B⁰ − B̄⁰ oscillations and measurements of $|V_{ub}|/|V_{cb}|$

at LEP

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Abstract

In this paper a review of the LEP analyses on $B^0 - B\bar{0}$ oscillations and on the measurement of $|V_{ub}|/|V_{cb}|$ is presented. These measurements are of fundamental importance in constraining the $\rho$ and $\eta$ parameters of the CKM matrix. A review of the current status of the $V_{CKM}$ matrix determination is also given.

Introduction

The data registration at the $Z^0$ pole has stopped at the end of 1995. The four LEP experiments (ALEPH, DELPHI, L3 and OPAL) have collected about 4M hadronic $Z^0$ decays per experiment.

In the past three years, the quality of the data analysis has continuously improved, thanks to a better understanding of the behaviour of all components of the detector. At the same time, new ideas, and then, new analyses have been tried. A more performant statistical treatment of the information has been also developed. As a result, the precision on the $\Delta m_d$ parameter has been improved and above all, the sensitivity for the $\Delta m_s$ parameter has been tremendously increased. The new and precise LEP analyses on $|V_{ub}|/|V_{cb}|$ are also a consequence of these improvements. Many analyses described in this paper have been presented at the last ’98 Summer Conferences and are still preliminary. This paper is organized as follows. The first sections are dedicated to the oscillations and $|V_{ub}|/|V_{cb}|$ analyses. In the last section the present status of the $V_{CKM}$ matrix is given with a special emphasis placed on the impact of the measurements presented in this paper.
The oscillation analyses

The probability that a $B^0$ meson oscillates into a $\bar{B}^0$ or stays as a $B^0$ is given by:

$$P_{B^0 \rightarrow \bar{B}^0} = \frac{1}{2} e^{-t/\tau_q} (1 \pm \cos \Delta m_q t)$$  \hspace{1cm} (1)

where the effect of CP violation has been neglected. $\tau_q$ is the lifetime of the $B^0_q$ meson, $\Delta m_q = m_{B^0_1} - m_{B^0_2}$ is the mass difference between the two mass eigenstates\(^1\) and gives the period of the time oscillations (the effect of a lifetime difference between the two states has been also neglected).

The Standard Model predicts:

$$\Delta m_d \propto A^2 \lambda^6 [(1 - \rho)^2 + \eta^2] f_{B_d}^2 B_d ; \Delta m_s \propto A^2 \lambda^4 f_{B_s}^2 B_s$$  \hspace{1cm} (2)

The difference in the $\lambda$ dependence of these expressions ($\lambda \sim 0.22$) implies that $\Delta m_s \sim 20 \Delta m_d$. It is then clear that a very good proper time resolution is needed to measure the $\Delta m_s$ parameter. A time dependent study of $B^0 - \bar{B}^0$ oscillations requires:

- the measurement of the decay proper time,
- to know if a $B^0$ or a $\bar{B}^0$ decays at $t = t_o$ (decay tag)
- to know if a $b$ or a $\bar{b}$ quark has been produced at $t = 0$ (production tag).

![Figure 1: The plot shows the time dependence behaviour of the $B_d^0 - \bar{B}_d^0$ oscillation. The points with error bars are the data. The curve shows the result of the fit using $\Delta m_d = 0.47$ ps\(^{-1}\).](image)

The precision on the $\Delta m$ measurement is given by the following relation:

$$\text{error} = \left( \sqrt{N f_{B_d^{(0)}} (2\bar{\varepsilon}_1 - 1)(2\bar{\varepsilon}_2 - 1)e^{-\left(\frac{\Delta m_d(t_o)\tau_q}{2}\right)^2}} \right)^{-1}$$  \hspace{1cm} (3)

---

\(^1\) $\Delta m_q$ is usually given in ps\(^{-1}\). 1 ps\(^{-1}\) corresponds to $6.58 \times 10^{-4}$ eV.
where $N$ is the total number of events in the sample; $f_{B_0^{d(s)}}$ is the fraction of events in which a $B_0^{d(s)}$ meson has been produced; $\varepsilon_2, \varepsilon_1$ are the tagging purities at the decay and production times respectively, defined as $\varepsilon = \frac{N_{\text{right}}}{N_{\text{right}} + N_{\text{wrong}}}$, where $N_{\text{right}}$ ($N_{\text{wrong}}$) are the numbers of correctly (incorrectly) tagged events and $\sigma_t$ is the proper time resolution given, approximately, as $\sigma_t = \sqrt{\left(\frac{m^2}{p^2}\right)\sigma_L^2 + \left(\frac{\sigma_p}{p}\right)^2 t^2}$, where $\sigma_L$ and $\sigma_p$ are the decay length and the momentum resolutions respectively.

**$\Delta m_d$ measurements**

A lot of analyses have been performed since 1994. A typical time distribution is shown in Figure 1. The time dependence behaviour with frequency $\Delta m_d \sim 0.470$ ps$^{-1}$, for the $B_d^0 - \bar{B}_d^0$ oscillation is clearly visible. This will be a textbook plot! The present summary of the results on $\Delta m_d$, as given by [1], is shown in Figure 2. Combining LEP/CDF and SLD measurements it follows that:

$$\Delta m_d = (0.477 \pm 0.017)\text{ps}^{-1}$$

(4)

$\Delta m_d$ is known with a precision of 3.4% relative error.

**Analyses on $\Delta m_s$**

Four types of analyses have been performed.

Table 1: The characteristics of the different analyses are given in terms of statistics ($N$), $B_s^0$ purity ($f_{B_s}$), tagging purity at the production and decay time ($\varepsilon_1, \varepsilon_2$) and time resolution in the first pico-second

<table>
<thead>
<tr>
<th>Analysis</th>
<th>N(events)</th>
<th>$f_{B_s}$</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\sigma_t(t &lt; 1\text{ps})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive lepton</td>
<td>~ 50000</td>
<td>~ 10%</td>
<td>~ 70%</td>
<td>~ 90%</td>
<td>~ 0.25 ps</td>
</tr>
<tr>
<td>$D_s^*h^\mp$</td>
<td>~ 3000</td>
<td>~ 15%</td>
<td>~ 72%</td>
<td>~ 90%</td>
<td>~ 0.22 ps</td>
</tr>
<tr>
<td>$D_s^*\ell^\mp$</td>
<td>~ 400</td>
<td>~ 60%</td>
<td>~ 78%</td>
<td>~ 90%</td>
<td>~ 0.18 ps</td>
</tr>
<tr>
<td>Exclusive $B_s^0$</td>
<td>~ 25</td>
<td>~ 70%</td>
<td>~ 78%</td>
<td>~ 100%</td>
<td>~ 0.08 ps</td>
</tr>
</tbody>
</table>

For all of them, the latest analyses make use of the combined tag method for tagging a $b$ or a $\bar{b}$ at production time. At LEP, the produced $b$ and $\bar{b}$ quarks fragment independently and the events can be divided in two separate hemispheres. If the measurement of the proper time is performed in one of those (same hemisphere), the other (opposite hemisphere) can be used to determine if a $b$ or a $\bar{b}$ quark was produced in that hemisphere. Several variables are considered in the opposite hemisphere:

- $Q = \frac{\sum_{i=1}^{n} q_i (\tilde{p}_i \cdot \tilde{e}_S)^0.6}{\sum_{i=1}^{n} (\tilde{p}_i \cdot \tilde{e}_S)^0.6}$, the hemisphere charge, defined as the charge of all ($n$) charged tracks ($q_i$) present in the hemisphere, weighted by their momentum ($p_i$) projected along the thrust axis ($\tilde{e}_S$) with a chosen value for the exponent (0.6),
• the hemisphere charge, considering only identified kaons,
• the charge of primary and secondary vertices,
• the presence of high $p_t$ leptons.

The use of these variables allow to have a tagging purity of the order of 70%.

Figure 2: Summary of the $\Delta m_d$ results from the LEP, SLD and CDF Collaborations are given. Details on how the different results have been combined are given in [1].

Tracks in the same hemisphere can be used also. This procedure is peculiarly clean if all the tracks from the $B^0_s$ have been reconstructed (as for $D^+_s \ell^- \bar{\nu}_\ell$ and exclusive $B_s$ analyses). In this case, tracks from the $B^0_s$ decay can be removed and the others, coming from the primary vertex can be used. The addition of informations from the same hemisphere allows to reach a tagging purity of 74%. Finally the use of all these informations on an event by event basis gives a purity of 78%.
The tagging of a B or a \( \overline{B} \) meson at decay time depends on the specific analysis and will be given in the following. Before describing the different analyses, the method used to measure or put a limit on \( \Delta m_s \) is briefly discussed.

**The amplitude method**

The method used to measure or to put a limit on \( \Delta m_s \) consists in modifying equation 1 in the following way: \( 1 \pm \cos \Delta m_s t \rightarrow 1 \pm A \cos \Delta m_s t \). \( A \) and \( \sigma_A \) are measured at fixed values of \( \Delta m_s \) instead of \( \Delta m_s \) itself. In case of a clear oscillation signal, at given \( \Delta m_s \), the value of the amplitude is compatible with \( A = 1 \) for this \( \Delta m_s \) and with \( A = 0 \) elsewhere. With this method it is also easy to set a limit. The values of \( \Delta m_s \) excluded at 95% C.L. are those satisfying the condition \( A(\Delta m_s) + 1.645 \sigma_A(\Delta m_s) < 1 \).

With this method, it is easy to combine different experiments and to treat systematic uncertainties in an usual way since, at each value of \( \Delta m_s \), a value for \( A \) with a gaussian error \( \sigma_A \), is measured. Furthermore, the sensitivity of the experiment can be defined as the value of \( \Delta m_s \) corresponding to \( 1.645 \sigma_A(\Delta m_s) = 1 \) (for \( A(\Delta m_s) = 0 \), namely supposing that the “true” value of \( \Delta m_s \) is well above the measurable value of \( \Delta m_s \)). The sensitivity is the limit which would be reached in 50% of the experiments.

**The inclusive lepton/combined tag analysis**

This analysis uses high \( p_t \) leptons which are mainly coming from direct b semileptonic decays (\( b \rightarrow \ell \)). The sign of the lepton tags the \( B^0_s \) at decay time. The initial sample consists in 80% leptons from B decays (and among those 90% \( b \rightarrow \ell \) (direct) and 10% \( b \rightarrow c \rightarrow \ell \) (cascade)) and of 20% leptons from charm decays or misidentification. The events \( b \rightarrow c \rightarrow \ell \) give the wrong tag for the \( B^0_s \) meson at decay time.

To reconstruct a B decay proper time, algorithms have been developed which aim at identifying charged (neutral) tracks which are more likely to come from the \( B^0_s \) decays. As result, in more than 50% of the cases, the error on the decay length is \( \sigma_{tL} \sim 250 \mu m \) and the relative error on the B energy is better than 10%, resulting in an error on the proper time of the order of 0.25 ps in the first pico-second.

A second crucial point for this analysis consists in trying to increase the \( B^0_s \) purity of the sample (the natural \( B^0_s \) purity of b events is around 10%) and to reduce the contribution from cascade decays. To enrich the sample in direct b semileptonic decays and, among those, in events coming from \( B^0_s \) decays, several variables have been used as the momentum and transverse momentum of the lepton, the impact parameters of all tracks in the opposite hemisphere relative to the main event vertex, the kaons in primary and secondary vertices in the same hemisphere, and the charge of the secondary vertex.

The result of this procedure is to increase the \( B^0_s \) purity by 30% and to reach more than 90% purity for the tagging at the decay time.

**D_±^sℓ∓/combined tag analysis**

The use of events in which a reconstructed \( D_s \) is accompanied by a high \( p_t \) lepton with an electric charge opposite in sign allows to select a sample having 60% \( B_s \) purity. The
proper time resolution benefits also from the fact that the only missing particle is the
neutrino: $B^0_s \to D^+_s e^- \bar{\nu}_e$. In the first pico-second the time resolution is about 0.18 ps in
more than 80% of the events.

The limiting factor is the available statistics because accessible $D_s$ branching fractions
are quite small (between $\sim 1\%$ and $\sim 5\%$). Several decay modes have to be selected.
Figure 3 shows an example in which six hadronic and two semileptonic decay modes have
been reconstructed.

![Figure 3: DELPHI $D^\pm \ell^\mp$ candidates. The figure on the left shows the $D_s$ mass
spectrum reconstructed from the following decay modes: $D^+_s \to \phi \pi^+, \phi \pi^+\pi^0, \phi \pi^+\pi^-\pi^+$,
$\bar{K}^0 K^+, \bar{K}^0 K^+$ and $K^0_S K^+$. The figure on the right shows the $\phi$ mass spectrum from the
decays $D^+_s \to \phi e^+\bar{\nu}_e$ and $\phi \mu^+\bar{\nu}_\mu$. The sum of the two samples gives $230 \pm 18 B^0_s$ candidates.](image)

**Exclusive $B^0_s$/combined tag analysis**

At the 1998 Moriond Conference, the DELPHI Collaboration has proposed the use of
exclusively reconstructed $B^0_s$ decays for $\Delta m_s$ analyses. These events have an excellent
proper time resolution $\sigma_t \sim 0.08$ ps and provide a gain in sensitivity at high values
of $\Delta m_s$ (equation 3). Figure 4 shows the $B^0_s$ mass spectrum using the decay modes: $B^0_s \to D_s \pi$ (or $a_1$) and $B^0_s \to D^0 K \pi$ (or $a_1$). The $D_s$ has been reconstructed in six hadronic
decay modes, as in the $D^\pm \ell^\mp$ analysis, and the $D^0$ is observed using $K \pi$ and $K \pi \pi \pi$ decay
modes. $17 \pm 8$ events have been reconstructed in the $B^0_s$ mass region. The combinatorial
background is estimated to be 35%.

**Summary of $\Delta m_s$ analyses**

The combined result of LEP/SLD/CDF [1] analyses is shown in Figure 5 and is:

$$\Delta m_s > 12.4 \text{ ps}^{-1} \text{ at } 95\% \text{ C.L.}$$

The sensitivity is at $13.8 \text{ ps}^{-1}$. LEP alone has a limit at $11.5 \text{ ps}^{-1}$ at $95\% \text{ C.L.}$, with a
sensitivity at $12.9 \text{ ps}^{-1}$. $\Delta m_s = 0$ is excluded between $14.5 \text{ ps}^{-1}$ and $16.5 \text{ ps}^{-1}$ with a $2\sigma$
significance at $15 \text{ ps}^{-1}$. The present summary of the results is given in Figure 5.
Figure 4: The $B_0$ mass spectrum obtained by the DELPHI Collaboration. The points with the error bars are the data with the fit superimposed. The contributions from non-$B_0$ decays, as given by the Monte Carlo simulation, are also shown.

Figure 5: The plot on the left shows the combined $\Delta m_s$ results from LEP/SLD/CDF analyses shown in an amplitude versus $\Delta m_s$ plot. The point with error bars are the data; the lines show the 95% C.L. curves (in dark the systematics have been included). The dotted curve shows the sensitivity. The plot on the right shows the summary of the $\Delta m_s$ results per experiment. The error are given at $\Delta m_s = 10$ $\text{ps}^{-1}$ (the sensitivity is also given). The way in which the combined value is obtained is described in [1].
The presence of leptons above the kinematical limit for those produced in the decay $B \rightarrow D \ell \nu_\ell$ ($b \rightarrow c$ transition proportional to the $|V_{cb}|$ CKM matrix element) is attributed to the transition $b \rightarrow u \ell \nu_\ell$ (proportional to the $|V_{ub}|$ CKM matrix element).

The CLEO and ARGUS Collaborations have been pioneers in this measurement. Nevertheless, as only a small fraction of the energy spectrum of these leptons is measurable, the systematic uncertainties in the modelling of the $b \rightarrow u$ transition to evaluate the ratio $|V_{ub}|/|V_{cb}|$ are quite large (of the order of 20%-25% relative error). Recently LEP experiments have shown their capabilities of measuring $|V_{ub}|$ with a statistical precision similar to the one from CLEO and with reduced systematic uncertainties. They use several kinematical variables, in events with an identified high transverse momentum lepton, which have a distinctive power to discriminate between $b \rightarrow c$ and $b \rightarrow u$ transitions. The first measurement has been performed by the ALEPH Collaboration by means of a neural network discriminating method.

The DELPHI measurement is simpler. With respect to the ALEPH analysis the information from the presence of a secondary vertex from the D decay is used. In $b \rightarrow u$ transitions, all tracks are coming from the B decay vertex. The presence of kaons at the D meson vertex is also used. The method is based on the fact that the hadronic system recoiling against the lepton in $b \rightarrow u \ell \nu$ decays is expected to have an invariant mass lower than the charm mass [2]. The sample is finally divided into a $b \rightarrow u$ enriched and a $b \rightarrow u$ depleted components and the energy of the lepton in the B rest frame is calculated. The result is shown in Figure 6 together with the summary of the results on $|V_{ub}|$.

![Figure 6](image_url)

Figure 6: The plots on the left show the energy of the lepton in the B rest frame after the background subtraction for the $b \rightarrow u$ enriched and $b \rightarrow u$ depleted samples. On the right the summary of $|V_{ub}|$ results is given.
### Status of the $V_{VCM}$ matrix

Table 2: The four constraints which allow, at present, to define the accessible region for the $\rho$ and $\eta$ parameters are listed in the first column. In the second column the dependence of these constraints relative to the different parameters is given. The last column gives the explicit dependence in terms of $\rho$ and $\eta$.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$V_{\text{CKM}} \times$ other</th>
<th>Constraint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b \to u$/ $b \to c$</td>
<td>$</td>
<td>(V_{ub}</td>
</tr>
<tr>
<td>$\Delta m_d$</td>
<td>$</td>
<td>V_{ud}</td>
</tr>
<tr>
<td>$\Delta m_d/\Delta m_s$</td>
<td>$</td>
<td>V_{td}</td>
</tr>
<tr>
<td>$\varepsilon_K$</td>
<td>$f(A, \eta, \rho, B_{K})$</td>
<td>$\sim \eta(1 - \rho)$</td>
</tr>
</tbody>
</table>

The $V_{\text{CKM}}$ matrix can be parametrized in terms of four parameters: $\lambda$, $A$, $\rho$ and $\eta$ (the Wolfenstein parametrization [3]). The Standard Model predicts relations between the different processes which depend on these parameters. The unitarity of the $V_{\text{CKM}}$ matrix can be visualized as a triangle in the $\rho-\eta$ plane. Several quantities which depend on $\rho$ and $\eta$ have to be measured and, if Standard Model is correct, they must define compatible values for the two parameters inside measurement errors and theoretical uncertainties. The measurement of $b \to u$/$b \to c$ transitions gives a constraint of the form $\rho^2 + \eta^2$. The measurement of $\Delta m_d$ gives a constraint of the form $(1 - \rho)^2 + \eta^2$. A measurement of the ratio $\Delta m_d/\Delta m_s$ gives the same type of constraint in the $\rho-\eta$ plane, as a measurement of $\Delta m_d$, but this ratio is expected to have smaller theoretical uncertainties since the ratio $f_{B_d}^2 B_{B_d}/f_{B_s}^2 B_{B_s}$ is better known than the absolute value $f_{B_d}^2 B_{B_d}$.

All details of the analysis presented here can be found in [4]. Using the available and most recent measurements and up to date theoretical calculations [4] the allowed region in the $\rho-\eta$ plane can be determined. It is shown in Figure 7 and corresponds to:

$$\rho = 0.189 \pm 0.074 \; ; \; \eta = 0.354 \pm 0.045$$

It is of interest to determine the central values and the uncertainties on the quantities $\sin 2\alpha$, $\sin 2\beta$ and $\gamma$ which will be directly measured at future B-factories or LHC experiments. The result is shown in Figure 8 and is:

$$\sin 2\beta = 0.73 \pm 0.08 \; ; \; \sin 2\alpha = -0.15 \pm 0.30 \; ; \; \gamma = (62 \pm 10)^0$$ (5)

The value of $\sin 2\beta$ is rather precisely determined with an accuracy already at the level expected after the first years of running at B factories. Finally it is possible to remove from the calculation the information of one of the constraint and to obtain its probability density function. The result for $\Delta m_s$ and $|V_{ub}|/|V_{cb}|$ is shown in Figure 9 and summarized in Table 3.
Figure 7: The $\bar{\rho} - \bar{\eta}$ allowed region. The contours at 68% and 95% C.L. are shown. The continuous lines correspond to the constraints obtained from the measurements of $|V_{ub}|$, $|V_{cd}|$, and $\varepsilon_K$. The dotted curve corresponds to the 95% C.L. limit obtained from the experimental limit on $\Delta m_s$.

Figure 8: The $\sin 2\beta$ and $\sin 2\alpha$ probability density distributions. The dark-shaded and the clear shaded intervals correspond to 68% and 95% C.L. regions respectively.
Figure 9: The left and the right plots show the probability density distributions for $\Delta m_s$ and $|V_{ub}|/|V_{cb}|$ respectively. The dark-shaded and the clear shaded intervals correspond to 68% and 95% C.L. regions respectively.

Table 3: The $\Delta m_s$ and $|V_{ub}|/|V_{cb}|$ measured values are compared with those obtained using the fitting procedure after having removed them from the fit.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Measured value</th>
<th>Fitted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta m_s$</td>
<td>$&gt; 12.4$ ps$^{-1}$ at 95% C.L.</td>
<td>$[9.5 - 17]$ ps$^{-1}$ 68% C.L.</td>
</tr>
<tr>
<td>$</td>
<td>V_{ub}</td>
<td>/</td>
</tr>
</tbody>
</table>

From these results the important impact of these two measurements in the determination of the allowed region for $\rho$ and $\eta$ is clear. Furthermore the expected probability distribution for $\Delta m_s$ shows that present analyses are exploring the one sigma region.

Conclusions

Important improvements have been obtained in the last two years in the analyses of $B^0 - \bar{B}^0$ oscillations. Combining LEP results with those from SLD and CDF, $\Delta m_d$ frequency is presently known with a 3.4% relative error ($\Delta m_d = 0.477 \pm 0.017$ ps$^{-1}$). The sensitivity on $\Delta m_s$ is at 13.8 ps$^{-1}$ and, the actual LEP/SLD/CDF combined limit, of 12.4 ps$^{-1}$ at 95% of C.L., is exploring the region where $\Delta m_s$ is expected to be according to the analysis [4]. The measurement of $\Delta m_s$ is still a challenge for LEP collaborations, $|V_{ub}|$ has been
measured at LEP with about the same experimental precision as the one obtained by CLEO and with a reduced dependence on theoretical models.

The phenomenological analysis presented in this paper gives:

\[ \rho = 0.189 \pm 0.074 \; ; \; \eta = 0.354 \pm 0.045 \]

and, in an indirect way:

\[ \sin 2\beta = 0.73 \pm 0.08 \; ; \; \sin 2\alpha = -0.15 \pm 0.30 \; ; \; \gamma = (62 \pm 10)^\circ \]

The situation will still be improved, at least until the next summer '99, before the starting of B-factories.

**Acknowledgement**

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   Parodi, F., Roudeau, P. and Stocchi, A. paper 586 contributed to the ICHEP98 Conference (Vancouver 23\textsuperscript{th}-29\textsuperscript{th} July 1998)