STATUS OF $\mathcal{CP}$ AND $\mathcal{CP}T$ VIOLATION IN THE NEUTRAL KAON SYSTEM

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Abstract

A phenomenological description of the neutral-kaon system is presented without assuming $\mathcal{CP}T$ conservation. The experimental methods and the underlying assumptions used to determine parameters of the neutral-kaon system ($\mathcal{CP}$-violating and non $\mathcal{CP}$-violating ones) are discussed. The experimental results are combined to test $\mathcal{CP}T$ conservation with as little prejudice as possible.

Invited Review Talk presented at the XVI International Conference on Physics in Collision, Mexico City, June 19-21, 1996
1 Introduction

1.1 CP Violation

$CP$ violation in the microscopic world has only been observed in the neutral-kaon system [1]. There, it is mainly due to different oscillation probabilities of $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$, causing a small admixture ($|\varepsilon| \approx 2.3 \times 10^{-3}$) of the wrong $CP$ eigenstate with the long-living neutral-kaon state:

$$|K_L\rangle = |K_2\rangle + \varepsilon|K_1\rangle.$$  \hspace{1cm} (1)

The observed $CP$ violation in the neutral-kaon system could be described in the Standard Model with 3 families of quarks and leptons [2]. However, the strength of $CP$ violation needed to explain baryogenesis [3] and the absence of antimatter in the universe seems to originate beyond the Standard Model [4].

In addition to the observed $CP$ violation in the mixing of neutral kaons ($\varepsilon$), one also expects $CP$ violation in their decay amplitudes ($\varepsilon'$). The theoretical calculations have rather large uncertainties because of nonperturbative strong interaction corrections. A recent estimate [5] predicts

$$\varepsilon'/\varepsilon = \left(3.1 \pm 2.5\right) \times 10^{-4},$$

where the small value is the result of the cancellation between gluonic and electroweak penguin operators. The experimental situation is ambiguous, two experiments with a similar sensitivity measured

$$\Re\left(\frac{\varepsilon'}{\varepsilon}\right) = \left(7.4 \pm 5.9\right) \times 10^{-4} \quad \text{E731}^6$$

$$\Re\left(\frac{\varepsilon'}{\varepsilon}\right) = \left(23 \pm 6.5\right) \times 10^{-4} \quad \text{NA31}^7.$$  

The disagreement will be resolved by three new experiments (KTEV, NA48 and KLOE), aiming for a $10^{-4}$ precision which will start soon. Looking into the neutral B-system is another way to answer the question of whether $CP$ violation is indeed described by the Standard Model.

1.2 CP T Violation

$CP T$ invariance for any Quantum Field Theory is a consequence of locality and Lorentz invariance [8]. $CP T$ invariance is questioned in the context of quantum gravity [9]. String theory as the most promising ‘theory of everything’ is not local and not necessarily Lorentz invariant and thus lacks the necessary conditions for the $CP T$ theorem. From $CP T$ invariance it follows that masses and lifetimes of particles and antiparticles are the same. Although the $CP T$ theorem is a very fundamental theorem, the experimental verifications are rather poor [10]:

$$\frac{m_{e^+} - m_{e^-}}{m_e} < 4 \times 10^{-8} \quad \frac{m_{p} - m_{\bar{p}}}{m_p} < 3 \times 10^{-9} \quad \frac{m_{n} - m_{\bar{n}}}{m_n} < 2 \times 10^{-4}.$$

$$\frac{\tau_{\mu^+} - \tau_{\mu^-}}{\tau_{\mu^+}} < 2 \times 10^{-4} \quad \frac{\tau_{\pi^+} - \tau_{\pi^-}}{\tau_{\pi^+}} < 2 \times 10^{-3} \quad \frac{\tau_{K^+} - \tau_{K^-}}{\tau_{K^+}} < 2 \times 10^{-4}.$$  

The best limit on a particle–antiparticle mass difference is obtained in the neutral-kaon system:

$$\frac{m_{\bar{K}^0} - m_{K^0}}{m_K} < 7 \times 10^{-10},$$

1
which approaches the interesting region of \(m_K/m_{\text{PL}}\), where quantum gravitational effects might be expected. However, the limit is based on assumptions about \(\mathcal{CPT}\) conservation in decay amplitudes. It increases by one order of magnitude if no assumptions about \(\mathcal{CPT}\) conservation are made. The calculations and the relevant measurements are discussed in this paper.

## 2 Neutral Kaon Phenomenology

Many different descriptions of the neutral-kaon system exist in the literature [11]. The basic concept is repeated here with specific emphasis on \(\mathcal{CPT}\) violation in order to have a consistent description for evaluating the experimental data in the later sections. No specific phase convention is applied, which makes it easier to recognize measurable parameters. Some published experimental results of parameters of the neutral-kaon system are based on assumptions about \(\mathcal{CPT}\) conservation or the observance of the \(\Delta S = \Delta Q\) rule. This has to be taken into account when testing \(\mathcal{CPT}\) conservation.

### 2.1 Time Evolution

In the absence of any strangeness-violating interaction, the stationary states, \(|K^0\rangle\) and \(|\bar{K}^0\rangle\), of a \(K^0\)-meson and \(\bar{K}^0\)-meson respectively, are mass eigenstates of the strong and electromagnetic interactions:

\[
(H_{\text{st}} + H_{\text{em}})|K^0\rangle = m_0|K^0\rangle \quad (H_{\text{st}} + H_{\text{em}})|\bar{K}^0\rangle = m_0|\bar{K}^0\rangle.
\]

Since the strangeness-violating interaction \(H_{\text{weak}}\) is much weaker than the strong and electromagnetic interaction, perturbation theory can be applied (Wigner–Weisskopf approach [12]). The time evolution of the neutral kaon wave function is then described by the following differential equation:

\[
i \frac{\partial}{\partial \tau} \begin{pmatrix} K^0(\tau) \\ \bar{K}^0(\tau) \end{pmatrix} = \Lambda \begin{pmatrix} K^0(\tau) \\ \bar{K}^0(\tau) \end{pmatrix} = (M - \frac{i}{2} \Gamma) \begin{pmatrix} K^0(\tau) \\ \bar{K}^0(\tau) \end{pmatrix},
\]

where the \(\bar{K}^0\)–\(K^0\) mixing matrix \(\Lambda\) can be split into two Hermitian matrices \(M\) and \(\Gamma\), called mass and decay matrices respectively. The matrix elements of \(\Lambda\) are given by:

\[
\Lambda_{ij} = m_0 \delta_{ij} + \langle i|H_{\text{weak}}|j\rangle + \sum_f \mathcal{P} \left( \frac{\langle i|H_{\text{weak}}|f\rangle \langle f|H_{\text{weak}}|j\rangle}{m_0 - E_f} \right) - i \pi \sum_f \langle i|H_{\text{weak}}|f\rangle \langle f|H_{\text{weak}}|j\rangle \delta(m_0 - E_f),
\]

where \(\mathcal{P}\) stands for the principal part and the indices \(i, j = 1\) and \(i, j = 2\) correspond to \(K^0\) and \(\bar{K}^0\) respectively. They can be calculated within the Standard Model, although with large uncertainties because of non-perturbative effects. Nevertheless, \(\mathcal{CP}\) violation in the \(K^0–\bar{K}^0\) system is an important input for constraining the CKM-matrix [13]. Since no direct \(K^0–\bar{K}^0\) transition exists within the Standard Model, the second term of Eq. 4 vanishes for ordinary weak interaction. We use the following parametrization of \(\Lambda\) with 8 real and positive parameters:

\[
\Lambda = \begin{pmatrix} m_{K^0} & m_{12} e^{i\phi_M} \\ m_{12} e^{-i\phi_M} & m_{\bar{K}^0} \end{pmatrix} - \frac{i}{2} \begin{pmatrix} \Gamma_{K^0} & \Gamma_{12} e^{i\phi_T} \\ \Gamma_{12} e^{-i\phi_T} & \Gamma_{\bar{K}^0} \end{pmatrix},
\]

where \(m_{K^0}\) and \(m_{\bar{K}^0}\) are equal to the ‘masses’, and \(1/\Gamma_{K^0}\) and \(1/\Gamma_{\bar{K}^0}\) to the ‘lifetimes’ of the \(K^0\) and \(\bar{K}^0\) states respectively. The relative phase between \(|K^0\rangle\) and \(|\bar{K}^0\rangle\) cannot be observed, which is the basis of the phase convention used by many authors.
The time evolution of initially pure $K^0$ and $\bar{K}^0$ states is given by

$$|K^0(\tau)\rangle = \left[f_+(\tau) + \frac{\Lambda_{22} - \Lambda_{11}}{\Delta \lambda} f_-(\tau)\right] |K^0\rangle - 2 \frac{\Lambda_{21}}{\Delta \lambda} f_-(\tau) |\bar{K}^0\rangle$$

$$|\bar{K}^0(\tau)\rangle = \left[f_+(\tau) - \frac{\Lambda_{22} - \Lambda_{11}}{\Delta \lambda} f_-(\tau)\right] |\bar{K}^0\rangle - 2 \frac{\Lambda_{12}}{\Delta \lambda} f_-(\tau) |K^0\rangle$$

with

$$f_\pm(\tau) = \frac{e^{-i\lambda_\pm}}{2}$$

$$\lambda_{L,S} = m_{L,S} - \frac{i}{2} \Gamma_{L,S} = \frac{\Lambda_{11} + \Lambda_{22}}{2} \pm \sqrt{\frac{(\Lambda_{22} - \Lambda_{11})^2}{4} + \Lambda_{12} \Lambda_{21}}$$

$$\Delta \lambda = \lambda_L - \lambda_S = \sqrt{(\Lambda_{22} - \Lambda_{11})^2 + 4 \Lambda_{12} \Lambda_{21}}$$

where $\lambda_S$ and $\lambda_L$ are the eigenvalues of the matrix $\Lambda$. The corresponding eigenvectors are given by:

$$|K_S\rangle = \frac{e^{i \varphi_S}}{\sqrt{1 + r_S^2}} \left(r_S |K^0\rangle + |\bar{K}^0\rangle\right)$$

$$|K_L\rangle = \frac{e^{i \varphi_L}}{\sqrt{1 + r_L^2}} \left(r_L |K^0\rangle + |\bar{K}^0\rangle\right)$$

with arbitrary phases $\varphi_S, \varphi_L$ and

$$r_S = \frac{2 \Lambda_{12}}{\Lambda_{22} - \Lambda_{11} - \Delta \lambda}, \quad r_L = \frac{2 \Lambda_{12}}{\Lambda_{22} - \Lambda_{11} + \Delta \lambda}.$$

### 2.2 Discrete Symmetries

$CP$ and $CP\mathcal{T}$ transformations change a stationary $K^0$ state into a $\bar{K}^0$ state and vice versa, whereas a $\mathcal{T}$ transformation does not alter the states except for an arbitrary phase:

$$CP|K^0\rangle = e^{i \phi_{CP}} |\bar{K}^0\rangle \quad CP|\bar{K}^0\rangle = e^{-i \phi_{CP}} |K^0\rangle$$

$$\mathcal{T}|K^0\rangle = e^{i \phi_{\mathcal{T}}}|K^0\rangle \quad \mathcal{T}|\bar{K}^0\rangle = e^{i \phi_{\mathcal{T}}}|\bar{K}^0\rangle$$

$$CP \mathcal{T}|K^0\rangle = e^{i (\phi_{CP} + \phi_{\mathcal{T}})} |K^0\rangle \quad CP \mathcal{T}|\bar{K}^0\rangle = e^{i (-\phi_{CP} + \phi_{\mathcal{T}})} |\bar{K}^0\rangle$$

By requiring $CP \mathcal{T}|K^0\rangle = \mathcal{T} CP|K^0\rangle$, it follows for the phases $\phi_t$ that

$$2 \phi_{CP} = \varphi_{\mathcal{T}} - \phi_{\mathcal{T}}.$$  

If $\Lambda$ is invariant under $\mathcal{T}$, $CP \mathcal{T}$ or $CP$ transformations, the following conditions must be satisfied:

$$\mathcal{T} : |\Lambda_{12}| = |\Lambda_{21}|$$

$$CP \mathcal{T} : \Lambda_{11} = \Lambda_{22}$$

$$CP : |\Lambda_{12}| = |\Lambda_{21}|$$

It is convenient to introduce the following $\mathcal{T}$ and $CP \mathcal{T}$ violation parameters:

$$\varepsilon_{\mathcal{T}} = \sin (\varphi_{SW}) \frac{|\Lambda_{12}|^2 - |\Lambda_{21}|^2}{\Delta \Gamma \Delta m} e^{i \varphi_{SW}}$$

$$\delta = \cos (\varphi_{SW}) \frac{\Lambda_{22} - \Lambda_{11}}{\Delta \Gamma} e^{i (\varphi_{SW} + \pi/2)}$$
with \( \varphi_{SW} = \arctan(2\Delta m/\Delta \Gamma) \). The lifetime difference \(^1\) \( \Delta \Gamma \equiv \Gamma_S - \Gamma_L \) is found to be about twice the mass difference \( \Delta m \equiv m_L - m_S \) and therefore \( \varphi_{SW} \approx 45^\circ \). Assuming small \( T \) and \( CPT \) violation, the time evolution of initially-pure strangeness states can be rewritten as

\[
|K^0(\tau)| = |f_+(\tau) - 2\delta f_-(\tau)| |K^0\rangle + (1 - 2\varepsilon_T) e^{-i\varepsilon_T} f_-(\tau)|\bar{K}^0\rangle
\]

\[
|\bar{K}^0(\tau)| = |f_+(\tau) + 2\delta f_-(\tau)| |\bar{K}^0\rangle + (1 + 2\varepsilon_T) e^{i\varepsilon_T} f_-(\tau)|K^0\rangle .
\]

2.3 Direct Measurements of the \( T \) and \( CPT \) Violation Parameters

Amplitudes which violate \( \Delta S = \Delta Q \) are expected to be very small within the Standard Model. Semileptonic decays might therefore be used to tag the strangeness of the neutral kaon at the time of its decay. The decay-rate asymmetry between a \( K^0 \) decaying as \( \bar{K}^0 \) and \( T \)-conjugated process (\( \bar{K}^0 \) decaying as \( K^0 \)) can be measured if we also know the initial strangeness. A similar asymmetry is constructed between \( CPT \)-conjugated processes. The CPLEAR experiment reports preliminary measurements of such time-dependent decay-rate asymmetries [14]:

\[
A_T(\tau) = \frac{R(\bar{K}^0 \to K^0)(\tau) - R(K^0 \to \bar{K}^0)(\tau)}{R(\bar{K}^0 \to K^0)(\tau) + R(K^0 \to \bar{K}^0)(\tau)} = \frac{|\Lambda_{21}|^2 - |\Lambda_{12}|^2}{|\Lambda_{12}|^2 + |\Lambda_{21}|^2} = 4\Re(\varepsilon_T)
\]  
\[
= (6.3 \pm 2.8) \times 10^{-3}
\]  

and

\[
A_{CPT}(\tau) = \frac{R(\bar{K}^0 \to K^0)(\tau) - R(K^0 \to \bar{K}^0)(\tau)}{R(\bar{K}^0 \to K^0)(\tau) + R(K^0 \to \bar{K}^0)(\tau)} \approx -4\Re(\delta) \quad \text{for } \tau \gg \tau_S
\]  
\[
= (0.3 \pm 2.8) \times 10^{-3}.
\]

The data is shown in Fig. 2. Previous experiments measured with a much better precision the semileptonic charge asymmetry in decays of the \( K_L \) state [10]:

\[
\hat{\delta}_L = \frac{R(K_L \to \pi^- l^+ \nu) - R(K_L \to \pi^+ l^- \bar{\nu})}{R(K_L \to \pi^- l^+ \nu) + R(K_L \to \pi^+ l^- \bar{\nu})} = 2\Re(\varepsilon_T + \delta)
\]  
\[
= (3.27 \pm 0.12) \times 10^{-3}.
\]

However, this measurement alone cannot determine whether the observed \( CPT \) violation is the result of \( CPT \) or \( T \) violation. More precise information about possible \( CPT \) violation in the neutral-kaon system is obtained by analysing hadronic decays.

In a search for a \( CPT \)-violating interaction, one cannot assume that this new interaction obeys the \( \Delta S = \Delta Q \) rule. Additional contributions to the neutral kaon decay rates, discussed in the following section, have to be taken into account [16].

\(^1\) The definition of \( \Delta m \) and \( \Delta \Gamma \) is based on the experimental result that \( m_L > m_S \) but \( \Gamma_L < \Gamma_S \)
Figure 2: Time-dependent decay-rate asymmetries measured by the CPLEAR collaboration.

2.4 \( \mathcal{CP} \) Violation in \( K \to \pi l \nu \) Decays

The following definitions are used for the \( \Delta S = \Delta Q \) allowed transitions:

\[
B_+ \equiv \frac{1}{2} \int d\Omega \sum_{\nu} |\langle \pi^- l^+ \nu | T | K^0 \rangle|^2, \quad B_- \equiv \frac{1}{2} \int d\Omega \sum_{\nu} |\langle \pi^+ l^- \bar{\nu} | T | K^0 \rangle|^2, \quad y \equiv \frac{B_+ - B_-}{B_+ + B_-},
\]

where the sum is over all spin (\( \tilde{s} \)) states and \( T \) stands for the T-matrix. For the \( \Delta S = \Delta Q \) forbidden transitions,

\[
\bar{x} \equiv \int d\Omega \sum_{\nu} e^{-i\nu T} \langle \pi^- l^+ \nu | T | \bar{K}^0 \rangle \langle \pi^- l^+ \nu | T | K^0 \rangle / B_+ \quad \text{and} \quad x \equiv \int d\Omega \sum_{\nu} e^{-i\nu T} \langle \pi^+ l^- \bar{\nu} | T | \bar{K}^0 \rangle \langle \pi^+ l^- \bar{\nu} | T | K^0 \rangle / B_- \quad \text{are used.}
\]

The four semileptonic decay rates of initially pure \( K^0 \) and \( \bar{K}^0 \) states are given in first order of the \( \mathcal{CP} \)- and \( \Delta S = \Delta Q \)-violation parameters by

\[
R_{K^0 \to \pi^+ \nu} (\tau) = B_+ \left\{ E_+ (\tau) - E_- (\tau) \Re (4\delta - 2\pi) + e^{-T\tau} \{ \cos(\Delta m\tau) + \sin(\Delta m\tau) \Im (4\delta - 2\pi) \} \right\} \tag{19}
\]

\[
R_{\bar{K}^0 \to \pi^- \bar{\nu}} (\tau) = B_- \left\{ E_+ (\tau) + E_- (\tau) \Re (4\delta + 2\pi) + e^{-T\tau} \{ \cos(\Delta m\tau) - \sin(\Delta m\tau) \Im (4\delta - 2\pi) \} \right\} \tag{20}
\]

\[
R_{K^0 \to \pi^- \bar{\nu}} (\tau) = B_- \left\{ E_+ (\tau) [1 - 4\Re(\varepsilon_T)] + E_- (\tau) \Re (2\pi) - e^{-T\tau} \{ \cos(\Delta m\tau) [1 - 4\Re(\varepsilon_T)] + \sin(\Delta m\tau) \Im (2\pi) \} \right\} \tag{21}
\]

\[
R_{\bar{K}^0 \to \pi^+ \nu} (\tau) = B_+ \left\{ E_+ (\tau) [1 + 4\Re(\varepsilon_T)] + E_- (\tau) \Re (2\pi) - e^{-T\tau} \{ \cos(\Delta m\tau) [1 + 4\Re(\varepsilon_T)] - \sin(\Delta m\tau) \Im (2\pi) \} \right\} \tag{22}
\]
The previously defined semileptonic decay-rate asymmetries become:

\[ A_{\text{CP}} = 2 \frac{E_-(\tau) [2\Re(\delta) - \Re(x_-)] + e^{-\tau_s} \sin(\Delta m\tau) [\Im(x_+) - 2\Im(\delta)]}{E_+(\tau) + e^{-\tau_s} \cos(\Delta m\tau)} - y \]

\[ = -4\Re(\delta) + 2\Re(x_-) - y \quad \text{for} \ \tau \gg \tau_s \]  

\[ A_T = 4\Re(\varepsilon_T) + y + 2 \frac{E_-(\tau)\Re(x_-) + e^{-\tau_s} \sin(\Delta m\tau)\Im(x_+)}{E_+(\tau) - e^{-\tau_s} \cos(\Delta m\tau)} \]

\[ = 4\Re(\varepsilon_T) - 2\Re(x_-) + y \quad \text{for} \ \tau \gg \tau_s \]  

\[ \delta_i = 2\Re(\varepsilon_T) + 2\Re(\delta) - 2\Re(x_-) + y . \]  

Measurements at short decay times are important for separating violations of the \( \Delta S = \Delta Q \) rule from \( \text{CPT} \) violation. The present limits of a violation of the \( \Delta S = \Delta Q \) rule [10] were all obtained assuming \( \text{CPT} \) conservation. New information will be added from the CPLEAR experiment.

### 2.5 \( \text{CP} \) Violation in \( K \to 2\pi \) Decays

Two \( \text{CP} \) eigenstates of a neutral kaon exist with the following properties: \( \text{CP}|K_1\rangle = +|K_1\rangle \) and \( \text{CP}|K_2\rangle = -|K_2\rangle \). The most favoured decay of a neutral kaon has the property \( \text{CP}|\pi \pi\rangle = +|\pi \pi\rangle \). If \( \text{CP} \) is conserved, one expects a fast \( (K_1) \) and a slow \( (K_2) \) decaying component because of the limited phase space (the other major decay channels have 3 particles). Indeed, it is found experimentally that \( \tau_L = 577 \times \tau_s \), indicating that \( \text{CP} \) violation in the neutral-kaon system is small. It is specific to the neutral-kaon system that \( \text{CP} \) is violated if decays into \( 2\pi \) are observed after decay times \( \tau \gg \tau_s \). The decay rates of initially pure \( K^0 \) and \( \bar{K}^0 \) mesons into \( 2\pi \) are given by

\[ R_{K^0 \to \pi \pi}(\tau) = B \left[ e^{-\tau/\tau_s} + |\eta_{\pi \pi}| e^{-\tau_s/\tau_L} + 2|\eta_{\pi \pi}| e^{-\tau_s} \cos(\Delta m\tau - \varphi_{\pi \pi}) \right] \]

\[ R_{\bar{K}^0 \to \pi \pi}(\tau) = \bar{B} \left[ e^{-\tau/\tau_s} + |\bar{\eta}_{\pi \pi}| e^{-\tau_s/\tau_L} + 2|\bar{\eta}_{\pi \pi}| e^{-\tau_s} \cos(\Delta m\tau - \varphi_{\pi \pi}) \right] \]  

with

\[ \eta_{\pi \pi} = -\frac{\langle \pi \pi |T|K_L\rangle}{\langle \pi \pi |T|K_S\rangle} \sqrt{1 + |r_L|^2} e^{i(\varphi_{\pi \pi} - \varphi_L)} , \quad \bar{\eta}_{\pi \pi} = -\frac{r_S}{r_L} \eta_{\pi \pi} \]

\[ B = |\langle \pi \pi |T|K_S\rangle|^2 \frac{1 + |r_L|^2}{|r_S - r_L|^2} , \quad \bar{B} = |r_L|^2 B . \]  

Within the limits of small \( \text{CP} \), \( \mathcal{T} \) and \( \text{CPT} \) violation

\[ \eta_{\pi \pi} = \bar{\eta}_{\pi \pi} , \quad \bar{B}/B = [1 + 4\Re(\varepsilon_T + \delta)] . \]  

The \( \text{CP} \)-violation parameter \( \eta_{\pi \pi} \) can be expressed through the isospin \( I = 0 \) and \( I = 2 \) decay amplitudes:

\[ A_{+-} = \sqrt{\frac{2}{3}} A_2 + \sqrt{\frac{2}{3}} A_0 \quad A_{00} = \sqrt{\frac{2}{3}} A_2 - \sqrt{\frac{1}{3}} A_0 . \]

It follows mainly from the measured ratio of the partial decay widths of \( \Gamma(K_S \to \pi^+\pi^-)/\Gamma(K_S \to \pi^0\pi^0) \approx 2 \) that the \( I = 2 \) amplitude is small \( (|A_2/A_0| \approx 0.045) \) [17]. Allowing for \( \text{CPT} \) violation in the decay amplitudes, it is common to write [18]

\[ A_I = (a_I + b_I) e^{i\delta_I} \quad \bar{A}_I = (a_I^* - b_I^*) e^{i\delta_I} e^{i(\phi_{\text{CP}} - \delta_T)} \]  

\[ (29) \]
Figure 3: Schematic representation in the complex plane of the relation between the \(CP\) and \(CP\, T\) violation parameter in the \(2\pi\) decay mode. Relative sizes are not in scale.

where \(a_I\) are the weak decay amplitudes, \(\delta_I\) the strong-interaction phase shifts and \(b_I\) possible \(CP\, T\)-violating amplitudes. The \(CP\)-Violation parameters in the \(2\pi\) decay mode are then given by

\[
\eta_{+-} = \varepsilon + \varepsilon' \quad \text{and} \quad \eta_{00} = \varepsilon - 2\varepsilon',
\]

with \(\varepsilon = \varepsilon_T + \delta + i \Delta \phi + \Delta A\). The relations between the different \(CP\)-violation parameters are illustrated in Fig. 3. In addition to \(CP\) violation in mixing we have:

- \(CP\) violation because of the interference between the \(2\pi\) decay and mixing amplitudes:

\[
\Delta \phi = \frac{1}{2} \left[ \varphi_T - \arg(A_0^* A_0) \right]
\]

(31)

- \(CP\) violation because of the interference between the \(A_0\) and \(A_2\) amplitudes:

\[
\varepsilon' = \frac{1}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \left[ i \Im \left( \frac{a_2}{a_0} \right) + \Re \left( \frac{a_2}{a_0} \right) \left\{ \Re \left( \frac{b_2}{a_2} \right) - \Re \left( \frac{b_0}{a_0} \right) \right\} \right].
\]

(32)

Since experimentally [19], \(\delta_2 - \delta_0 = -42^\circ \pm 4^\circ\), it follows that

- \(\varepsilon' \parallel \varepsilon_T\) is \(CP\, T\) conserving,
- \(\varepsilon' \perp \varepsilon_T\) is \(CP\, T\) violating.

- \(CP\, T\) violation in the \(I = 0\) amplitude: \(\Delta A = \Re \left( \frac{b_0}{a_0} \right)\)

2.6 \(CP\) Violation in \(K \to 3\pi\) Decays

The decay rates of \(K^0\) and \(\bar{K}^0\) into \(3\pi\) are given by

\[
\begin{align*}
R_{K^0 \to 3\pi}(\tau) & = D \left[ e^{-\tau/\tau_L} + \frac{1}{3\pi} 2^{e^{-\tau/\tau_L}} + 2 |\eta_{3\pi}| e^{-\Gamma_\tau \cos(\Delta m_\tau + \varphi_{3\pi})} \right] \\
R_{\bar{K}^0 \to 3\pi}(\tau) & = \bar{D} \left[ e^{-\tau/\tau_L} + \frac{1}{3\pi} 2^{e^{-\tau/\tau_L}} - 2 |\eta_{3\pi}| e^{-\Gamma_\tau \cos(\Delta m_\tau + \varphi_{3\pi})} \right].
\end{align*}
\]

(33)
Within the limits of small $\mathcal{CP}$ violation,

\[
\mathcal{D}/D = [1 + 4\mathcal{R}(\varepsilon_T - \delta)] \quad \left| \pi_{3\pi} \right|^2 = \left| \kappa_{3\pi} \right|^2 = \left| \frac{\int d\Omega \langle 3\pi | T | K_S \rangle^2}{\int d\Omega \langle 3\pi | T | K_L \rangle^2} \right|^2. 
\]

\[
\eta_{3\pi} = \eta_{3\pi} = -e^{i(\varphi_L - \varphi_S)} \left| \frac{\int d\Omega \langle 3\pi | T | K_S \rangle \langle 3\pi | T | K_L \rangle^*}{\int d\Omega \langle 3\pi | T | K_L \rangle^2} \right|^2. \tag{34}
\]

The decay $K_S \to 3\pi^0$ is always $\mathcal{CP}$ violating, since the $\mathcal{CP}$ eigenvalue of the $3\pi^0$ state is $-1$ ($I = 1$). In contrast, the decay $K_S \to \pi^+\pi^-\pi^0$ can proceed through the $\mathcal{CP}$-allowed [20] but angular-momentum-suppressed mode ($I = 0$ and angular momentum $l$ of $\pi^+\pi^-$ equals 1) or through the $\mathcal{CP}$-forbidden mode ($I = 1$ and $l = 0$). Other decay modes are either much suppressed by isospin arguments or forbidden by Bose statistics [21]. The $\mathcal{CP}$ conserving part of decay amplitude cancels in the numerator when integrating over the phase space (eq. 34). Only the $I = 1$ amplitude contributes to $\mathcal{CP}$ violation and therefore the $\eta_{000}$ and $\eta_{++-0}$ are expected to be equal:

\[
\eta_{000} = \eta_{++-0} = \varepsilon_T - \delta + \mathcal{R} \left( \frac{b_1}{a_1} \right) + i\Delta \phi_{3\pi}. \tag{35}
\]

An interference between the decay and mixing amplitudes or $\mathcal{CP}T$ violation in the $I = 1$ amplitude contribute in addition to $\mathcal{CP}$ violation from the neutral-kaon mixing to the $\mathcal{CP}$-violation parameter of the $3\pi$ decay mode.

### 2.7 Phase Difference between $\Gamma_{12}$ and the $I = 0$ Amplitude

According to Eq. 4, the matrix element $\Gamma_{12}$ is given by a sum over all final states common to $K^0$ and $\bar{K}^0$. Using the main contributions,

\[
\Gamma_{12} e^{i\varphi_T} = A_0^* A_0 + A_2^* A_2 + \int d\Omega A_{\pi\pi\pi}^* A_{\pi\pi\pi} + B_{\pi\pi} + B_{\pi x} + B_x + \ldots
\]

\[
+ \int d\Omega [A_{\pi^+\pi^-\pi^0}^* A_{\pi^+\pi^-\pi^0} + A_{\pi^0\pi^0\pi^0}^* A_{\pi^0\pi^0\pi^0} + \ldots] \tag{36}
\]

The amplitude $A_{\pi\pi\pi}$ is small [10]: $\text{BR}(K_S \to \pi^+\pi^-\gamma) \approx 2 \times 10^{-3}$. Its phase [22] is close to that of the $A_0$ amplitude because $\varphi_{\pi\pi\pi} = 43.6^\circ \pm 3.9^\circ$, and therefore its contribution to a phase difference between $\Gamma_{12}$ and $A_0$ can be neglected. For the rest we can write:

\[
\frac{1}{2} [\varphi_T - \arg(A_0^* A_0)] = \left| \frac{A_2}{A_0} \right|^2 \arg \left( \frac{A_2}{A_0} \right) + \frac{\Gamma_1}{\Gamma_3} \left[ 4\text{BR}(K_L \to l^+\pi^-\nu) \times \Im(x_+) \\
- \text{BR}(K_L \to 3\pi) \times \Im(\varepsilon_T - \delta - \eta_{++-0}) \right]. \tag{37}
\]

### 2.8 Tests of $\mathcal{CP}T$ Violation

Using precise measurements of the $\mathcal{CP}$-violation parameters in the $2\pi$, $3\pi$ and semileptonic decay modes, the following tests of $\mathcal{CP}T$ conservation are feasible [23]:

- A nonzero phase difference, $\varphi_{++-} - \varphi_{SW}$, is a measurement of $\mathcal{CP}T$-violating amplitudes perpendicular to $\varepsilon_T$ after correcting for $\Delta \phi$. By further assuming that no $\mathcal{CP}T$ violation in decay amplitudes exists, e.g. $\Gamma_{K^0} = \Gamma_{\bar{K}^0}$, a limit on the $\bar{K}^0 - K^0$ mass difference is obtained.
- By comparing $\Re(\eta_{++-})$ with the semileptonic charge asymmetry $\delta_l$, $\mathcal{CP}T$ violation in the $I = 0$ and semileptonic decay amplitudes is tested.
- The phase difference $\varphi_{00} - \varphi_{++-}$ tests $\mathcal{CP}T$ conservation in the $I = 2$ amplitude.
3 Experimental Methods

The following list outlines the most important methods used to measure C\(P\) violation in the neutral-kaon system. The phases of the C\(P\)-violation parameters \(\eta_{+-}\) and \(\eta_{00}\) are determined by studying the \(K_L - K_S\) interference in time-dependent decay rates. Because of the structure of the interference term, its determination is strongly correlated with the value of the \(K_L - K_S\) mass difference. Additional measurements of the \(K_L - K_S\) mass difference by studying the \(K_L - K_S\) interference in semileptonic decays are important. Two experiments report recent measurements of the phase of the C\(P\)-violation parameter \(\eta_{+-}\) and the \(K_L - K_S\) mass difference with improved precision compared with the world average values. Their methods and results will be discussed in more details in the following sections.

3.1 Using pure \(K_L\) and almost pure \(K_S\) beams

This was the original method of discovering C\(P\) violation in \(K_L\) decays [1]. It is sensitive to \(|\eta_{\pi\pi}|\) and currently used to search for C\(P\) violation in the \(2\pi\) decay amplitudes [6]:

\[
\frac{K_L \rightarrow \pi^0\pi^0}{K_S \rightarrow \pi^0\pi^0} / \frac{K_L \rightarrow \pi^+\pi^-}{K_S \rightarrow \pi^+\pi^-} = \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 = 1 - 6\Re \left( \frac{\varepsilon'}{\varepsilon} \right) .
\]

The charge asymmetry in semileptonic decays of a \(K_L\) beam yields information about the real parts of the C\(P\)-violation parameters \(\varepsilon_T\) and \(\delta\) [24].

3.2 Incoherent production of \(K^0\) and \(\bar{K}^0\)

The production rates of \(K^0\) and \(\bar{K}^0\) in proton collisions on a fixed target are different. Therefore, the \(K_L - K_S\) interference term does not cancel completely when adding \(K^0\) and \(\bar{K}^0\) decay rates. The phase of the C\(P\)-violation parameter \(\eta_{+-}\) can be extracted by measuring the time distribution of neutral-kaon decays close to the production target [25, 26]:

\[
D(p) \times |\eta_{\pi\pi}| \cos (\Delta m\tau - \varphi_{\pi\pi}) .
\]

However, only the product of the modulus of \(\eta_{+-}\) and the dilution factor \(D(p)\) can be determined from the interference term. The measurement of the interference term in semileptonic decays yields a direct measurement of the \(K_L - K_S\) mass difference [27].

3.3 Regeneration of \(K_S\)

By passing a pure \(K_L\) beam through a regenerator, the interference between the initial \(K_L\) amplitude and the regenerated \(K_S\) amplitude is measured:

\[
|\rho| \times |\eta_{\pi\pi}| \cos (\Delta m\tau - \varphi_{\pi\pi} + \varphi_\rho) ,
\]

where \(\rho = |\rho|e^{i\varphi_\rho}\) depends on the characteristics of the regenerator. This measurement requires knowledge of the regeneration amplitudes to determine the C\(P\)-violation parameter \(\eta_{+-}\). The experiments [28, 29] measure simultaneously the phase of the C\(P\)-violation parameter \(\eta_{+-}\) and the \(K_L - K_S\) mass difference.

3.4 Strangeness tagging

By tagging the strangeness at the time of creation of the neutral kaon, C\(P\)-violation parameters can be measured in different decay channels without the drawback of a dilution factor or a large regeneration correction. This method is directly sensitive to the \(K_L - K_S\) interference term [30]:

\[
|\eta_{\pi\pi}| \cos (\Delta m\tau - \varphi_{\pi\pi}) .
\]
Using semileptonic decays, the strangeness at the time of the neutral-kaon decay can also be tagged and, independent of $\mathcal{CP}$ violation in mixing, a precise measurement of the $K_L - K_S$ mass difference is obtained [31].

3.5 Two regenerator method

The decay rate into $\pi^+\pi^-$ is measured after a $K_L$ beam has passed through two slabs of matter with thicknesses $d_1$ and $d_2$. By varying the regenerator distance $d$, this method is sensitive to [32]:

$$\cos (\Delta m \times P[d_1, d_2, d]) \oplus \text{(Regeneration)},$$

where $P$ is a polynomial in $d_1, d_2$ and $d$. The regeneration correction is suppressed by $\mathcal{CP}$ violation.

3.6 Regeneration with initial tagged strangeness

None of the above experiments is able to determine the sign of $\Delta m$. This was done by measuring time-dependent decay rates of initial $K^0$ mesons after a regenerator. This method is sensitive to the interference between the original $K_S$ amplitude for decays to $2\pi$ and the $K_S$ amplitude regenerated from the $K_L$ component, since the rate equation contains a term

$$|\rho| \cos (\Delta m \tau + \varphi_\rho).$$

The regeneration phase $\varphi_\rho$ is determined from the difference between the forward-scattering amplitudes of $K^+$ and $K^-$ in the regenerator material, thus allowing the sign of $\Delta m$ to be determined [33]. CPLEAR uses this method in order to measure the regeneration amplitude in carbon.

4 The E773 Experiment

A schematic view of the E773 detector is shown in Fig. 4. Two regenerators are placed 117 m and 128 m downstream of the target and toggle between the two neutral-kaon beams. Photon Veto Counters are used for detecting particles leaving the fiducial volume of the detector. Four drift chambers and a magnet are used to reconstruct charged tracks and their momenta. An electromagnetic calorimeter consisting of lead-glass blocks is used for photon reconstruction. The detector is essentially the same as that used by E731 [34].
Figure 5: Background-subtracted, acceptance-corrected decay distribution of $K \to \pi^+\pi^-$ in the energy bin 40–50 GeV (crosses) and the predictions made with (solid line) and without (dotted line) the interference term.

The experiment measures time-dependent decay rates in vacuum after a regenerator. It is sensitive to the interference between the $K_L$ decay amplitude into the final state $\pi^+\pi^-$ ($\pi^0\pi^0$) and the regenerated $K_S$ decay amplitude:

$$R_f(\tau) \propto |\rho|^2 e^{-\tau/\tau_s} + |\eta|^2 e^{-\tau/\tau_L} + |\rho| |\eta| e^{-\tau/\tau} \cos(\Delta m\tau - \varphi + \varphi_\rho).$$  \hspace{1cm} (38)

An understanding of the detector acceptance and of the regeneration parameters ($|\rho|$ and $\varphi_\rho$) is crucial for the analysis. Monte Carlo distributions are compared with data sets of $K_L$ decays into $3\pi^0$ and $\pi e\nu$ in order to estimate a possible acceptance error. The regeneration parameter $\rho$ is assumed to follow a power law in the kaon momentum region of interest and its phase is determined through a dispersion relation:

$$\rho \propto p^{n-1} \exp[-i\pi(1 + \alpha)/2].$$  \hspace{1cm} (39)

A discussion about the accuracy of this procedure can be found elsewhere [35, 36].

4.1 Results of E773

The measured decay spectra (see Fig. 5) for different kaon momenta (20–160 GeV) are fitted with $\Delta m$ and $\varphi_{+-}$ ($\varphi_{00}$) floating and also with the power law exponent $\alpha$ and the modulus of the regeneration parameter $\rho$ at 70 GeV floating. They obtain the following results [29]:

$$\Delta m = (529.7 \pm 3.0_{\text{stat.}} \pm 2.2_{\text{sys.}}) \times 10^7 h$$
$$\varphi_{+-} = 43.53^0 \pm 0.58^0_{\text{stat.}} \pm 0.49^0_{\text{sys.}} \pm 0.52^0_{\Delta m} \pm 0.33^0_{\tau_s}$$
$$\varphi_{00} - \varphi_{+-} = 0.62^0 \pm 0.71^0_{\text{stat.}} \pm 0.75^0_{\text{sys.}},$$  \hspace{1cm} (40)

where a value of $\Delta m = (528.2 \pm 3.0) \times 10^7 h s^{-1}$ has been used in the estimation of $\varphi_{+-}$. The measurement of the phase difference $\varphi_{00} - \varphi_{+-}$ is independent of regeneration corrections.

5 The CPLEAR Experiment

The CPLEAR experiment measures time-dependent decay rates (Fig. 6) of initially-tagged neutral kaons. Physics parameters are extracted from time-dependent decay-rate asymmetries

$$A = \frac{R(K^0 \to f) - R(K^0 \to \bar{f})}{R(K^0 \to \bar{f}) + R(K^0 \to f)},$$  \hspace{1cm} (41)
to be less dependent on detector efficiencies. The neutral kaons are produced in the reactions

\[
p\bar{p} \text{ (at rest)} \rightarrow K^-\pi^+K^0 \\
K^+\pi^-\bar{K}^0
\]

with a branching ratio of \( \approx 2 \times 10^{-3} \) each. The strangeness of the neutral kaon at its creation is defined by the reconstructed charged kaon.

A detailed description of the experiment can be found elsewhere [37]. Antiprotons of 200 MeV/c are stopped inside a gaseous hydrogen target (about 10^6 per second) at high pressure. The cylindrical detector (Fig. 7) is placed inside a solenoid of 1 m radius, 3.6 m length, which provides a magnetic field of 0.44 T. The charged tracking system consists of two proportional chambers, six drift chambers and two layers of streamer tubes. The detector has been upgraded with a microvertex chamber to improve the trigger efficiency for events with neutral-kaon decays for the data taking in late 1994 and 1995. Fast kaon identification is provided by a threshold Čerenkov counter sandwiched between two scintillators. An electromagnetic calorimeter made of 18 layers of Pb converters and streamer tubes is used for photon detection and electron identification. Fast and efficient online data selection is achieved with a multi-level trigger system based on custom-made hardwired processors.

Kinematical constraints (energy-momentum conservation, \( K^0 \)-mass) and geometrical constraints (\( K^0 \) flight direction and vertex separation) are used in the analysis, not only to improve the lifetime resolution but also to suppress very efficiently the background from other \( K^0 \) decay channels or the remaining background from \( p\bar{p} \) annihilation events.

5.1 Tagging Efficiencies and Regeneration

The detection efficiencies of \( K^+ \) and \( K^- \) used to tag the strangeness of the neutral kaon are not identical because of their different interactions with the detector material. Any differences resulting from geometrical imperfections are reduced by reversing the magnetic field several times a day. Using the high statistics of the \( 2\pi \) decay mode, the differences are measured as a function of the kaon and pion momenta and used to correct not only the \( 2\pi \) data but also the semileptonic and \( 3\pi \) data. These corrections are necessary to keep the average
normalization constant as function of the kaon decay time since the distributions of kaon and pion momenta vary slightly with the kaon decay time.

Only the product $[1 + 4\Re(\varepsilon_T + \delta)] \times$ relative tagging efficiency is measured in the $2\pi$ decay mode (Eq. 28). The absolute value of the relative tagging efficiency is approximated using the measurement of the semileptonic charge asymmetry $\delta_1$ [10] whenever necessary. This procedure assumes $CP T$ conservation in semileptonic decay amplitudes and it has to be taken into account when using semileptonic results. In the analysis of the $2\pi$ and $3\pi$ channels, the absolute scale of the relative tagging efficiency is left floating in the fit of the asymmetry.

Since the decay volume is not in vacuum, small regeneration corrections have to be applied for neutral kaons travelling through the detector material. In the absence of experimental data for the difference between the forward-scattering amplitudes of $K^0$ and $\bar{K}^0$ in the momentum region of the present experiment ($< 800$ MeV/$c$), the values calculated by Eberhard and Uchiharam [38] are used. The data is corrected on an event-by-event basis depending on the measured momentum of the neutral kaon and on the detector materials traversed [39]. The regeneration corrections are only important for the $2\pi$ decay mode where the result of $\varphi_{+-}$ changes by $-3^\circ$ (old data set) and $-1.7^\circ$ (new data set). The present error of the corrections amounts to $20\%$. It will be considerably reduced when the 1996 run is evaluated, in which an additional piece of material (2.5 cm carbon) was inserted into the detector to measure directly the regeneration amplitudes with high precision.

### 5.2 Results of CPLEAR

The CPLEAR experimental programme includes: the determination of $|\eta_{+-}|$, $\varphi_{+-}$ and the $K_L - K_S$ mass difference; search for CP violation in decays to $3\pi$ and a violation of the $\Delta S = \Delta Q$ rule; the direct measurement of $T$ and $CP T$ violation. Intermediate results ($|\eta_{+-}|$, $\varphi_{+-}$, $\Delta m$, $\Re(\eta_{+-0})$, $\Im(\eta_{+-0})$) based on $\approx 30\%$ of the final statistics have been published. New data have been taken with the microvertex chamber active in the trigger. The measured decay rates are corrected for regeneration and normalization as described above.
5.3 Measurement of $\eta_{+-}$

A precise measurement of $\eta_{+-}$ is obtained by fitting the time-dependent decay-rate asymmetry (see Fig. 8) with its analytic expression of the physics parameter:

$$A_{+-}(\tau) = \frac{R(K^0 \rightarrow \pi^+\pi^-) - \alpha R(K^0 \rightarrow \pi^+\pi^-)}{R(K^0 \rightarrow \pi^+\pi^-) + \alpha R(K^0 \rightarrow \pi^+\pi^-)}$$

$$= -2|\eta_{+-}| e^{\frac{1}{\tau_s-1/\tau_L}} \cos(\Delta m \times \tau - \varphi_{+-}) \frac{\cos(m_s \times \tau - \varphi_{+-})}{1 + |\eta_{+-}|^2 e^{(1/\tau_s-1/\tau_L)^2}}. \quad (42)$$

The fit takes into account the residual background contribution, mainly from semileptonic decays, and the normalization factor $\alpha$. The published result [30] and the preliminary result for the new data [14] are in good agreement:

published

$$|\eta_{+-}| = (2.305 \pm 0.043_{\text{stat.}} \pm 0.031_{\text{sys.}}) \times 10^{-3}$$

$$\varphi_{+-} = 43.7^\circ \pm 0.9^\circ_{\text{stat.}} \pm 0.6^\circ_{\text{sys.}} \pm 0.4^\circ_{\Delta m} \quad (43)$$

preliminary

$$|\eta_{+-}| = (2.322 \pm 0.030_{\text{stat.}} \pm 0.027_{\text{sys.}}) \times 10^{-3}$$

$$\varphi_{+-} = 43.2^\circ \pm 0.7^\circ_{\text{stat.}} \pm 0.5^\circ_{\text{sys.}} \pm 0.4^\circ_{\Delta m} \quad (44)$$

In both cases the same value [40] of $\Delta m = (530.7 \pm 1.3) \times 10^{-7} m_s$ was used to allow comparison of the results. The main systematic error which is due to the regeneration corrections will be reduced to $\pm 0.1^\circ$ after evaluation of the 1996 data (Fig. 9).

5.4 Measurement of the $K_L - K_S$ Mass Difference

The strangeness of the neutral kaon at the time of its decay is tagged by the charge of the lepton. Comparing the rate of neutral kaons decaying with same-as-initial strangeness with the rate of those decaying with opposite-to-initial strangeness as a function of the decay time, yields a measurement of the $K^0 \rightarrow \bar{K}^0$ oscillation frequency. The $A_{\Delta m}$ asymmetry (Eq.45) measured by the CPLEAR experiment is shown in Fig. 10. The measurement takes into account a possible
violation of the $\Delta S = \Delta Q$ rule but assumes no $CP T$ violation in the decay amplitudes (e.g. $\Im(x_-) = 0$). The analytic expression of the $A_{\Delta m}$ asymmetry obtained from Eq. 19-22 is

$$A_{\Delta m} = \frac{[\mathcal{R}_+(\tau) + \bar{\mathcal{R}}_-(\tau)] - [\mathcal{R}_-(\tau) + \mathcal{R}_+(\tau)]}{[\mathcal{R}_+(\tau) + \bar{\mathcal{R}}_-(\tau)] + [\mathcal{R}_-(\tau) + \mathcal{R}_+(\tau)]}$$

$$\approx e^{-\tau} \cos(\Delta m \tau) + 2 \sin(\Delta m \tau) \Im(x_-) \frac{E_+(\tau) + 2E_-(\tau) \Re(x_+)}{E_+(\tau) + 2E_-(\tau) \Re(x_+)}.$$  (45)

The CPLEAR results are [31, 14]:

- published \quad \{ \Delta m \ = \ (527.4 \pm 2.9_{\text{stat.}} \pm 0.5_{\text{hyst.}}) \times 10^7 \text{hs}^{-1} \}  \quad (46)
- preliminary \quad \{ \Delta m \ = \ (530.3 \pm 2.3_{\text{stat.}} \pm 0.5_{\text{hyst.}}) \times 10^7 \text{hs}^{-1} \}. \quad (47)

The results are insensitive to regeneration and normalization corrections because of the way the asymmetry (Eq. 45) is constructed. The main systematic error is due to the background estimation of decays into $3\pi$, which is based on Monte Carlo simulations and measured branching ratios.

### 5.5 Search for Violations of the $\Delta S = \Delta Q$ Rule

The CPLEAR experiment currently assumes $CP T$ conservation when extracting values for the $\Delta S = \Delta Q$ rule violation parameter $x$. Nevertheless, their preliminary values for $\Re(x)$ and $\Im(x)$ are in a good approximation equal to $\Re(x_+)$ and $\Im(x_+)$. $\Re(x_+)$ is obtained from a fit of the semileptonic asymmetry $A_{\Delta m}$ and $\Im(x_+ - \delta)$ is obtained from the time-dependent
decay-rate asymmetry between $K^0$ and $\bar{K}^0$ in any semileptonic final state:

$$A_t = \frac{R(K^0 \rightarrow \pi l \nu) - R(\bar{K}^0 \rightarrow \pi l \nu)}{R(K^0 \rightarrow \pi l \nu) + R(\bar{K}^0 \rightarrow \pi l \nu)}$$

$$= 2\Re(\varepsilon_T) + \frac{1}{E_+(\tau)} \left\{ 2\Re(\delta) E_-(\tau) - e^{-\tau} \left[ (2\Re(\varepsilon_T) + y) \cos(\Delta m \tau) - 2\Im(x_+ - \delta) \sin(\Delta m \tau) \right] \right\}. \quad (48)$$

The CPLEAR results are

$$\Re(x_+) = (8.5 \pm 7.5_{\text{stat.}} \pm 6.9_{\text{syst.}}) \times 10^{-3} \quad (49)$$

$$\Im(x_+ - \delta) = (0.5 \pm 2.4_{\text{stat.}} \pm 0.6_{\text{syst.}}) \times 10^{-3}. \quad (50)$$

This is an improvement of almost one order of magnitude compared with the world average [10] for $\Re(x) = (-3 \pm 26) \times 10^{-3}$, and a factor of 2 for the world average [10] of $\Re(x) = (6 \pm 18) \times 10^{-3}$. It is also better than the limit on $\Delta S = \Delta Q$ violation from charged kaon decays [41]:

$$\frac{\Gamma(K^+ \rightarrow \pi^+ \pi^0 e^- \nu)}{\Gamma(K^+ \rightarrow \pi^+ \pi^- e^+ \nu)} < 3 \times 10^{-4} \rightarrow |x| < 17 \times 10^{-3}. \quad (51)$$

Important information about $CPT$ violation in semileptonic decay amplitudes can be obtained from the measurement of $A_T$ (Fig. 2). Owing to the normalization procedure used by CPLEAR, the measurements of $A_T$ and $A_{\text{CPT}}$ for decay times $\tau \gg \tau_S$ are

$$A_T^{\text{CPLEAR}} = 4\Re(\varepsilon_T) - 4\Re(x_-) + 2y \quad (52)$$

$$A_{\text{CPT}}^{\text{CPLEAR}} = -4\Re(\delta). \quad (53)$$

The measured values are $A_T = (6.3 \pm 2.1_{\text{stat.}} \pm 1.8_{\text{syst.}}) \times 10^{-3}$ and $A_{\text{CPT}} = (0.3 \pm 2.1_{\text{stat.}} \pm 1.8_{\text{syst.}}) \times 10^{-3}$, where the statistical errors will improve in the future by a factor of 2. The main systematic uncertainties at present result from the different reconstruction efficiencies for
\[ \Re(\delta) = -\frac{1}{8} \left( A_T^{\text{CPLEAR}} + A_{\text{CPT}}^{\text{CPLEAR}} - 2\delta_I \right) = -(0.1 \pm 3.7) \times 10^{-4}, \] (54)

where the systematic errors approximately cancel.

5.6 Search for CP Violation in \( K_S \rightarrow \pi^+ \pi^- \pi^0 \)

Using the time-dependent decay-rate asymmetry in the \( \pi^+ \pi^- \pi^0 \) final state, the CPLEAR experiment has reported a preliminary result based on their full statistics [14]:

\[ \Re(\eta_{+-0}) = \left( -4 \pm 8_{\text{stat.}} \pm 2_{\text{sys.}} \right) \times 10^{-3} \] (55)
\[ \Im(\eta_{+-0}) = \left( -4 \pm 10_{\text{stat.}} \pm 4_{\text{sys.}} \right) \times 10^{-3} \] (56)

with a correlation coefficient of 0.66. More details about this measurement can be found elsewhere [42], where the results of a part of the statistics are published.

The FNAL E621 experiment [43] published a result on \( \Im(\eta_{+-0}) \) by fixing \( \Re(\eta_{+-0}) = 1/2\delta_I \) and assuming \( \text{CP} \) conservation:

\[ \Im(\eta_{+-0}) = \left( -15 \pm 30 \right) \times 10^{-3}. \] (57)

Figure 11 shows the results of the two experiments in the complex plane. The values for \( \Re(\eta_{+-0}) \) and \( \Im(\eta_{+-0}) \) for E621 are taken from their publication.

6 Compilation of \( \Delta m \) and \( \eta_{+-} \) Measurements

The new measurements of \( \varphi_{+-} \) and \( \Delta m \) are in good agreement with previous measurements (see Fig. 12), and the precision of the physics parameter can be further increased by averaging all measurements. Using a previously described procedure [40] and the measurements of \( \varphi_{+-} \) and \( \Delta m \) listed in Table 1 and 2 respectively, the following world average values
for $\varphi_{+-}$ and $\Delta m$ are obtained:
\begin{align*}
\varphi_{+-} &= 43.56^\circ \pm 0.56^\circ \\
\Delta m &= (530.14 \pm 1.11) \times 10^7 h/s ,
\end{align*}

with a correlation coefficient $\rho$ of 0.657. The value of $\tau_S$ is left free in the fit, but constrained by the world average value [10]. The value of $\varphi_{+-}$ is in good agreement with $\varphi_{SW} = 43.47^\circ \pm 0.08^\circ$. It is interesting to see how the values of $\varphi_{+-}$ and $\Delta m$ have changed over the last few years (Fig. 13). A $2\sigma$ effect of $CP$ violation has completely disappeared with a lower value of $\Delta m$ and improved measurements of $\varphi_{+-}$.

Some of the experiments which use semileptonic decays to measure the $K_L - K_S$ mass difference assume $CP$ conservation in the semileptonic decay amplitudes. Without this assumption, the error on $\Delta m$ increases significantly [46]. If we ignore these experiments, we obtain a world average of
\begin{align*}
\varphi_{+-} &= 43.57^\circ \pm 0.66^\circ \\
\Delta m &= (530.19 \pm 1.54) \times 10^7 h/s ,
\end{align*}

with a correlation coefficient of 0.769.

Measurements of the modulus of the $CP$-violation parameter $\eta_{+-}$ and their average are shown in Table 3.
Figure 12: Measurements of $\varphi_{+-}$ and $\Delta m$ and their average value.

Figure 13: Historical perspective of $\Delta m$ and $\varphi_{+-}$. FIT95 refers to ref.39 and FIT96 are the results shown in Eq. 57.

Table 3: Measurements of the modulus of the $CP$-violation parameter $\eta_{+-}$.

| Experiment                  | $|\eta_{+-}|$ [$10^{-3}$] |
|-----------------------------|--------------------------|
| Geweniger [25]              | 2.30 ± 0.035             |
| BRFIT (PDG94)               | 2.266 ± 0.030            |
| CPLEAR prel.                | 2.316 ± 0.038            |
| average                     | 2.290 ± 0.020            |
7 Experimental Tests of CPT Violation

In the absence of precise measurements of all relevant $T$, $CP$ and $CPT$ violation parameters in the neutral-kaon system, mainly consistency checks of $CPT$ conservation can be done rather than determine limits on $CPT$-violation parameters. In a first approach, no $CPT$ violation in the decay amplitudes is assumed, which is motivated by the fact that a new $CPT$ violating interaction could enter in first order in the mass matrix, but only in second order in the decay matrix (Eq. 4). In a second approach, no assumptions about $CPT$ conservation in general are made. Only any conspiracy between $CPT$-violation parameters is ignored and the limit of a linear combination of $CPT$-violation parameters is also used as a limit of each individual parameter. In the third approach, a new measurement of CPLEAR is used to estimate the $K^0 - \bar{K}^0$ mass difference without any prejudice.

7.1 Evaluation of the Phase Difference between $\Gamma_{12}$ and the $I = 0$ Amplitude

A non-negligible contribution to the $CP$-violation parameter $\eta_{+-}$ can derive from the interference between the $2\pi$ decay amplitude and mixing amplitude even in the absence of any $CPT$ violation (Eq. 30). The different contributions to the phase difference according to Eq. 37 are estimated in the following paragraph.

The modulus of the amplitude $A_2$ is estimated from the measured partial decay widths of $K_S \to \pi^+\pi^-$, $K_S \to \pi^0\pi^0$ and $K^+ \to \pi^0\pi^+$ by neglecting $I = 5/2$ transitions [17]:

$$|A_2| = 0.045 \pm 0.002 . \quad (60)$$

The measurements [10, 19] of $\Re(\epsilon'/\epsilon) = (1.5 \pm 0.8) \times 10^{-3}$ and $\delta_2 - \delta_0 = -42^\circ \pm 4^\circ$ yield

$$\arg \left( \frac{A_2}{A_0} \right) \approx \sqrt{2} \Re \left( \frac{\epsilon'}{\epsilon} \right) \times \frac{|A_0|}{|A_2|} \times |\epsilon_T| = (1.1 \pm 0.6) \times 10^{-4} . \quad (61)$$

The contribution of the $I = 2$ amplitude to the sum in Eq. 37 is only $(4.5 \pm 2.4) \times 10^{-7}$ and can be neglected given the other terms. The contribution of the $3\pi$ decay mode can be estimated using the preliminary CPLEAR measurement of $\Im(\eta_{+-0})$ (Eq. 56):

$$\Delta \phi^{3\pi} \approx \frac{\tau_S}{\tau_L} \text{BR}(K_L \to 3\pi) \times \Im(\epsilon_T - \delta - \eta_{+-0}) \quad (62)$$

$$= (-2.4 \pm 6.4) \times 10^{-6}$$

The term $\Im(\epsilon_T - \delta)$ can safely be neglected given the present accuracy of $\Im(\eta_{+-0})$. Assuming lepton universality, we can use the result of CPLEAR obtained with electrons (Eq.50) to constrain the contribution of the semileptonic decay amplitudes:

$$\Delta \phi^{e\nu} = \frac{\tau_S}{\tau_L} \left[ 4 \text{BR}(K_L \to l^+\pi^-\nu) \times \Im(x_+) \right] \quad (63)$$

$$= (1.2 \pm 5.7) \times 10^{-6} .$$

Finally we obtain for the phase difference:

$$\Delta \phi = (-1.2 \pm 8.5) \times 10^{-6} , \quad (64)$$

which would correspond to a difference between $\varphi_{+-}$ and $\varphi_{SW}$ of $-0.02 \pm 0.16^\circ$. The uncertainty without the CPLEAR results has been $\pm 2.3^\circ$. 

20
7.2 $K^0 - \bar{K}^0$ Mass Difference

The precise measurement of the phase of the $CP$-violation parameter $\eta_{+^+}$ allows $CP\mathcal{T}$ violation which produces a phase difference between $\varphi_{+^+}$ and $\varphi_{SW}$ to be tested. The contributions to $\eta_{+^+}$ perpendicular to $\varepsilon_T$ are (according to Eq. 30)

\[
\delta_\perp = (\varphi_{+^+} - \varphi_{SW}) |\eta_{+^+}| - \Delta \phi \cos(\varphi_{SW})
\]

\[
\delta_\perp = \sin \varphi_{SW} \left[ \frac{m_{K^0} - m_{\bar{K}^0}}{2 \Delta m} + \mathcal{R} \left( \frac{b_0}{a_0} \right) \right] - \varepsilon_\perp'.
\]

Correcting for $\Delta \phi$ and using the world average values for $\varphi_{+^+}$ and $\Delta m$ (Eq. 58) we find

\[
\frac{\delta_\perp}{|\eta_{+^+}|} = (0.1 \pm 1.1)\% < 1.6\% \text{ with a } 90\% \text{ CL.}
\]

A very similar result is obtained with the values of Eq. 59. If we neglect $CP\mathcal{T}$ violation in decay amplitudes ($\varepsilon_\perp = 0$ and $\mathcal{R} \left( \frac{b_0}{a_0} \right) = 0$):

\[
\frac{m_{K^0} - m_{\bar{K}^0}}{m_{K^0}} = (1.0 \pm 4.3) \times 10^{-19} < 7.2 \times 10^{-19} \text{ with a } 90\% \text{ CL.}
\]

For estimations of the mass difference using approaches II and III see Table 4.

7.3 $CP\mathcal{T}$ Violation in the $I = 0$ Amplitude

By comparing $CP$ violation in the $2\pi$ decay mode with the charge asymmetry in semileptonic decays we find:

\[
\mathcal{R} \left( \frac{b_0}{a_0} \right) + \mathcal{R}(x_{-}) - \frac{1}{2} y = \frac{1}{3} (2 \eta_{+-} + \eta_{00}) - \frac{1}{2} \delta_t
\]

\[
= \mathcal{R}(\eta_{+-}) \left[ 1 - \mathcal{R} \left( \frac{\varepsilon_\perp'}{\varepsilon} \right) - (\varphi_{00} - \varphi_{+^+}) \tan(\varphi_{SW}) \right] - \frac{1}{2} \delta_t
\]

\[
= (0.9 \pm 6.4) \times 10^{-5}.
\]

Assuming no correlation of $CP\mathcal{T}$ violation in semileptonic and hadronic decays, we can use this result as an estimation of $CP\mathcal{T}$ violation in the $I = 0$ amplitude. Together with the result of $\varepsilon_\perp'$ (Eq. 71), this gives an estimate of the $K^0 - \bar{K}^0$ mass difference for approach II (Table 4).

The term $\mathcal{R}(x_{-}) - \frac{1}{2} y$ can be estimated based on the measurement of $A_T$ from CPLEAR, see section 7.5, which then gives $\mathcal{R}(b_0/a_0) = (-0.6 \pm 7.2) \times 10^{-4}$. At present, a large $CP\mathcal{T}$ violation in the $I = 0$ amplitude, which is cancelled by $CP\mathcal{T}$ violation in the semileptonic channel, cannot be excluded. Therefore, the limit on the $K^0 - \bar{K}^0$ mass difference is one order of magnitude higher than when assuming no cancellation (Table 4).

7.4 $CP\mathcal{T}$ Violation in the $I = 2$ Amplitude

Since the strong phase difference $\delta_0 - \delta_2$ is found to be close to the phase of $\varepsilon_T$, only $CP\mathcal{T}$ violating amplitudes can produce a nonzero component of $\varepsilon'$ perpendicular to $\varepsilon_T$ (Eq. 32):

\[
\varepsilon_\perp' = \frac{1}{3} (\varphi_{+^+} - \varphi_{00}) |\eta_{+^+}|
\]

\[
= \frac{1}{\sqrt{2}} \mathcal{R} \left( \frac{a_2}{a_0} \right) \left[ \mathcal{R} \left( \frac{b_2}{a_2} \right) - \mathcal{R} \left( \frac{b_0}{a_0} \right) \right].
\]
Table 4: $K^0 - \bar{K}^0$ mass difference for the different approaches.

<table>
<thead>
<tr>
<th>Approach</th>
<th>$\frac{m_{K^0} - m_{\bar{K}^0}}{m_K} [10^{-19}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>1.0 ± 4.3</td>
</tr>
<tr>
<td>II</td>
<td>-4.5 ± 10.2</td>
</tr>
<tr>
<td>III</td>
<td>10 ± 100</td>
</tr>
</tbody>
</table>

Using the combined result [29] of E731 and E773 of $\varphi_+ - \varphi_{00} = 0.30^\circ \pm 0.88^\circ$ one finds that

$$\epsilon'_\perp = (-0.4 \pm 1.2) \times 10^{-5}$$

(70)

$$\Re \left( \frac{b_2}{a_2} \right) - \Re \left( \frac{b_0}{a_0} \right) = (-2.0 \pm 7.1) \times 10^{-4}.$$  (71)

7.5 $\mathcal{CPT}$ Violation in Semileptonic Decays

The measurement of $A_T$ by CPLEAR limits possible $\mathcal{CPT}$ violation in semileptonic decay amplitudes (Eq. 52):

$$\Re(x_\perp) - \frac{1}{2}y = \Re(\epsilon_T) - \frac{1}{4}A_T^{\text{CLEAR}}.$$  (72)

The term $\Re(\epsilon_T) = |\epsilon_T| \cos(\varphi_{SW})$ can be approximated within the experimental errors of $A_T$ by (using only terms parallel to $\epsilon_T$ in Eq. 30)

$${\epsilon_T} \left[ 1 + \Re \left( \frac{\epsilon'_T}{\epsilon} \right) \right] = |\eta_{+-}| - \Delta \phi \sin(\varphi_{SW}) - \frac{\Gamma_{K^0} - \Gamma_{\bar{K}^0}}{2\Delta \Gamma} \cos(\varphi_{SW}) - \Re \left( \frac{b_0}{a_0} \right) \cos(\varphi_{SW}),$$

(73)

where the $I = 0$ amplitude cancels when expanding the lifetime difference of $K^0$ and $\bar{K}^0$ following Eq. 4:

$$\frac{\Gamma_{K^0} - \Gamma_{\bar{K}^0}}{2\Delta \Gamma} = \Re \left( \frac{b_0}{a_0} \right) - \frac{A_2}{A_0} \Re \left( \frac{b_2}{a_2} \right) - \frac{\Gamma_L}{\Gamma_S} \left[ 2BR(K_L \to \pi \nu) \Re \left( \frac{b_1}{a_1} \right) - BR(K_L \to \pi \nu) y \right].$$

(74)

Using Eq. 71, approximating $\Re(b_1/a_1)$ by the measurement of $\Re(\eta_{+-0})$ and inserting Eqs. 72-74 in Eq. 68 yields

$$\Re \left( \frac{b_0}{a_0} \right) = (-0.6 \pm 7.2) \times 10^{-4}$$

(75)

$$\frac{m_{K^0} - m_{\bar{K}^0}}{m_K} = (1 \pm 10) \times 10^{-18}.$$  (76)

A similar estimate has been obtained recently by Shabalin [47] based on the preliminary CPLEAR measurement of $A_{\mathcal{CPT}}$. Knowing the $K^0 - \bar{K}^0$ mass difference, the lifetime difference can be estimated using the direct measurement of $\Re(\delta)$ by CPLEAR (Eq. 54):

$$\frac{\Gamma_{K^0} - \Gamma_{\bar{K}^0}}{\Delta \Gamma} = (0.1 \pm 1.4) \times 10^{-3}.$$  (77)
8 Quantum Gravity

8.1 General Concept

There exists a possibility of $CPT$ violation in the context of quantum gravity [9], as a result of a possible modification of conventional quantum field theory. A framework for analysing this possibility is provided by the formulation [48] of open quantum-mechanical systems which are coupled to an unobserved environment. This would induce a loss of quantum coherence in the observed system, which should be described by a density matrix $\rho$ that obeys a modified quantum Liouville equation:

$$\dot{\rho} = i[\rho, H] + \xi H \rho,$$

(78)

where the extra term $\xi H$ may arise from quantum-gravitational effects. In the case of the neutral-kaon system, the open-system Eq. 78 introduces three $CPT$-violation parameters $\alpha, \beta, \gamma$ if energy and strangeness conservation are assumed [48]. Their magnitude is expected to be at most $O(m_K^2/m_{\text{Pl}})$, where $m_{\text{Pl}} = 1.2 \times 10^{19} \text{GeV}/c^2$. Unlike $CPT$ violation in the conventional quantum-mechanical framework, the new parameters $\alpha, \beta, \gamma$ do not introduce a mass or lifetime difference to the stationary neutral-kaon states. The loss of quantum coherence is best studied by precise measurements of the time evolution of neutral-kaon states.

8.2 Experimental Results

The time-dependent decay-rate asymmetries, $A_{++}$ (Fig. 8) and $A_{\Delta m}$ (Fig. 10), have been refitted by CPLEAR [49] with their new analytic expression [50]. Two other measurements of $CPT$ violation at large decay times have been used as additional constraints. The results obtained are:

$$\alpha = (-0.5 \pm 2.8) \times 10^{-17} \text{ GeV}/c^2 < 4.0 \times 10^{-17} \text{ GeV}/c^2$$
$$\beta = (2.5 \pm 2.3) \times 10^{-19} \text{ GeV}/c^2 < 2.3 \times 10^{-17} \text{ GeV}/c^2$$
$$\gamma = (1.1 \pm 2.5) \times 10^{-21} \text{ GeV}/c^2 < 3.7 \times 10^{-17} \text{ GeV}/c^2$$
$$|\varepsilon| = (2.32 \pm 0.06) \times 10^{-3},$$

(79)

where the 90% CL are obtained taking into account the positivity constraints $\alpha, \gamma > 0$ and $\alpha \gamma > \beta^2$.

No contribution of quantum-gravitational effects to the observed $CPT$ violation in the neutral-kaon system is observed. The precision of the parameters $\alpha, \beta, \gamma$ approaches the range of interest expected by dimensional arguments.

9 Summary

All measurements of $CPT$ violation in the neutral-kaon system, despite the NA31 measurement of $\Re(\varepsilon'/\varepsilon)$, are consistent with $CPT$ violation in the $K^0 - \bar{K}^0$ mixing alone. No deviation from $CPT$ conservation is found, although $CPT$ violation could still be quite large since $CPT$-violating terms can cancel in the measurements. Improvements can only be expected with future measurements of semileptonic decays. Nevertheless, the existing data from several experiments limits the $K^0 - \bar{K}^0$ mass difference in a region interesting for searching quantum-gravitational effects.

Acknowledgements

I would like to thank all of my colleagues and friends of the CPLEAR collaboration for the interesting work we have done together. I am especially grateful to M. Fidecaro and
T. Nakada for the many discussions on neutral-kaon phenomenology and to C. Touramanis for the discussions on the CPLEAR data analysis. I would also like to thank the organizers of this conference for such a pleasant stay in Mexico City.

References
and for a recent review:
[11] see for example:
[21] see T. Nakada in ref.[11].


[33] see for example:


[46] by a factor of 7, CPLEAR private communication.


