\( \alpha_S \): from DIS to LEP\(^*\)

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1. INTRODUCTION

The strong coupling \( \alpha_S \) is a fundamental parameter of the Standard Model. In comparison to parameters like \( \alpha_{em} \), \( M_Z \) and \( \sin^2 \theta_W \) it is relatively poorly known. However the precision of \( \alpha_S \) measurements has improved dramatically in recent years. More than twenty different types of process, from lattice QCD studies to the highest energy colliders, can be used to measure \( \alpha_S \) accurately. The most precise determinations now quote uncertainties in \( \alpha_S(M_Z^2) \) of less than 5%. There is also a remarkable consistency between the various measurements.

A comprehensive review of \( \alpha_S \) measurements, including detailed descriptions of the underlying physics for the most important processes, can be found in Ref. [1]. One year later, several of the measurements quoted in Ref. [1] have been updated, resulting in a slight shift in the overall ‘world average’ value. The purpose of the present review is to update the discussion on \( \alpha_S \) measurements given in Ref. [1], focusing on the new values reported in the last year. For more theoretical details, descriptions of other measurements and a full set of references, the reader is referred to the original review in Ref. [1].

The current situation is summarised in Fig. 1, which updates Table 12.1 of Ref. [1]. Before discussing the new measurements in detail, we begin with some

\(^*\) Based on a talk presented at the ‘New Non-Perturbative Methods and Quantisation on the Light Cone’ conference, Les Houches, France, March 1997
technical preliminaries. In perturbative QCD the dependence of the strong coupling on the renormalisation scale is determined by the $\beta$-function:

$$Q^2 \frac{\partial \alpha_S(Q^2)}{\partial Q^2} = \beta(\alpha_S(Q^2)),$$
\[ \beta(\alpha_S) = -b\alpha_S^2 \left( 1 + b'\alpha_S + b''\alpha_S^2 + \ldots \right), \]  

where \( b = (33 - 2n_f)/(12\pi) \) etc. The coefficients in the perturbative expansion depend, in general, on the renormalisation scheme (RS), although for massless quarks the first two coefficients, \( b \) and \( b' \), are RS independent. In essentially all phenomenological applications the \( \overline{\text{MS}} \) RS is used; see Ref. [1] for further discussion and explicit expressions for the known \( \beta \)-function coefficients.

At leading order, i.e. retaining only the coefficient \( b \), Eq. (1) can be solved for \( \alpha_S \) to give

\[ \alpha_S(Q^2) = \frac{\alpha_S(Q_0^2)}{1 + \alpha_S(Q_0^2) b \ln(Q^2/Q_0^2)}, \]

or

\[ \alpha_S(Q^2) = \frac{1}{b \ln(Q^2/\Lambda^2)}. \]

These two expressions are entirely equivalent – they differ only in the choice of boundary condition for the differential equation, \( \alpha_S(Q_0^2) \) in the first case and the dimensionful parameter \( \Lambda \) in the second. In fact nowadays \( \Lambda \) is disfavoured as the fundamental parameter of QCD, since its definition is not unique beyond leading order (see below), and its value depends on the number of ‘active’ quark flavours. Instead, it has become conventional to use the value of \( \alpha_S \) in the \( \overline{\text{MS}} \) scheme at \( Q^2 = M_Z^2 \) as the fundamental parameter. The advantage of using \( M_Z \) as the reference scale is that it is (a) very precisely measured [2], (b) safely in the perturbative regime, i.e. \( \alpha_S(M_Z^2) \ll 1 \), and (c) far from quark thresholds, i.e. \( m_b \ll M_Z \ll m_t \).

The parameter \( \Lambda \) is, however, sometimes still used as a book-keeping device. At next-to-leading order there are two definitions of \( \Lambda \) which are widely used in the literature:

\begin{align*}
\text{definition 1:} & \quad b \ln \frac{Q^2}{\Lambda^2} = \frac{1}{\alpha_S(Q^2)} + b' \ln \left( \frac{b'\alpha_S(Q^2)}{1 + b'\alpha_S(Q^2)} \right), \\
\text{definition 2:} & \quad \alpha_S(Q^2) = \frac{1}{b \ln(Q^2/\Lambda^2)} \left[ 1 - \frac{b' \ln \ln(Q^2/\Lambda^2)}{b \ln(Q^2/\Lambda^2)} \right].
\end{align*}

The first of these solves Eq. (1) exactly when \( b'' \) and higher coefficients are neglected, while the second (the ‘PDG’ definition [2]) provides an explicit expression for \( \alpha_S(Q^2) \) in terms of \( Q^2/\Lambda^2 \) and is a solution of Eq. (1) up to terms of order \( 1/\ln^3(Q^2/\Lambda^2) \). Note that these two \( \Lambda \) parameters are different for the same value of \( \alpha_S(M_Z^2) \), the difference being about one quarter the size of the current measurement uncertainty:

\[ \Lambda_1^{(5)} - \Lambda_2^{(5)} \simeq 15 \text{ MeV} \simeq \frac{1}{4} \delta_{\exp} \Lambda^{(5)}. \]

In this review we will be mainly concerned with measurements from \( e^+e^- \) colliders (in practice LEP and SLC) and from deep inelastic scattering. Both processes offer several essentially independent measurements, summarised in
Table I. — Summary of the most important processes for $\alpha_S$ determinations in $e^+e^-$ collisions and in deep inelastic lepton-hadron scattering.

<table>
<thead>
<tr>
<th>quantity</th>
<th>perturbation series</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^+e^-$</td>
<td>$R = R_0[1 + \alpha_S/\pi + \ldots]$</td>
</tr>
<tr>
<td>event shapes, $f_3$, ...</td>
<td>$1/\sigma d\sigma/dX = A\alpha_S + B\alpha_S^2 + \ldots$</td>
</tr>
<tr>
<td>$D^h(z,Q^2)$</td>
<td>$\partial D^h/\partial \ln Q^2 = \alpha_S D^h \otimes P + \ldots$</td>
</tr>
<tr>
<td>$N$ DIS</td>
<td>$F_i(x,Q^2)$</td>
</tr>
<tr>
<td>$\sigma(2+1\text{ jet})$</td>
<td></td>
</tr>
</tbody>
</table>

Note that all of these use the $q\bar{q}g$ vertex to measure $\alpha_S$, with the high $Q^2$ scale provided by an electroweak gauge boson, for example a highly virtual $\gamma^*$ in DIS or an on-shell $Z^0$ boson at LEP1 and SLC. There are two main theoretical issues which affect these determinations. The first is the effect of unknown higher-order (next-to-next-to-leading order (NNLO) in most cases) perturbative corrections, which leads to a non-negligible renormalisation scheme dependence uncertainty in the extracted $\alpha_S$ values. This is particularly true for the event shape measurements at $e^+e^-$ colliders. The exceptions here are the total $e^+e^-$ hadronic cross section (equivalently, the $Z^0$ hadronic decay width) and the DIS sum rules, which are known to NNLO. The second issue concerns the residual impact of $O(1/Q^n)$ power corrections. For some processes it can be shown that the leading corrections are $O(1/Q)$ (for example $O(1/M_Z)$ for the corrections to event shapes at LEP1 and SLC) which can easily be comparable in magnitude to the NLO perturbative contributions. In deep inelastic scattering, the higher-twist power corrections are $O(1/Q^2(1-x))$ and must be included in scaling violation fits especially at large $x$. Such power corrections (and their uncertainties) must be taken into account in $\alpha_S$ determinations, either using phenomenological parametrisations or theoretical models.

Before discussing the new high-energy collider measurements of $\alpha_S$ it is important to mention also determinations from lattice QCD, which have very small uncertainties. One of the simplest ways to define $\alpha_S$ on the lattice is to use the average value of the $1 \times 1$ Wilson loop (plaquette) operator:

$$\ln W_{1,1} = \frac{4\pi}{3} \alpha_P \left( \frac{3.4}{a} \right) \left[ 1 - (1.19 + 0.07n_f)\alpha_P \right],$$

(7)
\[ \alpha_{S} \text{ FROM DIS TO LEP} \]

where \( a \) is the lattice spacing. A variety of choices is available for determining \( a \), i.e. measuring the scale at which \( \alpha_{P} \) has the value measured in (7). Quarkonium level splittings, for example \( \Upsilon(S-P) \) and \( \Upsilon(1S-2S) \), are particularly suitable. Subsequently the plaquette \( \alpha_{P} \) can be converted to the standard \( \overline{\text{MS}} \) \( \alpha_{S} \) for comparison with other determinations:

\[
\alpha^{(\overline{\text{MS}}, n_f)}_{S}(Q^2) = \alpha^{(n_f)}_{P}(e^{5/3} Q^2) \left[ 1 + \frac{2}{\pi} \alpha^{(n_f)}_{P} + C_2(n_f) \left( \alpha^{(n_f)}_{P} \right)^2 + \ldots \right]. \tag{8}
\]

At present the two-loop coefficient is known only for \( n_f = 0 \) [3] – the shift in \( \alpha_{S} \) between using \( C_2(n_f = 0) \) and \( C_2 = 0 \) can be used to define a ‘conversion’ error. Several new lattice \( \alpha_{S} \) values have been obtained recently, see for example Ref. [4], and are included in Fig. 1. As an example of the high precision of these measurements, we quote the value obtained by the NRQCD collaboration [5] using the \( \Upsilon(S-P) \) splitting:

\[
\alpha^{(\overline{\text{MS}}, m_f)}_{S}(M_{Z}^2) = 0.1175 \pm 0.0011 \text{(stat. + sys.)} \pm 0.0013 \text{(conv.)}, \tag{9}
\]

where the first error is due to the lattice statistics and systematics, the second is from the extrapolation in the dynamical quark mass, and the third is the conversion error mentioned above.

In the following sections we will discuss new \( \alpha_{S} \) measurements from LEP/SLC and from deep inelastic scattering. Section 4 presents a new value for the \( \alpha_{S} \) world average.

2. \( \alpha_{S} \) FROM LEP AND SLD

In principle the most reliable determination of \( \alpha_{S} \) at the LEP and SLD \( e^{+}e^{-} \) colliders comes from the \( Z^0 \) hadronic width. In particular we have, for the ratio \( R_{Z} \),

\[
R_{Z} = \frac{\Gamma(Z^0 \rightarrow \text{hadrons})}{\Gamma(Z^0 \rightarrow e^+e^-)} = R_0 \left[ 1 + \frac{\alpha_{S}}{\pi} + C_2 \left( \frac{\alpha_{S}}{\pi} \right)^2 + C_3 \left( \frac{\alpha_{S}}{\pi} \right)^3 + \ldots \right], \tag{10}
\]

with \( R_0 = 3 \sum_{q} (v_{q}^2 + a_{q}^2) / (v_{u}^2 + a_{u}^2) \). The perturbative coefficients are known up to third order, see Ref. [1] for explicit expressions and references, and as a result the prediction is very stable with respect to variations in the renormalisation scale. In practice, since \( R_0 \) depends on the weak mixing angle and other electroweak parameters, it is more appropriate to perform a global fit to all relevant electroweak quantities, for example the (LEP and SLD) \( Z^0 \) partial widths and decay asymmetries, \( p\bar{p} \) collider measurements of \( M_{W} \) and \( m_{t} \), etc. Such analyses are performed regularly by the LEP Electroweak Working Group, and the results of a recent (1996) fit [6] are summarised in Table 2. An additional theory error from unknown higher-order corrections of \( \delta \alpha_{S}(M_{Z}^2) = \pm 0.002 \) has been estimated, see Ref. [6] and references therein. The resulting \( \alpha_{S} \) value,

\[
\alpha_{S}(M_{Z}^2) = 0.120 \pm 0.003 \text{(fit)} \pm 0.002 \text{(theory)}, \tag{11}
\]
Table II. — Values for the Standard Model parameters obtained from a global fit to LEP, SLD, $p\bar{p}$ and $\nu N$ data, from Ref. [6].

<table>
<thead>
<tr>
<th>parameter</th>
<th>fit value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_t$ [GeV]</td>
<td>$172 \pm 6$</td>
</tr>
<tr>
<td>$M_H$ [GeV]</td>
<td>$149^{+148}_{-82}$</td>
</tr>
<tr>
<td>$\alpha_S(M_Z^2)$</td>
<td>$0.120 \pm 0.003$</td>
</tr>
<tr>
<td>$\sin^2 \theta_{\text{eff}}$</td>
<td>$0.23167 \pm 0.00023$</td>
</tr>
<tr>
<td>$1 - M_W^2/M_Z^2$</td>
<td>$0.2235 \pm 0.0006$</td>
</tr>
<tr>
<td>$M_W$ [GeV]</td>
<td>$80.352 \pm 0.033$</td>
</tr>
</tbody>
</table>

is displayed in Fig. 1.

The other high-precision determination of $\alpha_S$ at LEP and SLC comes from event shapes, quantities which measure the relative contribution of the $O(\alpha_S)$ $e^+ e^- \rightarrow q\bar{q}g$ process to the total hadronic cross section, see Table 1. A typical example is the thrust distribution:

$$\frac{1}{\sigma} \frac{d\sigma}{dT} = \alpha_S A_1(T) + \alpha_S^2 A_2(T) + \ldots + O\left(\frac{1}{E_{\text{cm}}}\right). \quad (12)$$

Such quantities are known in perturbation theory to $O(\alpha_S^2)$, and the theoretical predictions in the $T \rightarrow 1$ region can be improved by resumming the leading logarithmic $A_n \sim \ln^{(2n-1)}(1-T)/(1-T)$ contributions to all orders, as discussed in Ref. [1]. Another important recent theoretical development has been an improved understanding of the leading $O(1/E)$ power corrections [7], which at LEP can be as numerically important as the next-to-leading perturbative corrections.

Event shapes have yielded $\alpha_S$ measurements over a wide range of $e^+ e^-$ collision energies, the most recent measurements being at the LEP2 energies $\sqrt{s} = 161$ and 172 GeV. Although the statistical precision of these measurements cannot match that obtained at the $Z^0$ pole, the results are consistent with the $Q^2$ evolution of $\alpha_S$ predicted by Eq. (1). For example, Fig. 2 shows the $\alpha_S$ values determined by the L3 collaboration [8] from event shape measurements at LEP1 and LEP2 energies. The solid line is the evolution predicted by perturbative QCD. Figure 1 contains a new ‘LEP1.5’ average value for $\alpha_S$ obtained from event shapes at $\sqrt{s} = 133$ GeV, taken from the 1996 review by Schmelling [9]:

$$\alpha_S(Q^2 = (133 \text{ GeV})^2) = 0.114 \pm 0.007 \Rightarrow \alpha_S(M_Z^2) = 0.121 \pm 0.008. \quad (13)$$

Another updated value in Fig. 1 is that obtained from the scaling violations of the fragmentation function measured in $e^+ e^- \rightarrow hX$ over a range of collision energies, the analogue of the scaling violations of structure functions in DIS.
The new value (an ALEPH/DELPHI average taken from Ref. [9]) corresponds to
\[ \alpha_S(M_Z^2) = 0.124 \pm 0.010. \] (14)

Finally, the CLEO collaboration have published [10] a new value for \( \alpha_S \) obtained from the relative decay rate of the \( \Upsilon(1S) \) into a single hard photon:
\[ \frac{\Gamma^{g\gamma}}{\Gamma^{gg}} = \frac{4}{5} \frac{\alpha}{\alpha_S(\mu^2)} \left[ 1 - (2.6 - 2.1 \ln(m_b^2/\mu^2)) \frac{\alpha_S(\mu^2)}{\pi} + \ldots \right]. \] (15)

The new value,
\[
\begin{align*}
\alpha_S(M_{\Upsilon(1S)}^2) & = 0.163 \pm 0.002(\text{stat.}) \pm 0.014(\text{sys.}) \\
\Rightarrow \alpha_S(M_Z^2) & = 0.110 \pm 0.001(\text{stat.}) \pm 0.007(\text{sys.}),
\end{align*}
\] (16)
is included in Fig. 1.

![Graph showing \( \alpha_S \) measurements](image)

Fig. 2. — Measurements of \( \alpha_S \) from event shapes at LEP1 and LEP2 from the L3 collaboration [8]. The errors correspond to experimental uncertainties.

3. \( \alpha_S \) from Deep Inelastic Scattering

The traditional method of measuring \( \alpha_S \) in deep inelastic scattering is from the strength of the structure function scaling violations predicted by the DGLAP
equations:

\[ Q^2 \frac{\partial q^{NS}}{\partial Q^2} = \frac{\alpha_S(Q^2)}{2\pi} P^{qq} \otimes q^{NS} \]

\[ Q^2 \frac{\partial q^S}{\partial Q^2} = \frac{\alpha_S(Q^2)}{2\pi} \left( P^{gg} \otimes q^S + 2n_f P^{gg} \otimes g \right) \]

\[ Q^2 \frac{\partial g}{\partial Q^2} = \frac{\alpha_S(Q^2)}{2\pi} \left( P^{gg} \otimes q^S + P^{gg} \otimes g \right), \]

(17)

where \( q^{NS} \) and \( q^S \) are respectively non-singlet and singlet combinations of quark distribution functions. The fixed target and HERA structure function data, spanning a large range in \( x \) and \( Q^2 \), are all consistent with NLO DGLAP evolution, and yield \( \alpha_S \) values which are in broad agreement. As an example, Fig.3 [11] shows the \( \chi^2 \) values for various DIS data sets as a function of the \( \alpha_S(M_Z^2) \) value in the evolution equations. With one exception, all data sets exhibit a \( \chi^2 \) minimum in the \( \alpha_S = 0.11 - 0.13 \) range. In fact the ‘best fit’ value for these data sets is \( \alpha_S(M_Z^2) = 0.118 \), exactly the world average value (see Section 4 below).

It is difficult to extract a proper error on \( \alpha_S \) from such global fit analyses. This requires a rigorous treatment of systematic errors and inclusion of higher-twist contributions in the fit. Several groups have performed such analyses. For example, the Milsztajn–Virchaux analysis of the SLAC/BCDMS (\( eN,\mu N \)) data [12] yields

\[ \alpha_S(M_Z^2) = 0.113 \pm 0.005, \]

(18)

where the error includes statistical, systematic and scale dependence uncertainties. Recently the CCFR collaboration have reported [13] a new value of \( \alpha_S \) from their \( F_{\nu N}^p, xF_{\nu N}^p \) high-precision data (see Fig. 3):

\[ \alpha_S(M_Z^2) = 0.119 \pm 0.002(\text{exp.}) \pm 0.001(\text{HT}) \pm 0.004(\text{scale}). \]

(19)

The second error is from an estimate of the higher-twist contribution using the model of Ref. [14], and the third is the scale dependence uncertainty implemented as in Ref. [12]. Note that the value in (19) is somewhat larger than the earlier (1993) CCFR value of \( \alpha_S = 0.111 \pm 0.004 \). The change is due to new energy calibrations of the detector [13].

Deep inelastic scattering structure functions satisfy a variety of sum rules, corresponding to the conservation of various nucleon quantum numbers. In general the parton model values of the sums have \( O(\alpha_S) \) corrections, which can be used to extract \( \alpha_S \) from measurements of structure function integrals at fixed \( Q^2 \). Two sums rules which have been used to obtain precision measurements are the Gross–Llewellyn Smith and Bjorken sum rules (see Ref. [1] for more discussion and references):

\[ \text{GLS : } \int_0^1 dx(F_3^{\nu p} + F_3^{\bar{\nu} p}) = 6 \left[ 1 + \frac{\alpha_S}{\pi} + \ldots \right] + \Delta_{\text{HT}}, \]

(20)
Fig. 3. — $\chi^2$ values for various DIS data sets obtained in a global fit to these and other hard scattering data [11].

BjS: \[ \int_0^1 dx (g_1^p - g_1^n) = \frac{1}{6} \frac{g_A}{g_V} \left[ 1 - \frac{\alpha_S}{\pi} + \ldots \right] + \Delta_{HT}, \] (21)

where $\Delta_{HT}$ represents $O(1/Q^2)$ higher-twist contributions. A new analysis [15] of polarised structure function measurements has produced an update of the $\alpha_S$ value from the Bjorken sum rule. In Ref. [15] Padé Summation is used to reduce the theoretical error from the choice of renormalisation scheme in the calculation of the perturbation series on the right-hand side of (21). The
resulting theoretical error in $\alpha_s(M_Z^2)$ is estimated at $\pm 0.002$:  

$$\alpha_s(M_Z^2) = 0.117^{+0.004}_{-0.007}(\text{exp.}) \pm 0.002(\text{theory}) \quad (22)$$

This new value is included in Fig. 1.
Finally, $\alpha_s$ can be obtained from jet fractions and event shapes in DIS, see Table 1. For example, NLO theoretical predictions are currently being used at HERA to extract $\alpha_s$ from the relative rate of ‘2+1’ jet production at high $Q^2$, the analogue of $f_3$ in $e^+e^-$ annihilation. No new results have been published since the review in Ref. [1].

4. Summary

The average value\(^{(1)}\) of the measurements presented in Fig. 1 is

$$\text{WORLD AVERAGE: } \alpha_s(M_Z^2) = 0.118 \pm 0.004. \quad (23)$$

Following Ref. [1], the error here is defined as ‘the uncertainty equal to that of a typical measurement by a reliable method’. In view of the recent improvements in the lattice, $Z^0$ hadronic width, and DIS ($\nu N$) determinations, it seems appropriate to decrease the uncertainty of $\pm 0.005$ in Ref. [1] to $\pm 0.004$. The central value in (23) has increased by $+0.002$ from that given in Ref. [1]. This is due primarily to (a) increases of $+0.003$ and $+0.004$ in the central values of the two lattice determinations, and (b) an increase in the CCFR $\nu N$ DIS scaling violation central value of $+0.008$. In view of the remarkable consistency of all the measurements, and in particular of those with the smallest uncertainties, it seems unlikely that future ‘world average’ values of $\alpha_s$ will deviate significantly, if at all, from the current value given in (23).

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   (1997).