UNIFIED MODELS OF WEAK AND ELECTROMAGNETIC INTERACTIONS

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1. INTRODUCTION

Spontaneously broken gauge theories provide a framework for the construction of renormalizable models of weak and electromagnetic interactions\(^1\)). In this talk I shall review the status of attempts to construct such models\(^2\)). First, for the benefit of those who are not expert in the field, I shall remind you of the ingredients of the models and the role they play. I shall then discuss the problems encountered in model building and the hopes which might be entertained for future models. Finally, I shall make some brief remarks about the implications of the evidence for neutral currents which has recently been reported\(^3\text{-}^5\)).

2. INGREDIENTS OF MODELS AND THEIR ROLE

A chain of argument which motivates the ingredients of renormalizable models of weak and electromagnetic interactions is illustrated in the accompanying chart\(^7\)); the rest of this section should be read in conjunction with this chart.

The data suggest that weak interactions are governed by vector currents and hence that they may be mediated by the exchange of an intermediate vector boson \(W\). Vector bosons lead to trouble when they are longitudinally polarized because the longitudinal polarization vector \(c_L\) behaves like \(k_\mu/M_W\) at high energy. Consider the reaction \(\nu\bar{\nu} \rightarrow W^+_L W^-_L\) in a high luminosity colliding \(\nu\bar{\nu}\) machine. Each of the longitudinal \(W\)'s introduces a factor \(c_L \sim k_\mu/M_W\) in the dimensionless amplitude, in which the resulting factor \(M_W^{-2}\) is compensated by a factor \(S\). This gives a Born cross-section which grows linearly in \(S\) at high energy. Such growth rapidly violates unitarity and leads to non-renormalizable divergences in the box diagram for \(\nu\bar{\nu} \rightarrow \nu\bar{\nu}\). This can be understood if we think of calculating the forward amplitude for \(\nu\bar{\nu} \rightarrow \nu\bar{\nu}\) in order \(g^6\) in terms of a dispersion integral over \(\sigma_B\). With \(\sigma_B \sim S\) the dispersion relation will diverge without two subtractions -- the subtraction constants are new arbitrary parameters and we have lost the ability to calculate. The theory is unrenormalizable. Alternatively, when we sum over all possible polarizations of the virtual \(W\)'s, the \(c_L\)'s give the notorious \(k_\mu k_\nu\) terms in the propagators which tend to constants as \(k \rightarrow \infty\) providing no convergence factors in the Feynman integral.

The source of all the trouble is the piece of \(c_L\) proportional to \(k_\mu M_W\). In a gauge theory the \(W\) couples to a conserved current and this piece is impotent. Can we see empirically how this might come about? The diagram which we considered must occur if the weak interactions are mediated by a \(W\); on its own this diagram would render the theory non-renormalizable. Since all diseases must be cured order by order in renormalizable perturbation theory, there must be other exchanges in the same order in the \(s\text{-}, t\text{-}\) or \(u\text{-}\)channels, i.e. these must be a neutral current and/or a heavy lepton.
Weak Int. Data → Vector Currents → Vector Bosons (W)

\[ \begin{align*}
\nu & \rightarrow W^+_
u, e^+ \rightarrow W^+_{e^+} \\
\overline{\nu} & \rightarrow W^-_{e^-}, e^- \rightarrow W^-_{e^-}
\end{align*} \]

etc.

Longitudinal Vector Bosons → Trouble:

\[ \left( \begin{array}{c}
\text{rest} \\
\text{frame}
\end{array} \right) \rightarrow \left( \begin{array}{c}
t \\
y \\
z
\end{array} \right) \rightarrow \text{boost along } OZ \rightarrow \left( \begin{array}{c}
\frac{k_x}{M_W} \\
0 \\
0
\end{array} \right) + \left( \begin{array}{c}
k_y \frac{k_x}{M_W} \\
0
\end{array} \right) \]

Consider:

\[ \begin{align*}
\nu_e & \rightarrow W^+_{\nu_e}, e^+ \rightarrow W^+_{e^+} \\
\overline{\nu}_e & \rightarrow W^-_{e^-}, e^- \rightarrow W^-_{e^-}
\end{align*} \]

\[ \text{Dimensionless Amp. } \sim \frac{g^2 S}{M_W^2} \]

\[ \sigma_B \sim \frac{g^2 S}{M_W^2} \]

\[ \rightarrow \text{Unitarity Violation} \]

\[ \rightarrow \text{Non-Renorm. Divergence in} \]

\[ \int \frac{d\sigma_B(s')}{s' - s} \rightarrow \text{2 subtractions} \]

In renorm. pert. th. must cure problems order by order.

Need:

\[ \begin{align*}
\text{Neutral Current} & \rightarrow \text{and/or} \\
\text{Heavy Lepton}
\end{align*} \]

\[ \text{can adjust couplings to cure worst problems.} \]

Generalization for all fermion-antifermion → WW

\[ \text{necessary and sufficient condition to cure worst problems: couplings must be those of a} \]

\[ \text{(Yang-Mills) theory having a generalized gauge invariance.} \]

Residual problems

\[ A(WW + WW) \sim \frac{g^2 S}{M_W^2} \]

\[ A(FF + WW) \sim \frac{g^2 S}{M_W^2} \]

\[ \rightarrow \text{exchange of scalar (Higgs) mesons (+ spont. symm. breaking)} \]

Systematic study of \( \phi \) couplings

"Derive"

Spontaneously broken gauge theories
It is a simple matter to work out the conditions which the coupling constants in these diagrams must satisfy in order that there is a cancellation and the sum of the amplitudes does not grow so rapidly at high energy. This is easily generalized to a systematic analysis of all processes of the type $FF \rightarrow WW$; a similar analysis can be made for $WW \rightarrow WW$. The result is that a necessary condition to ensure smooth high-energy behaviour is that the coupling constants are those of a Yang-Mills, or generally gauge-invariant, theory\(^8,9)\).

This is sufficient to remove the worst problems, but terms which grow like $g^2M_F^2/M_W^2$ remain in $FF \rightarrow WW$, and terms which grow like $g^2S/M_W^2$ remain in $WW \rightarrow WW$. These remaining difficulties must also be cured in the same order of perturbation theory. This forces us to introduce yet more particles -- the famous Higgs scalars ($\phi$), which are associated with spontaneous symmetry breaking in the formal approach. The $WW\phi$ coupling is proportional to some mass; since $\phi$ exchange must cancel the residual term in $WW$ scattering, it must be $M_W$. Similarly, the coupling to fermions must be of order $gM_F/M_W$ if $\phi$ exchange is to cancel the residual terms in $FF \rightarrow WW$.

It is possible to work out the conditions on the $\phi$ couplings which are necessary and sufficient to ensure that all Born amplitudes are well behaved at high energy. With the wisdom of hindsight, spontaneously broken gauge theories can be "derived" in this way\(^8,9)\).

Let me conclude this brief survey of the elements of models based on spontaneously broken gauge symmetries by reminding you that at present there is no direct evidence for the relevance of these models; many other strategies have been proposed to cure the divergence difficulties of weak interactions. Some of these alternative strategies have been reviewed in Ref. 10 where further references may be found.

3. MODELS -- PROBLEMS AND HOPES

3.1 Leptons

There are two minimal models of leptons -- the Weinberg-Salam model\(^11,12)\) with a neutral current but no new leptons, and the Georgi-Glashow model\(^13)\) with new leptons but no neutral current. Given the general strategy, it is easy to invent innumerable other models. The observation of neutral currents\(^3-4)\) excludes the Georgi-Glashow model; otherwise, not knowing why the muon exists and having no strong prejudices about the lepton spectrum, I personally see no strong reason to prefer one particular model.

3.2 Hadrons

For hadrons the situation is different -- most of us have many prejudices and there are plenty of experimental facts. The result is that we find it hard
to make convincing models. In my opinion, there are no plausible models at the moment. I shall review the problems of model building in a series of remarks:

(i) *Models with 3 quarks and the Cabibbo theory are hard to make*

Why should we want to make such a model? It seems to me that because of the success of SU(3), particularly the Cabibbo theory, and because we have acquired the habit of believing Gell-Mann's current algebra, it is natural to try to use quarks as the "fundamental hadronic fields" which must be specified if we wish to couple weak interactions renormalizably to hadrons [it could be argued that this is unnecessary and that strong interactions may damp divergences such as those we encountered in purely leptonic processes; current algebra/parton/light-cone arguments suggest that this does not occur\[18\]]).

Let us try to make a model using 3 quarks and the conventional Cabibbo current. The basic problem is that the amplitude corresponding to the diagram

\[
\text{N} \quad \overset{\text{P}}{\xrightarrow{\lambda}} \quad W^- \\
\bar{\lambda} \quad \overset{\bar{\text{P}}}{\xrightarrow{\bar{\lambda}}} \quad W^+
\]

(which must occur in this model) grows like \(S\) at high energy. This growth, which is incompatible with renormalizability, can be cured in four different ways:

a) We can change the Cabibbo theory, putting \(\bar{\lambda}Y_\mu(l - \gamma_5)p + \lambda Y_\mu(l + \gamma_5)p\) [this removes the worst high-energy growth since \((1 - \gamma_5)\) annihilates \((1 + \gamma_5)\) when we put the masses to zero to isolate the leading term which is helicity-conserving]. Such models\[15\] come into conflict with the measured sign of \(g_A\) in \(A_0\) decay and are almost certainly excluded by the data\[16\].

b) To the diagram

\[
\text{N} \quad \overset{\text{P}}{\xrightarrow{\lambda}} \quad W^- \\
\bar{\lambda} \quad \overset{\bar{\text{P}}}{\xrightarrow{\bar{\lambda}}} \quad W^+
\]

we can add

\[
\text{N} \quad \overset{\text{P}}{\xrightarrow{\lambda}} \quad W^- \\
\bar{\lambda} \quad \overset{\bar{\text{P}}}{\xrightarrow{\bar{\lambda}}} \quad Z^0 \xrightarrow{\text{W}^+} W^+
\]

and arrange a cancellation at high energy. We sense danger, however, since \(Z^0\) couples to a \(\Delta S = 1\) neutral current. To avoid conflict with experiment we must not only ensure that \(Z^0 \neq \mu\bar{\mu}\), \(ee\), \(\nu\bar{\nu}\) (to keep the rates for \(K_L^0 \rightarrow \mu\bar{\mu}\), \(K^+ \rightarrow \pi^+\nu\bar{\nu}\), \(K^+ \rightarrow \pi^+ee\), etc. sufficiently small) but also make \(Z^0\) a "schizon" \((Z^0 \neq \bar{Z}^0)\) in order to forbid the \(\Delta S = 2\) transition

\[
\text{N} \quad \overset{\text{P}}{\xrightarrow{\lambda}} \quad \bar{\lambda} \\
\bar{\lambda} \quad \overset{\bar{\text{P}}}{\xrightarrow{\bar{\lambda}}} \quad Z^0 \\
\text{N} \quad \overset{\text{P}}{\xrightarrow{\lambda}} \quad \bar{\lambda}
\]
which would lead to too large a value for $M_{K^0} - M_{\bar{K}^0}$ and $\Gamma(\Xi \to N\pi)$. It may be possible to construct such a schizon scheme (for example, in a model based on SU(3) there would be schizophrenic $\omega^0$'s with the quantum numbers of $K^0$ and $\bar{K}^0$, although there is no obvious simple way to introduce leptons and generate all observed decays correctly). However, I know of no satisfactory three-quark model of this type. I expect that such a model would have to be rather complicated. New leptons would certainly be needed. Furthermore, in order to cancel residual high-energy growth in $n\bar{n} \to W^+W^-$, there must be a Higgs scalar $\phi^0$ coupled to $n\bar{n}$; it may also have to be a schizon and decouple from known leptons unless the masses can be arranged in such a way that it makes suitably small contributions to $\Delta S = 1$ leptonic and $\Delta S = 2$ non-leptonic processes.

c) To the diagram

```
    n
   / \          \     \n  p  \   \  p'  \     \  
 W+  \    \   \    \  \  \  
  \   \  \  \  \    \  
  \  \  \  \  \  \  \  
    \bar{n}
```

we can add

```
    p
   / \          \     \n  p' \   \  p  \     \  
 W+  \    \   \    \  \  \  
  \   \  \  \  \    \  
  \  \  \  \  \  \  \  
    \bar{n}
```

or other new exchanges in the $t$- or $u$-channels, but in so doing we have increased the number of "quarks" to at least four. There must be new hadronic degrees of freedom beyond SU(3) -- 'charm' or new groups SU(3)' etc.

The simplest scheme of this sort is perhaps a four-quark model of the Glashow-Iliopoulos-Maiani type 17 in which there is a charmed $p'$ quark with quantum numbers $B = \frac{1}{3}$, $I = 0$, and $Q = \frac{2}{3}$ (the other quarks and the known stable hadrons having zero charm, charm being an additive quantum number conserved in strong interactions); this model can be married to the Weinberg-Salam model for leptons with Cabibbo doublets

\[
(1 - \gamma_5) \left( \begin{array}{c} p \\ n\cos \theta_c + l\sin \theta_c \end{array} \right) (1 - \gamma_5) \left( \begin{array}{c} p' \\ -n\sin \theta_c + l\cos \theta_c \end{array} \right)
\]

coupled to the $W$. If each quark comes in a red, a white, and a blue variety there are no anomalies (see below) and the model is renormalizable 18,19; it is the simplest renormalizable model known to me which, when combined with parton/light-cone ideas, can easily account for all deep inelastic data [except the CEA measurement of $\sigma(e^+e^- \to \text{hadrons})$].

Another possibility is to use the nine Han-Nambu quarks 20 but most of the schemes of this sort 21 which have been proposed rely on arbitrary
ad hoc assumptions to fit the data; they have another aesthetic drawback which will be discussed in remark (ii) below.

d) We can abandon "quark-like" schemes altogether. Examples of non-quark schemes are the ambitious models considered by Bars, Yoshimura and Halpern and by de Wit who attempt to make models of both strong and weak interactions. These models have the symbolic structure:

These schemes have the (dubious?) advantage that they make life easier for model builders by decoupling the group constraints on hadrons and leptons which are classified in representations of different groups $G_S$ and $G_W$ (if the W's interacted directly with leptons and hadrons, both would have to be classified in representations of $G_W$). Incidentally, we remark that this trick is of course quite independent of the strength with which $V_{\mu}$ couples; we can make it weak and construct models in which there are originally distinct sets of "hadronic" and "leptonic" weak bosons which get mixed and hence generate (universal) semileptonic process. If the V's couple strongly, obedient to the wishes of their begetters, some aesthetic drawbacks are encountered which will be discussed in remark (ii) below.

To summarize remark (i): the absence of $\Delta S = \pm 1$ neutral currents (plus our stupidity?) has led us to introduce charm, or other new (as yet unobserved) hadronic degrees of freedom, or to consider other very complicated schemes. A pessimist would say that nature is telling us to abandon gauge theories; an optimist would reply "No! Nature is giving us an important clue".

(ii) Unification of weak and electromagnetic interactions

An optimist would say "This is the greatest thing since Maxwell unified electricity and magnetism". A pessimist would reply "This unification seems unnatural and artificial to me. Electromagnetic interactions respect parity and charge-conjugation invariance which are violated by weak interactions. Furthermore, weak and electromagnetic interactions apparently have quite different
strengths. Will not unified models lead to parity violating corrections to strong interactions of order $\alpha$, from virtual $W$ loops?"

The answer to this question has already been reviewed by Wettlman in his talk\textsuperscript{1) so I shall only briefly repeat the conclusions in the literature:

If the strong interactions are mediated by vector mesons associated with a non-chiral gauge group, there can be no bad order $\alpha$ effects\textsuperscript{25}).

Otherwise order $\alpha$ effects can occur; often (perhaps always?) they can be renormalized to arbitrary values -- this is the case in the Bars-Yoshimura-Halpern, de Wit models\textsuperscript{26,27}). I find this ugly and tentatively reject such models.

It seems to me that the argument that there can be no bad order $\alpha$ effects might fail for non-abelian non-chiral gauge theories if it is possible to write parity-violating Yukawa couplings of Higgs scalars to fermions; luckily this may be impossible in many models because the diagonal Yukawa couplings necessarily conserve parity if we insist on CP-conservation. However, this same connection between the CP and P properties of Yukawa couplings may mean that if CP is conserved there can be no bad order $\alpha$ effects in some models in which the strong interactions are mediated by scalar mesons.

Putting aside these reservations, the criterion that the observed strength of parity violation in strangeness-conserving processes should have a natural origin would seem to lead to (abelian or non-abelian) non-chiral gauge theories of strong interactions. An optimist would say "We have learned something fundamental. Furthermore, it is very interesting that models in which the strong interactions are based on non-abelian gauge theories are the only ones which are asymptotically free\textsuperscript{28}) -- i.e. exhibit (quasi) EJorken scaling even according to field theorists. Some may view as an added bonus the fact that strongly interacting vector mesons "reggeize" in a spontaneously broken gauge theory\textsuperscript{29})."

A pessimist would reply "Soon there will be no models left. As pointed out by Pati and Salam\textsuperscript{30)}, this criterion removes all models based on SU(3)$'$ [Han Nambu\textsuperscript{21}) or coloured quarks\textsuperscript{31})] which exploit the SU(3)$'$ degree of freedom\textsuperscript{21,31}) in the construction of the weak current". The argument is that if the strong interactions are based on a gauge theory, the weak and strong groups must commute. Hence in these models the strong forces would be (colour blind) SU(3)$'$ singlets and SU(3)$'$ singlet, octet, decuplet, etc. states of these quarks would be degenerate\textsuperscript{32})

(iii) **Higher-order corrections**

Having ensured that lowest-order weak matrix elements always contain a factor $M_W^2$ to turn $e^2$ into $G_{\text{Fermi}}$, we must consider higher order effects which will generally be of order $\alpha G_F$ in unified models. However, it is an experimental
fact that the allowed second-order process $K^0_L \to \mu \bar{\mu}$ has a strength less than or of order $G_F^2$ and that the transition $K^0 \leftrightarrow \bar{K}^0$ has a typical "second order in $G_F$" strength $\sim \Gamma_{K^0}$. These induced second-order neutral current effects must therefore be suppressed. This is possible (e.g. with the GIM trick it is achieved by making $m_{\mu^*} - m_\mu$ "not too large" and hence charmed particles "not too heavy"), but in many models it is done in an unnatural ad hoc way\textsuperscript{33}). A pessimist would be shocked by the evasion of facts in many models in the literature but an optimist would regard the necessity of this suppression as another important clue for the construction of the "correct model".

(iv) Universality of weak and electromagnetic interactions

(i.e. Cabibbo universality and the quantization of charge) An optimist would observe that both kinds of universality could be understood in models based on semi-simple groups (i.e. groups which are not products containing abelian-U(1)-groups with corresponding transformations which are just phase changes, as in QED); consider a model with an "isospin-like" symmetry in which

$$L = \sum_i \bar{\psi}_i \gamma^\nu (i \partial_\nu + g \overline{T}_i \cdot W_\nu) \psi_i$$

is invariant under the infinitesimal transformation

$$\overline{W}_\nu \rightarrow \overline{W}_\nu + g \overline{T}_i \cdot \overline{W}_\nu + \partial_\nu \overline{\Lambda} + \ldots$$

$$\psi_i \rightarrow \psi_i + i g \overline{T}_i \cdot \overline{\Lambda} \psi_i + \ldots$$

Invariance requires that \( \overline{W}_\nu \) couples "universally", with the same strength \( g \), to all multiplets \( \psi_i \) (notice that this is not necessary if we only require "global isospin conservation" -- i.e. invariance when \( \overline{\Lambda} = \text{const.} \) -- nor would it be necessary if the transformation were abelian -- i.e. if it were just a phase transformation without an accompanying "rotation", as in QED where gauge invariance puts no constraint on charges).

A pessimist would reply that in fact universality of one sort or the other has been put in by hand in almost every model considered up to now. To take three examples:

a) The Weinberg-Salam model\textsuperscript{11,12} is based on the group SU(2) × U(1). The couplings of the vector meson associated with the abelian group U(1) are arbitrary -- there is nothing which prevents us from shifting the charges of all hadrons in the theory by an arbitrary amount \( \delta \), for example.
b) The Georgi-Glashow model\textsuperscript{13}) is based on the semi-simple group O(3). However, there is mixing in the leptonic triplet
\[
\begin{pmatrix}
E^+ \\
\sqrt{\cos \beta + E^0 \sin \beta} \\
e^-
\end{pmatrix}
\]
Consequently the weak coupling constant for leptonic interactions is proportional to \(\cos \beta\), which must be adjusted to ensure universality after the hadronic part of the model has been constructed.

c) Achiman\textsuperscript{21}) and Beg and Zee\textsuperscript{31}) have considered models based on Han-Nambu and/or coloured quarks which exploit the SU(3)' degree of freedom (and are therefore subject to Pati and Salam's criticism discussed above). After projecting out the SU(3)' singlet part of the current, which is the only operative part for matrix elements between singlets, they would obtain
\[
S_{\text{semileptonic}} = \frac{1}{3} S_{\text{leptonic}}
\]
if they used the Weinberg-Salam model for leptons. They therefore introduce a new lepton with a mixing angle precisely chosen to reduce \(S_{\text{leptonic}}\) by one-third -- the new leptonic doublet being:
\[
\begin{pmatrix}
\frac{\sqrt{3}}{\sqrt{8}} E^0 \\
\frac{3}{e^-}
\end{pmatrix}
\]

(v) Anomalies (of the Adler-Bell-Jackiw type)
The simplest anomalies\textsuperscript{34}) are associated with the diagrams

\[
\begin{tikzpicture}
\draw (0,0) -- (1,1) -- (2,0) -- cycle;
\draw (0,0) -- (1,1);
\draw (0,0) -- (2,0);
\draw (1,1) -- (2,0);
\node at (1,1) {$W^a$};
\node at (0,0) {$W^l$};
\node at (2,0) {$W^c$};
\node at (1,0.5) {fermion};
\end{tikzpicture}
\]
The loop integrals are divergent and it turns out that, in general, it is impossible to regulate them to make them finite while respecting all the symmetries of the theory; this is the anomaly. Since the renormalizability of the theory depends critically on the symmetries, we have to arrange that the singular (anomalous) contributions cancel when we sum the contributions from all fermions (hadrons and leptons)\(^{18,35}\). This is automatically satisfied for some "safe" groups\(^{36}\) (for which the contribution of each separate multiplet of fermions is necessarily anomaly-free); otherwise it restricts the choice of representations.

An optimist would be excited by this powerful constraint; in "non-safe" models it puts mutual constraints on the choice of hadrons and leptons -- e.g. a GIM model with four integrally charged quarks is only anomaly-free if \(\psi_{\mu}^i\) and \(\mu\) exist as well as \(\psi_{\nu}^e\) and \(\nu\); with four Gell-Mann/Zweig-like quarks, \(\psi_{\mu}^i, \psi_{\nu}^e, \psi_{\mu}^e, \) and \(\psi_{\nu}^e\) we need three colours\(^{18}\). A pessimist would point out that in many models the anomalies are removed in a totally ad hoc way (usually by introducing additional fermions in the last paragraph of the paper). An impartial observer might feel uneasy about the idea that such an obscure higher-order effect might dictate the structure of the world.

(vi) Many effects may be finite and calculable in gauge theories

Higher-order weak interactions are an obvious example but there are others. For example, if there can be T-violating electric dipole moments, they must be calculable\(^{37}\). No term with the structure of a dipole moment is allowed in a renormalizable Lagrangian. If, therefore, the calculated value were seemingly infinite there would be no "counter term" in the Lagrangian to cancel it. Hence in renormalizable theories it must be finite and well defined in terms of other quantities. Similarly, it may be impossible to write terms in the Lagrangian corresponding to certain mass difference in some theories; these mass differences must therefore be calculable\(^{38,39}\) [it is amusing to note that models can even be cooked up in which \(m_{\nu}\) is calculable and turns out to be of order \(m_{\mu}^{46}\)].

An optimist would be excited by this, while a pessimist would ask to see some successful calculation in a realistic model; these gentlemen would take the same attitude to our seventh and last remark.

(vii) Gauge theories might shed light on other problems

For example, the unconventional contributions to non-leptonic interactions due to \(Z\)'s and Higgs mesons might make it possible to understand the \(\Delta I = \frac{1}{2}\) rule\(^{41}\). It might be possible to understand the origin of the Cabibbo angle \(\theta_C\)\(^{23}\) expresses it as a function of other quantities in his model; Pais has a model\(^{42}\) in which the origin of \(\theta_C\) is necessarily linked to "maximal"
T-violation]. These models might explain the discrepancy between the size of parity violating effects in nuclei and the results of crude calculations using the conventional model\(^{13}\). These models might explain T-violation. T.D. Lee has stressed\(^{14}\) that when a field which is odd under \(T [\phi(r,t) \rightarrow -\phi(r,-t)]\) acquires a non-zero vacuum expectation value ((0|\(\phi|0) \neq 0) spontaneous T-violation occurs. This spontaneous T-violation is not only possible but may be inevitable in some gauge models; furthermore it has the desired order of magnitude in a large class of models\(^{15}\).

After these remarks, I shall end this section on model building with two questions (or speculations):

1) **Are we being too conservative?**

Perhaps really unconventional ideas are needed to make a satisfactory model, e.g. Pati and Salam\(^{16}\) want to unify hadrons and leptons, which they put in the same multiplet of a gauge theory. In such models there is a baryon-lepton-vector meson vertex which leads to baryon number non-conservation because of mixing of "baryonic" and "non-baryonic" vector mesons when these vector mesons are given mass by spontaneous symmetry breaking. They claim that this need not contradict experiment. With a proton made of three or more quarks, and no 1 or 2 quark states lighter than the proton, proton decay is sixth order in the basic coupling constant; they estimate that \(\Gamma \sim (G_B)^6\), where their \(G_B\) is less than or of the order of \(\alpha^2 G_F\) [in general, care may be necessary to avoid \(\Gamma \sim (G_B \alpha^2)^3\) in such models].

2) **Are "bare" Higgs mesons necessary?**

Could they be (\(\bar{\psi}\psi\)) bound states? Jackiw and Johnson\(^{16}\) and Cornwall and Norton\(^{17}\) have recently considered models of the Nambu-Jona-Lasinio type and have shown that it is possible to generate vector meson masses spontaneously without introducing Higgs mesons. This raises many questions, some of which we exhibit symbolically:

\[\begin{align*}
\mathcal{L}(V, \psi) & \xrightarrow{\text{Breaking}} M_V \neq 0 \quad ? \quad \mathcal{L}_{\text{eff}} \quad \text{for calc.} \quad ? \quad \text{renorm.} \\
\text{Spont. Symm.} & \quad \text{Physical Higgs mesons as bound states?} \\
\text{renorm.} & \quad \text{Non-perturbative \rightarrow no exact statements?} \\
& \quad \text{\rightarrow loose original motivation?}
\end{align*}\]
If these ideas are correct and lead to a renormalizable "effective Lagrangian" 
\( \mathcal{L}_{\text{eff}} \) of the Higgs form, it might mean that model builders should not reject 
models because they find the structure of the system of Higgs mesons unappealing 
(it might ultimately make it possible to fix the Higgs system uniquely for a 
given model of fermions). If this route does not lead to a renormalizable \( \mathcal{L}_{\text{eff}} \), 
we would presumably have lost the ability to calculate and much of the original 
motivation for studying spontaneously broken gauge theories.

4. NEUTRAL CURRENTS

Let us assume that the excess of muonless events in neutrino reactions on 
nuclei recently reported\(^{5,6}\) is due to a neutral current\(^{5,8}\) \([\text{If the target were} \]
a single nucleon at rest, it would, in principle, be possible to use the measured 
value of the final hadronic energy and momentum to calculate the four-momentum 
transfer \( q \) to the hadronic system and see if it is space-like (neutral current) 
or time-like (incoming neutron) -- with Fermi motion, measurement errors, 
unobserved neutrons, etc. this may be hard, but the reliability can be checked by 
trying to measure \( q^2 \) in charged current events from the observed hadronic energy 
and momentum alone\(^{4,9}\)\].

The theoretical interpretation of neutral current measurements requires a 
model; we assume here the Weinberg-Salam model for leptons. The hadronic neutral 
current then has the form

\[
J^Z_{\text{hadrons}} = \left( J^3 - 2 \sin^2 \theta_W J^{\text{em.}} \right)
\]

where

- \( \theta_W \) is the "Weinberg angle" which is constrained (at the 90% confidence level) 
  to satisfy \( 0.1 < \sin^2 \theta_W < 0.6 \) by the CERN results on leptonic interactions\(^3\) 
  and \( \sin^2 \theta_W < 0.3 \) by the experiment of Gurr, Reines and Sobel on \( \bar{\nu}_e + \nu_e \rightarrow \mu^- \).
- \( J^{\text{em.}} \) is the hadronic part of the electromagnetic current.
- \( J^3 \) is the third component of the "weak hadronic chiral isospin" current 
  which, in general, need not have any connection with the ordinary strong 
  isospin current.
Remarks:

1) The chiral current $J^3$ is built from pieces with the V-A form $[\bar{\psi}_\mu (1 - \gamma_5) \psi]$. Hence $J^3$ does not have the V-A form for $\Theta_W \neq 0$. According to parton/light-cone ideas the matrix elements of the V and A pieces of $J^3$ will therefore be different (for $\Theta_W \neq 0$) and the V-A interference term cannot reach its kinematically allowed maximum (which requires $\langle V \rangle = \langle A \rangle$). This implies

$$\frac{\sigma (\bar{\nu} + A \rightarrow \bar{\nu} + \cdots)}{\sigma (\nu + A \rightarrow \nu + \cdots)} > \frac{1}{3}.$$ 

However, experimentally

$$\frac{\sigma (\bar{\nu} + A \rightarrow \mu^+ + \cdots)}{\sigma (\nu + A \rightarrow \mu^- + \cdots)} \approx \frac{1}{3}.$$

Hence we expect

$$\frac{\sigma (\bar{\nu} + A \rightarrow \bar{\nu} + \cdots)}{\sigma (\bar{\nu} + A \rightarrow \mu^+ + \cdots)} > \frac{\sigma (\nu + A \rightarrow \nu + \cdots)}{\sigma (\nu + A \rightarrow \mu^- + \cdots)}$$

as is observed.

2) More quantitative results require more specific assumptions about the model for hadrons. What is usually done is to abstract the connection of "weak" and strong isospin which holds in the four quark GIM model, hoping that it might be true more generally, and use it to obtain bounds. The lower bounds for inclusive processes were compared to the data by Nyatt in his talk. The experimental results are only just compatible with the bounds (which require $\Theta_W \approx 35^\circ$ and hence $M_W \approx 65$ GeV, $M_Z \approx 80$ GeV in this model); is this embarrassing? I think the answer is no, since the lower bounds would be saturated if both:

a) the isospin zero contribution to the neutral current cross-section were negligible compared to the I = 1 contribution; this would not be surprising since:

$$\sigma_{I=0} < \frac{1}{10} \sigma_{I=1}$$

both in photoproduction and also in deep inelastic electroproduction according to a comparison with neutrino scattering (at least in conventional models), and
\[
\frac{\sigma(\bar{\nu} + A \rightarrow \mu^+ \ldots)}{\sigma(\nu + A \rightarrow \mu^- \ldots)} = \frac{1}{3}
\]

which is roughly true experimentally.

This is exemplified in Sehgal's model calculation\(^{52}\), which has already been discussed by Myatt\(^6\). Sehgal considered the four (ncn-integrally charged) quark model discussed in (i) c) of Section 3 above, taking the nucleon to consist of (ppn) plus a small quark-antiquark sea (colour, which is necessary to remove anomalies, has no effect on these calculations). This model accounts for the charged current data very easily; the neutral current results are displayed in a figure in Myatt's talk. Using the CERN result\(^4\)

\[
\frac{\sigma(\nu A \rightarrow \nu \ldots)}{\sigma(\bar{\nu} A \rightarrow \bar{\nu}^- \ldots)} \bigg|_{E_{\text{Hadronic}} > 1 \text{ GeV}} = 0.23 \pm 0.03
\]

the model gives

\[
\sin^2 \theta_W = 0.38 \pm 0.07
\]

and then predicts

\[
\frac{\sigma(\bar{\nu} + A \rightarrow \bar{\nu} \ldots)}{\sigma(\bar{\nu} + A \rightarrow \mu^+ \ldots)} \bigg|_{E_{\text{Hadronic}} > 1 \text{ GeV}} = 0.65 \pm 0.12
\]

not in contradiction with the CERN result which is 0.46 \pm 0.09 (note that the errors on the CERN result are purely statistical).

3) Detailed models, such as Sehgal's, predict the form of the double differential neutral current cross-section in the scaling region. With a stationary target (no Fermi motion) this can, in principle, be obtained by measuring the momentum and energy of the final hadrons, which give q and hence \(\nu\) and \(q^2\). This may be too ambitious, but \(\nu = E_{\text{final hadrons}} - M_p \pm 0\) (Fermi energy is comparatively simple to measure\(^{53}\)) and hence \(d\sigma/d\nu\) averaged over the neutrino spectrum could be compared to calculations such as Sehgal's\(^{52}\) (Sehgal also gives flux independent predictions for \(\langle \nu \rangle\) and \(\langle Q^2 \rangle\)). This would be very interesting.
4) Theorists can take their favourite model and very easily re-do all the old charged current phenomenology (sum rules, structure function relations, Adler PLAC relation, isospin tests, etc.), thus providing tests of the models.

5) Theorists have already examined the (parity and charge-conjugation violating) effects of $\gamma-Z$ interference in $e^+e^- \rightarrow ?^{54}$, $\mu^+ \rightarrow \mu + \ldots^{55}$ and in atomic physics$^{56}$ in their favourite models [the effects are of order $10^{-6} \times Q^2$ in (GeV)$^2$]. They can now re-examine the results with the parameters fixed by the neutral current neutrino data.

The impact of neutral currents on gauge theories in general might be summarized in the following dialogue:

**Optimist:** "The beautiful neutral current experiments show that we are on the right track".

**Pessimist:** "Not at all; neutral currents are logically quite independent of gauge theories. In fact, in their classic 1960 Phys. Rev. Letter, Lee and Yang give the search for possible $\Delta S = 0$ neutral currents as the third of their nine reasons for doing neutrino experiments".

**Optimist:** "You must be impressed by the fact that the most simple-minded four quark model fits the data so easily".

**Pessimist:** "Hypocrite! you keep telling me that in your opinion there are no plausible models".

5. **FINAL CONCLUSIONS**

The **optimist** would conclude that a good model could explain almost everything (unify weak and e.m. interactions; dictate the form of the strong interactions; explain weak universality and the quantization of charge; allow the calculation of higher-order effects, mass differences, and possibly other quantities; explain the $\Delta I = \frac{1}{2}$ rule, T-violation, the Cabibbo angle, ...). If asked why there is no good model at present, he might reply that it is because we are too stupid or that there is not enough evidence, which would be the case if there are many still undiscovered leptons (discovering the "ultimate" model now may be as hard as inventing SU(3) in 1946 would have been).

A **pessimist** would reply that spontaneously broken gauge theories have not yet shed new light on any unexplained fact. However, he might have to admit that as a new class of renormalizable field theories they are of mathematical interest and also that they have served as an "imagination stretcher" (to borrow Bjorken's phrase) by focusing attention on the many untested hypotheses/prejudices embodied in the conventional theory of weak interactions; they have also been a stimulus to experiment. A **cynic** would ask why an additional "imagination stretcher" or
or stimulus was needed; it was noted long ago that the number of muonless events in neutrino experiments is surprisingly large. In his thesis, E.C.M. Young estimated\textsuperscript{57}) that the number of muonless events was three or four times as large as expected from background in the neutrino experiments on freon in the CERN 1.2 m bubble chamber; the background in experiments on propane was discussed by Myatt\textsuperscript{58}). These experiments could not have established neutral currents, due to limited statistics and the small size of the chamber, but it is amusing to note that if the excess of muonless events had been attributed to neutral currents it would have given\textsuperscript{59})

\[
\frac{\sigma(\nu + A \rightarrow \nu + \ldots)}{\sigma(\nu + A \rightarrow \mu + \ldots)} = 0.17 \pm 0.06
\]

in good agreement with the recently published result.

The optimist can go home, ponder the neutral current data and produce the "ultimate model". Meanwhile even a cynical pessimist might concede that, whatever the final outcome, the net impact of gauge theories has been beneficial.
REFERENCES AND FOOTNOTES

1) Formal aspects of gauge theories have been reviewed by M. Veltman in his Invited Paper presented to this Conference (CERN-TH. 1742, 1973).

2) A very personal view is presented here. I have made no attempt to give a systematic survey of all the models which have been proposed. Some further references may be found in:
   B.W. Lee in Proc. XVI Int. Conf. on High-Energy Physics, NAL 1972.
   E.S. Abers and B.W. Lee, to be published in Physics Reports.
   C.H. Llewellyn Smith, CERN-TH 1710, 1973 (to be published in Proc. of the 1973 Scottish Universities Summer School and in Proc. of the Fifth Hawaii Topical Conference on Elementary Particle Physics, 1973). These lecture notes contain a review of many experimental aspects of gauge theories which will not be covered in this talk because of lack of time.

4) F.J. Hasert et al., CERN preprint, to be published in Physics Letters.
5) A. Benvenuti et al. (Harvard-Pennsylvania-Wisconsin Collaboration), paper contributed to this Conference.
6) The data have been reviewed by G. Myatt, Invited Paper presented to this Conference.
7) Most of the elements of the argument are known to many people; they are spelled out in great detail in my lecture notes quoted in footnote 2 above.
14) J.D. Bjorken, Phys. Rev. 148, 1467 (1966), and subsequent work stemming from this paper.
19) With strong interactions mediated by a "colour octet" of vector mesons, this model satisfies the aesthetic criterion discussed in Remark 2. A model with this structure was, I believe, first discussed by J.C. Pati and A. Salam (Maryland Technical Report 73-085, 1973; Phys. Rev., in press). This paper deals explicitly only with models with integrally-charged constituents. The first discussion of this model with non-integrally-charged constituents was given (independently) by C. Itoh, T. Mamiikawa, K. Miura and T. Watanabe, Tokyo preprint, 1973.


    R.N. Mohapatra, Maryland preprint 73-022 (1972).

22) I. Bars, M.B. Halpern and M. Yoshimura, Phys. Rev. Letters 29, 969 (1972);


24) A model in which this idea is exploited has been considered by P.G.O. Freund,

25) This result was derived by S. Weinberg [Phys. Rev. D8, 605 (1973)] for
    abelian groups. The generalization to non-abelian groups was carried out
    first by R.N. Mohapatra, J.C. Pati and P. Vinciarelli ("Canonical Estimates
    of Weak Radiative Corrections in Unified Gauge Theories and Selection
    Letters 31, 494 (1973)], and D.V. Nanopoulos, Sussex University preprint,

26) I. Bars, "Parity Violation and Comparisons between Quark-Schemes and

27) B. de Wit (Ref. 23) noted the parity problem in his paper.

28) Asymptotic freedom has been discussed by M. Veltman in his talk (Ref. 1)
    where references to the original literature may be found.

    The vector mesons and fermions "reggeize" but not the Higgs mesons.

30) J.C. Pati and A. Salam, Trieste Internal Report IC/73/81.

31) M.A.B. Beg and A. Zee, Ref. 21 above.

32) The degeneracy can be lifted by introducing quark mass differences, but
    Pati and Salam conclude that a sufficiently big effect cannot be obtained
    without introducing large (few GeV) mass splitting inside SU(3)' [and/or
    SU(3)] multiplets; there is an SU(9) symmetry which cannot be very badly
    broken down to SU(3) X SU(3)' by mass terms without equally bad breaking
    of SU(3) and/or SU(3)'.

33) This problem has been reviewed by J.R. Primack in Proc. XVI Int. Conf. on
    High-Energy Physics, NAL 1972.
34) See S.L. Adler in Lectures in Particles and Quantum Field Theory, Vol. 1 (M.I.T. press, 1970) for a review of basic ideas about anomalies. The importance of removing anomalies in gauge theories was pointed out by M. Veltman (private communication). The way to remove them was pointed out in Refs. 18 and 35.


37) A. Pais and J.R. Primack, Rockefeller preprint C00-2232B-21, 1973, have discussed the calculation of dipole moments in a variety of models.


41) The role of the Higgs mesons in this context has been discussed by B.W. Lee and S.B. Treiman, Phys. Rev. D7, 1211 (1973). Examples of models in which the $\Delta I = \frac{1}{2}$ rule is satisfied may be found in A. Pais, Phys. Rev. D8 (in press); M.A.B. Beg, Phys. Rev. D8, 664 (1973).

42) A. Pais, Phys. Rev. Letters 29, 1912 (1973), and in Ref. 41 above.


48) If the excess is real it might have some other origin. A heavy lepton is one possibility which has been suggested in the literature; this seems unlikely for two reasons:

a) the effects observed at CERN (Ref. 4) and NAL (Ref. 5) have the same magnitude despite the enormous energy difference; if a "heavy" lepton were responsible then, as stressed by E. de Rafael (private communication), it would have to have a modest production threshold on the scale of CERN neutrino energies -- i.e. to be rather light (a few hundred MeV);

b) if such a lepton coupled universally to electrons and muons, it would have to decay overwhelmingly into hadrons (plus neutrinos), which would be very surprising for any light (less than or of order a few GeV) lepton according to conventional ideas about weak decays. Decays to channels containing $e^+$ and $\mu^+$ would have to be very rare since no $e^+$'s were seen in the CERN $\nu$ film (H. Deden et al., CERN preprint submitted to Physics Letters, August 1973). For $e^-$ and $\mu^-$ decays, universality would give

\[
\frac{\sigma(\nu_e + A \rightarrow e^+ \cdots)}{\sigma(\nu_\mu + A \rightarrow \mu^- \cdots)} \propto \frac{\phi(\nu_e) [\sigma(\nu_e A \rightarrow e^+ \cdots) \phi(\nu_e) + \sigma(\nu_e A \rightarrow L^+ \cdots) \chi]}{\phi(\nu_\mu) [\sigma(\nu_\mu A \rightarrow \mu^- \cdots) \phi(\nu_\mu) + \sigma(\nu_\mu A \rightarrow L^- \cdots) \chi]}
\]
where

$$X = \Phi(\nu_e) \frac{\Gamma(L_{\text{lepton}}, L \rightarrow e + \cdots)}{\Gamma(L \rightarrow H)}$$

and we have set $m_\mu = m_e = 0$ and used the fact that the fluxes satisfy $\Phi(\nu_\mu) \approx 100 \Phi(\nu_e)$. With this interpretation of the muonless events

$$\frac{\Gamma(L \rightarrow \text{hadrons})}{\Gamma(L \rightarrow H)} \frac{\sigma(\nu_e A \rightarrow L^+ \cdots)}{\sigma(\nu_e A \rightarrow \nu^- \cdots)} \approx 0.23 \pm 0.03.$$ 

Using

$$\left| \frac{\sigma(\nu_e A \rightarrow e^+ \cdots)}{\sigma(\nu_e A \rightarrow \nu^- \cdots)} \right|_{\text{apparent}} = 1.26 \pm 0.23,$$

(H. Deden et al., loc. cit.) we would conclude that

$$\frac{\Gamma(L \rightarrow e + \cdots)}{\Gamma(L \rightarrow H)} \lesssim 0.02.$$ 

If such a lepton is responsible for the effect, the measured hadronic energy and momentum would give $q^2 = m_\nu^2 = 0$.

49) Events of the type

$$n + A \rightarrow n \ (\text{unobserved}) + \cdots,$$

would also give a space-like $q^2$. Presumably, however, the $q^2$ distribution of these events would fall much more rapidly with increasing $q^2$ than the distribution for charged current or genuine neutral current events.

50) H.S. Gurr, F. Reines and H.W. Sobel, Phys. Rev. Letters 28, 1046 (1972). An improved result was reported by Reines at the Trieste Conference on Weak Interactions, 1973; see the discussion in Myatt's talk, Ref. 6.

51) Bounds for various processes may be found in the following papers:

In all these papers it is assumed that the hadronic neutral current has the isospin structure of the four-quark GIM model or of the "simple Weinberg model", in which there are no strange particles (we stress again that without some assumptions about the hadronic model the Weinberg-Salam scheme implies nothing about semileptonic neutral current processes). Note that although Paschos and Wolfenstein imply that their Eq. (16)
(which gives \( \sin^2 \theta_W \) directly in terms of neutrino cross-sections) holds in both types of model, it actually requires that the isoscalar part of the neutral current is either pure vector or pure axial vector (which is not generally true with the GIM scheme), or else that its contribution to \( \sigma^\nu \mu^+ \cdots - \sigma^\nu \mu^+ \cdots \) and to \( \sigma^{\nu \mu^+} \cdots - \sigma^{\nu \mu^+} \cdots \) vanishes for some other reason (this happens in the four-quark model if we set the Cabibbo angle to zero and use parton/light-cone ideas; with no cuts in the variable \( x = Q^2/2\lambda \), the isoscalar contributions to these cross-section differences are zero because they are proportional to combinations of the difference of the numbers of \( \lambda \) and \( \bar{\lambda} \) quarks and of \( q' \) and \( \bar{q}' \) quarks in the nucleon.


53) This has been stressed by A. de Rujula (private communication).

R. Rudny and A. McDonald, Oxford preprint 37/73.
See also J.D. Bjorken, Invited Paper presented to this Conference, for a discussion and references.


56) C. Bouchiat (unpublished).

