the tree level) matrix elements need to be used to properly evaluate quantum correlations. Parton-level Monte Carlos are event generators for multi-parton final states, which incorporate the exact tree-level matrix elements. They can be used for parton-level simulations of multi-jet processes, under the assumption that each hard parton will be identified with a final-state physical jet with momentum equal to the momentum of the parent parton. Selection and analysis cuts can be applied directly to the partons. In some cases, the partonic final states can be used as a starting point for the shower evolution performed using a shower MC such as HERWIG, PYTHIA, or ISAJET. For a discussion of the problems involved in ensuring the colour-coherence of the shower evolution when dealing with multi-parton final states, see [294].

In the following, we collect some information on the most frequently used parton-level MCs used in connection with top quark studies.

### D21 VECBOS\(^{29}\)

VECBOS [150] is a Monte Carlo for inclusive production of a \(W\)-boson plus up to 4 jets or a \(Z\)-boson plus up to 3 jets. VECBOS is therefore a standard tool used in the simulation of backgrounds to \(t\bar{t}\) production. The matrix elements are calculated exactly at the tree level, and include the spin correlations of the vector boson decay fermions with the rest of the event. Various parton density functions are

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available and distributions can be obtained by using the kinematics of the final state, available on an event-by-event basis together with the corresponding event weight. The code and its documentation can be obtained from:

http://www-theory.fnal.gov/people/giele/vecbos.html

Documentation on the use of VECBOS within ATLAS can be found in [295].

D22 CompHEP\textsuperscript{30}

CompHEP is a package for the calculation of elementary particle decay and collision properties in the lowest order of perturbation theory (the tree approximation). The main purpose of CompHEP is to generate automatically transition probabilities from a given Lagrangian, followed by the automatic evaluation of the phase-space integrals and of arbitrary distributions. The present version has 4 built-in physical models. Two of them are the versions of the Standard Model (SU(3)xSU(2)xU(1)) in the unitary and t'Hooft-Feynman gauges. The user can change the models or even create new ones.

The symbolic part of CompHEP allows the user to perform the following operations:

1. to select a process by specifying incoming and outgoing particles for the decays of $1 \rightarrow 2, \ldots, 1 \rightarrow 5$ types and the collisions of $2 \rightarrow 2, \ldots, 2 \rightarrow 4$ types,
2. to generate Feynman diagrams, calculating the analytical expressions for the squared matrix elements,
3. to save the algebraic symbolic results and to generate the optimized Fortran and C codes for the squared matrix elements for further numerical calculations.

The numerical part of CompHEP allows to convolute the squared matrix element with structure functions and beam spectra, to introduce various kinematic cuts, to introduce a phase space mapping in order to smooth sharp peaks of a squared matrix element, to perform a Monte Carlo phase space integration by VEGAS, to generate events and to display distributions for various kinematic variables. Recently, an interface with PYTHIA has been created [283]. This allows to perform realistic simulations of the process including hadronisation effects as well as the effects of the initial and final state radiation.

The CompHEP codes and manual are available from the following Web sites:

http://theory.npi.msu.su/~comphep
http://www.ifh.de/~pukhov

D23 ALPHA\textsuperscript{31}

ALPHA is an algorithm introduced in [296] for the evaluation of arbitrary multi-parton EW matrix elements. This algorithm determines the matrix elements from a (numerical) Legendre transform of the effective action, using a recursive procedure which does not make explicit use of Feynman diagrams. The algorithm has a complexity growing like a power in the number of particles, compared to the factorial-like growth that one expects from naive diagram counting. This is a necessary feature of any attempt to evaluate matrix elements for processes with large numbers of external particles, since the number of Feynman diagrams grows very quickly beyond any reasonable value.

An implementation of ALPHA for hadronic collisions was introduced in [294], where the algorithm was extended to the case of QCD amplitudes (see also [297]). The main aim of the hadronic version of ALPHA is to allow the QCD parton-shower evolution of the multi-parton final state, in a way consistent with the colour-coherence properties of the soft gluon emission dynamics. This is achieved by evaluating the QCD amplitudes in an appropriate colour basis [294], such that the assignment of a specific colour flow configuration on an event-by-event basis. The pattern of colour flow defines the colour currents

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required to implement the angular ordering prescription which embodies, at the leading order in the $1/N_c$ expansion, the quantum coherence properties of soft-gluon radiation, as discussed in Appendix D1. A version of the code is being completed [298], which incorporates the evaluation of $W^+b\bar{b} + n$ jets ($n \leq 4$), with all $b$-mass effects included. This program will allow a complete evaluation of the $W^+$ multijet backgrounds to single top and $t\bar{t}$ production. The code contains 3 modules: the first for the generation of parton-level events, with the assignment of partonic flavours, helicities and colour flows. The second for the unweighting of the events, and the third for the parton-shower evolution of the initial and final states, done using the HERWIG MC. The code will soon be available from the URL:

http://home.cern.ch/˜mlm/alpha

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