INTRODUCTION

Much of what I shall speak about in these lectures has happened during the last year. Moreover, the production rate of physicists these days is so great that it has been quite impossible for me even to follow the literature, let alone appreciate it or pass critical judgement on it\(^1\). Instead of attempting to review these developments, I therefore propose to give a brief sketch of the basic ideas underlying them. I shall do this only at a general level, ignoring most of the details. My reason for so doing is simply that of necessity -- I cannot hope to do otherwise.

The topics I shall discuss include, in particular: (i) finite energy sum rules, (ii) duality, and (iii) the Veneziano model. These are closely connected subjects, and together they offer an exciting new approach to strong interactions. Although the first results obtained so far are extremely encouraging, there still remain many unanswered questions of great importance. It will probably be quite some time before all these ideas can be knitted together to form a consistent whole. In my lectures, the topics are therefore kept separate, although I shall try my best to point out the relationship between them.

1. **FINITE ENERGY SUM RULES**

Finite energy sum rules are just another method of exploiting the analytic properties of scattering amplitudes. As such, they are thus little different in principle from ordinary dispersion relations which have been in common use for over ten years. In practice, however, they seem to be particularly useful for relating the asymptotic behaviour of scattering amplitudes to their values at low energies. They are thus a handy tool for checking the consistency of asymptotic models, such as the Regge model.
Consider for concreteness the case of $\pi N$ scattering. In the following we shall ignore the complications due to the nucleon spin, which are not essential for illustrative purposes. The amplitude, as represented by Fig. 1

![Fig. 1](image)

is a function of the momenta $p_\tau$ and isospin indices $i_\tau$ of the four external lines. Notice that for convenience we have taken all the lines as incoming. Bose statistics require that the amplitude be asymmetric under the interchange of the two pions, namely under the simultaneous interchange of the momenta $p_1 \leftrightarrow p_4$ and the isospin indices $i_1 \leftrightarrow i_4$.

Because of relativistic invariance and isospin symmetry, it is usual to express the amplitude in terms of invariant quantities: thus, instead of $p_\tau$, we have $s = (p_1 + p_2)^2$, $t = (p_2 + p_3)^2$, and $u = (p_2 + p_4)^2$, and instead of the isospin indices $i_\tau$ we shall use the total isospin in the $t$-channel $I_t = 0,1$.

The variables $s$, $t$, and $u$ are not independent, being related by the linear condition

$$s + t + u = 2m^2 + 2\mu^2 \quad . \quad (1)$$

We shall choose as our independent variables, $t$ and $\nu = s - u$, the advantage of which will be apparent later. The analytic properties of the amplitude in the variables $t$ and $\nu$ are easily seen. For example, for fixed $t$, the amplitude is analytic in $\nu$ apart from singularities due to bound states and scattering thresholds in the $s$- and $u$-channels.

Now both the $s$- and $u$-channels correspond to $\pi N$ elastic scattering; we have thus poles at both $s$ and $u = m^2$ corresponding to the nucleon state, and branch points at $s$ and $u = (m + \mu)^2$ corresponding to the $\pi N$ threshold. On the $\nu$-plane for fixed $t$, therefore, the situation is as shown in
Fig. 2, where $\nu_1 = t - 2\mu^2$ and $\nu_0 = t + 4m\mu$. The points $\nu = \nu_1$ ($\nu = -\nu_1$) and $\nu = \nu_0$ ($\nu = -\nu_0$) correspond to $s = m^2$ ($u = m^2$) and $s = [m + \mu]^2$ ($u = [m + \mu]^2$), respectively.

The following symmetry properties of the amplitude are useful later for our derivation of the finite energy sum rules:

i) Under the interchange of the momenta $p_1 \leftrightarrow p_4$, we have $s = (p_1 + p_2)^2 \leftrightarrow (p_4 + p_2)^2 = u$; or in other words, $\nu \leftrightarrow -\nu$. Now the whole amplitude has to be invariant under the simultaneous interchange $p_1 \leftrightarrow p_4$ and $i_1 \leftrightarrow i_4$. This means that for $I_\tau = 0$, which is even under $i_1 \leftrightarrow i_4$, we have (even under crossing)

$$A_0(\nu) = A_0(-\nu),$$

whereas for $I_\tau = 1$, which is odd under $i_1 \leftrightarrow i_4$, we have (odd under crossing)

$$A_1(\nu) = -A_1(-\nu).$$

In particular, this explains why the singularities on the $\nu$-plane as exhibited in Fig. 2 are symmetrically situated about the origin.

ii) For both $I_\tau = 0$ and 1, the 'reality' property of the amplitude implies

$$A(\nu^*) = A^*(\nu).$$

This is a consequence of hermiticity.
Consider now the amplitude \( A_1(\nu, t) \) for \( I_t = 1 \). In the Regge model, this is supposed to be dominated at high energy and small momentum transfer by the exchange of the \( \rho \) trajectory. Now for fixed \( t \), and \( s \to \infty \), one sees from Eq. (1) that \( \nu \sim 2s \). Hence we have for the asymptotic form of the amplitude

\[
A_1(\nu, t) \to \beta_\rho(t) \left( \frac{s}{s_0} \right)^{\alpha_\rho} \sin \frac{\pi \alpha_\rho}{\pi \alpha_\rho} \left( \frac{s}{s_0} \right)^{\alpha_\rho}. \tag{5}
\]

Let us assume that there exists a certain value \( N \) such that for all \( |\nu| \geq N \), the asymptotic form (5) is already a good approximation to the amplitude. We then apply the Cauchy theorem to the function \( A_1 \) integrating along the semicircle of radius \( N \) in the upper-half \( \nu \)-plane, as shown in Fig. 2. Since the function \( A_1 \) is analytic inside the semicircle, we have

\[
\int_{-N}^{N} A_1(\nu + i\epsilon, t) \, d\nu = \int_{\gamma} A_1(\nu, t) \, d\nu, \tag{6}
\]

where the right-hand integral is carried along the arc of the semicircle, namely \( |\nu| = N \). Since, by assumption, the asymptotic form (5) is already a good approximation to \( A_1 \) for \( |\nu| = N \), we can replace the integrand on the right by the asymptotic form, obtaining

\[
\int_{-N}^{N} A_1(\nu + i\epsilon, t) \, d\nu = 2i \frac{\beta(t) N^{\alpha_\rho + 1}}{\alpha_\rho(t) + 1}. \tag{7}
\]

Using now the symmetry (3) of \( A_1 \) under crossing, we can rewrite the left-hand integral as

\[
\int_{-N}^{N} A_1(\nu + i\epsilon, t) \, d\nu = \int_{0}^{N} [A_1(\nu + i\epsilon, t) - A_1(\nu - i\epsilon, t)] \, d\nu. \tag{8}
\]

The reality condition (4) then gives for Eq. (7) the equation

\[
\int_{0}^{N} \text{Im} \, A_1(\nu, t) \, d\nu = \frac{\beta(t) N^{\alpha_\rho + 1}}{\alpha_\rho(t) + 1}, \tag{9}
\]

which is the simplest example of a finite energy sum rule.\(^2\)
The physical content of Eq. (9) is nothing more than just the analytic properties of \( A_1(\nu, t) \) and its assumed asymptotic behaviour. The only new point is that these properties have been expressed in a manner particularly convenient for phenomenological exploitation. The left-hand side is an integral over the 'low-energy region' from 0 up to the energy \( N \), while the right-hand side is expressed in terms of those parameters that are supposed to characterize the amplitude for high energies, i.e. \( \nu > N \). Equation (9) therefore can be regarded as a consistency relation connecting scattering at high and low energies imposed by analyticity.

The finite energy sum rule (9) can be generalized in various ways. We mention those generalizations that have been particularly useful:

i) The function \( \nu^{2m} A_1(\nu, t) \) for integral \( m > 0 \) has the same analytic properties and symmetries (3) and (4) as the original \( A_1(\nu, t) \). One can thus apply the Cauchy theorem to \( \nu^{2m} A_1(\nu, t) \) instead of \( A_1(\nu, t) \) in the same manner, obtaining

\[
\int_0^N \nu^{2m} \text{Im} A_1(\nu, t) \, d\nu = \beta_\rho(t) \frac{N \alpha_\rho(t+2m+1)}{\alpha_\rho(t) + 2m + 1} . \tag{10}
\]

This will be an additional consistency relation to be satisfied by the \( A_1 \) amplitude. In principle, one has an infinite sequence of such integral moment sum rules. In practice, however, only the first few of these are useful; with the higher moment sum rules degenerating rapidly into approximate identities.

ii) By using odd moments instead of even moments as in point (i), one can obtain sum rules also for amplitudes that are even under crossing. Thus for \( A_0(\nu, t) \), for example, assuming that for \( \nu > N \) the amplitude is dominated by the \( P \) and \( P' \) trajectories, one would have for integral \( m > 0 \)

\[
\int_0^N \nu^{2m-1} \text{Im} A_0(\nu, t) \, d\nu = \beta_P(t) \frac{N \alpha_P(t+2m)}{\alpha_P(t) + 2m} + \beta_{P'}(t) \frac{N \alpha_{P'}(t+2m)}{\alpha_{P'}(t) + 2m} . \tag{11}
\]
iii) With slight modifications, sum rules can also be derived for moments $\gamma A(\nu,t)$, where $\gamma$ is not an integer. These so-called continuous moment sum rules involve, in general, both the real and imaginary parts of the amplitude.

iv) Obviously, the methods discussed above are not restricted to $\pi N$ scattering, or to the $\rho$, $P$, and $P'$ Regge poles alone. Indeed, such sum rules can be derived for any assumed asymptotic form of the amplitude, such as sums of Regge poles or even Regge cuts. The sum rules derived in each case will afford a means for checking phenomenologically whether the assumed high-energy behaviour is consistent with the low-energy scattering data.

The applications of finite energy sum rules to phenomenological analyses can be divided into three types differing only by emphasis:

a) using low-energy data to predict high-energy parameters, such as Regge intercepts and residue functions;

b) resolving ambiguities in the low-energy region by means of high-energy data;

c) making simultaneous fits of high- and low-energy data in a manner consistent with analyticity.

Obviously, since high-energy data are in general less accurate than those at lower energies, points (a) and (c) far outweigh point (b) in importance.

The usefulness and accuracy of finite energy sum rules depend mainly on the available data. A particularly favourable case is the sum rule at $t = 0$, where by means of the optical theorem, the imaginary part of the amplitude is simply related to the total cross-section. With the accurate measurements of total cross-sections already available, one can make very accurate predictions of Regge parameters at $t = 0$. Thus, for example, the Pomeron intercept $\alpha_p(0)$ has been determined by such means\(^b\) in $\pi p$ scattering to be $\alpha_p(0) = 1 \pm 0.02$.

For $t \neq 0$, the imaginary part of the amplitude is no longer given directly in terms of measured cross-sections. One then has to rely on phase-shift analysis, which for $\pi N$ scattering is already quite reliable up to 2 GeV/c. Feeding this into the left-hand side of Eq. (9), one can evaluate the integral and obtain the $\rho$-exchange parameters for various
values of \( t \). In this way, Dolen, Horn and Schmid\(^2\) were able to predict the so-called 'wrong-signature nonsense' dip in \( d\sigma/dt \) for \( \pi N \) charge-exchange scattering: \( \pi^- p \rightarrow \pi^0 n \).

The art of exploiting the finite energy sum rules is now highly developed. The sensitivity of the method is often enhanced by judicious use of the various moments. As an example, I wish to quote the work on photoproduction of charged pions by the Rome-Trieste group\(^5\). Using the low-energy phase-shift analysis obtained previously, they were able to determine the Regge parameters for both the pion and its 'conspirator' to remarkable accuracy, and to predict successfully the high-energy data up to 16 GeV. Their result is shown in Figs. 3 and 4.

In those reactions where reliable phase-shift analyses have not been performed, such as \( K N \) scattering, it is often found possible to make qualitative predictions by assuming that the imaginary part of the amplitude at low energy is completely dominated by resonances. This will not be a direct check of analyticity and Regge behaviour since it involves the further assumption of resonance dominance. However, its qualitative success has led to the important new concept of 'duality', which will be the subject of our next lecture.

2. **Duality and Exchange Degeneracy**\(^6\)

Finite energy sum rules, as discussed in the previous section, are based only on two theoretical concepts, namely (i) analyticity, and (ii) asymptotic behaviour (not necessarily Regge) of the scattering amplitude. Clearly, being so general, they can have few predictions, and will remain only as a phenomenological tool for data analysis, unless supplemented by further assumptions.

We now propose the following:

(A) down to fairly low energies, \( \sim 2 \text{ GeV/c} \), scattering amplitudes are already well-approximated by the exchange of a few Regge poles;

(B) the imaginary part of a scattering amplitude is dominated entirely by direct channel resonances;

both statements being subject to an important exception which we shall later specify.
Neither of these assumptions are new; they have frequently been made both in phenomenological analysis and in theoretical studies. However, when coupled with the finite energy sum rules, they take on an additional significance. Consider again as an example the \( I_t = 1 \) amplitude in \( \pi \pi N \) scattering, namely \( A_1(v, t) \) in Section 1, which satisfies the sum rule (9). On the right, we have the \( \rho \)-exchange amplitude, which we shall assume to be approximately valid down to energies of \( \sim 2 \text{ GeV/c} \).

On the left, we shall assume that \( \text{Im} A_1 \) is dominated by the direct channel resonances, which occurs in \( \pi N \) scattering. Equation (9) then becomes a relation between the masses and widths of nucleonic resonances and the Regge parameters \( \alpha \) and \( \beta \) of \( \rho \)-exchange. Since the resonance parameters are well-known from phase-shift analysis and \( \alpha_\rho \) and \( \beta_\rho \) from high-energy Regge fits, the relation is subject to a direct check with existing data, and is found to be approximately valid. Thus, for example, by feeding in the resonance parameters on the left-hand side of Eq. (9), one can predict with some certainty the dip in \( d\sigma/dt \) at \( t \sim -0.6 \text{ GeV}^2 \) for the reaction \( \pi^- p \rightarrow \pi^0 n \).

This relation between resonances in the direct channel and the exchanged Regge poles is of great theoretical significance since the Regge poles themselves are supposed to be connected to resonances in the exchange channels. This significance is best appreciated in reactions such as \( \pi \pi \rightarrow \pi \pi \), where the direct and exchange channels are identical. Equation (9) then becomes a consistency requirement involving the \( \rho \)-trajectory on both sides, which can be used to restrict the trajectory parameters. This is then the so-called FESR bootstrap.

The implications of finite energy sum rules plus the assumptions (A) and (B) do not, however, stop there. As we have seen, analyticity implies, in addition to Eq. (9), further sum rules for various moments of the amplitude. In each case, the contributions of resonances on the left must add in such a way as to build up the Regge exchange or the right. Now the higher moment sum rules will emphasize the higher mass resonances. The only way then for all the sum rules to be satisfied will be to have the integrand itself approximately equal to the Regge amplitude.

Clearly, this 'duality' or equivalence between the direct channel resonances and Regge-pole exchange should not be taken too literally, at least in the low-energy region where the resonance amplitude shows large
fluctuations as a function of the energy. It is supposed to hold only in the average sense when the resonance amplitude is integrated over a small ($\sim 1$ GeV) interval. It is this 'semi-local average' over the resonance contribution which is supposed to be approximately equal to the Regge amplitude.

A dramatic demonstration of this, at first sight, amazing fact has been given by Schmid\textsuperscript{7}). He took the Regge parameters as determined from fits at high energy to extrapolate the $\rho$-exchange amplitude in $\pi N$ scattering down to energies $\sim 2$ GeV. Then, performing a partial wave analysis on this, he obtained for each partial wave a loop on the Argand diagram very similar to those obtained by phase-shift analysis as evidence for nucleon resonances. Moreover, these 'pseudo-resonances' were shown to lie approximately on a linearly rising trajectory! Indeed, on closer examination it was found that such a behaviour of partial wave phases is an almost automatic consequence of the Regge form of the amplitude, for any exchanged trajectory with finite slope\textsuperscript{8}).

At this point, we should turn back to specify the important exception mentioned at the beginning of this section. This concerns what is known as the Pomeranchuk trajectory in the theory of Regge poles. Now, it has long been accepted by theoreticians that elastic scattering at high energy is dominated by the diffractive mechanism, or in other words, by the shadow effects of multiparticle channels via unitarity. If one insists on representing this by a Regge trajectory carrying the quantum numbers of the vacuum, namely, the Pomeron, then its parameters can be determined phenomenologically from scattering data. It was found that the trajectory thus obtained is much flatter than all other known trajectories. Indeed, apart from perhaps the recent results reported from Serpukhov, existing data are not inconsistent with $\alpha_p' = 0$. If this is true, then the $P$ trajectory cannot be dual to resonances in the sense of Schmid, since a flat trajectory will not give rise to loops in the Argand plot. Another reason for this belief is as follows. Since the Pomeron carries the quantum numbers of the vacuum, its exchange has the same contributions in all isospin states in the direct channel. If it is dual to resonances then one expects resonances in all isospin states which are more or less degenerate. This is certainly contradictory to everything we know experimentally.
It is not yet clear in what way the Pomeron is going to affect our previous discussion of duality. Indeed, no answer to this question is likely until one understands more about diffractive scattering. However, as a first approximation, one may assume that the Pomeron contribution is additive to the dual part of the amplitude; namely, we write for the full amplitude: \( A = A_{\text{Dual}} + A_{\text{Pomeron}} \), where \( A_{\text{Dual}} \) has then all the properties prescribed by Schmid. This hypothesis has been popularized by Harari and is found to be qualitatively valid in the cases where it has been checked\(^9\). Nonetheless, it should be accepted only as a first approximation and not as the final word on the Pomeranchuk problem.

This 'principle of duality', as formulated by Schmid, must rank with 'bootstrap' and 'maximal analyticity' as one of the most loosely defined principles in the history of physics. Nonetheless, like the others, it has proved extremely fruitful as a basis for the understanding of strong interaction dynamics\(^{10}\). I shall try here to summarize a few of its main consequences.

The first implications of 'duality' are negative. It destroys two accepted concepts which have been in common use for several years.

2.1 *Interference models*

This concerns the intermediate energy region, say 2-6 GeV/c in \( \pi N \) scattering. Below 2 GeV/c incoming energy, phase-shift analysis tells us that the scattering amplitude is dominated by a large number of resonances. In fact, many aspects of \( \pi N \) reactions can be qualitatively understood by taking account of these resonances alone, and neglecting everything else. This is the basic premise of the so-called isobar models. Whereas at energies \( > 6 \) GeV/c, the large number of Regge fits performed seem to show that the amplitude is dominated by the exchange of a few Regge poles. The question then arises: What about the intermediate energy region? One obviously needs here some sort of an interpolation between the wild fluctuations of the resonances below 2 GeV/c to the smooth behaviour of Regge amplitudes at higher energy. A natural assumption, at first sight, would seem to be the following: one just adds the resonance contribution to the Regge amplitude, thus:

\[
A = A_{\text{Res}} + A_{\text{Regge}}. \tag{12}
\]
At low energies, $A_{\text{Res}}$ would dominate and $A_{\text{Regge}}$ would be small, while at higher energies, $A_{\text{Res}}$ will diminish and $A_{\text{Regge}}$ will take over. In the intermediate region, say 2-6 GeV/c, $A_{\text{Res}}$ and $A_{\text{Regge}}$ are comparable and interfere. This is thus the so-called Interference Model, which for some years has enjoyed a fair amount of success. On closer examination, however, this assumption cannot be strictly correct in view of the discussion given above. According to Schmid, $A_{\text{Res}}$ and $A_{\text{Regge}}$ both represent one and the same thing; they cannot therefore possibly interfere. And, true enough, a closer scrutiny of the early successes of the interference model reveals that they are indeed independent of the interference assumption.

There is one point here which should be clarified and which has led to some misunderstanding. In our discussion, $A_{\text{Regge}}$ represents only the contribution of a 'small number' of leading poles. Otherwise the discussion would be completely meaningless. Had we allowed many non-leading poles in $A_{\text{Regge}}$, then

$$A_{\text{Regge}} = \sum_i \beta_i(t) \zeta_i(t) \alpha_i(t)$$

would itself already be a complete expansion of the amplitude. We shall not need Schmid to tell us that Eq. (12) is unfeasible. It is only when a small number of poles are concerned (e.g. the $p$-contribution in $A_1$ for $\pi N$ scattering) that the distinction between 'duality' and the 'interference model' takes on a meaning.

2.2 Subtraction of background from resonances in data analysis

A resonance in hadron collision data usually appears as a peak or hump in the effective mass distribution of its decay products, with the peak standing above some sort of smooth background. The question then arises: How much of the peak should be regarded as the resonance and how much as the background? The problem is accentuated where the 'background' itself has a hump at around the resonance position because of some 'kinematic enhancement'. A classic example of this is the $A_1$ hump in the reaction $\pi p \rightarrow \pi pp$; it is still not clear whether the $A_1$ is a true resonance or just a kinematic enhancement called the Deck effect. In the Regge language, the Deck effect arises as follows. Consider the
double-Regge diagram of Fig. 5. The fact that both the momentum transfers \( t_1 \) and \( t_2 \) are restricted to small values implies that the diagram is appreciable only in one corner of the Dalitz plot\(^{11}\), as shown in Fig. 6. It is clear that on the effect mass plot, the distribution will be enhanced at low values of the \( \pi\rho \) mass. If indeed there is a resonance called the \( A_1 \), it will show up as a little peak on top of the hump. Formerly, one may have been tempted to consider the Deck effect as the background, and as such to be subtracted from the resonance. However, with the new concept of duality, this is no longer clear. Since the Regge amplitude and the resonance contribution are essentially one and the same thing, the subtraction of the Deck effect will necessarily remove most of the resonance as well. Indeed one becomes a little confused as to what one can actually call a resonance.

The two consequences of 'duality' just discussed, though of great importance both theoretically and experimentally, are destructive rather than creative. Duality claims the interference model to be wrong and background subtraction from resonances to be dangerous, but it does not yield a better method in either case. To do this, one has to have a more specific framework than just the loose statement given earlier of 'duality'. A good example of such is the Veneziano model, which we shall discuss later.

Even in its loose form adopted in this section, however, 'duality' is able to give some definite predictions of great significance. One of the most interesting of these concerns the exchange degeneracy of Regge
trajectories. The concept of exchange degeneracy was first introduced by Arnold from other considerations. What 'duality' does is to give the concept a somewhat sounder basis.

Consider again a specific example, say, KN scattering\textsuperscript{12} for K mesons with $S = +1$. Since elastic KN scattering admits the exchange of vacuum quantum numbers, there will be a Pomeron contribution, which we assume to be additive as discussed above. What concerns us here is only the dual part of the amplitude. Now, one outstanding feature of KN scattering is the empirical fact that no resonance with $B = +1$ and $S = +1$ are known to exist. This means that in the direct channel for KN scattering, the contributions of resonant states are negligible. Since 'duality' suggests that the resonance contributions and Regge exchanges are essentially the same, the absence of resonances would imply that the imaginary part of the Regge amplitude is also zero. This statement is true for both Kp and Kn scattering, and for both forward and backward scattering; it must therefore hold separately for each of the amplitudes with $I_t = 0,1$ and $I_u = 0,1$. Now for these amplitudes, the following trajectories are expected to contribute:

\begin{align*}
I_t = 0 & : \quad \omega \quad f_0 \\
I_t = 1 & : \quad \rho \quad A_2 \\
I_u = 0 & : \quad \Lambda \quad \chi^*(1520) \\
I_u = 1 & : \quad Y_1^*(1385) \quad \chi^*(1765),
\end{align*}

where the two trajectories occurring in each case have opposite signatures. For the imaginary part of each Regge amplitude to cancel for all $t$ and $u$, the trajectories must therefore be degenerate in pairs in both the trajectory function $\alpha$ and the residue $\beta$. The fact that the meson trajectories are indeed approximately degenerate is well known. For the baryons, also, it appears that the prediction is equally valid, as can be seen in Figs. 7 and 8.

Similar arguments have been applied with fair success to other reactions. However, there also exist some cases, such as baryon-antibaryon scattering, for which the predictions of 'duality' appear not to be valid. On the whole, I would say that the results in this direction have been quite impressive though not yet entirely overwhelming.
Fig. 7 (a) exchange-degenerate sequence of $Y_0^*$.  
(b) the residues of the sequence also approximately degenerate (Ref. 12).
Fig. 8  (a) exchange-degenerate sequence of $Y_1^*$.  
(b) the residues of the sequence also approximately degenerate (Ref. 12).
3. THE VENEZIANO MODEL

As a result of both theoretical and phenomenological studies carried out in recent years, especially those based on the finite energy sum rules and 'duality' that I have described in the last two sections, we have now quite a good knowledge of the properties possessed by two-body collision amplitudes. I shall list below some of these that are relevant for our present discussion:

i) analyticity,

ii) crossing symmetry,

iii) Regge asymptotic behaviour,

iv) resonances on linear rising trajectories,

v) 'duality' in the sense of Schmid.

The Veneziano model\textsuperscript{13} is a particular example that satisfies all these properties. It is thus a good theoretical laboratory in which to study hadron collision amplitudes. Moreover, since it already has so much in common with physical amplitudes, one may reasonably hope that it may serve also as a phenomenological model for the description of experimental data.

Again, instead of introducing the subject in full generality, I prefer to begin by giving a specific example. For this, I have chosen \( \pi\pi \) scattering\textsuperscript{14}, which is particularly suitable for our illustrative purpose.

Let \( i_\pi \) and \( p_\pi \) (\( r = 1, 2, 3, 4 \)) denote the isospin indices and the four-momenta of the external pions, respectively. The isospin indices \( i_\pi \) can take the values 1, 2, or 3, where as usual \( \pi_1 = \frac{1}{2}(\pi^+ + \pi^-) \), \( \pi_2 = (1/2i)(\pi^+ - \pi^-) \), \( \pi_3 = \pi^0 \). To each value of \( i_\pi = 1, 2, 3 \) we shall associate the \( 2 \times 2 \) Pauli matrix \( \tau_{i_\pi} \). We shall take all the momenta as ingoing, so that the Mandelstam variables \( s, t, \) and \( u \) are given as

\[
    s = (p_1 + p_2)^2 = (p_3 + p_4)^2
\]
\[
    t = (p_2 + p_3)^2 = (p_1 + p_4)^2
\]
\[
    u = (p_1 + p_3)^2 = (p_2 + p_4)^2.
\]

(14)

The only known trajectories that are strongly coupled to the \( \pi - \pi \) system are: (i) the \( \rho \)-trajectory with isospin \( I = 1 \) and negative signa-
ture; and (ii) the $f_0$-trajectory with isospin $I = 0$ and positive signature. The two trajectories are empirically almost degenerate on the Chew-Frautschi plot. Moreover, 'duality' arguments similar to those given in the last lecture, plus the fact that no meson resonances with double charge are known, imply that the $\rho$ and $f_0$ trajectories are exchange degenerate partners. We shall therefore introduce a common trajectory function $\alpha$ for both the $\rho$ and $f_0$, and assuming $\alpha$ to be linear, we have $\alpha(x) = \alpha_0 + \alpha'x$, where as usual $\alpha_0$ is the intercept and $\alpha'$ the slope of the trajectory.

In this notation then, the Veneziano amplitude for $\pi\pi$ scattering, as first given by Lovelace\textsuperscript{14}), takes the following form:

$$ T = V(s,t) + V(t,u) + V(u,s), \quad (15) $$

where

$$ V(s,t) = \beta \, \text{Tr}(\tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4}) \times \frac{\Gamma[1 - \alpha(s)] \Gamma[1 - \alpha(t)]}{\Gamma[1 - \alpha(s) - \alpha(t)]} \quad (16) $$

$$ V(t,u) = \beta \, \text{Tr}(\tau_{i_1} \tau_{i_3} \tau_{i_2} \tau_{i_4}) \times \frac{\Gamma[1 - \alpha(t)] \Gamma[1 - \alpha(u)]}{\Gamma[1 - \alpha(t) - \alpha(u)]} \quad (17) $$

$$ V(u,s) = \beta \, \text{Tr}(\tau_{i_1} \tau_{i_3} \tau_{i_4} \tau_{i_2}) \times \frac{\Gamma[1 - \alpha(u)] \Gamma[1 - \alpha(s)]}{\Gamma[1 - \alpha(u) - \alpha(s)]} \quad (18) $$

In the formulae, $\beta$ is a constant, $\text{Tr}$ stands for trace, and $\Gamma$ is the gamma-function as defined by Euler. We shall show that this function $T$ does have all the desired properties of the $\pi\pi$ amplitude listed at the beginning of the section.

Since all the lines are identical, crossing symmetry here is equivalent to the statement that the amplitude is symmetric under any of the
4! = 24 permutations of the external lines. It is convenient to distinguish between (i) cyclic permutations, e.g. \((1234) \rightarrow (2341)\), and reversals, e.g. \((1234) \rightarrow (4321)\); and (ii) other permutations, such as \((1234) \rightarrow (2134)\). Those orderings of the four external lines which are related by type (i) permutations we shall call equivalent orderings.

It can then readily be checked that the 24 different orderings are divided into three disjoint equivalent classes, which can be represented by the diagrams in Fig. 9. One notes that these three diagrams correspond exactly to the three terms in Eq. (15). Indeed, the two variables occurring in each term are just the c.m. energies for the two Mandelstam channels which can be formed in the diagram without changing the ordering of the external lines. Thus, for example, for the first diagram, the variables are just \(s\) corresponding to 12-34 and \(t\) corresponding to 23 \(\rightarrow\) 41, as required. From the properties of traces of matrices, it is then seen that each term in Eq. (15) is invariant under a permutation of type (i). Moreover, a permutation of type (ii) transforms a term in Eq. (15) into another, leaving the sum invariant. In other words, one has then fully established the crossing symmetry of the amplitude.

![Fig. 9](image)

Next, to study the analytic properties of the amplitude \(T\) in Eq. (15), we first note that the gamma-function \(\Gamma(z)\) is analytic in \(z\) on the whole complex \(z\)-plane apart from simple poles at \(z = 0, -1, -2, \ldots\), and that \(\Gamma(z)\) has no zeros anywhere. Consider then \(V(s,t)\) in Eq. (16); being a ratio of \(\Gamma\)'s, \(V(s,t)\) must be analytic except where the \(\Gamma\) functions in the numerator have poles, i.e. \(\alpha(s) = 1, 2, 3, \ldots\) and \(\alpha(t) = 1, 2, 3, \ldots\). It does not have double poles, however, since when \(\alpha(s)\) and \(\alpha(t)\) are both positive integers, the denominator \(\Gamma[1 - \alpha(s) - \alpha(t)]\) will also have a pole to cancel it. Similar properties will also hold for \(V(t,u)\) and \(V(u,s)\), and eventually also for \(T\).
What then are these poles in $T$? They must represent the resonances lying on our degenerate $\rho-f_0$ trajectory. Since the amplitude is completely symmetric, we need consider only one Mandelstam channel; say, for example, the $s$-channel. At $\alpha(s) = 1$, which should give us the $\rho$-meson, we have no pole in $V(t,u)$, while the poles in $V(s,t)$ and $V(u,s)$ give us together for $T$ the residue

$$\text{Res } T = -\delta \left[ \text{Tr} \left( \tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4} \right) \alpha(t) \right. + \left. \text{Tr} \left( \tau_{i_1} \tau_{i_3} \tau_{i_4} \tau_{i_2} \right) \alpha(u) \right],$$

where we have used the relation $\Gamma(1+z) = z \Gamma(z)$. Remembering that $\alpha(x) = \alpha_0 + \alpha' x$, namely a linear function in $x$, and that $t = -2k_s^2(1 - \cos \theta_s)$, $u = -2k_s^2(1 + \cos \theta_s)$, one sees that the residue of $T$ at $\alpha(s) = 1$ is a linear function of $\cos \theta_s$. Hence the pole must represent some particle with maximum angular momentum 1. Collecting the leading terms in $\cos \theta$, one has

$$\text{Res } T \sim 2\beta k_s^2 \alpha' \cos \theta_s \times$$

$$\times \left[ \text{Tr} \left( \tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4} \right) - \text{Tr} \left( \tau_{i_1} \tau_{i_3} \tau_{i_4} \tau_{i_2} \right) \right].$$

(20)

Projecting into the isospin states in the $s$-channel, one obtains for:

$I_s = 0$:

$$2\beta k_s^2 \alpha' \cos \theta_s \left[ \text{Tr} \left( \tau_{i_1} \tau_{i_2} \right) \text{Tr} \left( \tau_{i_3} \tau_{i_4} \right) - \text{Tr} \left( \tau_{i_1} \tau_{i_3} \right) \text{Tr} \left( \tau_{i_4} \right) \right]$$

$$= 2\beta k_s^2 \alpha' \cos \theta_s \left[ \delta_{i_1 i_2} \delta_{i_3 i_4} - \delta_{i_1 i_3} \delta_{i_2 i_4} \right] = 0$$

$I_s = 1$:

$$2\beta k_s^2 \alpha' \cos \theta_s \left[ \sum_x \text{Tr} \left( \tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4} \right) \text{Tr} \left( \tau_{i_1} \tau_{i_3} \tau_{i_4} \right) \right. - \left. \sum_x \text{Tr} \left( \tau_{i_2} \tau_{i_1} \tau_{i_3} \tau_{i_4} \right) \text{Tr} \left( \tau_{i_1} \tau_{i_3} \tau_{i_4} \right) \right]$$

$$= 2\beta k_s^2 \alpha' \cos \theta_s \left[ \sum_x (i \epsilon_{i_1 i_2 x}) (i \epsilon_{i_3 i_4}) - \sum_x (i \epsilon_{i_2 i_1 x}) (i \epsilon_{i_3 i_4}) \right]$$

$$= 2\beta k_s^2 \alpha' \cos \theta_s (-2) \sum_x \epsilon_{i_1 i_2 x} \epsilon_{i_3 i_4} \neq 0.$$
We have then shown that the pole at $\alpha_s = 1$ has maximum spin 1, and that the spin 1 part has pure isospin 1, namely the same quantum number of the $\rho$ meson as required.

Repeating the arguments for $\alpha(s) = 2$, one easily sees that the leading term in $\cos \theta_s$ in the residue now becomes

$$\text{Res} \quad T \sim -\beta(2k_s^2)^2 \alpha'^2 \cos^2 \theta_s \times$$

$$\times \left[ \text{Tr}(\tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4}) + \text{Tr}(\tau_{i_1} \tau_{i_3} \tau_{i_4} \tau_{i_2}) \right]$$  (21)

with a + sign between the two traces. The pole having now a residue of second order in $\cos \theta_s$ must represent a particle of maximum spin 2, and the spin 2 part, because of the change in sign, has now $I_s = 0$, i.e. the same quantum numbers of the $f_0$ meson.

In general, at $\alpha(s) = \ell$, the residue of the pole is a polynomial in $\cos \theta_s$ of degree $\ell$, thus representing a particle of maximum spin $\ell$. The leading term in $\cos \theta_s$ has isospin 1 if $\ell$ is odd, but isospin 0 if $\ell$ is even. In other words, our amplitude $T$ contains two linear trajectories in the $s$-channel, an odd signature trajectory with $I = 1$, and an even signature trajectory with $I = 0$. The trajectories are degenerate with the same intercept $\alpha_0$ and the same slope $\alpha'$. We shall identify them with the $\rho$ and $f_0$ trajectories, respectively.

Next we wish to show that the amplitude (15) has the proper Regge behaviour. Again because of symmetry, we need only establish this for one channel, say for $s \to \infty$ and fixed $t$. One must, however, be careful in taking this limit, for, as we have just shown, the function $T$ has an infinite sequence of poles on the real $s$-axis so that the limit $s \to \infty$ on the real axis cannot strictly exist. We shall therefore define instead as the 'Regge limit' the following: $\sigma \to \infty$ for $s = \sigma + i\varepsilon \sigma$ and some infinitesimal $\varepsilon > 0$. Namely, instead of approaching $\infty$ on the $s$-plane strictly along the real axis, we shall approach $\infty$ along a ray at an infinitesimal angle to the real axis. That this can indeed be taken as the Regge limit is non-trivial and needs some justification. I shall return to this later when we discuss the problem of unitarity for the Veneziano model.
Accepting for the moment our new definition of the Regge limit, we return now to the amplitude $T$ in Eq. (15). We shall consider only the first two terms, for, as can be shown, the third term $V(u,s)$ vanishes exponentially as $s \to \infty$. Projecting out the isospin states, this time however in the $t$-channel, we obtain:

$$T_0 = \frac{\Gamma[1 - \alpha(s)] \Gamma[1 - \alpha(t)]}{\Gamma[1 - \alpha(s) - \alpha(t)]} + \frac{\Gamma[1 - \alpha(t)] \Gamma[1 - \alpha(u)]}{\Gamma[1 - \alpha(t) - \alpha(u)]}$$  \hspace{1cm} (22)

$$T_1 = \frac{\Gamma[1 - \alpha(s)] \Gamma[1 - \alpha(t)]}{\Gamma[1 - \alpha(s) - \alpha(t)]} - \frac{\Gamma[1 - \alpha(t)] \Gamma[1 - \alpha(u)]}{\Gamma[1 - \alpha(t) - \alpha(u)]}.$$  \hspace{1cm} (23)

Notice the difference in sign for $T_0 = 0$ and $1$, which, as we shall see, will give us the different signature of the $\rho$ and $f_0$ trajectories.

To go further, we need the following properties of the $\Gamma$ function:

$$\Gamma(z) \Gamma(1 - z) = \pi \sin \pi z \Gamma(1 - \frac{\pi}{2})$$  \hspace{1cm} (24)

$$\lim_{z \to \infty} \frac{\Gamma(z + a)}{\Gamma(z + b)} = z^{a-b} \left[ 1 + \frac{2}{2a} (a - b)(a + b - 1) + O(z^{-2}) \right]$$  \hspace{1cm} (25)

which can be found in standard text-books.

Using Eq. (24), we can rewrite Eqs. (22) and (23) as

$$T_1 \approx \frac{\pi}{\Gamma[\alpha(s)] \sin \pi \alpha(t)} \cdot \left\{ (-1)^\alpha \frac{\Gamma[1 - \alpha(u)]}{\Gamma[1 - \alpha(t) - \alpha(u)]} + \frac{\sin \pi [\alpha(t) + \alpha(s)]}{\sin \pi \alpha(s)} \cdot \frac{\Gamma[\alpha(t) + \alpha(s)]}{\Gamma[\alpha(s)]} \right\}.$$  \hspace{1cm} (26)

Then using Eq. (25), we have

$$\lim_{s \to \infty} \frac{\Gamma[\alpha(t) + \alpha(s)]}{\Gamma[\alpha(s)]} = [\alpha(s)]^\alpha(t) = (\alpha_s)^\alpha(t).$$  \hspace{1cm} (27)
Moreover, since \( s + t + u = 4\mu^2, s + \infty \) above the real axis for fixed \( t \) implies \( u \to -\infty \) below the real axis. This gives again by Eq. (25)

\[
\lim \frac{\Gamma[1 - \alpha(u)]}{\Gamma[1 - \alpha(t) - \alpha(u)]} = \left[-\alpha(u)\right]^{\alpha(t)} = (\alpha's)^{\alpha(t)}. \tag{28}
\]

Using then the identity \( \sin (A + B) = \sin A \cos B + \cos A \sin B \), we can rewrite:

\[
\frac{\sin \pi[\alpha(t) + \alpha(s)]}{\sin \pi \alpha(s)} = \cos \pi \alpha(t) + \sin \pi \alpha(t) \cot \pi \alpha(s), \tag{29}
\]

where on taking the limit as previously defined, \( \cot \pi \alpha(s) \to -i \).

Altogether then for \( T^I \), one has the limit

\[
\lim T^I \approx \frac{\pi}{\Gamma[\alpha(t)]} \cdot \frac{(-1)^I + e^{-i\pi \alpha(t)}}{\sin \pi \alpha(t)} \cdot (\alpha's)^{\alpha(t)} \tag{30}
\]

which is identical to the Regge limit, e.g. Eq. (5), provided we identify \( \beta(t) \) to \( 1/\Gamma[\alpha(t)] \) and \( s_0 \) to \( \alpha^{-1} \). Notice we do have negative signature for \( I = 1 \) (the \( \rho \)-trajectory) and positive signature for \( I = 0 \) (the \( f_0 \)-trajectory) as anticipated.

Having now proved the Regge behaviour and also the existence of resonance poles in the amplitude, 'duality' becomes an automatic consequence, since the same function has been shown explicitly to contain both the \( s \)-channel resonance poles and \( t \)-channel Regge exchange amplitude. Indeed, the loosely formulated 'principle of duality' receives in the Veneziano model the first concrete realization.

Besides the properties just proved, the Veneziano-Lovelace formula (15) has, in addition, the following attractive features:

i) **Absence of exotic states.** The term \( V(s,t) \) in Eq. (16) has poles only in the \( s \)- and \( t \)-channels and not in the \( u \)-channel. In the \( s \)-channel, the complete isospin structure can be seen by the identity

\[
\frac{1}{4} \text{Tr}(\tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4}) = \frac{1}{2} \text{Tr}(\tau_{i_1} \tau_{i_2}) \frac{1}{4} \text{Tr}(\tau_{i_1} \tau_{i_3} \tau_{i_4}) +
\]

\[+ \sum_x \left[ \frac{1}{2} \text{Tr}(\tau_{i_1} \tau_{i_2} \tau_x) \frac{1}{2} \text{Tr}(\tau_{i_3} \tau_{i_4} \tau_x) \right], \tag{31}
\]
which shows that the s-channel poles can have only isospin 0 or 1 and no higher. A similar statement will hold also for the other terms in Eq. (15) and for all these Mandelstam channels. As we have already seen in the last lecture on 'duality', the absence of exotic states is closely connected with exchange degeneracy. The facts that in Eq. (15) no resonance occurs with \( I > 1 \) and that the \( \rho \) and \( f_0 \) trajectories are degenerate are connected, and represent just a particular realization of the arguments presented earlier.

ii) The Adler consistency condition. A condition derived by Adler from current algebra states that the \( \pi \pi \) scattering amplitude has to vanish at the symmetry point \( s = t = u = \mu^2 \) when one pion has zero mass. One notes from Eq. (16) that \( V(s,t) \) does indeed have a zero at

\[
1 - \alpha(s) - \alpha(t) = 0 ,
\]

which can be identified with the Adler zero if

\[
\alpha(\mu^2) = \frac{1}{2}.
\]

Assuming the \( \rho \) mass to be 764 MeV, this gives \( \alpha(0) = 0.48 \), which is not far from the intercept of the \( \rho \)-trajectory determined empirically from Regge fits. Once Eq. (33) is satisfied, one sees that the whole amplitude \( T \) vanishes at the Adler point.

iii) Existence of daughter trajectories. As we have seen above, the residue at \( \alpha_s = \lambda \) is in general a polynomial of order \( \lambda \) in \( \cos \theta_s \). The maximum spin content is thus \( \lambda \), as already stated. However, the polynomial being in general not identical to the Legendre polynomial \( P_\lambda(\cos \theta_s) \), there will also be components of lower spins. These are called daughter states which are degenerate with the parent states. Thus, for example, the \( \rho \) meson in Eq. (15) will be degenerate with another spin zero state with \( I = 0 \) called the \( \epsilon \), while the \( f_0 \) is degenerate with a spin 1 state with \( I = 1 \) called the \( \rho' \). Now although there is no clear evidence for the existence of these states, it has long been conjectured that the \( \epsilon \) can explain the asymmetry in \( \rho^0 \) decay, while the \( \rho' \) has often been invoked to improve Regge fits and to 'explain' the \( 1/q^4 \) dependence of the proton electromagnetic form factors. The occurrence of daughter trajectories seems to be a general feature of dual resonance models. Whether the daughter states can be regarded as real resonances, however, remains to be clarified.
In spite of its many beautiful features, the Veneziano model is not free from diseases. By far the most serious of these are (i) the problem of ambiguities, and (ii) the question of unitarity.

i) We have shown that the amplitude (15) does possess all the properties listed at the beginning of the lecture. The question naturally arises whether there are other functions satisfying the same condition, and if so how different they can be from (15). Unfortunately, the question has not been completely answered. A wide class of such solutions are indeed known, which however are not very different in structure from Eq. (15). Consider the modification

\[ V(s,t) \rightarrow V(s,t) + V'(s,t) \]

\[ V'(s,t) = \beta' \text{Tr}(\tau_{i_1} \tau_{i_2} \tau_{i_3} \tau_{i_4}) \frac{\Gamma[m - \alpha(s)] \Gamma[n - \alpha(t)]}{\Gamma[\ell - \alpha(s) - \alpha(t)]} \],  \hspace{1cm} (34) \]

with \( m, n > 1, \ell > m + n \). It can readily be seen that the new \( V(s,t) \) when substituted in Eq. (15) will also possess all the required properties. It follows then that any convergent series of such terms as \( V' \) when added to \( V \) will give an equally good amplitude as far as the properties (i) to (v) are concerned. These are the so-called satellite terms. Such terms in general only modify the coupling of trajectories at the daughter levels. Thus, they may not be too important for a first approximation. Also, there are some weak theoretical arguments connected with factorization, which favour the original form (15) with no satellites. Nonetheless, the possibility of introducing satellites represents such a large degree of ambiguity, that the predictive power of the model is drastically weakened.

ii) The model amplitude (15) is not unitary. This can easily be seen since the function has only poles on the real axis, whereas a unitary amplitude should have cuts corresponding to thresholds of elastic and inelastic channels, while its poles corresponding to resonances should move off the real axis onto the unphysical Riemann sheet. For this reason, when taking the Regge limit of (15), we had to simulate this by artificially approaching infinity along a ray at an angle to the real axis. A connected question is that of the Pomeranchuk 'trajectory'.

On the one hand, we have good reason to believe that the Pomeron really represents the shadow effect of inelastic channels via unitarity. On the other, we know that the Pomeron is typically non-dual and therefore cannot be contained in such an amplitude as (15). Attempts have indeed been made to 'unitarize' the Veneziano model, but have as yet met with no success. For phenomenological purposes, one may simulate the effects of unitarity by artificially introducing an imaginary part to the trajectory function α. The poles of (15) will then move off the real axis, but their residues will now, in general, not be polynomials in cos θ, and our arguments given above will be only approximately valid.

In addition to the two main problems discussed above, there are further problems in the Veneziano model such as the parity-doubling of certain trajectories, which are not realized in nature. Though serious in themselves, they may not be quite as overwhelming in comparison with the two preceding ones.

In view of these difficulties, it is perhaps not clear to many people why some theoreticians are so enthusiastic about the model. The reason for this enthusiasm is twofold:

i) At least for certain idealized cases, the Veneziano model can be generalized to processes with any number of external lines (15). This generalized model, in addition to all the properties possessed by the original Veneziano model for four-line processes discussed above, has the further attractive feature of being consistent with the bootstrap hypothesis in the following sense: If one takes a bound state between two particles in the amplitude for an N-line process, the residue at the pole reduces to just the amplitude for an (N - 1)-line process. Although the difficulties mentioned previously still remain, it is nonetheless highly non-trivial to have a model that consistently treats all hadronic processes on the same basis. For this reason, the theoretical interest in the model is understandable.

ii) In the few cases where the Veneziano model has been applied seriously in phenomenology, the success has been impressive. It is true that in all such applications, the difficulties mentioned above were only either ignored or artfully avoided. One may thus dispute the ultimate significance of the agreement found with experiment. However, at this
stage of utter confusion in our knowledge of hadronic processes, the value of a concise formula such as this, which can summarize so much information in terms of so few parameters, should not be underestimated. As an example, I quote the fit of Petersson and Törnqvist\textsuperscript{16}) to the reaction $K^- p \rightarrow \pi^+ \pi^- \Lambda$ in terms of only one parameter, the normalization. Two diagrams from their paper are quoted in Figs. 10 and 11.

![Diagram]

Fig. 10 The energy dependence of the total cross-section and of the partial cross-sections for $Y_1^{\pm}$ production in the reaction $K^- p \rightarrow \pi^+ \pi^- \Lambda$ (Ref. 16).
Fig. 11 The percentage effective mass distributions at 3 GeV/c (Ref. 16).
FOOTNOTES AND REFERENCES

1) On the Veneziano model alone, the list compiled by Lovelace contains more than 400 references between Aug. 1968 and July 1969.


3) e.g. A. Della Selva, L. Masperi and R. Odorico, Nuovo Cimento 54 A, 979 (1968).

4) A. Della Selva et al., contribution to the 14th Int. Conf. on High-Energy Physics, Vienna (1968), paper 621.


6) For a general review of duality, see M. Jacob, lecture notes at the Schladming Winter School; CERN preprint, TH 1010 (1969).


8) C.B. Chiu and A. Kotanski, Nuclear Phys. 7, 615 (1968);
   ibid. 8, 553 (1969).


10) This may be just a reflection of the special mentality of physicists working on strong interactions.

11) See, for example, Chan Hong-Mo, K. Kajantie and G. Ranft, Nuovo Cimento 49, 157 (1967).


15) See, for example, Chan Hong-Mo, CERN preprint, TH 1057 (1969).