A MATCHED SPIN ROTATOR FOR LEP

A. Blondel and F. Keil
CERN, Geneva, Switzerland

ABSTRACT. This report describes a spin rotator installed in a straight section of LEP, its optical properties and its effect on polarization. It is found that a depolarization as small as $2 \times 10^{-3}$ is feasible.

Introduction

This report describes a Richter-Schwitters spin rotator [1] for LEP, which satisfies all the optical and the most important spin matching conditions. Chapter Layout of the LEP spin rotator gives the layout in detail. Chapter Optical and spin-matching describes how the spin matching conditions can be formulated such that standard beam-optical matching programs can be used for finding a solution. The matched spin rotator for LEP obtained by this method is discussed in Chapter Spin-matched solution.

Layout of the LEP spin rotator

The schematic layout of half of the spin rotator is shown in Figure 1. The other half is antisymmetric with respect to the interaction point IP. The string of bending magnets B1 between the quadrupoles QS2 and QS3 bends the beam vertically by $18.3459$ mrad, and the bending magnet string B2 between the quadrupoles QS9 and QS10 bends the beam by $\psi = -3.46054$ mrad. The sum of the bending angles is $\phi = 14.88536$ mrad, such that the spin is rotated from the vertical direction into the longitudinal one at the $Z_{\eta}$ energy of $46.5$ GeV. Compared to an earlier version [2], the bending magnet string B2 is one half-cell closer to the IP. Therefore, the focusing direction of the quadrupole in front of B2 changes. This is thought to ease the spin matching.

![Figure 1: Layout of the Richter-Schwitters Spin Rotator](image)

The maximum vertical displacement of the design orbit with the spin rotator from the median plane in the standard LEP configuration occurs in the B1 dipoles. It amounts to $\pm 564$ mm and is caused by the distance of these dipoles from the IP. However, the B1 dipoles are inserted into the LEP lattice at the nearest place to the IP where a straight section of adequate length is available. But,
a greater distance from the IP makes it easier to shield the experiments from the synchrotron radiation emitted in the B1 dipoles. The properties of the dipoles in the spin rotator are summarized in Table 1.

Because of the vertical slope of the design orbit, each spin rotator increases the LEPI circumference by \( d = 8.7 \) mm. This effect has consequences on the exact positions where the bunch collisions occur along the beam axis. The four bunches in each beam are equidistant under all circumstances. Therefore, if the distance between two IP’s becomes longer by the insertion of a spin rotator, the actual collision points are shifted from the IP’s. With one spin rotator in LEPI, the collision points at the rotator and at the diametrically opposite IP are not shifted, while the other two collision points are shifted by \( \pm d/4 \) which is small enough to be neglected. With two diametrically opposite spin rotators, the collisions occur at the IP’s.

<table>
<thead>
<tr>
<th>Table 1: Properties of Spin Rotator Dipoles at 46.5 GeV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dipole</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>Eff. length</td>
</tr>
<tr>
<td>Bending angle</td>
</tr>
<tr>
<td>Field</td>
</tr>
<tr>
<td>Radiation loss</td>
</tr>
<tr>
<td>Radiation power at 3 mA</td>
</tr>
<tr>
<td>Critical energy</td>
</tr>
</tbody>
</table>

**Optical and spin-matching**

Since the spin rotator replaces a section of the standard LEPI lattice, it has to be matched optically to the LEPI lattice. The optical matching conditions are given in Section Optical matching conditions. In addition, the spin rotator has to satisfy spin-matching conditions. In Section Spin-matching conditions they are formulated such that they can be solved by standard beam-optics programs. The procedure and the computer files used for finding the spin-matched rotator for LEPI are described in Section Spin matching.

**Optical matching conditions**

The spin rotator is inserted into the straight part of the LEPI lattice occupied by the low-\( \beta \) and RF insertions. Therefore the insertion must be matched to the dispersion suppressor such that the values of \( \alpha_x, \alpha_y, \beta_x, \beta_y \) at the entrance of the dispersion suppressor remain unchanged when the spin rotator is installed. This yields four optical conditions. The new bending magnets bend the beam only in the vertical plane. Therefore there are no conditions on the horizontal dispersion \( D_x \) and its derivative \( D'_x \). Because of the vertical bending, the vertical dispersion \( D_y \) and its derivative \( D'_y \) do not vanish any longer. Because of the antisymmetry of the spin rotator, the dispersion at the IP vanishes, but its derivative does not. Without matching, the vertical dispersion will propagate through all the arcs. This is undesirable because of the increase in the vertical beam size due to vertical quantum excitation, and because of the coupling for off-momentum particles. Looking from the IP towards the arc, vertical dispersion matching implies 2 conditions, \( D_y = 0 \) and \( D'_y = 0 \), with a non-zero starting value for \( D'_y \) at the IP. Looking from the arc towards the IP, one can start with \( D_y = D'_y = 0 \) at the end of the insertion, and impose the condition \( D_y = 0 \) at the IP. This is one condition less than needed for matching in the other direction. Hence there are five or six conditions for the beam optics alone.
Spin-matching conditions

The spin matching conditions which have to be satisfied by a Richter-Schwitters spin rotator in LEPI were given by Buon [3] and Blondel [4]. The three extra conditions are caused by the quadrupoles between the dipole strings of the LEPI spin rotator:

- The vertical phase advance $\mu_y$ from the IP to the centre of the B1 dipole string should be a multiple of $\pi$.
- The vertical phase advance $\mu_y$ between the centres of the B1 and B2 dipole strings should be a multiple of $2\pi$.
- Spin matching of the horizontal betatron oscillations implies the following condition [4]:

$$\int_{\beta_0}^{\beta_1} K \sqrt{\beta_s \cos \mu_s} \, ds = \sin \xi \int_{\beta_0}^{\beta_2} K \sqrt{\beta_s \cos \mu_s} \, ds$$

(1)

Here $\sin \xi = 0.3571154$ is the projection of the spin vector onto the orbit between B1 and B2, and $K$ is the quadrupole strength. As noted in [4], the integrals in (1) are proportional to the change of the slopes of a particle starting at the IP with $x \neq 0$ and $x' = 0$.

The two conditions on the vertical phase advance are familiar to standard beam-optics programs. The horizontal spin matching condition can be expressed as a relation between the $R_{21}$ elements of the 6x6 TRANSPORT [5] matrices $R$ from the IP to the centres of the dipole strings B1 and B2 which are routinely calculated by standard beam-optics programs:

$$R_{21}(B2) = \frac{1 + \sin \xi}{\sin \xi} R_{21}(B1)$$

(2)

Thus, eight or nine conditions must be satisfied in total. One condition is on matrix elements, the other conditions are on orbit functions, e.g. $\alpha$, $\beta$, $\mu$, $D$, or $D'$. The variables available to satisfy these matching conditions are the six gradients of the six independently powered quadrupoles in the low-$\beta$ insertion, and the two gradients of the four quadrupoles in the RF insertions which are powered in pairs. Hence, there are enough variables to satisfy all the conditions when matching towards the IP.

Spin matching

The MAD program [6] does not currently allow constraints on matrix elements. During matching, MAD only "knows" the matrix of the current element and the orbit functions, but not the accumulated matrix. Therefore, only the conditions on the orbit functions were satisfied and the quadrupole gradients obtained were used as starting values for matching with TRANSPORT [5] which allows simultaneous conditions on matrix elements and orbit functions. First matching attempts with TRANSPORT started at the IP and matched towards the arcs. However, when the orbit functions were launched with a non-zero value of $D_s'$ at the IP, TRANSPORT did not understand the constraints on $D_s$ and $D_s'$ at the end of the insertion.

There are relations between the elements of the $R$ matrix for a beam line and the elements of the $R$ matrix for its reflection. In particular, the $R_{11}$ and $R_{21}$ matrix elements are the same for a beam line and its reflection. Therefore, TRANSPORT can be used for matching from the dispersion suppressor towards the IP. This is advantageous because the number of constraints is smaller, and one gets around the TRANSPORT problem of launching the matching with a non-zero initial value of $D_s'$. Entering the constraints on the optical functions $\alpha$ and $\beta$, and on the dispersion $D_s$ is straightforward [7]. The conditions that the vertical phase advances are multiples of $\pi$ become conditions that the $R_{41}$
elements vanish for a matrix which is initialized at the centre of the B2 dipole string. The constraint on the matrix elements $R_{21}(B1)$ and $R_{21}(B2)$ is formulated as follows:

- Since the matching is done towards the IP, the linear matrices $A$ from B2 to B1 and $C$ from B2 to the IP are available.
- Using the properties of reflected and inverse $2 \times 2$ matrices with unity determinant, the spin matching condition (2) is expressed in terms of elements of the matrices $A$ and $C$:

$$C_{21} = \frac{1 + \sin^2 \gamma}{\sin^2 \gamma} (C_{21}A_{22} - C_{22}A_{21})$$  \hspace{1cm} (3)

- The elements of the $A$ and $C$ matrices needed are stored in TRANSPORT registers [7]. Arithmetic in those registers is used to obtain an expression which vanishes when the horizontal betatron oscillations are spin matched and which is also stored in a TRANSPORT register. The value in that register is constrained during the matching.

As a first step towards spin matching, a MAD data file was set up in which a spin rotator is inserted into the second superperiod, i.e. in Pit 4. After painful experience, none of the elements or beam lines in the IEP definition were redefined. Instead, all definitions of modified elements and beam lines were added using previously unused names. The new beam line definitions start at the very top with a definition of a modified line called LEPM, and then continue with shorter and shorter beam lines. All modified elements are defined by first copying the unmodified definitions from the IEP database [8], and then changing them by hand. Proceeding in this manner ensures that no changes are made involuntarily to beam lines and elements which are not involved in the spin rotator. The second step towards spin matching consists in preparing the data for TRANSPORT. The lattice description is translated by a program [9] from the TWISS file generated by MAD. The TRANSPORT 'vary codes' and constraints are inserted into the TRANSPORT data by hand.

**Spin-matched solution**

Using the procedure described above, an optically and spin-matched spin rotator was found with MAD and TRANSPORT. Below, its optical properties are described first, followed by a discussion of the results of a spin simulation.

**Optical properties of the solution**

The solution found with MAD and TRANSPORT satisfies all optical conditions and all spin-matching conditions. In particular, it has $R_{y}(B1) = -0.0600$ and $R_{y}(B2) = -0.2280$ which fulfils the spin matching condition (2) to the accuracy given. The quadrupole strengths in the standard IEP configuration and after spin matching as well as their relative changes are shown in Table 2. The largest change occurs in Q83M. The strength of the strongest MQA quadrupole, Q84M, is still low enough that the maximum gradient of the MQA quadrupoles, 10.9 T/m, is adequate for operation up to 78 GeV. Each spin rotator increases the tunes by $\Delta Q_\ast = 0.1135$ and $\Delta Q_t = 0.2441$, since the matching does not impose a constraint on the phase advance in a superperiod with a spin rotator. Such constraints can be satisfied later by varying quadrupoles in the neighbouring dispersion suppressor.

The orbit functions through half the spin rotator are shown in Figure 2. Despite the extra conditions due to the spin matching, the orbit functions do not differ much from those in the standard IEP configuration [10], apart from the vertical dispersion. The vertical dispersion is small in the vertical dipoles of the spin rotator because of the first two spin-matching conditions. Therefore the increase of the vertical emittance due to the emission of synchrotron radiation in these dipoles is also small. The vertical emittance was computed with the PETROC program [11] for the unlikely case of spin rotators near all four experimental pits. The ratio of the vertical and horizontal emittances was found to be

253
Table 2: Quadrupole Strengths for Spin Rotator

<table>
<thead>
<tr>
<th>Strength</th>
<th>After</th>
<th>Before</th>
<th>Rel. Change(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>KQSCM</td>
<td>-0.16373560</td>
<td>-0.16462428</td>
<td>-0.5</td>
</tr>
<tr>
<td>KQSIM</td>
<td>0.03392751</td>
<td>0.03522834</td>
<td>-3.7</td>
</tr>
<tr>
<td>KQSIM</td>
<td>0.02612732</td>
<td>0.01752874</td>
<td>49.1</td>
</tr>
<tr>
<td>KQS4M</td>
<td>-0.04215348</td>
<td>-0.0348306</td>
<td>21.0</td>
</tr>
<tr>
<td>KQS5M</td>
<td>0.02028132</td>
<td>0.0213431</td>
<td>-5.0</td>
</tr>
<tr>
<td>KQS6M</td>
<td>-0.03255052</td>
<td>-0.02796097</td>
<td>16.4</td>
</tr>
<tr>
<td>KFSM</td>
<td>0.03543275</td>
<td>0.03284922</td>
<td>7.9</td>
</tr>
<tr>
<td>KDSM</td>
<td>-0.03333682</td>
<td>-0.03284922</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Figure 2: Orbit Functions in the Spin Rotator
$2.58 \times 10^{-3}$, about a factor of two smaller than that expected from the misalignments in LEP [10], and more than an order of magnitude smaller than that needed for optimum luminosity. With fewer spin rotators, the emittance ratio should be smaller in proportion.

**Spin simulation results**

The magnitude of the depolarizing effects in LEP with one of the spin-matched Richter-Schwitters spin rotators was obtained by simulation with SLIM [12]. The results are displayed in Figure 3 and Figure 4. The MAD optics is calculated with the thick lenses, whereas SLIM works with thin lenses. Hence, the resulting optical matching is not perfect. In particular, a small ($\approx 1$ cm) vertical dispersion propagates through the lattice. It may be seen, however, that the spin rotator does not excite integer resonances, and the horizontal betatron resonances very little. A small excitation of the vertical betatron and synchrotron resonances is present, because it is impossible to make the vertical dispersion vanish as well as to impose a spin matching condition over the entire length of the vertical dipoles B1 (23.1 m) and B2 (11.55 m).

![Diagram](image_url)

**Figure 3:** Polarization degree in LEP with one spin rotator. The tunes are: $Q_x = 70.5018$, $Q_y = 78.4713$, $Q_z = 0.0855$. Machine equipped with polarization wiggles such that $r_x = 0.36$ minutes.
Altogether the result is almost perfect and depolarizing effects arising from the presence of the spin rotator are much smaller than those arising from lattice imperfections, as studied elsewhere in this report [13] [14]. A depolarization of a few per thousand or less can be obtained at nearly any energy given the flexibility offered by the optics to modify the vertical and horizontal tunes.

One should be cautious of the fact that neither solenoids nor twisted quadrupoles have been included in these simulations. The spin being parallel to the magnetic field in the solenoids, one would not need here to compensate the effect of the experimental solenoid by a mini-spin-rotator as suggested by Rossmanith [15] for the case of transverse polarization. The effect of the solenoidal field on particles travelling on betatron oscillations will very likely result in a modification of the above mentioned spin-matching conditions. This clearly remains to be analyzed in detail.

**Figure 4:** Depolarizing effects. Decomposition of the depolarizing effects in the same machine as Figure 3: dashed line: depolarization due to synchrotron oscillations, dash-dotted line: depolarization due to vertical betatron oscillations, dotted line: depolarization due to horizontal betatron oscillations, full line: their vector sum (note interference effects).
Conclusions

The Richter-Schwitters spin rotator is the simplest one that can be designed for LEP, and installed in the straight section(s) surrounding the interaction point(s). The small slope at the interaction point of about 15 mrad can presumably be handled by the experiments at little extra cost. A solution to the optical and spin-matching conditions has been found and is described in this paper. Its depolarizing effects are very small. The simplicity of this scheme makes it very attractive.

Bibliography

(13) J.-P. Koutchouk, this report.
(14) A. Blondel and J.M. Jowett, this report.