Rescattering Effects, Isospin Relations and Electroweak Penguins in $B \rightarrow \pi K$ Decays

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Abstract

It is argued that, in the presence of soft final-state interactions, the diagrammatic amplitude approach adopted in many analyses of hadronic $B$ decays into light mesons can be misleading when used to deduce the unimportance of certain decay topologies. With the example of $B \rightarrow \pi K$ decays, it is shown that the neglect of so-called annihilation and colour-suppressed amplitudes (including electroweak penguins), as well as penguin contributions involving an up-quark loop, is not justified. The implications for the Fleischer–Mannel bound on the angle $\gamma$ of the unitarity triangle, and for the CP asymmetry in the decays $B^\pm \rightarrow \pi^\pm K^0$, are pointed out.

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The study of CP violation in the rare decays of $B$ mesons is the main target of present and future "$B$ factories". It is hoped that this will shed light on the origin of CP violation, which may lie outside the standard model of strong and electroweak interactions. Whereas at present only a single measurement of a CP-violating asymmetry exists (the quantity $\epsilon_K$ in $K$ decays), the measurements of several CP asymmetries in $B$ decays will make it possible to test whether the CKM mechanism of CP violation is sufficient to account for the data, or whether additional sources of CP violation are required (for some excellent recent reviews, see Refs. [1, 2]). In the latter case, this would directly point towards physics beyond the standard model.

In order to achieve this goal, it is necessary that the theoretical calculations of CP-violating observables in terms of standard model parameters are, at least to a large extent, free of hadronic uncertainties. This can be achieved, for instance, by measuring time-dependent asymmetries in the decays of neutral $B$ mesons into CP eigenstates, such as $B \rightarrow J/\psi K_S$. In many other cases, however, there are nontrivial strong-interactions effects affecting the CP asymmetries. In the absence of a reliable theoretical approach to calculate these effects, strategies have been developed that exploit the isospin symmetry of the strong interactions, or its approximate SU(3) flavour symmetry, to derive relations between various decay amplitudes, which can be used to eliminate hadronic uncertainties (see Refs. [3]–[8] for some early applications of this approach). A comprehensive review of these methods can be found in Ref. [1].

In this note, we question the theoretical approximations underlying some of these analyses, which need to rely on “plausible” dynamical assumptions such as the neglect of colour-suppressed or annihilation topologies. To be specific, we consider the decay amplitudes for the various $B \rightarrow \pi K$ modes and analyse the relations among them imposed by isospin symmetry. The effective weak Hamiltonian governing these transitions has the structure [9]

$$\mathcal{H}_{\text{eff}} = \sum_{i=1,2} \left[ \lambda_u Q_u^i + \lambda_c Q_c^i \right] + \lambda_t \sum_{i=3}^{10} Q_i + \text{h.c.},$$  

(1)

where $\lambda_q = V_{qs}V_{qb}^*$ are products of elements of the CKM matrix, satisfying the unitarity relation $\lambda_u + \lambda_c + \lambda_t = 0$, and $Q_i$ represent the products of local four-quark operators with short-distance coefficient functions. Relevant for our purposes are only the isospin quantum numbers of these operators: the current–current operators $Q_{1,2}^u \sim \bar{b}s\bar{u}u$ have components with $\Delta I = 0$ and $\Delta I = 1$; the current–current operators $Q_{1,2}^c \sim \bar{b}s\bar{c}c$, as well as the QCD penguin operators $Q_{3,...,6} \sim \bar{b}s\sum \bar{q}q$, have $\Delta I = 0$; the electroweak penguin operators $Q_{7,...,10} \sim \bar{b}s\sum e_q\bar{q}q$, where $e_q$ are the electric charges of the quarks, have components with $\Delta I = 0$ and $\Delta I = 1$. Thus, we may write $\mathcal{H}_{\text{eff}} = \mathcal{H}_{\Delta I=0} + \mathcal{H}_{\Delta I=1}$ with

$$\mathcal{H}_{\Delta I=0} = \sum_{i=1,2} \frac{\lambda_u}{2} \left( Q_u^i + Q_d^i \right) + \lambda_c \sum_{i=3}^{10} Q_i - \lambda_t \sum_{i=7}^{10} Q_i^{\Delta I=1} + \text{h.c.},$$

$$\mathcal{H}_{\Delta I=1} = \sum_{i=1,2} \frac{\lambda_u}{2} \left( Q_u^i - Q_d^i \right) + \lambda_t \sum_{i=7}^{10} Q_i^{\Delta I=1} + \text{h.c.},$$  

(2)
where the $\Delta I = 1$ components of the electroweak penguin operators are defined as $Q_{7, 7}^{\Delta I = 1} \sim \frac{1}{3} \bar{b}s(\bar{u}u - \bar{d}d)$. Taking into account that the initial $B$-meson state has $I = \frac{1}{2}$, whereas the final states ($\pi K$) can be decomposed into states with $I = \frac{1}{2}$ and $I = \frac{3}{2}$, the Wigner–Eckart theorem implies that the physical decay amplitudes can be described in terms of three isospin amplitudes, which are defined as $[5, 6]$

\[
\begin{align*}
A_{3/2} &= \sqrt{\frac{1}{3}} \langle \frac{3}{2}, \pm \frac{1}{2} | H_{\Delta I = 1} | \frac{1}{2}, \pm \frac{1}{2} \rangle, \\
A_{1/2} &= \pm \sqrt{\frac{2}{3}} \langle \frac{1}{2}, \pm \frac{1}{2} | H_{\Delta I = 1} | \frac{1}{2}, \pm \frac{1}{2} \rangle, \\
B_{1/2} &= \sqrt{\frac{2}{3}} \langle \frac{1}{2}, \pm \frac{1}{2} | H_{\Delta I = 0} | \frac{1}{2}, \pm \frac{1}{2} \rangle.
\end{align*}
\]  

(3)

From the decomposition of the effective Hamiltonian in (2) it is obvious which operator matrix elements and weak phases enter the various isospin amplitudes. The resulting expressions for the physical $B \to \pi K$ decay amplitudes are given by

\[
\begin{align*}
A_{-+} &= A(B^0 \to \pi^- K^+) = A_{3/2} + A_{1/2} - B_{1/2}, \\
A_{+0} &= A(B^+ \to \pi^+ K^0) = A_{3/2} + A_{1/2} + B_{1/2}, \\
A_{00} &= \sqrt{2} A(B^0 \to \pi^0 K^0) = 2A_{3/2} - A_{1/2} + B_{1/2}, \\
A_{0+} &= \sqrt{2} A(B^+ \to \pi^0 K^+) = 2A_{3/2} - A_{1/2} - B_{1/2}.
\end{align*}
\]  

(4)

Instead of expressing the isospin amplitudes in terms of operator matrix elements, many practitioners prefer to analyze the $B$ decay amplitudes in terms of a diagrammatic notation, in which complex amplitudes are associated with certain flavour-flow topologies $[7, 8]$. If we neglect electroweak penguin diagrams for the moment (we will come back to them later), the topologies relevant to our discussion are the so-called “tree topology” $T$, the “colour-suppressed tree topology” $C$, the “annihilation topology” $A$, and the “penguin topology” $P$ shown in the upper plots in Figure 1. In terms of these quantities, the decay amplitudes take the form $[8]$

\[
\begin{align*}
A_{-+} &= -(T + P), & A_{00} &= -C + P, \\
A_{+0} &= A + P, & A_{0+} &= -(T + C + A + P),
\end{align*}
\]  

(5)

while the isospin amplitudes are given by $A_{3/2} = -\frac{1}{2}(T + C)$, $A_{1/2} = -\frac{1}{6}T + \frac{1}{3}C + \frac{1}{2}A$ and $B_{1/2} = P + \frac{1}{2}(T + A)$. If electroweak penguin contributions are neglected, the amplitudes $T$, $C$ and $A$ are proportional to $\lambda_u$, whereas the penguin amplitude $P$ has contributions proportional to all three $\lambda_q$, and we define $P = \lambda_u P_u + \lambda_c P_c + \lambda_t P_t$.

The diagrammatic approach provides a redundant parametrization of the decay amplitudes in that there are more flavour-flow topologies than isospin amplitudes. We stress that, whereas the isospin amplitudes can be defined in a transparent way in terms of operator matrix elements using the decomposition (2), this is not the case for the individual amplitudes in the diagrammatic approach. For instance, the matrix elements
of the current–current operators $Q^{\alpha_1\alpha_2}_u$ contribute to $T$, $C$, $A$ and $P_u$. Still, the approach is perfectly legitimate in a mathematical sense, as long as one works with exact expressions for the physical decay amplitudes. However, the main virtue of the diagrammatic approach is claimed to be the fact that it would allow one, by making “plausible” dynamical assumptions, to simplify the relations between decay amplitudes. In particular, it is usually argued that annihilation diagrams are suppressed relative to tree diagrams by a factor of $f_B/m_B \sim \text{few } \%$ stemming from the fact that in order to have the quarks inside the initial $B$ meson annihilate each other through a weak current they have to be at the same point, implying a suppression proportional to the wave-function at the origin. This argument is used to conclude that $|A| \ll |T|$, and hence $A$ is often neglected. Similarly, it is argued that colour-suppressed tree diagrams are suppressed relative to colour-allowed ones by a factor $a_2/a_1 \sim 0.2$, and hence $|C| \ll |T|$. Finally, it is often assumed that penguin diagrams are dominated by the contributions from heavy-quark loops, whereas the contribution from the up-quark loop is neglected (i.e. $|\lambda_u P_u| \ll |T|$). Even the charm-penguin contribution has been neglected in some applications; however, its importance has been stressed recently by several authors [10, 11]. With these assumptions, the two amplitudes in (5) which correspond to processes that have been observed experimentally simplify to

$$A_{-+} \approx -(T + P), \quad A_{+0} \approx P,$$

(6)

where $P \approx \lambda_c(P_c - P_t)$. Since there is no nontrivial weak phase in $\lambda_c$, one does not expect a CP asymmetry in the decays $B^\pm \rightarrow \pi^\pm K^0$, and these equations can be used to derive a bound on the weak phase $\gamma$ of the tree amplitude $T$. Defining $T/P = re^{i\gamma}e^{i\delta}$, where $\delta$ is an unknown strong phase, one finds for the ratio of the branching ratios for the two processes $B_d \rightarrow \pi^\pm K^\pm$ and $B^\pm \rightarrow \pi^\pm K^0$, averaged over CP-conjugate modes,

$$R = \frac{\text{Br}(B_d \rightarrow \pi^\pm K^\pm)}{\text{Br}(B^\pm \rightarrow \pi^\pm K^0)} = 1 + 2r \cos \gamma \cos \delta + r^2 \geq \sin^2 \gamma.$$

(7)

This is the Fleischer–Mannel bound, which excludes a region of parameter space around $\gamma = 90^\circ$ provided that $R < 1$ [12]. Given that the current experimental value $R_{\text{exp}} = 0.65 \pm 0.40$ [13] indicates that this may indeed be the case, this bound has received a lot of attention. Its implications for CP phenomenology in the standard model and beyond have been analyzed in Refs. [14, 15].

The purpose of this note is to stress that soft rescattering effects, which have been shown to be potentially significant even in the decays of heavy hadrons [16], may invalidate the assumptions about the relative size of the amplitudes in the diagrammatic approach discussed above, and thus may invalidate the bound in (7). Our main point is that the topologies $C$, $A$ and $P_u$ contain contributions corresponding to final-state rescatterings of the leading tree amplitude $T$, as shown in the lower plots in Figure 1. If the (unknown) final-state phases happen to be large, it is thus natural to assume that $T$, $C$, $A$ and the up-penguin $P_u$ are all of a similar magnitude. Note, in particular, that the naive arguments in favour of a suppression of $A$ and $C$ relative to $T$ no longer apply. For
instance, the “wave-function suppression” of the annihilation amplitude is absent in the “soft” annihilation process shown in the figure. Therefore, although the diagrammatic approach was originally designed to provide a model-independent parametrization of decay amplitudes including all strong-interaction effects, in its practical form, in which certain dynamical approximations are adopted, it does not provide an appropriate representation of the amplitudes unless final-state rescattering effects are negligible. On the other hand, the similar importance of the various contributions \((T, C, A\) and \(P_u\)) involving the CKM parameter \(\lambda_u\) emerges naturally in an approach where the different isospin amplitudes are related to operator matrix elements.

It is instructive to illustrate our point in the context of a simple model for the weak decay amplitudes. For this purpose, we shall adopt the generalized factorization prescription [17] to calculate the short-distance contributions to the matrix elements of the current–current operators \(Q^1, Q^2\), neglect “hard” (short-distance) annihilation contributions and electroweak penguins, and neglect the imaginary parts of the charm- and up-quark penguin diagrams, which reflect long-distance contributions from physical intermediate states. Once the short-distance contributions are calculated, long-distance effects are accounted for by introducing elastic rescattering phases for the two isospin channels of the final-state mesons. In this model, the short-distance \(B \to \pi K\) amplitudes are given by \(A_{\text{SD}+} = -(M_T + M_P),\ A_{\text{SD}0} = M_P,\ A_{\text{SD}00} = -M_C + M_P,\) and \(A_{\text{SD}0+} = -(M_T + M_C + M_P),\) where \(M_P\) represents the short-distance penguin contributions, and

\[
M_T = \frac{G_F}{\sqrt{2}} \lambda_u a_1 f_K (m_B^2 - m_\pi^2) F_0^{B \to \pi}(m_K^2) \approx (2.5 \pm 0.4) V_{ub}^* \times 10^{-6} \text{GeV},
\]

\[
M_C = \frac{G_F}{\sqrt{2}} \lambda_u a_2 f_\pi (m_B^2 - m_K^2) F_0^{B \to K}(m_\pi^2) \approx (0.20 \pm 0.06)M_T,
\]  

\[(8)\]

Figure 1: Flavour-flow topologies relevant to \(B \to \pi K\) decays. In the lower plots, the topologies \(C, A\) and \(P\) are redrawn as soft final-state rescatterings from the tree amplitude. The dots indicate the quark fields contained in the operators of the effective weak Hamiltonian. The shaded blobs represent (intermediate) hadronic states.
are the factorized matrix elements of the current–current operators $Q^u_{1,2}$ in the notation of Ref. [17], from which we also take the values of the hadronic form factors with conservative errors. The ratio of the hadronic parameters $a_1$ and $a_2$ is taken as $a_2/a_1 = 0.22 \pm 0.05$. From a naive comparison with (5), one would conclude that $|T| = |M_T|$, $|C| = |M_C|$, $A = 0$, and $|P| = |M_P|$. These results are indeed often used to estimate the magnitudes of $T$ and $C$. This identification is not justified, however. Instead, we must calculate the short-distance contributions to the different isospin amplitudes and then account for the final-state phases. This gives

$$A_{3/2} = -\frac{1}{3} (M_T + M_C) e^{i\phi_{3/2}},$$
$$A_{1/2} = -\frac{1}{6} (M_T - 2M_C) e^{i\phi_{1/2}},$$
$$B_{1/2} = \left(M_P + \frac{1}{2} M_T\right) e^{i\phi_{1/2}}. \quad (9)$$

Inserting these results into the general expressions (4) yields

$$A_{-+} = -(M_T + M_P) e^{i\phi_{1/2}} - X,$$
$$A_{+0} = M_P e^{i\phi_{1/2}} - X,$$
$$A_{00} = -(M_C + M_P) e^{i\phi_{1/2}} - 2X,$$
$$A_{0+} = -(M_T + M_C + M_P) e^{i\phi_{1/2}} - 2X, \quad (10)$$

where

$$X = \frac{1}{3} (M_T + M_C) \left(e^{i\phi_{3/2}} - e^{i\phi_{1/2}}\right). \quad (11)$$

We stress that, even in a factorization approach, it is important to include final-state rescattering effects in the way outlined above, unless it is experimentally known that such effects are negligible (i.e. that $|\phi_{3/2} - \phi_{1/2}| \ll 1$). Comparing the result (10) with the relations (5) of the diagrammatic approach, we now obtain

$$T = M_T e^{i\phi_{1/2}} + X - \Delta P,$$
$$A = -X - \Delta P,$$
$$C = M_C e^{i\phi_{1/2}} + 2X + \Delta P,$$
$$P = M_P e^{i\phi_{1/2}} + \Delta P, \quad (12)$$

where $\Delta P$ is arbitrary and cancels in the predictions for the physical decay amplitudes. This reflects the redundancy in the parametrization of three isospin amplitudes in terms of four diagrammatic amplitudes. By choosing $\Delta P$ appropriately, it is possible to redistribute the rescattering contributions between the various amplitudes, leaving the physical decay amplitudes unchanged. We stress that the strong phases entering the diagrammatic amplitudes are not governed by the isospin of the final states fed by these amplitudes, the reason being that the diagrammatic amplitudes are not isospin amplitudes. For instance, we see that at least one of the amplitudes $A$ or $P$ must contain the
phase $\phi_{3/2}$, although they both lead to final states with $I = \frac{1}{2}$ only. If we choose to set $\Delta P = 0$, for instance, we get

\[
\frac{T}{MT e^{i\phi_{1/2}}} = 1 + \frac{1}{3} \left( 1 + \frac{MC}{MT} \right) (e^{i\Delta\phi} - 1),
\]
\[
\frac{C}{MT e^{i\phi_{1/2}}} = \frac{MC}{MT} + \frac{2}{3} \left( 1 + \frac{MC}{MT} \right) (e^{i\Delta\phi} - 1),
\]
\[
\frac{A}{MT e^{i\phi_{1/2}}} = -\frac{1}{3} \left( 1 + \frac{MC}{MT} \right) (e^{i\Delta\phi} - 1),
\]

(13)

where $\Delta\phi = \phi_{3/2} - \phi_{1/2}$. Unless $|\Delta\phi| \ll 1$, it is not justified to assume that $|C| \ll |T|$ or $|A| \ll |T|$. In the presence of soft final-state interactions, there is no colour suppression of $C$ with respect to $T$, and there is no intrinsic smallness of the annihilation topology $A$. For a phase difference of 45°, for instance, we find $|T| : |C| : |A| \approx 1 : 0.61 : 0.33$. If we choose instead $\Delta P = -X$ so as to keep the annihilation amplitude small, we would find $|T| : |C| : |\Delta P| \approx 1 : 0.31 : 0.32$. With this choice, the up-quark penguin receives a large rescattering contribution, which is of a similar magnitude as the tree amplitude.

Although our model is too simple to provide for a trustworthy calculation of the decay amplitudes, we believe it illustrates nicely the potential pitfalls of the diagrammatic method. A more realistic analysis would have to include inelastic rescattering contributions [16]. Also, there are important contributions to the strong phase of the charm penguin $P_t$ from rescattering through intermediate states such as $D_sD$ or $J/\psi K$. As a consequence, the two $I = \frac{1}{2}$ amplitudes $A_{1/2}$ and $B_{1/2}$ will, in general, acquire different phases. To get a more realistic model, we thus modify the last relation in (9) to read

\[
B_{1/2} = |MP| e^{i\phi_P} + \frac{1}{2} MT e^{i\phi_{1/2}}.
\]

(14)

$M_P = \lambda_c(P_c - P_t)$ has no nontrivial weak phase, whereas $MT$ is proportional to $e^{i\gamma}$. Before we outline some of the implications of our results, we come back to the discussion of electroweak penguin operators, which according to (2) and (3) contribute to all three isospin amplitudes. As far as isospin (but not SU(3) flavour) symmetry is concerned, the $\Delta I = 0$ contributions of electroweak penguins can be absorbed into a redefinition of the top-quark penguin amplitude $P_t$ contained in $B_{1/2}$. However, electroweak penguins with $\Delta I = 1$ cause problems, since they induce terms proportional to $\lambda_t$ in the amplitudes $A_{3/2}$ and $A_{1/2}$. Using Fierz identities to rewrite the current–current operators, and adopting the notation of Ref. [9], we find

\[
\mathcal{H}_{\Delta I = 1} = \frac{G_F}{2\sqrt{2}} \left\{ \left[ \lambda_u C_1(\mu) - \frac{3}{2} \lambda_t C_9(\mu) \right] (\bar{b}_\alpha s_\alpha)_{V-A} (\bar{u}_\beta u_\beta - \bar{d}_\beta d_\beta)_{V-A} \\
+ \left[ \lambda_u C_2(\mu) - \frac{3}{2} \lambda_t C_{10}(\mu) \right] (\bar{b}_\alpha s_\beta)_{V-A} (\bar{u}_\beta u_\alpha - \bar{d}_\beta d_\alpha)_{V-A} + \ldots \right\} + \text{h.c.},
\]

(15)

where $C_i(\mu)$ are Wilson coefficients, and the ellipses represent the contributions from the operators $Q_7$ and $Q_8$, which have a different Dirac structure. To give an idea about the
relative importance of the various contributions, we quote the values of the coefficients at $\mu = m_b$ (in the NDR scheme): $C_1 \approx -0.185$, $C_2 \approx 1.082$, $C_7 \approx -10^{-3}$, $C_8 \approx 0.4 \cdot 10^{-3}$, $C_9 \approx -9.4 \cdot 10^{-3}$, $C_{10} \approx 1.9 \cdot 10^{-3}$. The values of $C_7$ and $C_9$ are so tiny that it should be a good approximation to neglect the contributions the operators $Q_7$ and $Q_8$. However, the other two electroweak penguins are important. Using $\lambda_u/\lambda_t = -\lambda^2 R_6 e^{i\gamma}$ with $\lambda = 0.22$ and $R_6 \approx 0.36$ [1], we find $[\lambda_u C_1 - \frac{3}{2} \lambda_t C_9] \approx -|\lambda_u|(0.19 e^{i\gamma} + 0.81)$ and $[\lambda_u C_2 - \frac{3}{2} \lambda_t C_{10}] \approx |\lambda_u|(1.08 e^{i\gamma} + 0.16)$. This proves, without any assumption about hadronic matrix elements, that electroweak penguins give an important contribution to the $\Delta I = 1$ amplitudes $A_{3/2}$ and $A_{1/2}$. In the generalized factorization scheme, their effects can be included by replacing the hadronic parameters $a_1$ and $a_2$ in (8) with the new values

$$a_1^{\text{eff}} \approx a_1 + (0.025 a_1 - 0.740 a_2) e^{-i\gamma} \approx a_1 (1 - 0.14 e^{-i\gamma}),$$

$$a_2^{\text{eff}} \approx a_2 + (0.025 a_2 - 0.740 a_1) e^{-i\gamma} \approx a_2 (1 - 3.28 e^{-i\gamma}),$$

(16)

where in the last step $a_2/a_1 \approx 0.22$ has been used. To derive (16), we have only used the ansatz $a_1 = C_2 + \xi C_1$ and $a_2 = C_1 + \xi C_2$ [17] as well as the values of the Wilson coefficients quoted above; the hadronic parameter $\xi$ does not enter in this result. We observe that electroweak penguins give a large contribution to the matrix element $M_C$ in (8), whereas their effect on $M_T$ is moderate. Note that the notion of “colour suppression” of the electroweak penguin contributions in the decays $B^0 \to \pi^- K^+$ and $B^+ \to \pi^+ K^0$ [12, 15], which is sometimes employed as an argument in favour of their smallness, rests on a naive cancelation of the contributions of $M_C$ to the sum $A_{3/2} + A_{1/2}$, which according to (9) does not take place in the presence of final-state interactions.

We are now ready to work out the consequences of our results. A model-independent analysis, which allows for the possibility of having significant final-state interactions, must assume that the amplitudes $T, C, A$ and $P_a$ entering in (5) all have a similar magnitude. As discussed above, it must also include the contributions of electroweak penguin operators. Therefore, we parametrize the isospin amplitudes in the most general form

$$\frac{A_{3/2} + A_{1/2}}{\lambda_c(P_c - P_t)} = -\frac{1}{2} e^{i\gamma} \left(r e^{i\delta} - s e^{i\eta}\right) + t e^{i\zeta},$$

$$\frac{B_{1/2}}{\lambda_c(P_c - P_t)} = 1 + \frac{1}{2} e^{i\gamma} \left(r e^{i\delta} + s e^{i\eta}\right),$$

(17)

where $r, s$ and $t$ are real parameters expected to be of a similar magnitude, and $\delta, \eta$ and $\zeta$ are unknown strong phases. The electroweak penguin contribution with $\Delta I = 0$ is included in the definition of $P_t$, while that with $\Delta I = 1$ defines the term proportional to $t$. For completeness, we note that in the model discussed above

$$r e^{i\delta} = e^{i(\phi_{1/2} - \phi_T)} \left|\frac{M_T}{M_P}\right| \left[1 + \frac{1 + x}{3} \left(e^{i\Delta\phi} - 1\right)\right].$$
\[ -se^{in} = e^{i(\phi_{1/2} - \phi_P)} \left| \frac{M_T}{M_P} \right| 1 + \frac{x}{3} \left( e^{i\Delta \phi} - 1 \right), \]
\[ te^{i\kappa} \approx e^{i(\phi_{1/2} - \phi_P)} \left| \frac{M_T}{M_P} \right| \left[ 0.07 + (0.05 + 1.09x) \left( e^{i\Delta \phi} - 1 \right) \right], \tag{18} \]

where \( x = M_C/M_T \approx 0.2 \). It is apparent that rather significant rescattering effects can arise if the strong phases of the two isospin amplitudes \( A_{1/2} \) and \( A_{3/2} \) are different from each other, i.e. if \( \Delta \phi = O(1) \). The exact theoretical expression for the ratio of branching ratios in (7) becomes
\[
R = \frac{1 + 2r \cos \gamma \cos \delta - 2t \cos \zeta + r^2 + t^2 - 2rt \cos \gamma \cos(\delta - \zeta)}{1 + 2s \cos \gamma \cos \eta + 2t \cos \zeta + s^2 + t^2 + 2st \cos \gamma \cos(\eta - \zeta)}, \tag{19} \]

which is the correct generalization of the Fleischer–Mannel result. Clearly, without additional information about the hadronic parameters no model-independent bound on the angle \( \gamma \) can be derived. A related question is that about the expected size of the CP asymmetry in the decays \( B^\pm \to \pi^\pm K^0 \), for which we find
\[
A_{\text{CP}} = -\frac{2s \sin \gamma \left[ \sin \eta + t \sin(\eta - \delta) \right]}{1 + 2s \cos \gamma \cos \eta + 2t \cos \zeta + s^2 + t^2 + 2st \cos \gamma \cos(\eta - \zeta)} \tag{20}.
\]

In order to evaluate these results, some information about the parameters \( r, s \) and \( t \) is required. Model estimates, combined with the known hierarchy of CKM elements, suggest that \( r, s, t = O(0.1) \) (see also the estimate below, where we show that \( \left| \frac{M_T}{M_P} \right| = 0.14 \pm 0.04 \)), in which case it is a good approximation to work with the linearized expressions
\[
R \approx 1 + 2 \cos \gamma \left( r \cos \delta - s \cos \eta \right) - 4t \cos \zeta, \quad A_{\text{CP}} \approx -2s \sin \gamma \sin \eta. \tag{21} \]

From these results, it follows that \( (R - 1) \) and \( A_{\text{CP}} \) can naturally be of order 10–20%, in contrast to claims that the standard model would not allow for a sizable CP asymmetry in \( B^\pm \to \pi^\pm K^0 \) decays [12, 15, 18]. In linear approximation, the CP asymmetry is insensitive to electroweak penguin contributions. In our model, using the relations in (18) and setting \( x = 0.2 \), we find
\[
R \approx 1 + \left| \frac{M_T}{M_P} \right| \left\{ 2 \cos \gamma \left[ 0.2 \cos(\phi_{1/2} - \phi_P) + 0.8 \cos(\phi_{3/2} - \phi_P) \right] + \left[ 0.8 \cos(\phi_{1/2} - \phi_P) - 1.1 \cos(\phi_{3/2} - \phi_P) \right] \right\}, \]
\[ A_{\text{CP}} \approx 0.8 \left| \frac{M_T}{M_P} \right| \sin \gamma \left[ \sin(\phi_{3/2} - \phi_P) - \sin(\phi_{1/2} - \phi_P) \right]. \tag{22} \]
Note, in particular, the potentially large contribution to $R$ from electroweak penguins, given by the second term in parenthesis. Unless $|\phi_{3/2} - \phi_{1/2}|$ is small, this contribution may well dominate over the term involving the angle $\gamma$. We thus disagree with Ref. [12], where it was argued that the electroweak penguin contribution to $R$ is generally very small, i.e. of order 1%, and can be neglected.

Some information about the magnitude of the isospin amplitudes can be obtained by employing SU(3) flavour symmetry to relate the $B \rightarrow \pi K$ with $B \rightarrow \pi\pi$ decay amplitudes. In the process, the CKM parameters for $b \rightarrow s$ transitions have to be replaced by those for $b \rightarrow d$ transitions. Since $\lambda_u^{b-s}/\lambda_u^{b-d} = O(\lambda^{-1})$ while $\lambda_t^{b-s}/\lambda_t^{b-d} = O(\lambda)$, where $\lambda = 0.22$ is the Wolfenstein parameter, it follows that electroweak penguin contributions in $B \rightarrow \pi\pi$ decays are much smaller than in $B \rightarrow \pi K$ decays. Thus, in the SU(3) limit, the following triangle relations hold [3, 8]:

\[
3A_{3/2} = A(B^+ \rightarrow \pi^+ K^0) + \sqrt{2} A(B^+ \rightarrow \pi^0 K^+) = A(B^0 \rightarrow \pi^- K^+) + \sqrt{2} A(B^0 \rightarrow \pi^0 K^0) = \frac{V_{us}}{V_{td}} \sqrt{2} A(B^+ \rightarrow \pi^+\pi^0) + \text{electroweak penguins.} \tag{23}
\]

Using the CLEO measurement $\text{Br}(B^+ \rightarrow \pi^+\pi^0) = (1.0^{+0.6}_{-0.5}) \times 10^{-5}$ [13], we find that $|A_{3/2}|_{t=0} = (2.3^{+0.7}_{-0.6}) \times 10^{-4}$ (in “branching ratio units”, where $|A| = \text{Br}^{-1/2}$). This is in good agreement with our model prediction for the factorized decay amplitudes in (8), which yields $|A_{3/2}|_{t=0} = (2.0 \pm 0.5) \times 10^{-4}$ and $|A_{1/2}|_{t=0} = (0.4 \pm 0.1) \times 10^{-4}$, where we have assumed $|V_{ub}| = (3.5 \pm 0.5) \times 10^{-3}$. The subscript “$t = 0$” indicates that these numbers do not include electroweak penguin contributions. Furthermore, the CLEO measurements $\text{Br}(B^0 \rightarrow \pi^- K^+) = (1.5^{+0.5}_{-0.4}) \pm 0.1 \pm 0.1 \times 10^{-5}$ and $\text{Br}(B^+ \rightarrow \pi^+ K^0) = (2.3^{+1.1}_{-1.0} \pm 0.3 \pm 0.2) \times 10^{-5}$ [13] imply that $|B_{1/2} - A_{3/2} - A_{1/2}| = (3.9^{+0.6}_{-0.5}) \times 10^{-3}$ and $|B_{1/2} + A_{3/2} + A_{1/2}| = (4.8^{+1.3}_{-1.1}) \times 10^{-3}$. This is a strong indication that the isospin amplitude $B_{1/2}$, which contains the top- and charm-penguin contributions, dominates, i.e. $|B_{1/2}| \approx (4.1 \pm 0.5) \times 10^{-3} \gg |A_{1/2}|, |A_{3/2}|$. This information can be used to estimate the magnitude of one of the hadronic parameters entering the prediction for the ratio $R$ in (21):

\[
|r \cos \delta - s \cos \eta| < 2 \left| \frac{A_{3/2} + A_{1/2}}{\lambda_c (P_t - P_D)} \right|_{t=0} < 2 \left( \frac{|A_{3/2}| + |A_{1/2}|}{|B_{1/2}|} \right)_{t=0} = 0.13 \pm 0.03. \tag{24}
\]

Moreover, in the context of our model, we find from (9) that

\[
\frac{M_T}{M_P} \approx \frac{3}{1 + x} \left| \frac{A_{3/2}}{|B_{1/2}|} \right|_{t=0} = 0.14 \pm 0.04. \tag{25}
\]

These numerical estimates confirm that the effects parametrized by $r$, $s$ and $t$ are indeed of order 10%. We thus conclude that the linearized relations in (21) are reliable.

To summarize, we have argued that the diagrammatic approach to derive approximate isospin or flavour SU(3) relations between weak decay amplitudes is misleading in
the presence of soft final-state rescattering effects. There is no theoretical justification to neglect annihilation topologies, colour-suppressed topologies or up-quark penguin topologies relative to the colour-allowed tree topology. A classification scheme based on using isospin amplitudes defined in terms of operator matrix elements avoids these problems, since all the above-mentioned amplitudes are contained in the same matrix elements of the current–current operators $Q_{n,2}^u$, and thus it is natural that they all have a similar magnitude. On the other hand, many analyses of CP asymmetries in $B$ decays rely on neglecting such “suppressed” contributions. In view of our results, a careful reinvestigation of these analyses is necessary in order to judge whether amplitude relations that have been claimed to be “almost model independent” are really theoretically trustworthy. In the present work, we have shown that the Fleischer–Mannel bound for $\gamma$ [12] relies on such unjustified assumptions. With a realistic treatment of final-state interactions, no useful bound can be obtained. In particular, we have shown that the effects of electroweak penguins are severely enhanced in the presence of different strong phases for the isospin amplitudes $A_{1/2}$ and $A_{3/2}$ and, in principle, can yield a contribution to $R$ of as much as 10–20%. We have also shown that, in the context of the standard model, the CP asymmetry in $B^{\pm} \rightarrow \pi^{\pm}K^0$ decays can be of order 10%.

The problem that rescattering effects could invalidate the results derived using a diagrammatic amplitude analysis has been pointed out previously by Wolfenstein [19] and by Soni [20]. The issue has also been discussed recently in Ref. [21], where conclusions different from ours have been reached. In that paper, the authors perform an isospin analysis and absorb possible soft rescattering contributions into the amplitudes $T$ and $P$, corresponding to the choice $\Delta P = -X$ in our notation in (12). However, then they neglect these effects by implicitly assuming that the rescattering contribution $\Delta P$ to the up-quark penguin is much smaller than the tree amplitude $T$. As we have shown, this assumption is not justified.

While this paper was in writing, we became aware of a letter by Gérard and Weyers [22], in which conclusions similar to ours are reached. In particular, in a model with quasi-elastic rescattering these authors derive the first two relations in (10).

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References
