PHYSICS OF COOLED BEAMS

I. Hofmann and G. Kalisch
GSI Darmstadt, Postfach 110552, D-64220 Darmstadt, Germany

Abstract

Various aspects of diagnostics and its interpretation for coasting and bunched beams cooled by electrons are presented. Schottky spectra and their modification by impedances are central subjects in this discussion. We show that the noise generated by numerical simulation is closely related to the noise due to the graininess of a real beam, which helps interpreting coasting and in particular bunched beam spectra. Noise spectral analysis is used for determining the phase space density and the vicinity to instability. For bunched beams at higher currents some not yet understood instability phenomena are described by means of streakcamera images.

1. INTRODUCTION

For low phase space density beams the momentum spread can be obtained directly from the frequency span of the Schottky spectrum. This is not possible for cooled high phase space density beams, where the observed spectrum can be modified by collective effects and also by the friction force as was made evident in the first electron cooling experiments [1, 2]. It is thus necessary to use theoretical tools in order to obtain the desired information from measurements. In this context a close comparison between experiment and computer simulation is necessary. We confine the studies presented here to phenomena in the longitudinal phase space, where space charge effects can be significant. In transverse direction there is a limitation set by the maximum tune shift, which is determined by nonlinear resonances. Experimental observations are made at the GSI electron cooling storage ring ESR [3].

2. INTERPRETATION OF SCHOTTKY SPECTRA

2.1 Experimentally Obtained Spectra

The origin of the Schottky spectrum is the noise from the statistical distribution of particles. This gives rise to current fluctuations, which induce a voltage on a pick-up. Schottky spectra for a Na$^{10+}$ beam at 250 MeV/u are shown in Fig. 1, where we compare the spectrum with electron cooler current nearly zero and increased in steps up to 0.5 A [4]. The Schottky power is proportional to the square of the current fluctuations, which is proportional to the momentum distribution function only in the low phase space density case [5], where particles are completely uncorrelated in phase around the machine. With cooling there are collective effects (waves), which correlate particle phases. This leads to peaks at the frequencies of the collective motion.

2.2 Analytical Support and Interpretation

For high phase space densities the beam responds collectively on the field excited by the statistical fluctuations of the uncorrelated particles ("source field"). This results in a total electric field, which is obtained as usual by dividing the source electric field by the dielectric
Fig. 1: Schottky spectra for increasing cooler current to maximum of 0.5 A.

function $\epsilon$ (see also Ref. [6]). Using Vlasov’s equation (see Ref. [1, 2, 7, 8]) one thus obtains the modified expression for the Schottky power spectrum for harmonic $n$ according to

$$P(\Omega, p) = \frac{q^2 e^2 N}{\pi} \frac{\Psi_0(\Omega/n)}{|\epsilon|^2}$$  \hspace{1cm} (1)

with the plasma dielectric function given by

$$\epsilon(\Omega, n) = 1 - iZ_|| \frac{\eta q I}{\beta^2 \gamma Amc^2} D(\Omega, n)$$  \hspace{1cm} (2)

where $Z_||$ is the coupling impedance, $\eta = 1/\gamma^2 - 1/\gamma_t^2$ (transition energy $\gamma_t >> 1$), and $D$ a dispersion integral involving the distribution function of frequencies:

$$D = \int \frac{\partial \Psi_0}{\partial \omega} \frac{I}{\Omega - n\omega} d\omega = PV \int \frac{\partial \Psi_0}{\partial \omega} \frac{I}{\Omega - n\omega} d\omega + i \frac{\pi}{n} \frac{\partial \Psi_0}{\partial \omega}$$  \hspace{1cm} (3)

For the low-intensity limit $\epsilon \approx 1$, Eqn. 1 yields directly the distribution function. For high phase space density $\epsilon$ can be close to zero for frequencies, which correspond to longitudinal "plasma waves". They are excited by the noise and can propagate in beam direction or opposite to it (fast and slow waves). This is responsible for the sharp peaks in the spectrum at these frequencies. The peaks would be infinite, if Landau damping were absent. At the band center the power is suppressed strongly, which is due to the shielding of the noise by the collective motion. This phenomenon is analogous to the well-known Debye shielding of a test particle in a plasma.

If the coupling impedance were known at sufficient accuracy, the distribution function could be determined in an iterative way: assuming a first guess for $\Psi_0$ one can calculate $\epsilon$. Inserting this approximate $\epsilon$ into Eqn. 1 and using the measured $P$ one obtains a new $\Psi_0$, with which one can repeat the iteration. With unknown impedance the method still works if a second independent measurement exists, which is the beam transfer function method (see below), hence both impedance and distribution function can be determined consistently (see Ref. [8, 6]).
With only Schottky spectra and an uncertainty in the impedance - noting that the space charge impedance depends logarithmically on the beam size - it is thus not straightforward to obtain the correct $\Delta p/p$. A comparison with analytically derived spectra by calculating $\epsilon$ can be useful to determine $\Delta p/p$ in an approximate way [9, 6].

2.3 Simulation Noise

The above outlined interpretation of collectively distorted noise spectra on the basis of analytical calculations of a dielectric function can be strongly supported by calculating "real noise" by means of computer simulation. As we shall see below this is of particular interest for bunched beams. In simulation working with discrete particles one has natural fluctuations in the line density, which are analogous to the fluctuations in real beams. In Fig. 2 we show such an instantaneous line density for a coasting beam simulated by the "particle-in-cell" program SCOP-RZ. "Particle-in-cell" refers to the technique of calculating the self-interaction of the beam by creating each time-step a density function on a grid in $r, z$ and solving Poisson’s equation for an infinitely conducting pipe. This procedure takes care of what is usually described by the space charge impedance. In Fig. 2 we also show a projection of the beam into the $x, z$ plane (different scales in $x$ and $z$). By Fourier analyzing the line density we can also include the measured machine impedances and thus obtain a full description of the electromagnetic interaction of the beam.

The simulation Schottky noise is obtained by recording the line density over a large number of revolutions and carrying out a Fourier transformation. Smooth spectra (high "confidence level") can be obtained only by carrying out an averaging over a number (typically several ten) of independent spectra, which is the same procedure as in the experiment.

In Fig. 3 we show spectra for large (uncooled) and 10 times smaller (cooled) momentum spread with no external machine impedance (i.e. only space charge active), and with a resistive component. The uncooled case corresponds to $\Delta p/p = 10^{-3}$ for a 20 mA Ne$^{10+}$ beam at 250 MeV/u. The momentum distribution is Gaussian in all cases and remains unchanged during the simulation. The different noise spectra are related to longitudinal correlations between particles due to the collective interaction. The latter is absent for large momentum spread due to Landau damping.

Simulation noise is particularly valuable to study directly the effect of extra contributions to the coupling impedance, which can be turned on and off separately, while the momentum distribution is well-known as input. This is also shown in Fig. 3 for an additional real part of the coupling impedance $Z_{\parallel}/u$ of 50 $\Omega$, which could be due to a low impedance rf cavity. The left peak corresponding to the slow wave is strongly enhanced, whereas the right peak is reduced. A further enhancement of the resistive impedance (at constant $\Delta p/p$) would lead to a growing left peak.

![Fig. 2: Fluctuations of line density (left) and scatter plot (right) of coasting beam simulation.](image-url)
3. STABILITY AND INSTABILITY

3.1 Transition to Instability

The limit of instability in Fig. 3 would be characterized by an infinitely large left peak of the Schottky noise. This happens at a sufficiently large resistive part of the coupling impedance depending on the inductive part. For a more detailed discussion of stability one introduces the beam transfer function (BTF), which is the inverse response function [11]:

$$\frac{1}{\tau_{||}} = \frac{1}{\tau_{||,0}} + Z_{||}$$

(4)

with (using Equ. 3)

$$\tau_{||,0} = -i \frac{eZ \eta I}{\beta^2 \gamma Am_p c^2 D}$$

(5)

A measured BTF diagram for an electron cooled beam is shown in Fig. 4 [10]. According to Eq. 4 the effect of the impedance is a shift of the origin, which must lie in the interior of the curve to ensure stability. Due to the equilibrium with the cooling mechanism (Coulomb scattering) the momentum distribution is very close to a Gaussian. The tails of the Gaussian lead to a largely extended stable area in the direction of inductive impedance (space charge). This is beneficial for energies far below the transition energy. There the space charge impedance (inductive) is large, which has the effect of shifting the coherent frequencies into the tails of the distribution function.

We note that the worst case in the sense of stability is a parabolic distribution, for which the boundary of stability is described by a small circle in the BTF diagram. The small size of this area is due to the absence of Landau damping in the tails. This circle is described by the "Keil-Schnell criterion":

$$I_{th} = \beta^2 \gamma \frac{Am_p c^2}{Ze} |\eta| \left( \frac{\Delta p}{p} \right)^2 \frac{f_{whm}}{|Z_{||}|}$$

(6)

According to Fig. 4 the allowed impedance (or in an equivalent sense beam current, following Equ. 6) is much larger for a Gaussian distribution as long as it is mainly in the inductive direction.
Fig. 4: Measured stability diagram (BTF measurement) for cooled beam, with Keil-Schnell circle indicated; arrow corresponding to cooled case of Fig. 3 (center), dashed arrow to unstable case of Fig. 5.

For the simulation example of Fig. 3, $I/I_{th}$ has been as large as 7 (arrow in Fig. 4) without loss of stability. Measurements in various cooler rings have shown that $I/I_{th}$ can indeed exceed 5 due to such Gaussian-like momentum distributions [13, 14, 12].

Obviously an additional resistive component of 1 kΩ (i.e. comparable with the space charge impedance) in the simulation example of Fig. 3 would shift the origin far beyond the boundary of stability. To show the onset of instability we have simulated such a case in Fig. 5. Such a resistive impedance could stem, for instance, from a ferrite loaded rf cavity tuned at resonance with the beam.

It is noted that the coasting beam develops a self-bunching effect, which leads to a trapping of particles in the longitudinal self-field. The instability saturates at a broadened momentum distribution, which is almost a factor 3 broader than the initial distribution. Due to the quadratic dependence of the threshold current on the momentum spread (Equ. 6) it is clear that this broadening leads to a final distribution, where $I/I_{th} \approx 1$. We thus confirm that $I/I_{th} \gg 1$ is only possible for small resistive compared with inductive impedance.

A different behaviour of the longitudinal microwave instability for a resistive impedance much smaller than the space charge impedance, but with $I/I_{th}$ as large as 50 was studied by simulation in connection with heavy ion fusion storage rings [15]. In this case the instability was found to develop extended "stabilizing tails", which would suppress further growth and leave the small initial momentum spread for most of the beam unchanged.

3.2 Role of Intrabeam Scattering

The question can be raised what the maximum $I/I_{th}$ is in a given cooler ring. Why are we not approaching the limit of stability, which would be accompanied by a large growth of the "slow wave" peak in the Schottky spectra as indicated by the simulation experiment of Fig. 3? It must be assumed that intrabeam scattering (IBS) plays a crucial role in this context. A possible consequence of IBS could be that the beam is longitudinally colder only if it is made transversely hotter.

In the lack of systematic measurements such a statement can be supported by calculations of IBS equilibria, which we have carried out for the ESR lattice. In Fig. 6 we plot $\Delta p/p$ as a function of the horizontal emittance under the assumption of a constant longitudinal cooling rate. The curve shows theoretical equilibria starting from an experimentally observed equilibrium at $\Delta p/p = 4.1 \cdot 10^{-5}$ and $\epsilon_h = 0.08 \cdot 10^{-6}$ m-rad for Au$^{79+}$ at 250 MeV/u and $N = 2.7 \cdot 10^6$. It is recognized that considerably smaller momentum spreads can be expected if the transverse
Fig. 5: Simulation of longitudinal microwave instability for $C^6+$ at 250 MeV/u, $\Delta p/p = 3 \cdot 10^{-5}$, $I=50 mA$ and 1k$\Omega$ resistive impedance at fundamental harmonic.

emittances are blown up. This can be achieved by an extra transverse heating source (rf noise) or by spoiling the transverse cooling optimisation. We can understand this result by noting that the transfer of heat from the transverse to the longitudinal degree of freedom happens at a rate, which is the slower the larger the velocity mismatch is between both degrees of freedom.

3.3 Influence of Cooling Force

It is necessary to examine whether the observed Schottky noise is only a consequence of the ion beam properties and the impedance, or whether it is also influenced by the cooling friction force. This is conceivable if one notices that the stability behaviour depends on the Landau damping provided by the tails of the momentum distribution. Electron cooling is also a damping mechanism with a rate given by the cooling rate $\nu_{cool}$. The friction force can be ignored, if the ratio of cooling rate to phase mixing rate (given by the spread of angular frequency) is small. This criterion can be approximately written as (for a more detailed model and further references see Ref. [8])

$$\frac{\nu_{cool}}{n \delta \omega} \ll 1$$

(7)

It is noted that the influence of the friction force is largest for small $n$. By comparing spectra for small and large $n$ it should be possible to identify such an influence, if the cooling rate is large enough; no experimental evidence seems to exist yet for such an observation.
Fig. 6: Calculated IBS equilibria in the vicinity of experimental data.

4. BUNCHEO BEAMS

4.1 Effect of Space Charge

We are interested in an appropriate measure for the phase space density of cooled bunches. The analogue of space charge in the center of the Schottky band for coasting beams is the reduction of the applied rf potential well due to space charge ("potential well flattening"), which results in an effective potential reduced by a factor $\alpha$:

$$V_{\text{eff}} = V_{rf}/\alpha$$  \hspace{1cm} (8)

We can compare this expression with Eqn. 1 for a coasting beam locally equivalent to the bunched beam center. This comparison suggests that $\alpha$ is equivalent to the dielectric function $\epsilon$, which describes the Schottky current ($\propto (\text{power})^{1/2}$) suppression in the band center. Provided that in both cases space charge is the dominant contribution to the impedance one thus has - ignoring a geometry factor of order unity depending on the bunch shape - :

$$\alpha \approx 1 + I/I_{\text{th.b.c.}} \approx \epsilon_{\text{b.c.}}$$  \hspace{1cm} (9)

with the understanding that $I_{\text{th.b.c.}}, \epsilon_{\text{b.c.}}$ are for coasting beams equivalent to the bunched beam center. $\alpha$ is thus an an appropriate measure for the longitudinal phase space density. $\alpha \gg 1$ would indicate strongly space charge dominated bunches, which have been observed for cooled proton bunches [16].

The potential well flattening leads to a bunch lengthening effect and a reduction in synchrotron frequency $\omega_0$. In harmonic approximation one immediately has the following relation:

$$\frac{\omega_{\text{eff}}}{\omega_0} = \alpha^{-1/2}$$  \hspace{1cm} (10)

From an experimental point of view an accurate measurement of $\alpha$ is desirable. We suggest that the most reliable way is to directly measure $\omega_{\text{eff}}$ and $\omega_0$.

4.2 Frequency Spectra

As an example we have measured in the ESR bunched beam spectra of a Ne$^{10+}$ beam of 250 MeV/u and $3 \times 10^8$ particles at an rf voltage of 100 V (Fig. 7).

Such measurements have been successful only for sufficiently stable operation of the electron cooler and relatively low intensity. The spectrum contains a central line a multiple of the revolution frequency and a large number of sidebands. Some of the sidebands are at multiples of 50 Hz; we assume they are due to the power supply of the cooler or rf. To obtain a more
solid ground for a physical interpretation we have calculated the simulation noise for a bunched beam, which allows a direct comparison with the experimental observation and helps interpreting them. Simulation results are shown in Fig. 8 for zero space charge and finite space charge of comparable strength as in the above experiment. The band width has been increased to reduce computer time. For zero space charge one obtains simply the multiples of the synchrotron frequency.

For finite space charge each sideband is split into several lines for coherent and incoherent (low-level signals) frequencies. The coherent frequencies refer to dipole (bunch center, \( m=1 \)), quadrupole (bunch length, \( m=2 \)) and sextupole (shape unsymmetry, \( m=3 \)) oscillations. The height of these lines reflects the amplitude of excitation of a particular mode. The notion of multipole oscillations in this context is applied to the shape of the distribution in longitudinal phase space. The dipole oscillation is unaffected by space charge since the bunch center oscillates carrying along its own space charge. The relevant frequency is thus the zero space charge synchrotron frequency \( \omega_0 \) (230 Hz in Fig. 7). We note that for each observed line there are also higher harmonics, which are just multiples. This explains the increasing number of lines at higher sidebands. The incoherent frequency peaks of single particles are much less pronounced.

**Fig. 7:** Split Schottky lines for bunched beam with moderate space charge (relevant satellites filled black).

**Fig. 8:** Simulation noise for bunched beam without (left) and with moderate space charge (right).
Fig. 9: Streakcamera pictures of 0.6 mA (left) and 1.9 mA (right, unstable) bunches of Ne$^{10+}$ at 250 MeV/u, with time running downwards.

due to the random summation of signals from individual particles. This incoherent synchrotron frequency $\omega_{\text{eff}}$ (185 Hz in Fig. 7) is thus a direct measure of the effective potential.

From these frequencies we calculate with Equ. 10, $\alpha = 1.55$, which indicates that 35% of the applied rf voltage is compensated by space charge. We assume that intrabeam scattering heating of the momentum distribution is the main reason why the observed $\alpha$ was not any larger.

4.3 Bunches at Higher Intensities

Experiments in the ESR with bunches at larger intensities have so far not lead to sufficiently stationary conditions to measure Schottky spectra. For total currents exceeding typically 1 mA (i.e. 10% of the maximum stable stored coating beam currents for Ne, C beams) it was found that bunches were not in a stable equilibrium with the electron cooling. Periodically (in intervals of the order of a second) a phase of cooling to short bunch length was followed by a disintegration of the bunch profile. In Fig. 9 we show streakcamera pictures of a low intensity stable bunch and a snapshot during the unstable phase at three times higher intensity, both at an rf voltage of 1 kV. Each picture is for a single bunch extracted from the ring and hitting a plastic scintillator (NE102) with ns time resolution. The camera records the light through a horizontal slit centered on the spot. At subsequent times the light is deflected (vertically downwards in Fig. 9) which yields a longitudinal - horizontal cut through the bunch in real space. The overall time resolution is 1-2 ns.

The horizontal size of the image gives the instantaneous horizontal beam size ($\propto \epsilon_h^{1/2}$) as a function of position along the bunch. Shown are equi-intensity contours of the scintillator light, which are converted alternatively into black and white. In the low-intensity picture the unsymmetry of the image is due to a scintillator afterglow. For the three times higher intensity it is seen that the bunch has suffered an unstable behaviour both in longitudinal and in horizontal direction. In this case peak intensity is in the center and in the half-moon shaped area at the head of the bunch. This instability phenomena might be connected with a de-tuning between cooler energy and rf frequency, which requires careful study in the future.

Acknowledgment: The authors are indebted to B. Franzke and the ESR-group for their support in the measurements.

References


