In preparing this report, I was faced with two main difficulties. First of all the extreme wealth of experimental and theoretical information combined with the lack of any fully accepted guiding principle makes it difficult to draw general conclusions. Secondly the superspecialization and the use of different languages and formalisms makes it harder and harder to be equally competent in all domains of research in our field.

What allowed me to perform in some way my task, is the fact that, this year, some general trends are emerging, which seem to point towards some simple and unified pictures of particle physics.

On the other hand, I was greatly helped in overcoming my ignorance and my lack of information by numerous illuminating discussions with members and visitors of the CERN Theoretical Study Division and with participants to this Conference. This allowed me to follow, I hope, with profit the beautiful rapporteur talks given at this Conference.

In addition I wish to thank the secretaries of our Theory Division for the beautiful slides that, I am sure, you will appreciate.

First of all, let me list a few facts that have emerged during the last years and which confirm things which have already been discussed in previous conferences.

1. No evidence for $\Delta Q \neq \Delta S$.
2. Very good verification of Cabibbo theory of weak interaction with equal vector and axial angles.
3. CP violation seems to indicate the superweak model. Final confirmation will come with the measurement of the neutron dipole moment.

On the other hand, experimental verification of some theoretical predictions in strong interaction physics are amazingly accurate. For example:

1. Charge-exchange processes are completely dominated by the exchange of a single $p$ trajectory which is almost perfectly linear with intercept at 0.5:

2. $SU_3$ and universality predictions for the coupling of the $p$ trajectory are very accurately verified by the data.

Let me finally report an experimental finding which may play an important role in future conferences. The production of two $\pi$ pairs in high energy neutrino production has been reported. If other events of this kind will be found, this might be the beginning of some new phenomenon whose interpretation will keep many of us busy.
I now proceed to the main object of this talk: to discuss the most important theoretical and experimental trends which have emerged in this conference.

1. Spontaneous Symmetry Breaking

In my opinion the most fundamental fact which has emerged during the last decade is that a fully symmetric Lagrangian can give rise to non-symmetric results.

This phenomenon which goes under the name of "spontaneous symmetry breaking" is related to the following circumstance. If a Lagrangian is symmetric under a certain symmetry transformation there must be a symmetric equilibrium configuration; however, this configuration might be unstable and there might be other stable non-symmetric equilibrium configurations.

To my knowledge, the first illustration of such a situation is due to the French philosopher Jean Buridan (1300-1350). His famous example is illustrated in Fig. 1.

The position of the donkey at equal distance from two equal amounts of food is indeed an unstable equilibrium whereas (as shown in Fig. 1) there are two non-symmetric stable equilibrium positions at the locations of the food. An infinitesimal displacement from the middle position will cause the donkey to break macroscopically the reflection invariance. At this point one is tempted to generalize Buridan's model to a case in which the symmetry involved is a rotational one. This is shown in Fig. 2 where we have used a "point donkey approximation".

Again the donkey position will be an unstable equilibrium, whereas the potential will have a minimum on the circle where the food is placed. Once the donkey is on this circle he can move indifferently around it. This circular motion will be uniform: i.e., a zero frequency motion which would involve in the quantum language the presence of a zero mass boson: the Goldstone boson.

Let us now leave the donkey world and consider in more detail what happens in quantum field theory. The most common situation is when we have to deal with a scalar field \( \phi \). This field belongs to a certain symmetry multiplet, for example in chiral symmetry, the scalar field \( \phi \) and the pion field \( \pi \) form a quartet.

Now if we attribute a bare negative (mass)\(^2\) to the scalar field, we immediately see that the usual vacuum becomes unstable. The new stable vacuum state will give rise to a non-zero expectation value of \( \phi \)

\[
\langle 0 | \phi | 0 \rangle \neq 0
\]

In this case the symmetry will be spontaneously broken. Table I shows what happens both in the cases of symmetric and unsymmetric vacuum.
From the preceding Table, we see that the invariance property of the Lagrangian still retains all its predictive power. Since the vacuum takes an active part in the transformation, one is led to predictions which are very different from the customary symmetry properties of the particle spectrum.

It is not surprising that the last two years have witnessed a large interest in the fundamental properties of spontaneous symmetry breaking. In particular there have been very interesting investigations concerning the dynamical mechanism of spontaneous symmetry breaking and on the dependence of the phenomenon both on temperature and on density.

I shall discuss in more detail the last point because of its important implications for the study of nuclear matter and probably also for particle physics. In a series of recent investigations, T.D. Lee and G.C. Wick have studied in the framework of Lagrangian theory the properties of the ground state in the case of dense nuclear matter. Special emphasis has been given to the case of chiral symmetry.

<table>
<thead>
<tr>
<th>Symmetric vacuum</th>
<th>Unsymmetric vacuum</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \langle \phi \rangle = 0 )</td>
<td>( \langle \phi \rangle \neq 0 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Equal mass multiplets</th>
<th>Zero mass particle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Goldstone boson</td>
<td>Goldstone bosons</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Algebraic relations</th>
<th>Exact theorems for encoding and absorption of soft (zero momentum) Goldstone bosons</th>
</tr>
</thead>
<tbody>
<tr>
<td>between coupling constants of multiplet</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>For some groups some particles must be massless</th>
<th>Those particles acquire mass</th>
</tr>
</thead>
</table>

In the case of chiral invariance

<table>
<thead>
<tr>
<th>Fundamental multiplet formed by ( \pi ) and ( \sigma )</th>
<th>The pion (Goldstone boson) is massless. Exact soft pion theorems</th>
</tr>
</thead>
<tbody>
<tr>
<td>The nucleon mass vanishes</td>
<td>The nucleon acquires mass</td>
</tr>
</tbody>
</table>

TABLE I

From the preceding Table, we see that the invariance property of the Lagrangian still retains all its predictive power. Since the vacuum takes an active part in the transformation, one is led to predictions which are very different from the customary symmetry properties of the particle spectrum.

The very important result of this analysis is that, for very large values of the nuclear density, the chiral symmetric vacuum becomes stable, whereas for low density the usual non-symmetric vacuum is present. This can lead to a new kind of abnormal state of nuclear matter in which the effective mass of the nucleon is zero. As a consequence, one finds a boundary between abnormal and normal states of matter, i.e., some kind of bag which follows from the chiral invariant Lagrangian. This suggests that extension of those ideas to particle physics might lead to important breakthrough in our field and some fundamental explanation of the "bag mechanisms" which have been put forward in order to explain quark confinement. Of course, a three-quark structure (as suggested by experiment) does not certainly correspond to high density. Probably a very dense neutral gluon field is needed for the overall success of the mechanism. This last related point will certainly play a central role in future investigations.

2. Spontaneously Broken Gauge Invariance and Unified Field Theories

A very interesting new situation takes place in the case of gauge theories. Let us consider the situation in which the scalar field possesses some charge and thus transforms like

\[ \phi = e^{i\theta} \phi \]

under some gauge transformation. If \( \langle \phi \rangle \neq 0 \) that invariance under that gauge transformation is spontaneously broken. Table II shows what happens in the cases of symmetric and unsymmetric vacuum.

In the case of the non-symmetric vacuum, the transformed Lagrangian contains several terms. Those terms are strongly correlated in such a way that in
In principle it should have three helicity components:

\[ +1 \quad (0) \quad -1 \]

Because of gauge invariance the helicity zero "particle" is decoupled and does not correspond to a physical particle.

The tree approximation the over-all asymptotic contribution is better than that of each separate term. Indeed, it was shown by t Hooft that the previous mechanism allows to construct renormalizable theories of massive vector particles. The fundamental principle is that: renormalizability depends on the form of the Lagrangian and not on the form of the vacuum state.

All this development gave rise to an important breakthrough: the construction of unified renormalizable theories of weak and electromagnetic interactions. It is not my task to go in detail since this was already discussed in the last conference. I wish to recall that the simplest version, due to A. Salam and S. Weinberg involves four vector bosons \( W^+, W^0, W^- \) and \( B \). Because of the presence of a non-symmetric vacuum \( W^0 \), \( W^- \) and a combination of \( W^0 \) and \( B \) acquire mass, whereas the other neutral combination remains massless and is identified with the photon. The Fermi coupling is constantly related to the electromagnetic coupling constant by the relation:

\[
\frac{g}{\sqrt{2}} = \frac{e \sin^2 \beta}{2 M^2_W}
\]

where \( M_W \) is the mass of the intermediate boson and \( \beta \) is the weak angle, which is the only free parameter in the model.

This important theoretical development triggered an intensive experimental search for neutral non-strange weak currents.

Although details have still to be settled, published results at CERN and FNAL in addition to the new results of CALTECH, FNAL, ARGONNE and BNL seem to be in favour of the presence of neutral currents.

The absence of strange neutral currents is still a big challenge for theoreticians. The most popular explanation involves the presence of more than three fundamental quarks with at least a new kind of "charm" quantum number. The search for charmed particles will be one of the important tasks for the next years.

The most recent developments in this field go beyond unification of weak and electromagnetic interactions but aim to include strong interactions as well.

The physical idea is illustrated in the next figure.
At smaller distances, comparable with the inverse of the intermediate boson mass one obtains unification between weak and electromagnetic interactions. It is tempting to speculate that at even smaller distances related to the inverse masses of super-heavy vector bosons, weak electromagnetic and strong interaction will all have the same strength.

This will involve a single fundamental symmetry group of nature with a single basic multiplet involving both quarks and leptons. Work is going on on this exciting program. As first pointed out by Pati and Salam, a new fascinating possibility is the decay processes involving hadronic and leptonic members of the multiplet which will lead to non-conservation of baryonic number. This is not necessarily in contrast with the present experimental evidence in favour of stability of matter. Indeed the decay of the nucleon will imply the simultaneous decay of three quarks. The rate of this triple decay process can be of the order of $10^{30}$ years. We see that here the still unknown mechanism for quark confinement is also responsible for stability of ordinary matter.

Of course, observation of such rare decay would represent a fundamental break-through. Professor Reines has told me that measurements of lifetimes $10^{30}$ to $10^{31}$ years are within the present experimental possibilities. Work in that direction should be strongly encouraged. As a theoretician, I wish to say that conservation of baryonic number has not a fundamental theoretical reason as for example conservation of electric charge. It appears in most of our schemes because of pure experimental reasons. It is indeed possible that removal of this selection rule will help us in understanding unified interactions in the same way as non-conservation of parity has helped us in understanding the structure of weak interaction.

3. Supersymmetries

During the last years a tendency has emerged towards a greater unification of interactions in the framework of renormalizable theories. In this development, the introduction of super-symmetries, involving supermultiplets containing both bosons and fermions, is playing an important and very promising role. This new idea has been introduced by Wess and Zumino under the inspiration of the Neveu and Schwarz dual model and has received important contributions by Salam and Strathdee.

The idea of combining particles of different spin in larger supermultiplets and at the same time to imbed internal and Poincare symmetries in a larger algebra is not new. However, all previous attempts have encountered very serious theoretical difficulties. Because of that, schemes like $SU(6)$ have not managed to become anything more than successful phenomenological models.

The requirements for an acceptable theoretical approach involving supermultiplets are the following.

(1) Combination of fields in supermultiplets should give rise to conserved (or partially conserved) charges, i.e., it should give rise to new constants of motion.

(2) The new charges should obey simple algebraic relations, possibly leading to unification between internal and Poincare symmetries.

(3) Cancellations should take place in such a way that the tree approximation should have the best possible asymptotic behaviour. This would make the theory renormalizable with a minimum number of counter terms.
(4) The whole scheme should not make Sidney Coleman
unhappy.

Supersymmetries satisfy all those requirements. The
key element which did overcome the no-go theorem
barrier is that the new algebra involves both
\textit{commutators} and \textit{anticommutators}.

The basic element of supersymmetries is the "super-
field" which is a combination of boson and fermion
fields. Several models have been proposed. A common
feature is the important role of scalar and pseudo-
scalar fields. This, of course, suggests exploitation
of such fields for the purpose of spontaneous symmetry
breaking.

A fundamental property is that, beyond \textit{scalar constants}
of motion (weak and electromagnetic charges) and
\textit{vector constants of motion} (four momentum) also
\textit{new spinor constants of motion} \( Q_a \) \((a = 1, 2, 3, 4)\) are
present. The algebra of the \( Q_a \) is in terms of anti-
commutators and, amazingly, involves the four-
momentum operators. This gives a direct link between
internal and Poincare symmetries. A particular
impressive expression for the Hamiltonian is the
following:

\[
H = \frac{1}{4} \sum Q_a^2
\]

suggesting that maybe energy and momentum are not the
simplest most fundamental operators.

Work in applying the idea of supersymmetries to the
world of elementary particles is still in a preliminary
stage. One could speculate about a supermultiplet
containing the only two genuine zero mass particles:
photon and neutrino. In the hadron world, the
question of a possible relation of supersymmetries
with the quark structure of bosons and fermions is
still open. Supersymmetries indeed exhibit

\begin{enumerate}
\item mathematical rigour, beauty, and some mystery. In
my opinion they shall be an important component of
future theories.
\end{enumerate}

\textbf{Small distance behaviour}

We have already seen the very important role which the
concept of renormalizability has played in recent
theoretical developments. In a renormalizable theory,
small distance behaviour is such that all expressions
for observable quantities are finite. Only \textit{non-
observable} quantities like bare masses and coupling
constants are infinite.

Experimental investigation of the small distance
behaviour of physical amplitudes is thus of
fundamental importance. Of course, the most reliable
observations are made by means of incoming leptons
whose interaction (electromagnetic or weak) is known,
at least at lowest order in perturbation theory. The
reactions which have been most thoroughly studied are:

\begin{enumerate}
\item \textbf{lepton} + N \rightarrow \textbf{lepton} + \text{anything}, which (in
the case of the electron) is interpreted as
\((\gamma) + N \rightarrow \text{anything}\)
\((\gamma)\) is a virtual \textit{spacelike photon}
\item \textbf{e}^+ + e^- \rightarrow \text{anything}, which can be interpreted
as \[\gamma\] \rightarrow \text{anything}
\((\gamma)\) is a virtual \textit{timelike photon}
\end{enumerate}

If we consider large values of all momenta, masses
become unimportant and (since coupling constants are
dimensionless) the only quantities with dimensionless
are the energies and momenta of the particles. This
suggests that a dimensional study of the physical
quantities involved should give definite predictions
on asymptotic behaviour. Let us briefly recall this
well-known analysis.
The $(\gamma) - N$ total cross-sections can be expressed in terms of dimensionless structure functions

$$\Sigma_{[\gamma N]}(v,u) = \frac{1}{u} F(v,u)$$

It is useful to introduce the dimensionless integrals

$$\int F(v,u) \left( \frac{v}{u} \right)^n \frac{dv}{v} = F_n(u)$$

where $G(u)$ is again a dimensionless quantity.

At this point, the theoretical analysis depends on whether we wish to rely on an elementary picture on what is going on, or whether we wish to use the full complexity and strength of renormalization theory.

(1) Elementary theory (parton model, superrenormalizable theory, free particle light plane commutators)

Since the only dimensional quantities at high energy are the kinematical variables $v$ and $u$, one can use elementary dimensional argument leading to the predictions:

$$F(v,u) \to f \left( \frac{v}{u} \right)$$

i.e.,

$$F_n(u) \to f_n$$

$$G(u) \to g$$

(2) Highbrow theory

In the framework of a renormalizable theory, elementary dimensional arguments are not completely correct. Indeed, such theories are only mathematical meaningful in the presence of a cut-off. Renormalizability is the statement that observable quantities have a finite limit when the cut-off momentum $k_{\text{max}}$ tends to infinity. Since observable and non-observable quantities enter together in the field theoretical equations a complete dimensional analysis has to take into account the presence of $k_{\text{max}}$.

The method of performing this more advance investigation is the renormalization group of Peterman and Stueckelberg which is now being used in the form given by Gell Mann and Low and by Callan and Symanzik. Application of renormalization group to small distance behaviour leads to an interesting and significant result.

A definite subclass of renormalizable theories (excluding all non-gauge theories) exhibits ultra-violet asymptotic freedom, which in simple words means that small distance behaviour does not differ appreciably from that of a free theory. The predictions of naive dimensional analysis are corrected in the sense that the constants appearing in the asymptotic limits become now slowly varying functions of $u$. This is shown in the following table.
If one tries to compare these predictions with experiment one is faced with a very puzzling situation. On one side, Bjorken scaling seems to be consistent with most experimental findings, so that the logarithmic corrections do not seem to be needed at the moment. Probably the present range of energy and momentum transfers is such that those logarithmic asymptopia do not yet become effective.

On the other hand, recent $e^+e^-$ data suggest a form of $G(u)$ which does not show any tendency of becoming constant in the present experimental range. Those extremely important experimental results will be discussed later.

Let me finally recall that renormalizable theories cannot resemble free theory both at small and large distances. Asymptotically free theories, in which the sum of ultra-violet terms leads to a quasi-free behaviour, exhibit, at large distances, trends which are radically different from that of a free theory. This circumstance can be useful in order to search for an explanation, in the framework of conventional theory, of why free quarks are not experimentally seen (quark confinement).

5. $e^+e^-$ Annihilation

Recent experimental results on $e^+e^-$ annihilation were totally unexpected by the great majority of theoreticians and their interpretation constitutes a fundamental challenge of our ideas about elementary particles and their interactions.

Let me start by listing some of the most important experimental findings.

(A) The dimensionless ratio $R(q^2) = \frac{G_{\text{hadrons}}(\sqrt{q^2})}{G_{\gamma\mu\nu}(\sqrt{q^2})}$ seems to increase linearly with $q^2$.

(B) Charged secondaries are produced with low momentum (~ 500 MeV). The average momentum is roughly independent on the initial energy.

(C) Inclusive single particle spectra scale only for large momenta of the observed secondary.

Clearly the presence of a rising $R(q^2)$ constitutes a definite theoretical difficulty. It seems to be in contrast with what was predicted on the basis of the most popular models (like partons) and also by means of the renormalization group. The questions are:

(a) why does $R(q^2)$ rise?

(b) why, on the other hand, deep inelastic scattering exhibits early scaling?

It is not surprising that these questions have called a very great amount of attention by theoreticians. In this respect two main point of view emerge.

(A) The phenomenon of rising $R(q^2)$ is a transient and we should expect constant behaviour at higher energies. This, for example, could be caused by the presence of non-leading power terms which have not yet died out in the present energy range. This could also be caused by the opening of new physical thresholds.

Certainly if $R(q^2)$ will finally approach a constant value at higher energy the whole question will lose its explosive character. I am convinced that in this case the technical question of why scaling in deep inelastic scattering takes place earlier than in $e^+e^-$ annihilation will find an adequate answer.

(B) If the increase of $R(q^2)$ will continue for higher values of $q^2$, then we shall be in for real trouble. Let us try to see where this
TRENDS IN PARTICLE PHYSICS

Trouble really lies.

Let us use the parton model in order to study in more detail the relation between deep inelastic scattering experiments and $e^+ e^-$ annihilation. The parton model certainly predicts scaling in deep inelastic scattering. For $e^+ e^-$ annihilation it predicts that the secondaries in the process should be energetic particles and that this process should give rise to scaling.

This suggests that:

1. Production of fast secondaries in $e^+ e^-$ annihilation should scale in complete analogy with deep inelastic scattering. This is clearly supported by experiment.

2. The real deviation from theory is the fact that the production of fast secondaries is not the most important phenomenon. Experimentally secondaries are mainly produced at low momentum. This dominant contribution which does not scale (why should it?) is responsible for the "anomalous" behaviour of $R(u)$.

It is important to notice that the 500 MeV cut-off in momentum in $e^+ e^-$ annihilation is of the same order as the cut-off in transverse momenta in high energy hadronic interactions.

In conclusion if the present trends persist at high energies, the fundamental problem will be: why is the momentum of secondaries confined below 500 MeV? Is this upper bound a manifestation of some very general law of physics? Clearly theoretical work should be addressed towards schemes (far from renormalizable theories) which will introduce in a way or in another some kind of fundamental length. The situation in this domain is still very preliminary and will be discussed later in this report. Certainly most theoretical approaches predicting a steady rising of $R(q^2)$ involve at some point some fundamental length. For example, the following possibilities have been considered: an anomalous magnetic moment for quarks, a non-renormalizable interaction with vector particles and a new fundamental lepton-hadron interaction. Time will tell. Let me express the hope that new high energy data will soon be available.

6. Quarks and Partons

As time passes, more and more indirect evidence is accumulating in favour of the existence of point structures inside hadrons. The trend is definitely towards the original spin $\frac{1}{2}$, non-integer charge, three quark model. Let us list the main experimental indications.

(A) Scaling in lepton inclusive reactions

$e^+ e^- + \text{hadrons} \to \text{anything}$: The ratio between cross-sections for longitudinal and transverse "photons" definitely indicate spin $\frac{1}{2}$ for the quarks. The spectral sum rules give definite indications in favour of the original three quark model.

(B) Asymptotic behaviour of electromagnetic form factors: experiments are in reasonable agreement with the parton model prediction

$$F_1(u) = u^{-n_1^+ 1}$$

where $n_1$ is the number of quarks contained in the particle.

(C) Asymptotic behaviour of fixed angle scattering amplitude. Here again the data are consistent with the parton prediction:

$$\frac{d\sigma}{dt} = s^{2-N} f(t)$$

where $N$ is the number of quarks involved in the reaction (for example, in $pN$ scattering $N = 2 + 3 + 2 + 3 = 10$).
(D) Finally, the large transverse momentum single particle inclusive distribution indicate a power law, again in agreement with the parton model.

We see that the overall picture is quite good even if there are many fine points to be clarified. It is important to notice that the number $n^i$ and $N$ appearing in the asymptotic exponents (which are in reasonable experimental agreement) are those evaluated on the basis of the original quark model which lead to a $qqq$ structure for the lowest fermions and a $q\bar{q}$ structure for the lowest bosons.

Let us now discuss in more detail the problem of quarks and of their dynamics.

The two main sources of evidence in favour of presence of quarks are the following.

(1) The existence of point structures in hadrons as revealed by high energy experiments. The most direct and reliable information comes from high energy deep inelastic lepton scattering which can be interpreted in terms of light plane commutators of local currents.

(2) The structure of low lying levels of hadrons is strongly suggesting a $qqq$ model for fermions and the $q\bar{q}$ model for bosons. The simplest and most successful interpretation of hadron structure requires quarks obeying parastatistics. The application of the exclusion principle to quarks is best described by attributing a new colour quantum number to quarks and by requiring that only colour singlets are observable.

It is important to know the relation between the quark wave function as revealed by (1) (current quarks) and by (2) (constituent quarks). The canonical transformation connecting current quarks to constituent quarks has been the object of numerous investigations. It has led to remarkable progress in the study of the structure of hadrons and their strong and electromagnetic properties.

The beautiful success of the quark model in explaining many features of hadron physics gives a somewhat urgent touch to the question: why have quarks never been seen?

It is unfortunate that until now no completely satisfactory answer has been suggested. Some possibilities are the following.

(A) In the framework of gauge theories, strong deviation of infra-red behaviour from free theory may give rise to a barrier responsible for complete confinement of quarks inside hadrons. Examples supporting this view have been given in the framework of two dimensional field theoretic models. It is still unclear whether realistic four-dimensional Lagrangian (which exhibit a much weaker infra-red behaviour) will also lead to a similar situation.

(B) It is possible to introduce confinement in a more direct way by starting from new field theories with confined fields. This can either be introduced as a new dynamical starting point or derived from a conventional Lagrangian by a variational approach. This will be discussed later in this talk.

(C) One can finally reverse the problem and interpret strong interactions as a pure consequence of the boundary conditions responsible for confinement.

We indeed see that the theoretical situation on this question is still in a very preliminary stage. It is unclear whether the present theories will lead at
all to confinement or whether this will require in the future some important changes in our outlook.

7. Fundamental Length in Particle Physics

Some of the previous discussions might suggest the possibility that future theories of elementary particles could strongly differ from what is presently available and that renormalizable theories might at best be first order approximation of the new approach. Although it is hard to talk about things which are only unperfectly known, one feature which, in my opinion, should be incorporated in future schemes is the presence of a new universal constant with the dimension of a length.

This idea is by no means new, it is around from many decades. At present, however, there are both experimental and theoretical arguments for its revival. I shall first list some of the reasons why a universal length should be introduced and then discuss how people have tried to implement this program.

Why?

(A) In order to make present field theory mathematically meaningful one has to introduce an ultraviolet cut-off $k_{\text{max}}$ so that all integrals are convergent. We know that in renormalizable theories observable quantities tend to a finite limit as $k_{\text{max}} \to \infty$.

However, people interested in a theory and not in the limit of a theory, might desire to keep $k_{\text{max}}$ finite: this has led to the early development of non-local field theories.

(B) An experimental evidence favouring strongly some kind of universal length is the presence of straight line Regge trajectories with a universal slope. This evidence comes both from timelike values of $s$ (i.e., from the masses of particles) and from spacelike $s$ (i.e., from high energy scattering). This experimental input is the starting point of dual theories.

(C) One of the most fundamental questions in particle physics is that of quark confinement. In order to reconcile the presence of quarks inside hadrons, together with the absence of free quarks one is led to introduce new theories in which particles are naturally confined inside finite regions in space (bags). In this case the fundamental length (i.e., the size of the bag) will be directly related with large distance behaviour of the quark wave functions.

(D) Another universal quantity appearing in experiments is the universal cut-off of transverse momenta of secondaries in high energy hadron reactions. This, together with the similar cut-off of three-dimensional momentum in $e^+e^-$ annihilation, points out towards a single dynamical origin.

How?

(A) Non-local or non-renormalizable theories. The oldest way of introducing a fundamental length is to start from a Lagrangian with a non-local interaction term. Non-renormalizable theories have essentially the same physical content than non-local theories. Indeed higher order counter terms contain an ever increasing number of gradients. Their sum will be essentially equivalent to a non-local interaction. Unfortunately, non-local theories give rise to all sort of troubles, so that they have received little interest in recent times. It is hard to avoid violation of macroscopic causality and of
VI-12  S Fubini

gauge invariance. It is of course possible that all this trouble is due to misuse of perturbation theories and that those theories might be somewhat healthier than one could argue on the basis of their present appearance.

(B) Dual theory. The starting point of dual theory is indeed closer to physical facts. It is based on the presence of parallel linear trajectories, which seems to be supported by experimental evidence. Although it has received a beautiful field theoretical interpretation in terms of interacting strings, dual theory has started and, in my opinion, still essentially remains an S matrix theory. Dual amplitudes miraculously manage to satisfy almost all the axioms that a reasonable scattering theory should satisfy. They exhibit the correct Regge and multi-Regge behaviour in all channels, and satisfy a rudimentary unitarity in terms of a consistent factorization of all poles. In addition, all difficulties due to the presence of ghost states have been successfully overcome. The overall mathematical machinery exhibits remarkable beauty and consistency. The overall constraints are so stringent that only a few solutions are possible. It is thus extremely hard to change any of the empirically unsatisfactory features like, for example, some of the zero intercepts of the trajectories. An unpleasant feature is the exponentially decreasing asymptotic energy dependence at fixed angle. This in disagreement with experiments and shows that dual models do not imply the existence of point constituents inside the nucleons.

On closer investigation we thus see that dual models strongly resemble non-local theories. This is probably the reason why nobody has ever succeeded to introduce a satisfactory treatment of local currents in the dual framework. The interpretation of the dual model as a new kind of non-local theory is also very strongly implied by the string version of the models.

An optimistic note is given from the fact that, after all, the essentially non-local dual theories lead to a very reasonable and analytic S matrix. This might suggest that maybe the difficulties of non-local field theories are of our own making.

On the other hand it is possible that the difficulties of dual models in dealing with lepton-hadron interaction will disappear when we shall learn how to treat leptons and hadrons on the same ground, as it is implied by the modern forms of unified theories.

(C) MIT bag. This approach was directly inspired by the problem of quark imprisonment. It aims to give a fundamental theory of confined quark fields. This approach has already been applied at the classical level to important examples. Work is now in progress in order to introduce a consistent quantization procedure. The first results in the framework of this approach are indeed encouraging.

In analogy with the string model the Hagedorn exponential increase of the number of levels has been derived. Using an approximation which strongly resembles the Fermi Thomas model for atoms and the independent particle model for nuclei the static properties of particles have been studied. Both the level structure and the ground state matrix elements (like magnetic moments and $\frac{J}{E}$) are obtained in a very satisfactory way. This suggests that one is indeed close to Nature. At this stage it is
hard to predict whether this point of view will lead to an approach with the same amount of generality and self-consistency as conventional field theory, or whether it will develop into a successful phenomenological approach to particle physics. In any case it will constitute a very interesting field for future investigations.

(D) Variational approach to confinement. This approach which has been developed by the Stanford group and by Y. Yamaguchi in Tokyo aims to obtain confinement from conventional field theory. This work seems like the natural extension of Lee and Wick results to particle physics. The ground state wave functions is obtained by means of a variational approach with a trial function which resembles in some way that used by S. Tomonaga a few decades ago. The idea is very interesting and, of course, its results resemble those of the MIT bag. The main difference is that in this last case the effects at the surface of the bag are more important. The empirical predictions are at the moment somewhat less satisfactory. One should not forget, however, that everything is still on a very preliminary stage.

In conclusion, let me say that approaches including a fundamental length are not yet strong competitors, for what concerns rigour and consistency, with ordinary field theories. However, it is possible that they contain a new physical element absent from conventional theories. In this case they will constitute a main avenue for future progress.

8. Progress in High Energy Hadron Phenomenology

During the last years we have witnessed very interesting new experimental results both at ISR and at NAL. Particularly striking is the discovery of the rising behaviour of total cross-sections. The recent experimental findings, which triggered a beautiful theoretical revival, are not in contradiction with any of our general ideas about high energy hadron collisions. Theoreticians knew all the time that phenomena connected with high energy cross-sections and with diffraction scattering are complicated ones. The present situation does not require us to abandon any of our models; it just forces us to make a better job with them.

Let me immediately say that some people have indeed done a better job, even before rising cross-sections had been discovered. I am referring to the pioneering work of Cheng and Wu, who, under the inspiration of their analysis of Feynman graphs, and using an eikonal approximation have predicted a rising cross-section of the form

$$\sigma_T = A(\log s)^2$$

which is in reasonable agreement with experiment and represents, by the way, the largest rate of increase allowed by the celebrated Froissart bound.

At present a large amount of work is being done on detailed theoretical analysis of high energy hadron physics. Lack of time does not allow me to cover all interesting investigations on the subject. I shall thus concentrate on a particularly exciting development: the new work done in the Soviet Union as well as in the United States on the application of Gribov Reggeon calculus and of the renormalization group to high energy hadron reactions.

Let me start by recalling a few well-known properties of the 1962 multiperipheral model whose results are still in rough agreement with experiment and which constitute the zero order approximation of the new schemes.
The multiperipheral model is based on the consideration of graphs of the form

where each blob represents a low energy interaction. The integral equation which sums up these contributions exhibits in the high energy region invariance under the scaling transformation

\[ s \rightarrow e s . \]

All well-known hadronic scaling properties are a simple consequence of this fundamental invariance property of the multi-peripheral equation. In particular the following results have been found:

(A) factorized power behaviour of total cross-sections

\[ \sigma(T) \propto s^{(\alpha-1)} \]

(B) uniform distribution in rapidity of secondaries

\[ dN \propto \frac{ds'}{s'} \]

(C) logarithmic increase of multiplicity

\[ < N > \propto \alpha \log s \]

The shadow of multiperipheral scattering leads to a contribution equivalent to the exchange of a single Regge trajectory.

It is important to have an estimate of the order of magnitude of the corrections to simple multiperipheral scaling. This is particularly relevant in the case of total cross-sections: experimental agreement requires appreciable corrections.

Investigation of various rescattering correction in the framework of the multiperipheral picture leads to non-scaling corrections whose size depends critically on the value of \( \alpha(0) \). For the value \( \alpha(0) = 1 \), suggested by an experiment, non-scaling corrections (Regge cut) only by factors of the order of \( q'/\log s \). We are thus faced by the following problems:

(1) can we understand theoretically the experimental value \( \alpha(0) = 1 \)?

(2) accepting the experimental value \( \alpha(0) = 1 \), can we find a reliable method of summing up the Regge cut contributions?

I do not think a satisfactory answer to the first question has yet been found. On the other hand the Gribov-Reggeon calculus is a very powerful tool towards the solution of the second problem.

Starting from consideration of asymptotic behaviour of Feynman graphs, Gribov has indeed given a set of general rules in order to evaluate the effect of any complicated exchange of Regge poles in high energy interactions.

An important result is that, given an appropriate dictionary, the rules for combining Reggeons can be translated into rules for combining particles in ordinary perturbation theory. This suggests that a formal summation of all poles and cut contributions should be made by means of an effective field theoretical Lagrangian, whose perturbation expansion corresponds to the different possible combinations of Reggeons.

Some of the most important rules relating Regge theory to particle theory are given in the next table:
Since the actual value of $\alpha(0)$ is indeed around one, the problem of summation of poles and cut contributions is the Regge equivalent of the infra-red problem in field theory. This has suggested to apply to the effective Reggeon Lagrangian the techniques of the renormalization group. The final results are theoretical predictions whose difference from the elementary multiperipheral ones consists in the fact that constant quantities now become slowly varying functions of energy. Comparison between some of the old and new results is shown in the next table:

**TABLE IV**

<table>
<thead>
<tr>
<th>Particle theory</th>
<th>Regge theory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single particle state</td>
<td>Single Regge pole</td>
</tr>
<tr>
<td>Multiparticle state</td>
<td>Regge cut</td>
</tr>
<tr>
<td>Physical particle with $m^2 &gt; 0$</td>
<td>Intercept $\alpha(0) &lt; 1$ pole dominates on cut</td>
</tr>
<tr>
<td>Zero mass particle $m^2 = 0$, Infra-red problem</td>
<td>Intercept $\alpha(0) = 1$ Regge pole and branch point coincide at $t = 0$</td>
</tr>
<tr>
<td>Tachion $m^2 &lt; 0$ vacuum becomes unstable. This eliminates the elementary tachion</td>
<td>$\alpha(0) &gt; 1$ Froissart bound violated. Also here a similar shift in the ground state should restore Froissart bound</td>
</tr>
</tbody>
</table>

The fundamental exponent $\beta$ can be theoretically evaluated, for example, by means of a Wilson $\epsilon$ expansion. The important question for experimental comparison is at what energy range the multiperipheral regime is still an acceptable approximation and at what energy logarithmic asymptopia becomes important. This and many other problems will be the object of future investigation.

But now it is time to close this summary talk. I shall do that by risking a forecast on what might be the most important future trends, both experimental and theoretical.

**Experiment:**

The remaining disagreement about the size of neutral weak currents should hopefully be settled. On the other side the situation about strange neutral currents is still very open and might give rise to surprises. It is strongly hoped by theoreticians that charmed particles will soon be observed. The question of the existence of heavy intermediate vector bosons will sooner or later come to the forefront. Before such a particle is observed we will not be sure about the validity of our ideas on weak and electromagnetic interactions.

It will be very important to collect more evidence about the existence of point structures inside hadrons. If the present trends persist and if no free quarks will be observed, the challenge for theoreticians will become even more dramatic.

Finally, we are eagerly awaiting experiments of $e^+ e^-$ annihilation at higher energies, again our present theories might be in the future subject to a very rude test.
The program of introducing unified theories of leptons and hadrons in order to have a single scheme for strong electromagnetic and weak interactions will play a fundamental role in the future. In my opinion the present attempts of unification are at a very preliminary stage. Very important improvement of our understanding of particle dynamics will be required before we can hope to extrapolate with success from our present knowledge into a complete unexplored energy range.

However, I have no doubt that unified theories are a main highway for future progress and that the new approach based on supersymmetries will be of great importance in those developments.

Understanding the composite nature of hadrons will constitute a great challenge for theoreticians. Future work should clarify whether the still unsolved problems of colour and of confinement will find their answer within the present schemes or whether they should require some essential change in our way of thinking.

The very important role which the structure of the vacuum state plays in particle dynamics is now being recognized. It is likely that this role will be even more fundamental in future theories.

In conclusion I have a feeling that our field is evolving towards a situation where there shall be a sharp boundary between those things which are well understood and those things which are not understood at all. I thus believe that future has in store strong difficulties and, maybe, some disappointments but I am convinced that it will also bring novelty, beauty and excitement.