LOW-ENERGY GRAVITINO INTERACTIONS∗

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I discuss the low-energy limit of several processes involving only ordinary particles and gravitinos. Astrophysical and laboratory applications are briefly addressed.

It is well known that the solution of the hierarchy problem in supersymmetric extensions of the Standard Model (SM) requires a mass splitting $\Delta m \sim 1$ TeV between ordinary particles and their superpartners. This requirement, however, leaves largely undetermined the supersymmetry-breaking scale $\sqrt{F}$, or, equivalently, the gravitino mass $m_{3/2} = F/(\sqrt{3}M_P)$, $M_P$ being the Planck mass. The ratio $\Delta m^2/F$ is given by the coupling of the goldstino to the matter sector under consideration. If this coupling is gravitational, of order $m_{3/2}/M_P$, then $\Delta m$ is of order $m_{3/2}$ and supersymmetry breaking takes place at the intermediate scale $\sqrt{F} \sim 10^{10}$ GeV. On the other hand, if the goldstino coupling to matter is of order 1, then $\sqrt{F}$ is comparable to the mass splitting $\Delta m$, and the gravitino becomes superlight, with a mass of about $10^{-5}$ eV. In the absence of a theory of supersymmetry breaking, $F$ should be treated as a free parameter.

If the gravitino is superlight, then one expects a substantially different phenomenology from that characterizing the Minimal Supersymmetric Standard Model (MSSM). In this case, only the $\pm 3/2$ gravitino helicity states can be safely omitted from the low-energy effective theory, when gravitational interactions are neglected. The $\pm 1/2$ helicity states, essentially described by the goldstino field, should instead be accounted for at low energy, because of their non-negligible coupling to matter. The lightest supersymmetric particle is the gravitino and peculiar experimental signatures can arise from the decay of the next-to-lightest supersymmetric particle into its ordinary partner plus a gravitino [1].

Moreover, even when all superymmetric particles of the MSSM are above the production threshold, interesting signals could come from those processes where only ordinary particles and gravitinos occur. As soon as the typical energy of the process is larger than $m_3/2$, a condition always fulfilled in the applications discussed below, one can approximate the physical amplitudes by replacing external gravitinos with goldstinos, as specified by the equivalence theorem [2]. If the masses of the ordinary particles involved are negligible with respect to the energy of interest, these processes are controlled by just one dimensionful parameter, the supersymmetry-breaking scale $\sqrt{F}$, entering the amplitudes in the combination $(\tilde{G}/\sqrt{2}F)$, $\tilde{G}$ denoting the goldstino wave function.

This class of processes includes $\gamma\gamma \rightarrow \tilde{G}\tilde{G}$, $e^+e^- \rightarrow \tilde{G}\tilde{G}$, which may influence primordial nucleosynthesis, stellar cooling and supernovae explosion [3, 4]. Of direct interest for LEP2 and for the future linear colliders is the reaction $e^+e^- \rightarrow \tilde{G}\tilde{G}\gamma$. Partonic reactions such as $q\bar{q} \rightarrow \tilde{G}\tilde{G}\gamma$, $q\bar{q} \rightarrow \tilde{G}\tilde{G}g$ and $qg \rightarrow \tilde{G}\tilde{G}q$ can be indirectly probed at the Tevatron collider or in future hadron facilities. In the absence of experimental signals, one can use these processes to set absolute limits on the gravitino mass. At variance with other bounds on $m_{3/2}$ discussed in the literature [5], these limits have the advantage of not depending on detailed assumptions about the spectrum of supersymmetric particles. Finally, the study of these processes can reveal unexpected features of the low-energy theory, which were overlooked in the standard approach to goldstino low-energy interactions.

The natural tools to analyse the above processes are the so-called low-energy theorems [6]. According to these, the low-energy amplitude for the scattering of a goldstino on a given target is controlled by the energy–momentum tensor $T_{\mu\nu}$ of the target. To evaluate the physical amplitudes, it is more practical to make use of an effective Lagrangian, containing the goldstino field and the matter fields involved in the reactions, and providing a non-linear realization of the supersymmetry algebra [7]. For instance, in the non-linear construction of [8], the goldstino field $\tilde{G}$ and the generic matter field $\varphi$ are incorporated into the following superfields:

$$\Lambda_\alpha \equiv \exp(\theta Q + \bar{\theta} Q) G_\alpha = \frac{G_\alpha}{\sqrt{2}F} + \theta_\alpha + \frac{i}{\sqrt{2}F}(\tilde{G}\sigma^\mu \bar{\theta} - \theta\sigma^\mu \tilde{G})\partial_\mu \frac{\tilde{G}_\alpha}{\sqrt{2}F} + \ldots,$$

$$\Phi \equiv \exp(\theta Q + \bar{\theta} Q) \varphi = \varphi + \frac{i}{\sqrt{2}F}(\tilde{G}\sigma^\mu \bar{\theta} - \theta\sigma^\mu \tilde{G})\partial_\mu \varphi + \ldots.$$  \hspace{1cm} (1)

The goldstino–matter system is described by the supersymmetric Lagrangian:

$$\int d^2\theta d^2\bar{\theta} \Lambda^2 \bar{\Lambda}^2 \left[ 2F^2 + \mathcal{L}(\Phi, \partial\Phi) \right],$$  \hspace{1cm} (3)
where $\mathcal{L}(\varphi, \partial \varphi)$ is the ordinary Lagrangian for the matter system. This non-linear realization automatically reproduces the results of the low-energy theorems, in particular the expected goldstino coupling to the energy-momentum tensor $T_{\mu\nu}$ associated to $\varphi$.

An alternative approach consists in constructing a low-energy Lagrangian, starting from a general supersymmetric theory defined, up to terms with more than two derivatives, in terms of a Kähler potential, a superpotential and a set of gauge kinetic functions. The effective theory can be obtained by integrating out, in the low-energy limit, the heavy superpartners [3].

When applied to the process $\gamma\gamma \to \tilde{G}\tilde{G}$, the two procedures yield the same result. The only independent, non-vanishing, helicity amplitude for the process is:

$$a(1, -1, 1/2, -1/2) = 8 \sin \theta \cos^2 \frac{\theta}{2} \frac{E^4}{F^2}, \quad (4)$$

where $(1, -1)$ and $(1/2, -1/2)$ are the helicities of the incoming and outgoing particles, respectively; $E$ and $\theta$ are the goldstino energy and scattering angle in the centre-of-mass frame. The total cross section is $s^3/(640\pi F^4)$.

In earlier cosmological and astrophysical applications, with a typical energy range from about 1 keV to 100 MeV, a cross-section scaling as $\Delta m^2 s^2 / F^4$ was assumed, giving rise to a lower bound on $m_{3/2}$ close to $10^{-6}$ eV. When the correct energy dependence is taken into account, this bound is reduced by at least a factor 10, and becomes uninteresting compared to those obtainable at colliders.

When considering $e^+e^- \to \tilde{G}\tilde{G}$, in the limit of massless electron, one has to face an unexpected result [9]. On the one hand, by integrating out the heavy selectron fields, one finds the following helicity amplitude:

$$a(1/2, -1/2, 1/2, -1/2) = 4(1 + \cos \theta)^2 \frac{E^4}{F^2}, \quad (5)$$

all other non-vanishing amplitudes being related to this one. On the other hand, by using the non-linear realization of [8], one obtains:

$$a(1/2, -1/2, 1/2, -1/2) = 4\sin^2 \theta \frac{E^4}{F^2}. \quad (6)$$

The amplitudes of eqs. (5) and (6) scale in the same way with the energy, but have a different angular dependence. We should conclude that the low-energy theorems of ref. [6] do not apply to the case of a massless fermion. A particularly disturbing aspect is that the non-linear realization of eq. (3) is supposed to provide the most general parametrization of the
amplitude in question, independently of any considerations about the low-energy theorems. In the case at hand, the Lagrangian of eq. (3) reads:

\[\mathcal{L}_e = \int d^2\theta d^2\bar{\theta} \Lambda^2 \bar{\Lambda}^2 \left[ 2F^2 + iE\sigma^\mu \partial_\mu \bar{E} + iE^c\sigma^\mu \partial_\mu \bar{E}^c \right], \tag{7}\]

where \(E\) and \(E^c\) are the superfields associated to the two Weyl spinors \(e\) and \(e^c\) describing the electron, according to eq. (2). The solution to this puzzle \([9]\) is provided by the existence of an independent supersymmetric invariant that has been neglected up to now in the literature:

\[\delta\mathcal{L}_e = \int d^2\theta d^2\bar{\theta} (\Lambda E \bar{\Lambda} \bar{E} + \Lambda E^c \bar{\Lambda} \bar{E}^c). \tag{8}\]

The amplitudes of eq. (5) are reproduced by the combination \(\mathcal{L}_e + 8 \delta\mathcal{L}_e\).

On the other hand, there is no reason to prefer either the result of eq. (5) or that of eq. (6). The process \(e^+e^-\rightarrow \tilde{\mathcal{G}}\tilde{\mathcal{G}}\) does not have a universal low-energy behaviour. In the framework of non-linear realizations, this freedom can be described by the invariant Lagrangian \(\mathcal{L}_e + \alpha \delta\mathcal{L}_e\), where \(\alpha\) is a free parameter of the low-energy theory.

The process \(e^+e^-\rightarrow \tilde{\mathcal{G}}\tilde{\mathcal{G}}\gamma\) suffers from a similar ambiguity \([10]\). Indeed the soft and collinear part of the cross-section, which is the dominant one, is associated to the initial-state radiation and hence is determined by the cross-section for \(e^+e^-\rightarrow \tilde{\mathcal{G}}\tilde{\mathcal{G}}\). The total cross-section, with appropriate cuts on the photon energy and scattering angle, scales as \(\alpha_{em}s^3/F^4\). The photon energy and angular distributions are not universal, as for the case of the goldstino angular distribution in \(e^+e^-\rightarrow \tilde{\mathcal{G}}\tilde{\mathcal{G}}\). They could be completely determined only through a computation performed in the fundamental theory. From the non-observation of single-photon events above the SM background at LEP2, one can roughly estimate a lower bound on \(\sqrt{F}\) of the order of the machine energy \([10]\). The precise value of this limit would require the analysis of the relevant background, as well as the inclusion of the above mentioned theoretical ambiguity. However, in view of the quite strong power dependence of the cross-section on the supersymmetry-breaking scale, one expects only a small correction to the limit obtained by a rough dimensional estimate.

The partonic processes \(q\bar{q}\rightarrow \tilde{\mathcal{G}}\tilde{\mathcal{G}}\gamma\), \(q\bar{q}\rightarrow \tilde{\mathcal{G}}\tilde{\mathcal{G}}g\) are also expected to have cross-sections scaling as \(\alpha_{em}s^3/F^4\) and \(\alpha_gs^3/F^4\) respectively. The agreement between data and SM expectations in \(p\bar{p}\rightarrow \gamma + \not{E}_T + X\) and \(p\bar{p}\rightarrow \text{jet} + \not{E}_T + X\) at the Tevatron collider could then be used to infer a lower limit on \(\sqrt{F}\). This is expected to be around the typical total energy of the partonic subprocess, about 600 GeV.
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REFERENCES