1992 – 1993 ACADEMIC TRAINING PROGRAMME

LECTURE SERIES FOR POSTGRADUATE STUDENTS

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TITLE : Symmetry and symmetry breaking in particle physics
TIME : 26, 27, 28, 29 January & 1 February, 11.00 to 12.00 hrs
PLACE : Auditorium

ABSTRACT

We will discuss how the constraints on symmetries can be implemented in the calculation of particle properties. Using the notion of "effective field theories" we will show how symmetry can be used to parameterize in interactions of even very complicated systems in relatively simple ways.

For examples we will discuss the pions and kaons of QCD and radiative corrections in the weak interactions. Topics to be covered:

1) Symmetries and constraints on interactions
2) Low energy effective interactions
3) Mesons and chiral symmetry breaking
4) Weak interactions and radiative corrections
Over the next 5 lectures, we want to discuss how symmetries can be applied to understanding physics. Our approach is going to be a look not at what the symmetries of the world are (we have been lectures about this) but how we go about applying symmetry principles -- the techniques for implementing these symmetries in our theories and getting observable consequences out.

Slow and Pedagogical -- a formal at the beginning with S.M as the principle example.
Over 50 years ago Fermi wrote his theory of Weak Interactions

\[ \mathcal{H}_F = \frac{G_F}{\sqrt{2}} \, J^+_\mu J^-_\mu + h.c. \]

\[ J^+_\mu = J^+_{\mu h} + J^+_{\mu e} \]

\[ J^+_{\mu h} = \bar{\nu}_e \, \gamma^\mu (1 + \gamma_5) \nu_e \]

\[ J^+_{\mu e} = \sum_{i=1}^{3} \bar{\nu}_i \, \gamma^\mu (1 + \gamma_5) U_{ei} \]

which described, among other things

\[ \pi^- \rightarrow e^- \, \bar{\nu}_e \]

Using lowest order perturbation theory we can compute the amplitude

\[ i \mathcal{A} (\pi^-(p) \rightarrow e^- \, \bar{\nu}_e) \]

\[ = \frac{G_F}{\sqrt{2}} \left\langle e^- \bar{\nu}_e | J^+_{\mu h} | \pi^- \right\rangle \]

\[ = \frac{G_F}{\sqrt{2}} \left\langle e^- \bar{\nu}_e | J^+_{\mu e} | 0 \right\rangle \left\langle 0 | J^-_{\mu h} | \pi^- \right\rangle \]
It is conventional to write
\[ \langle 0 | \bar{u} \gamma^\mu (1 + i\gamma_5) d | \Pi^- \rangle = i f_\pi p^\mu \]
\[ \langle e^- \bar{\nu}_e | e^- \gamma^\mu (1 + i\gamma_5) \nu_e | 0 \rangle \equiv E^\mu \]
(Standard Feynman Rules for $E^\mu$)

Questions

\[ \rightarrow 2 \] Why product of currents?
\[ \rightarrow 1 \] Why these currents?
\[ \rightarrow 3 \] What does $\langle 0 | \bar{u} \gamma^\mu (1 + i\gamma_5) d | \Pi^- \rangle = i f_\pi p^\mu$ mean?
\[ \rightarrow 4 \] How does this relate to the Standard Model?

To understand the answers to these questions we need to understand

Symmetries and Effective Field Theories.

What are symmetries and how are they reflected in the operators that we wrote?
Global Symmetries in Field Theory

\[ S = \int d^4x \mathcal{L}(\phi, \partial\phi) \]

Imagine some set of 

Initial Value Data (IVD) = \{ \phi(x, t_0), \phi(x, t) \}

Now construct a continuous path from this IVD to some other IVD:

\[ \text{VD}(\lambda) : \text{VD}(0) = \{ \phi(x, t_0), \phi(x, t) \} \]

Now use equations of motion to find FVD

If the diagram is true (i.e., diagram is commutative) we have a symmetry.

:: Symmetry relates the time evolution of different sets of IVD.
Let \( \mathcal{D}\Phi = \frac{\partial \Phi}{\partial \lambda} \bigg|_{\lambda=0} \)

Symmetry \( \iff \) Equations of Motion

Unchanged

\[ \delta \mu^\mu = \frac{\delta L}{\delta (\partial_\lambda \Phi)} \]

\[ \mathcal{D}S = 0 \quad \Downarrow \quad \mathcal{D}L = 0 \]

\[ \mathcal{D}L = \frac{\delta L}{\delta \Phi} \mathcal{D}\Phi + \frac{\delta L}{\delta (\partial_\mu \Phi)} \partial_\mu \mathcal{D}\Phi \]

\[ = \partial_\mu \mu^\mu \mathcal{D}\Phi + \mu^\mu \partial_\mu \mathcal{D}\Phi \]

\[ = \partial_\mu [\mu^\mu \mathcal{D}\Phi] \]

So \( \partial_\mu J^\mu = 0 \quad \Downarrow \quad J^\mu = \pi^\mu \mathcal{D}\Phi \)

\[ \frac{\delta Q}{\delta \lambda} = \int d^3x J^0 = 0 \]

What if we have 2 symmetries?

\[ \text{IVD} \xrightarrow{\alpha_1} \text{IVD}' \xrightarrow{\alpha_2} \text{IVD}'' \]

or

\[ \text{IVD} \xrightarrow{\alpha_2} \text{IVD}'' \xrightarrow{\alpha_1} \text{IVD}''' \]
Combine \( \lambda \lambda_2 \rightarrow \) Must also be a symmetry
\[ \lambda_2 \lambda_1 \rightarrow \]

\# or taking the difference

If \( Q_1, Q_2 \) are conserved charges
\[ [Q_1, Q_2] = 0 \]
is also conserved

Let
\[ (D^2 D_1 - D^2 D_2) \phi = D_3 \phi \]
if
\[ \Box J^u_1 = 0 = \Box J^u_2 \quad \Rightarrow \Box J^u_3 = 0 \]

Ex
2 free massless fermions

\[ L = \overline{\Psi}_1 i \gamma \Psi_1 + \overline{\Psi}_2 i \gamma \Psi_2 \]

\[ = (\overline{\Psi}_1, \overline{\Psi}_2) i \gamma (\Psi_1, \Psi_2) \]

Look at
\[ (\psi_1) \rightarrow e^{-i \lambda T} (\psi_1) \]

\[ T = 2 \times 2 \text{ Matrix} \]

\[ D(\psi_1) = -i \lambda T (\psi_1) \]
\[ D_L = -i(\psi_1, \psi_2) [T^+ - T] \psi \]

So \( T = \text{Hermitean Matrix for symmetry} \)

There are 8 such matrices:

\[ H^a = \delta^a_0 = i \psi \gamma^a \]

Currents \( J^a = \psi \gamma^a T^a \psi \)

Check theorem \( T^1 = \frac{\sigma^1}{2}, \quad T^2 = \frac{\sigma^2}{2} \)

\[ \left( D_{\alpha} \mathbb{1} - D_{\alpha} D_{\beta} \right) \psi = \frac{1}{4} (\sigma^1 \sigma^2 - \sigma^2 \sigma^1) \psi \]

\[ = \frac{i}{2} \sigma^3 \psi \equiv D_3 \psi \]

Note that \( T^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \) is not a consequence of the other three.

\[ \Rightarrow \quad SU(2) \times U(1) \]

Generalize \( SU(N) \times U(1) \quad (N^2 \text{ currents}) \)
Actually has a bigger symmetry.

Recall $\lambda$ matrix nonsense

\[ \psi = \psi_L + \psi_R \]

\[ \{ \psi_L, \psi_R \} = 2g \psi \]

\[ \{ \psi_L, \psi_L \} = \{ \psi_R, \psi_R \} = 0 \]

\[ \lambda^2 = 1 \quad \lambda^+ = \lambda^- \]

\[ \mathcal{P}_+ = \frac{1}{2} (1 + \lambda_5) \]

so \[ \mathcal{P}_+^2 = \mathcal{P}_+ \quad \mathcal{P}_-^2 = \mathcal{P}_- \quad \mathcal{P}_+ \mathcal{P}_- = \mathcal{P}_- \mathcal{P}_+ = 0 \]

also \[ \mathcal{P}_+ + \mathcal{P}_- = 1 \]

so \[ (\mathcal{P}_+ + \mathcal{P}_-) \]

\[ \mathcal{L} = \overline{\psi} \gamma^\mu (\mathcal{P}_+ + \mathcal{P}_-) \psi \]

\[ = \overline{\psi}_L \gamma^\mu \psi_L + \overline{\psi}_R \gamma^\mu \psi_R \]

\[ = \overline{\psi}_L \gamma^\mu \psi_L + \overline{\psi}_R \gamma^\mu \psi_R \]

and we see that we can make a symmetry transformation separately on $\psi_L$ and $\psi_R$.

\[ \rightarrow 2N^2 \text{ currents} \quad J_{\mu}^a_L, J_{\mu}^a_R \]

SU$_L$(N) x SU$_R$(N) x U(1)$_L$ x U(1)$_R$ chiral
We have our currents in \( Y_P \). Do they come from some symmetry?

\[
J_u^- = \frac{1}{2} \bar{u}_L \gamma_\mu T^u u_L \quad \text{and} \quad J_u^+ = \frac{1}{2} \bar{u}_L \gamma_\mu \gamma_5 U
\]

\[
= 2 \bar{u}_L \gamma_\mu \gamma_5 u_L = 2 d_L \gamma_\mu u_L
\]

Let \( q_L \equiv (u_L, d_L) \) \( \Psi \equiv (\psi_e L, e^-_L) \).

Then \( J_u^\pm = 2 \bar{q}_L \gamma_\mu T^\pm q_L \) \( J_\mu^\pm = 2 \bar{\Psi} \gamma_\mu T^\pm \Psi \).

with \( T^+ = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \)

\( T^- = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \)

Look like "raising" and "lowering" operators.

\( \to 2 \) of the currents for \( SU(2)_L \).

But they aren't hermitian!

However,

\[
J_u^+ J_u^- = (J_u^1 + i J_u^2)(J_u^1 - i J_u^2)
\]

\[
= J_{\mu_1} J_{\mu_0} + J_{\mu_2} J_{\mu_2}
\]

\[
J_u^{\mu_1} = \bar{\Psi} \gamma_\mu T^{\mu_1} \Psi \quad T^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \]

\[
T^{\mu_2} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}
\]
By our theorem we should have another current—

\[ T^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \Rightarrow \begin{pmatrix} \gamma^u_3 & \gamma^d_3 \\ \gamma^d_3 & -\gamma^u_3 \end{pmatrix} \]

This current is neutral (no electric charge).

We might expect a neutral current interaction as well as the charged current interaction of Fermi.

\[ \mathcal{H}_F = \frac{G_F}{\sqrt{2}} \sum_a \gamma^a \cdot \sum_a (\gamma^a J^{\mu}_+ J^{\mu}_- + \gamma^a J^{\mu}_3 J^{\mu}_3) \]

We (almost) see this! There is also a part involving the neutrino and electron, and neutral currents definitely occur in nature. We got this as a consequence of symmetry. We also have

\[ \frac{\text{strength of charged current}}{\text{strength of neutral}} \]

Symmetry \( \iff \) Interaction Operators

But is this exactly right? NO
But there is a problem. What about the rest of $L_3$? e.g. $m_e \bar{e} e = m_e (\bar{e}_e L + \bar{e}_e R)$

Not invariant \textarrow{bad news}. $SU(2)$ symmetry is 

\textbf{BROKEN}

But we need this symmetry. Maybe it's still around but hidden

\underline{HIDDEN SYMMETRY}

Consider $L = \frac{1}{2} (\bar{\psi} u \Phi) \bar{\psi} \Phi + \bar{\psi} i \gamma_5 \psi - \frac{1}{\sqrt{2}} \bar{\psi} L \Phi R + h.c.$

This has a chiral symmetry

\[ \psi_L \rightarrow e^{i\phi} \psi_L, \quad \psi_R \rightarrow e^{-i\phi} \psi_R, \quad \phi \rightarrow e^{2i\phi} \phi \]

(or $D \psi_L = i \psi_L, \quad D \psi_R = -i \psi_R, \quad D \phi = 2i \phi$).

Note that $m (\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$ 

VIOLATES this symmetry

If the electron mass were not a constant but transformed under the $\bar{\psi} \psi$ $SU(2)$ symmetry, then we could make the mass term invariant.

In order for $m_e$ to transform it must
But the electron mass can't transform, it's a number! Only fields transform (values of fields are a IVD)

so we might replace the mass term with

\[ g_e \bar{\Psi}_L \phi e_R + \text{h.c.} \]

In order for this to be invariant under

\[ SU(2) \]

\[ \Psi_L \rightarrow L \Psi_L \quad L = 2 \times 2 \text{ Unitary Matrix} \]

\[ \phi \rightarrow L \phi \]

\[ \phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix} \]

Then

\[ \bar{\Psi}_L \phi e_R \rightarrow \bar{\Psi}_L L\phi e_R = \bar{\Psi}_L \phi e_R \]

is invariant as we want! Great!

But

\[ \bar{\Psi}_L \phi e_R = \bar{\gamma}_e \phi_1 e_R + \bar{e}_L \phi_2 e_R \]
And this isn't a mass term at all! It's an interaction with a new particle.

What if this new particle is very heavy?
Then we don't see it (can't be produced)
Ok, but then what does this thing do?
Think about a magnet (bunch of spins).
The spins can fluctuate (up \rightarrow down) but there are forces which between neighboring spins which make them want to align.
Dynamics are rotationally invariant, but the minimum energy configuration is not!
How does this give a mass?

\[ \phi_2 \rightarrow m_e \]

Can we arrange the dynamics of \( \phi \) to do this?

Yes. The potential could be

\[ V(\phi) = \lambda (|\phi|^2 - u^2)^2 \]

\[ |\phi|^2 = |\phi_1|^2 + |\phi_2|^2 \]

\[ V'(\phi) = 0 \implies |\phi_1|^2 + |\phi_2|^2 = u^2 \]

Certainly satisfied by \( \phi_2 = u, \phi_1 = 0 \).

But other solutions are possible, e.g.
\( \phi_2 = 0, \phi_1 = u \).

This is obvious by symmetry. If \( \phi_0 \) is a solution, \( |\phi_0|^2 = u^2 \).

Then so is \( \phi' = U \phi_0 \).
What does this mean?

By symmetry, they are all equivalent.

Pick one \( \phi_0 = (\psi) \) say.

Then let's expand about this

\[
\phi = \phi_0 + \phi_1 + \phi_2
\]

Look at potential for \( |\phi_1| \) and \( |\phi_2| \)

"Mexican Hat".

Clearly the "angular" direction has no curvature

\[
|\phi_1|^2 + |\phi_2|^2 = \rho^2 \quad \tan \frac{\phi_1}{\phi_2} = 0.
\]

Then \( V = 0 \Rightarrow \rho = v \) \( \text{degeneracy} \)

\( \rho = 0 \) (our choice)

Then at \( \theta = \theta_0 \)
$$\phi = \begin{pmatrix} \phi^r \\ \phi^i \end{pmatrix} = \begin{pmatrix} (\phi_3 + i \phi_4)/\sqrt{2} \\ (\phi_1 + i \phi_2)/\sqrt{2} \end{pmatrix}$$

Then our potential is minimized by

$$\sum_i \phi_i^2 = u^2$$

This is a dummy notation $\rightarrow$ re-write in angular coordinates.

Let

$$\begin{pmatrix} u_R \\ d_R \end{pmatrix} \equiv \psi_R$$

$$\Sigma = \rho e^{i \sum \phi^a T^a}$$

$\Sigma = 1,2,3.$

Complicated relationship between $\rho, \phi^a$ and $\psi_R$.

But under $SU(2)$

$$\Sigma \rightarrow L \Sigma$$ (easy to remember)

Then

$$\bar{\psi}_L \Sigma \psi_R \text{ is invariant.}$$

Potential is $(\Sigma^+ \Sigma = -u^2)^2 \rightarrow$ take ground state

$$\Sigma_0 = u \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$ ($\rho = u$, $\phi^a = 0$)
Then we get a term
\[ g \bar{U}_L U_R + \bar{d}_L d_R \] + h.c.
\[ \rightarrow m_u = g U, \quad m_d = g U. \]
We have gotten \( m_u \neq m_d \). Why?

Symmetry again \( \rightarrow \)
\[ p = v \] hid the \( SU(2) \) symmetry.
But our interaction has an \( SU(2)_\rho \) symmetry as well
\[ \Sigma \rightarrow \Sigma R^+ \]
\[ \psi_R \rightarrow R \psi_R \]
and the term
\[ \Sigma \bar{U}_L \psi_R + h.c. \]
has the "diagonal" symmetry
\[ \left\{ \begin{array}{l}
\psi_L \rightarrow U \psi_L \\
\psi_R \rightarrow U \psi_R
\end{array} \right. \]
This symmetry is sometimes called the "custodial" \( SU(2) \) and is very useful.
In fact this custodial SU(2) is crucial (17) in understanding why $p=1$.

If we are able to hide symmetries, why can’t we “hide” the symmetry in the FERMİ theory $\Rightarrow p \neq 1$.

But the custodial symmetry isn’t hidden, it’s explicitly realized and this ensures $p=1$.

But the custodial symmetry isn’t exact — for example $m_{\mu} \neq m_{d}$ and (not very important) $(m_{\mu} - m_{d}) \ll (250 \text{ GeV})$.

But electromagnetism also breaks this symmetry (if $q$ has different charges in different components).

So we expect $p=1$ if $\alpha_{\text{em}} = 0$.

So \[ \frac{\Delta p}{p} \approx \alpha_{\text{em}} \approx 1\% \]

which is just about right!
Why did we get $\frac{3}{2} \neq GB$?

$SU(2) \rightarrow 3 \text{ "Generators" } T^a$

we say these symmetries are “broken”.

In fact they are merely hidden.

Our “ground state” (minimum of the potential) has a degeneracy $|\Sigma=0\rangle \equiv |\Sigma_0\rangle$

$\hat{U} |\Sigma_0\rangle = |U\Sigma_0\rangle \equiv |\Sigma_0\rangle$.

Which is also a “ground state”.

But then $e^{iT^a \theta} |\Sigma_0\rangle = |\Sigma_0 + \text{mesons}\rangle$

has the same energy as $|\Sigma_0\rangle$.

→ Mesons are massless.

True for every charge which corresponds to a symmetry that is hidden.
So we have discovered how to keep the symmetry but have a symmetry breaking mass.

What's the difference?

1. At tree level in this process, no difference

2. Goldstone Bosons $\rightarrow$ massless scalars, which we don't see in this case.

3. Radiative Corrections are different (new particles) Higgs

In this case the differences are enormous -- massless particles which couple like this would be easy to see!

Are there massless particles?

No! The only things close are the pions, but they aren't massless.

(and Goldstone's Thom says exactly massless)

Also they don't couple properly.

(Wrong symmetry properties)
As long as we are talking about GBs.

**Strong Interactions (QCD)**

Long before the theory of color was understood (at least the dynamics) a great deal of progress was made possible through symmetry and effective field theories. We would like to understand the behavior of the products of QCD at low energies. (The 3rd question we raised)

We will begin by considering the lightest quarks, $[u,d]$, for simplicity.

There is one obvious fact about these quarks more important than any other $\rightarrow$

we don't see them.

What do we see?

We see "hadronic resonances"

$P, N, \Lambda, \ldots$

all of mass $\sim 1$ GeV
But we see a few light resonances.

\[
\begin{align*}
\pi^\pm, \pi^0 & \quad K^+ K^0 \bar{K}^0 \eta \\
& \quad 140 \text{ MeV} \quad 500 \text{ MeV}
\end{align*}
\]

In principle we should be able to understand these objects from QCD -- but this is too hard.

But whatever force binds quarks into hadrons we can try to understand as much as possible.

As bound states, shouldn't we expect the pions to be heavier? Why not 1 GeV?

Let's try to use symmetry:

Start with the simplest Lagrangian possible.

\[
L = \bar{q} i\gamma \cdot \gamma q \quad q = (u, d)
\]

\[
= \bar{q}_L i\gamma \cdot \gamma q_L + \bar{q}_R i\gamma \cdot \gamma q_R
\]

Of course this theory is free quarks -- no binding force at all.
It has a large set of symmetries:

\[ SU_L(2) \times U(1) \times SU_R(2) \times U(1) \]

If we add an interaction (hopefully a confining one) it's not going to probably going to reduce this symmetry. But let's be optimistic and suppose that it leaves as much of this symmetry as possible, and look at the consequences.

1. \( U(1)_V = U(1)_L \times U(1)_R \)

\[ q_L \rightarrow e^{i\frac{\pi}{3}} q_L, \quad q_R \rightarrow e^{i\frac{\pi}{3}} q_R \]

This symmetry is called “Baryon Number” and is observed to be an exact symmetry except for some fancy non-perturbative quantum effects which are very small.

It has usual consequences:

\[ p \rightarrow 0 e^+ \pi^0, \quad \tau > 550 \times 10^3 \text{ years} \]

So we won't say much about it.
\[ \mathbf{SU}_L(2) \times \mathbf{SU}_R(2) \times \mathbf{U}_A(1) \quad A = L - R. \] (23)

If these symmetries were true, we would expect consequences.

\( \mathbf{SU}_V(2) \) called "isospin". (Like \( \mathbf{SU}_c(2) \)

except only acts on (\( \nu \)) not leptons.)

It's a pretty good symmetry →

\[
\frac{m_p - m_N}{m_p + m_N} \sim 10^{-2}.
\]

Lots of good nuclear stuff.

what about \( \mathbf{SU}_A(2) \) ? and \( \mathbf{U}_A(1) \).
As we have discussed, we have basically 3 choices

1. Not a symmetry -- unpleasant since we can't say anything

2. Explicit symmetry (like SU(2)). We would then be able to classify particle according to their "Axial" isospin -- just like (p, N) are an "iso-doublet". We would expect new particles which are axial iso-doublet. \[ \rightarrow \]
don't see them.

3. Hidden Symmetry -- in this case we wouldn't expect to classify particles according to SU(2) since its consequences are hidden. Looks like 1 except \[ \rightarrow \] Goldstone Bosons

How many G.B.? Should be one massless particle for each symmetry we hide \[ \rightarrow \]. In this case 3 for SU(2) (or four if we try SU(2) × U(1)).

Because the symmetry is axial, we expect pseudo scalar \[ \rightarrow \].
$SU(2)_V$ is called "isospin" and works pretty well.

$SU(2)_A$ is violated by the mass term.

(N.B. Some of these symmetries are gauged. But gauge couplings are small (weak and electromagnetic, so we ignore this for now)

We know how to get massless, spinless particles

→ Hidden (Global) Symmetry

3 pions → pseudo scalar

Maybe \[
\begin{cases}
SU(2)_V & \text{unbroken} \\
SU(2)_A & \text{hidden}
\end{cases}
\]

To answer this we have to solve QCD (hard)

But we can look at the consequences (easy)
Let's write an "effective" theory just for the pions, at low momentum.

We know that we should have the pions as "angular" variables, so let's write

\[ \Sigma = \epsilon e^{i \pi f} \]

\[ \Pi = \text{ dimension one matrix of pion fields } \equiv \tilde{\Pi} T^a \]

\[ J = \text{ dimension one constant to turn } \frac{\tilde{\Pi}}{J} \rightarrow \frac{\tilde{\Pi}}{2} \]

into an "angle".

What about "Higgs"? (The radial part of \( \Sigma \)?)

We don't necessarily have it. All we know are the GB.

Called "non-linear sigma model"

How does this transform under \( SU(2)_L \times SU(2)_R \)?

\[ \Sigma \rightarrow L \Sigma R^+ \quad L, R \in SU(2) \]

(If \( L = R \) this is Vector; \( L = R^+ \) "Axial")

What can the Lagrangian be? Must be invariant under this transform (must have symmetry)

\[ L = \frac{F^2}{4} \text{Tr} \epsilon \Sigma^+ \gamma^\mu \Sigma \]
Why this? The $\frac{f^2}{4}$ is a convention; it gives the canonical normalization for the pion kinetic energy term.

Why 2 derivatives? If we work at low momenta, we expect more derivatives have powers of momenta suppressed by some scale. What is this scale?

Set by QCD. Beautiful argument by Weinberg suggests $4\pi f \equiv \Lambda_x \sim 1$ GeV.

This theory has non-trivial interactions.

E.g. four pions:

$$\Sigma = 1 + 2i\pi f - \frac{1}{2} \cdot 4 \pi^2 \frac{f^2}{f^2}$$

So

$$\mathcal{L} \equiv \frac{f}{\Lambda_x} \left( \phi \phi \phi \phi \cdots \text{Tr} \left[ \phi \phi^\dagger \right]^2 \right)$$

This predicts the $\pi-\pi$ scattering length at threshold! (Weinberg) Just do normal Feynman–Dyson perturbation theory for this interaction. Gives things in terms of $f$ which we don't yet know.
How do we find $f$?

Let's go back to QCD, and include the Weak Interaction. Just gauge $SU(2) \times U(1)$. But we are at low momenta, so use Fermi theory.

$$L = \bar{q} i \not{D} q + \cdots + \frac{G_F}{\sqrt{2}} J^u_+ J^-_+$$

With the currents as before. But we now have an interaction of the form

$$L_I = \cdots + \frac{G_F}{\sqrt{2}} \left( J^u_+ J^-_+ h.c. \right)$$

$$J^u_+ = \bar{q} 8^u (1 + 
\xi_5) T^+ q$$

But this is the current associated with $SU(2)$ rotations! Including this interaction in the effective theory is a snap.

$$L_{\text{eff}} = \frac{f^2}{4} \text{Tr} \xi_3 \not{u} \not{b} + \frac{G_F}{\sqrt{2}} \left( J^u_+ J^-_+ h.c. \right)$$

What is $J^u_+$ in terms of the quarks?
Use Noether procedure: \[ \Sigma \to L \Sigma \Omega; \]

\[ L = 1 + i \varepsilon T^a \]

\[ D_a \Sigma = i \varepsilon a T^a \Phi + \ldots \]

\[
J_{\mu a} \bigg|_{L = 1} = \frac{f^2}{2} \text{Tr} \left[ \Sigma^+ T^a \Sigma \right] + \frac{1}{3} \text{Tr} T^a [\Pi, [\Pi, \Sigma]]
\]

G.B. has only derivative coupling \[ + \frac{1}{3} \text{Tr} T^a [\Pi, [\Pi, \Sigma]] \]

so

\[ \langle 0 \mid J_{\mu a} \big| \Pi^- \rangle = \frac{i f}{\rho^a} \]

This can be measured in \( \Pi \) decay!

\[ f = 93 \text{ MeV} \]

- Relation between \( \Pi \) decay \& into leptons and \( \Pi^+ - \Pi^- \) scattering from

\[ \text{Hidden Chiral Symmetry} \]


What about the strange quark?

If it's light enough, $SU(2) \times SU(2) \rightarrow SU(3) \times SU(3)$

All formulae are the same (2x2 matrices become 3x3 matrices).

To get the $K^+ \rightarrow \mu^+ \nu$ decay, or

$$K^+ \rightarrow \pi^0 \mu^+ \nu$$

we need to include the semileptonic part of the $\Delta S = 1$ weak Hamiltonian

$$L = \frac{G_F}{\sqrt{2}} \sum_3 J_{\ell}^+ J_{\mu}^- (\Delta S = 1) + h.c.$$

$$J_{\mu}^- (\Delta S = 1) = \bar{u} \gamma_\mu (1 + \gamma_5) s$$

$$= 2 \bar{q}_L \sigma_\mu \gamma_5 h q_L$$

where $q_L = \begin{pmatrix} u \\ d \\ s \end{pmatrix}_L$ and

$h = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Usual trick: pretend it's a field. How does it transform?

$$h \rightarrow L h L^+$$

So in our effective theory.
As usual we need this current in \( \rho \). 

\[
\frac{j_{\mu^-}(\Delta s=1)}{h} = 2 \frac{j_{\mu^-}(\Delta s=1)}{h L} \\
= i f^2 \text{Tr} \left( \Sigma^+ h \Theta^\mu \Sigma \right) \\
= -\sqrt{2} f \sum K^- \bigg\{ \frac{1}{\sqrt{2}} \bigg( \frac{1}{\sqrt{2}} K^- \Theta^\mu I^+ \pi^0 \Theta^\mu K^- \bigg) \\
+ \frac{2}{3} f \sum \bigg( \Theta K^- \pi^- \Theta^\mu \pi^- \Theta^\mu \pi^+ \bigg) \\
+ \frac{1}{\sqrt{2}} \pi^+ \pi^- \Theta^\mu K^- \bigg\} 
\]
This little trick has allowed us to include a term in \( \text{Jeff} \) which breaks the symmetry exactly as \( \Omega \rightarrow \Omega \). However, the coefficient of this operator is not fixed this way:

\[
\mathcal{L} = \frac{f^2}{4} \text{Tr} \, \bar{\psi} \Sigma^+ \Sigma + \alpha f^2 \text{Tr} (M \Sigma + \text{h.c.})
\]

\[\alpha = \text{unknown dimensionless constant.}\]

\[M = \begin{bmatrix} m_u & 0 \\ 0 & m_d \end{bmatrix}\]

\[
\therefore \text{We can't determine } m_u \text{ and } m_d \text{ this way.}
\]

Just the ratios.

Expand to two pions:

\[
\text{Jeff} = \text{Tr} \, \bar{\psi} \Sigma^+ \Sigma + \bar{\psi} \Gamma \psi
\]

\[
m^2_{\pi^0} = 8a \Lambda \text{Tr} \begin{bmatrix} m_u \\ m_d \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 2a \Lambda (m_u + m_d)
\]

\[
m^2_{\pi^\pm} = 2a \Lambda (m_u - m_d)
\]

This introduces \textbf{NEW} \ 4 \text{ pion interactions} \rightarrow \text{corrections to } \pi-\pi \text{ scattering.
Try Another Approach

why is $J^\mu J_\mu$ the interaction?

Maybe comes from the exchange of some particle?

$L_I = A_\mu J^\mu$ then we would get

$$J^\mu \langle 0 | A_\mu A_\nu | 0 \rangle \to J^\nu$$

$$\propto g_{\mu\nu}.$$

Can we have $\langle 0 | A_\mu A_\nu | 0 \rangle \propto g_{\mu\nu}$?

- Problem $\langle 0 | A_0 A_0 | 0 \rangle \equiv || A_0 | 0 \rangle ||$
  $\langle 0 | A_1 A_1 | 0 \rangle \equiv || A_1 | 0 \rangle ||$

  Ratio $= -1 \implies$ Negative Norm State
  $\implies$ Negative Probability
How do we get rid of this negative norm state?

Need to eliminate degrees of freedom

N.B. The photon (which is a spin one, or "vector" particle)

only has 2 physical degrees of freedom — the 2 polarizations of the photon (L circ. or R circ.)

How did this happen?

"Gauge Theory"

\[ L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \]

We can make a transform

\[ A_\mu \rightarrow A_\mu + \partial_\mu \epsilon \]

\[ F_{\mu\nu} \rightarrow F_{\mu\nu} \]

for any function \( \epsilon (x, t) \)

This looks like a symmetry — but it's not
How many degrees of freedom is this theory? How do we develop a canonical formulation?

Set up Initial Value Problem.

Claim $\mathcal{A}_1, \mathcal{A}_2, F^{01}, F^{02}$ are good set of initial value data.

To see this let's write the EOM

$$\begin{aligned}
\underline{0} & \quad \partial \mu F^{\mu \nu} = 0 \\
F^{\mu \nu} &= \partial \mu A^\nu - \partial \nu A^\mu
\end{aligned}$$

But we can always choose $A_3 = 0$ by virtue of a gauge transformation. Then

$$\begin{aligned}
\o & \partial \mu F^{\mu 0} + \partial \nu F^{\nu 0} + \partial \gamma (-\partial^3 A_0) = 0
\end{aligned}$$

$\implies$ This is not an EOM, it gives $A_0$ at $t=0$ in terms of IVD

But then

$$\begin{aligned}
F^{01} + \partial^1 A_0 &= \dot{A}_1 \\
F^{02} + \partial^2 A_0 &= \dot{A}_2
\end{aligned}$$

so we know the time derivative of our canonical "coordinates."
The remaining equations give
\[ \partial_0 F_{0i} + \partial_j F_{0j} = F_{0i} + \partial_\perp \left( \partial_0 A_i - \partial_j A_i \right) \]
\[ + \partial_2 \left( \partial_0 A_i - \partial_j A_i \right) \]
\[ + \partial_3 \left( \partial_3 A_i \right) = 0 \]
gives us \( F_{0i} \).

\textit{Mutatis Mutandis for } F_{02}

The last equation gives
\[ \partial_\perp \]

So, given \( \{ A_1, A_2, F_{01}, F_{02} \} \) as IVD,
EOM allow \( \partial_\perp \) time evolution \( \checkmark \)

In fact these objects are canonically conjugate

\[ L = -\frac{i}{4} F_{\mu \nu} F^{\mu \nu} + \frac{1}{4} F_{\mu \nu} (\partial_\mu A_\nu - \partial_\nu A_\mu) \quad \text{(First Order Formalism)} \]

\[ = -\frac{i}{4} F_{\mu \nu} F^{\mu \nu} + F^{i j} (\partial_\mu A_\nu - \partial_\nu A_\mu) 
+ F^0 i \left( \partial_0 A_i - \partial_i A_0 \right) 
+ F^0 0 \left( -\partial_0 A_0 \right) 
+ F^{i 3} (-\partial_3 A_i) \]
The reason is that the part of the gauge field that looks like the derivative of a scalar is not a true degree of freedom; it doesn't appear in the Lagrangian at all: neither kinetic nor potential.

This transformation is called a "gauge transformation." Rather than taking us from one point in configuration space to another, it changes the description of the point, but doesn't move it. It's not a symmetry.

(Example → transform sponge.)

Configuration space is smaller than we thought.
But now we have FEWER things than we thought. (38)

We might have guessed 3, for a massive spin 1 particle which should have 2 transverse polarizations plus one longitudinal pol.

BUT our states are massless \Rightarrow 2 pol.

How could we get mass?

\[ m^2 \delta_{\mu\nu} \]

No good since this doesn't have gauge invariance. This would re-introduce our negative probability!

But remember our discussion of hidden symmetry.

Maybe we can hide the symmetry in the mass term.
We have discovered a trick for removing degrees of freedom. Let's go back to our hidden symmetry

$$\Sigma = \text{Higgs} = \rho e^{i\phi} T^a$$

$$L = \text{Tr}(\bar{\Theta}_\mu \Sigma^+ \Theta^\mu \Sigma) - V + \cdots - g_\mu \bar{\Phi}_L \Sigma q_R + \cdots$$

Let's be simple, for a moment, and make $\Sigma$ a complex number instead of a matrix

$$\Sigma = \rho e^{i\phi}$$

$$\rightarrow \quad \Theta_\mu \Sigma \Theta^{\mu \Sigma^+} \cdots - g \bar{\Phi}_L \Sigma q_R + \text{h.c.}$$

Now our symmetry is

$$\Sigma \rightarrow e^{i\frac{\lambda}{\rho}} \Sigma$$

$$\psi_L \rightarrow e^{i\frac{\lambda}{\rho}} \psi_L$$

$$\psi_R \rightarrow \psi_R$$

What if we tried rewriting things by a coord.

Transform

$$\chi_L = \theta^I(x)$$

Then

$$\chi_L \equiv e^{-i\theta(x)} \psi_L \quad \chi_R \equiv \bar{\psi}_R$$

Then

$$L = \Theta_\mu \Sigma \Theta^{\mu \Sigma^+} + \bar{\chi} i \not\partial \chi - \bar{\chi}_L (i\not\partial \theta) \chi_L + g \bar{\chi}_L \not\partial \chi_R + \text{h.c.}$$
We haven't done anything, just rewritten this

But now we have.

\[ L = \ldots + (\partial \mu \partial \nu) \bar{J}^{\mu} + \alpha m \bar{\chi} \chi + \gamma h \bar{\chi} \chi \]

So everything is manifest \( \partial = GB \)
\[ h = h_{\text{Higgs}}. \]

We still have to get rid of \( \partial \). But \( \partial \) only appears as \( \partial \partial \), the derivative of \( \partial \).

(Important G.B. only \( \partial \partial \) has derivative coupling.)

But this derivative of a scalar field looks like the piece of the \( \gamma_0 \) vector field that went away when we used gauge invariance.

Hidden symmetry has extra mode \( \partial \chi \)

Gauge invariance removes mode \( \partial \chi \).

\[ \text{COMBINE} \]
How do we introduce the gauge field? We want it to couple like \( A_\mu \).

\[ \Theta_\mu \rightarrow \mathcal{D}_\mu \equiv \partial_\mu + iA_\mu. \]

Then

\[ \mathcal{D}_\mu \Sigma^+ e_{\nu} \Sigma \rightarrow \mathcal{D}_\mu \Sigma^+ e_{\nu} \Sigma \]

and

\[ \bar{\Psi} i \gamma^4 \rightarrow \bar{\Psi} i \gamma^4. \]

Now we still have our gauge invariance. This means that we can choose any gauge we like. For example, let's choose a take our original \( \Sigma \).

\[ \mathcal{L} = \bar{\Psi}_L i \gamma^\mu \gamma^5 \Psi_L + \bar{\Psi}_R i \gamma^\mu \gamma^5 \Psi_R + \mathcal{D}_\mu \Sigma^+ e_{\nu} \Sigma - \nabla(\Sigma) \]

\[ + g \bar{\Psi}_L \Sigma \Psi_R + \text{h.c.} \]

And perform a gauge transform

\[ \Sigma \rightarrow \rho \]

we can get rid of \( \Theta \) entirely!

This has

\[ U = e^{-i\omega} \text{ so } \Psi_L \rightarrow e^{-i\omega} \Psi_L \]

\[ A_\mu \rightarrow A_\mu \]
\[ D_{\mu} \psi_L \rightarrow [\partial_{\mu} i A_{\mu} + i A_{\mu} \theta] [e^{-i \theta} \psi_L] \]

\[ = e^{-i \theta} D_{\mu} \psi_L \]

and

\[ \bar{\psi}_L i D \psi_L \rightarrow \bar{\psi}_L i D \psi_L. \]

\[ \bar{\psi}_L \Sigma \psi_R \rightarrow \rho \bar{\psi}_L \psi_R = (\Sigma + h) \bar{\psi}_L \psi_R \]

and

\[ D_{\mu} \Sigma^+ D_{\mu} \Sigma \rightarrow \rho^2 A_{\mu} A^\mu = (\Sigma + h) A_{\mu} A^\mu \]

\[ L = \bar{\psi}_L i D \psi_L + \frac{1}{4} F^2 + (\Sigma + h)^2 A^2 \]

\[ + \rho (\Sigma + h) \bar{\psi}_L \psi_R + h.c. \]

And we have EVERYTHING.

1. No GB
2. Massive Vector field
3. Massive fermion
4. Interaction \( A_{\mu} J^\mu \) with \( m \ll a g \mu \).

Note: I never said "Break" the gauge "symmetry". There was never any gauge "symmetry" to break. There \underline{WAS} a global symmetry.
This little device is usually called the Higgs (43) phenomena; two bits of physics which were thought to be mere curiosities combined to solve one of the most challenging puzzle in particle physics.

This trick of eliminating the (now) unphysical "GB" is called going to Unitary gauge. But it isn't the most convenient for calculations; we can use any set of coordinates that are not redundant (any gauge)
So far we have used symmetries to look at the possible effective theories at low energies →

\[ p^2 \ll 1 \text{ GeV}^2 \] pions as Goldstone Bosons

\[ p^2 \ll (100 \text{ GeV})^2 \] Fermi Theory of Weak Interactions

In both these cases we didn't need to know what the exact theory was → symmetries and low momenta allowed construction of an effective theory.

What about \[ p^2 \sim (100 \text{ GeV})^2 \ll 1 \text{ TeV}^2 \] Full weak interactions → physical Intermediate Vector Bosons

As an effective theory → gives information about energies above 100 GeV.

Only include what we know, and no more; ignore fermions for now.
Should write most general theory consistent with symmetries with smallest # of derivatives.

Should include All physical particles.

\[ \text{SU}_L(2) \text{ Symmetry} \]

\[ \text{SU}_C(2) \text{ custodial (broken by electromagnetism)} \]

\[ \Sigma = e^{i \frac{g^a T^a}{\sqrt{2}}} \]

\[ \xi = \text{Angular Variables for SU}_L(2) \quad T^a = \frac{g^a}{\sqrt{2}} \]

\[ \upsilon = 250 \text{ GeV} \quad (\text{to get } G_F \text{ right}) \]

Under \[ SU_L(2) \quad \Sigma \rightarrow L \Sigma \]

\[ SU_R(2) \quad \Sigma \rightarrow \Sigma R^+ \]

Theory should have \[ SU_L(2) \times SU_R(2) \times U(1)_L \times U(1)_R \]

if we turn off Electroweak Interactions

\[ L = \frac{v^2}{4} \text{ Tr } \xi \Sigma^+ \Sigma \]

But we need to include \[ W^+, Z^0, \gamma \text{ (Transverse Parts)} \]

We want to "gauge" four of the symmetries

\[ SU_L(2) \times SU_R(2) \times U(1)_L \times U(1)_R \]
\[ SU(2) \rightarrow 3 \text{ gauge bosons} \]

Need one more \( \alpha \) \( \star \) one part of

\[ SU(2)_L \times U(1)_R \times U(1)_L \]

\[ \rightarrow Y = T^3_R + \frac{1}{2}(B-L) \]

(Recall \( U(1)_L \propto B \) on quarks
\( \propto -L \) on leptons)

Under \( B-L \) \( \Sigma \rightarrow \Sigma \)

So \( U(1)_y \) \( \Sigma \rightarrow \Sigma \epsilon^{\frac{i \sigma^3 \alpha}{2}} \)

\[ \partial_\mu \Sigma \rightarrow D_\mu \Sigma = \partial_\mu \Sigma + ig W^a_{\mu} \Gamma^a \Sigma - ig' \Sigma T^3 \rho_{\mu} \]

(\( D_\mu \psi_L = \partial_\mu \psi_L + ig W^a_{\mu} \Gamma^a \psi_L - ig' B_{\mu} \frac{1}{2}(B-L) \psi_L \)

eetc.)

\[ W_{\mu} = \sum_{a=1}^{3} W^a_{\mu} T^a \]

\[ \mathcal{L} = \frac{\mu^2}{4} \text{Tr} D_\mu \Sigma^+ D^\mu \Sigma - \frac{1}{2} \text{Tr} W_{\mu \nu} W^{\mu \nu} - \frac{1}{4} B_{\mu \nu} B^{\mu \nu} \]
But we can choose our gauge so that the "GB" aren't around at all → unitary gauge

\[ \Sigma = 1 \]

Then:

\[ \Phi \rightarrow \frac{\kappa^2}{4} \text{Tr} \, \partial_\mu \Sigma^+ \partial^\mu \Sigma \]

\[ = \frac{\kappa^2}{4} \left\{ \frac{g^2}{2} W^+_\mu W^-\mu + \frac{g^2}{2} W^+ \cdot W^- + \frac{g^2}{2} B_\mu B^\mu \right\} \]

\[ = \frac{\kappa^2}{4} \left\{ \frac{g^2}{2} W^+ \cdot W^- + (g^2 + g'^2) Z_\mu Z^\mu - \right\} \]

where

\[ Z_\mu = \cos \theta \, W^3_\mu - \sin \theta \, B_\mu \]

\[ A_\mu = \sin \theta \, W^3_\mu + \cos \theta \, B_\mu \]

\[ \tan \theta = \frac{g'}{g} \]

\[ M_W^2 = \frac{g^2 \sin^2 \theta}{4} \]

\[ H^2 = \frac{M_W^2}{\cos^2 \theta} \]

How does the Fermi Theory Arise?

1) Charged current

\[ \Phi_F = \frac{\bar{J}^+ \, J^-}{2} \frac{g^2}{\kappa^2 M_W^2} \]

at low momenta \( \kappa^2 \gg M_W^2 \)

\[ \Rightarrow \frac{\bar{J}^+ \, J^-}{2} \frac{g^2}{M_W^2} \rightarrow \frac{4 \, G_F}{\sqrt{2}} = \frac{g^2}{M_W^2} \]
\[ \frac{g^2}{\cos^2 \theta} \left( J_{\mu}^3 - \sin^2 \theta J_{\mu \text{em}} \right) \left( J_{\mu}^3 - \sin^2 \theta J_{\mu \text{em}} \right) \]

When \( k^2 \ll M_Z^2 \)

\[ \Rightarrow \quad \frac{g^2}{\cos^2 \theta} \frac{1}{M_Z^2} = \rho \frac{4G_F}{\sqrt{2}} = \rho \frac{g_z^2}{M_W^2} \]

\[ \Rightarrow \quad \rho = 1 \] (Custodeal SU(2) !)

3. EM. \[ g_s \sin \theta \frac{J_{\mu \text{em}}}{k^2} \]

But our real Question --

How do we talk about corrections??

From these interactions we get several things →

\[ k^2 \sim 0 \quad G_F \]

But also \( k^2 \rightarrow M_W^2, M_Z^2 \) → poles (physical particles)
How do we parameterize these changes?

A changed relationship between FERMILAB (high energy) and LEP (low energy)

What kinds of things can we say?

Then can compute corrections (e.g., $\rho$)

They show structure

How do they show up?

$G_\ell$, $m_\ell$, $\rho$

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$G_\ell$, $m_\ell$, $\rho$
Our theory is an "effective" theory valid only at low momentum -- whatever physics exists at higher energies will reflect itself as other operators we have not included.

What do we mean by other operators?

We have constructed the most general theory at 2 derivatives which had the SU(2) x SU(2) symmetry. But we could violate the custodial symmetry if we want. We need to preserve SU(2)_{R|T^3} (since we intend to gauge it).

But things like \( \mathcal{A}^+ \bar{\Phi} \gamma^5 \Phi \Sigma^+ \delta^3 \Sigma^+ \) are ok. For example,

\[
\mathcal{L}_1 = c_1 \alpha^2 \left( \text{Tr} \left( \frac{\delta^3 \Sigma}{2} \right)^2 \right)^2
\]

\( \Sigma^+ \)

What does this do? Should break SU(2)

\[ \rightarrow \ p \neq 1 \]
\[ z \rightarrow 1 \]

\[ z \equiv \frac{c_u v^2}{4} \left( g W^2 - g' B^2 \right)^2 \]

\[ = \frac{c_u v^2 (g^2 + g'^2)}{4} \frac{Z^2 z^u}{2} \]

This changes the ratio of

\[ w^+ \rightarrow \pi^+ (0) \quad \text{to} \quad \pi^0 \pi^0 (0) \rightarrow \eta \eta \]

\[ \rightarrow \text{this is the } p \text{ parameter.} \]

The physics above 1 TeV will affect the \( p \) parameter, which we can measure. For us, this is an ARBITRARY input parameter but any (good) model of physics beyond \( 1 \text{ TeV} \) should predict this parameter.

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**Higgs Particle**

**Technicolor**

(For no very good reason people take the "standard" model as the one with a Higgs, and measure corrections wrt it.)
By measuring this accurately we get information about higher energies.

What might we expect for $C_1$?

We know $(\rho - 1) \leq 1\%$.

so $C_1 \leq 1\%$

This is actually a pretty good constraint on new physics—it can't violate the custodial $SU(2)$ by more than a few percent. (ex. triplet Higgs).

Other operators?

$L_2 = C_2 \frac{g^2}{4} \left( \text{Tr} \Sigma \tau^3 \Sigma^+ W_{\mu\nu} \right)^2$

$\rightarrow C_2 \frac{g^2}{4} \left( \frac{1}{2} \right) (W^3_{\mu\nu})^2$

Doesn't affect $K^2 \to 0$, but changes $\frac{M_W}{M_Z}$.
\[ L_3 = \frac{g g'}{2} B_{\mu\nu} \text{Tr} \Sigma^2 \Sigma^+ W_{\mu\nu} \]

\[ \rightarrow \frac{g g'}{4} B_{\mu\nu} W^3 \]

Changes "mixing" between \( \delta \) and \( \Theta^0 \).

That's it at this order for the

(There are others, but by suitable manipulations they become the same as these)

These three changes (tests of "new"

we can do these

physics) appear in different parameterizations

with different names, but they are all the same physics

\[
\left\{ \begin{array}{c}
s, t, u \\
\epsilon_1, \epsilon_2, \epsilon_3 \\
\delta_1, \delta_2, \delta_3
\end{array} \right\}
\]
We can do similar things for other operators at 4 derivatives which affect $Z^0$ decays, 3 gauge boson interactions, etc.

The key message is the idea of effective theory: the best we can do is construct an expansion in momenta consistent with symmetry; theorists can then suggest how their personal theories show up:

1. New Particles
2. Symmetry Violation
3. Higher Derivatives

Our job is to suggest and search...