THEORETICAL ASPECTS OF HIGH ENERGY PHENOMENOLOGY

L. Bertocchi
CERN - Geneva

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I. REGGEIZATION OF HELICITY AMPLITUDES AND RELATED PROBLEMS

A. Amplitudes free from kinematical singularities; constraints

Probably most of the topics which, in my opinion, have gone through important developments in the theory of strong interactions at high energy during the last year are related to the problem of generalizing the Regge formalism to the general spin and mass situation.

Whenever one describes a high energy scattering by means of the exchange of objects having definite quantum numbers in the t channel, it is customary to use the helicity amplitudes, in terms of which it is simple both to discuss the t channel quantum numbers and express the density matrix (if, for instance, one of the final particles is a resonance).

Now the central problem in Reggeizing helicity amplitudes comes out: find a combination of t channel helicity amplitudes, which is free from kinematical singularities, namely possesses only singularities of dynamical origin, as for instance those required by the Mandelstam representation.

This problem has been partially solved by Hara 2) and Ling-Lie Wang 3); firstly, from the t channel helicity amplitudes, \( f_{\lambda_1, \lambda_3; \lambda_2, \lambda_4}^{(t)}(s, t) \), as defined by Jacob and Wick 4), one factorizes out a term, which is singular on the boundary of the physical region

\[
\frac{f_{\lambda_1, \lambda_3; \lambda_2, \lambda_4}^{(t)}}{s, t} = \left( \frac{\sin \Theta_t}{2} \right)^{|\lambda - \mu|} \left( \frac{\cos \Theta_t}{2} \right)^{|\lambda^* - \mu^*|} \frac{f_{\lambda, \lambda_3; \lambda_2, \lambda_4}}{s, t}
\]

where \( \lambda = \lambda_1 - \lambda_3, \mu = \lambda_2 - \lambda_4 \), and \( \Theta_t \) is the angle in the t channel. The rest of the kinematics is defined in Fig. 1.

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Then \( \bar{f}(t) \) has no kinematical singularities on zeroes in \( s \). The next step is to construct from \( \bar{f}^{(t)}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} \) the so-called "parity conserving" amplitudes

\[
\bar{f}^{(t)}(\pm \xi) \quad = \quad \bar{f}^{(t)}_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} + \bar{f}^{(t)}_{\lambda_1, \lambda_2, -\lambda_3, -\lambda_4}
\]

which get contributions only either from natural or unnatural parity exchanges in the \( t \) channel.\( ^6 \).

The \( \bar{f}^{(t)}(\pm \xi) \) still have kinematical singularities at \( t = 0 \) and at the thresholds and pseudo-thresholds of the \( t \) channel

\[
\mathbb{T}_{ij}^{(t)} = \left( m_i - m_j \right)^2
\]

Wang took care of those singularities writing

\[
\bar{f}^{(t)}(\pm \xi) = \mathbb{K}^{(t)} \bar{f}^{(t)}(\pm \xi)
\]

and gave a prescription for \( \mathbb{K}^{(t)} \). The new amplitudes \( \bar{f}^{(t)}(\pm \xi) \) are now free from any kinematical singularities both in the \( s \) and \( t \) channels, and are therefore suitable for Reggeizing. The point is that she correctly removed all the kinematical singularities in \( t \), and some, but not all, the kinematical zeroes in \( t \). If then one expresses the invariant amplitudes (which are supposed to be free from kinematical singularities) in terms of the \( \bar{f}^{(t)}(\pm \xi) \), these zeroes induce singularities in the invariant amplitudes. In order to remove these singularities, one has to impose some constraint equations, at the corresponding values of \( t \), between different \( t \) channel helicity amplitudes.

The general solution to this problem has been recently given by Cohen-Tannoudji, Morel and Navelet \( ^7 \) (CTMN); starting from the invariant spinor amplitudes, which have been shown by Joos \( ^6 \), Williams \( ^9 \).
and Hepp 10) to be free of kinematical singularities (on the basis of
axiomatic field theory), they have given a general recipe to construct
what they call "regularized helicity amplitudes", RHA, denoted by
\( \tilde{F}(s,t) \) (s, t) 11) (\( \{ \lambda \} \) denotes here all the set of the helicity
indexes), which they prove to be completely free from any kinematical
branch point, pole or even zero at the boundary of the physical region,
at the thresholds, pseudo-thresholds and \( t = 0 \) 12). The RHA are not,
however, all linear independent for any \( t \). This can be shown in the
following way.

One can relate the RHA in the \( t \) channel with the RHA in
the \( s \) channel; CTMN have shown that, if one writes

\[
\tilde{F}(s) = \sum_{\{ \lambda \}} \chi_{\{ \lambda \} \lambda'} \tilde{F}(t) 
\]

(1.5)

then the crossing matrix \( \chi_{\{ \lambda \} \lambda'} \) is a rational function of \( s \) and \( t \),
\( \chi = P/Q \), \( P \) and \( Q \) polynomials. But, since \( \tilde{F}(s) \) has no kinematical
singularities or zeroes in \( t \), the right-hand side has to be regular at
the zeroes of \( Q \); in order to be so, there must exist relations between
different \( t \) channel RHA at those zeroes (constraints).

CTMN have given a general solution also to this problem, relating
the RHA to the transversity amplitudes introduced by Kotanski 14),
which are amplitudes defined by the condition that the spins are quanti-
zed along the normal to the scattering plane. The transversity ampli-
tudes have the nice property of being quasi-diagonal under crossing; in
fact, one has

\[
F^{(t)}(s,t) = \varepsilon^{\alpha \{ \tau \} \{ \tau' \}} F^{(s)}(s,t) 
\]

(I.6)

where \( \{ \tau' \} \) denotes the set of transversity indices, and \( \alpha^{\{ \tau \} \{ \tau' \}} \)
is a (known) phase factor 15). Once related the RHA to the transversity
amplitudes, it is easy (but cumbersome!) to find the crossing relation
for the RHA.
From the general analysis of CTMN it follows that, in order to satisfy crossing and analyticity in each channel, there must exist definite linear relations (constraints) among t channel RHA and their derivatives at the thresholds and pseudo-thresholds of the t channel; these relations are listed in the most general case in the paper of CTMN 16).

It has been shown moreover very recently by Frautschi and Jones 18) that if one looks to the parity conserving helicity amplitudes, \( F_{(\pm)}^{(t)} \) 11), there exist constraints at \( t = 0 \) also when all the four masses are different, so that it does not coincide with a pseudo-threshold. The constraints in this particular case always connect two parity conserving helicity amplitudes corresponding to opposite natural parity.

We can give here some simple, well-known, examples of these constraints.

In the \( \bar{T} N \) scattering, the invariant amplitudes \( A \) and \( B \) are connected to the t channel helicity amplitudes by the relations 21)

\[
A = \frac{m}{p^2} \left[ \frac{1}{4} F_{++} + \frac{m}{2} (t - u) F_{+-} \right] \quad B = -2 m \ F_{+-} \tag{I.7}
\]

where \( p = (t - 4m^2)^{\frac{1}{2}} \) is the momentum of the \( \bar{N}N \) pair. Since \( A \) is regular at \( t = 4m^2 \), the relation

\[
F_{++} + \frac{m}{2} (t - u) F_{+-} = 0 \tag{I.8}
\]

must hold at \( t = 4m^2 \), for any \( s \).
Another well known example is the already old discovery by Volkov and Gribov \(^{22}\) that in nucleon-nucleon scattering there exist two constraints at \( t = 0 \) between the (in general independent) five \( t \) channel helicity amplitudes. The physical reason of these constraints is also clear: the value \( t = 0 \) is the forward direction of the \( s \) channel, where, in order to conserve angular momentum, only three amplitudes are non vanishing. In order for the remaining two to vanish, there must exist two constraints among the five \( t \) channel helicity amplitudes.

Other simple examples concerning the discussion of the vector meson production and of the photoproduction are given in recent papers by Hogaasen and Salin \(^{23}\) and by Diu and Le Bellac \(^{24}\).

Now, what is the relevance of these constraints for Regge theory? In general, the constraint equations on the \( t \) channel amplitudes connect amplitudes with different quantum number in the \( t \) channel, and which are therefore dominated by different Regge poles. A possible solution of the constraint equations is therefore that at some particular points there must exist relations between different Regge poles (conspiracy).

B. - Constraint equations and Regge theory

Let us now come to the problem of the implications of the constraint equations at the threshold and pseudothreshold on Regge theory.

This problem has been studied by many people \(^{25}\), and in a general way, by Leader \(^{26}\).

Leader classifies three main types of solutions of the constraint equations:
a) conspiracy: there exist relations between trajectories of different Regge poles with different quantum numbers, valid at the constraint points;

b) evasion: the constraint equations are satisfied enforcing conditions on different residuum functions, but not trajectories;

c) daughters: the constraints are satisfied by requiring the existence of sequences of Regge poles with the same quantum numbers, but different (and related) trajectories.

In the evasive solution, in order to avoid singularities, in general a large number of amplitudes do vanish at the constraint points, so that the general kind of predictions is that many helicity amplitudes do not contribute there. (Of course, the prediction is directly testable only if the constraint point is on the boundary of the physical region, as in the equal mass problem.)

Generally speaking, Leader has shown that, for boson Regge trajectory, there is never the necessity of conspiracy, and one can always have an evasive solution; on the contrary in the Fermion case, there is always the necessity of a conspiracy at $t = 0$, which is nothing else that the well-known Gribov phenomenon that in the $t < 0$ region the fermion trajectories have to occur in pairs of opposite parity complex conjugate trajectories.

Which mechanism is actually chosen in a particular case has to be decided from the analysis of the experimental prediction of the different possibilities.
C. Unequal masses; daughter trajectories

The existence of daughter trajectories is, however, a more general phenomenon, and is not only connected with the problem of the constraint equations. In fact, in the general mass problem, their presence is required even if the particles are spinless, again in order to satisfy analyticity.

The root of this new difficulty lies in the fact that, in the unequal mass case, the cosine of the angle in the crossed channel does not always grow with \( s \), but in some interval it remains bounded.

Historically, the first problem in which this difficulty was recognized is the backward pion-nucleon scattering, but the problem is much more general.

We shall consider as an example the scattering of two particles of mass \( m_a \) and \( m_b \),

\[
A + B \rightarrow C' + A'
\]

for a small value of the momentum transfer \( t = (p_B - p_A)^2 \) the relation between \( \cos \Theta_t \) and the Mandelstam variables is

\[
\cos \Theta_t = - \left[ 1 + \frac{1}{t^2 - 2 t (m_a^2 + m_b^2) + (m_a^2 - m_b^2)^2} \right]
\]

(I.9)

It is easy to verify that, for \( 0 \leq t \leq t_0 = (m_a^2 - m_b^2)^2/s \) one has \( |\cos \Theta_t| \leq 1 \) (this interval lies inside the physical region); in particular

\[
\cos \Theta_t (t = 0) = 1
\]

(I.10)

for any value of \( s \).
If now one writes down the contribution of a Regge pole to the scattering amplitude, expanding the Legendre polynomial of $\cos \theta_t$ and ordering the terms in powers of $s$, one finds that the leading term of this expansion is well of the form $\int \beta_0(t) \, s^{\alpha(t)}$, with $\beta_0(t)$ regular at $t = 0$, but the next-to-the-leading terms $\int \beta_n(t) \, s^{\alpha(t)-n}$ for $n = 1, 2, \ldots$ have residues $\beta_n(t)$ which are singular at $t = 0$. However, the complete amplitude is supposed to satisfy Mandelstam analyticity, and therefore has to be regular at $t = 0$.

The solution of this puzzle, given by Freedman and Jiung-Ming Wang [27], is that, in order to recover analyticity, there must exist other trajectories, passing through $\alpha_n = \alpha(0) - n$ at $t = 0$, whose contribution just cancels the singular behaviour of the next-to-the-leading terms which have the same power in $s$.

These trajectories are called daughter trajectories (the trajectory passing through $\alpha(0)$ at $t = 0$ being the parent); they enjoy the following properties:

a) their angular momentum differs by an integer $n$ from the angular momentum of the parent at $t = 0$; this integral spacing is not necessarily true for $t \neq 0$;

b) they have all the internal quantum numbers, as isospin, baryon number, hypercharge, of the parent trajectory;

c) the signature and the parity of the $n$-th order daughter are related to those of the parent by the relations

\[ \tau_n = (-1)^n \tau_0 \quad \rho_n = (-1)^n \rho_0 \]

(I.11)

namely, they have alternating signature and parities.

Now, since the properties of the Regge trajectories are independent on the external masses to which they are coupled, this means that the daughter trajectories can be always present, also in the equal mass case, even if their residuum is no longer singular (unless they are not coupled to a particular two-particle state, since the relevant residuum vanishes).
D. Lorentz symmetry at $t = 0$; Toller families

The general problem of constraints, daughters and conspiracies can be also looked from a much more general point of view, at least in the pairwise equal mass case $(m_1 = m_3, m_2 = m_4)$, where at $t = 0$ the four momentum exchanged in the crossed channel vanishes identically $(P \mu = 0)$.

From a group theoretical point of view, this point has a particular symmetry, since the scattering amplitude at $P \mu = 0$ is invariant under the little group of the general Poincaré group belonging to $P \mu = 0$, which is (isomorphic to) the homogeneous Lorentz group $O(3,1)$. One can then expand the scattering amplitude into irreducible representations of this group [as in the usual $O(3)$ invariance one develops in Legendre polynomials or in three-dimensional rotation functions]. Then one "Reggeizes" this expansion by means of a sort of Sommerfeld-Watson transform.

This theory has been developed in a series of papers by Toller et al. [29] for the group $O(3,1)$; the same results have been later obtained by Freedman and Wang [30] and by Domokos [31] using the simpler compact group $O(4)$ as the symmetry group; all the results which have been explicitly obtained are the same [32].

As in the three-dimensional problem one assumes that the asymptotic behaviour of the scattering amplitude when $\cos \theta \to \infty$ is governed by a pole in the complex angular momentum plane (Regge pole), whose residuum is proportional to the Legendre polynomial of complex index $\alpha$, one assumes now that, at $t = 0$, the asymptotic behaviour of the scattering amplitude is governed by a pole in the complex plane of a sort of "four-dimensional angular momentum", which will again be denoted by $\alpha$ (Toller pole), whose residuum is proportional to the continuation to complex $\alpha$ of the irreducible representations of $O(3,1)$.
Then the first immediate result of this analysis is the existence of daughter trajectories. In fact, any irreducible representation of integer order $\mathbb{N}$ of the larger group $\mathfrak{g}$ we shall refer, for simplicity, the $O(4\mathbb{N})$, which are the Gegenbauer polynomials, can be expressed in terms of the normal Legendre polynomials of all the integer orders from $0$ up to $N$. If $\alpha$ is not integer, then the Gegenbauer polynomial corresponds to an infinite sequence of Legendre polynomials or orders

$$j_n = \alpha - n, \quad n = 0, 1, 2, \ldots$$

Therefore any Toller pole at $\alpha$ contains an infinite family of Regge poles, with $j_n = \alpha - n$. The Regge pole at $j = \alpha$ can be called the parent trajectory of the Toller family, the Regge poles at $j_n = \alpha - n$ the daughter trajectories of order $n$.

We shall now summarize here the main results of Toller's analysis.

The asymptotic behaviour of the scattering amplitude for $s \to \infty$ at $t = 0 \ (P \mu = 0)$ is governed by the exchange of Toller poles. Toller poles are characterized by the following set of quantum numbers (apart from internal quantum numbers, as isospin, hypercharge, and so on):

1) A Lorentz quantum number, $M$, which can take the values

$$M = 0, \frac{1}{2}, 1, \frac{3}{2}, 2, \ldots \quad (I.12)$$

Integer values of $M$ correspond to boson trajectories, half-integer to fermions. The possible values of $M$ for the Toller poles which can couple to the external particles are restricted by the selection rule

$$M \leq \min \left( j_1 + j_3, j_2 + j_4 \right) \quad (I.13)$$
where \( j_1, j_2, j_3, j_4 \) are the spins of the initial (final) particles of the scattering in the \( t \) channel. The physical meaning of \( M \) is related to the helicity change between particles 1 and 3 or 2 and 4. We can easily check that for fermion trajectories both \( j_1 + j_3 \) and \( j_2 + j_4 \) are half-integer so that \( M \) half-integer corresponds well to fermion exchange.

2) The Lorentz signature \( \Upsilon_\ell \), which is related to the behavior of the amplitude with respect to the CPT operation, and can assume the values \( \Upsilon_\ell = \pm 1 \).

3) In the case of \( M = 0 \), a quantum number \( \sigma \) related to the natural parity \( (\sigma = +1 \) corresponds to the natural parity, \( \sigma = -1 \) to the unnatural parity); for \( M \neq 0 \), \( \sigma \) does not commute with the other quantum numbers.

4) A complex parameter \( \alpha \), which can be called the "four-dimensional angular momentum", and is such that the dominant asymptotic behavior of the amplitude is \( s^\alpha \).

Always using the group properties one can derive the following results:

a) any Toller pole at \( \alpha \) gives rise to an infinite family of Regge poles at \( j_n = \alpha - n \) (daughter trajectories of order \( n \)); they can have different quantum numbers from the parent, as the parity and charge conjugation; the Lorentz signature \( \Upsilon_\ell \) coincides with the usual Regge signature of the parent, \( \Upsilon_\ell = \Upsilon_\ell(0) \); moreover the Regge signatures of the daughters are given by \( \Upsilon_\ell(n) = (-1)^n \Upsilon_\ell \).

b) all the residuum functions of the daughter trajectories do satisfy factorization, if the parent Toller trajectory does \( \Upsilon_\ell \).

c) some definite relations exist among different Regge trajectories corresponding to the same Lorentz pole (conspiracy). These relations follow from the fact that one unique Toller pole determines all the properties (trajectories and residuum functions) of all the Regge trajectories in the family. These relations satisfy the general constraints found by CTMN, at \( t = 0 \), which is a pseudo-threshold of the problem (remember that in order to have \( P = 0 \) one needs pairwise equal masses).
From Toller's analysis one can then conclude the existence of daughters and conspiracies, at least at $t = 0$.

Let us now consider in more detail Toller's results. We shall consider, to be definite, only processes in which at least the 1-3 vertex is a nucleon-nucleon vertex, since this is the actual situation in all the feasible experiment.

Since $J_1 + J_2 = 1$, only Toller families with $M = 0$ and $M = 1$ can be coupled with the NN pair at $t = 0$. One finds therefore the following three classes of Toller families (remember that for $M = 0$, $\varphi$ is a good quantum number; moreover, in each family we can have the two possibilities for the Lorentz signature, $\tau = \pm 1$).

**CLASS I : $M = 0$, $\varphi = +1$**

By definition, the family contains only particles of natural parity.
A Toller pole of this family yields only one series of Regge poles, spaced by two units of angular momentum $34)\,$

$$J_n = \chi, \chi - 1, \chi - 2, \ldots$$

All these poles have the same parity, signature and charge conjugation, given by

$$\tau = \zeta = \rho = \tau_l$$

(1.14)

For the daughters of odd order $n$, which are not coupled to the equal mass system, the relation would be

$$\tau_n = -\zeta_n = \rho_n = (-1)^n \tau_l,$$

namely, for $n$ odd, $\zeta = \tau_l = -\rho = -\tau_l$.

The ratio of the residuum function between the daughter of order 2 and the parent turns out to be (at $t = 0$)

$$\frac{\beta^{(1)}}{\beta^{(0)}} = \frac{2 \alpha + 1}{\chi \alpha}$$

(1.15)

This relation, in Leader's language, would be a daughter-like one.
All the particles which couple to an equal boson-boson pair at 
\( t = 0 \) necessarily belong to this class, since \( j_2^1 + j_4^1 = 0 \) which 
forces \( M = 0 \), and moreover the parity of the boson-boson pair is 
\((-1)^{l+1}\), namely the natural parity, and hence \( \sigma = +1 \).
Moreover, also in the case in which also particles 2 and 4 have 
spin \( \frac{1}{2} \), only particles in this class can contribute, through the 
optical theorem, to the spin-averaged total cross section.
Regge trajectories such as the \( P, P^1, g, \omega, A_2 \), which do con-
tribute to the forward coherent amplitude of the \( B + F \rightarrow B + F \) pro-
cesses, have therefore to be classified in Class I. They have all 
the same \( M = 0, \sigma = +1 \), and differ either by \( \tau \) (as \( g \) and \( A_2 \)), 
by isospin (as \( g \) and \( \omega \)) or simply by \( \alpha \) (as \( P \) and \( P^1 \)).

**CLASS II : \( M = 0, \sigma = -1 \)**

This class contains therefore only families of unnatural parity.
A Toller pole at \( \alpha \) gives rise to two different series of Regge 
poles, one with \( n \) even
\[
\mathcal{J}_n = \alpha, \alpha^{-1}, \alpha^{-2}, \ldots
\]
with quantum numbers
\[
\zeta = \rho = -\tau = -\tau_{\ell} \quad \text{(I.16)}
\]
the second one for \( n \) odd
\[
\mathcal{J}_n = \alpha^{-1}, \alpha^{-3}, \alpha^{-5}, \ldots
\]
with quantum numbers
\[
\zeta = -\rho = \tau = -\tau_{\ell} \quad \text{(I.16')}
\]
The relation between residuum functions relates Regge poles of the 
first series \((n \text{ even})\) with the Regge poles of the second series 
\((n \text{ odd})\). The relation is therefore between Regge poles of the 
opposite Regge signature and parity, but same charge conjugation. 
The first relation is
\[
\beta^{(4)} / \beta^{(0)} = \frac{2 \alpha + 1}{\alpha (\alpha + 1)} \quad \text{(I.17)}
\]
This relation is a conspiracy, which connects Regge trajectories of angular momentum spaced by a unity, opposite Regge signature and parity but same charge conjugation.

A possible assignment of existing particles in Class II is a particle like the $A_1$, which has $P=+1$, $\mathcal{C}=-1$, $C=+1$. It would be the parent of a Toller family of $\mathcal{C}=1$ whose first daughter would have the quantum numbers $P=-1$, $\mathcal{C}=+1$, $C=+1$, and would therefore be a $J^{P_c} = 0^{-1}$ particle [the $\pi'(1640)$ meson?].

Another possibility is the B meson ($J^{P_c} = 1^{+1}$), which has $P=+1$, $C=G(-1)^I=-1$, $C=-1$, so it would be an odd daughter of a family with Lorentz signature $\mathcal{C}=+1$.

**CLASS III: $M=1$**

Now $\mathcal{C}$ is no more diagonal with $M$, and the class contains parity doublets, namely every Toller pole contains two families of Regge poles with all the same quantum numbers except opposite parity (and with the same $\mathcal{C}$ at $t=0$). The complete assignments of quantum numbers for $\mathcal{C}=\pm 1$ and for even and odd $n$ is shown in Table I.

Moreover, Class III, $c$ (odd $n$ with natural parity and $P=-C$) cannot be coupled to the NN pair.

The constraint equations are again conspiracies, which connect in general both Regge trajectories inside the same parity doublet with $n$ even, and trajectories in the other parity doublet with $n$ odd. The first relation contains only the two Regge trajectories in the parent parity doublet, and is

$$\beta^{(o)}_\alpha/\beta^{(o)}_\alpha = \frac{\alpha^{+1}}{\alpha}$$  (I.18)

The second relation connects the parity doublet with $n=2$ and the trajectory with $n=1$, $\mathcal{C}=-1$.

A possible assignment of particles in Class III is that the pion ($\pi$) belongs to a parity doublet, belonging then to the Subclass III, $b$. This implies the existence of a trajectory ($\Pi_a$) with $Q_{\Pi_a}(0) = Q_{\Pi}(0)$, one unit of isospin and quantum numbers corresponding to Subclass III, $b$, namely $P=C=\mathcal{C}=+1$. 

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The fact that this partner of the pion in the parity doublet is not known, at least with a mass near to $\mu^2$, is of course a difficulty, but could possibly be explained \(^{35}\) either by assuming that at $\gamma^a = 0$ the $\Gamma^a$ chooses nonsense (see later section II), or by supposing that the $\Gamma^a$ trajectory is very flat, so that $\gamma^a = 0$ is reached only at a very big mass \(^{2}\) the $\Pi^\nu(1030)$ meson \(^{2}\) we stress again that the $0(3,1)$ symmetry relates $\gamma^a$ to $\gamma^a$ only at $t = 0$.

Apart from the difficulty of the non-existence of a light $\Pi^a$, there is another argument against the assignment of the pion to Class III, \(^{36}\).

The argument is that the pion mass is so small, that one could possibly put $\gamma^a = 0$ at $t = 0$, namely one could neglect the pion mass, as in the usual soft pion philosophy in current algebra.

But then the pion, being a particle of mass 0 at $t = 0$, would not be classified in $M = 1$, since this representation of the $0(3,1)$ group does not contain the spin 0 representation of the $0(3)$ rotation group.

If this argument is valid, the pion could then be classified in Class II, $M = 0, \Omega^- = 1$. However, also this assignment meets a difficulty, since the quantum numbers of the pion

$$\zeta = \gamma = -\rho = 1$$

correspond to those of an odd daughter of Class II, so the pion should be at least the first daughter of a particle with quantum numbers

$$J = 1, \quad \gamma = -1, \quad \epsilon_0 = \rho = 1$$

which should have approximatively the same mass as the pion, which again is not known.

There is therefore a doubt whether the pion should be classified in a Toller family at all, or if it decouples from any particle at $t = 0$.  

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Another possible assignment is that the B particle, instead of belonging to Class II as discussed before, belongs to Class III, b, having \( \gamma = C = P = -1 \). Then his partner in the parity doublet would have the quantum numbers \( \gamma = C = P = -1 \), namely the same quantum numbers as the \( \xi \), so a possible candidate could be the \( \xi' \), whose presence is often invoked to explain the polarization in \( \eta^- p \rightarrow \eta^0 n \) charge-exchange.

It has to be stressed, however, that if the \( \xi' \) belongs to Class III, b, it cannot be coupled to the bosons at \( t = 0 \), but only at \( t \neq 0 \), due to the \( O(3,1) \) symmetry breaking. It could then explain the \( \eta^- p \rightarrow \eta^0 n \) polarization, but would not contribute to the total cross-sections.

E. - Phenomenological implications of conspiracies and daughters

Till now, we have only introduced conspiracies and daughters in order to preserve analyticity and crossing.

However, very important physical consequences can be derived from the existence of daughters and conspirators.

The most striking feature is to allow certain helicity amplitudes to be important in the forward direction, while usually kinematical factors require them to be small or to vanish.

The first example we shall consider is the \( \eta^- \) exchange contribution to \( np \) charge exchange scattering \(^{35}\).

From its quantum numbers, the pion can contribute to the following s channel helicity amplitudes (see Fig. 2)

\[
\phi_\perp = \langle ++ | --- \rangle \\
\phi_\parallel = \langle + - | - + \rangle \\
\text{(I.19)}
\]
Moreover, the $\Pi$ contribution is such that

$$\phi_2^{\Pi} = \phi_4^{\Pi} \quad (1.20)$$

Now, the amplitude $\phi_4$ is an helicity flip amplitude, so it has to vanish at $t=0$ from angular momentum conservation. The other amplitude, $\phi_2$, which corresponds to no net helicity flip, is not constraint to vanish at $t=0$; however, the previous equality says that, since $\phi_4^{\Pi} = 0$, also $\phi_2^{\Pi} = 0$. As a conclusion, the complete $\Pi$ contribution vanishes at $t=0$.

Suppose, however, that there exists a parity doubling brother of the pion, the $\Pi_a$; it would again contribute only to $\phi_2$ and $\phi_4$, but these contributions are now related by

$$\phi_2^{\Pi_a} = -\phi_4^{\Pi_a} \quad (1.21)$$

due to the positive parity of the $\Pi_a$.

The combined contributions of $\Pi$ and $\Pi_a$ are now

$$\phi_2 = \phi_2^{\Pi} + \phi_2^{\Pi_a} \quad \phi_4 = \phi_4^{\Pi} + \phi_4^{\Pi_a} = \phi_2^{\Pi} - \phi_2^{\Pi_a} \quad (1.22)$$

One can then impose $\phi_4 = 0$ by choosing $\phi_2^{\Pi} = \phi_2^{\Pi_a}$, without obtaining $\phi_2 = 0$.

Of course, in order to have an effective cancellation between the $\Pi$ and the $\Pi_a$ contribution at all the energies, one must have an equality between the $\Pi$ and the $\Pi_a$ trajectories at $t=0$ and a relation between the residuum functions, namely a conspiracy.

It is interesting to analyse in detail how the conspiracy allows a non-zero contribution at $t=0$. 
The constraint equation (Volkov-Gribbov)\(^{22}\) is \(\phi_4 = 0\). If we have \(\phi_4 = \phi_4^{-}\)\(_\alpha\), then \(\phi_4^{-}\)\(_\alpha\) must vanish when \(t \to 0\), let us say as \(t\); the constraint is evaded. As a consequence, also \(\phi_2 \sim t\). We have

\[
\phi_2^{-\alpha} = \phi_4^{-\alpha} = C^{-\alpha} t
\]

(1.23)

\(C^{-\alpha}\) constant for \(t \to 0\).

If, however, we choose the parity doublet, the constraint can be satisfied by a conspiracy between different \(\tau\) channel amplitudes (\(\tau_\alpha\) and \(\tau_\alpha^\dagger\) correspond to different quantum numbers in the \(\tau\) channel). Then, defining as before the contribution of the \(\tau_\alpha\) as

\[
\phi_2^{-\alpha} = -\phi_4^{-\alpha} = C^{-\alpha} t
\]

we can choose a singular behaviour \(C \sim K/t\) of both \(C^{\tau_\alpha}\) and \(C^{\tau_\alpha^\dagger}\) when \(t \to 0\), and satisfy the constraint, provided that only \(K^{-\alpha} = K^{\tau_\alpha}\). The conspiracy allows separate amplitudes in the \(\tau\) channel to behave in a "more singular way" at the conspiracy point.

We shall come back on the problem of np charge-exchange later. We can notice, however, that in this problem conspiracy involves only some of the helicity amplitudes which could be present at \(t = 0\) (as the non-flip-non-flip amplitude), so that in order to test experimentally the conspiracy one would need difficult polarization measurements.

There are, however, other striking predictions of the conspiracy theory, which could be tested more directly; they are:

a) In absence of conspiracy, and if it is dominated by \(\tau\) channel exchanges, the pion photoproduction differential cross-section must have a dip in the forward direction. This result, which was discussed first by Dill and Le Bellac\(^{24}\), and then derived in quite a general way by Drell and Sullivan\(^{37}\), both for separate natural and unnatural parity exchanges, is valid whenever one can neglect s-channel contributions but would not be true if different natural parities are exchanged at the same time in the \(\tau\) channel.\(^{38}\)
The origin of this property is that one cannot transmit to the pion the helicity $|\lambda| = 1$ of the photon, which, in the forward direction, has to be absorbed by the proton. The only amplitude which, from angular momentum conservation, could be different from zero at $\theta_s = 0$ is therefore the "flip-flip" amplitude, which, however, is depressed near the forward direction by kinematical factors.

Again conspiracy at $t = 0$ (which at finite energy does not correspond to $\theta_s = 0$) can allow the flip-flip amplitude to survive in the forward direction, allowing different trajectories to satisfy a constraint equation (for instance, in charged pion photoproduction they could be a conspiring $\pi^- \pi^0$ parity doublet).

Photoproduction is a happy situation in this respect, since the "non-flip-non-flip" amplitude is not present, due to the transverse character of the photon, and so cannot mask the effect of conspiracy on the "flip-flip" amplitude.

The situation is, however, complicated by the gauge invariance; it is well known that gauge invariance imposes definite relations between the otherwise independent amplitudes which describe photoproduction, and so induce additional complications in the Reggeization of pion photoproduction. This problem has been fully analysed in a recent paper by Ball and Jacob, who have explored all the combined effects of gauge invariance, transversity of the photon and constraint relations on photoproduction.

In the framework of the $O(4)$ symmetry, the precise mechanism of conspiracy in pion photoproduction has been recently studied by Mitter (see also Ref. 19). Strictly speaking, $O(4)$ symmetry applies only to $P_M = 0$, which does not correspond to $t = 0$ in the unequal mass case. However, it is possible to show 41) that the conspiracy in the $\overline{NN}$ vertex (where $O(4)$ applies, since the masses are equal), just satisfies the constraints at $t = 0$ in any process in which a $\overline{NN}$ vertex is present, as in photoproduction.
Mitter was able to show that in forward photoproduction the only possible conspiracy is a Class III conspiracy (parity doublet); this can be also simply understood noticing that the quantum number \( M \) is connected with the helicity change, and here

\[
|\lambda' - \lambda''| = 1
\]

(1.24)

Very recently\(^\text{42}\), experimental evidence was presented that at high energy (till 10 GeV) and at extremely small momentum transfer (till \( 10^{-4}\) GeV/c\(^2\)) there is no dip in charged pion photoproduction near the forward direction (except possibly at the highest energy); moreover, the differential cross-section at fixed \( t \) goes down as \((d\sigma/dt) \propto s^{-2} \alpha(t)^{-2}\) with \( \alpha(t) \sim 0 \) for all \( t \) (which means a very flat "Regge trajectory", resembling a fixed pole at \( j=0 \)).

This absence of a dip can always be described as the effect of a conspiracy in a general sense, namely that different parities are exchanged at the same time in the \( t \) channel; this does not mean necessarily that there is conspiracy among different Regge trajectories, since a \( s \) channel contribution will produce of course the same effect. This problem will be fully discussed in the report by Panofsky at this Conference.

b) A similar problem is the forward production of vector mesons from pseudoscalar mesons in the reaction

\[
\pi^+ + N \rightarrow \gamma^+ + N
\]

(1.25)

Here again, in absence of conspiracies, only transverse helicities of the vector mesons survive at \( \Theta_s = 0 \), all the other helicity amplitudes being either zero (when there is a net helicity flip) or depressed by the kinematical half-angle factors \((\sin \frac{\Theta_t}{2}) |\lambda' - \lambda''|\) or \((\cos \frac{\Theta_t}{2}) |\lambda' + \lambda''|\), as the flip-flip amplitude.
It has been shown moreover by Frautschi and Jones \(^{18}\) (see also \(^{23,24}\)) that the helicity amplitude \(\rho(t)_{0,0;1,1}^{B}\), which is not directly depressed by the half-angle factors, is linked to the reduced helicity amplitude \(\rho(t)_{0,0;1,1}^{\frac{1}{2},-\frac{1}{2}}\) by the constraint

\[
\rho(t)_{0,0;1,1}^{\frac{1}{2},-\frac{1}{2}} = 2 \rho(t)_{0,0;1,1}^{\frac{1}{2},\frac{1}{2}}
\]  

(1.26)

and that, in absence of conspiracy, both \(\rho(t)_{0,0;1,1}^{\frac{1}{2},\frac{1}{2}}\) and \(\rho(t)_{0,0;1,1}^{\frac{1}{2},-\frac{1}{2}}\) vanish at \(t = 0\) as \(t^\frac{1}{2}\). This result comes from the fact that the two amplitudes receive contributions from different quantum numbers exchanges in the \(t\) channel, so that in absence of conspiracy each of them has to vanish separately. Therefore, in absence of conspiracy, all amplitudes are depressed near the forward direction.

The conspiracies in this problem have been studied by Sawyer \(^{43}\), using again \(O(4)\) symmetry. Since \(j_{\pi} + j_{\nu} = 1\), but the vector meson is not necessarily transverse, both Class II and Class III can be present (Class I does not contain conspirators, and so is not considered).

Since Class II contains only unnatural parity trajectories, and the natural parity of the produced vector meson is opposite to that of the pion, parity conservation allows to have helicity amplitudes with \(\lambda_{\nu} = 0\) \(^{44}\); this amplitude is predicted to be the dominant one at \(\Theta_{S} = 0\) when \(s \to \infty\). On the contrary, if the conspiracy is of Class III, one predicts the dominance near \(\Theta_{S} = 0\) of the amplitudes with \(\lambda_{\nu} = \pm 1\). Sawyer \(^{45}\) has also computed a table of contributions, at the leading order in \(s\), of the possible conspiratorial families with isospin equal to one and of both Lorentz signature in different forward reaction of nucleons (Table II).
From the analysis of this table, one can conclude:
- the analysis of the angular distribution of the vector meson production allows to conclude that, if no dip is present near $t = 0$, a conspiracy is effective;
- in the case of no dip, the analysis of the density matrix elements at very small $t$ allows to distinguish between Class II conspiracy ($g_{00} \sim 1$, Re $g_{01}$ and $g_{1,-1} \sim 0$) or Class III ($g_{00}$ and Re $g_{01} \sim 0$, $g_{1,-1} \sim 1$), and to choose the Lorentz signature of the conspiring family.

That Class II and Class III conspiracies give such different results in these reactions can also be understood from a more general point of view. The constraint equations to be satisfied are the following \(^{18),23),24)}\)

\[
\begin{align*}
\mbox{I.26} & \quad i f_0,0; t, -t = 1 f_0,0; t, t, \frac{1}{2} \\

f_{t,0; \frac{1}{2}, \frac{1}{2}} & = f_{t,0; \frac{1}{2}, -\frac{1}{2}} - i f_{t,0; \frac{1}{2}, \frac{1}{2}} - i f_{t,0; \frac{1}{2}, -\frac{1}{2}} \\
\end{align*}
\]

Only unnatural parity poles can contribute to the amplitudes in the first relation, which is therefore satisfied either by an evasion or by a Class II conspiracy. The second relation, in which the vector meson has helicity $\pm 1$, is satisfied either by an evasion or by conspiracy of Class III.

Let us look in more detail the first relation (I.26). The maximum singular behaviour at $t = 0$ allowed by the analyticity (Wang's factor) is \(^{18),24)}\) $t^{-\frac{3}{2}}$ for both $f_{0,0; \frac{1}{2}, \frac{1}{2}}$ and $f_{0,0; \frac{1}{2}, -\frac{1}{2}}$. However, in absence of conspiracy, (I.26) is satisfied requiring that both $f_{0,0; \frac{1}{2}, \frac{1}{2}}$ and $f_{0,0; \frac{1}{2}, -\frac{1}{2}}$ vanish at $t = 0$ as $t^{\frac{3}{2}}$.

Let us now consider the conspiratorial solution, which is obtained by requiring
\[ f_{\omega, \omega; \frac{1}{2}, \frac{1}{2}}(t) = \frac{a}{t^{\frac{3}{2}}} \quad j \cdot f_{\omega, \omega; -\frac{1}{2}, -\frac{1}{2}}(t) = -2i \frac{a}{t^{\frac{3}{2}}} \quad t \to 0 \]  

(I.28)

This conspiracy is exactly a Class II conspiracy. In fact, we remember first of all that the relation between the \( f(t) \) and the complete t channel helicity amplitudes is

\[ f_{\omega, \omega; \frac{1}{2}, \frac{1}{2}}(t) = \frac{f_{\omega, \omega}(t)}{f_{\omega, \omega; 0, 0}} \quad j \cdot f_{\omega, \omega; -\frac{1}{2}, -\frac{1}{2}}(t) = \sin \Theta_c \cdot f_{\omega, \omega; 0, 0}(t) \]  

(I.29)

Moreover, the quantum numbers of the two amplitudes are such that, for instance, a particle with the quantum numbers of the \( A_1 \) can contribute to \( f_{\omega, \omega; 0, 0; \frac{1}{2}, -\frac{1}{2}}(t) \) and a particle with the quantum numbers of the pion to \( f_{\omega, \omega; 0, 0; \frac{1}{2}, \frac{1}{2}}(t) \). These are just the quantum numbers of the parent \( (A_1) \) and first daughter \( (\pi) \) in Class II.

Remembering that the amplitude \( f_{\omega, \omega; 0, 0; \frac{1}{2}, -\frac{1}{2}}(t) \) is a spin-flip amplitude, we can reggeize \( f_{\omega, \omega; 0, 0; \frac{1}{2}, \frac{1}{2}}(t) \) and \( f_{\omega, \omega; 0, 0; \frac{1}{2}, -\frac{1}{2}}(t) \) writing

\[ f_{\omega, \omega; \frac{1}{2}, \frac{1}{2}}(t) = \beta_{\pi}(t) \quad j \cdot f_{\omega, \omega; -\frac{1}{2}, -\frac{1}{2}}(t) = \sin \Theta_c \cdot \beta_{A_1}(t) \quad \alpha_{\omega A_1}(t) \]  

(I.30)

where \( \alpha_{A_1} \) is the sense-nonsense factor. The constraint equation is satisfied near \( t = 0 \) taking

\[ f_{\omega, \omega; \frac{1}{2}, \frac{1}{2}}(t) = -2 \sin \Theta_c \cdot i \cdot \alpha_{A_1}(t) \quad j \cdot f_{\omega, \omega; -\frac{1}{2}, -\frac{1}{2}}(t) = \alpha_{A_1}(t) \quad \omega_\pi(0) = \alpha_{A_1}(0) - 1 \]  

(I.31)

\[ \omega_\pi(0) = \alpha_{A_1}(0) - 1 \]  

(I.31')
From this formula, it seems that these contributions are at high s only of the order of \( s^{\alpha A_1^{-1}} \). How can then be that Sawyer finds contributions of the order \( \alpha A_1 s^{\alpha A_1} \)? The answer is that one has to perform the limit \( s \to \infty, t \to 0 \) along the curve \( \theta_s = 0 \), namely the forward direction. From the kinematical relations one finds that \( \lim_{s \to \infty} \sin \theta_t = 0 \), but along this curve \( s \sim t^{-\frac{1}{2}} \), so that the factors \( t^{-\frac{1}{2}} \) in both \( \frac{f_0(t)}{t} \) and \( \frac{f_0(t)}{t} \) give an enhancement factor \( \sim t \), which allows both of these amplitudes to grow at \( \theta_s = 0 \) like \( \alpha A_1 s^{\alpha A_1} \); however, from angular momentum conservation, \( f_0(t) \) vanishes at \( \theta_s = 0 \) from the presence of the \( \sin \theta_t \) factor. Since in the forward direction in the unequal mass case the crossing matrix is diagonal, the s channel amplitude \( \langle 0^\frac{1}{2} | T | 0^\frac{1}{2} \rangle \) will grow as \( \alpha A_1 s^{\alpha A_1} \). It is interesting to notice that Frautschi and Jones have verified that the exchange of an elementary \( A_1 \), whose propagator contains a part which behaves as \( 1^+ \) and another which behaves as \( 0^- \), satisfies the constraint (I.26) by conspiracy and contributes fully to the non-spin flip amplitude in the forward direction.

The predictions of the conspiracies of Class III (parity doublets) have been studied, using the factorization principle, by Fox, Leader and Rogers, in a paper presented to this Conference, for the reactions of the kind

\[
\bar{B} + N \to V + N, \quad \bar{B} + N \to V + A
\]  

(I.32)

where \( B \) is a pseudoscalar meson, \( N \) a nucleon, \( V \) a vector meson and \( A \) a spin \( \frac{3}{2} \) resonance. They have given detailed predictions and fits for what concerns both the angular distributions and the density matrix elements near \( t = 0 \).

An interesting application of the pion parity doublet to the reaction

\[
\pi + N \to \gamma + A
\]

(I.33)
has been also recently proposed by Le Bellac 47). He has noticed
that one can consider together the reactions

\[ a) \; \mathcal{N}\mathcal{N} \rightarrow \mathcal{N}\mathcal{N} \quad b) \; \pi \pi \rightarrow \mathcal{N}\mathcal{N} \quad c) \; \pi \mathcal{N} \rightarrow \mathcal{N}\mathcal{N} \quad (I.34) \]

The pion contributions to these reactions are linked by
factorization

\[ \left( \beta_{c,\frac{1}{2};0,0}^2 \right) = \beta_{b,0;0,0} \cdot \beta_{a,\frac{1}{2};\frac{1}{2},\frac{1}{2}}^b \quad (I.35) \]

In the conspiratorial solution, \( \beta_{\frac{1}{2},\frac{1}{2};0,0}^c \) is finite at
\( t=0 \). In the same hypothesis (parity doublet conspiracy, Class III),
\( \beta_{\frac{1}{2},\frac{1}{2};0,0}^c \rightarrow \sqrt{t} \). Therefore, to satisfy factorization, \( \beta_{b,0;0,0} \rightarrow t \).
In other words, looking to the \( \beta_{\frac{1}{2},\frac{1}{2};0,0}^c \) residuum, in the non-conspira-
torial solution the factor \( \sqrt{t} \) comes from the \( \mathcal{N}\mathcal{N}\pi \pi \) vertex,
in the conspiratorial solution from the \( \pi \mathcal{N}\pi \pi \) vertex. Then one
considers the reactions

\[ a) \; \mathcal{N}\Delta \rightarrow \mathcal{N}\Delta \quad b) \; \pi\pi\pi \rightarrow \mathcal{N}\mathcal{N} \quad c) \; \pi\mathcal{N} \rightarrow \mathcal{N}\Delta \quad (I.36) \]

Using again factorization, one has

\[ \left( \beta_{\frac{1}{2},\frac{1}{2};0,0}^c \right) = \beta_{b,0;0,0} \cdot \beta_{d,\frac{1}{2},\frac{1}{2}} \quad (I.37) \]

The amplitude \( \beta_{\frac{1}{2},\frac{1}{2};0,0}^c \) is analytic at \( t=0 \), and therefore
cannot have branch points; hence the product \( \beta_{b,0;0,0} \cdot \beta_{d,\frac{1}{2},\frac{1}{2}} \)
should behave as an even power of \( t, \sim t^{2n} \). We remember that
\( \beta_{b} \sim t \).

The solution with \( n=0 \) (\( \beta_{d} \sim 1/t \)) gives a singularity in
\( \beta_{d} \), which is not allowed by the Wang prescription; so the con-
clusion is that \( \beta_{\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{1}{2}} \) should go at least as \( t \), and therefore
the amplitude

\[ T_{\frac{1}{2},\frac{1}{2};0,0} (n\mathcal{N} \rightarrow \mathcal{N}\Delta) \quad (I.38) \]
should vanish as \( t \rightarrow 0 \) as \( t \).
Therefore a pion parity doublet predicts a dip at $t = 0$ in the reaction $\bar{\Upsilon} + N \rightarrow \zeta + \Delta$.

This dip is actually observed.

c) The np charge-exchange

As we have already said, another process in which conspiracy could be effective is the np charge exchange.

$$\rho \ n \Rightarrow n \ \rho$$  \hspace{1cm} (I.39)

This process should be explained together with its crossed reaction

$$\bar{\rho} \ \rho \Rightarrow \bar{n} \ n$$  \hspace{1cm} (I.39')

in which the same Regge poles are exchanged, but in different combinations.

We shall discuss in detail this process, as an example of a complicated use of conspiracies.

The experimental features one has to explain are 48)

a) the exceptionally sharp peak in the differential cross-section of process I, with a width of about $0.02 \text{(GeV/c)}^2$;

b) the fact that this sharp peak persists to very low energies and the width is almost energy independent;

c) the large difference in the magnitudes of the cross-sections for processes (I.39) and (I.39') at the same value of energy and momentum transfer [for $|t| > 0.02 \text{(GeV/c)}^2$];

d) the energy dependence of $\bar{p}p \rightarrow \bar{n}n$ data.

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Since the $\xi$ and $R$ trajectories are rather similar in the region of interest, their interference is small, and cannot explain the difference $c)$. This means that $\xi$ and $R$ are not the only trajectories $^50)$. 

A small contribution at $t=0$ in reaction I of the $\pi^-$ and $R$ is moreover consistent with the small values within large errors of the differences $\mathcal{F}_{pp} - \mathcal{F}_{pn}$ and $\mathcal{F}_{pp} - \mathcal{G}_{pn}^-$ $^52)$. 

On the other hand, it is rather appealing to attribute the extremely narrow peak at $|t|<0.02$ to the exchange of the pion, which has such a tiny mass.

The pion contribution can be exactly computed at $t=-\rho^2$; the extrapolation of this contribution in the physical region indicates that the pion effect is important.

However, as we said, without conspiracy, the contribution of the pion vanishes at $t=0$.

Arbab and Dash $^4$) have discussed various conspiracies; mechanisms:

1. The pion parity doublet

The pion is assumed to belong to Class III. In order to overcome the difficulty of the assignment of the pion in the limit $\rho^2\to 0$, it is assumed to decouple from the amplitude at $t=0$ for $\rho^2=0$. The effect of the finite pion mass is to shift the position of this zero from $t=0$ to some value $t=t_0$, so that the pion contribution is actually proportional to $(1-t/t_0)$. 

In order to explain the feature $c)$, a $B$ trajectory ($J^P=1^{++}$) is also included. As we have already noticed, such a $B$ trajectory could also belong to a parity doublet together with a particle as the $\zeta^-$ $^53)$. This choice, barring the difficulty of the assignment of the $\pi^-$ to Class III, gives a possible, although not exciting, fit.
2 - Class II interferences

The pion contribution vanishes at $t = 0$, but in np scattering it interferes destructively with another family of Class II (unnatural parity); the first daughter of this family actually interferes with the pion, while the parent contributes to other helicity amplitudes, but is necessary in order to satisfy the Volkov-Gribov constraint. Possible assignments of quantum numbers of this Class II family are:

$$\alpha$$

<table>
<thead>
<tr>
<th>Parent</th>
<th>$t$</th>
<th>$P$</th>
<th>$G$</th>
</tr>
</thead>
</table>
| +      | +   | -   | -   | ($A_1$?)

1st daughter + - -

or

$$\beta$$

<table>
<thead>
<tr>
<th>Parent</th>
<th>$t$</th>
<th>$P$</th>
<th>$G$</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>-</td>
<td>+</td>
<td></td>
</tr>
</tbody>
</table>

1st daughter - + +

There is however a serious difficulty; the analysis of the constraint equation requires $\alpha(0)_{parent} \sim 0.1$, so that $\alpha(0)_{daughter} \sim -1$, and the interference effect would be terribly energy dependent.

Moreover, in alternative $\alpha()$, the contribution of the daughter which has to interfere with the pion is almost imaginary ($\bar{t} = +, \alpha \sim -1$), while the pion contribution is almost real ($t = +, \alpha \sim 0$), so that there is very little interference. In alternative $\beta()$, this difficulty is not present, but the daughter has $G = +1$, so it changes sign from $pn \rightarrow np$ to $p\bar{p} \rightarrow n\bar{n}$; if in the first reaction it has to interfere with the pion to produce a sharp peak, in the second it would interfere positively, giving a large enhancement at $|t| > 0.02$. So the results with these choices are rather bad.

3 - Pion interference with a Class III doublet

The third possibility is that again the pion contribution is zero at $t = 0$, and it interferes negatively with a parity doublet of Class III. The possible quantum numbers assignments are now:
The alternative $S$ has the same difficulty as $B$, but has neither the difficulty of $\alpha$ (different energy behaviour and phases in quadrature) nor that of $B$ (opposite $G$ parities). Arbab and Dash conclude that also this conspiracy plus interference model, although quite involved, can give a possible fit to the data.
- Table I -

<table>
<thead>
<tr>
<th>Class</th>
<th>M</th>
<th>n</th>
<th>$\sigma$</th>
<th>$\bar{\tau}$</th>
<th>$C$</th>
<th>$P$</th>
<th>Possible Particles</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>even</td>
<td>+1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>odd</td>
<td>+1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
</tr>
<tr>
<td>II</td>
<td>0</td>
<td>even</td>
<td>-1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = -\bar{\tau}_e$</td>
<td>$\bar{\tau} = -\bar{\tau}_e$</td>
<td>$\bar{\tau} = -\bar{\tau}_e$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>odd</td>
<td>-1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = -\bar{\tau}_e$</td>
<td>$\bar{\tau} = -\bar{\tau}_e$</td>
<td>$\bar{\tau} = -\bar{\tau}_e$</td>
</tr>
<tr>
<td>III</td>
<td>a)</td>
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<td>even</td>
<td>+1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
</tr>
<tr>
<td></td>
<td>b)</td>
<td>1</td>
<td>even</td>
<td>-1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
</tr>
<tr>
<td></td>
<td>c)</td>
<td>1</td>
<td>odd</td>
<td>+1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
</tr>
<tr>
<td></td>
<td>d)</td>
<td>1</td>
<td>odd</td>
<td>-1</td>
<td>$\bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
<td>$\bar{\tau} = \bar{\tau}_e$</td>
</tr>
</tbody>
</table>

In the Table, the Regge signature $\bar{\tau}$, the charge conjugation $C$ and the parity $P$ of a Regge trajectory, daughter of order $n$ in the Toller family, are given in terms of the Lorentz signature $\bar{\tau}_e$. 
<table>
<thead>
<tr>
<th>Class of reaction</th>
<th>( II ) (even)</th>
<th>( II ) (odd) ( (A_1 ) conspiracy)</th>
<th>( III ) (even) ( (\pi ) conspiracy)</th>
<th>( III ) (odd) ( (B ) conspiracy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Sigma N \to \Sigma N )</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>( \Sigma N \to \omega N )</td>
<td>yes</td>
<td>no</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>( \Upsilon N \to \Xi N )</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( \Upsilon N \to \Upsilon N )</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>( \Upsilon N \to \Lambda )</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>( \Lambda N \to \Lambda N )</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

\( \Sigma \) means a particle with \( J^{\text{PG}} = 0^+ \); yes means that a forward cross-section goes as the leading power, no means that it goes down more quickly. In parenthesis is written the Lorentz signature.
II. - THE DIP MECHANISM

Let us now discuss in some detail the problem of the dips and ghost-killing mechanism in Regge theory.

As it is well known, the dip in $\pi^- p \to \pi^0 n$ charge-exchange at $t = -0.6$ (GeV/c)$^2$ is usually associated in Regge theory with the fact that $\alpha'_3 (-0.6) = 0$. The same kind of explanation holds for the dip in the backward $\pi^+ p$ scattering, which is associated with the value $\alpha'_1 = \frac{1}{2}$ for the nucleon trajectory.

These explanations of the dips are therefore related to the behaviour of the Regge residue functions near an integer value of the trajectory (half-integer for baryon trajectories). In order to be definite, we shall always consider the behaviour near the value $\alpha = 0$; for the behaviour near a value $\alpha = n$, it is just enough to replace everywhere $\alpha$ with $\alpha - n$.

If the external particles have non-zero spin, we shall define the transition between the Regge pole, which is exchanged in the $t$ channel, and a pair of external particles as:

a) sense, if the net helicity change, $(\lambda_1 - \lambda_3)$, between the external particles, is 0;

b) nonsense, if the net helicity change, $(\lambda_1 - \lambda_3)$, between the external particles, is $> 0$.

If the relevant value of $\alpha$ is different from 0, one has to replace these conditions by $|\lambda_1 - \lambda_3| \leq \alpha'$ (sense), $|\lambda_1 - \lambda_3| > \alpha$ (nonsense).

Passing now to the amplitude, a $t$ channel transition between particles 1-3 and 2-4 can be:

a) a sense-sense amplitude, $T_{ss}$, if both the transitions 1-3 $\to$ Regge and 2-4 $\to$ Regge are sense transition;

b) a sense-nonsense amplitude, $T_{sn}$, if one of the transitions is sense, the other nonsense;

c) a nonsense-nonsense amplitude, $T_{nn}$, if both of the transitions are nonsense.
A simple example of sense-sense and sense-nonsense amplitudes is given by boson-fermion scattering (0 and \( \frac{1}{2} \) spins), near \( \alpha = 0 \), then the boson pair-Regge transition is always sense, while the Regge-fermion-antifermion transition is sense for the non-helicity flip amplitude \(| \lambda_1 - \lambda_2 | = 0 \), nonsense for the helicity flip amplitude \(| \lambda_1 - \lambda_2 | = 1 \). A simple example of sense-nonsense and nonsense-nonsense amplitude is the pion photoproduction on nucleons (always near \( \alpha = 0 \)), where the \( \gamma - T \) - Regge transition is always nonsense \(| \lambda_\gamma - \lambda_T | = 1 \), while the nucleon-antinucleon Regge transition can again be either sense or nonsense. An example in which all the three kinds of amplitudes are present is elastic nucleon-nucleon scattering; here, both vertices can be either sense (non-helicity flip) or nonsense (helicity flip), and so they can be combined to give the three kinds of amplitudes. In this case, examples of \( ss, sn, nn \) amplitudes are

\[
\begin{align*}
\langle s^+ | T | s^+ \rangle & \rightarrow T_{ss} \quad \text{(no helicity flip in both vertices)} \\
\langle s^+ | T | s^+ \rangle & \rightarrow T_{sn} \quad \text{(helicity flip in one vertex only)} \\
\langle s^+ | T | s^+ \rangle & \rightarrow T_{nn} \quad \text{(double helicity flip)}.
\end{align*}
\]

The behaviour of a scattering amplitude near an integer (half-integer) value of \( \alpha \), where one can have a nonsense transition, is determined by the behaviour of its residuum function.

The three kinds of amplitudes behave near \( \alpha = 0 \) as \(^{54}\)

(we do not include factors which are regular at \( \alpha = 0 \))

\[
\begin{align*}
T_{ss} & \propto \beta_{ss} \xi_\alpha S^\alpha \\
T_{sn} & \propto \beta_{sn} \xi_\alpha S^\alpha \\
T_{nn} & \propto \beta_{nn} \xi_\alpha S^\alpha
\end{align*}
\]

(II.1)
where \( \mathcal{F}_\alpha = \left( 1 + e^{-i \pi \alpha} \right) / (2 \sin \pi \alpha) \) is the signature. The kinematical factors \( \alpha^{\frac{1}{3}} \) and \( \alpha^{\frac{1}{3}} \) in \( T_{sn} \) and \( T_{nn} \) come from the behaviour near \( \alpha = 0 \) of the rotation functions, which are the appropriate generalization to non-zero spin of the Legendre polynomials, and which have different behaviours in the \( ss \), \( sn \) and \( nn \) regions.

Since the residuum functions \( \beta_{ss} \), \( \beta_{sn} \), and \( \beta_{nn} \) are factorizable, namely they are the product of two terms referring to the two different vertices 1-3 Regge and 2-4 Regge, they are constraint to satisfy the factorization equation

\[
(\beta_{sn})^2 = \beta_{ss} \beta_{nn}
\]  

(II.2)

Let us now distinguish a right signature integer point, where \( \mathcal{F}_\alpha \rightarrow 1/\alpha \), from a wrong signature integer point, where \( \mathcal{F}_\alpha \rightarrow \text{const.} \)

We remember that a right signature integer means an integer value of \( \alpha \) which has the same parity as the values of the angular momentum for which actually the physical particles are present on the trajectory. Wrong signature points are those which have the wrong parity (odd if the particles have even spin and vice versa).

Since we are considering always the value \( \alpha = 0 \), this is a right signature point for example for the \( P' \) trajectory, a wrong signature point for the \( G \) trajectory. Generalizations to other cases are obvious.

A. - Right signature points

Since, as seen in formula (II.1), the \( T_{sn} \) amplitude contains a factor \( \alpha^{\frac{1}{3}} \), the \( \beta_{sn} \) residuum function has to be proportional to a half-integer power of \( \alpha \), otherwise one would have a branch point in \( T_{sn} \) at a value \( t = t_0 \) [such that \( \alpha(t_0) = 0 \)], where there is no general reason to expect a singularity. If \( t_0 \neq 0 \), the scattering amplitude cannot have a pole at \( t = 0 \). Since the signature factor already contains this pole, the product \( \alpha^{\frac{1}{2}} \beta_{sn} \) should at least behave in such a way to cancel this pole.
The simplest choice is therefore \(55\)

\[
\beta_{sn} \propto \alpha^{\frac{1}{2}}
\]  

(II.3)

Then, always discarding poles, the factorization can be satisfied by the two choices

1) \(\beta_{ss} \propto \text{const}, \quad \beta_{sn} \propto \alpha^{\frac{1}{2}}, \quad \beta_{nn} \propto \alpha \) (choosing sense mechanism)

2) \(\beta_{ss} \propto \alpha, \quad \beta_{sn} \propto \alpha^{\frac{1}{2}}, \quad \beta_{nn} \propto \text{const} \) (choosing nonsense or Gell-Mann \(56\) mechanism)

(II.4)

The first alternative is impossible for a right signature point, since the \(T_{ss}\) amplitude would still have the pole of the signature factor, and one would have a particle of imaginary mass, \(\sqrt{t_0}\) (ghost).

In the second alternative, one would have a pole in the non-asymptotic term \(54\), which is supposed to be removed by the compensating trajectory.

One can further choose a less simple assumption, namely

\[
\beta_{sn} \propto \alpha^{\frac{1}{2}}
\]  

(II.5)

One could then have four possible assignments of the factors \(\alpha\) in \(\beta_{ss}\) and \(\beta_{sn}\), (one leading again to a ghost in \(T_{ss}\)), but only two have been considered in the literature

3) \(\beta_{ss} \propto \alpha, \quad \beta_{sn} \propto \alpha^{\frac{1}{2}}, \quad \beta_{nn} \propto \alpha \) (Chew \(57\) mechanism)

4) \(\beta_{ss} \propto \alpha^2, \quad \beta_{sn} \propto \alpha^{\frac{1}{2}}, \quad \beta_{nn} \propto \alpha \) (non-compensating \(58\) mechanism)

(II.6)
These two possibilities correspond just to the multiplication by a factor $\alpha$ of all the terms in the choosing-sense or Gell-Mann mechanisms. The last mechanism is called non-compensating, since with this choice there is not a ghost anymore even in the non-asymptotic term, and the compensating trajectory is not required to kill the ghost. Still, however, if the compensating trajectory is not there, the non-leading term, which behaves as $s^{-\alpha-1}$, is bigger than the leading term $s^\alpha$ at $\alpha' = 0$ (the non-leading term in $T_{nn}$ is finite, while all the other amplitudes $T_{ss}$, $T_{sn}$ and the leading term of $T_{nn}$ are zero).

The two remaining possibilities

\[
\beta_{ss} \propto \alpha \beta_{sn} \propto \alpha^{\frac{1}{2}}, \beta_{nn} \propto \alpha^3
\]

(II.7)

have never been considered, neither a higher half-integer power in $\beta_{sn}$. We can only remark that if the alternatives are multiplied by higher powers of $\alpha$, all the amplitudes will vanish at $\alpha' = 0$, as it already happens in alternative 4), so there will be actually no different predictions between those given by these more complicated choices and alternative 4), apart the way in which the amplitudes tend to zero.

From the phenomenological point of view, the dips in $\eta^{\pm}p$ and $\bar{p}p$ elastic scattering have been analysed by Chiu et al. (58) under the assumption that it is due to the fact that for some value of $t$ near the dip one has $\alpha(p^2) = 0$. They have tried to distinguish alternative 3) (Chew) from alternative 4) (non-compensating), looking also to the polarization data in $\eta^{\pm}p$ scattering. The measurable difference between the two mechanisms is that in 3) the cross-section coming from the non-helicity flip amplitude due to $p'$ exchange is non-zero, while in 4) it vanishes. In both the $p'$ helicity flip amplitude is zero. Both mechanisms gave a possible fit, but on a statistical basis Chiu et al (58) have concluded that the non-compensating mechanism 4) should be preferred.
In an analysis of the reaction $K^+p\rightarrow K^0\Delta^{++}$, Krammer and Moor have concluded that the $A_2$ trajectory probably follows the mechanism (Gell-Mann) near $\alpha = 0$, since there is no evidence of a dip. This conclusion is consistent with the behaviour of the angular distribution of the reaction $\Xi^-p\rightarrow \eta n$, where again no dip is seen.

B. - Wrong signature points

At a wrong signature point, all the mechanisms (1)-4), listed before, could be present. In fact, since at a wrong signature point the signature factor is constant, there is never a ghost in the theory.

In fact, alternative 1) (choosing-sense) was taken for the $\zeta$ trajectory in order to explain, in the $\pi^-p\rightarrow \eta n$ angular distribution, at the same time the dip at $t = -0.6$ (GeV/c)$^2$ and the finite non-zero cross-section at the dip. With this mechanism (see Table III), the amplitude $T_{\theta n}$ (helicity flip) vanishes, while $T_{\theta s}$ (helicity non-flip) is finite.

Another interesting application of the theory of the dips is the reaction

$$\pi^-p\rightarrow \omega n$$

(II. 8)

in the case in which it is supposed to go through the $\zeta$ exchange. Since the pion has unnatural parity and both $\omega$ and $\zeta$ have natural parity, the $\omega$ cannot be produced in the helicity zero state. Therefore, at $\alpha_\zeta = 0$, the transition $\pi^- - \omega - \zeta$ is always nonsense. If the $\zeta$ chooses sense, as it is implied by the behaviour of $\pi^-p\rightarrow \eta n$, (see however), one will predict a zero in the angular distribution. However, even if the $\zeta$ chooses nonsense, and therefore $\beta_{nn} \propto \text{const.}$, still the $T_{nn}$ amplitude would be zero at $t = -0.6$ (GeV/c)$^2$.  

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One has therefore the general prediction that, what ever, the mechanism is, one should find a zero in the angular distribution of (II.8), except for terms which do not correspond to the γ exchange.

Another well-known example of a dip at a wrong signature point is the \( \pi^+ p \to p \pi^+ \) backward scattering, for which the value \( \alpha = -\frac{1}{2} \) for the nucleon trajectory corresponds to nonsense-nonsense transitions, and one expects a dip near \( u = -0.2 \text{ (GeV/c)}^2 \) which is actually seen in the experiment.

The situation at the wrong signature points is, however, more complicated. In fact, it has been shown by Mandelstam and Wang (see also Mueller and Trueman, Drechsler) that at wrong signature integer points the partial wave scattering amplitude has a fixed pole in the complex angular momentum plane.

The argument is essentially the old one of Gribov and Pomeranchuk; in a relativistic theory (in contrast with potential theory), where the third Mandelstam spectral function \( g_{su}(s,t) \) is different from zero, the discontinuity across the left-hand cut in the \( t \) plane of the scattering amplitude is singular at unphysical wrong signature integer points, and this singularity generates a fixed pole in the partial wave scattering amplitude.

The difference between the Mandelstam-Wang and the Gribov-Pomeranchuk arguments is that, following Gribov-Pomeranchuk, the right-hand cut discontinuity has an infinite accumulation of poles around the value of \( j \) in question, which contributes to the asymptotic behaviour of the amplitude.

Mandelstam and Wang were able to show that this argument is not valid in presence of cuts in the angular momentum plane (see also), whose origin is again due to the third spectral function. Moreover, the presence of cuts in \( j \) has the effect that the fixed pole at \( j = N \) does not contribute to the asymptotic behaviour.
(there is no \( a^N \) term in the amplitude) \(^{75} \)). The effect of the fixed pole is just to modify the behaviours of the \( \beta_{sn} \) and \( \beta_{nn} \) residuum functions, which become singular at the unphysical wrong signature points, and the singularity is of the form (we always consider the point \( \alpha = 0 \))

\[
\beta_{ss} \propto \text{const} \quad \beta_{sn} \propto \alpha^{-\frac{1}{2}} \quad \beta_{nn} \propto \alpha^{-1}
\]

(II.9)

This is the behaviour of that part of the \( \beta \)'s which comes from the effect of the third spectral function (the fixed pole effect) and is the same whether at zero order in \( \mathcal{J}_{su} \) the trajectory chooses sense or nonsense \([\text{alternatives 1 and 2}]\). Remembering the kinematical factors in front of \( \beta_{sn} \) and \( \beta_{nn} \) one realizes that the behaviour of the amplitudes is

\[
T_{ss} \propto \text{const} \quad T_{sn} \propto \text{const} \quad T_{nn} \propto \text{const}
\]

for alternatives 1 and 2

\[
T_{ss} \propto \alpha \quad T_{sn} \propto \alpha \quad T_{nn} \propto \alpha
\]

for alternatives 3 and 4

(II.10)

In general, the amplitudes will contain both terms which are of zero order in \( \mathcal{J}_{su} \) for which alternative 1)-4) give different results, and "singular terms", which do come from the fixed pole effect, and have the same behaviour for \( T_{ss}, T_{sn}, T_{nn} \) as discussed below. Of course, the only way to save the explanation of the dip mechanism is to suppose that the effect of the fixed pole on the residuum function is small, so that the \( T_{sn} \) scattering amplitude would actually behave as \( \alpha \mathcal{A}_{sn} = (\alpha C_1 + C_2) \). In that case, if \( C_2 \) is small, its only effect would consist in displacing slightly the position of the dip. If \( C_2 \) is large, on the one hand the eventual dip would be pushed very far so that it would have no immediate connection with the value to where \( \alpha (t_0) = 0 \), and would possibly disappear, on the other hand one would expect that all the effects of
the third spectral function become important, as the effect of the
cuts in the angular momentum plane. In that case, the amplitude
would have not an asymptotic Regge behaviour anymore.

Therefore the Regge behaviour of the $\pi^- p \rightarrow \pi^0 n$ differen-
tial cross-section with the energy, and the existence of the dip at
t = t₀, where $\alpha_q(t_0) = 0$, should be considered as related effects,
both suggesting that the effect of the fixed pole is not very impor-
tant 76).

The question, however, whether the third spectral function
effect is big or small is a dynamical one, which can have different
answers in different problems.

This should be kept in mind, when one tries to understand
why the reaction

$$\pi^+ p \rightarrow \omega N^{++}$$

where one should expect a zero 64) in the angular distribution
near t = -0.6 (GeV/c)², shows actually no structure. There is in
fact always the possibility that in this reaction the fixed pole
effect be important, in contrast to what happens in $\pi^- p \rightarrow \pi^0 n$.
This possibility is, however, a very ugly one, since one would not
understand why the third spectral function in one case is important
and in the other is not. Moreover, in this situation, all the
theory of dips would loose any predictive power. So it is probably
more reasonable to assume that the lack of dip in the $\pi^+ p \rightarrow \omega N^{++}$
is due to a strong effect of the $B$ exchange 65).

A problem where the fixed pole has a very important effect is
the Compton scattering in the forward direction, t = 0.

The question is whether the Pomeranchuk trajectory can con-
tribute to the Compton scattering coherent amplitude at t = 0. The
answer is no, in Regge theory, if there is no fixed pole effect.
This result follows from the crossing properties of the photon. In fact, the $s$ channel coherent amplitude is defined as the amplitude for which neither the photon nor the target particles do flip their helicities

$$\lambda_S^s \equiv \lambda_S^s, \quad \lambda_2^s = \lambda_4^s$$

In order to cross one photon and one of the target particles from the $s$ to the $t$ channel, one has to do the following replacements

$$\lambda_4 \rightarrow \lambda_4 \quad \text{but} \quad \lambda_3^s \rightarrow -\lambda_3^s$$

(II.11)

This result follows from the different crossing properties of a particle with zero mass, in contrast with a particle with non-zero mass $70), 77).$

Therefore,

$$\lambda = \lambda_S^t - \lambda_4^t = 0 \quad \text{but} \quad \mu = |\lambda_S^t - \lambda_3^s| = |\lambda_S^t - \lambda_3^s| = 2$$

(II.12)

As a consequence, the coherent $s$ channel amplitude is a sense-nonsense amplitude, as viewed from the $t$ channel, if one exchanges an angular momentum $\alpha'(0) = 1$. If there is no fixed pole effect, the $t$ amplitude contains a factor $(\alpha' - 1)$ and the signature factor is constant (the point $\alpha' = 1$ is a wrong signature point for the Pomeranchuk trajectory).

Therefore the contribution of the Pomeranchuk pole to the total $\gamma$ induced cross-section (related to the imaginary part of the coherent amplitude via the optical theorem) vanishes identically.

This selection rule works only at $t = 0$, where $\alpha = 1.$
For non-zero angles the Pomeranchuk contribution to the elastic coherent amplitude is different from zero. Moreover, one can exchange the Pomeranchuk pole also in the photoproduction of vector mesons even in the forward direction (which at \( s \to \infty \) coincides with \( t = 0 \)), since the helicity of the vector meson is not changed by crossing. One has therefore the set of absurd results:\(^{77}\)

\[
\begin{align*}
\text{a)} \quad & \mathcal{G}_{\text{ind}} \quad (\text{vector meson production}) \propto \frac{1}{L_s} \\
\text{b)} \quad & \mathcal{G}_t \\
\text{c)} \quad & \mathcal{G}_{\text{tot}} \quad \rightarrow \quad 0 \quad \text{as an inverse power of} \quad s \\
\end{align*}
\]

and therefore

\[
\mathcal{G}_m > \mathcal{G}_t > \mathcal{G}_{\text{tot}}
\]

\(^{II.13}^{1}\)

This contradictory result is no more valid if one allows a fixed pole, which yields a factor \((\alpha - 1)^{-1}\) in the residuum function, which just removes the zero of \( T_m \), allowing the Pomeranchuk pole to contribute fully in the forward direction. One should notice that in this process it is not necessary to require the presence of cuts in the angular momentum to protect the fixed pole, since in any weak or electromagnetic process one must not use bilinear unitarity, so that a fixed pole will never violate unitarity\(^ {78}\).

The problem of choosing sense or nonsense can also be seen from the point of view of \( O(3,1) \) symmetry. Of course, the Lorentz symmetry, strictly speaking, applies only at \( P_\mu = 0 \), namely \( t = 0 \), and in general the nonsense point is for \( t \neq 0 \). However, one can imagine to vary the strength of the interaction, which generates the "bound states" on the Regge trajectory, in such a way to drive
the unphysical sense point at \( t = 0 \). If the amplitude has enough analyticity properties in the coupling constant, the results obtained at \( t = 0 \) would be also true at the actual point \( t = t_0 \).

From the use of Lorentz symmetry one can derive a number of interesting results \(^{79}\); among them, we quote the two following ones:

a) in a Toller pole, the sense-choosing property of the parent pole determines the properties of all the daughters, but this does not mean that the property is the same for all the daughters; definite rules exist \(^{79}\) which allow to fix the sense-choosing properties of the daughters as a function of that of the parent; moreover, alternatives \( (1, \alpha^3) \) and \( (\alpha^3, 1) \) for the distribution powers we mentioned below, are never found;

b) on a wrong signature point, one does not find always the behaviour which corresponds to the fixed pole effect. This result can be possibly understood by supposing that the residuum of the fixed pole just vanishes for that value of the coupling constant for which \( \alpha(0) = \text{integer} \), this is due to the particular symmetry of the point \( t = 0 \); this should then not be true in general when the wrong signature unphysical point is located at \( t \neq 0 \).
<table>
<thead>
<tr>
<th>Residuum functions (1)</th>
<th>Amplitudes - RIGHT SIGNATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{ss}$</td>
<td>$\beta_{sn}$</td>
</tr>
<tr>
<td>1</td>
<td>$(\alpha - j_0)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\alpha - j_0)$</td>
<td>$(\alpha - j_0)^{\frac{1}{2}}$</td>
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<td></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\alpha - j_0)$</td>
<td>$(\alpha - j_0)^{\frac{1}{2}}$</td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$(\alpha - j_0)^2$</td>
<td>$(\alpha - j_0)^{\frac{1}{2}}$</td>
</tr>
<tr>
<td></td>
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</tr>
</tbody>
</table>

(1) At zero order in the third spectral function for the wrong signature point.
(2) All these results are for the leading asymptotic term; the non-leading term is obtained dividing these terms by $(\alpha - j_0)$.

Behaviour of the residuum functions and the amplitudes near an integer nonsense values of $\alpha = j_0$. $t_0$ is value of $t$ such that $\alpha(t_0) = j_0$. 

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<table>
<thead>
<tr>
<th></th>
<th>Residuum functions (1)</th>
<th>Amplitudes - WRONG SIGNATURE (1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) Choosing</td>
<td></td>
<td></td>
</tr>
<tr>
<td>sense mechanism</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{ss}$</td>
<td>$\beta_{sn}$</td>
<td>$\beta_{nn}$</td>
</tr>
<tr>
<td>1</td>
<td>$(\alpha - j_0)^{1/2}$</td>
<td>$(\alpha - j_0)$</td>
</tr>
<tr>
<td>$T_{ss}$</td>
<td>$T_{sn}$</td>
<td>$T_{nn}$</td>
</tr>
<tr>
<td>const.</td>
<td>$(\alpha - j_0)$</td>
<td>$(\alpha - j_0)^2$</td>
</tr>
<tr>
<td>$T_{sn}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$T_{nn}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2) Gell-Mann choosing nonsense mechanism

|                  |                                        |                                   |
| 1                |                                        |                                   |
| $\beta_{ss}$     | $\beta_{sn}$                           | $\beta_{nn}$                      |
| $(\alpha - j_0)$ | $(\alpha - j_0)^{3/2}$                 | 1                                 |
| $T_{ss}$         | $T_{sn}$                               | $T_{nn}$                          |
| $(\alpha - j_0)$ | $(\alpha - j_0)$                      | $(\alpha - j_0)^2$               |
| $T_{sn}$         |                                       |                                   |
| $T_{nn}$         |                                       |                                   |

3) Chew mechanism

|                  |                                        |                                   |
| 1                |                                        |                                   |
| $\beta_{ss}$     | $\beta_{sn}$                           | $\beta_{nn}$                      |
| $(\alpha - j_0)^2$ | $(\alpha - j_0)^{3/2}$                | $(\alpha - j_0)^2$               |
| $T_{ss}$         | $T_{sn}$                               | $T_{nn}$                          |
| $(\alpha - j_0)^3$ | $(\alpha - j_0)^2$                | $(\alpha - j_0)^2$               |
| $T_{sn}$         |                                       |                                   |
| $T_{nn}$         |                                       |                                   |

4) Non-compensating mechanism

|                  |                                        |                                   |
| 1                |                                        |                                   |
| $\beta_{ss}$     | $\beta_{sn}$                           | $\beta_{nn}$                      |
| $(\alpha - j_0)$ | $(\alpha - j_0)^{3/2}$                | $(\alpha - j_0)^2$               |
| $T_{ss}$         | $T_{sn}$                               | $T_{nn}$                          |
| $(\alpha - j_0)^2$ | $(\alpha - j_0)^2$                | $(\alpha - j_0)^2$               |
| $T_{sn}$         |                                       |                                   |
| $T_{nn}$         |                                       |                                   |

<table>
<thead>
<tr>
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<th>Residuum functions (3)</th>
<th>Amplitudes - WRONG SIGNATURE (3)</th>
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<td>$\beta_{sn}$</td>
<td>$\beta_{nn}$</td>
</tr>
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<tr>
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<td>$T_{nn}$</td>
</tr>
<tr>
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<td>const.</td>
<td>const.</td>
</tr>
<tr>
<td>$T_{sn}$</td>
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<td></td>
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<tr>
<td>$T_{nn}$</td>
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3 and 4

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<thead>
<tr>
<th></th>
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<th>Amplitudes - WRONG SIGNATURE (3)</th>
</tr>
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<tbody>
<tr>
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<td></td>
<td></td>
</tr>
<tr>
<td>$\beta_{ss}$</td>
<td>$\beta_{sn}$</td>
<td>$\beta_{nn}$</td>
</tr>
<tr>
<td>$(\alpha - j_0)$</td>
<td>$(\alpha - j_0)^{1/2}$</td>
<td>1</td>
</tr>
<tr>
<td>$T_{ss}$</td>
<td>$T_{sn}$</td>
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</tr>
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<td>$T_{nn}$</td>
<td></td>
<td></td>
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</table>

(1) At zero order in the third spectral function for the wrong signature point.

(2) All these results are for the leading asymptotic term; the non-leading term is obtained dividing these terms by $(\alpha - j_0)$.

(3) At first order in the third spectral function for the wrong signature point.
III. - REGGE POLES AND SU(3) SYMMETRY

The SU(3) symmetry can be used in conjunction or without specific high energy models in order to relate together different reactions at high energy. One obtains in this way a number of useful results. Some of them are model independent, in the sense that they are consequences only of the assumption that some definite SU(3) multiplets are exchanged in the crossed channel, others are obtained by the combined assumption of SU(3) plus a specific model \(^{80}\), as the Regge pole model, or by more specific assumptions inside the model, as the exchange degeneracy.

The simplest result of the application of SU(3) symmetry plus the peripheral idea, is the prediction that there must exist forward and/or backward peaks in some reactions, but that they should be absent in other reactions \(^{81}\). These results follow from the assumption that in the crossed channel (t channel for meson exchange, u channel for baryon exchange) \(^{82}\) one can exchange only those SU(3) multiplets which correspond to the observed existing particles. Since the observed mesons belong only to \(1\), \(8\) and \(10\) SU(3) multiplets, one would expect forward peaks, due to meson exchange, only when the reactions are such that the quantum numbers of the t channel correspond to those of the \(1\), \(8\) or \(10\) SU(3) multiplets.

In the same way, the well established baryons have been classified in \(1\), \(8\) and \(10\) SU(3) multiplets, one should expect therefore backward peaks only when those multiplets can be exchanged.

One should remark here that if there exist other particles which fall outside these multiplets, but are weakly coupled to mesons and nucleons, it is likely that their effect will be small also when they are exchanged in the crossed channel.

Following Barger \(^{81}\), I will report here few examples.
The reaction
\[ \pi^+ p \rightarrow \kappa^+ \Sigma^+ \]
can have both a forward and a backward peak, since meson octets and \( \xi^{1\frac{3}{2}}, \xi^{8\frac{3}{2}} \) or \( \xi^{10\frac{3}{2}} \) baryon multiplets can be exchanged. One observes at 3 GeV/c incident momentum a forward peak of \( \sim 250 \mu b/st \) and a backward peak of \( \sim 10 \mu b/st \).

In the reaction
\[ \kappa^- \rho \rightarrow \kappa^0 n \]
there exists a sharp forward peak, but no backward peak; this absence of backward peak can be interpreted by saying that there does not exist a baryon with \( Q = S = +1 \), or equivalently that such a baryon would not be classified in \( \xi^{1\frac{3}{2}}, \xi^{8\frac{3}{2}} \) or \( \xi^{10\frac{3}{2}} \) SU(3) representations.

On the contrary, in the reaction
\[ \bar{n}^- \rho \rightarrow \kappa^+ \Sigma^- \]
there exists a backward peak, but no forward peak, which would correspond to an exchange of a doubly charged strange meson.

One can proceed further, using the concept of peripheral interaction (reggeized or not), where the amplitude is assumed to be dominated by the exchange of definite quantum numbers in the crossed channel, and try to get relations between different processes, which are based only upon the symmetries of the amplitudes. Assuming that the crossed channel exchanges correspond to the existing SU(3) multiplets, one can obtain relations between different cross-sections in the form of sum rules.

Let us take as an example the meson-baryon reactions with charge-exchange but no strangeness exchange
\[ \pi^- p \rightarrow \pi^0 n ; \quad \bar{n}^- \rho \rightarrow \gamma n ; \quad \bar{k}^- \rho \rightarrow \kappa^0 n ; \quad k^+ p \rightarrow \kappa^0 n \]

(III.1)
The external mesons belong to an octet representation of $SU(3)$; since moreover the proton-neutron vertex is the same in all the reactions, the symmetry of the meson vertex can be evaluated independently of the baryon vertex, and no $F/D$ ratio enters into the problem. Since $G$ parity is not diagonal in the reactions involving strange mesons, we have two amplitudes, corresponding to the exchange of the two octets with $G = \pm 1$, which we shall denote by $f (G = +1)$ and $A_2 (G = -1)$.

We shall therefore have only two independent amplitudes, and so two relations between the four reactions, which are

\[
\left[ \mathcal{G}_t^t (K^\mp p) - \mathcal{G}_t (K^- n) \right] - \left[ \mathcal{G}_t^t (K^\mp p) - \mathcal{G}_t (\pi^+ n) \right] = 1
\]

\[
\left[ \mathcal{G}_t (\pi^- p) - \mathcal{G}_t (\pi^+ p) \right]
\]

(III.2)

and

\[
\frac{d\sigma}{dt} (K^\mp p \to \bar{K}^0 n) + \frac{d\sigma}{dt} (K^\mp n \to K^0 p) = \frac{d\sigma}{dt} (\pi^- \to \bar{K}^0 n) + 3 \frac{d\sigma}{dt} (\pi^- \to \eta n)
\]

(III.3)

Relation (III.2) is in very good agreement with experiment \(^{81},^{84}\); for relation (III.3) there are no good high energy data for the $K^+ n \to K^0 p$ charge-exchange.

Similar relations can be obtained by replacing the outgoing neutron with a baryon $N^*_8$ resonance; one gets in this way the relation \(^{86}\)

\[
\frac{d\sigma}{dt} (K^+ p \to K^0 N^{*+}) + 3 \frac{d\sigma}{dt} (K^- p \to K^- N^{*+}) =
\]

\[
\frac{d\sigma}{dt} (\pi^+ p \to \eta N^{*+}) + 3 \frac{d\sigma}{dt} (\pi^- p \to \eta N^{*+})
\]

(III.4)
One can proceed further, and assume now the exchange of two Regge trajectories, a vector octet with $G=+1$ (the $G$ octet) and a tensor octet with $G=-1$ (the $A_2$ octet). The difference between the previous hypothesis is that now one describes simultaneously all the energies, while the relations previously obtained were valid at fixed energy.

All the charge-exchange reactions (III.1) can be expressed now, at $t=0$ where no spin flip is present, in terms of two trajectory intercepts and two independent residuum functions. Therefore there exist two energy-independent relations between the four residuum functions describing the four forward scattering amplitudes (III.1).

A statistical analysis of all the existing experimental data gave

\[
\begin{array}{ccc}
\frac{\mathcal{Y}_G \, \pi^+ \pi^-}{\mathcal{Y}_G \, n \, n} & = & 1.12 \pm 0.12 \\
\frac{\mathcal{Y}_{A_2} \, \pi^+ \pi^-}{\mathcal{Y}_{A_2} \, n \, n} & = & 1.07 \pm 0.15
\end{array}
\] (III.5)

One has to stress that the use of SU(3) in conjunction with Regge poles allows already a simple way of introducing a symmetry breaking in the masses, since one takes the actual values of $\mathcal{Y}(0)$ for different particles in the same SU(3) multiplet. This form of symmetry breaking has no effect on reactions (III.1), where only one trajectory in each octet is exchanged, but is important when for instance one compares reactions with $G$ exchange and reactions with $K^*$ exchange.

A general fit of all the hypercharge and/or charge exchange reactions, where the vector and tensor octets are exchanged in the t
channel, has been recently attempted by Salin [88]; he has fitted the angular distributions of all the existing data, (at energies larger than 3 GeV) for the reactions

\[ \pi^- \rho \rightarrow \pi^0 n; \quad \pi^- \rho \rightarrow \eta n; \quad \pi^+ \rho \rightarrow K^+ \Sigma^+; \quad \pi^- \rho \rightarrow K^0 \Lambda^0 \]

\[ K^- \rho \rightarrow \bar{K}^0 n; \quad K^- \rho \rightarrow \pi^0 \Lambda^0; \quad K^- \rho \rightarrow \pi^- \Sigma^-; \quad K^- \rho \rightarrow \eta \gamma^0 \]  

(III.6)

Taking the $K^*$ and $K^{**}$ trajectories to be parallel to the $\xi$ and $\Lambda_2$ trajectories [so allowing SU(3) breaking for the masses], but using exact SU(3) for the vertices, he has obtained a rather good fit to all these reactions.

The free parameters are the two F/D ratios for the vector $1^-$ and the tensor $2^+$ octets. The best fit gave

\[ \left( \frac{F}{D} \right)_V \simeq -11 \quad \left( \frac{F}{D} \right)_T \simeq 0.25 \]  

(III.7)

More detailed assumptions, which use the notion of exchange degeneracy, both in the form of "strong exchange degeneracy" [90], [91] (equality between both couplings and trajectories of the $1^-$ and $2^+$ octets) or "weak exchange degeneracy" [92] (equality only for the $1^-$ and $2^+$ couplings, but not trajectories), give in general results which are not in good agreement with experiment [85], [87], [93].
IV. - POLARIZATION PHENOMENA

We would like to discuss here the polarization phenomena in terms of Regge pole theory. One should immediately stress that the polarization, being due to an interference between different amplitudes, is much more sensitive to details of Regge parameters or to the presence of non-leading singularities in the \( j \) plane and other non-Regge contributions than the total or differential cross-sections.

Therefore, even if all the experimental data on total cross-sections, differential elastic cross-section and \( \Pi^- p \rightarrow \Pi^- n \) or \( \Pi^- p \rightarrow \eta n \) charge-exchange are rather well fitted using only five Regge poles, \( P, P', \xi, \omega \) and \( A_2 \) (see, for instance, the paper by Harita et al. 94) presented to this Conference), the polarization data can be used in order to test the reliability of the parameters or to find the limits of the model.

We shall discuss here two problems, in which there is disagreement with the simple five-pole fit.

A - The polarization in \( \Pi^\pm p \) elastic scattering

The experiment 95) shows that the polarization changes sign going from \( \Pi^+ p \) to \( \Pi^- p \), which has two consequences:

i) The interference between the \( P \) and \( P' \) amplitudes is small, since this interference would give a contribution to the polarization which does not change sign going from \( \Pi^+ \) to \( \Pi^- \). Therefore the ratio between the amplitudes \( A(s,t) \) and \( B(s,t) \) for the Pomeronchuk and the \( P' \) exchange is almost the same

\[
\left( \frac{A}{B} \right)_\rho \approx \left( \frac{A}{B} \right)_{\rho'}
\]

(ii) The polarization is due to the interference between the combined \( (P,P') \) amplitude, which is predominantly non-spin-flip and the spin flip amplitude due to \( \xi \) exchange.
However, a weak point has been raised by Höhler and Eisenbeiss in a paper contributed to this Conference.

Let us define

\[ \sigma^\pm = \frac{d \sigma^\mp}{d \Omega} \quad \text{for } \pi^\pm p \text{ elastic scattering} \]

and introducing as usual the non-spin flip and spin flip amplitudes as

\[ t^\pm (k, \omega, \Theta) = c^\pm (k, \omega, \Theta) + i \sigma^\pm \cdot q^\pm (k, \omega, \Theta) \]

One can then form the combinations

\[ G_+ \rho_+ + G_- \rho_- = 4 \gamma_m \left( f^{(+)g^{(+)}} + f^{(-)g^{(-)}} \right) \]

\[ G_+ \rho_+ - G_- \rho_- = 4 \gamma_m \left( f^{(+)g^{(-)}} + f^{(-)g^{(+)}} \right) \]

(IV.2)

where

\[ f_\pm = f^{(+)} \pm f^{(-)} , \quad g_\pm = g^{(+) - 1} g^{(-)} \]

(IV.3)

and

\[ \sigma_\pm = |f_\pm|^2 + |g_\pm|^2 \]

\[ G_\pm \rho_\pm = 2 \gamma_m \left[ f^\pm g^\pm \right] \]

(IV.4)

One can now use the information coming from the charge-exchange \( \pi^- p \rightarrow \pi^0 n \), which, from charge independence, depends only on the antisymmetric combinations \( f^{(-)} \) and \( g^{(-)} \). At high energies (\( > 3 \text{ GeV} \), for instance), it is experimentally known that the charge-exchange cross-section is much smaller than the elastic \( \pi^\pm p \) cross-sections. Therefore in the first relation one can neglect \( f^{(-)} g^{(-)} \).
in comparison with $f^{(+)} g^{(+)*}$. They now use a further approximation, namely that $g^{(+)} \ll f^{(+)}$, so that $4|f^{(+)}| \approx \sqrt{\delta (\bar{u}_+ + \bar{v}_-)}$. This hypothesis is for instance true in the Regge pole model, and corresponds to the situation in which the forward elastic peak is mainly due to non-spin flip effects.

Then dividing the first relation by $4|f^{(+)}|$, one gets

$$\sqrt{\delta (\bar{u}_+ - \bar{v}_-)} g^{(+)} \sin \phi = |q^{(+)}|,$$

where $\phi = \arg [\bar{f}^{(+)}] - \arg [\bar{g}^{(+)*}]$.

The experimental results are shown in Fig. 3, together with the Regge pole model predictions. They show that $\sin \phi$ is positive at 6 GeV/c, and becomes negative at 12 GeV/c. A similar behaviour cannot be reproduced by a Regge pole model including only the $P$ and $P'$ trajectories. A similar argument is obtained from the second relation, if one neglects $f^{(-)} g^{(+)*}$ as compared with $f^{(+)} g^{(-)*}$.

B - The polarization in $\pi^- p \rightarrow \pi^0 n$ charge-exchange

We remember that this polarization has been measured at 5.9 and 11.2 GeV/c; its mean value in a given momentum transfer interval turns out to be

$$\langle \rho_{5.9} \rangle = (14.4 \pm 3.0)\% \quad \langle \rho_{11.2} \rangle = 0.8 \pm 0.2 \quad \frac{\langle \rho_{11.2} \rangle}{\langle \rho_{5.9} \rangle}$$

$$\langle \rho_{11.2} \rangle = (13.0 \pm 2.3)\%$$

(IV.6)
This non-zero polarization is a difficulty in a pure Regge pole theory including only the $\gamma'$ exchange, since when only one Regge pole is exchanged the non-spin flip and spin flip amplitudes are in phase so that the polarization vanishes.

Many explanations have been offered to explain the experimental situation, always remaining more or less in the framework of a modified Regge theory.

We shall briefly review some of them, in which always a new contribution interferes with the dominant Regge exchange.

a) The existence of another trajectory with the quantum numbers of the $\gamma'$, the $\gamma'$ \cite{98}. This trajectory was originally introduced in order to explain the experimental features of the pn and $pp$ total cross-sections and of the $np$ charge-exchange, using only $\gamma$ and $\bar{\pi}$ exchange (without $\bar{\pi}$ and conspiracies). The value of $\alpha(\gamma')(0)$ obtained in this way was $\alpha(\gamma')(0) = -0.63$, which gives a too quick decrease of the polarization with the energy. However, probably it is not reasonable to neglect the $\bar{\pi}$ contribution in np charge exchange, and therefore this value of $\alpha(\gamma')(0)$ is not reliable. In order to fit the actual energy behaviour of the polarization as given before, an intercept difference $\alpha(\gamma') - \alpha(\gamma) \approx 0.45$ would be necessary, resulting in a value $\alpha(\gamma')(0) \approx 0.1$ \cite{99}.

One has to notice, however, that Rarita and Schwarzschild \cite{101} claim the necessity of a $\gamma'$ with $\alpha(\gamma')(0) \approx -0.48$, in order to fit the $K^+n$ charge exchange at 2.3 GeV/c.

Independent evidence for a $\gamma'$ comes from the analysis of the finite energy sum rules \cite{102}.

A different possibility is that the $\gamma'$ belongs to a Toller pole of Class III (parity doublet, see Section I.d), in which case it would not be coupled to the $\bar{\pi}^-\pi^0$ system at $t = 0$, but only at $t \neq 0$. It will therefore contribute to the
polarization, but not to the total cross-section differences. This possibility has been explored by Sertorio and Toller\textsuperscript{103}, who found that a "conspiring" $g'$ should be preferred on a statistical basis to a "non-conspiring Class I" $g'$.

b) The effect of tails of low energy resonances at high energy\textsuperscript{104,105}. This model gives probably a fit to the polarization at 5.9 GeV/c, but the extrapolation of the results of the model to an energy as high as 11.2 GeV/c requires the computation of the effect of the resonance tails very far from the position of the resonances themselves, of doubtful physical meaning. Moreover, the model gives a too rapid decrease of the polarization with the energy.

c) The effect of infinite Regge recurrences\textsuperscript{106} in the $s$ channel. This model has been studied by Altarelli et al.\textsuperscript{107}, and in more or less the same way by Desai et al.\textsuperscript{108}. One assumes that the baryon resonances, present in the $s$ channel, will continue to show up with a Regge recurrence of the form

$$\mathcal{J} = a \, s + b$$

(IV.7)

namely the spin is linearly related to $s$.

The crucial assumption is then the energy dependence of the elasticities and widths of the resonances. It was shown in Ref.\textsuperscript{107} that using simple reasonable forms for the functional dependence of these parameters and adjusting some constants in order to fit the known constants of the first resonances in the recurrences, it is possible to reproduce the $s$ and $t$ dependence of the polarization, even if those constants were varied in a rather wide range\textsuperscript{109}. In general, the direct channel contribution is almost imaginary, coming mostly from the partial waves in the neighbourhood of $J\sim \text{Re} \, \alpha(s)$. 

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However, both this model and the previous one are subjected to the general criticism against the Regge plus resonances interference model, which will be discussed in more detail later (Section VI).

d) Of course, a mixture of the previously discussed effects can be also considered \(^{100}\), introducing therefore more parameters, and the agreement of the theory with experiment is in general improved.

e) The effect of a cut in the angular momentum plane. This possibility was first discussed by de Lany et al. \(^{110}\) and reconsidered in more detail by Chiu and Finkelstein \(^{111}\).

The basic assumption is that the amplitude due to the \(\xi\) exchange interferes with that generated by a Regge cut due to the successive exchange of a \(\xi\) and a \(\rho\).

Then the position of the branch point generated by this double Regge pole exchange is \(^{112}\), assuming linear trajectories for the Regge poles

\[
\alpha_{\xi}(t) = \alpha'_{\xi} \frac{t}{t'_{\xi} + t'_{\rho}} \quad \alpha_{\rho} = \frac{\alpha'_{\rho} \alpha'_{\xi}}{t'_{\xi} + t'_{\rho}}
\]  \((IV.8)\)

The crucial point is now the form of the discontinuity across the cut. Chiu et al. \(^{111}\) assume that also the discontinuity has a branch point at \(J = \alpha_{\xi}\), and they are then able to show that, up to terms of the order of \(1/\ln E\), the phases of the contributions of the cut to both the helicity non-flip and the helicity flip amplitudes are the same, and given by

\[
\frac{\pi}{2} (1 - \alpha_{\xi}) + (\gamma + 1) \ell_{\xi}^{-1} \left( \frac{\pi}{2} \ln \left( \frac{E}{E_0} \right) \right)
\]  \((IV.9)\)

where \(E_0\) is the scale factor, and \(\gamma\) is a real constant, characterizing the order of the singularity at \(\alpha' = \alpha'_{\xi}\).
Within this approximation, the cut behaves as an effective pole, but with an energy dependent phase. One can then easily compute the polarization, which turns out to be

\[ P = \left( \frac{E}{E_0} \right) \alpha'_c - \alpha'_g \frac{\ln \left[ \frac{\pi}{2} \left( \alpha'_g - \alpha'_c + \frac{2}{\pi} (g+1) \frac{\ln \left( \frac{E}{E_0} \right)}{\ln \left( \frac{E}{E_0} \right)} \right) \right]}{\left[ \ln \left( \frac{E}{E_0} \right) + \left( \frac{\pi}{2} \right)^2 \right] \left( \frac{g+1}{2} \right)} \]  

(IV.10)

What matters in the polarization is therefore not only \( \alpha'_c \), which gives the bulk of the energy dependence, but also the "effective phase"

\[ \frac{\alpha'_c}{\alpha'_g} = \frac{\gamma}{\alpha'_c} - \frac{2}{\pi} (g+1) \frac{\ln \left( \frac{E}{E_0} \right)}{\ln \left( \frac{E}{E_0} \right)} \sim \alpha'_c - \frac{\gamma+1}{\ln \left( \frac{E}{E_0} \right)} \]  

(IV.11)

for \( E \gg E_0 \).

One finds then an interesting conclusion: there is a zero in the polarization at a value of \( t \) such that \( \alpha'_g = \alpha'_c \). This zero moves with the energy, and is given by

\[ t_0(E) = \frac{1}{\ln \left( \frac{E}{E_0} \right)} \frac{\gamma+1}{\alpha'_c - \alpha'_g} < 0 \]  

(IV.12)

since \( \alpha'_c < \alpha'_c \).

For \( |t| < |t_0|, \) \( \alpha'_g > \alpha'_c \). In this interval of \( t \) the polarization is positive, and if one varies the energy at fixed \( t \) \( \alpha'_c - \alpha'_g \) becomes smaller, and the polarization decreases; for \( |t| > |t_0|, \) the polarization is negative, and increases (in absolute value) with the energy.
Other explanations, which are in a certain sense related to the Regge cut model, have been proposed by Cohen-Tannoudji et al.\textsuperscript{113} and by Henzi\textsuperscript{114}.

Cohen-Tannoudji et al. try to enforce the unitarity condition in the Regge pole theory, including absorption corrections. Their parametrization of the amplitude, when reinterpreted in terms of singularities in the angular momentum plane, corresponds to the existence of poles and branch points. With this model they were able to give a fit to all the data on meson-baryon scattering at high energy, and in particular to the $\pi^- p \to \pi^0 n$ polarization.

Henzi uses an overlap function method, and tries to explain the existing data on $\pi^0 p$ elastic scattering and $\pi^- p \to \pi^0 n$ differential cross-section in terms of simple forms on the overlap function. This simplicity of the overlap function, when translated back to the scattering amplitude, yields corrections to the asymptotic (Regge) behaviour of the charge-exchange amplitude, which explain the polarization data.

Different explanations were given by Le Bellac\textsuperscript{115} using the coherent droplet model and by E. Bia\textsuperscript{116} using only the asymptotic expansion of the scattering amplitude.
V. - THE LARGE ANGLE SCATTERING

The most striking features of the new experimental data in the large angles region are (see Di Lella's report):
1) the break found in the fixed angle proton-proton angular distribution at $P_{inc} \sim 6 \text{ GeV}/c$;
2) the fact that the $pp$ angular distribution can be fitted as
   $$\frac{d\sigma}{dt} = \beta \ e^{-\frac{5 s \ tan^2 \theta}{g}}$$
   where $\beta$ and $g$ are different constants in the regions before and after the dip.

Many explanations have been tried for point 1), and point 2) has also been discussed in connection with the lower bound on the scattering amplitude. We shall list briefly here the relevant models.

a) Kokkedee and Van Hove $^{117}$ have explored in detail the possible implications of the fact that the energy at which the break is present corresponds to the threshold for the baryon-antibaryon pair production. This coincidence was already pointed out by the authors of the experiment $^{118}$.

The physical idea is that, in contrast with the peripheral mechanism of resonances production, which involves many partial waves, the $\bar{p}\bar{p}$ production is a rather central process, which therefore involves only few low partial waves.

The part of the overlap function which comes from the $\bar{p}\bar{p}$ production should therefore be a slowly varying function of the angle (isotropic if only the $s$ wave is present) and generated by unitary an elastic amplitude which, although very small inside the diffraction peak as compared with other contributions, shows up at large angles.

They have also tried an estimate of the $\bar{p}\bar{p}$ production cross-section, which is necessary to generate the effect, and which turns out to be of the correct order of magnitude.
Huang et al. have proposed an explanation based on the simultaneous of a single Pomeranchuk Regge trajectory and of a Regge cut, due to a double Pomeranchuk exchange. They argue that, since the slope of the Pomeranchuk trajectory is rather small, its trajectory would cross the point $\alpha = -1$ roughly at the value $t \approx 7 \text{(GeV/c)}^2$, which corresponds to the position of the observed break at $90^0$. At $\alpha = -1$ all the transitions are nonsense, on a right signature point. As we have discussed in Section II, the trajectory cannot choose sense here, however, the mechanisms 3 and 4 predict that an amplitude nonsense-nonsense vanishes at this point. In our case, this means that both the non-spin flip and the spin flip coupling of the Pomeranchuk to the $\bar{p} p$ pair are zero at $\alpha = -1$. Since the contribution of the pole vanishes at $\alpha = -1$, at this value the cut contribution, otherwise small, dominates the amplitude.

However, Chiu et al. have noticed that this model predicts a wrong fixed $s$ angular dependence near the break region. This fact is connected with the assumption of the model that the position of the break, being due to a property of the Pomeranchuk trajectory $\alpha(t)$ depends only upon $t$, while in the CERN experiment the relevant variable is $s \cdot \sin \theta$.

Sakmar and Wojtasek have instead used only a Pomeranchuk trajectory with $\alpha'(0) \approx 0.2$. If this trajectory crosses the value $\alpha = 0$ at $t = -7 \text{(GeV/c)}^2$, then now the break should be connected with this fact. The flatter behaviour of the cross-section beyond the break is interpreted now as being due to the fact that both the Legendre polynomial and the other $\alpha$ dependent terms which do appear in the amplitude are slowly varying functions of $\alpha$ for $\alpha < 0$.

The same criticism which can be raised against the previous model can also be considered here, namely the model predicts that the position of the peak is only a function of $t$.  

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Both this model and the previous one are based on the badly known slope of the Pomeranchuk trajectory. The first assumes \( \alpha' = -1 \) at \( t = -7 \, (\text{GeV}/c)^2 \), the second \( \gamma = 0 \) at the same \( t \), and therefore their conclusions are extremely model dependent.

d) Chiu et al. \(^{120}\) have proposed a model, in which the angular region before the break is dominated by some \( t \) channel exchange, as Regge pole exchange, while beyond the break the scattering amplitude saturates its lower bound, and the Regge-like contribution has already disappeared. This is a quite interesting point, since the fixed \( \theta \) behaviour found in the experiment is not in agreement with the Cerulus-Martin lower bound

\[
|f(s, \cos \theta)| \geq \exp \left[-a \, s^{\frac{1}{2}} \ln s \right]
\]

where \( a \) is a constant (in \( s \)), whose value depends upon \( \theta \).

However, Chiu and Tan \(^{122}\) were able to show that the actual fixed angle lower bound is related to the upper bound in \( t \) which is assumed for the amplitude. If one takes a different upper bound in \( t \) than that corresponding to the Cerulus-Martin bound, then the more general lower bound becomes

\[
|f(s, \cos \theta)| \geq \exp \left[-a \, (\gamma) \, s^{\gamma} \ln s \right]
\]

with \( \gamma \geq \frac{1}{2} \).

In particular, when the Regge trajectories are linearly rising in \( t \), \( \alpha' = \alpha_0 + \alpha_1 t \), for all \( t \), one has \( \gamma = 1 \).

This model predicts that, as the angle is decreased from 90°, the value of \( |t| \) at which the break appears should also decrease, having so a better chance to agree with experiment. However, no detailed analysis of the new CERN data \(^{118}\) were performed along the lines of this model.
A weak point of this model is to explain why the cross-section before the break, which is supposed to follow a different mechanism from that which operates beyond the break, has the same functional dependence \( e^{-\frac{s \sin \theta}{a}} \), and seems therefore to saturate already a lower bound, although with a different value of the constant \( a \).

e) Finally, Islam and Rosen \(^{123}\) have used a relativistic optical model, with three complex energy dependent potentials, each representing different regions of the spatial proton distribution. The most external potential is responsible for the diffraction, the other gives effects in the large angle region. The break occurs just when the most inner potential becomes dominating over the intermediate potential. The ranges of these two potentials turn out to be \( \sim 0.4F \) and \( 0.26F \).

More than a model, this can be considered as a parametrization of the data; moreover, the energy behaviour at fixed \( t \) is not consistent with the observed behaviour, if the potentials are Yukawa-like, as taken in this paper.

As a general comment, we can notice that till now none of the proposed models have given any explanation of the rather unusual variable \( s \cdot \sin \theta \) which seems to fit in a simple way the experiment. The connection of the actual form of the lower bound with the asymptotic properties in \( |t| \) is also very interesting.
VI. - BACKWARD SCATTERING AND RELATED PROBLEMS

As discussed in the report of di Lella, the high energy data on $\pi^+ p$ backward scattering are consistent with a baryon Regge pole exchange picture, the nucleon trajectory only contributing to $\pi^+ p$ with $\alpha_N(0) = -0.24 \pm 0.05$, and the $\Delta$ trajectory contributing to $\pi^- p$ with $\alpha_\Delta(0) = -0.14 \pm 0.06$.

We would like to discuss here three points which are connected with backward scattering:

A) The criticism against the interference model (Regge + resonances in the direct channel), recently raised on the basis of finite energy sum rules;

B) The evidence for a parity doubling in the baryon trajectories;

C) The application of baryon-exchange models to the actual explanation of the proton-antiproton annihilation in two pseudoscalar mesons.

A. Interference model

It is well known since a long time that a scattering amplitude should satisfy the analyticity properties, namely the dispersion relations, puts severe constraints on the Regge pole parameters.

In general, these constraints are imposed in the form of sum rules. The first historical application of a sum rule to a Regge pole theory is due to Igi \(^{124}\), and led him to the discovery of the $P^+$ trajectory. Very recently, the interference model of Berger and Cline \(^{125}\) was criticized on the basis of sum rules \(^{102},^{126}\).

Essentially, the interference model writes the scattering amplitude as the sum of Regge pole terms, exchanged in the crossed channel ($t$ channel for meson exchange, $u$ channel for baryon exchange), plus resonance terms, coming from the contribution of the direct $s$ channel.

$$\mathcal{T}(s, t) = \mathcal{T}_{\text{Regge}}(s, t) + \mathcal{T}_{\text{Res}}(s, t)$$  \hspace{1cm} (VI.1)
The fact that such an assumption can lead to results which contradict analyticity can be simply shown if one considers the imaginary part of the symmetric pion-nucleon amplitude

$$\tilde{T}(+)^{1} = \frac{T^+_n + T^-_n}{2}$$

at \( t = 0 \), expressed by the sum \((\tilde{G}^+_n + \tilde{G}^-_n)/2\) through the optical theorem.

Then the dispersion relation for \( \sigma^{(+)}/2 \) can be cast in the form of a sum rule

$$\frac{f^2}{2} \int_1^\infty \sigma^{(+)}(\gamma) - \sum_i \beta_i \nu \sigma^{(+)}(\nu) \, d\nu = 0 \quad (VI.2)$$

where \( f^2 \) is the \( \pi N \) coupling constant, and \( \beta_i \nu \sigma_i^{+} \) are the contributions of all the Regge poles with \( \sigma_i^{+} \geq -2 \).

However, following the interference model

$$\sigma^{(+)} - \int_{\Delta} f_{\mu\nu}^+ = \int_{\Delta} f_{\mu\nu}^- \geq 0$$

and therefore the integral extends over a positive definite function, which can never give the negative result \((-f^2\Delta^2)/2M\). Therefore the interference model, as defined below, is not compatible in the entire energy range with the dispersion relations.

The sum rule (VI.2) teaches us however that if we include also the nucleon pole in the dispersive integral, the (smooth) Regge behaviour should be equal to the averaged value of the complete amplitude.

This property can be also traced out in the analyticity condition, expressed in the form of finite energy sum rules (FESR); again using analyticity and expressing the asymptotic behaviour of the amplitude down to an energy \( N \) as the sum of all possible Regge poles, one can obtain the FESR \( 102 \).
\[ \frac{1}{N} \int_{0}^{N} F(\nu) d\nu = \sum_{\text{all } i} \frac{\beta_i \alpha_i}{1 - (\alpha_i + 2)} \]

which again says that the Regge terms express the mean value of the amplitude.

As a substitute of the interference model, Dolen et al. 102 have proposed the prescription

\[ T(s,t) = T_{\text{Regge}}(s,t) + T_{\text{Res}}(s,t) - \langle T_{\text{Res}}(s,t) \rangle \]

where the last term is the locally averaged value of the resonant amplitude.

One can therefore have to deal with different situations, which can be illustrated by different physical examples.

a)

The contribution of the resonances enter into the amplitude with opposite signs, so that \( \langle T_{\text{res}} \rangle \neq 0 \). In this case the prescription of the interference model and of formula (VI.4) are almost the same. This is the case in the antisymmetric pion-nucleon cross-section \( \sigma^{(-)} = (\sigma_{\pi^- p} - \sigma_{\pi^+ p})/2 \), related to the amplitude \( A^{(-)} \), in the symmetric spin flip amplitude \( B^{(+)} \), and in the backward \( \pi^- p \rightarrow p \pi^- \) scattering.

In this case both \( \langle T_{\text{res}} \rangle \) and \( T_{\text{Regge}} \) are very small as compared with the oscillations of \( T_{\text{res}} \). Therefore the results of any model in the region where the resonances are important is very sensitive to the resonance parameters. It is therefore possible to have a fit of the backward \( \pi^- p \rightarrow p \pi^- \) scattering using resonances alone 127, with slightly different resonance parameters such that \( T \sim \langle T_{\text{res}} \rangle \neq 0 \) and \( T_{\text{Regge}} = 0 \), or by assuming Regge alone 102, and taking \( T_{\text{res}} \sim \langle T_{\text{res}} \rangle = 0 \). This fact means that also the resonance parameters extracted from the interference model are doubtful 126.
b) The resonances enter in the amplitudes all with the same sign, and their oscillations are small as compared with the mean value of the amplitude. This is the case in the symmetric \( \bar{\eta} \ p \) total cross-section, as discussed below, namely in the amplitude \( A'(+), \) in the antisymmetric \( \bar{\eta} \ p \) spin flip amplitude \( B(-), \) and in the \( \bar{\eta}^+ p \to p \bar{\eta}^+ \) backward scattering. In this case, \( T_{\text{res}} \sim \langle T_{\text{res}} \rangle, \) but also \( T_{\text{Regge}} \sim \langle T_{\text{res}} \rangle, \) so that both the resonances alone and the Regge alone are good representations of the scattering amplitude. This is in fact the case also for \( \bar{\eta}^+ p \to p \bar{\eta}^+ \) scattering, where the resonances \( ^{128,129,130} \) alone saturate the backward cross-section in the intermediate energy region, but where the Regge amplitude \( ^{102,126} \) as extrapolated from higher energy data, gives also the magnitude of the measured cross-section. For the same reason it is possible to describe the dip in the \( \bar{\eta}^- p \to \bar{\eta}^- n \) (which is due to the amplitude \( B(-) \)) both by the Regge theory \(^{61} \) and using resonances alone, since all the dominant resonances have a zero in a range of \( |t| \) around \( 0.6 \) (GeV/c)\(^2 \) \(^{102} \).

In all these cases, the prescription of the interference model would give a double counting of the amplitude.

The same argument applies also to the explanation of Hoff \(^{131} \) of the elastic \( \bar{\eta}^- p \) angular distribution as due to a resonance effect alone.

The FESR have been recently used by Dolen et al. \(^{102} \), in order to determine the parameters of Regge trajectories, both in the form (VI.3) and in the form of higher moment sum rules

\[
\zeta_m \equiv \frac{1}{N^m+1} \sum_{0}^{N} \gamma \mu \int_{\mu} \gamma d \gamma = \sum \frac{\beta \gamma \mu}{\omega_{\gamma} \mu} \cdot \frac{N^\alpha}{\mu \omega_{\gamma} \mu + 1} \cdot \frac{\Gamma_{\mu} \omega_{\gamma} \mu + 1}{\mu \omega_{\gamma} \mu + 1} \tag{VI.5}
\]

These sum rules have been used not only in the forward direction, but also for \( t \neq 0 \). In order to use them, one has therefore to have a knowledge of \( \text{Im} F(\gamma, t) \) for \( t \neq 0 \). In general, this requires an analytic continuation in \( t \) (or \( \cos \theta \)), since one needs the amplitude at fixed \( t \) for all the energies from \( N \) down to the threshold, and for \( t \neq 0 \) there is an energy interval in which this value of \( t \) is outside the physical region.
This analytic continuation has been actually performed using the partial wave expansion, namely continuing a finite number of Legendre polynomials for $|\cos \theta| > 1$.

Although this procedure is difficult to be justified in a general way, the results obtained are encouraging. In fact, it was found that the amplitudes $A^{(-)}(\nu, t)$ and $B^{(-)}(\nu, t)$ change sign respectively at $t \approx -0.1$ and $t \approx -0.6$, as it is expected from the phenomenological analysis of the cross-over effect $^9$ and of the dip in $\pi^- p \rightarrow \nu^0 n$ $^{51)}$. This change of sign in $B^{(-)}$ is due to the fact that the neutron pole contribution is the only negative contribution to the FESR, and its importance becomes bigger and bigger when $|t|$ increases, overwhelming all the other resonance contributions. The same procedure was recently applied to $\pi^+ p$ backward scattering $^{152}$, and it was found that the residuum of the $N_\alpha$ trajectory changes sign around $u \approx -0.2$ (as expected from the observed dip in $\pi^- p$ backward scattering), that the residuum of the $\Delta$ trajectory does not change sign, (as expected from the smooth behaviour in $u$ of the $\pi^- p$ backward scattering), and that the residuum of the $N_\gamma$ trajectory is quite smaller than that of the $N_\alpha$ trajectory.

**B. - The evidence for a parity doubling in the baryon trajectories**

The evidence for an approximate parity doubling of baryonic SU(3) multiplets has been recently discussed by Barger and Cline $^{133)$. The foundation of these considerations is the Mac Dowell reflection symmetry $^{13)$, as applied to linearly rising Regge trajectories.

Mac Dowell symmetry asserts that in pseudoscalar-nucleon scattering, the partial wave scattering amplitude of total angular momentum $J$, signature $\tau$, and parity $\rho$, taken at a given value of the total energy $W = \sqrt{s}$, is connected with the partial wave amplitude of total angular momentum $J$, signature $\tau$, and parity $-\rho$ at the value $-W$ by the relation

$$\frac{J}{\tau, \rho} (\omega) = - \frac{J}{\tau, -\rho} (-\omega)$$

(VI.6)
Suppose now that there exists a Regge pole at \( J = \alpha(w) \) of signature \( \tau \) and parity \( \sigma \); then \( \mathbb{h}_\tau^J_\sigma(w) \) has the form

\[
\mathbb{h}_\tau^J_\sigma(w) = \frac{\beta_\sigma(w)}{J - \alpha_\sigma(w)}
\]

(VI.7)

Then the Mac Dowell symmetry imposes the existence of another Regge trajectory, of signature \( \bar{\tau} \) and parity \( -\sigma \), passing through \( \alpha_{-\sigma} = J \) for the (unphysical) value \(-w\) of the energy:

\[
\mathbb{h}_\tau^J_{\bar{\tau}}(-w) = \frac{\beta_{-\sigma}(-w)}{-J - \alpha_{-\sigma}(-w)}
\]

(VI.8)

with

\[
\alpha_\sigma(w) = \alpha_{-\sigma}(-w), \quad \beta_\sigma(w) = -\beta_{-\sigma}(-w)
\]

(VI.9)

We stress that this second trajectory satisfies this equality at a negative energy \(-w\); in order to find the physical particles lying on this opposite parity trajectory, one must be able to continue \( \alpha_{-\sigma}(-w) \) from a negative to a positive value of the energy. This continuation requires of course a knowledge of \( \alpha(w) \) as a function of \( W = \sqrt{u} \) (we define \( u \) = square of the total energy, since we will be interested in the case in which the baryon trajectories are exchanged in the \( u \) channel).

Suppose now that \( \alpha(w) \) is an even function of \( W \) (as happens in particular if \( \alpha(w) \) is a straight line in \( W^2 = u \)); then \( \alpha(-W) = \alpha(w) \), and the analytic continuation is trivial. Then one concludes that the trajectories with parity = \( \sigma \) and parity = \( -\sigma \) will coincide, or almost coincide, if \( \alpha(w) \) is almost symmetric.
For the nucleon SU(3) octet, the octet $N_\alpha$ and the octet $N_\beta$ have opposite parities, and are related by the Mac Dowell symmetry. In Fig. 4 taken from Ref. 33, are reported all the known resonances belonging to those multiplets. For those resonances whose quantum numbers are experimentally known, as the $N_\alpha(1688, \frac{5}{2}^+)$ and $N_\beta(1650, \frac{3}{2}^-)$ the symmetry is remarkable. Moreover, together with the settled resonances as $\Sigma_\beta(1765, \frac{5}{2}^-)$ and $\Lambda_\beta(1827, \frac{5}{2}^-)$, a $\Lambda(1810)$ and a $\Sigma(1930)$ have been found, which could belong to the $\frac{5}{2}^+$ recurrence of the $\Lambda$ and $\Sigma$; their mass difference is certainly compatible with the effect of the symmetry breaking.

Similarly, a $\Sigma(2260)$ and a $\Lambda(2340)$ have been found, which fit nicely as the $\frac{5}{2}^+$ recurrence. On the basis of the measured elasticities, Barger and Cline have tentatively assigned the $\Sigma(2260)$ to the $\Sigma_\beta$ and the $\Lambda(2340)$ to the $\Lambda_\beta$; if this assignment is correct, this would constitute a further proof of this (near exact) parity doubling also for $J = \frac{9}{2}$.

An indirect proof that for the nucleon trajectory the odd terms in $W$ of $\mathcal{V}_N(w)$ are small is given by the existence of the dip in backward $\Pi^+p$ scattering. In fact, at $u \sim 0.2$ (GeV/c)$^2$ Re $\mathcal{V}(w) = -\frac{1}{2}$, but this condition almost coincides with $\mathcal{V} + \frac{1}{2} = 0$ only if Im $\mathcal{V}(w)$ is small, which means that, at $w^2 < 0$, the odd terms in $W$ are small.

In this scheme, a weak point is the non-existence of $\frac{1}{2}^-$ particles on the $\beta$ trajectory. However, the Mac Dowell symmetry, as applied also to the residuum function, implies that, if $\mathcal{V}(w) = \mathcal{V}(-w)$, $\beta_p(w) = -\beta_{-p}(-w)$, and therefore $\beta(w)$ should change sign somewhere passing from the $N_\alpha$ to the $N_\beta$ trajectory; since the non-existence of the $\frac{1}{2}^-$ octet imposes that $\beta_{-p}(m_{1\frac{1}{2}}) = 0$, one can assume that $\beta(w)$ changes sign just at the value $\frac{1}{2}$ of $J$.

From the interference model, other $N_\alpha$ and $N_\beta$ resonances are found, with the following quantum numbers (Table IV).
It is then tempting \(^{132}\) to assume that they all belong to a Teller family, the \(N^\alpha\) and \(N^\beta\) being the parent trajectory, symmetric in the \(W\) plane, the \(N^\gamma\) the first daughter, whose \(\alpha(W)\) is now antisymmetric in \(W\), and the other \(S_{11}\) and \(P_{11}\) resonances lying on a second daughter trajectory (Fig. 5). The mechanism of killing the \(\frac{1}{2}^-\) resonance would be only effective on the parent trajectory. One has to notice, however, that not all the resonances listed in Table IV are classified here, and moreover that in some case the spin-parity or the family assignment is different. The \(N^\gamma(2640, \frac{11}{2}^-)\) of Table IV classified as \(N^\alpha (2640, \frac{9}{2}^-)\) in Fig. 5, and the \(N^\gamma(3020, \frac{15}{2}^-)\) is now \(N^\alpha(3020, \frac{11}{2}^-)\).

The puzzling situation is now that of the \(\Delta\) trajectory, which is very well represented by a straight line in \(W^2\), passing through the \(\Delta(1238, \frac{3}{2}^+), \Delta(1924, \frac{7}{2}^+)\) and the \(\Delta(2450), \Delta(2840)\), \(\Delta(3220)\) of proposed \(\frac{11}{2}^+, \frac{15}{2}^+\) and \(\frac{19}{2}^+\) spin-parities. A nice straight line indicates that the odd terms in \(\alpha(W)\) are extremely small, so that one should expect an almost exact parity doubling.

The apparent absence of the negative parity resonances could be explained with the ugly assumption that \(\beta(W)\) vanishes on all the relevant values of \(W\), where the resonances should be present.

One can, however, notice that the recently found \(\Delta(2020)\) \(^{135}\) could well be the \(\left(\frac{7}{2}^-\right)\) Mac Dowell reflected of the \(\Delta(1924, \frac{7}{2}^+)\), so that there is also the possibility of the existence of the odd parity partners of the \(\Delta\) trajectory.

C. Proton-antiproton annihilation into two pseudoscalar mesons

It was already noticed a long time ago \(^{136}\) that, at high energy, the backward \(\pi^+p\) scattering

\[\pi^+p \rightarrow p \pi^+ / \pi^-p \rightarrow p \pi^-\]  \(\text{(VI.10a)}\)
and the annihilation

\[
\bar{\rho}\rho \rightarrow \pi^+\pi^-
\]  \hspace{1cm} (VI.10b)

for small angles respectively of the final \(
\pi^+
\) or \(\pi^-
\) with respect to the proton direction should be connected together.

In fact, these two reactions are connected by crossing, and a Pomeranchuk like argument \(^{137,138}\) gives that, in the limit of infinite \(s\)

\[
\left( \frac{d\sigma}{du} \right)_a = 2 \left( \frac{d\sigma}{du} \right)_b ,
\]

\(s \rightarrow \infty\) \hspace{1cm} (VI.11)

At high, but finite energy, the kinematic relation is more complicated, and moreover it is possible that non-asymptotic terms also enter into the game.

In order to compare these two reactions at finite \(s\), one needs therefore a model which allows to relate explicitly the reactions (VI.10a) and (VI.10b).

Barger and Cline \(^{139}\) have recently attempted to derive the cross-section for reaction (VI.10b) from the existing knowledge of the \(\bar{\eta}^+p\) backward scattering (VI.10a).

Under the simplifying assumptions that the \(\bar{\eta}^+p\) backward scattering can be represented by the only \(N\alpha\) exchange with a straight line trajectory and a constant residuum, and \(\bar{\eta}^-p\) backward scattering by the \(\Delta\) exchange, with the same assumptions, then the reactions (VI.10a) and (VI.10b) are connected by

\[
\left( \frac{d\sigma}{du} \right)_b = \frac{1}{2} \left( \frac{d\sigma}{du} \right)_a \cdot \frac{\left[ s - (m_\pi^+)^2 \right] \left[ s - (m_\pi^-)^2 \right]}{s \left[ s + u + 2 \left( m^2 - m_\Delta^2 \right) \right]} \]
\hspace{1cm} (VI.12)
In order to give an estimate of the integrated cross-section for (VI.10b), one has, however, to take care of another kinematical fact.

The relation (VI.12) connects the two reactions at the same value of \( u = (p_p - p_{\pi^-})^2 \); however, the two physical regions in \( u \) of the two reactions are quite different.

In fact, in backward scattering, the physical region extends from the positive value \( u_B^a = (M^2 - \mu^2)^2/s \) through \( u = 0 \) down in the negative \( u \) region; for annihilation, the physical region starts only at the negative value \( u_B^b = -(\sqrt{s/4 - \mu^2} - \sqrt{s/4 - \mu^2})^2 \).

Of course, when \( s \to \infty \), both \( u_B^a \) and \( u_B^b \) tend to \( u = 0 \), one from the right and the other from the left; however, at finite energy, there is a finite interval, extending from \( u_B^a \) to \( u_B^b \), allowed for backward scattering but forbidden for the annihilation (Fig. 6).

Therefore to estimate the integrated annihilation cross-section one has to integrate the measured backward scattering angular distributions only from the (negative) value of \( u_B^b \) down in the negative \( u \) region.

This kinematical restriction is extremely important, since the backward \( \pi^+p \) is extremely peaked near \( u = u_B^a \), so that, in the few GeV range, at \( u = u_B \) most of the backward cross-section has already disappeared.

An estimation of the \( \bar{p}p \to \pi^+\pi^- \) integrated cross-section, based on such a model, gave \( 1.5 \mu \) b at \( \sqrt{s} = 2.9 \) GeV, while a recent measurement \(^{140}\) gave at the same energy \( 6 \pm 3 \mu \) b, (on the basis of three events), which is of the correct order of magnitude.
**- Table IV -**

<table>
<thead>
<tr>
<th>Resonance (Mass in MeV)</th>
<th>Spin-parity $J^P$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \delta$ (1236)</td>
<td>$3/2^+$</td>
</tr>
<tr>
<td>$\Delta \delta$ (1924)</td>
<td>$7/2^+$</td>
</tr>
<tr>
<td>$\Delta \delta$ (2450)*</td>
<td>$11/2^+$</td>
</tr>
<tr>
<td>$\Delta \delta$ (2640)*</td>
<td>$15/2^+$</td>
</tr>
<tr>
<td>$\Delta \delta$ (3220)*</td>
<td>$19/2^+$</td>
</tr>
<tr>
<td>$N \gamma$ (1512)</td>
<td>$3/2^-$</td>
</tr>
<tr>
<td>$N \gamma$ (2210)*</td>
<td>$7/2^-$</td>
</tr>
<tr>
<td>$N \gamma$ (2640)*</td>
<td>$11/2^-$</td>
</tr>
<tr>
<td>$N \gamma$ (3020)*</td>
<td>$15/2^-$</td>
</tr>
<tr>
<td>$N \gamma$ (3350)*</td>
<td>$19/2^-$</td>
</tr>
<tr>
<td>$N \alpha$ (938)</td>
<td>$1/2^+$</td>
</tr>
<tr>
<td>$N \alpha$ (1688)</td>
<td>$5/2^+$</td>
</tr>
<tr>
<td>$N \alpha$ (2220)*</td>
<td>$9/2^+$</td>
</tr>
<tr>
<td>$N \alpha$ (2610)*</td>
<td>$13/2^+$</td>
</tr>
<tr>
<td>$N \alpha$ (2970)*</td>
<td>$17/2^+$</td>
</tr>
<tr>
<td>$N \alpha$ (~1470)</td>
<td>$1/2^+ (P_{11})$</td>
</tr>
<tr>
<td>$N \rho$ (~1560)</td>
<td>$1/2^- (S_{11})$</td>
</tr>
<tr>
<td>$\Sigma \rho$ (~1550)</td>
<td>$5/2^- (D_{15})$</td>
</tr>
<tr>
<td>$\Delta \rho$ (~1690)</td>
<td>$1/2^- (S_{31})$</td>
</tr>
<tr>
<td>$N \beta$ (~1715)*</td>
<td>$1/2^- (S_{11})$</td>
</tr>
</tbody>
</table>

*) In this table, the "conjectured" resonances have been indicated by an asterisk.
VII. - THE DOUBLE REGGE EXCHANGE MODEL AND THE THREE-BODY FINAL STATES

During the last year, many attempts have been put forward of producing a theory which could explain the production processes with three bodies in the final state, and which do not belong to the "quasi-two body" class 141)-145).

The idea is a particularization of the old multiperipheral model 146) as applied to the actual three-body final state, and with stronger restrictions on the momentum transfers, which come from the Reggeized exchange and from the t dependence of the vertices.

Essentially one observes that, since any two-body reaction is at high energy dominated by peripheral exchanges, which are well described by a Regge theory, it is probable that a double Regge model (Fig. 7) would be useful when each pair of relative energies $s_{ij}$ is large.

In order to compare the model with experiment, one has therefore to select only the events which are of "high energy" in all the three relative energies of the final particles, cutting out from the Dalitz plot of the three final particles the low-energy resonance region in all the three channels (Fig. 8, regions I, II, III).

For the remaining central region IV, one can give general predictions, which do not depend upon detailed properties of the model, but only upon the "diffractive" behaviour of the vertices, namely the fact that, as in elastic scattering, they decrease very rapidly with the momentum transfer. Those properties are:

a) the centre of the Dalitz plot is depopulated;

b) a diagram with a given central particle (particle 4 in Fig. 7) gives an appreciable contribution only in one corner of the Dalitz plot (Fig. 8). The three corners correspond therefore to different classes of diagrams, having a different particle as a central one.
As a consequence, if one class of diagrams is forbidden by some selection rule, as for instance when the quantum numbers of the exchanged objects a or b do not correspond to any existing particle, the corresponding corner of the Dalitz plot will be empty.

This prediction is clearly verified experimentally in the reaction \((\pi^- \rho \rightarrow K^0 \bar{K}^0 n)\).

In fact, the only possible diagram is that of Fig. 9. All the other possible diagrams with different distributions of the final particles are forbidden, since they correspond to exchanges of objects which have a wrong relation between strangeness, charge and baryonic number. Therefore only one corner should be populated.

However, since it is not possible to distinguish experimentally between the \(K^0\) and \(\bar{K}^0\), and only the long-lived \(K^0\) are observed, both the corners corresponding to \(K^0\) and \(\bar{K}^0\) will be populated, but the neutron corner will be empty. This prediction is in agreement with experiment (Fig. 10).

The model, parametrized as a double exchange of Regge poles, allows to predict the distribution in all the kinematical variables in terms of a restricted number of parameters, some of which are taken from the two-body experiments as the Regge trajectories \(\alpha_1'(t)\). The remaining parameters have been also fitted \((142)\), but the statistics is still now too low to allow definite conclusions.

From these results, one can safely say that features as those shown in Fig. 10 (the "cornering effect") are an experimental proof that the production at high relative energies is mediated by a double peripheral mechanism, with vertices strongly damped in the momentum transfer.
Owing to a lack of statistics, there is not yet a direct proof that this double peripheral mechanism is actually a double Regge exchange.

The best experimental proof of the double Regge exchange would be the energy-dependence of the differential cross-section in one of the \( s_{ij} \) relative square energies as a function of the corresponding momentum transfer, taking all the other variables fixed.

This proof would have the same relevance as the measured shrinking with the energy of the \( \pi^- p \rightarrow \pi^0 n \) (two body) charge-exchange.

A test like that requires an experiment with both high incident energy and statistics, in order to allow a large enough range of \( s_{ij} \) and a large number of bins with a significant number of events in \( s_{ij} \) and in the momentum transfer. Existing bubble chamber exposures are already in position to allow such analysis.

Finally we would like to mention that have been recently studied at CERN the asymptotic properties of the model \(^{148}\), which come from its different features as compared with the old multiperipheral model, and the properties of the density matrix elements of the decay of a resonance in a process like

\[
\begin{align*}
1 + 2 & \rightarrow 3 + 4 + 5 \\
\end{align*}
\]

when one of the final particles is a resonance \(^{149}\).
VIII. - COHERENT SCATTERING AND PRODUCTION ON LIGHT NUCLEI

The problem of coherent interactions of high energy elementary particles with light nuclei has received a lot of attention in the last year, both theoretically and experimentally, as a tool for obtaining elementary particle scattering amplitudes from nuclear scattering.

A series of experiments of elastic scattering on a variety of light nuclei targets (deuteron, helium, oxygen) has been recently performed with 1 GeV protons at the Cosmotron $^{150}$. We can notice two striking features of the elastic angular distributions:

i) The angular distribution $d \sigma / dt$ decreases very fast from $t = 0$, falling by three or four order of magnitudes; then suddenly it changes its behaviour and flattens; the change of slope happens at $t_0 \approx -0.35 (\text{GeV/c})^2$ for pD scattering, $t_0 \approx -0.25 (\text{GeV/c})^2$ for $p - ^4\text{He}$ scattering and $t_0 \approx 0.07 (\text{GeV/c})^2$ for $p - ^{16}\text{O}$ scattering. The value of the slope of $d \sigma / dt$ in the flatter region is, for $p - ^{4}\text{He}$ scattering, roughly half the value of the slope of the elastic proton-proton scattering in the same range of energy.

ii) Around the value $t = t_0$, there is a dip in the angular distribution; this dip is clearly seen in the $^{16}\text{O}$ and $^4\text{He}$ cases (Fig. 11), while the experimental situation, although compatible with a small dip, is not precise enough for the deuteron (Fig. 12).

Both of these features can be easily explained in terms of the Glauber shadow theory of multiple scattering $^{151}$.

Let us consider for simplicity the deuteron. The scattering amplitude in the laboratory system of a projectile $x$ on a deuteron target, at high incident momentum $\vec{k}$ and small momentum transfer $t = - (\vec{k} - \vec{k}')^2$, is expressed as (neglecting spin complications)
\[ f_{xd}(\frac{k}{2}, (\frac{k}{2}, -\frac{k}{2}), t) = \left[ G((\frac{k}{2}, -\frac{k}{2}), t) f_{xp}(\frac{k}{2}, (\frac{k}{2}, -\frac{k}{2}), t) + f_{xn}(\frac{k}{2}, (\frac{k}{2}, -\frac{k}{2}), t) \right] + \]
\[ + \frac{i}{2 n} \int d_{q} \mathcal{G}(q^2) \int \mathcal{D}(q, x) f_{xp}(\frac{k}{2}, (\frac{k}{2}, -\frac{k}{2}), t) f_{xn}(\frac{k}{2}, (\frac{k}{2}, -\frac{k}{2}), t) \]

(VIII.1)

where \( G(q^2) = \int d_{q} x e^{i q \cdot x} | \mathcal{D}(x) |^2 \) is the deuteron form factor, and \( q \) is a two-dimensional vector, orthogonal to the beam direction \( \bar{x} \).

\( f_{xp} \) and \( f_{xn} \) are the xp and xn scattering amplitudes.

The first two terms correspond to the single scattering amplitudes on proton or neutron, the third to double scattering.

One can easily verify that:

a) at \( t=0 \), the single scattering contribution is much bigger than the double scattering contribution \[ \text{the shadow correction to the total cross-section, obtained from the imaginary part of (VIII.1), is in general only a few per cent}\];

b) the single scattering amplitude decreases with \( |t| \) much faster than the double scattering amplitude; while the single scattering terms decrease essentially by the form factor effect, which is related to the deuteron size, the double scattering term, if for instance one parametrizes \( f_{xp} \) and \( f_{xn} \) as \( f_{xp}, f_{xn} \propto e^{(b/2)t} \), decreases only as \( e^{(bt/4)} \), independently of the form factor which only fixes the normalization of the double scattering amplitude at \( t=0 \); therefore at \( |t| \) larger than the value \( t_0 \), where the absolute values of the single and double scattering amplitudes are equal, the differential cross-section will behave as \( e^{(bt/2)} \); the values of \( t_0 \), estimated from the currently used wave functions and the known values of the parameters of the elastic nucleon-nucleon scattering, turn out to correspond correctly to the measured values of \( t_0 \) for \( p, ^4\text{He} \) and \( ^{16}\text{O} \).
c) if $f_{xp}$ and $f_{xn}$ are purely imaginary, then both the single and double scattering amplitudes are purely imaginary, but of opposite signs, so that they tend to cancel. At $t = t_0$ one would therefore expect a zero in the differential cross-section; this zero becomes a dip, which is filled very rapidly if there are small real parts in the scattering amplitudes; it turns out that the value of the cross-section at the dip is extremely sensitive to the value of the ratios

$$\alpha_{p,n} = \frac{\text{Re} \ f_{xp}(x,n)}{\text{Im} \ f_{xp}(x,n)}$$

taken at $t = t_0$; therefore a measure of the cross-section at the dip gives information about the variation of $\alpha$ with $t$ (at $t = 0$ $\alpha$ is known from the optical theorem and Coulomb interference and/or forward dispersion relations).

From the analysis of the $p$-$d$ and $p$-$^4$He experiments at 1 GeV, the following values of $\alpha_{pp}$, $\alpha_{pn}$ have been obtained

<table>
<thead>
<tr>
<th>$t \left(\left(\frac{\text{GeV}}{c}\right)^2\right)$</th>
<th>$\alpha_{pp}$</th>
<th>$\alpha_{pn}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (exp. rel.)</td>
<td>-0.05</td>
<td>-0.6</td>
</tr>
<tr>
<td>-0.24 ($^4$He)</td>
<td>-0.33</td>
<td>-0.33</td>
</tr>
<tr>
<td>-0.35 (D)</td>
<td>-0.6</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

(VIII.2)

and the $t$ variation can be represented as $\alpha_{n}^{p} = -\left(\frac{0.05}{0.6}\right) + 0.6t - 6.3t^2$ for $|t| \leq 0.4 \text{ (GeV/c)}^2$. 

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These numbers show quite a strong variation with $t$, but the result at $t = -0.35$ is rather uncertain, since the eventual dip is not accurately measured. It is also difficult to assign errors to those numbers.

An over-all fit to all the experimental elastic scattering measurements on all the available targets has been carried out by Franco \(^{152}\), choosing a form $f = f_0 e^{i \gamma t}$ for the scattering amplitude; he was able to fit all the cross-sections in the dip regions with $\gamma \approx 1$ (GeV/c)\(^2\).

Experiments in order to measure the dip and the subsequent shoulder in $\pi^+D$ and $\pi^+\text{He}$ elastic scattering are in preparation at CERN \(^{153}\).

The same kind of interference effects, with a dip-bump structure, are also expected to be present in the angular distribution of the coherent production of resonances in reactions like

$$\chi + A \rightarrow \chi + A$$

(VIII.3)

where $\chi$ is a resonance, and $A$ the target nucleus, which after the reaction has to remain in its ground state.

It has been shown that \(^{154},^{155}\), for reaction (VIII.3)

- the position of the dip is sensitive to the total $\chi$ nucleon cross-section;
- the differential cross-section at the dip is sensitive to the phase of the elastic $\chi$ nucleon scattering amplitude;
- the slope of the cross-section after the dip, in the case of deuteron scattering, is sensitive to the slope of the elastic $\chi$ nucleon differential cross-section.

Experiments of this kind are also planned in the near future, both with incident $\pi^+$ mesons \(^{156}\) and photons.
ACKNOWLEDGEMENTS

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REFERENCES


6) It is customary to say that a particle has "natural parity" if the relation between its parity and angular momentum is $P = (-1)^J$, that has "unnatural parity" if the relation is $P = (-1)^{J+1}$. Sometimes we shall speak of "different natural parities", and then we mean natural versus unnatural parity.


11) The relation between $\tilde{f}(t)$ and the previously defined parity conserving regular amplitudes is

$$\tilde{f}_{\pm}^{(t)}(\lambda_1, \lambda_2; \lambda_3, \lambda_4) = \tilde{f}_{\lambda_1, \lambda_2; \lambda_3, \lambda_4}^{(t)}(t) \pm \tilde{f}_{\lambda_1, \lambda_2; -\lambda_3, -\lambda_4}^{(t)}(t)$$

namely a relation similar to that between the $\tilde{f}_{\pm}(t)$ and $\tilde{f}(\pm)(t)$.

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12) Except a kinematical square root branch point at $s=0$ in a boson-fermion channel, which is connected with the existence of the Mac-Dowell symmetry 13).


15) This simple property under crossing is a consequence of the fact that in both channels one quantizes the spin along the same direction (the normal to the scattering plane), which is invariant under crossing.

16) The difference between Wang's and CTMN's results can be summarized as follows:
   a) CTMN have proved that RHA are free from kinematical singularities, while Wang has only shown that the removal of the singularities she proposes is consistent with crossing.
   b) Wang does not impose the constraint conditions, and so does not remove all the kinematical zeroes.

Moreover, since the definition of the normal direction used by CTMN is different from that used by Trueman and Wick 17) in obtaining the crossing matrix, there is the possibility of some different phases in the crossing matrix, which, however, do not affect the final results.


All the problems of the Reggeization of helicity amplitudes and constraints have been reconsidered in a simple and comprehensible (although not always rigorous) way by these authors in a series of recent papers 18), 19), 20).


28) In the unequal mass case, for t=0 one has $P_\mu P^\mu=0$, but the components of the four-vector $P_\mu$ are not all zero.


32) To use $O(4)$ symmetry instead of $O(3,1)$ means that one performs an analytic continuation from the real to the imaginary axis in the energy variable (Wick rotation). The fact that the results are the same can be possibly understood by noticing that both groups are contained in the complex Lorentz group; if the amplitude enjoys enough analyticity properties, then the invariance group would be the complex Lorentz group. I thank Dr. M. Toller for a discussion on this point.

33) This property has been also proved in Ref. 24) using standard arguments.

34) A set of poles with $\frac{1}{2}$ odd and quantum numbers $C = \sum_{\ell}$, $P = \sum_{\ell} (-1)^n \sum_{\ell}$ would appear in the unequal mass case, as shown in Ref. 27).


36) G.F. Chew, quoted in Ref. 30).


38) s channel contributions are always "conspiratorial" when looked from the t channel, in the sense that they always mix different parity exchanges in the t channel.


41) G. Cosenza, A. Sciarrino and M. Toller - Private communication.

42) For a review of the experimental situation of $\pi^-$ and $K$ mesons photoproduction, see the report of Panofsky at this Conference.

44) L. Jones - Preprint CALT. 68-126 (1967).
   The helicity amplitudes \( f_{0,0}^{(t)} \), \( \lambda_1 \), \( \lambda_2 \) are zero for all \( t \)
   from parity conservation if the reaction goes through
   natural parity exchange.


46) G.C. Fox, E. Leader and T.W. Rogers - Cambridge Preprint (1967),
    submitted to this Conference.

47) M. Le Bellac - "Pion Conspiracy in \( \pi^- N \rightarrow \eta \Delta \)", Argonne Preprint
    (1967).

48) See Ref. 49), where the experimental situation is summarized by
    these words, and Ref. 35).


50) Although it is, in principle, possible to fit the np charge
    exchange alone using only the \( g \) and \( R \), in order to
    explain the peak at very small \(|t|\), one needs the physically
    ridiculous assumption that the \( g \) residuum function
    varies as \((m^2/s_g)^{(\alpha_g(t)-1)/2}\), with
    \( s_g = 4.5 \text{ (MeV)}^2 \) \( 51 \).


52) We do not think that the procedure of getting the \( g \) and \( R \)
    residuum functions from total cross-section differences only,
    and then predict the np forward charge-exchange amplitude
    \( 53 \) is a meaningful procedure, due to the very
    big errors in the cross-section differences. If one esti-
    mates the \( g \) and \( R \) contribution by extrapolating smoothly
    to \( t=0 \) the np→pn differential cross-section for
    \(|t|>0.02 \text{ (GeV/c)}^2\), the result is again consistent with the
    total cross-section differences.

54) This result is correct for the $T_{nn}$ amplitude only if one considers the leading term in the expansion in $s$. The mirror reflected term around $\alpha = -\frac{3}{2}$, which behaves as $s^{-\alpha - 1}$, does not contain the factor $\alpha$. This non-asymptotic term is supposed to be cancelled by a compensating trajectory, passing through $-N - 1$ for $\alpha = 0$, at least when its presence generates a ghost.

55) If we want to avoid a ghost in a right signature point for $t < 0$, the products $\alpha J_{mn}^*$, $\frac{\alpha}{2} J_{sn}$ and $J_{ss}$ should vanish at $\alpha = 0$ in order to compensate the zero of $\sin \theta \alpha$. Here we choose always a multiplicative ghost killing, taking the $J_{ss}^*$ to be proportional to an ad hoc power of $\alpha$. An alternative possibility is that the finite part of (for instance) $\frac{1}{2} J_{ss}^* \alpha^0$ is cancelled by a fixed pole $s^0$. I thank Professor D. Amati for a discussion on this point.


59) There exists an argument by Chew 57), based on a N/D approach, which forbids the two assignments $1, \alpha^3$ and $\alpha^3, 1$. The only possibilities would consist in multiplying by the same power of $\alpha$ all the residuum functions in the choosing sense or choosing nonsense alternatives. The same result is also obtained using the Lorentz invariance at $P \alpha = 0$ (see later).
60) The presence of other contributions which are not connected with the \( P' \) exchange, as the \( P \) exchange, and which do not show any peculiar behaviour near the point \( t = t_0 \) where \( \varphi_{P'}^-(t_0) = 0 \) could displace the position of the dip from \( t = t_0 \). The first to propose that the dip in \( \tilde{\eta}^\pm \tilde{p} \) elastic scattering should be connected with a zero in the \( P' \) trajectory was: S. Frautschi - Phys.Rev.Letters 17, 722 (1966).


62) F. Arbab and C.B. Chiu - Phys.Rev. 147, 1045 (1966);

63) F. Arbab, N.F. Bali and J. Dash - UCRL Preprint 17 325 (1967), have made a detailed analysis of the \( \sigma \) and \( R \) sense-choosing mechanisms. Their result is that they are not able to distinguish between different mechanism, looking to the experimental data for the

\[
\tilde{\eta}^\pm \rightarrow n^0 n, \quad n^\pm \rightarrow \eta n, \quad k^\pm \rightarrow \bar{k}^0 n
\]

differential cross-sections, and to the

\[
\sigma(n^+ p) - \sigma(n^- p), \quad \sigma(k^+ n) - \sigma(k^- n), \quad \sigma(k^+ p) - \sigma(k^- p)
\]

total cross-section differences.

More precisely, they have tried the following possibilities for the \( \sigma \) and \( R \) trajectories at \( \alpha' = 0 \) (the number denotes the kind of mechanism 1 - 4)

\[
\begin{array}{ll}
\sigma & R \\
a) & 13 \\
b) & 12 \\
c) & 22
\end{array}
\]

The fit could hardly distinguish between alternatives a) and b), in both of which the \( \sigma \) chooses sense. The third possibility, in which the \( \sigma \) chooses nonsense, has the
unpleasant feature that the $\pi^- p \to \pi^0 n$ angular distribution should have a zero at $t = -0.6 (\text{GeV}/c)^2$. In order to reproduce the non-zero observed cross-section, one is forced to assume a further contribution to $\gamma$ exchange. In Ref. 63) a background contribution of the form

$$\gamma \sigma (a + bt) \gamma$$

was supposed, and the fit was obtained with $\gamma = -0.2$.


65) The measured values of the decay density matrix elements suggest, however, that also an unnatural parity exchange should be present ($\rho_{00} \neq 0$). For the suggestion of a $B(1^{++})$ exchange see, e.g., Ref. 66).


73) The difference between right and wrong signature points is that the third spectral function contributes to the left-hand cut discontinuity with two singular terms, which add at the wrong signature points, but cancel exactly at the right signature points.

75) A fixed pole of the form $s^N$, where $N$ is greater than 1 and independent on $t$ would violate the Froissart bound. If the external spins are high enough, $J=N>1$ could be an unphysical nonsense value.

76) "Les éléments mobiles sont aussi importants que les éléments fixes" (Jean-Luc Godard "Deux ou trois choses que je sais d'elle"). It is likely that, on the contrary, the effect of the moving pole is more important than the effect of the fixed pole.


79) M. Toller, Private communication.

80) We shall discuss here only the results which come from the combined assumptions of SU(3)+Regge pole models; other models are discussed in Lipkin's talk at this Conference.


82) We refer here always to meson-nucleon scattering.

83) This relation follows from

$$\langle k' \rho | k^0 \pi \rangle - \langle k' \pi | k^0 \rho \rangle = -\sqrt{2} \langle \pi \rho | \pi^0 \pi \rangle$$

using charge-independence and the optical theorem.

85) V. Barger and D. Cline - Phys. Rev. 156, 1522 (1967).
88) Ph. Salin - CERN Preprint TH. 762 (1967); see also Ref. 89).
99) The value quoted by Beaupre et al. 100 is slightly higher ( \( \Delta \gamma ' \approx 0.17 \)), but they used older polarization data, where the energy dependence was less marked.


109) Actually, the predicted energy dependence of the polarization found in Ref. 107) is in better agreement with the new experimental data 97) than with the other polarization results of the same group.


114) R. Henzi - CERN Preprint TH. 762 (1967), submitted to this Conference, and private communication.


116) E. Biaxas - Preprint TPJU 3-67 (1967).


126) C.B. Chiu and V. Stirling - CERN Preprint TH. 840 (1967).


130) S. Minami - Osaka Preprint (1967).


139) V. Barger and D. Cline - "NN Annihilation into Two Pseudoscalar Mesons at High Energy" Wisconsin Preprint (1967).


147) The essential difference between the model of Ref. 141) and the old multiperipheral model of Ref. 146) is that in the old model the momentum transfer damping was given by an elementary particle propagator, while in 141), a much stronger exponential damping is introduced through the Regge residuum functions.


V. Franco and R.J. Glauber – Phys.Rev. 142, 1195 (1966);
V. Franco – Phys.Rev.Letters 16, 944 (1966);
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For a relativistic generalization in the deuteron case, see:
E.S. Abors, H. Burkhardt, W.L. Toplitz and C. Wilkin – Nuovo Cimento 42, 365 (1966);
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V. Franco and E. Coleman – Phys.Rev.Letters 17, 827 (1966);
152) V. Franco - Private communication from H. Palevski.


J. Combe, M. Querrou, J. Gardès and J.P. Alard - "Proposition d'Expérience sur l'Etude de la Section Efficace Différentielle de Diffusion $\pi^- - ^4$He à une Energie Incidente de 1.12 GeV", PH III-67/18 (1967);


156) On isospin 0 nuclei, the production from incident $\pi$ beams of resonance of positive G parity and natural parity, as the $\rho$ is in general depressed since it cannot go via $\pi^-\omega$ exchange from isospin conservation and the $\omega$ exchange is depressed near the forward direction, since it cannot contribute to helicity zero $\rho$'s. The production of $\rho$'s on isospin zero targets near $t=0$ would be therefore a proof of the exchange of a meson with $I=0$, $G=-1$ and $P=(-1)^{J+1}$. On the contrary, the production of a resonance with $G=-1$ and $P=(-1)^{J+1}$, as the $A_1$ meson, can go via the Pomeranchuk exchange, so it is not depressed near $t=0$ and would be the best application of this theory.

One can notice moreover that the approximate selection rule $^{157)}$ (which we propose to call depression rule), saying that from a $0^-$ incident particle one can produce coherently only the series $0^-, 1^+, 2^-, \ldots$, is only valid at small $t$, and is probably no more true already in the dip region. In any case, however, it will completely alter the pattern of the angular distribution before the dip.

FIGURE CAPTIONS

Figure 1: A schematical definition of the kinematics.

Figure 2: Momenta and helicities in the proton-neutron charge exchange:
   a) the amplitude $\langle ++ | T | -- \rangle$
   b) the amplitude $\langle -- | T | ++ \rangle$

Figure 3: The value of $|g^{(+)}| \sin \varphi$ as a function of $t$ for the three energies of 6, 10 and 12 GeV/c; the continuous curves are the Regge pole predictions, the dots with errors are the experimental results (taken from Ref. 96).

Figure 4: The parity doublets of the octet baryon resonances (taken from Ref. 133).

Figure 5: A proposed nucleon Teller family, containing the parent trajectory ($N_\alpha$ and $N_\beta$), the first daughter ($N_\gamma$) and the second daughter (taken from Ref. 126).

Figure 6: The kinematical allowed regions for the backward scattering and the annihilation.

Figure 7: A double Regge pole exchange diagram. 1 and 2 are the initial particles, 3, 4, 5 the final particles, a and b the exchanged Regge poles.

Figure 8: The different regions in the Dalitz plot: I, II and III are the resonance strips.

Figure 9: The double Regge exchange diagram for the reaction $\pi^- p \rightarrow K^0 \overline{K}^0 n$.

Figure 10: The actual Dalitz plot for the region IV of the reaction $\pi^- p \rightarrow K^0 \overline{K}^0 n$ (taken from Ref. 142).

Figure 11: The differential cross-section $d\sigma/dt$ for the reaction $p + ^4He \rightarrow p + ^4He$ at 1 GeV. The continuous curve is the fit done with the use of the Glauber theory (taken from Ref. 150).

Figure 12: Same as Fig. 11, for the reaction $p + D \rightarrow p + D$. 

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Fig. 3
Fig. 5
Fig. 9
\[
\begin{align*}
\rho_{p,n} &= -0.6 \\
\rho_{p,p} &= -1.2 \\
\rho_{p,p} &= \rho_{n,n} = -0.33
\end{align*}
\]

Fig. 12