THE STRESS ANALYSIS FINITE ELEMENT PROGRAM

- FINIMINI -

(ISR VERSION OF FINESSE)

by

I. H. Wilson

Abstract

This report describes the capabilities of FINIMINI and gives some examples.

Resumé

Cette note décrit le programme FINIMINI et présente quelques exemples.

Geneva, 1 October, 1975
FINIMINI

This program is a modified version of the FINESSE program obtained from the National Research Development Corporation (NRDC) in England and adapted to the CERN CDC 7600 by the PS Division\(^1\). The modifications to FINESSE resulting in FINIMINI (ISR version of FINESSE) are outlined at the end of this report.

1. INTRODUCTION

The finite-element method is now accepted as a powerful and general method for the solution of problems in engineering and the physical sciences.

The theory is well-documented\(^2,3\). Reference\(^2\) in particular treats in some detail the theoretical aspects associated with the type of element used in FINESSE, and will therefore not be reproduced here.

This report serves to advertise the presence of FINIMINI within the ISR Department and to describe its main characteristics. Essentially it is a general-purpose program for the solution of linear problems in stress analysis.
2. PROGRAM OUTLINE

Following the technique of analysis, the structure to be analysed is subdivided into a set or grid of finite elements. The boundaries and junctions of these elements are defined by nodes.

The problem of stress analysing the structure can be formulated in several ways; here use is made of the displacement method, that is to say the basic unknown is the displacement at any point within an element. This displacement is described in terms of the nodal displacements using 'shape functions'. (These vary with element type).

Resolution of the problem is obtained by imposing the condition that the total stored potential energy should be a minimum. The mathematical operations finally reduce to the inversion of a banded symmetric stiffness matrix 'K' which relates the applied forces to the resulting nodal deflections. The order of 'K' is approximately equal to (number of nodes x number of degrees of freedom of each node). Inversion is carried out using Gaussian reduction and the final solution obtained by subsequent back substitution.

3. TYPES OF STRESS ANALYSIS

Four types of stress analysis are available:

(a) Plane stress \((\sigma_z = 0, \epsilon_z \neq 0)\)

(b) Plane strain \((\sigma_z \neq 0, \epsilon_z = 0)\)

(c) Axisymmetric

(d) General three-dimensional.

Stress output can be in the form of cartesian stresses either at nodes or for elements, with the added option of printing the numerical values of the principal stresses and their direction cosines. If a stress plot is required, the principal stress data can be written directly onto a CDC 6600 permanent file.

The overall distortion of the structure is output in the form of nodal cartesian displacements.

*In practice at least one degree of freedom must be zero to avoid a singular matrix
4. **ELEMENT TOPOLOGY**

FINESSE uses the isoparametric type of element, that is to say, the shape function relating the position of a particle within an element to the positions of the element nodes, is the same as that relating its displacement to the displacements of the nodes.

The choice of element depends on the particular problem to be solved; the range of elements available in FINIMINI is shown in Fig. 1. The flexibility of the different elements is reflected by their associated 'shape functions'. Listed below are the assumed displacement functions in the x-y plane for the elements available in FINESSE.

\[ U, \text{ the displacement in the x direction for linear triangle (} U_{LT} \text{)} = a_1 + a_2x + a_3y \]

\[ U_{LQ} \text{ (linear quadrilateral)} = U_{LT} + a_4xy \]

\[ U_{QT} \text{ (quadratic triangle)} = U_{LQ} + a_5x^2 + a_6y^2 \]

\[ U_{QQ} \text{ (quadratic quadrilateral)} = U_{QT} + a_7x^2y + a_8y^2x \]

\[ U_{CT} \text{ (cubic triangle)} = U_{QQ} + a_9y^3 + a_{10}x^3 \]

\[ U_{CQ} \text{ (cubic quadrilateral)} = U_{CT} + a_{11}x^3y + a_{12}y^3x \]

The shape functions for the displacement in the y direction, \( v \), are similar.

From the above it is clear that the linear triangle corresponds to a constant stress and strain element since \( \frac{\partial U}{\partial x} = a_2 \).

Use of this element to model a structure having a high stress gradient field would therefore necessitate a very large number of elements, that is to say a fine mesh. Savings in time and money could easily be made by using a more flexible element in a coarser grid (see example no. 1).

Random nodal numbering is possible and in no way affects the resultant size of the semi-bandwidth of the stiffness matrix. Elements, however, must be sequential starting from 1.
5. NODAL GEOMETRY

The geometry of the structure is defined by a list of node numbers and their corresponding geometric coordinates. Data may be input in the following local coordinate systems:

(a) Cartesian \((x,y,z)\)

(b) Cylindrical \((r,\theta,z)\)

or

(c) Spherical \((\rho,\phi,\theta)\).

After being input these coordinate data are transformed and stored with reference to a local cartesian coordinate system \((x,y,z)\).

6. NODAL CONSTRAINTS

Each node is allowed 2 degrees of freedom for two-dimensional analyses and 3 for three-dimensional analyses. Three types of constraints may be imposed on these degrees of freedom:

(a) Earthed nodes

This constraint permits one or more degrees of freedom at a node to be set to zero.

(b) Coupled nodes

This constraint permits a 'master' node to impose the same displacements of all its degrees of freedom on a 'slave' node.

(c) Prescribed variables and reactions

This constraint permits one or more degrees of freedom at a node to be set equal to prescribed values. For nodes constrained in this manner, the reaction due to the constraint is calculated and printed by the system.

To facilitate the prescription of nodal constraint data, the local coordinate system \((x,y,z)\) may be orientated within the global cartesian system \((x_0,y_0,z_0)\) at any node by means of a transformation matrix of direction cosines.
7. **LOADING**

Loading may be of the following types:

(a) Temperature
(b) Pressure
(c) Concentrated point loads
(d) Body forces due to acceleration
(e) Initial strain

Several load cases may be run at any one time with combinations of the above as long as the constraints remain unchanged.

8. **MATERIALS**

The following material values can be input:

(a) Coefficient of linear thermal expansion
(b) Density
(c) Elastic modulus
(d) Rigidity modulus
(e) Poisson's ratio

The Elastic modulus ($E$) is permitted to vary with temperature according to the relationship

$$E_1 = a + bT + cT^2$$

where

- $E_1$ is Young's modulus
- $T$ is Temperature
- $a, b, c$ are constants.

The data input consists of up to 3 values of Young's modulus at three given temperatures.
9. **OUTPUT**

Before any calculation is begun, the system reads and prints an image/echo of each card. This helps during 'post mortens'.

Cartesian \((x_o,y_o,z_o)\) stresses and displacements are output for all nodes for each run.

An option exists for printing principal stresses based on the nodal (averaged) stresses.

Failure runs are accompanied by ERROR DIAGNOSTICS.

10. **MODIFICATIONS TO FINESSE RESULTING IN FINIMINI**

The main modifications are as follows:

a) The upper limit on the maximum number of elements has been increased from 100 to 250 and the size of the COMMON Block increased from 5600 to 20,000. This enables a more realistic size of problem to be handled by the computer.

b) An option has been incorporated to accept either Poisson's ratio \(\nu\) or the Rigidity Modulus \(G\) for two-dimensional analysis. This modification, where the relationship \(E = 2G (1+\nu)\) is not necessarily obeyed, helps in the solution of problems involving sliding surfaces. When the \(G\) input option is chosen, a value of \(\nu = 0.3\) is assumed.

c) Changes in format and added printing options have allowed the amount of output to be substantially reduced to an efficient minimum.

d) Small changes to the program allow it to be run with a new ISR plotter program for mesh and eventually output data display.

e) The sub-routines for the simple linear triangular elements have been incorporated.

N.B. This program is subject to the same conditions of use as outlined in the memorandum ISR-DI/FF/cc - 5th July, 1974.
EXAMPLES

1. A simple steel cantilever beam was idealised in different ways as shown in fig. 2. The results obtained from the computer program are compared with the deflection ($\delta_{\text{max}}$) at the point of application of the load, and the root stress ($\sigma_{\text{max}}$), obtained from conventional engineering bending theory. It is evident that the flexibility of the beam is seriously limited by the use of simple triangular elements. Using only two of the more sophisticated elements however, the error in deflection is zero and in stress 2.5%.

A copy of both INPUT and OUTPUT is given for the above two-element cantilever test problem.

2. Input and output data used and produced during the analysis of the stresses in the coils of a prototype superconducting magnet were fed to a modified version of a PS plotter program to give, in fig. 3, the idealised structure and in fig. 4, the plot of principal stresses at nodes. This facility is useful in data checking and in output digestion but will be superseded by the new ISR plotter program.

3. Comparison of F.E. results and experimental results for a new type of detector for R 806T are given in fig. 5. The loading is due to an internal pressure of 1.5 kg/cm$^2$. 

REFERENCES

1. R. Holsinger, PS Division.


4. **Conditions specified by the NRDC**

That the programs when received by the ISR Division will be kept confidential and will be used for R & D purposes only and not for commercial purposes, and that the originals and any further copies made will be returned on request. That no copies will be provided for use outside the ISR Division and that details of the programs will not be published without the consent of the National Research Development Corporation (NRDC).

(Initialled F.A.F.)

5. Written by R. Holsinger, PS Division, for FINESSE.
<table>
<thead>
<tr>
<th>Linear Triangle</th>
<th>Cubic Triangle</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadratic Triangle</td>
<td>Cubic Quadrilateral</td>
</tr>
<tr>
<td>Quadratic Quadrilateral</td>
<td>Cubic Quadrilateral</td>
</tr>
<tr>
<td>Linear Quadrilateral</td>
<td>Quadrative</td>
</tr>
<tr>
<td>Hexahedron</td>
<td>Linear Quadrilateral</td>
</tr>
</tbody>
</table>

Fig: 1
RESULTS OBTAINED FROM DIFFERENT IDEALISATIONS OF SIMPLE TWO-DIMENSIONAL STEEL CANTILEVER BEAM USING TRIANGULAR AND QUADRATIC ELEMENTS

\( W = 6.25 \text{ kg} \)

\[
\begin{align*}
\sigma_{\text{max}t} &= 15 \text{ kg/mm} \\
\delta_{\text{max}t} &= 0.64 \text{ mm}
\end{align*}
\]

\( L = 160 \text{ mm} \)

<table>
<thead>
<tr>
<th>IDEALISATION</th>
<th>( \frac{\sigma_{\text{max}}}{\sigma_{\text{max}t}} ) (%)</th>
<th>( \frac{\delta_{\text{max}}}{\delta_{\text{max}t}} ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>44%</td>
<td>44%</td>
</tr>
<tr>
<td></td>
<td>71%</td>
<td>70%</td>
</tr>
<tr>
<td></td>
<td>83%</td>
<td>84%</td>
</tr>
<tr>
<td></td>
<td>79%</td>
<td>73%</td>
</tr>
<tr>
<td></td>
<td>100%</td>
<td>97.5%</td>
</tr>
</tbody>
</table>

Fig: 2
EXAMPLE No. 1—CANTILEVER BEAM TEST PROBLEM

W = 6.25 Kg

PRINT ECHO OF CARD INPUT DATA STREAM

1 1 EXAMPLE CANTILEVER
1 2 RUN FIVE DEUX ELEMENTS RECTANGULAIRES VINGT NŒUDS
2 1 1 1 900
2 2 2 0 1 0 0
7 1 1 + 0.200E+05 20.000 + 0.200E+05 30.000 + 0.200E+05 40.000
7 2 1 0.000 0.000 0.000 300
8 1 110000 1 2 3 4
443 1 1 1 2 3 4 6 8 12 11 10 9 7 5
443 2 9 - 10 11 - 12 - 14 - 16 - 20 - 19 - 18 - 17 - 15 - 13
5 4 1 0.00 20.00
5 4 2 0.00 13.33
5 4 3 0.00 6.67
5 4 4 0.00 0.00
5 4 5 26.67 20.00
5 4 6 26.67 0.00
5 4 7 53.33 20.00
5 4 8 53.33 0.00
5 4 9 80.00 20.00
5 4 10 80.00 13.33
5 4 11 80.00 6.67
5 4 12 80.00 0.00
5 4 13 106.67 20.00
5 4 14 106.67 0.00
5 4 15 133.33 20.00
5 4 16 133.33 0.00
5 4 17 160.00 20.00
5 4 18 160.00 13.33
5 4 19 160.00 6.67
5 4 20 160.00 0.00
9 1 16 1
9 2 20 -6.250
9 9 = 0
**INPUT explanation**

Card 11  Title
Card 12  Commentary
Card 21  Program control parameters
Card 22  Choose options - plane stress - principal stress output at nodes - input Poisson's ratio
Card 71  Three values of Young's Modulus at three temperatures
Card 72  Density, coefficient of thermal expansion and Poisson's ratio
Card 81  x and y degrees of freedom constrained for nodes 1, 2, 3 and 4
Card 442  Topology
Card 54  Geometry
Card 91  Type of loading
Card 92  Point load of 6.25 Kg. in -Y direction

**OUTPUT**

---

**DISPLACEMENTS FOR LOAD CASE NUMBER 1**

<table>
<thead>
<tr>
<th>NODE</th>
<th>COMPONENT 1</th>
<th>COMPONENT 2</th>
<th>COMPONENT 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0000000</td>
<td>-0.0000000</td>
<td>0.0000000</td>
</tr>
<tr>
<td>2</td>
<td>0.0000000</td>
<td>-0.0000000</td>
<td>0.0000000</td>
</tr>
<tr>
<td>3</td>
<td>-0.0000000</td>
<td>0.0000000</td>
<td>0.0000000</td>
</tr>
<tr>
<td>4</td>
<td>-0.0000000</td>
<td>0.0000000</td>
<td>-0.0000000</td>
</tr>
<tr>
<td>5</td>
<td>0.01695359</td>
<td>-0.02444597</td>
<td>0.02444597</td>
</tr>
<tr>
<td>6</td>
<td>0.01694715</td>
<td>-0.02444571</td>
<td>0.02444571</td>
</tr>
<tr>
<td>7</td>
<td>-0.01331917</td>
<td>0.09065550</td>
<td>0.09065550</td>
</tr>
<tr>
<td>8</td>
<td>0.0133203</td>
<td>0.09066131</td>
<td>0.09066131</td>
</tr>
<tr>
<td>9</td>
<td>0.04316339</td>
<td>-0.19186212</td>
<td>0.19186212</td>
</tr>
<tr>
<td>10</td>
<td>0.1428972</td>
<td>0.19107199</td>
<td>0.19107199</td>
</tr>
<tr>
<td>11</td>
<td>0.1428559</td>
<td>-0.19105647</td>
<td>0.19105647</td>
</tr>
<tr>
<td>12</td>
<td>0.04314072</td>
<td>-0.19181526</td>
<td>0.19181526</td>
</tr>
<tr>
<td>13</td>
<td>0.0518359</td>
<td>0.32016042</td>
<td>0.32016042</td>
</tr>
<tr>
<td>14</td>
<td>0.05185079</td>
<td>0.32017279</td>
<td>0.32017279</td>
</tr>
<tr>
<td>15</td>
<td>0.05697025</td>
<td>0.46680343</td>
<td>0.46680343</td>
</tr>
<tr>
<td>16</td>
<td>0.05697524</td>
<td>0.46683907</td>
<td>0.46683907</td>
</tr>
<tr>
<td>17</td>
<td>0.05846199</td>
<td>0.62276176</td>
<td>0.62276176</td>
</tr>
<tr>
<td>18</td>
<td>0.01942749</td>
<td>0.62258295</td>
<td>0.62258295</td>
</tr>
<tr>
<td>19</td>
<td>0.01940678</td>
<td>0.62275176</td>
<td>0.62275176</td>
</tr>
<tr>
<td>20</td>
<td>-0.05862279</td>
<td>0.62283696</td>
<td>0.62283696</td>
</tr>
</tbody>
</table>

**THEORETICAL VALUE = 0.64 mm**
### Nodal Stresses for Load Case Number 1

<table>
<thead>
<tr>
<th>NODE</th>
<th>COMPONENT 1</th>
<th>COMPONENT 2</th>
<th>COMPONENT 3</th>
<th>THEORETICAL STRESS 15 kg/mm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>15.024</td>
<td>4.507</td>
<td>-.377</td>
<td>15.04 4.49 (.99936 -.03581)</td>
</tr>
<tr>
<td>2</td>
<td>4.984</td>
<td>1.495</td>
<td>-.360</td>
<td>5.01 1.47 (.99816 -.03519)</td>
</tr>
<tr>
<td>3</td>
<td>-4.975</td>
<td>-1.493</td>
<td>-.298</td>
<td>-1.47 -5.00 (.08466 -.99641)</td>
</tr>
<tr>
<td>4</td>
<td>-15.011</td>
<td>-4.503</td>
<td>-.370</td>
<td>-4.49 -15.02 (.03516 -.99938)</td>
</tr>
<tr>
<td>5</td>
<td>12.534</td>
<td>2.601</td>
<td>-.222</td>
<td>12.54 2.60 (.99976 -.02232)</td>
</tr>
<tr>
<td>6</td>
<td>-12.533</td>
<td>-2.600</td>
<td>-.237</td>
<td>-2.59 -12.54 (.02379 -.99972)</td>
</tr>
<tr>
<td>7</td>
<td>10.039</td>
<td>.699</td>
<td>-.083</td>
<td>10.04 .70 (.99996 -.00866)</td>
</tr>
<tr>
<td>8</td>
<td>-10.031</td>
<td>-.683</td>
<td>-.101</td>
<td>-10.03 .68 (.01076 -.99994)</td>
</tr>
<tr>
<td>9</td>
<td>7.465</td>
<td>-1.284</td>
<td>.155</td>
<td>7.47 -1.20 (.99984 .01766)</td>
</tr>
<tr>
<td>10</td>
<td>2.461</td>
<td>.471</td>
<td>-.442</td>
<td>2.53 -.54 (.98929 -.14594)</td>
</tr>
<tr>
<td>11</td>
<td>-2.524</td>
<td>.360</td>
<td>-.467</td>
<td>-2.60 .43 (.15604 -.98775)</td>
</tr>
<tr>
<td>12</td>
<td>-7.539</td>
<td>1.173</td>
<td>.080</td>
<td>-7.54 1.17 (.09914 -.99996)</td>
</tr>
<tr>
<td>13</td>
<td>4.998</td>
<td>-.648</td>
<td>.048</td>
<td>5.00 -.65 (.99996 .00851)</td>
</tr>
<tr>
<td>14</td>
<td>-4.970</td>
<td>.686</td>
<td>.122</td>
<td>-4.97 .69 (.02148 -.99977)</td>
</tr>
<tr>
<td>15</td>
<td>2.448</td>
<td>-.172</td>
<td>-.031</td>
<td>2.45 -.17 (.99993 .01187)</td>
</tr>
<tr>
<td>16</td>
<td>-2.518</td>
<td>.087</td>
<td>.027</td>
<td>-2.52 .03 (.01051 .99994)</td>
</tr>
<tr>
<td>17</td>
<td>.032</td>
<td>1.026</td>
<td>.017</td>
<td>1.03 .03 (.01754 .99985)</td>
</tr>
<tr>
<td>18</td>
<td>.071</td>
<td>.719</td>
<td>-.356</td>
<td>.88 -.09 (.40438 -.91459)</td>
</tr>
<tr>
<td>19</td>
<td>.058</td>
<td>.399</td>
<td>-.421</td>
<td>.68 -.23 (.55872 -.82936)</td>
</tr>
<tr>
<td>20</td>
<td>.068</td>
<td>.091</td>
<td>-.177</td>
<td>.26 -.10 (.68414 -.72935)</td>
</tr>
</tbody>
</table>
Idealised structure of a 1/8th symmetrical cross-section of a super-conducting magnet
COMPUTER PLOT OF MAGNITUDE AND DIRECTION OF PRINCIPAL STRESSES IN THE COIL DUE TO MAGNETIC AND PRESTRESS FORCES

Fig: 4
DISTRIBUTION OF TANGENTIAL STRESS ($\sigma_t$) ALONG THE OUTSIDE WALL OF THE CROSS-SECTION FOR TANK GEOMETRY N°1

Fig. 5