An elementary introduction to the phenomenology of ultra relativistic heavy ion collisions will be given. It is focused on the expected features of the quark-gluon-plasma in terms of thermodynamical behaviour, hydrodynamic expansion and of its detectable signatures. Selected data obtained with high energy light projectiles (p, α) and lower energy heavy ions will serve as a reference for the subsequent review of current experiments on high energy collisions. Finally, a look at future experiments at CERN and with RHIC will be attempted.
Heavy Ion Collisions at High Energy
0. Introduction

Why does one need a new type of strong interaction experiments?

After all, QCD is in good shape:

- QCD successful, where $\sigma_s$ small
- Limitations due to hadronization
- Confinement: true vacuum such that colored objects do not move more than $1/fm$
- New approach to vacuum/confinement create region of free quarks by phase transition in $\Lambda\Lambda'$ coll.
Is there a chance for a phase transition in A'N?

- free atoms, mol. → gas <-> e.g. steam
  ↓
- free A⁺, e⁻ → e.m. plasma <-> sun
  ↓
- free q, g? → QGP <-> early universe
  ↓
- QCD thermodynamics?

- A'N at high energy:

Problem & motivation: little overlap with previous experiments
Contents

I. Introduction

A. Conventional features of A+A Collisions
   - Cross sections, nuclear effects
   - Inclusive distributions for meson & baryons
   - Collective effects
   - El. magn. processes

B. Phenomenology of QGP
   - Phase transitions & thermodynamics
   - Space-time history:
     Kinetic theory / hydrodynamics

C. Search for QGP
   - Current experiments
   - Theory & practice of signatures
   - Look into future
1. Cross sections

Mean free path $\lambda_N \approx 1.7$ fm: radii: $^{12}$C: 2.7 fm, $^{208}$Pb: 7 fm

$\Rightarrow$ nucleus = black disc

$\sigma_T = \pi R^2 = \pi r_0^2 \left( A_B^{1/3} + A_T^{1/3} - 5 \right)^2$

$r_0 \approx 1.2$ fm, $\sigma \approx 1.3$

Exp. problem:

$\sigma_T = \sigma_{(e, \text{coh.+ incoh.})} + \sigma_{(nuc. break up)} + \sigma_{(prod.)} + \sigma_{(e.m.)}$

$\sigma_{in}$ (A III)

\[ 160 \text{ 200 GeV/c/N} \]

- This experiment: NA36 prel.
- IRIS [12] ?ausart

+ $\{\text{NA35 prel.} \}$

- $\sigma_{(prod.)}$

\[ \sigma_{in} \]

\[ \text{H}_4 \text{He} \quad \text{Ne} \quad \text{Cu} \quad \text{Pb} \quad \text{Au} \]

**Fig. 2**

$\sigma_{in} (^{16}O^{208}Pb) \approx 3.6 \div 426$; $\sigma_{in} (pp) \approx 34 \text{ mb}$ (PDG)

$\sigma_{in} \approx \text{indep. of } T_f$

$\sigma_{in} (^{16}O \text{ Em}) \approx 1015 \text{ mb}$ between $T_{lab} = 2.1 \rightarrow 200 \text{ GeV/c}$

2. Event Structure and Geometry

- Simple picture: Abrasion-Ablation:
  assuming straight line trajectories
  i.e Glauker framework = geom. optics

- # spectators given by $b$ measure violence of collision
  $b$ very small: central cell.

- Check of idea:
  $\bar{E}(P)/E_B = A_T^{1/3}/(A_B^{1/3} + A_T^{1/3})^2$
  $Q = \text{charge yield of spec.}$
  $Q = (\bar{E} - \bar{V}) \cdot \pi r_0^2 (A_B^{1/3} + A_T^{1/3})^2$
  but: $r_0 < 1.2$ fm
  Reason? Glauker for $> 2$ GeV/c only

![Graph showing total yields of nuclear charge for projectile fragments]
3. Nuclear Fragmentation
(consequences for design of expt.)

Heavy and p beams:
- $dN/dA \sim A^{-2}, \tau = 2.6$
- geom. approach: no $V$ dependence
- $E_{\text{excit}} < 1 \text{ GeV} \Rightarrow$ no complete break-up

Berthier, PL 1988, 417

target fragmentation

\[ \frac{16}{8} O + \frac{197}{79} Au \quad \text{CERN (NA41)} \]

YIELD vs. $Z$ of target fragments

Type of mechanism:
- $A = 10 - 50$: Multi. fragmentation. $A \approx 50 - \frac{2}{3}A_T$ \(\not\rightarrow\) fission
- $A > \frac{2}{3}A_T$: Spallation

Note: for $E_{\text{lab}} > 3 \text{ GeV}/A$ (mb)
yield indep. of $V$
(for p Au at least, Kaufmann, PRC 22, 16?)
4. Fermionmation of Fragments

- boost into projectile rest syst.

Expectation [Goldhaber, PLSB, 206]:

\[ \sigma_{11} = \frac{P_F}{V^5} \sqrt{\frac{A_F(A - A_F)}{A - 1}}, F: \text{fragment} \]

\[ P_F = \text{Fermion momentum} \]

\[ = (3\pi^2 n)^{\frac{1}{3}}, n = \text{nuclear density} \]

[Landau - Lifshitz]

- \( \Delta \): probably due to binding [E. Gregorot, PRC 30, 6545]

- Measured widths consistent with \( P_F \)

Note:

- \( \frac{dN}{dA} (1S) \) & \( \frac{dN}{dP} (A) \) & known for fragments in average collisions

- Simulation possible for eqpt. design & analyses (e.g. MC Fint, NPA 404, 551)

- From \( \sigma_{11} \): fragments are expected at polar angle \( \theta_{lab} = (0 \pm 0.3^\circ) \), \( P_{lab} \geq 60 \text{ GeV/c} \)
Geometrical optics: straight light rays
High energy elastic scattering: strong maximum at small angles

$\Psi_{out}(b) = \Psi_{in}(b) S(b) = \Psi_{in}(b) (1 - \Gamma(b)), \ b \perp \bar{z}$

$\Gamma(b) = 0 \quad \Gamma(b) = 1$

$\Gamma$ = profile function

- Multiple scattering in nucleus A

$S_{A}(\bar{z}) = \int s_{i}(\bar{b} \pm \bar{z}) g_{A}(\bar{z} \cdots \bar{z}) d\bar{z}$

$\Gamma$ = wave function

Scattering amplitude $F(\Delta \bar{z}) \sim \int d\bar{b} [1 - S_{A}(\bar{b})] \exp(i \bar{b} \cdot \Delta \bar{z})$

- From $F(\Delta \bar{z})$ one can calculate probabilities for multiple interactions in A'A collisions
- Probabilities of this type are needed in production models for A'A collisions
Dual Parton Model (Capella, PL 81B, 68)

- In A'A' collisions strings can be formed between diq-q and q-q.
- Structure functions are needed for all partons.
- Strings hadronize; q-q strings fit e+e- data
- The number of strings is determined via chamber weights.

Lund-Fritiof (Andersson, NP B241, 289)

- $N \rightarrow \leftarrow N$
- \[ \rightarrow \leftarrow \] the collision leads to a momentum transfer

- Shape of momentum transfer chosen
- Strings break into hadrons
- A'A': multiple scattering probabilities are generated based on straight line trajectories
6. Machinism Distribution

\[ \psi(z) = <n> \frac{\sigma_{\text{ch}}}{\sigma_{\text{ch}}} \]


\( \sigma_{\text{ch}} \geq 4.5 \text{ GeV/c} \)

- \((p,d,He)+Ta\)
- \((p,He)+C \rightarrow \pi^0+x\)
- \((p,d,He)+Ta \rightarrow \pi^0+x\)
- \(p+p \rightarrow \pi^0+x\)

geometry: \[ 40^0 \text{ Ar} + \text{UO}_2 \]

Sandoval, PRL 54, 674

Charged-particle cross section, \(\sigma_{\text{ch}}\) (mb)

- Minimum bias trigger
- Central collision trigger
- Central no trigger

Total charged particles, \(n_{\text{ch}}\)

- \(O\text{Em} \text{ at } 60 \text{ GeV/c}\):
  - \(\langle n_r \rangle = 27.8 \pm 0.3\)

- \(O\text{Pb} \text{ at } 200 \text{ GeV/c}\)
  - \(\langle n_s \rangle = 88\)

- Capella, Z. Phys. C 10, 249:
  - \(\langle n_{\text{ch}} \rangle \approx 88\)

- All data consistent:
  - DPM fits
\[ \Delta \approx 80 - 120 \quad \text{WA80} \approx \text{NA35 in same } \Delta \alpha \]
\[ \Rightarrow 80 - 120 \text{ extra charged particles in target region?} \]
\[ n_{\text{ch}} (\text{plateau/10}) \sim A_T^{1/3} : \text{diameter of target} \]
\( y \leq 1.8 \) strong increase of multiplicity: \textit{cascading} formation time \( T_0 \rightarrow y_{T_0} = T_0 \cosh y = \beta e \approx e \)

\[ e = \lambda_p \Rightarrow y_c = \cosh^{-1}(\lambda_p/T_0), \quad T_0 = 1 \mu \mathrm{m} \]

\( y_{c} \approx 1.12 \)

Exp. results consistent with \textit{parton} picture:

- \( y < y_c \): \textit{partons} hadronize \textit{cascading} inside nucleons
- \( y > y_c \): \textit{multiple} parton collisions in nucleus

Should have experiment with \textit{particle ident. in target region}

\textit{Note:} \textit{coulion} collisions:

\[ \text{less cascading in dilute matter?} \]
\[ R_{\text{bin}}(y) \sim \frac{\sigma(p \rightarrow \pi^-) + \sigma(\kappa \rightarrow \pi^-)}{\sigma(p \rightarrow \pi^-)} \]

\[ = 1 + \frac{\sigma(\kappa \rightarrow \pi^-)}{\sigma(p \rightarrow \pi^-)} \] isospin

\[ = 1 + \frac{\sigma(p \rightarrow \pi^+)}{\sigma(p \rightarrow \pi^-)} \] increases with \( x_f \) due to valence quarks

- \( |y| \leq 2.5 \) or \( x_f \leq 0.4 \):
  - no evidence for valence quarks

Note:
\[ A = \text{Au} | \text{Ag} | \text{Au} | \text{Pb} \]
\[ \varepsilon/A = 0.48 | 0.46 | 0.43 | 0.4 | 0.4 \]
at SPS/ISR: proton identification on statistical basis
\[ s^+ - s^- = s(p^-) \]

- \[ n_{ch} \approx \langle n_{ch} \rangle \]
- \[ \langle \Delta y(pp) \rangle \approx \langle \Delta y(\omega\omega) \rangle \]
- \[ n_{ch} \gtrsim 1.5 < n_{ch} > \]

\[ \langle \Delta y(\omega\omega) \rangle \approx 1.45 \]

**Jezabek, PL B175, 206**

**Blosel, NP B135, 379**

- \[ \langle n_X \rangle at x_F = 0 \]
  - [30.6, 44.9, 52.8, 62.7 GeV]
  - Flugger and Mann (3)
  - [6.84 GeV], data signed
  - [4.93 GeV]

- \[ pp \rightarrow n, 12/24 \]
- \[ pp \rightarrow p, 40/25 \text{ GeV/c} \]

- \[ n_{ch} \approx \langle n_{ch} \rangle \]
- \[ 25 < n_{ch} \]
- \[ X9 < n_{ch} < X7 \]

- \[ \Delta y > \] smaller than earlier results: more work

\[ \text{Motivation:} \]

One can choose \[ \sqrt{s} \] such that \[ \gamma_{beam} = \Delta y \]

\[ \Rightarrow \text{baryons come to rest in c.m.s.} \ (p_{lab} \approx 2-25 \text{ GeV/c}) \]
S. Incidence Transverse Momentum Distributions

Cronin
PRD 11, 3105
\[ \sigma(pA) \sim A^{\alpha(p_A)} \]
a naively: \( \alpha = 1 \)
\( \alpha > 1 \quad \frac{A^2}{A_{\text{p}}^2} \)
mult. scatt.?  
Fermi motion?

\( P_T (\text{GeV/c}) \)

\[ P_T (\text{GeV/c}) \]

\[ P (\text{GeV/c}) \]

\[ \text{pointlike scatt. (9,9)} \]

\[ \text{SFM: Bell, PL 1148, 213} \]
\[ \text{AFS: Akerlof} \]
\[ \text{NP B 209, 309} \]

\[ \text{mult. 2 Fermi} \]

\[ \text{Staszcz} \]
2. Phys. C 19, 7

\[ \text{multiple scatt.} \]
\[ \text{Sukhatme PR 25, 1978} \]
\[ \text{only Fermi mom.} \]
\[ \text{2. Phys C 21, 155} \]

\[ \text{mult. scatt.} \]
\[ \text{Lev} \]

\[ \text{multiple scatt.} \]
\[ \text{AFS: Akerlof} \]
\[ \text{NP B 209, 309} \]

\[ \text{small } y_{CM} \]
Conclusion:

- Very large $E_T$ due to cascading?
- Why does WNM fit the data?
Measured total neutral energy $E_{\text{tot}}^0$

$pp$ data:

$$\frac{d\sigma}{dE} \sim \frac{\alpha}{\Gamma(p)} (\Delta E)^{p-1} e^{-\mu E}$$

$\Rightarrow$ expect for $n$ indep. coll.:

$$\frac{\alpha}{\Gamma(n)} (\Delta E)^{n p-1} e^{-\mu E}$$

i.e. $p \rightarrow n p$

$<E_{\text{tot}}^0>$

$E_{\text{tot}}^0 / <E_{\text{tot}}^0>$

R110-BCMOR

$\sqrt{s_{NN}} = 31$ GeV

- $o = p-p$
- $x = d-d$
- $* = \alpha-\alpha$

$\Delta y \approx 0.9$

$\Rightarrow$ DPM

[Capella, prc comm.]

$<E_{\text{tot}}^0>$

$E_{\text{tot}}^0$ (GeV) $E_{\text{M}}^0$ (GeV)

$\Rightarrow$ found:

- good fit to $\otimes$
- with same $p$, $\alpha \rightarrow \frac{<E_{\text{tot}}^0>_{pp}}{<E_{\text{tot}}^0>_{d-d}}$

if indep. coll.:

$F = f$ and $F = f$

Here one has found same shapes for $F$ and $f$

$\Rightarrow$ correlations
15. Isotropic Events and Collective Flow

Gustafsson PL 142 B, 141 (Stöbele PRC 27, 1349)

\[ \frac{2}{N_i} \langle p_T \rangle \]

\[ \frac{1}{N_i} \sum_{n=1}^{N_i} |p_n| (\text{MeV/c}) \langle p_n \rangle \rightarrow \]

\[ 1 = \frac{\langle p_x \rangle}{\langle p_{\parallel} \rangle} = \frac{2}{\pi} \frac{\langle p_T \rangle}{\langle p_{\parallel} \rangle} \]

\( \rightarrow \) isotropy \( \Rightarrow \) isotropic central coll. events

Conclusion: \( A > 40 \)

Coll. of He\( \rightarrow \) Ne at 4.5 GeV/c: Gadzicki, Z. Phys. C31, 549

\( \alpha_E \) measures isotropy

\( \alpha_E = 1 \): Like pp

\( \alpha_E = 0 \): isotropic

- if \( \Lambda \) outside NN kin. limits

\( \Rightarrow \Lambda \) and \( \bar{\Lambda} \) isotropic

\( \Lambda_{\text{out}}: \sim 10^{-3} \) central \( \sim 10^{-4} \) \( \bar{\Lambda}_{\text{in}} \)

'fully stopped and thermalized hot source'
stepping of nuclei \rightarrow compression

\rightarrow collective flow after expansion?

(Carl Yaf PRL 52, 1590)

Ritter, NP A 447, 3c

\[ E/A = 400 \text{ MeV} \]

\[ A = \begin{array}{c}
40 \\
93 \\
197
\end{array} \]

\[ \text{Ca + Ca, Nb + Nb, Au + Au} \]

\[ \text{E/A} = 150 \text{ MeV, 250 MeV, 400 MeV, 650 MeV, 800 MeV} \]

\[ \langle \theta \rangle \text{ decreases for E/A} \geq 400 \text{ MeV} \]

finite flow angle should give a handle

on compressional effects in nuclear matter

(see section B tomorrow)
II. Electromagnetic Ionization

- \[ b > R_1 + R_2 \]
  - no nuclear interaction
  - but \( z_1^2 \), \( z_2^2 \) large!

Olsen, PR C24, 1529

\[ \sigma(\gamma, n) \]

\[ \sigma(\gamma, n) \]

Weizsächer-Williams:

\[ \sigma_{\text{cond}} \sim \int \frac{dN}{dE_\gamma} \sigma(\gamma, n) \ dE_\gamma \]

\[ \sim z_T^{1.8} \]

Remember:
- \( \sigma(17O\gamma n) \propto 4.6 \) at 100 GeV/c
- expect strong plus dependence
\[ \sigma_{\text{EMD}} \rightarrow \sigma_{\text{Emb}} \]

\[ \text{Bevalac} \]

\[ {^{197}}\text{Au}({}^{197}, x){^{196}}\text{Au} \]

\[ {^{197}}\text{Au}({}^{197}, x){^{196}}\text{Au} \]

\[ \sqrt{s} \]

\[ \sigma_{\text{EmD}} \]

\[ \sigma_{\text{EmB}} \]

\[ \text{Pb} \rightarrow \text{GeV/c} \]

- consequence: \( \text{Au + Au at 100 \times 100 \text{ GeV/c:}} \)

\[ \sigma_{\text{EmD}} (\text{Au+Au}) = 30 \div 60 \, 6 \rightarrow \sigma_{\text{EmB}} \approx 9.46 \]

Limits beam lifetime and/or luminosity.

- If these cross sections are large, what about

\[
\begin{align*}
\text{Au} & \rightarrow O \\
\text{Au} & \rightarrow O
\end{align*}
\]

\[ e = e, \mu, \tau \ldots e, \mu \]

Do physics with ion beams? (Theoretical predictions soon)
II. Phase Transitions

Experimental limits:

\[ \frac{dN}{dp_T^2} \sim e^{-aE_T} \]

\[ E_T = \sqrt{m^2 + p_T^2} \]

- Boltzmann:
  \[ \frac{dN}{d\nu^2} \sim e^{-E/\hbar T} \]

\[ \Rightarrow \text{thermodyn.:} \quad \langle p_T \rangle \sim T \]

- Complete formula
  \[ \text{[Raukauh, Adv. Phys. 31(8), 551]} \]

\[ \langle p_T^2 \rangle \]

\[ \pi K P \]

\[ T \]

Water \(+\) steam / steam (free mol.)

Water (bound mol.) \(\rightarrow\) energy

\[ \langle p_T \rangle \quad [\text{MeV/c}] \]

\[ \uparrow 400 \]

\[ \uparrow \]

\[ \uparrow \]

\[ \text{pp} \rightarrow \pi + X \]

'Hadronic matter at the boiling point'

\[ T \approx 160 \text{ GeV} \quad \text{Hagedorn, N. Cim. 56, 1027} \]

\[ 200 \]

\[ \text{[GeV/c]} \]

\[ 1 \quad 10 \quad 10^2 \quad 10^3 \]

\[ \text{pt} \quad \text{[GeV/c]} \]
2. Definition

- El. insulator

\[ V_{el} \sim -\frac{e}{r} \]

El. conductor

\[ V_{ac} \sim -\frac{\mu}{r} + \sigma r \]

\[ V_{ec} = V_{ac} \text{ at very small } r \Rightarrow \text{ also Debye screening?} \]

\[ V_{\text{Debye}} \sim \frac{\mu}{r} e^{-r/\lambda_d} \]

Debye screening

\[ \xi(0, T) \]

1.2

\[ T \approx 160 \text{ MeV} \]

\[ T \approx 300 \text{ MeV} \]

Increasing distance

- Color insulator

Baryons overlap: stopping
- Color conductors
- Abundant \( \pi \) production
- \( \pi \) overlap

Joins better than \( N \):
- More effective screening
- Better equilibration
- More space
3. Thermodynamics in a Nutshell

- \( N_q \approx 240 \) ("O" \text{amu}), \( N_q \approx 1400 \) (\text{U})
- no heat bath: thermodynamics: justified?

- 1st Law of Thermodyn.: (Heat = energy, energy conservation)
  \[ dE = TdS - PdV + \mu dN \]
  \( E = \) energy, \( T = \) temperature, \( P = \) pressure
  \( V = \) volume, \( N = \) \# particles
  \( S = \) entropy: measure \# of states accessible to system
  \( \mu = \) chem. potential: governs flow of particles between systems
  
  \[ \text{equiv.: } dS = \frac{1}{T} dE + \frac{1}{P} dV - \frac{1}{\mu} dN \]

- equilibrium 2 phases:
  \[ \Delta (S_a + S_b) + \alpha \Delta (E_a + E_b) + \beta \Delta (V_a + V_b) + \gamma \Delta (N_a + N_b) = 0 \]
  \( \Rightarrow \begin{align*}
  T_1 &= T_2 \\
  \mu_a &= \mu_b \\
  P_a &= P_b \\
  \text{thermal} &\quad \text{chemical} &\quad \text{mechan.} \\
  \text{equilibrium.}
  \end{align*} \]

- 2 variables independent: i.e. for \( P_a = P_b \Rightarrow T_C = T_C (\mu_a) \)
  \[ \text{phase diagram} \]

- Generally:
  \[ P = P (T, V, \mu) \]
  
  \[ \text{equation of state EOS} \]
  (remember ideal gas: \( p \approx \frac{RT}{V} \))

- Equiv.: \( \Delta U = E - TS - \mu N = -pV \) \( \Rightarrow \) minimum
  \( (\equiv F \text{ free energy for } \mu = 0) \)
  \( \Rightarrow \) system goes into state of \( P_{\text{max}} \)

- If \( \Delta U \) known \( \Rightarrow \) EOS known (see below)
- **Statistical Mechanics**

\[ S = -T \ln Z \]

\( Z = \text{grand partition function} \)

- classical \( Z = \sum \exp \left( \frac{E_i}{k_B T} \right) \)

  \( \text{sum over occupied states} \)

- QCD: \( Z \) integral over field \((q, g)\) configurations

  \( \Rightarrow \) QCD

- From \( Z \) one gets:
  
  energy density \( \varepsilon = \frac{E}{V} = \frac{T}{V} \frac{\partial \ln Z}{\partial T} + \mu N \)
  
  particle density \( n = \frac{N}{V} = \frac{T}{V} \frac{\partial \ln Z}{\partial \mu} \)
  
  entropy density \( s = \frac{S}{V} = \frac{1}{V} \frac{\partial (T \ln Z)}{\partial T} \)
  
  pressure \( P = \frac{\partial (s \ln Z)}{\partial \ln Z} \)

  \( \Rightarrow \) one can calculate EOS etc. from \( Z \),

  e.g. properties of phase transitions

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Landau - Lifshitz V

Kittel: Thermal Physics

Landsberg: Thermodynamics and Stat. Mechanics

Clemmans: Phys. Rev. 130, 217
4. Thermodynamic Approaches to Phase Transitions

\[ T \text{ [MeV]} \]
\[ n_0 \quad n_0 \quad n_b \quad \text{or } \mu \]
\[ \text{nuclei} \quad \text{Hadronic phase:} \]
\[ \text{QGP EOS: } \varepsilon_\alpha = \varepsilon_\alpha (T, V, \mu) \]
\[ P_\alpha = P_\alpha (T, V, \mu) \]

\[ \text{Gibbs: e.g. } P_H (T_c) = P_\alpha (T_c) \quad \text{need EOS for } H, Q! \]

a) EOS extracted from data: [e.g. Stock, Phys. Rep. 135, 259]

at fixed \( V, S, <u_n> \)
smaller than predicted

\[ \Rightarrow \text{evidence for compressional energy?} \]

\[ \varepsilon_H = \varepsilon_0 + \varepsilon_{\text{comp}} + \varepsilon_{\text{therm}}. \]

Note: compression should cause flow (sect. A.10)

b) EOS of ideal hadron gas:

no \( N, \) for \( \frac{m_\pi}{T} \to 0: \]

\[ \varepsilon_H = \varepsilon_\pi (\text{free } \pi) = 3 \frac{\pi^2}{30} T^4 \text{ Stefan-Boltzmann.} \]

\[ P_H = \frac{1}{3} \varepsilon_\pi \quad \text{(remember pressure of black-body radiation)} \]
assumptions: \( n_s = 0, \, m = 0, \, \mu = 0 \)

\[ n = \frac{\pi}{(2\pi)^3} \int dp \left[ \exp \left( \frac{\pi}{\hbar} (E - \mu) \right) \right]^{-1}, \text{ } n \text{: particle density} \]

\[ E = p \text{ and } \mu = 0 \]

\[ n = \frac{4\pi}{(2\pi)^3} \int p^2 dp \left[ \exp \left( \frac{\pi}{\hbar} p \right) \right]^{-1} \]

\[ \text{Fermi} \]

\[ L = \frac{4\pi}{(2\pi)^3} T^3 \left\{ \frac{1}{3} \right\} \rightarrow \frac{3.6}{4\pi^2} T^3 = n_q \text{ per atom} \]

\[ \text{Bose} \]

\[ \frac{\Gamma(3) \zeta(3)}{\zeta(3)} = \frac{2.4}{2\pi^2} T^3 = n_g \text{ per atom} \]

\[ \text{Strangeness: } \mu_s = 0 \]

\[ n_\frac{1}{2} = n_s = \frac{1}{2\pi^2} n_s^2 T K_2 \left( \frac{m_s}{T} \right) \]

\[ \text{for } T \uparrow \text{ or } m_s \downarrow \]

\[ \frac{5}{q} \uparrow \]

\[ m \uparrow \text{ or } m \downarrow \]

Pauli principle

\[ \frac{\Gamma(3) \zeta(3)}{\zeta(3)} \]

\[ n_q = n_g = \frac{2.4}{2\pi^2} T^3 \]

\[ \text{Koch, Phys. Rep. 142, 167} \]

\[ E = \frac{1}{(2\pi)^2} \int p^2 dp \left[ \right] \text{ } J = \frac{\pi}{(2\pi)^3} T^4 \left\{ \begin{array}{c} (2^3 - 1) n^4 B_4 \text{ } \text{Fermi} \\ \frac{(2\pi)^4}{8} B_2 \text{ } \text{Bose} \end{array} \right. \]

\[ \Rightarrow \text{per dof} \quad E_q = \frac{\pi^2}{8} T^4 \quad E_g = \frac{\pi^2}{30} T^4 \]
total \[ E_\text{tot} = \frac{\pi^2}{30} \left( g_g + \frac{2}{3} g_q \right) T^4 = \frac{\pi^2}{30} \alpha T^4 \quad \text{(black body)} \]

for dof: \[ g_g = 2 \times 8_c \quad g_q = 2 \times 3_c \times 2.5_e \quad \text{lots of gluons} \]

From above: \[ \frac{E}{\text{parton}} = \frac{E}{n} \quad (m=0) \]

\[ g_\ell: \quad \frac{s}{n} = 2.71 T \quad \frac{E}{E} = 2.13 T \]

\[ g_{\mu\nu}: \quad \frac{s}{n} = 3.16 T \quad \frac{E}{E} = 2.48 T \]

also: \[ \text{entropy/parton} = \frac{s}{n} \]

\[ s = \frac{\mu^2}{v_s^2} \frac{\sqrt{s}}{T} = \frac{1}{3} \frac{\sqrt{s}}{T} \quad (v_s = \text{speed of sound}) \]

\[ \Rightarrow \text{gluons:} \quad \frac{s}{n} = g_\ell = 3.61 \]

\[ \text{quarks:} \quad \frac{s}{n} = g_q = 4.21 \]

\[ \text{Ledlik, PRL 37, 3747} \]

Numerical examples:

\[ E_N = 0.5 \text{ GeV/fm}^3 \quad E_{\pi} (m=0) = 12 \cdot T^4 \text{ GeV/fm}^3 \quad \text{factor} \]

\[ E_A = 0.12 \text{ GeV/fm}^2 \quad E_{\pi} \propto T^4 \text{ GeV/fm}^2 \times 10 \]

Absolute prediction: \[ T_c = (200 \pm 40) \text{ MeV} \quad (T^4!) \]

\[ \text{LQCD} \]

(for \( m=0 \), see Cleymans, Phys. Rep. 120, 241)
Bag EOS: \( E_\Omega = E_\Omega + B \)

\( P = \frac{1}{3} (E_\Omega - 4B) \)

\( P_H = P_Q \Rightarrow \frac{n^2}{120} \left( \frac{1}{2} - \frac{3}{2} \right) \frac{T^4}{e} = B \)

\( \Delta \text{def QGP} \)

Bag EOS

\( \frac{\epsilon}{T^4} \)

\( \frac{\beta}{T^4} \)

\( \beta \text{ Stefan-Boltzmann} \)

\( \beta \text{ latent heat} \quad \text{1st order} \)

\( \beta \text{ order} \)

\( \epsilon / T^4 \)

\( S / T^3 \)

\( \beta \text{ order} \)

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Evidently, predictions depend on EOS assumed.

- Important work at Bevalac and LACD

Empirical EOS

Cserepes

\[ T(\text{MeV}) \]

\[ Q \]

\[ H \]

\( \mu \rightarrow \)

\( 0 \rightarrow 1.5 \text{ GeV} \)

\( 0.9 \text{ GeV} \rightarrow \)

\( \mu \rightarrow \)

\[ \text{Density [fm}^{-3}\text{]} \]

\[ n_b \rightarrow \]

\[ T_c = 235 \text{ MeV}, \ n_b = 0.16 \text{ fm}^{-3} \]

\[ K = 240 \text{ MeV} \]

\[ \text{OCD Plasma} \]

\[ T_c \approx (160 \div 220) \text{ MeV} \]

\[ \Sigma_q(T=0) \approx 2 \div 7.5 \text{ GeV/fm}^3 \]
In this produce & **π** production model, a high T π production was

\[ E_{\pi}^{T} = \frac{\Delta E}{\Delta y} \cdot \frac{1}{V} \text{ with } V = v \Delta y A_{l} \text{ & } v = 1 \text{ fm} \] 

Recall: \[ E_{\pi}^{T} = \frac{2}{\sqrt{s}} \text{ GeV} \]

\[ E_{\pi}^{T} = \frac{1}{v} \frac{dN}{dy} \text{ at } v = 1 \text{ fm} \]

\[ \text{pp } \sqrt{s} = 50 \text{ GeV/c } E_{\pi}^{T} = 0.35 \text{ GeV/fm}^{2} \]

Remember: \[ E_{\pi} > 2 \text{ GeV/fm}^{2} \]

For \( \Lambda' \Lambda: \frac{dN}{dy} = \Lambda'(dN)/pp \text{ (GeV/c)} \]

\[ \Rightarrow E_{\pi}^{T} \sim \Lambda'/\Lambda^{2} \Rightarrow \text{cons!} \]

\[ E_{\pi} \text{ ~ isoelastic process} \]

<table>
<thead>
<tr>
<th>pp → N(<em>{1}) N(</em>{2}) + pions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \frac{\Delta \sigma}{\Delta E_{1} \Delta E_{2}} \text{ mb/GeV}^{2} ]</td>
</tr>
<tr>
<td>Distribution over CMS energies of the two nucleons, normalized to 37 mb</td>
</tr>
</tbody>
</table>

\[ 19 \text{ GeV/c, Brookhaven NP 89/765} \]

\[ E > E_{Q} \Rightarrow \text{ p+p } \geq 10 \text{ GeV/c} \]

1. Stopping: \( M = 0 \)

\[ \Rightarrow \text{ p+p} \]

\[ E = 2E_{Q}y_{f} \]

\[ E_{T} \text{ into cms Lorentz contraction} \]

\[ E > E_{Q} \Rightarrow \text{ p+p } \geq 10 \text{ GeV/c} \]

19 GeV/c, Brookhaven NP 89/765
2) Order of phase transition (Landau-Lifshitz II)
   - Latent heat corresponds to 1st order phase transition
   - 1st order: coexistence of 2 phases possible
     also supercooling/superheating
   - 2nd order phase transition
     no latent heat, no phase coexistence

b) Chiral phase transition
   above: \( u_q = 0 \) assumed

   c) in conductor: solid: \( u_e + u_{vac} \)
      due to boundary conditions [Kivel, Solid State Physics]
   \( u_q \neq 0 \) maybe because of confinement?
   If \( u_q = 0 \) \( \Rightarrow \) large, include \( u \) in \( J_f \)

   QCD: (Satz, Phys. 127, 167)
   chiral phase trans. at deconfinement?

   c) \( x_s = 0 \) assumed throughout

   \( x_s = 0 \Rightarrow \Sigma_{4GF} \Rightarrow 0.6 \Sigma_{4GF} (x_s = 0) \)
   reduce effective \# dof
   lower \( T_c \)?

   e.g. [Rafelski, Phys. Rep. 81, 231]
I. Space-Time History of A+A Collisions

1. Introduction

- Typical A'A cell at SPS: \( \sigma_0 \) no symmetry
- central collisions \( \sigma_0 \) cylindrical sym.
- now \( A' = A \) \( \sigma_0 \) symmetric
- \( T_s \) very high \& \( \mu = 0 \), i.e. \( T \to \infty \)
  \[ \Rightarrow \] will be discussed in the following
- SPS: probably \( \mu > 0 \)
  but simple scenario \( T_s \to \infty, \mu = 0 \)
  useful to develop intuition

\( T = 0 \) occasionnally \( \sigma_0 \)

- formation of
  - on shell partons (initial conditions)
  - towards thermal (kinetic theory)
  - thermal QGP (EOS)

- expansion (hydrodynamics)
  - hadronization and expansion (kinetics & hydrodynamics)
  - freeze out of hadrons
  - detection
Dimensional arguments: (Blaschot, Act. Phys. Pol. B18, 659)

- No of partons $\sim A$
- Volume comoving $V \sim A^{3/2}$, only variable $k_T$ of partons: $L \sim \frac{1}{k_T}$
- $2r_{\text{part}} < L$ to equilibrate
  $r \sim \frac{1}{\sigma n}$
- Assume (QCD): $s \sim \frac{1}{3} \sim \frac{4}{k_T^2}$
  and $n = \frac{A}{V} \sim A^{3/2} k_T$

From $2r_{\text{part}} < L \Rightarrow k_T \sim A^{1/6}$ for partons creating QGP

- Consequences
  - $E = \frac{E}{V} \sim \frac{A k_T}{V} \sim A^{3/2}$ (A^{3/2} in sect. I.5)
  - $2 \sim t(\text{formation}) \sim k_T^{-1} \sim A^{-1/6}$
  - $L/R_A \sim A^{-1/6}/A^{1/3} \sim \frac{1}{\sqrt{A}}$ (measure of multiple scattering)

  i.e. if $A$ small $\Rightarrow$ no equilibration

$\Rightarrow$ use ions

See also: Blaschot, NP B19, 857; Kerman, PRL 56, 219
Hua, PRL 56, 616
2.1 Approach to Equilibration: Kinetic Theory

a) Collision of particles may yield equilibrium
\[ f(\mathbf{r}, \mathbf{p}, t) = \text{phase space density} \]

Boltzmann eqn. for gas:
\[ \frac{df}{dt} = \frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f + \mathbf{p} \cdot \nabla f = -\frac{\partial (df/dE)}{dE} \text{c骰} \]

at equilibrium: \[ \frac{df}{dt} = 0 \]

near equilibrium:
\[ \frac{df}{dt} = -\frac{f - \bar{f}}{\tau} \]
\( \bar{f} \) : equilibrium
\( \tau \) : relaxation approx.

b) In principle \( \nabla \) holds here as well

for \( df_i / dt \), \( i = u, \bar{u}, d, \bar{d}, s, \bar{s}, g \)

\( \frac{df}{dt} \) includes creation & annihilation

of \( q \) and \( \bar{q} \)s.

- It has to be modified, however: color is exchanged

\( \Rightarrow \) color is dynamic variable \( \Rightarrow \) term: \( \nabla_q f \cdot \mathbf{q} \),

also quantum mechanical framework.

b) Simple example: \( \bar{s} \bar{s} \) production by \( gg \rightarrow \bar{s} \bar{s} \), \( g \bar{g} \rightarrow \bar{s} \bar{s} \)

with \( \nabla f_i = 0 \) and \( S_s = \int f_s \, d^3 \mathbf{p} \)

\[ \Rightarrow \frac{ds_s}{dt} = \frac{S_s}{\tau} [1 - S_s / S_s] \]

Conclusion:
- \( gg \) dominates \( \bar{s} \bar{s} \) production (16 dof)
- \( \bar{s} \bar{s} \) produced fast

Koch, Phys. Rep. 142

\[ A \sim \frac{1}{t} \]

\[ t \sim 10^{-11} \text{sec} \]

\[ 10^2 \text{MeV} \]
Transition to hydrodynamics of fluids

Start from \( \frac{df}{dt} = \frac{df}{dx} + \frac{v}{x} \frac{df}{dx} = -\frac{df}{dx}/c_v \), i.e. \( F = 0 \)

\[ \frac{df}{dx} \]  

This term is source of dissipative effects giving transport coefficients:

heat conductivity, viscosity.

One can show that: [Hedin, Phys. Part. Nucl. Phys. 13]

1) \[ \int \frac{df}{dt} d\bar{p} = 0 = \frac{df}{dx} + \text{div}(\bar{p} \bar{u}) \]

\[ \bar{p} = \text{const}, \bar{u} = \bar{v} \]

continuity eqn.

2) \[ \int \frac{df}{dt} d\bar{p} = 0 = \frac{df}{dx} + \text{div}(\bar{p} \bar{u} + \bar{u}) + \nabla \bar{P} \]

\[ \text{Euler eqn.} \]

3) \[ \int \frac{df}{dt} \bar{v} d\bar{p} = 0 = \frac{df}{dx} + \text{div} (\bar{v} \bar{u}) + \nabla (\bar{v} \bar{P}) \]

\[ \text{energy conservation} \]

Hence: by integration (loss of information) one arrives at hydrodynamic's equations for viscous fluids (stress tensor \( \bar{P} \) includes transport coeff.)

For viscous fluids Euler equation = Navier-Stokes eqn.

From kinetic theory transport coefficients can be calculated to be used in hydrodynamics.

Kinetic theory may suggest that the e+e- plasma does not consist of free gl. Eq. [Heine, NP A 461, 490] e.g. [Heine, NP A 461, 490]

Dissipative phenomena: Daniel wired, PR D 31, 57, Hosoya, NP B 250, 666
(ii) Assume: relativistic case, no viscous (dissipative) effects

To remember: \( \frac{\partial \varepsilon}{\partial t} + \nabla (\varepsilon \mathbf{u}) + \nabla (\mathbf{u} \cdot \mathbf{P}) = 0 \) (previous page energy conservation).

Relativistic hydrodynamics starts from an energy-momentum tensor \( T^\mu{}^\nu = (E+P) u^\mu u^\nu - g^\mu{}^\nu \rho \)

\( u^\mu = (1, \mathbf{v}) \)

\( \frac{1}{c} x^\mu = u^\mu \)

Energy momentum conservation

\( \partial_\mu T^\mu{}^\nu = 0 \Rightarrow u^\mu \partial_\mu T^\mu{}^\nu = 0 \)

\( \Rightarrow u^\nu \partial_\nu \varepsilon + (E+P) \partial_\nu u^\nu = 0 \) (see above!)

Now: from \( E+P = Ts + \mu u \) \( \rightarrow \) \( dE = Tds + \mu du \)

one has \( u^\nu \partial_\nu S + u^\nu \mu \partial_\nu u + (Ts + \mu u) \partial_\nu u^\nu = 0 \)

or \( T \partial_\nu (u^\nu S) + \mu \partial_\nu (u^\nu u^\nu) = 0 \)

Particle conservation: = 0

\( \Rightarrow \partial_\nu (u^\nu S) = 0 \)  Entropy conservation

C.f. [Landau, Lifshitz \&i; Cleymans, Phys. Rep. 130, 217]
Scaling Hydrodynamics:

\[ \frac{dN}{dy} \propto \text{const.} \]

\[ \Rightarrow \text{invariance under Lorentz boost along } \widehat{z} \text{ beam} \]

\[ \Rightarrow \text{physics quantities depend on the only Lorentz scalar } z, \]

\[ z = \sqrt{\frac{E^2}{c^2} - 1}, \]

- longitudinal flow \( u = \frac{z}{c} \)

- from previous page one finds with \( u v = 0 \), \( \partial u^\mu = \frac{\partial}{\partial z} \)

\[ \frac{\partial E}{\partial z} = - \frac{E + P}{c} \]

\[ \Rightarrow s \cdot z = \text{const.} \text{: isentropic} \]

\[ s \sim \frac{1}{z} \text{ 1-dim. expansion} \]

Consequences:

- \( dN(\text{partons}) = \int dV n = \int dy z d^2x_\perp \frac{3}{c} \) (remember \( c = \frac{3}{4} \))

\[ \Rightarrow \frac{dN}{dy} = \frac{1}{c} A_{LFT} \tau_0 s_0 = \frac{1}{c} A_{\perp} \tau_0 s_0 \quad f = \text{final} \]

One can relate observed particle density to initial entropy density \( s_0 \) (or \( \frac{5}{3} = s_0 T_0 ) \)
\begin{align*}
\frac{dN}{d\eta} & \sim \frac{1}{\tau} A_{L} \to s_0, \quad s \sim \frac{2\pi}{S_T} \sim T^3 \\
& \sim \frac{1}{\tau} A_{L} \to T_0^3 \\
& \sim \Lambda^3 \Lambda^{-\nu} (\Lambda^\nu)^3 = \Lambda^1 \quad (\text{cosmics!})
\end{align*}

\[ \tau T^3 \sim \tau s = \text{const} \quad \text{with} \quad \tau \sim \Lambda^{-\nu} \quad \Rightarrow \quad T \sim \Lambda^{\nu}
\]

\[ \Rightarrow \text{use } \tau \text{ units}
\]

\[ \tau = \sqrt{\frac{e^2 - e_t^2}{c}} = \text{const}
\]

Physics depends only on \( \tau \)

\[ \Rightarrow \text{slow particles} = (\nu = \frac{2}{16}) \text{ are created first}
\]

(inside-outside cascade)

\[ S \cdot \tau \sim \tau T^3 = \text{const} \Rightarrow \tau_Q = \tau_i \left( \frac{T_i}{T_c} \right)^3 \sim \Lambda^{-\nu} (\Lambda^\nu)^3 = \Lambda^0
\]

\[ T_H = \tau_Q \frac{\sigma_0}{\sigma_H} \quad T_F = \tau_M \left( \frac{T_c}{T_F} \right)^3
\]

\[ \Rightarrow \tau_i : \tau_Q : \tau_H : \tau_F \approx 1 : 16 : 80 : 240
\]
Scaling high, long flow

Baym NPA 407, 541

$T/T_c \uparrow$

$T/T_0 \uparrow$

$r/R \uparrow$

$T_c \uparrow$

$T_c$ \rightarrow

very similar: cooling governed by long expansion

radial velocity $v_r \uparrow$

arrives late at $r=0$!

Baym NPA 407, 541

lines of $T/T_c =$ const.

radial exp. $\uparrow$ wave fraction wave
Since shock front reaches late at $r = 0$, there is no plasma any more $\Rightarrow$ does not affect strongly previous arguments ($T_i, T_e$)

No phase transition:

[shocks: Landau-Lifshitz $\square$]
2) Recombination

\[ \text{Problem: } S = 5V = \text{const} \]

\[ c = \frac{S}{n} = 4.2 \quad \Rightarrow \quad \frac{S}{n} = 3.6 \]

\[ \# \text{ dof } 6 \quad \# \text{ dof } 3 \]

\[ \Rightarrow \text{entropy not conserved} \]

2 ways out: a) increase \( V \) b) fragmentation

Consequence of recombin. for \( p_{20} \):

- it is easier for a quark to find other quark q than to find \( \bar{q} \)

\[ \Rightarrow \] easier to produce e.g. \( K^+ = 5u \) than \( K^- = 5\bar{u} \)?

easier to produce \( \Lambda = s\bar{u}n \) than \( \bar{\Lambda} = 5u\bar{u}\)?

Therefore, 5 quarks might be accumulated in QGP \( \Rightarrow \) strange matter?

3) Fragmentation

\[ \text{new } q\bar{q} \text{ pairs i.e. entropy created} \]

\[ \text{Note: this changes flavor composition in } H \]

\[ \text{See also: van Hove, Phys. C 27, 1351} \]
Kinetic theory for chemical equilibration in hadron phase (Koch, Phys. Rep. 142)

\[ \pi K N Y \rightarrow \pi K N Y \]  
\[ \text{at hadronization} \quad \text{at freeze out} \]

For rate equations one needs

\[ \mathcal{G} \left( \frac{\pi K N Y}{\Delta} \right) \rightarrow \frac{\pi K N Y}{\Delta} + x \]  
including resonances!
...search for the CEP

**Experiments**

'Conceptual Design'

\[ A_B \rightarrow A_T \]

\[ S_T \rightarrow P_B \]

\[ P_B = \frac{A_B}{A_T} \]

Transverse \( \frac{1}{2} \)

Central Coll. beam spect.

'fireball'

Consequence: Common features of (nearly) all exps:

\[ \rightarrow I \rightarrow T \]

Cal. \( \pm 0.3^\circ \) central coll. 'no' spect.
Experiment NA34: Lepton production

≈ all components except close to NA34/i
Side view of NA6 set-up (compressed EN75)
using NA10 $\mu$-pair spectrum.

high rate, high mean $\mu\mu \geq 1/4$
II. Experimental Arrangement

1) Introduction

a) Do central collisions occur?

\[ \text{fireball?} \]

b) Is there evidence for an isotropic fireball?

c) Characteristics of $E_T$ production: geometry and energy density?

Warning: nearly all results unpublished => preliminary
200 GeV: very often projectile stopped in target. 120 GeV: beam not present. 80 GeV: no projectile present.
5) (Pseudo-)Rapidity distributions for central events

$E_{\text{lab}}$ 1

0 $\text{Ag/Br}$
200 GeVc
$N_s > 200$

[Holycross, HEA-55-82-04 (KLM)]

- $E_{\text{lab}}$ & KLM data agree
  if $N_A = A_t^{0.3}$ (1st lecture) $\Rightarrow$ via $0\text{Pb}$ data: $2 \approx 3$ central

\[ \frac{dN}{dy} \]

\[ \text{negatives} \]

NA 35 Preliminary
200 GeV $^1\text{O} + \text{Au}$
Central vetto

- One finds
  $Y_{\text{lab}}$ (max) $\approx 2.4$
  $Y_{\text{lab}}$ (max, $pp$) $= 3$

- Note: $Y_{\text{lab}}$ (max) = $Y_{\text{lab}}$ (max, $pp$)
  $\frac{1}{2} \ln \left( \frac{P_t}{A_t^2} \right)$

$\Delta \approx 0.6$ for 16 + 55 nuclei!

\[ \Delta \]

\[ \frac{\Uparrow}{\text{beam target/geometry}} \]

Rafelski
UCT-TP 81/1987
Production of transverse energy $E_T$

Abbott, et al., 10 PL
E802 / BNL

- Overall shape probably determined by geometry
- But, obvious that independent pAu cell
  fit 160A cell 22

Bamberger PLC184, 271

$^{40}$Pb 200 GeV/nucleon NA35

$2.2 < q < 3.8$
The data exceed MC predictions at large $E_T$.

No 60 A GeV

No 200 A GeV

Fig. 2
\(-0.1 \leq \eta \leq 2.9\)

\(^{160}\text{A}, 60 \text{ GeV/nucleon} - \eta > -0.1, \text{Ag} > 2.9\)

\(^{160}\text{A}, 200 \text{ GeV/nucleon} - \eta > -0.1, \text{Ag} > 2.9\)

\(7\% \, E_T \text{ scale}\)

\(\text{IRIS (DPM)}\)

\(dE_t/da\)

\(\eta \rightarrow 2\pi \frac{E_t}{\text{GeV}}\)

\(\text{small number of } \eta\text{ spect.}\)

\(\text{cascade?}\)

\(\langle E_{\eta} \rangle \text{ WABO!}\)
Size of Source

Pion Interferometry

Correlation function

\[ C(\Delta q) \sim \frac{N_{\pi\pi}}{N_{86}} \sim 1 + \alpha f(\Delta q_1) g(\Delta q_2) \]

\[ \pi_1 \rightarrow R_1, \quad \pi_2 \rightarrow R_2 \]

Result: \( R_1 = (8.9 \pm 1.6) \text{ fm} \), \( R_\parallel = (5.6 \pm 1.3) \text{ fm} \), \( \alpha = 0.77 \pm 0.19 \)

\( R_{100} \approx 3 \text{ fm} \)

consistent with isotropy

\[ \text{Bailey, CERN-EP/87-42} \]

\[ \text{expected from isotropy at 1y126} \]

\[ \text{pp} \rightarrow h^+ \text{ at } 360 \text{ GeV/c} \]
2. QGP Signatures of Thermalization & T from $\gamma, \bar{\gamma}$

In QGP: $e^+ e^-$ and $\gamma$ are expected from

\[ \text{e.g. } q\bar{q} \rightarrow e^+ e^- \\ q\bar{q} \rightarrow \gamma \gamma \]

\[ \text{Heinem. PRD 31, 545} \]

\[ \text{Cugnans, PRD 35, 2153} \]

Since $E/\text{parton} \sim T$ (lecture 2):

\[ \gamma, e^+ e^- \rightarrow T \]

In rest frame:

\[ \begin{align*}
\text{Rate for } e^+ e^- \text{ production: } & \quad \frac{dN}{d^4x} = \frac{10}{2\pi^2} \alpha^2 T^4 \quad \text{[Kajanskie, PRD 34, 2746]} \\
\text{Rate for } \gamma \text{ production: } & \quad \sim T^4 \\
\end{align*} \]

$\Rightarrow \gamma, e^+ e^-$ good kinematics; escape without reinteraction.

![Graph showing $dN/d^4x$ for different processes]

- Prediction:
  - Yield may be low

- There may be a way out (in principle):
  - So far, expansion neglected:

\[ \text{New: } dN(p) \sim \int d^2x_1 \theta(x) \int dy \theta(y) \]

\[ \uparrow \text{rate long. scaling} \]
Remember: scaling hypothesis:

\[ \frac{dN(\pi)}{dy} = \frac{1}{2} a_1 T^2 \sim \frac{1}{2} a_1 T^2 \]

\[ \Rightarrow \frac{d^2 T}{dTdy} \sim \frac{dN(\pi)}{dy} T^{-3} \sim \frac{dN(\pi)}{dy} T^{-4} \]

Therefore:

\[ \int R d\tau dy \sim \left( \frac{dN(\pi)}{dy} \right)^2 \frac{dT}{T^2} \]

i.e.

\[ \not= \frac{d\sigma(p+p)}{dy} \sim \left[ \frac{dN(\pi)}{dy} \right]^2 \]

or

\[ \frac{d\sigma(p+p)}{dy} \sim \frac{dN(\pi)}{dy} T_c \]

\[ \Theta \Theta \]

- for hard $\pi^+$ production: yield $\sim F_2(x_1) F_2(x_2)$
- structure fem

for thermal $\pi^+$ production: yield $\sim S \cdot S$

\[ \Theta \Theta \]

The normalized $\pi^+$ yield should give a handle on $T_c$, $dN(\pi)/dy$ should yield $\frac{T_c}{\sqrt{s}}$!

Comments:
- in pp collisions at 158
  low mass events depend on $\left[ \frac{dN(\pi)}{dy} \right]^2$
  "AFS, Åkesson, prelim. / Åkeren, Phys. Lett. 192B, 463"
- Potential danger from semileptonic decay of charm:
  $D_s \overline{D}_s$ events may give very large
  $e^-$-meson from $D_s \overline{D}_s$ Fäkeler, Vig, Z. Phys C 10, 159]
  Rate $\Rightarrow$
- $\gamma^* \rightarrow e^-$ unpolarized [Kuyzer, CERN-TH 459/85]
2nd lecture: \( \langle p_T \rangle \propto T \) in rest frame

\[ \langle p_T \rangle \]

This is neglecting long/radial expansion.

\[ T(T) \text{ determined by } T_c, T_c \text{ and type of phase tran.} \]

\[ \text{i.e. } T(T) \text{ cooling } (T(T) \text{ flow } \rightarrow p_T) \]

As a consequence measured \( \langle p_T \rangle \) cannot easily be related to any \( T \), \( \langle p_T \rangle \) reflects whole history.

\( \langle p_T \rangle \) vs \( dN/dy \) depends on details of hydrodynamical treatment:

Kataya, PRD 39, 2755

Wang, PRD 35, 2404

\[ \text{• Still: large } dN/dy \text{ should receive larger } p_T \text{ contribution from flow (flow } \rightarrow \text{ velocity).} \]
Guidance from experiment needed

**D + Au 200 GeV/A**

NA 35 Preliminary $n^-$ negatives

$1 < y < 3.8$

$(p_T) = 320 \pm 20$ MeV/c

**Explanation?**

$1/\text{n} \, \frac{dN}{dp_T}$ (arb. units)

**SFH/15R: Bell 2.6pnp. C27, 197**

**NA34**

Preliminary Data

$200$ GeV/n $1.0 < y < 2.0$

$\frac{1}{p} \rightarrow w$

$^{16}O \rightarrow w$

$1/\text{p} \, \frac{dN}{dp_T}$ (GeV$^{-2}$) arbit. units

**p$_T$** [GeV/c]

$0 \rightarrow 1.0$

$1/\text{n} \, \frac{dN}{dp_T}$ (arb. units)

$0 \rightarrow 5$

$p_T$ (GeV/c)

$0 \rightarrow 1.6$
Chemical equilibrium: 

- **Charge:** we probably to heavy 

- **Strangeons:** From $g$ to hadrons:
  - $g, \bar{g}, g \rightarrow s, \bar{s}$ (chondic equation)
  - equilibration & expansion
  - hadronization:
    - recombination and/or fragmentation
    - changes flavor content
  - approach to equilibrium in the hadron phase

Predictions have to be done including all these steps and also for a scenario without QCP formation:

- **Conclusion** (probably): [Koch, Phys. Rep. 142] 

  $K_{\pi}$ not the most sensitive quantity to QCP any way: $K^* \rightarrow K_{\pi}$ 'covers' $K_{\pi}$  
  from $\lambda = \frac{\# s}{\# g}$

$\bar{\Lambda}, \Xi, \Omega$ probably best signatures, especially for $\mu \rightarrow 0$

*caution*: $s$-retention in QCP $\rightarrow$ strange matter?

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Experimentally via polarization: Jacob, Phys. Lett. 190B, 173
**Very conservative estimate:**

- includes: o gluons
- slow long, expansion
- assumes chem.

Equilibration of hadron phase:

Thus, $g$ for no CQGP overestimated $10^3$.

**AFS/ISR**

- $p + \bar{p}$ $E_T = 30$ GeV
- $p + \bar{p}$ $E_T = 40$ GeV
- $x \times \sqrt{s}$ $E_T = 30$ GeV
$^{16}O$ Am
200 GeV/c
NA 35
π⁺π⁻

$M_{\pi\pi}$ [GeV]
NA36 TPC  CENTRAL  RUN  568
             TAPE  3722
             DATE  12 Oct 87
             TIME  01 01 05
             EVENT  22940

Top view

on-line

drift

time slice pointing
to target position

target

beam

A36 TPC  CENTRAL  RUN  568
             TAPE  3722
             DATE  12 Oct 87
             TIME  01 01 05
             EVENT  22940

Side view

selected time slice

target

beam
If Debye screening radius > radius of particle

\[ \Rightarrow \text{no building: suppression} \]

- cannot reappear at
  hadronization: \[ c + \bar{c} \rightarrow J/\psi \text{ since } \gamma \text{ small} \]

- \[ J/\psi \] not absorbed in nuclear matter around QGP: \[ S_{NP} \sim 1 \text{mb} \] [Schoeck PR157,3008]

- Cannot be formed abundantly before QGP \( (\tau \sim 1 \text{ fm/c, too slow}) \)

- However, if \( c\bar{c} \) formed in QGP
  \[ \begin{array}{ccc}
  9 \\
  \bar{9} \\
  \hline
  m \leftrightarrow c \\
  \bar{m} \leftrightarrow \bar{c}
  \end{array} \]

  have to move into the correct relative position: \( J/\psi \)

\[ \Rightarrow \text{if } \bar{p} (c + \bar{c}) \text{ small:} \]

\[ \frac{1}{J/\psi} \text{ reached inside QGP but no building} \]

\[ \text{if } \bar{p} (c + \bar{c}) \text{ large (large } x_F / p_T) : \]

\[ \frac{1}{J/\psi} \text{ reached outside QGP: building less suppression} \]

[ Karch, CERN-TH 4699/87]
$E_T^0 < 28$ GeV (no QGP?)

$E_T^0 > 50$ GeV (QGP?)

**Left Panel:**
- $p_T(J)$
- $M_{μμ}$ vs. $N_{μμ}$
- $N_{μμ} = 9.325 \pm 0.594$
- $N_{μμ} = 0.008 \pm 0.004$
- $\chi^2/\text{ND} = 1.78$
- $M_{μμ} = 3.097 \pm 0.003$
- $σ_{μμ} = 0.160 \pm 0.003$
- $ω_{μμ} = 1.280 \pm 0.103$
- $N_{μμ} = 4745 \pm 4745$
- Total Entrees: 4745

**Right Panel:**
- $p_T(J)$
- $M_{μμ}$ vs. $N_{μμ}$
- $N_{μμ} = 5.890 \pm 0.39$
- $N_{μμ} = 0.000 \pm 0.005$
- $\chi^2/\text{ND} = 0.97$
- $M_{μμ} = 3.097 \pm 0.003$
- $σ_{μμ} = 0.160 \pm 0.003$
- $ω_{μμ} = 1.353 \pm 0.112$
- $N_{μμ} = 2751.4 \pm 2751.4$
- Total Entrees: 2751.4
For events with $2.7 < \eta < 3.5$ GeV/c:

$$dR/dP_t = a \cdot \beta P_t$$

where:
- $a = 0.66 \pm 0.06$
- $\beta = 0.31 \pm 0.06$

$\chi^2/\text{NDF} = 0.43$

For $E_T \leq 28$ GeV, $p_T(\psi) \leq 1$ GeV:

- $w = 3.097 \pm 0.000$
- $c = 0.150 \pm 0.000$
- $b = 1.011 \pm 0.095$
- $N/N_0 = 8.68 \pm 0.785$
- $N/N_0 = 0.018 \pm 0.006$
- $\chi^2/\text{NDF} = 1.64$

For $E_T > 50$ GeV, $p_T(\psi) < 1$ GeV:

- $w = 3.097 \pm 0.000$
- $c = 0.150 \pm 0.000$
- $b = 1.326 \pm 0.163$
- $N/N_0 = 4.628 \pm 0.482$
- $N/N_0 = 0.001 \pm 0.019$
- $\chi^2/\text{NDF} = 0.50$

MAXIMUM LIKELIHOOD

$R = \frac{\psi(h; E_T)}{\psi(k; E_T)}$

Graph showing suppression at $E_T$ and $p_T(\psi)$.
no evidence for suppression
Other potential signatures of deconfinement

- In case of recombination:
  - $q$ and $\bar{q}$ recombine randomly
  - $\Rightarrow$ no $\pi^+\pi^-$ short range correlations

- At ISR: high $p_T$ production via 'valence dip'

\[
p^+ \rightarrow p^+ p^+ \quad \frac{\text{# of } p}{p_T^2} \sim 5 \quad p_T \gg 4 \text{ GeV}
\]

\[\text{Cronin, Phys. Lett. 47B, 1973}\]

- While propagating through QGP, $p_T$ remains:\
  \[
  \Rightarrow \frac{p_T}{\text{GeV}} \gg 1 \text{ GeV}
  \]

2. Mean free path:

- Informative to have $L_q, L_{\bar{q}}$ in QGP

- If hard parton scattering at $t \approx 0$

\[
N \rightarrow E \rightarrow N
\]

\[
\text{QGP}
\]

\[
\text{Jets} \leftarrow \text{Jet}_1 \rightarrow \text{Jet}_2
\]

- Jets non-collinear
- Jets may get stopped
- Maybe one can flavor by the jets: $L_q$ vs $L_{\bar{q}}$

[Geist, CERN-EP/85-192]
Fluctuations

explosive hadronization of QGP bubbles

→ rep. of hadrons close to rep. of bubble

→ fluctuations in $\frac{dN}{dy}$ of width $\Delta y = 1$

---

$\frac{dN_{ch}}{dy} \text{ c.m.s.}$

Jacce

$\text{Si + Ag Br}$

$N_{ch} = 1010$

---

Look into Future

RHIC at BNL: ∼1995

\[ \frac{dN_{ch}}{dy} \approx 2.5 \times 200 = 500 \text{ at } y=0 \]

\[ \frac{dN_{ch}}{dy} \text{ (central)} \geq 3 \times 500 = 1500 \text{ at } y=0 \]

Compare:

\[ \text{pp: } \langle n_{ch} \rangle \approx 22 \]

\[ \text{LHC: } \langle n_{ch} \rangle \approx 50 \text{ (?)} \]

\[ e^+e^-2\text{TeV: } \langle n_{ch} \rangle \approx 70 \text{ (?)} \]

From disussion of signatures:

- tracking
- lepton pairs
- calorimetry

Comment: 

- Tracking: extremely ambitious.

- Calorimetry: general flow measurements and \( d \rho / d y \) fluctuations.

  ⇒ jet spectroscopy

  ⇒ "Kuai calorimeters"
PHOTON & HADRON SPECTROMETER FOR RHIC

IRONYOKE

HADRONIC CALORIMETER

TRACKING CHAMBER

ELECTROMAGN.CAL.
Conclusions

- Search for UCP of fundamental interest
  (also cosmology ...)
- Theoretical & experimental work
  very challenging but feasible
- NA38 result seems to contradict
- Good pp data needed (and pA)
- Experiments are well on their way
- Ideas for RHIC experiments exist

Anaxagoras: 'Phenomena are views of the Invisible'