MEASUREMENT OF THE LUMINOSITY FOR BEAMS CROSSING AT SMALL ANGLES IN A LOW-θ SECTION

by

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Abstract

It is shown that the luminosity for beams crossing at small angles in a low-θ section can be obtained by measuring its variation when the beams are displaced with respect to each other. This holds, provided a correction factor is applied which is defined in general and evaluated numerically for a few typical cases. In practice, it turns out to be close to unity.

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1. INTRODUCTION

A very convenient method of measuring the luminosity in a storage ring has been proposed by Van der Meer \(^1\). It is widely used in the ISR. In connection with design studies for larger storage rings \(^2\) in which the beams collide at small angles in low-\(\beta\) sections the question arises whether the same method still gives the correct luminosity. It will be shown below that this is only true if one includes a correction factor which is calculated for a few typical cases. It turns out that the correction factor is very close to unity under most conditions found in practice. The calculation below is done for vertical crossings of the beam. They can be applied to horizontal crossings by rotating the coordinate system and interchanging the words horizontal and vertical in the text.

2. ANALYTIC DERIVATION

The luminosity can be calculated in a straightforward manner if the shapes and intensities of the crossing beams are known. For small crossing angles \(\alpha\) such that \(\sin \alpha = \alpha\) and \(\cos \alpha = 1\), the expression is

\[
\mathcal{L} = 2c \int x_1(x, y, s) x_2(x, y, s) \, dx \, dy \, ds
\]  

Here, \(c\) is the velocity of light, and \(x_1\) and \(x_2\) are the (number) densities of the protons in the two beams. The integration goes over the whole interaction volume.

Van der Meer's method measures the luminosity \(\mathcal{L}\) as a function of the horizontal beam displacement \(a\) which for vertical crossing is given by

\[
\Lambda(a) = 2c \int x_1(x - a, y, s) x_2(x, y, s) \, dx \, dy \, ds
\]
If one integrates \( A(a) \) over \( a \) and divides it by \( A(0) \), one obtains an effective width \( a_{\text{eff}} \)

\[
a_{\text{eff}} = \frac{\int A(a) \, da}{A(0)} = \frac{\int A(a) \, da}{\mathcal{L}}
\]

(3)

It is sufficient experimentally to observe a quantity which is proportional to the luminosity, e.g. a beam-beam counting rate in a given detector. Clearly, \( A(0) \) is equal to the luminosity defined in (1). The merit of this definition of \( a_{\text{eff}} \) is that - in the case of the ISR - it can be inserted into the luminosity formula

\[
\mathcal{L} = \frac{2c}{a_{\text{eff}} a} \lambda_1 \lambda_2
\]

(4)

and yields the correct luminosity. In (4), \( \lambda_1 \) and \( \lambda_2 \) are the line (number) densities in the two beams, and \( a \) is the crossing angle. When the transverse beam profiles vary within the length of the intersection region (4) cannot be expected to be accurate.

In order to simplify the definition of \( a_{\text{eff}} \), we introduce the integrated distribution functions \( \sigma_i \)

\[
\sigma_i(y,s) = \int \rho_i(x,y,s) \, dx
\]

(5)

and write explicitly

\[
\int A(a) \, da = 2c \int \rho_1(x - a,y,s) \rho_2(x,y,s) \, dx \, dy \, ds \, da
\]

(6)

Since \( a \) appears only inside \( \rho_1 \), we may perform this integration and find

\[
\int A(a) \, da = 2c \int \sigma_1(y,s) \rho_2(x,y,s) \, dx \, dy \, ds
\]

(7)
Repeating the same operation on $\rho_2$ yields

$$\int A(s) \, da = 2c \int \sigma_1(y,s) \sigma_2(y,s) \, dy \, ds$$  \hspace{1cm} (8)$$

We shall show below that $a_{\text{eff}}$ is equal to the width $w$ of hypothetical beams with the following properties:

i) their density distributions $\rho_i^h(x,y,s)$ are uniform in $x$ and have the same dependence on $y$ and $s$ as $\rho_i(x,y,s)$ and the same currents.

ii) the width $w$ is adjusted such that these hypothetical beams have the same luminosity as the actual beams.

The first condition is satisfied by choosing $\rho_i^h$ as follows:

$$\rho_i^h(x,y,s) = \begin{cases} 
\frac{1}{w} \int \rho_i(x,y,s) \, dx = \frac{\sigma_i(y,s)}{w} & \text{for } |x| < \frac{w}{2} \\
0 & \text{for } |x| > \frac{w}{2} 
\end{cases}$$  \hspace{1cm} (9)$$

The hypothetical luminosity is

$$L^h = 2c \int \rho_1^h \rho_2^h \, dx \, dy \, ds$$

$$L^h = \frac{2c}{w} \int \sigma_1 \sigma_2 \, dy \, ds$$  \hspace{1cm} (10)$$

Imposing the condition $L^h = L$ yields for $w$

$$w = \frac{\int \sigma_1 \sigma_2 \, dy \, ds}{\int \rho_1 \rho_2 \, dx \, dy \, ds}$$  \hspace{1cm} (11)$$

which may be seen to agree with (3), by using (1) and (8).
We have thus demonstrated that Van der Meer's method allows to determine a parameter $a_{\text{eff}}$ which is equal to the width of a hypothetical beam which has the same luminosity as the actual beam.

In order to find that luminosity we still have to do the integral (1) for the hypothetical beams. This implies that we have eliminated the actual beam profile in the $x$ direction, but not yet those in the $y$ and $s$ directions. Since the latter are trivial in the case of the ISR, Van der Meer's method works there without further calculation.

3. **SPECIFIC EXAMPLES**

Below, we shall calculate the luminosity for a few specific beam profiles, and derive a correction factor which relates the actual luminosity to the simple formula (4).

3.1. **Gaussian beams without dispersion**

We assume for the hypothetical beams that $\beta_y$ has a minimum $\beta_0$ at the crossing point, that the rms beam half height is given by $\sigma_0$, that the vertical dispersion and its derivative vanish there, and that the density distribution is given by a Gaussian. The distribution functions are then defined as follows

$$
\sigma_1(y,s) = \frac{\lambda_1}{\sigma_0 \sqrt{2\pi(1 + s^2 \beta_0^{-2})}} \exp\left[ -\frac{y^2}{2\sigma_0^2(1 + s^2 \beta_0^{-2})} \right]
$$

$$
\sigma_2(y',s') = \frac{\lambda_2}{\sigma_0 \sqrt{2\pi(1 + s'^2 \beta_0^{-2})}} \exp\left[ -\frac{y'^2}{2\sigma_0^2(1 + s'^2 \beta_0^{-2})} \right]
$$

(12)

Here, the two coordinate systems $(y,s)$ and $(y',s')$ belong to the two beams and are related by
\[ y' = y \cos \alpha + s \sin \alpha \]
\[ s' = s \cos \alpha - y \sin \alpha \]

Since we are only interested in the case with \( \alpha \ll 1 \), and also \( y \ll s \), we may simplify the transformation and write

\[ y' = y + s \alpha \]
\[ s' = s \]

The luminosity \( \mathcal{L}^h \) becomes

\[
\mathcal{L}^h = \frac{2c \lambda_1 \lambda_2}{2\pi \sigma_0} \int_{-\lambda/2}^{+\lambda/2} \exp\left[-\frac{s^2 \alpha^2}{4\sigma_0^2(1 + s^2 \beta_0^{-2})}\right] \frac{ds}{(1 + s^2 \beta_0^{-2})^\frac{1}{2}}
\]  
(13)

The length over which the luminosity is integrated is \( \lambda \). Comparing this to the luminosity formula \( \mathcal{L}^f \) (4) gives for the ratio

\[
\frac{\mathcal{L}^h}{\mathcal{L}^f} = \frac{\alpha}{2\sigma_0 \sqrt{\pi}} \int_{-\lambda/2}^{+\lambda/2} \exp\left[-\frac{s^2 \alpha^2}{4\sigma_0^2(1 + s^2 \beta_0^{-2})}\right] \frac{ds}{(1 + s^2 \beta_0^{-2})^\frac{1}{2}}
\]  
(14)

The ratio (14) is the correction factor \( F \) which has to be applied to the simple formula (4) in order to find the actual luminosity. If we introduce two parameters \( \xi \) and \( \eta \) used previously 3) and given by

\[
\xi = \frac{\lambda}{2\beta_0} \quad \eta = \frac{a\sigma_0}{\sigma_0}
\]  
(15)

we may write

\[
F(\xi, \eta) = \frac{\eta}{2\sqrt{\pi}} \int_{-\xi}^{+\xi} \exp\left[-\frac{\eta^2}{4} \frac{u^2}{1 + u^2}\right] \frac{du}{(1 + u^2)^{\frac{1}{2}}}
\]  
(16)
\( \xi \) expresses the length of the intersection region in units of \( \beta_0 \), and \( \eta \) is the ratio between the crossing angle \( \alpha \) and the rms beam half divergence.

The correction factor \( F \) has been evaluated numerically for several values of \( \xi \) and \( \eta \) and is shown in Table I.

3.2. Gaussian beams with dispersion

We assume that the beam profile comes from two contributions, betatron oscillations and momentum spread, and that both have Gaussian distributions with rms radius \( \sigma \) and \( \tau \), respectively. The overall profile is then given by the convolution of the two distributions and becomes

\[
\sigma_i(y,s) = \frac{\lambda_i}{[2\pi(\sigma^2 + \tau^2)]^{\frac{3}{2}}} \exp\left[-\frac{y^2}{2(\sigma^2 + \tau^2)}\right]
\]

(17)

We further assume that \( \beta_Y \) has a minimum \( \beta_0 \) at the crossing point, and that the vertical dispersion there is \( \tau_0 \), and its derivative \( \tau_0' \). Then \( \sigma \) and \( \tau \) depend on \( s \) in the following way

\[
\sigma(s) = \sigma_0(1 + \frac{\sigma^2}{\beta_0 c^2})^{\frac{3}{2}}
\]

(18)

\[
\tau(s) = \tau_0 + \tau_0' s
\]

Note again that it is not necessary to know the horizontal beam profile and its variation due to betatron oscillations and momentum spread. Using (10), (17) and (18), the luminosity of the hypothetical beams becomes

\[
\mathcal{L}^h = \frac{2e}{w} \int \frac{\lambda_1 \lambda_2}{2\pi(\sigma^2 + \tau^2)} \exp\left[-\frac{y^2}{2(\sigma^2 + \tau^2)} - \frac{(y + s \alpha)^2}{2(\sigma^2 + \tau^2)}\right] dy \, ds
\]

(19)

\[
\mathcal{L}^h = \frac{4e}{w} \int \frac{\lambda_1 \lambda_2}{\sqrt{\pi}(\sigma^2 + \tau^2)} \exp\left[-\frac{\sigma^2 \alpha^2}{4(\sigma^2 + \tau^2)}\right] ds
\]
Dividing this by the luminosity formula (4) yields for the correction factor $F$

$$F(\xi, \eta, \frac{\tau_0}{\sigma_0}, \frac{\tau_0'}{\sigma_0}) = \frac{n}{2\sqrt{\pi}} \int_{-\xi}^{+\xi} \exp \left\{ -\frac{n^2}{4} \frac{u^2}{\left[1 + u^2 + \left(\frac{\tau_0 + \tau_0' \beta_0}{\sigma_0} u\right)^2\right]^2} \right\} \frac{du}{1 + u^2 + \left(\frac{\tau_0 + \tau_0' \beta_0}{\sigma_0} u\right)^2}$$

(20)

Two special cases give simpler solutions

i) $\tau_0 = 0$

When the vertical dispersion vanishes at the crossing point, we have

$$F(\xi, \eta, 0, \frac{\tau_0'}{\sigma_0}) = \frac{n}{2\sqrt{\pi}} \int_{-\xi}^{+\xi} \exp \left\{ -\frac{n^2}{4} \frac{u^2}{1 + \left(\frac{\tau_0'}{\sigma_0} u\right)^2} \right\} \frac{du}{\left[1 + \left(\frac{\tau_0'}{\sigma_0} u\right)^2\right]^2}$$

(21)

By a change of variables this can be shown to yield:

$$F(\xi, \eta, 0, \frac{\tau_0'}{\sigma_0}) = F\left(\xi \left(1 + \left(\frac{\tau_0'}{\sigma_0} \right)^2\right)^{\frac{1}{2}}, \frac{n}{\left(1 + \left(\frac{\tau_0'}{\sigma_0} \right)^2\right)^{\frac{1}{2}}} \right)$$

(22)

where $F(\xi, \eta)$ has been defined previously.

ii) $\tau_0' = 0$

When the derivative of the vertical dispersion vanishes at the crossing point we find
\[ F(\xi, \eta, \frac{\tau_0}{\sigma_0}, \phi) = \frac{\eta}{2\sqrt{\pi}} \int_{-\xi}^{+\xi} \exp\left[ -\frac{\eta^2}{4} \frac{u^2}{1 + \frac{\tau_0^2}{\sigma_0^2} + u^2} \right] \frac{du}{\left(1 + \frac{\tau_0^2}{\sigma_0^2} + u^2\right)^{1/2}} \]  \hspace{1cm} (23)

By a change of variables this can be shown to give

\[ F(\xi, \eta, \frac{\tau_0}{\sigma_0}, \phi) = F\left(\frac{\xi}{\left(1 + \frac{\tau_0^2}{\sigma_0^2}\right)^{1/2}}, \eta\right) \]  \hspace{1cm} (24)

4. DISCUSSION

An inspection of Table I shows that for the distribution functions (12) and \( \xi > \frac{1}{2} \) and \( \eta > 10 \) the correction factor differs from unity by less than 2.3%. Different distribution functions - in \( y \) and \( s \) only! - will have different correction factors, which can also be evaluated numerically.

However, it can be shown for any distribution that at large enough \( \xi \) the correction factor \( F \) can be expanded into a power series in \( \eta \) as follows

\[ F = 1 + \frac{2}{\eta^2} + \frac{c_4}{\eta^4} + \ldots \]  \hspace{1cm} (25)

where the coefficients \( c_4, \ldots \) depend on the details of the distribution functions. Hence, if \( 2/\eta^2 \ll 1 \) the higher order terms in \( \eta \) must have a very small effect on the correction factor.

The intersection region where Van der Meer's scheme is least likely to work, is the high-luminosity intersection 2). It has the following characteristics:
\begin{align*}
\beta_o &= 5 \text{ m} \\
\sigma_o &= 0.3 \text{ mm} \\
\alpha &= 0.86 \text{ mrad} \\
\lambda &= 20 \text{ m} \\
\alpha_p &= 0 \\
\alpha_p' &= 0.063 \\
\Delta p/p &= \pm 1.8 \times 10^{-3}
\end{align*}

Converting these parameters into those necessary for calculating \( F \) yields

\begin{align*}
\xi &= 2 \\
\eta &= 14.3 \\
\tau_o' &= 1.13 \times 10^{-4} \\
\tau_o' \beta_o/\sigma_o &= 1.89
\end{align*}

We find that the correction factor becomes \( F(4.28, 6.69) = 1.06 \).

A determination of the luminosity to an accuracy of a few percent requires the following quantities to be known well enough

- the circulating currents \( I_1 \) and \( I_2 \)
- the effective width \( a_{\text{eff}} \)
- the crossing angle \( \alpha \)
- the correction factor \( F \)

Measuring a crossing angle of about 1 mrad to 1\% accuracy requires measuring the beam positions to 100 \( \mu \)m at 10 m distance. Thus the crossing angle may well be the most inaccurate ingredient in the luminosity measurement.
This difficulty can be avoided by calibrating the luminosity monitors while the beams cross at a much larger angle than usually. This makes the measurement of the crossing angle easier and brings the correction factor $F$ even closer to unity.

5. **CONCLUSIONS**

We have shown that Van der Meer's method of measuring the effective beam width by displacing the two crossing beams with respect to each other and using that width in a simple luminosity formula can be used in high-luminosity insertions with small $\beta$-values and crossing angles, provided a correction factor $F$ is applied. This correction factor has been evaluated for a few typical cases. It turns out that it differs from unity by only a few percent if the crossing angle is large compared to the rms beam divergence.

**REFERENCES**

1) S. van der Meer; CERN/ISR-PO/68-31 (1968).


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