The study of the differences of the fragmentation of quarks and gluons to jets of hadrons gives insight into the fundamental structure of QCD. Results from different approaches to properties of quarks and gluons are shown. The colour factor ratio $C_A/C_F$ is measured in agreement with the QCD prediction. Identified particles in quark and gluon jets are investigated, revealing no overproduction of isoscalar $\eta^0$ and $\phi(1020)$ in gluon jets, but an excess of protons.

1 Introduction

In QCD there are three fundamental vertices representing the three and the four gluon coupling and the coupling of a fermionic current to a gluon. Neglecting the four gluon coupling which is of $O(\alpha_s^2)$ there are three fundamental processes described by these vertices: The radiation of a gluon by a quark ($q \to qg$) or by another gluon ($g \to gg$) and the splitting of a gluon into a quark-antiquark pair ($g \to q\bar{q}$). The probability of these processes is given by the splitting kernels which take the kinematics into account and are proportional to the colour factors $C_F$, $C_A$ and $T_F$ respectively. The colour factors are the structure constants of the $SU(3)$ colour symmetry group of QCD, vividly speaking, they take care of the bookkeeping of undetermined colour flows in these processes. As the main contribution to particle production in hadronic events is due to the radiation of gluons, it is to be expected that the differences of the properties of jets initiated by quarks to those of jets initiated by gluons are in terms of the ratio $C_A/C_F$. The assumption of local parton-hadron duality (LPHD) gives rise to the hope that these perturbatively calculated differences can be observed in the hadronic final state.

2 Experimental access

To get information about gluons, events with three jets are used, as they contain exactly one gluon jet. The jets of a hadronic three jet event are numbered according to their energy with jet 1 being the most energetic one while angles between jets are numbered according to the jet opposite of the angle. There are three different ways of getting information about quarks and gluons out of hadronic three jet events presented here. In the jet analysis\cite{2,5}
single jets taken from three jet events are looked at. Gluon jets are identified using an anti-tagging technique in events with primary $b$-quarks. Information about light quark jets is obtained by subtracting the properties of gluon jets from a sample of unidentified jets. Approximately 142,000 identified gluon jets enter this analysis. As the dynamical scale for a single jet the so called hardness scale $\kappa_h = E_{\text{jet}} \cdot \sin \theta_{\text{min}}$ is used, where $\theta_{\text{min}}$ represents the smaller of the two angles adjacent to a jet. Although this is a problem which, in principle, involves two scales, $\kappa_H$ proves to be an appropriate choice to reduce this to a one scale problem. The jets entering this analysis cover the dynamical range of approximately $\kappa_h = 6 GeV \ldots 29 GeV$. In the hemisphere analysis events are selected with a hard gluon initiating the most energetic jet. The gluon jet is defined by all particles in the hemisphere of the leading jet. This type of event requires both subleading jets to be tagged as $b$-quark jets successfully. Light quark jets to compare with are defined by a whole hemisphere of an event. This procedure gives 439 gluon jets at an energy of 40.1GeV and quark jets at 45.6GeV. Corrections for the difference in energy are applied to the quark jets. In the symmetric event analysis\textsuperscript{2,3} complete three jet events are investigated. Only events with $\theta_2$ being equal to $\theta_3$ within small tolerances ($\pm 1.25^\circ$) are used so the whole event can be described by giving only $\theta_1$. No assignment of particles to jets and no jet tagging is needed. A total of approximately 162,000 symmetric events enter this analysis.
3 Results

As stated above, the gluon bremsstrahlung is expected to be $C_A/C_F$ times higher in gluon than in quark jets. This contrasts to the experimental observation of the ratio $r_{ch}$ of the charged multiplicity in gluon jets over quark jets to be below 1.5 as found over the whole scale range of the jet analysis and $1.514 \pm 0.019 \pm 0.034$ for the hemisphere analysis. This difference to the naïve expectation can be explained by higher order corrections and effects of finite energy. Soft partons emitted at large $p_T$ cannot resolve single partons within the shower-evolution of a jet, so the effective colour charge emitting these particles is the one of the primary parton. Figure 1 shows the $p_T$ distribution of soft particles ($|p| < 4\text{GeV/c}$) in quark and gluon jets and the ratio of both in the lower plot as taken from the hemisphere analysis. For higher $p_T$ the ratio is in the expected range of $C_A/C_F = 2.25$. Monte-Carlo studies confirmed that this ratio in the range of $0.8\text{GeV/c} < p_T < 3\text{GeV/c}$ reflects the colour factor ratio, yielding $r_{ch}(|p| < 4\text{GeV/c}, 0.8\text{GeV/c} < p_T < 3\text{GeV/c}) = 2.29 \pm 0.09 \pm 0.15$ in good agreement with the QCD expectation for $C_A/C_F$. The same idea is used in figure 2 from the event analysis. This plot shows the multiplicity within a cone with fixed opening angle of $30^\circ$ perpendicular to the event plane. The abscissa is the angle $\theta_1$ between jet 2 and 3. The plotted line is derived from a prediction by V.Khoze, W.Ochs and S.Lupia and parametrised by

$$N_{qqg}(\theta_1) \propto 2 + \cos \frac{\theta_1}{2} - \cos \theta_1 - \frac{1}{N^2_c} \left(1 + \cos \frac{\theta_1}{2}\right)$$

(1)

with only the normalisation left free. The prediction describes the data well, giving evidence of coherent radiation within a three jet event. This provides a direct test of a perturbative calculation independent of fragmentation models. Both approaches are complementary, as in the hemisphere analysis the topology and therefore the colour charge separation is fixed while partons of different wavelengths and resolution capabilities have been investigated, while in the symmetric event analysis the cone and therefore the phase space of the partons remains fixed and the separation of the colour charges is varied with the topology.

For the multiplicity of a hadronic three jet event there is a MLLA-prediction of the simple form

$$N_{qgq}(\theta_1) = N_{qg}(2E^*, p_T^1) + \frac{1}{2} N_{gg}(p_T^1)$$

(2)
where coherence effects are taken into account by the used scales $2E^*$ and $p_T^1$. As in the symmetric event analysis the event is described by only giving $\theta_1$, the two scales $2E^*$ and $p_T^1$ can be expressed as functions of this angle. The contributions $N_{q\bar{q}}$ and $N_{gg}$ are correlated by a differential equation including the colour factor ratio with corrections. Taking the parametrisation of $N_{q\bar{q}}(\sqrt{s})$ from events at various beam energies and fixing the only free parameter left by a measurement of the $gg$-multiplicity by CLEO$^1$, one gets an absolute prediction of the topology dependence of the three jet multiplicity. This prediction is found to be in very good agreement with the data from the symmetric event analysis down to an opening angle of $\theta_1 \simeq 30^\circ$. The lower curve in figure 3 shows the parametrisation $N_{q\bar{q}}(\sqrt{s})$ of the data. The upper curve is $N_{gg}$ derived from $N_{q\bar{q}}$, the corresponding data points from the DELPHI symmetric event analysis are calculated from three jet event multiplicities by subtracting $N_{q\bar{q}}$. The $N_{gg}$ data point at $10\, \text{GeV}$ from CLEO has been used to fix the prediction while the point at $40\, \text{GeV}$ is taken from OPAL’s hemisphere analysis$^8$. The lowest three points from the CLEO analysis are without systematic errors$^1$. The overall agreement between data and prediction is good. The result for $C_A/C_F$ gained by the symmetric event analysis is $C_A/C_F = 2.246 \pm 0.062 \pm 0.080 \pm 0.095$ where an ansatz has been used which takes care of non-perturbative effects by an added offset$^2$. Numeric results from a fit with equation 2 are to be expected soon.

While the multiplicity is dominated by soft particles, the study of scaling violation in quark and gluon jets offers the possibility to obtain information

![Figure 4. The fragmentation function of the gluon as a function of $\kappa_H$.](image1)

![Figure 5. Strength of the scaling violation in quark and gluon jets as a function of $x_F$.](image2)
from the other end of the spectrum. The energy dependence of the fragmentation functions is described by the DGLAP-equations. As the splitting kernels occur in this set of integro-differential equations, again the colour factor ratio $C_A/C_F$ is expected to be found in the comparison of gluon to quark jets. In the jet analysis the fragmentation functions of quarks and gluons have been investigated\( ^5 \). Figure 4 shows the fragmentation function of the gluon as a function of $\kappa_H$ and $x_E$ as a parameter. $\kappa_H$ is used as the dynamical scale as we are dealing with single jets. The curves are the result of an evolution of the DGLAP-equations to first order. The description of the data is very good and extrapolates well to the data taken from the hemisphere analysis at the rightmost\( ^8 \). Fitting a power-law ansatz through the measured fragmentation functions yields the result shown in figure 5 where the double logarithmic slopes found in gluon and quark jets are plotted. Obviously the scale dependence is more pronounced in gluon than in quark jets. Again the lines represent the results of the evolution of the DGLAP equations. A fit with this evolution yields for the colour factor ratio $C_A/C_F = 2.25 \pm 0.09 \pm 0.06 \pm 0.11$ where the last error reflects the dependence on the jet finding algorithm and choice of scale. This is in excellent agreement with the QCD prediction. Also identified particles in quark and gluon jets have been studied, especially $\pi^\pm$, $K^\pm$, $p\bar{p}$, $\pi^0$ and $\eta$ as well as the resonances $K^*(892)$ and $\phi(1020)$\( ^4,9 \). These analyses both have dealt with single jets similar to the jet analysis described above. The investigation of the multiplicity of these particles in quark in gluon jets is of interest especially for the $\eta$ and $\phi(1020)$, as these mesons are isospin singlets as the gluons are. Despite this community, no overproduction of these two particles has been found in gluon jets. The multiplicity of $\eta$ particles has been studied at three different values for $\kappa_H$ between 7GeV and 25GeV. The multiplicity found in quark and in gluon jets as well as the ratio of both always agreed with the expectations from charged particles\( ^9 \). For the resonances $K^*(892)$ and $\phi(1020)$ the normalised ratio $R_X = [n_q(X)/n_q(X)]/[n_g(ch)/n_q(ch)]$ with $n_q,g(X)$ being the multiplicity of particle $X$ in quark or gluon jets and $ch$ denoting all charged particles has been investigated. The results obtained are $R_{K^*} = 1.7 \pm 0.5$ and $R_\phi = 0.7 \pm 0.3$ revealing especially no excess of $\phi$ in gluon jets\( ^4 \). While also pion and kaon production have been found in agreement with the results for all charged particles, a significant excess of protons in gluon jets has been found with $R_p = 1.205 \pm 0.041 \pm 0.02$\( ^4 \). Also the momentum spectra and $\xi_p$ distributions...
of these particles have been studied showing a good agreement with the predictions of Jetset and Ariadne and more pronounced deviations from Herwig predictions. The position of the maximum of the $\xi_p$ distributions, $\xi^*_p$, is shown in figure 6 for protons, pions and kaons. The data points denoted with Y and MERC are taken from a jet analysis in which only two symmetric event topologies where used. The behaviour of the quark $\xi^*_p$ values from this jet analysis extrapolates well to the measurements from events at higher energies. For pions and protons the $\xi^*_p$ values found in quark and gluon jets are in good agreement, while for kaons the $\xi^*_p$ values found in gluon jets are significantly higher than in quark jets. This observation does not agree with the assumptions of LPHD.

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References

3. DELPHI Collaboration, DELPHI 99-115 CONF 302
4. DELPHI Collaboration, DELPHI 99-116 CONF 303
5. DELPHI Collaboration, CERN-EP/99-144
8. OPAL Collaboration, CERN-EP/99-028
9. OPAL Collaboration, OPAL Physics Note, PN407