THE COMPOUND NUCLEUS AND THE RANDOM PHASE APPROXIMATION

T. Ericson

CERN - Geneva

Varenna, August 7 - 26, 1961

2513/TH.229
30 October 1961
It is the purpose of these lectures to discuss the implications which follow from the naïve old definition of the compound nucleus reaction as a reaction for which formation and decay are independent and uncorrelated but for the conservation of exact quantum numbers, in particular of energy and angular momentum. We will show that the independence hypothesis can only be valid as an average property of the compound system, as soon as we leave the energy region in which the width of resonances is small compared to the level distance in the intermediate nucleus. In addition, in the same energy region of overlapping levels, the so-called "continuum region", the average cross-section is smoothly varying with energy; again there are special phenomena, fluctuation effects, in the cross-sections to be expected, if the resolution of the incident energy is made good enough, and these variations have nothing to do with ordinary resonances. We will see that such fluctuations both will provide the means of measuring the average width, i.e., the lifetime of very highly excited compound nuclei, and the means of obtaining the compound elastic cross-section outside the ordinary resonance region.

In order to discuss how this can be done, we will start very qualitatively and intuitively and become more and more precise as we go along, so as to finish by a calculation of fluctuations in the total cross-section, a case in which we have to separate a non-random part of a matrix element from a random part.

The nuclear reaction can be looked at in terms of successive collisions: the incident particle first enters the target nucleus, hits one of the nucleons which may or may not be ejected. If it is not ejected in this first collision we may have it ejected after two collisions, three collisions and so on. This means essentially that we view the reaction analyzed in different orders of perturbation theory. In such a development the direct interaction (D.I.) processes are the simple one-collision processes, the Born approximation, while the compound nuclear (C.N.) processes stand on the other side as the
"many"-collision processes which result in decay from a thoroughly mixed up system. We can regard these collision processes on a time scale: the D.I. occurs typically within a time of the order of the passage time of a particle through the nuclear matter, \( t_0 \sim 2R/v \), which is a few times \( 10^{-22} \) sec. The C.N. reaction is quite a bit slower than this, and will take times of the order of \( 100 \ t_0 \) or more.

We usually study a nuclear reaction not on a time scale but on the complementary energy scale by looking at cross-section as a function of energy. By the uncertainty relation we can then immediately conclude that a D.I. contribution can only show major variations over an energy range \( \hbar/t_0 \sim 5\text{–}10\text{MeV} \) while the C.N. contribution may vary over an energy region determined by its width \( \Gamma \), the energy uncertainty corresponding to its lifetime. Therefore, a rapid energy variation of a cross-section is associated with reaction processes which take long time.

There is no "a priori" separation of the slowly varying D.I. part of a matrix element from the rapidly varying C.N. part. The two extreme processes will therefore in general interfere with each other. On the other hand, they take place on a very different time scale, which should ensure that it be possible to separate the two. This is in fact the very reason why we expect the longlived C.N. contribution to be independent of the formation process; the time scale is entirely different.

It is easy to see qualitatively when interference between D.I. and C.N. should occur (shape elastic scattering behaves of course just as D.I. in this respect, since both are on the same time scale).

Consider a wave train incident on a nucleus. It has a length \( vT \), where \( v \) is the particle velocity and \( T \) the passage time of the wave train (Fig. 1).
Schematic description of wave train of length $vT$ incident on a nucleus

When this wave train hits the nucleus, the elastic and D.I. contributions are emitted immediately; the C.N. contributions are delayed by the lifetime $\tau$ of the compound system. Therefore, if $T \ll \tau$, the direct part of the outgoing wave is already far from the nucleus when the compound part of the wave is emitted (Fig. 2).

Schematic picture of non-interference of D.I. and C.N. scattering at low incident energy resolution

There will then not be any overlap of the two outgoing waves and thus no interference between them. However, a necessary condition for this is $T \ll \tau$. The incoming wave has then an uncertainty in energy $\Delta E = \hbar / T$.

If we now try to define the incident wave better and better in energy, we have to make the incident wave train longer and longer, i.e., $T$ larger. When we reach $T \gg \tau$, the end of the incident wave still produces D.I., while the front of the wave has already caused a simultaneous compound emission (Fig. 3).
Fig. 3

Schematic picture of interference of D.I. and C.N. scattering at high incident energy resolution.

There is, therefore, an overlap between the D.I. wave and the C.N. wave. In this case interference effects are natural; indeed, it is practically unavoidable not to have them. In this last case, we cannot even expect the pure C.N. decay to be independent of the formation process, since the incident wave still falls in on the nucleus, while the C.N. wave is being emitted. In the above figures, these effects have been illustrated on elastic scattering, but the general situation is much the same. We conclude from this that interference effects occur when we use energy resolution of the incident beam, $\Delta E$, with $\Delta E \leq \Gamma$, the C.N. width. Notice: we have not invoked that there should be very sharp resonances of a width much smaller than the level spacing; these arguments apply equally well in the continuum region of overlapping levels.

We have to distinguish carefully in the following between the case of isolated, non-overlapping resonance levels and that of overlapping resonance levels. The first is characterized by $\Gamma \ll D$, where $D$ is the spacing of the resonances; the latter has $\Gamma \gg D$, so that a large number of resonances is contained within the width $\Gamma$. In the first case of isolated levels of small width, it is simple to assert the clean separation of formation and decay of the compound nucleus, and of non-interference of direct and compound processes. This is due to the nearly infinite lifetime of the compound nucleus.
which permits us to regard these resonances as sharp delta-functions on the D.I. background. The independence of formation and decay shows up, for example, in the well-known fact that the emission from a sharp level has an angular distribution symmetrical around 90° to the beam direction, a simple consequence of the loss of "forwardness" and "backwardness" during the long lifetime of the system. These statements are to a good approximation true as long as $\Gamma << D$.

The situation is quite different when $\Gamma >> D$. In this region, instead of sharp discontinuous resonances, the C.M. gives rise to a continuous cross-section which, however, fluctuates with energy. We can see what happens if we rely somewhat on reaction theory; we will rather arbitrarily make use of the Feshbach formulation and work in terms of the transition matrix $\mathcal{T}$.

We will furthermore neglect the influence of angular momentum conservation, since this is a messy, but qualitatively an unessential complication. We thus always assume all particles to be spinless and that they both enter and leave the nucleus as $s$-waves.

In terms of the $\mathcal{T}$-matrix

$$\mathcal{S}_{\text{tot}}^-(E) = 2\pi \hbar^2 \text{Im} \mathcal{T}_{oo}^-(E)$$

$$\mathcal{T}_{if}^-(E) = \pi \hbar^2 \left| \mathcal{T}_{of}^-(E) \right|^2$$

(1)

Feshbach now shows that the $\mathcal{T}$-matrix can be divided into two parts

$$\mathcal{T}_{of}^-(E) = \mathcal{T}_{of}^\text{p}-(E) + \sum_i \frac{\delta_{oi} \delta_{if}}{E - E_i} = \mathcal{T}_{of}^\text{p}-(E) + \mathcal{T}_{of}^\text{CN}-(E)$$

(2)
where the first part, $I_{of}^{(p)}(E)$, is slowly varying with $E$ and represents the elastic or D.I. part of the matrix element, and where the second part is the resonant C.N. part with intermediate resonant states $i$ of complex energy $E_i = \text{Re}E_i - i\Gamma_i/2$. The $\Gamma_i$'s are the total widths of the resonant state $i$, in general varying from state to state. The terms $\gamma_{oi}$ and $\gamma_{if}$ are the width amplitudes for ground state to resonance and resonance to final state.

Since the cross-section $\sigma_{if}^{(p)}(E) \propto \bigg| I_{of}^{(p)}(E) \bigg|^{2} + 2 \text{Re} I_{of}^{(p)}(E) I_{of}^{(CN)}(E) + \bigg| I_{of}^{(CN)}(E) \bigg|^{2}$, we clearly see the interference between the D.I. amplitude and the C.N. amplitude. If we want to make them independent we have to average over an energy interval. Since $I_{of}^{(p)}$ is nearly a constant, a separation requires $\langle I_{of}^{(CN)}(E) \rangle = 0$, which thus amounts to a random phase hypothesis for $I_{of}^{(CN)}(E)$. The random phase can hardly have its origin in the smoothly varying function $1/E - E_i$, but must come from the width amplitude factor $\gamma_{oi} \gamma_{if}$.

It is, therefore, clear that in order to separate D.I. and C.N. from each other it is necessary to assume a random phase approximation for the C.N. matrix elements; we would have been led to the same conclusion considering compound nuclear angular distributions forgetting the D.I. altogether; the average independence of formation and decay is equivalent to a random phase approximation. Since we always have to be careful with random phase approximations, let us first examine the ideas behind it and the evidence for it; this is so much more necessary since the D.I. contributions to the matrix element occur from the addition of coherent contributions from far-away resonances.

For example, study $\gamma_{oi} = \langle 0 | T_{i} | 1 \rangle$, the matrix element for a transition between the initial state and the resonant state $i$. $T_{i}$ is the corresponding transition matrix. This matrix element contains a few kinematical factors; we will assume that we are not at such low incident energy.
that these vary appreciably from resonance to resonance. We know that the resonant states \( i \) in a highly excited nucleus are exceedingly complex mixtures of states if we use a shell model description, for example. The mixing of states is very thorough, since the spacing of the resonances is much smaller than the matrix elements of the shell model residual interactions. In such a situation there is no particular reason to believe the resonant state \( i \) to have neither a particularly good, nor a particularly bad, overlap in \( \langle 0 | \mathcal{F}_1 | i \rangle \). It is convenient to look at \( \mathcal{F}_1 | 0 \rangle \) as a vector in a Hilbert space. This vector is projected out on a subspace of resonant states \( i \), say resonances in an energy region of \( \sim 100 \) keV. The resonant states are orthogonal and provide a set of base vectors for this vector space. The statement that they have nothing to do with \( \mathcal{F}_1 | 0 \rangle \) means simply that this vector has random direction with respect to the axes, the resonant states.

The projection of \( \mathcal{F}_1 | 0 \rangle \) on an axis \( i \), \( \gamma_{i0} = \langle i | \mathcal{F}_1 | 0 \rangle \), will then have a probability distribution \( f(\gamma_{i0}) \), which we can easily show to be close to Gaussian. For, since no axis \( i \) is specially designated, the probability distribution is the same for any axis. Furthermore, with a random direction of \( \mathcal{F}_1 | 0 \rangle \) both signs of a projection are equally probable, and the mean value of \( \gamma_{i0} \), \( \langle \gamma_{i0}^2 \rangle = 0 \). Its mean square \( \langle \gamma_{i0}^2 \rangle = \text{const} = a^2 \).

Let us now rotate the axes of our subspace to another random direction, which can be done by an ordinary unitary transformation taken to be real for simplicity. Then

\[
\gamma'_{i0} = \sum_k \gamma_{ik} \gamma_{ko}
\]  

(3)

Of course, this cannot change the probability distribution which now is \( f(\gamma'_1) \).

\[
f(\gamma'_{i0}) = f(\gamma_{i0})
\]  

(4)
On the other hand \( \gamma'_{1o} \) is the sum of a large number of terms \( \alpha_{ik} \gamma'_{ko} \) each with a mean value 0 and random sign. There is a well-known theorem of statistics that if you add numbers of random sign like this you get a Gaussian distribution; we could have interpreted \( \alpha_{ik} \gamma'_{ko} \) as the error of a measurement. The mean square of \( \gamma'_{oi} \) is

\[
\langle \gamma_{oi}'^2 \rangle = \sum_k \alpha_{ik}^2 \langle \gamma_{ko}'^2 \rangle = a^2 \sum_k \alpha_{ik}^2 = \sigma^2
\]

(5)

as it naturally has to be. Therefore, we have shown that asymptotically

\[
f(\gamma_{io}) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{\gamma_{io}^2}{2\sigma^2}\right)
\]

(6)

It is particularly noteworthy that it plays a very little role that \( \int_{1}^{0} \) has a constant length as long as it has to be split randomly on a large number of axes.

There is experimental evidence that such a random distribution of \( \gamma_{1o} \) really occurs approximately. This is obtained from the distribution of the widths \( \Gamma_{oi} = \gamma_{oi}^2 \) in slow neutron resonances. If there is a random distribution of \( \gamma_{oi}' \), then we can easily predict the corresponding distribution for \( \Gamma_{oi} \). We observe first that \( \langle \Gamma_{oi} \rangle = \langle \gamma_{oi}'^2 \rangle = \sigma^2 \). If the distribution of \( \Gamma_{oi} \) is \( P(\Gamma_{oi}) \), then obviously

\[
P(\Gamma_{oi}) d\Gamma_{oi} = 2f(\gamma_{oi}) d\gamma_{oi} \quad \gamma_{oi} > 0
\]

(7)

Here we write \( d\gamma_{oi} = \frac{1}{2} \frac{d(\gamma_{oi}^2)}{\sqrt{\gamma_{oi}^2}} \) and use Eqn. (6) in Eqn. (7)

\[
P(\Gamma_{oi}) d\Gamma_{oi} = 2 \cdot \frac{1}{\sqrt{2\pi} \langle \gamma_{oi}'^2 \rangle} e^{-\frac{\Gamma_{oi}^2}{2\langle \gamma_{oi}'^2 \rangle}} \cdot \frac{1}{\sqrt{\Gamma_{oi}}} d\Gamma_{oi}
\]

(8)
so that

$$P(G_o) = \frac{1}{\sqrt{2.\pi} \langle G_o \rangle^2} \left( \frac{\langle G_o \rangle}{\langle G_o \rangle} \right)^\frac{1}{2} \exp \left\{ -\frac{1}{2} \frac{G_o}{\langle G_o \rangle} \right\}$$

This is the well-known Porter-Thomas distribution \(2\) which gives a good description of the experimental distribution of neutron widths. (In practice, the reduced neutron width, since the energy variation of the kinematical factors are important for slow neutrons). Its qualitative behaviour is shown in Fig. 4.

![Fig. 4](https://example.com/fig4.png)

Qualitative form of the Porter-Thomas distribution

Therefore we have quite good reasons to believe that the \(\gamma_{oi}\) and \(\gamma_{if}\) the width amplitudes which occur in Equ. (2), have a very important component of random nature, at least as long as we study them in not too large energy intervals. We will, therefore, explore the consequences of the random hypothesis in the region of overlapping levels \(3\): this is the region several MeV or more above the neutron threshold, where many neutron exit channels are available. In order to make considerations easier we will make the following assumptions which can be experimentally fulfilled to a good approximation.
1) $\Gamma_i \gg D$, i.e., in practice, exit channels leading to neutron emission to a rather large number of final states. This implies excitation larger than several MeV above neutron binding.

2) $\Gamma_i \approx \Gamma = \text{const.}$; this is a consequence of the assumption of many final states, since $\Gamma_i = \sum_f \Gamma_{if}$, i.e., a sum of many squared terms of similar magnitude. It is plausible and it can be shown that the deviation of $\Gamma_i$ from its mean value is of the order $1/\sqrt{n}$, where $n$ is the number of final neutron states.

3) energy resolution better than $\Gamma$, since we can average later over energy if wanted.

4) cross-section taken for transition to one isolated final state, since averages again can be performed afterwards if wanted.

Going back to the expression (2) for the matrix element we observe that the compound part has a denominator $1/(E-ReE_1+i\Gamma/2)$. Consequently, the C.N. contributions to the matrix element of an energy $E$ come nearly entirely from compound states $i$ of an energy $E-\Gamma/2 \ll ReE_1 \ll E+\Gamma/2$, since outside this region the denominator just mentioned becomes large, so that contributions are small. To see the qualitative consequences of the random approximation let us replace the denominator by a constant for $E-\Gamma/2 \leq ReE_1 \leq E+\Gamma/2$ and with infinity outside. The compound contribution is then simply

$$J^{(CN)}_{of} \sim \frac{\sum_i \gamma_{oi} \gamma_{if}}{i\Gamma/2}; \quad E-\Gamma/2 \leq ReE_1 \leq E+\Gamma/2$$

(10)

According to hypothesis, $\gamma_{oi}$ and $\gamma_{if}$ are random numbers. Since $\Gamma >> D$, a large number of states $i$ is included in the sum. We have thus a sum of a large number of terms of random sign which must give a Gaussian probability distribution, as already shown in Equs. (3) - (6).
From this result we can immediately make the following qualitative prediction on the behaviour of the cross-section: if we study the cross-section at two different energies \( E \) and \( E' \), so that \( |E-E'| > \Gamma \), the intermediate states will be quite different; therefore, \( \mathcal{J}_{\text{of}}^{(\text{CN})}(E) \) and \( \mathcal{J}_{\text{of}}^{(\text{CN})}(E') \) are uncorrelated random numbers and have accordingly a Gaussian distribution. They will thus interfere independently with the slowly varying direct contribution \( \mathcal{J}_{\text{of}}^{(\text{DI})} \), so that the cross-section can be quite different at \( E \) and \( E' \).

On the other hand, if \( |E-E'| < \Gamma \), the intermediate states will be largely the same; the matrix element and thus the cross-section hardly changes, for energy changes of this order.

We must thus conclude that the cross-section will exhibit fluctuations over an energy region of the order of \( \Gamma \). If there is no D.I. contribution of importance, the cross-section taken at energy intervals \( |E-E'| > \Gamma \) will be a random number squared. It will then, like neutron widths, have a Porter-Thomas distribution.

Since the "width" of the cross-section fluctuations is \( \Gamma \), the average compound nuclear width, this quantity can thus be determined even at high excitation (Fig. 5).

![Fig. 5](image)

Qualitative behaviour of cross-section as a function of energy in the "continuum"
We may ask: how much do we have to average such a cross-section in order to wash out the fluctuations? This is simple. Suppose we average over an energy region $\Delta E$. The intrinsic fluctuations in the Porter-Thomas distribution are such that $< (\sigma - <\sigma> )^2 > = 2<\sigma>^2$. By averaging over $\Delta E$ we add $\Delta E/\Gamma$ independent variables (since in energy intervals $<\Gamma$ the cross-section is strongly correlated). The fluctuation is thus

$$\sigma \sim <\sigma> \left\{ 1 \pm \sqrt{2\Gamma/\Delta E} \right\} ; \Delta E > \Gamma. \quad (11)$$

We can similarly average over $n$ final states; this means averaging over $n\Delta E/\Gamma$ independent variables and is thus

$$\sigma \sim <\sigma> \left\{ 1 \pm \sqrt{2\Gamma/n\Delta E} \right\} ; \Delta E > \Gamma. \quad (12)$$

The fluctuations can thus be damped either by summing over many final states or by averaging over energy.

At this point it is useful to make a short digression to discuss the effects of possible residual phase relations between the matrix elements, i.e., the breakdown of the random phase approximation. It is well-known that the phase relations between far-away resonances give rise to the D.I. contributions and to optical model resonances; furthermore, at very high energies of the incident particle only coherent scattering occurs. How this works is now obvious: if the coherent part of the matrix element is small compared to the random part it becomes important in averages over large energy regions, while the fluctuations dominate locally. The small coherent part is then only important as an interference from far-away levels, while it can be
completely neglected for close resonance states. At very high incident energy another phenomenon sets in. With increasing excitation \( \Gamma \) increases also. It reaches eventually such a large value that it will include the full strength of single particle shell model states which the residual interactions have mixed into a large number of resonant states. At this point we get approximate sum rules in Eq. (10), so that if we sum over all the resonant states of a single particle state \( a \)

\[
\mathcal{F}_{\text{of}} \frac{\delta a \gamma_{af}}{1 \Gamma^{2}} ; \quad \Gamma \text{ large} \quad (13)
\]

which will be a very simple type scattering of very non-random nature. All the preceding results on the fluctuations must therefore specifically relate to the region of sufficiently small \( \Gamma \), let us say \(<100 \text{ keV}\) which is of the order of the residual interaction matrix elements in a shell model Hamiltonian. For the small \( \Gamma \) there is no contradiction between D.I. and random matrix elements. In general one should of course make a deliberate separation between the random and non-random part of the compound matrix element, as we will see in a moment.

In order to obtain a quantitative and not only a qualitative description of the fluctuation effects it is useful to introduce a correlation function \( F(\varepsilon) \) defined in terms of mean values as

\[
F(\varepsilon) = \langle (\sigma(\varepsilon + \varepsilon) - \langle \sigma \rangle)(\sigma(\varepsilon) - \langle \sigma \rangle) \rangle = \langle \sigma(\varepsilon + \varepsilon) \sigma(\varepsilon) \rangle - \langle \sigma \rangle^{2} \quad (14)
\]

This function is thus positive for \( \varepsilon = 0 \); it then falls off and becomes zero for large \( \varepsilon \), since the fluctuations are uncorrelated over large energy intervals. The fall-off, as we previously discussed, must be over an energy region of the order \( \Gamma \) as in Fig. 6.

![Fig. 6](image_url)

Qualitative variation of the correlation function \( F(\varepsilon) \) with \( \varepsilon \).
We will now sketch the calculation of such a correlation function using the matrix element of Eq. (2). However, since the inelastic cross-section itself involves a matrix element squared, the correlation function will involve the matrix element to the fourth power, which becomes a bit complicated, though it can be done. It is much simpler to do the calculation for the total cross-section which is linear in and matrix element. This has two other advantages:

1) that we automatically have an important non-random part of the matrix element since final and initial states are the same: \( \gamma_{oi} \gamma_{if} \) is \( \gamma_{oi}^2 > 0 \). We can thus see how the random approximation has to be used in a general case.

2) that we obtain an approximate procedure to determine the compound elastic cross-section which in the continuum is a rather elusive quantity.

Thus, since the total cross-section is \( \sigma_{\text{tot}} = 2\pi \chi^2 \text{Im} \int_0^\infty \), its mean value is (we assume for simplicity, though it is not necessary, equidistant level spacing \( D \))

\[
\langle \sigma_{\text{tot}} \rangle = 2\pi \chi^2 \langle \text{Im} \int_0^\infty (E) \rangle = 2\pi \chi^2 \text{Im} \int_0^\infty (E) + \langle \sum_i \frac{\chi_i^2}{(E - \text{Re}E_i)^2 + (\Gamma/2)^2} \rangle = \\
= 2\pi \chi^2 (\text{Im} \int_0^\infty (E) + \frac{\chi_i^2}{\Delta E})
\]

(15)

Similarly, the calculation of the correlation function is, to a large extent, straightforward using complex contour integration

\[
\langle \sigma_{\text{tot}} (E + \varepsilon) - \langle \sigma_{\text{tot}} \rangle (\sigma_{\text{tot}} (E) - \langle \sigma_{\text{tot}} \rangle) \rangle = \\
(2\pi \chi^2)^2 \langle \left( \int_{\text{Im}} \sum_i \frac{\chi_i^2}{E + \varepsilon - \text{Re}E_i + i\eta_i} - \frac{\pi \langle \chi_i^2 \rangle}{\Delta E} \right) \left( \int_{\text{Im}} \sum_i \frac{\chi_i^2}{E - \text{Re}E_i + i\Gamma/2} + \frac{\pi \langle \chi_i^2 \rangle}{\Delta E} \right) \rangle = \\
(2\pi \chi^2)^2 \left\{ \frac{\Delta E}{\text{Im}} \sum_i \frac{\chi_i^2}{\text{Re}E_i - \text{Re}E_j + i\Gamma/2} - \left( \frac{\pi}{\Delta E} \langle \chi_i^2 \rangle \right) \right\}
\]

(16)
where $\Delta E$ is the energy region over which the average is performed. At this point it is necessary to make explicit use of the fact that the width amplitude $\gamma_{oi}^2$ is non-random. We write it as

$$\gamma_{oi}^2 = \langle \gamma_{oi}^2 \rangle + \delta \gamma_{oi}^2$$  \hspace{1cm} (17)$$

where the $\delta \gamma_{oi}^2$ is the fluctuation, and by definition has mean value zero. Introducing this in Equ. (16) we can neglect all terms linear in $\delta \gamma_{oi}^2$, since their mean value is zero; similarly terms containing $\delta \gamma_{oi}^2 \delta \gamma_{oj}^2$ with $i \neq j$ are on the average negligible compared to the diagonal terms $\langle \delta \gamma_{oi}^2 \rangle^2$. We therefore obtain

$$F(\epsilon) = (2\pi \hbar^2)^{\frac{3}{2}} \int \frac{\pi}{\Delta E} \sum_{i,j} \frac{\langle \gamma_{oi}^2 \rangle^2}{\Re E_i - \Re E_j + \epsilon + i\Gamma} + \frac{\pi}{\Delta E} \sum_{i,j} \left( \frac{\langle \gamma_{oi}^2 \rangle^2}{i\Gamma + \epsilon} - \left( \frac{\pi}{D} \langle \delta \gamma_{oi}^2 \rangle \right)^2 \right)$$  \hspace{1cm} (18)$$

Replacing the first double sum by an integration we have finally

$$F(\epsilon) = (2\pi \hbar^2)^{\frac{3}{2}} \frac{\pi}{D} \langle \delta \gamma_{oi}^2 \rangle \frac{\Gamma}{\Gamma^2 + \epsilon^2}$$  \hspace{1cm} (19)$$

This very simple result shows clearly that the correlation function falls off within an interval $\Gamma$; the correlation function can thus clearly be used to determine the average width. The Breit-Wigner type form of the correlation function is natural, since the Fourier transform of $F(\epsilon)$ to time variables yields a factor $e^{-t/\hbar}$. It is thus characteristic of the decay time $\hbar/\Gamma$ of the compound nucleus.
We can obtain an estimate of the compound elastic scattering from Equ. (19). For, with isolated resonances,

\[
\langle \mathbf{t} \cdot \mathbf{d} \rangle = \pi \frac{\Delta}{D \Gamma} \frac{\pi \langle \mathbf{y}_{0i}^4 \rangle}{D \Gamma} = \pi \frac{\Delta}{D \Gamma} \frac{\pi \langle \mathbf{y}_{0i}^2 \rangle^2}{D \Gamma} + \pi \frac{\Delta}{D \Gamma} \frac{\langle (\delta \mathbf{y}_{0i}^2)^2 \rangle}{D \Gamma} \tag{20}
\]

If \( y_{0i} \) is random, we know that its square \( y_{0i}^2 \) has strong fluctuations; it varies like a Porter-Thomas distribution, which would have \( \langle (\delta y_{0i}^2)^2 \rangle = 2 \langle y_{0i}^2 \rangle \). Something close to that fluctuation will occur in general. Therefore, we can obviously write Equ. (19) as

\[
\mathcal{F}(\varepsilon = 0) = \langle \mathbf{t}^2 \mathbf{E} \rangle - \langle \mathbf{t}^2 \rangle = \alpha \cdot \frac{\Delta}{\mathbf{y}} \pi \frac{\Delta}{D \Gamma} \langle \mathbf{t} \cdot \mathbf{d} \rangle, \tag{21}
\]

where \( \alpha \) is a constant of the order of unity. The study of correlation functions in this way should thus provide a check on the magnitude of the compound elastic cross-section which can be estimated theoretically. The results obtained here remain essentially valid when angular momentum is included, but the fluctuations are somewhat damped.

As a conclusion: the study of cross-sections with high resolution above the "isolated resonance" region can yield important information both on the average width \( \Gamma \) and on the cross-section for the compound reaction as compared to that of the direct reaction. The latter has its importance therein that it makes use of an interference effect, which permits very small compound cross-sections to be determined. It is also very important and interesting to clarify to what extent the random phase approximation itself is valid at different excitation energies; this yields direct information on the structure of the excited nucleus. In this way it can be hoped that we eventually will be able to understand the difficult intermediate region of reactions which are neither fully compound nor pure direct interaction reactions.
REFERENCES

1) H. Feshbach, Ann. of Physics 5, 357 (1958)


3) Part of the following considerations are published see T. Ericson, Adv. in Phys. 2, 425 (1960), Sect. 11, T. Ericson, Phys. Rev. Letters 5, 430 (1960);
   D. Brink and R. Satchler [private communication (R.S.)] have studied the random interference between D.I. and C.N. along essentially the same lines as here.